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Green-Schwarz action for Type IIA strings on $AdS_4 \times CP^3$

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Abstract

We present the Green-Schwarz action for Type IIA strings on $AdS_4 \times CP^3$. The action is based on a $\mathbb{Z}_4$ automorphism of the coset $OSp(4|6)/(SO(1,3) \times SU(3) \times U(1))$. The equations of motion admit a representation in terms of a Lax connection, showing that the system is classically integrable.

1 Introduction

Following the papers of Bagger and Lambert [1] (see also [2]) there has been much interest in $\mathcal{N} = 8$ $d = 3$ supersymmetric theories.¹ Recently it has been suggested that certain $\mathcal{N} = 6$ $D = 2+1$ $U(N) \times U(N)$ Chern-Simons theories at level k with matter have a dual description as M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$ [4]. We may view the S^7 as a Hopf fibration with the radius of CP^3 proportional to Nk and the radius of the Hopf circle proportional to $N^{1/6}k^{-5/6}$. The authors of [4] then identify a limit of the N - k parameter space in which the M-theory is described by a weakly coupled string theory on a $AdS_4 \times CP^3$ background

$$ds_{\text{str}}^2 = \frac{R^3}{k} \left(\frac{1}{4} ds_{AdS_4}^2 + ds_{CP^3}^2 \right), \quad (1.1)$$

together with a four-form RR flux on AdS_4 and a two-form RR flux on $CP^1 \subset CP^3$. In this note we propose a GS action for type IIA string theory (1.1). We follow the approach developed in a series of papers in the study of GS actions in Minkowski and $AdS_5 \times S^5$ spacetimes [5]. This approach has proven to be particularly useful in the studies of integrability of string theory; indeed we are able to construct a Lax connection for this action in analogy with the results of [6]. In section 2 we present an explicit realisation of the $\mathbb{Z}_4$ automorphism and in section 3 we present the action, discuss κ -symmetry for the model and construct the Lax pair for it; conclusions are presented in section 4.

2 A $\mathbb{Z}_4$ automorphism

The GS action presented in this paper will be based on the super-coset

$$\frac{OSp(4|6)}{SO(1,3) \times SU(3) \times U(1)}. \quad (2.1)$$

The Lie super-algebra can be represented by a super-matrix M written in block-matrix form

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \quad (2.2)$$

where A is 4×4 , B is 4×6 , C is 6×4 and D is 6×6 . In order to belong to the super-algebra $OSp(4|6)$ we require that M satisfies

$$\Omega M + M^t \Omega = 0, \quad (2.3)$$

¹Earlier work on such theories with less supersymmetry can be found in [3] and references therein.

where the supertranspose of M is given by

$$M^t = \begin{pmatrix} A^t & C^t \\ -B^t & D^t \end{pmatrix}, \quad (2.4)$$

while the fixed matrix Ω is

$$\Omega = \begin{pmatrix} \omega & 0 \\ 0 & 1 \end{pmatrix}, \quad (2.5)$$

and

$$\omega = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}. \quad (2.6)$$

In terms of the block matrices this reduces to

$$A^t \omega + \omega A = 0, \quad D + D^t = 0, \quad C = B^t \omega. \quad (2.7)$$

In particular A is a $Sp(4)$ matrix, D is a $SO(6)$ matrix. We also require the reality condition

$$M^\dagger = - \begin{pmatrix} C^{-1} & 0 \\ 0 & 1 \end{pmatrix} M \begin{pmatrix} C & 0 \\ 0 & 1 \end{pmatrix}, \quad (2.8)$$

with C defined in appendix A. The bosonic sub-algebra of M then generates $Sp(2, R) \times SO(6) \sim SO(2, 3) \times SU(4)$ (see appendix A for more detail).

Next we define the following automorphism

$$G(M) \equiv g^{-1} M g \equiv \begin{pmatrix} K & 0 \\ 0 & -iL \end{pmatrix} M \begin{pmatrix} K & 0 \\ 0 & iL \end{pmatrix} = \begin{pmatrix} KAK & iKBL \\ -iLCK & LDL \end{pmatrix}, \quad (2.9)$$

where

$$K = \begin{pmatrix} 1_2 & 0 \\ 0 & -1_2 \end{pmatrix}, \quad (2.10)$$

and

$$L = \begin{pmatrix} 0 & i & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & -i & 0 \end{pmatrix}. \quad (2.11)$$

It is easy to see that $G(M) \in OSp(4|6)$ since

$$K\omega = \omega K, \quad K^t = K, \quad L^t = -L, \quad K^2 = 1_4, \quad L^2 = 1_6, \quad (2.12)$$

and so we may verify equation (2.7) explicitly

$$(KAK)^t\omega + \omega KAK = K(A^t\omega + \omega A)K = 0, \quad (2.13)$$

$$(LDL)^t + LDL = L(D^t + D)L = 0, \quad (2.14)$$

$$(iKBL)^t\omega = -iLB^t\omega K = -iLCK. \quad (2.15)$$

It is also easy to show that

$$G^2(M) = \begin{pmatrix} A & -B \\ -C & D \end{pmatrix}. \quad (2.16)$$

which shows that G generates a $\mathbb{Z}_4$ action on $OSp(4|6)$. Since $G(M) = g^{-1}Mg$ this is an automorphism of the Lie superalgebra, in other words

$$[G(M_1), G(M_2)] = G([M_1, M_2]). \quad (2.17)$$

Notice also that while $iL \in SO(6)$, $K \notin Sp(4)$, and so $g \notin OSp(4|6)$; as a result G is an outer automorphism. As shown in appendix A the sub-algebra left invariant by G is $SO(1, 3) \times SU(3) \times U(1)$.

3 GS action for Type IIA string theory on $AdS_4 \times CP^3$

In this section we construct the spacetime supersymmetric action for Type IIA string theory on $AdS_4 \times CP^3$. We first present the form of the action then discuss κ -symmetry and finally show that the action is supersymmetric.

3.1 The action

Let us briefly recall the construction of the GS action on a super-coset G/H . We require that:

(i) H be bosonic and, (ii) G admit a $\mathbb{Z}_4$ automorphism that leaves H invariant, acts by -1 on the remaining bosonic part of G/H , and by $\pm i$ on the fermionic part of G/H . The currents $j_\alpha = g^\dagger \partial_\alpha g$ can then be decomposed as

$$j_\alpha = j_\alpha^{(0)} + j_\alpha^{(1)} + j_\alpha^{(2)} + j_\alpha^{(3)}, \quad (3.1)$$

where $j^{(k)}$ has eigenvalue i^k under the $\mathbb{Z}_4$ automorphism. The +1 eigenspace is

$$M^{(0)} = \left\{ \begin{pmatrix} A^{(0)} & 0 \\ 0 & D^{(0)} \end{pmatrix} \mid A^{(0)} = K A^{(0)} K, D^{(0)} = L D^{(0)} L \right\}, \quad (3.2)$$

where $A^{(0)} = K A^{(0)} K$ implies

$$A^{(0)} = \begin{pmatrix} a_1 & a_2 \\ a_3 & -a_1 \end{pmatrix} \oplus \begin{pmatrix} a_4 & a_5 \\ a_6 & -a_4 \end{pmatrix} \sim SO(1, 3), \quad (3.3)$$

where $A^{(0)}$ satisfies the reality condition (2.8) and hence spans $SO(1, 3)$ as shown in appendix A. Similarly we may describe the solutions of $D = LDL$ in terms of linear combination of anti-symmetric matrices $(M^{ij})_{kl} = \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}$. We find the condition $D = LDL$ implies

$$\begin{aligned} D^{(0)} = & \{M^{12}, M^{34}, M^{56}, M_{15} \mp iM_{16} \pm iM_{25} + M_{26}, \\ & M_{35} \mp iM_{36} \pm iM_{45} + M_{46}, M_{13} \pm iM_{14} \mp iM_{23} + M_{24}\} = SU(3) \times U(1). \end{aligned} \quad (3.4)$$

The explicit form of the -1 eigenspace is

$$M^{(2)} = \left\{ \begin{pmatrix} A^{(2)} & 0 \\ 0 & D^{(2)} \end{pmatrix} \mid A^{(2)} = -K A^{(2)} K, D^{(2)} = -L D^{(2)} L \right\}, \quad (3.5)$$

where

$$A^{(2)} = \sum_{\mu=0}^3 a_{\mu}^{(2)} \gamma^{\mu 5}, \quad D^{(2)} = \sum_{\mu=4}^9 d_{\mu}^{(2)} S^{\mu}, \quad (3.6)$$

with the $\gamma^{\mu 4}$ and S^{μ} defined in Appendix A

In terms of the $\mathbb{Z}_4$ -graded currents the GS action can be written as

$$\mathcal{L}_{\text{GS}} = \int d^2\sigma \sqrt{-g} g^{\alpha\beta} \text{Str}(j_{\alpha}^{(2)} j_{\beta}^{(2)}) + \epsilon^{\alpha\beta} \text{Str}(j_{\alpha}^{(1)} j_{\beta}^{(3)}). \quad (3.7)$$

When restricted to bosonic fields (by setting $j^{(1)}$ and $j^{(3)} = 0$) the above action reduces to the sigma model on $Sp(4)/SO(1, 3) \times SO(6)/SU(3) \times U(1)$, in other words to $AdS_4 \times CP^3$. By construction this action has 24 supersymmetries, and has local κ symmetry. We take this as strong evidence that this action describes Type IIA string theory on the background (1.1).

3.2 κ -symmetry

Actions of the form (3.7) constructed over cosets with a $\mathbb{Z}_4$ automorphism typically have κ -symmetry. The fermions in this theory live in the $\mathbf{4} \times \mathbf{6}$ of the $Sp(4) \times SO(6)$ and so naively

there are 24 real fermionic fields. In flat space and in $AdS_n \times S^n$ κ symmetry can be used to gauge away half of them. In other, less conventional, models it has been recently shown that κ symmetry is trivial on-shell and cannot be used to gauge away any fermionic degrees of freedom [8]. In the present model κ symmetry will allow us to remove 8 real fermionic degrees of freedom - leaving us with 16 physical fermions - the required number for Type IIA string theory on $AdS_4 \times CP^3$. In order to see that κ symmetry can indeed be used to gauge away 1/3 of the fermions we consider for simplicity the superparticle.² In this case the action is simply

$$\int d\tau e \text{Str}(j_\tau^{(2)} j_\tau^{(2)}), \quad (3.8)$$

where e is the worldline metric. Picking the coset representatives as

$$g = g_{\text{ferm}} g_{\text{bos}}, \quad (3.9)$$

where g_{ferm} (g_{bos}) is the exponential of a purely fermionic (bosonic) Lie-algebra element, it is a simple exercise (see Appendix B) to expand the above action to quadratic order in fermions. One finds that the two-fermion term is of the form

$$\mathcal{L}_{2 \text{ ferm}} = \int d\tau e K^{ai, bj} \theta_{ai} \partial_\tau \theta_{bj} = \int d\tau e \sum_{\mu=0, \dots, 3} p_\mu \gamma_{ac}^{\mu 4} \omega^{cb} \theta_{ai} \partial_\tau \theta_{bi} + \sum_{\mu=4, \dots, 9} p_\mu S_{ij}^\mu \omega^{ab} \theta_{ai} \partial_\tau \theta_{bj}, \quad (3.10)$$

where $\mu = 0, \dots, 3$ and $\mu = 4, \dots, 9$ is the spacetime AdS_4 and CP^3 index with p_μ the spacetime momentum; $a, b, c = 1, \dots, 4$ and $i, j = 1, \dots, 6$ are $Sp(2)$ and $SO(6)$ indices; θ_{ai} are the 24 real fermionic fields in the coset (2.1). The on-shell condition comes from the variation of e in the above action and is simply

$$\text{Str}(j_\tau^{(2)} j_\tau^{(2)}) = \eta^{\mu\nu} p_\mu p_\nu = 0. \quad (3.11)$$

We shown in appendix B that on-shell $K_{ai, bj}$ has rank 16. This indicates that κ -symmetry of the action can be used to gauge away 8 real fermions leaving us with 16 physical fermions.

²We defer a more detailed analysis of κ -symmetry and gauge fixing to a forthcoming paper [10].

3.3 Integrability

The equations of motion that follow from (3.7) are ³

$$0 = \partial_\alpha(\sqrt{-g}g^{\alpha\beta}j_\beta^{(2)}) - \sqrt{-g}g^{\alpha\beta} [j_\alpha^{(0)}, j_\beta^{(2)}] + \frac{1}{2}\epsilon^{\alpha\beta} \left([j_\alpha^{(1)}, j_\beta^{(1)}] - [j_\alpha^{(3)}, j_\beta^{(3)}] \right), \quad (3.12)$$

$$0 = (\sqrt{-g}g^{\alpha\beta} + \epsilon^{\alpha\beta}) [j_\alpha^{(3)}, j_\beta^{(2)}], \quad (3.13)$$

$$0 = (\sqrt{-g}g^{\alpha\beta} - \epsilon^{\alpha\beta}) [j_\alpha^{(1)}, j_\beta^{(2)}]. \quad (3.14)$$

As is by now well known these equations follow from the zero curvature condition of a Lax connection [6] (we follow the notation of [7])

$$\partial_\alpha \mathcal{L}_\beta - \partial_\beta \mathcal{L}_\alpha - [\mathcal{L}_\alpha, \mathcal{L}_\beta] = 0, \quad (3.15)$$

where

$$\mathcal{L}_\alpha = l_0 j_\alpha^{(0)} + l_1 j_\alpha^{(1)} + l_2 \gamma_{\alpha\beta} \epsilon^{\beta\rho} j_\rho^{(2)} + l_3 (j_\alpha^{(1)} + j_\alpha^{(3)}) + l_4 (j_\alpha^{(1)} - j_\alpha^{(3)}). \quad (3.16)$$

The parameters l_i are constants which depend on the spectral parameter λ and are

$$l_0 = 1, \quad l_1 = \frac{1 + \lambda^2}{1 - \lambda^2}, \quad l_2 = \frac{2\lambda}{1 - \lambda^2}, \quad l_3 = \pm \frac{1}{\sqrt{1 - \lambda^2}}, \quad l_4 = \pm \frac{\lambda}{\sqrt{1 - \lambda^2}}. \quad (3.17)$$

An infinite tower of local commuting charges may then be constructed using the monodromy matrix of the above Lax connection in the standard fashion.

4 Conclusions

In this paper we have constructed the Green-Schwarz action for Type IIA string theory on $AdS_4 \times CP^3$. This construction relies on the presence of a $\mathbb{Z}_4$ automorphism for the coset (2.1). We have argued that the action has an unconventional form of κ -symmetry which can be used to gauge-fix 1/3 of the fermionic fields. We have also shown that the model is classically integrable. Since the construction is purely algebraic it may also be used to construct the GS action for the background $AdS_2 \times CP^1$, in this case the coset being $OSp(2|3)/(SO(2) \times U(1))$. We expect that it should be possible to construct the corresponding Berkovits string for this background as well; which presumably will also be integrable. It would be interesting to see the precise form of the ghost sector of that theory. Given the classical integrability of this string theory and the

³Note that since we are discussing classical integrability, the result of [6] is completely general for all actions of the form (3.7) and does not depend on the specifics of the coset under investigation. This is reflected in the equations presented in this sub-section.

proposed [4] dual descriptions in terms of three-dimensional Chern-Simons theories coupled to matter it seems likely that these dualities may be investigated within the framework developed over the last few years in the study of $AdS_5 \times S^5$ - $\mathcal{N} = 4$ Super-Yang Mills duality [9]; indeed progress in this direction has already begun to appear [11].

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A Matrix representations of $Sp(2)$ and $SO(6)$

A.1 The $Sp(2, R) \sim SO(2, 3)$ algebra

Using 4×4 -dimensional γ matrices we may construct the $SO(2, 3)$ Lie lie algebra. Indeed define

$$\gamma^0 = \begin{pmatrix} 0 & i1_2 \\ i1_2 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & i\sigma^1 \\ -i\sigma^1 & 0 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & i\sigma^2 \\ -i\sigma^2 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & i\sigma^3 \\ -i\sigma^3 & 0 \end{pmatrix}, \quad (A.1)$$

and $\gamma^4 = \gamma^0 \gamma^1 \gamma^2 \gamma^3$. These satisfy the anti-commutation relations

$$\{\gamma^\alpha, \gamma^\beta\} = 2\eta^{\alpha\beta}, \quad \alpha, \beta = 0, \dots, 4 \quad (A.2)$$

where $\eta^{\alpha\beta} = \text{diag}(-1, 1, 1, 1, -1)$. We may pick the charge conjugation matrix to be $C = \gamma^0 \gamma^4$; the γ matrices then satisfy

$$\gamma^{\alpha\dagger} = C^{-1} \gamma^\alpha C. \quad (A.3)$$

The Lie algebra $SO(2, 3)$ is generated by

$$\gamma^{\alpha\beta} = \frac{1}{2} (\gamma^\alpha \gamma^\beta - \gamma^\beta \gamma^\alpha), \quad (A.4)$$

with the $\gamma^{\alpha\beta}$ satisfying the reality condition

$$\gamma^{\alpha\beta\dagger} = -C^{-1} \gamma^{\alpha\beta} C. \quad (A.5)$$

It is easy to check that the $\gamma^{\alpha\beta}$ satisfy the symplectic condition

$$\gamma^{\alpha\beta t} \omega + \omega \gamma^{\alpha\beta} = 0, \quad (A.6)$$

showing that $SO(2, 3) \sim Sp(2, R)$. We may also easily show that G the $\mathbb{Z}_4$ automorphism defined in section 2 acts as $+1$ on the $SO(1, 3)$ subgroup generated by $\gamma^{\alpha\beta}$ with $\alpha, \beta = 1, \dots, 4$.

A.2 The $SO(6) \sim SU(4)$ Lie algebra

The Lie algebra $SO(6)$ contains an $SU(3) \times U(1)$ subalgebra (see equation (3.4)). We label the remaining 6 generators as S^μ , $\mu = 4, \dots, 9$. A convenient parametrisation of these is given by

$$\begin{aligned} S^4 &= M_{14} + M_{23}, & S^5 &= M_{24} - M_{13}, & S^6 &= M_{16} - M_{25}, \\ S^7 &= M_{26} - M_{15}, & S^8 &= M_{36} - M_{45}, & S^9 &= M_{46} - M_{35}. \end{aligned} \quad (\text{A.7})$$

B Fermionic quadratic terms

If we pick the group element g as in equation (3.9), then the superparticle action (3.8) to quadratic order in fermionic fields is

$$\mathcal{L}_{2 \text{ ferm}} = \int d\tau e \text{Str}(g_{\text{bos}} j_{\text{bos}}^{(2)} g_{\text{bos}}^{-1} \Theta^t \partial_\tau \Theta), \quad (\text{B.1})$$

where the 24 real fermions θ_{ai} are grouped into an $OSp(2|6)$ matrix

$$\Theta = \begin{pmatrix} 0 & \theta_{ai} \\ (\theta^t \omega)_{jb} & 0 \end{pmatrix}, \quad (\text{B.2})$$

and $g_{\text{ferm}} = \exp(\Theta)$. Explicitly $\mathcal{L}_{2 \text{ ferm}}$ is given in equation (3.10) where

$$p_\mu = \text{Str}(g_{\text{bos}} j_{\text{bos}}^{(2)} g_{\text{bos}}^{-1} \Gamma^\mu) / \text{Str}(\Gamma^\mu \Gamma^\mu) \quad (\text{B.3})$$

where

$$\Gamma^\mu = \gamma^{\mu 4} \otimes 1, \quad \mu = 0, \dots, 3 \quad \Gamma^\mu = \omega \otimes S^\mu, \quad \mu = 4, \dots, 9. \quad (\text{B.4})$$

For generic on-shell momenta p_μ (*i.e.* those that satisfy equation (3.11)) the matrix $K_{ai, bj}$ may be brought into the form

$$p_0 \gamma_{ac}^{04} \omega^{cb} \otimes \delta_{ij} + p_5 \omega^{ac} \otimes S_{ij}^5 \sim \begin{pmatrix} p_1 & 0 & 0 \\ 0 & p_5 & -p_1 \\ 0 & p_1 & -p_5 \end{pmatrix}^{\oplus 8}, \quad (\text{B.5})$$

by a global $Sp(2) \times SO(6)$ rotation. Since the on-shell condition reduces to $p_5 = \pm p_0$, we see that for generic momenta $K_{ai, bj}$ has rank 16.

References

- [1] J. Bagger and N. Lambert, Phys. Rev. D **75** (2007) 045020 [arXiv:hep-th/0611108]; J. Bagger and N. Lambert, Phys. Rev. D **77** (2008) 065008 [arXiv:0711.0955 [hep-th]]; J. Bagger and N. Lambert, JHEP **0802** (2008) 105 [arXiv:0712.3738 [hep-th]].
- [2] A. Gustavsson, arXiv:0709.1260 [hep-th]. A. Gustavsson, JHEP **0804** (2008) 083 [arXiv:0802.3456 [hep-th]].
- [3] J. H. Schwarz, JHEP **0411** (2004) 078 [arXiv:hep-th/0411077]. D. Gaiotto and X. Yin, JHEP **0708** (2007) 056 [arXiv:0704.3740 [hep-th]].
- [4] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, arXiv:0806.1218 [hep-th].
- [5] R. R. Metsaev and A. A. Tseytlin, Nucl. Phys. B **533** (1998) 109 [arXiv:hep-th/9805028]; M. Henneaux and L. Mezincescu, Phys. Lett. B **152** (1985) 340. N. Berkovits, M. Bershadsky, T. Hauer, S. Zhukov and B. Zwiebach, Nucl. Phys. B **567** (2000) 61 [arXiv:hep-th/9907200]. M. B. Green and J. H. Schwarz, Phys. Lett. B **136** (1984) 367. W. Siegel, Phys. Lett. B **128** (1983) 397.
- [6] I. Bena, J. Polchinski and R. Roiban, Phys. Rev. D **69**, 046002 (2004) [arXiv:hep-th/0305116].
- [7] L. F. Alday, G. Arutyunov and A. A. Tseytlin, JHEP **0507**, 002 (2005) [arXiv:hep-th/0502240].
- [8] B. Stefański, jr. JHEP **0707** (2007) 009 [arXiv:0704.1460 [hep-th]].
- [9] A very incomplete list of references is: S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Nucl. Phys. B **636** (2002) 99 [arXiv:hep-th/0204051]. S. Frolov and A. A. Tseytlin, Nucl. Phys. B **668** (2003) 77 [arXiv:hep-th/0304255]. M. Staudacher, JHEP **0505** (2005) 054 [arXiv:hep-th/0412188]. R. A. Janik, Phys. Rev. D **73** (2006) 086006 [arXiv:hep-th/0603038]. N. Beisert, R. Hernandez and E. Lopez, JHEP **0611** (2006) 070 [arXiv:hep-th/0609044]. N. Beisert, B. Eden and M. Staudacher, J. Stat. Mech. **0701** (2007) P021 [arXiv:hep-th/0610251].
- [10] B. Stefański, jr. *to appear*.
- [11] M. Benna, I. Klebanov, T. Klose and M. Smedback, arXiv:0806.1519 [hep-th]. J. A. Minahan and K. Zarembo, arXiv:0806.3951 [hep-th]. D. Gaiotto, S. Giombi and X. Yin, arXiv:0806.4589 [hep-th]. A. Armoni and A. Naqvi, arXiv:0806.4068 [hep-th].