hep-th/9812088
DAMTP-1998-166

Gravitational Couplings of D-branes and O-planes

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December 1998

Abstract

An explicit calculation is performed to check all the tangent bundle gravitational couplings of Dirichlet branes and Orientifold planes by scattering q gravitons with a $p+1$ form Ramond-Ramond potential in the world-volume of a $D(p+2q)$-brane. The structure of the D-brane Wess-Zumino term in the world-volume action is confirmed, while a different O-plane Wess-Zumino action is obtained.

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1 Introduction

Properties of D-branes [1] have provided new insights into the structure of string theory. The effective world-volume action of a D-brane has two terms. The Dirac-Born-Infeld (DBI) term describes the world-volume vector potential and scalars coupling to the Neveu-Schwarz-Neveu-Schwarz (NS⊗NS) fields. The Wess-Zumino (WZ) term gives the coupling of the vector potential and the pullback of the curvature fields to the Ramond-Ramond (R⊗R) potentials. D-branes have no intrinsic gravitational dynamics. However, a D-brane is affected by the gravitational field in the background space in which it is embedded.[1]

The DBI part of the action, at least for abelian gauge fields, has been known for some time [2]. The WZ action is known precisely, at least at long wavelengths, as it may be determined by an anomaly cancelling mechanism. The gauge part of the WZ action was first determined for Type I theory in [3], while a general D-brane gauge WZ action was obtained in [4, 5, 6]. In particular in [6] a compact way of writing the action was given. The first gravitational term of the WZ action, obtained in [7], was related using a chain of dualities to a one-loop term discovered in [8]. This term provided further evidence for the Heterotic/Type IIA duality first suggested in [9]. Later the whole WZ action was determined by an anomaly cancelling argument [10] (for a review of the WZ couplings see [11]). While D-branes themselves are non-chiral, a configuration of intersecting D-branes which contained in its worldvolume theory chiral fermions was found [10]. Using the anomaly inflow mechanism [12] the relevant anomaly cancelling terms in the action were obtained. This was generalised to include gravitational couplings to a non-trivial normal bundle in [13] and [14]. The WZ D-brane coupling can be summarised as

$$I^D_{WZ} = T_p \int_{\mathcal{B}_p} C \wedge \mathrm{tr}\exp\left(iF/(2\pi)\right) \wedge \sqrt{\hat{A}(R)}, \tag{1}$$

where $\mathcal{B}_p$ is the world-volume of a Dirichlet p-brane, $T_p^2 = 2\pi(4\pi^2\alpha')^{3-p}$ as in [1][2] and F is the world-volume gauge field (more generally when the NS⊗NS potential B is non-zero F is replaced by $\mathcal{F} = F - B$). $\hat{A}$ is the Dirac or A-roof genus, and R is the pull-back of the curvature from the space-time to the D-brane world-volume. C is a sum over the R⊗R potentials as differential forms. The worldvolume integral will pick out terms which appear as $p+1$ forms. The gauge part of this action was computed by several authors [6, 4, 5] and can be obtained by constructing an appropriate boundary state and requiring it to be BRST invariant. In this paper the gravitational part of the WZ term is discussed, so F is set to zero. Thus for example the couplings of the R⊗R two form are

$$I_{\text{D-brane}} = T_1 \int_{\mathcal{B}_1} C^{(2)} + \frac{T_5}{24(4\pi)^2}\int_{\mathcal{B}_5} C^{(2)}\wedge \mathrm{tr}(R^2) + T_9 \int_{\mathcal{B}_9} C^{(2)}\wedge\left[\frac{1}{1152(4\pi)^4}(\mathrm{tr}R^2)^2 + \frac{1}{720(4\pi)^4}\mathrm{tr}(R^4)\right]. \tag{2}$$

Compactifications of Type II theories involving gauging worldsheet orientation produce orientifold planes (O-planes) [15, 16, 17]. For a review of some such compactifications see [18]. Worldvolume actions of D-branes and O-planes exhibit some similarities. Both objects carry R⊗R charge and have WZ gravitational couplings. An important difference is that D-branes have a worldvolume gauge field not present on O-planes. The gravitational couplings of O-planes were obtained by considering Type I theory as an orientifold of Type IIB [15].[3] While this paper was in its final stages of preparation [33] appeared which has substantial overlap with the content of this paper. The form of the orientifold WZ coupling is given by

$$I^O_{WZ} = -2^{p-4} T_p \int_{\mathcal{B}_p} C \wedge \sqrt{L(R/4)}, \tag{3}$$

where L is the Hirzebruch polynomial. In [33] the normal bundle couplings were also obtained. Expanding the above action for the O9-plane gives

[1] Throughout this paper we consider a trivial normal-bundle.
[2] In the notation of [1] we set $\alpha_p = 1$.
[3] The ten dimensional case is somewhat degenerate as branes and planes fill spacetime.

$$I_{\text{O9-plane}} = T_9 \int_{\mathcal{B}_9} -32C^{(10)} + C^{(6)}\wedge \frac{2}{3(4\pi)^2}\mathrm{tr}(R^2) + C^{(2)}\wedge \left[\frac{-1}{144(4\pi)^4}(\mathrm{tr}R^2)^2 + \frac{7}{180(4\pi)^4}\mathrm{tr}(R^4)\right]. \qquad (4)$$

The first term in the above is simply the 10 dimensional tadpole [19, 20]. The eight form coupling is different to the one obtained in [21], but agrees with [33]. The four form coupling is as postulated by [21] which is particularly reassuring since it is needed for consistency with M-theory [22].

Apart from [7], very little evidence has been presented in the literature for the above terms (in particular for the eight form gravitational couplings). So it is interesting to check their presence explicitly. We confirm both sets of terms given in the equations above by calculating the relevant amplitudes in superstring theory. Other R⊗R potentials will have similar couplings to the curvature two forms, however these follow from the above couplings by T-duality.

A recent paper made some initial steps in calculating the gravitational couplings of D-branes and O-planes [23] by calculating the four form term of the Dirac genus. In this paper we shall verify the complete structure of both the D-brane and O-plane actions by considering appropriate scattering amplitudes in the world-volume of D-branes and O-planes. More precisely we shall scatter q gravitons and $C^{(p+1)}$ in the worldvolume of a D$(p+2q)$-brane. This paper is organised as follows. In Section 2 we discuss the calculations that are to be performed as well as the conventions used. In sections 3 and 4 the four and eight form amplitudes for D-branes and crosscaps are calculated. The D-brane and O-plane amplitude results are compared with the actions given above in sections 5 and 6, respectively. In section 6 we discuss also the Green-Schwarz anomaly canceling mechanism [24].

2 Preliminaries

We intend to confirm the gravitational couplings (2) and (4) by calculating scattering amplitudes of the R⊗R two-form with two or four gravitons on either a D-brane or O-plane. This will be done by considering the worldvolume scattering of an appropriate number of gravitons with one R⊗R potential in the presence of a boundary or crosscap. The boundary state corresponds to a D-brane while the crosscap to an orientifold plane. The boundary state formalism was developed in [20, 19, 3] for purely Neumann boundary conditions. Dirichlet boundary conditions were discussed in [25]. Boundary states with Dirichlet boundary conditions were initially constructed for D-instantons in [26]. Mixed boundary states were first discussed by Polchinski [1], and in [4, 27, 6, 5] and have been used extensively since, see for example [28, 29, 30].

We shall be using vertex operators for gravitons in the $(-1,0)$ and $(0,0)$ pictures, and the R⊗R two form in the $(-\frac{3}{2},-\frac{1}{2})$ picture. These are

$$\begin{aligned}
V^g_{(-1,0)}(k,\zeta,z,\bar z) &= \zeta_{\mu\nu}\psi^\mu(z)(\bar\partial\tilde X^\nu + \frac{k}{2}\tilde\psi\tilde\psi^\nu)(\bar z)e^{ik\cdot X(z,\bar z)}, && (5)\\
V^g_{(0,0)}(k,\zeta,z,\bar z) &= \zeta_{\mu\nu}(\partial X^\mu + \frac{k}{2}\psi\psi^\mu)(z)(\bar\partial\tilde X^\nu + \frac{k}{2}\tilde\psi\tilde\psi^\nu)(\bar z)e^{ik\cdot X(z,\bar z)}, && (6)\\
V^{C^{(2)}}_{(-3/2,-1/2)}(k,C^{(2)},z,\bar z) &= \frac{1}{n!}C^{(2)}_{\mu\nu}S^A\tilde S^B\Gamma^{\mu\nu}_{AB}e^{ik\cdot X(z,\bar z)}, && (7)
\end{aligned}$$

where ghost and superghost dependence has been suppressed.

The mode expansions for the fields on a cylindrical world-sheet are

$$\begin{aligned}
X^\mu(\tau,\sigma) &= x^\mu + 2\pi p^\mu\tau + i\sum_{n\neq 0}\frac{1}{n}(\alpha^\mu_n e^{-in(\tau-\sigma)} + \tilde\alpha^\mu_n e^{-in(\tau+\sigma)}),\\
\psi^\mu(\tau,\sigma) &= \sum_r \psi^\mu_r e^{-in(\tau-\sigma)}, \qquad \tilde\psi^\mu(\tau,\sigma) = \sum_r \psi^\mu_r e^{-in(\tau+\sigma)}, \qquad (8)
\end{aligned}$$

where $0 \leq \sigma \leq 2\pi$. The boundary state is taken to be the end-state at $\tau = 0$ of a semi-infinite cylinder, and satisfies

$$\begin{aligned}
(\partial_\tau X^\mu(\tau=0,\sigma))\,|B\rangle &= 0 \quad \mu = 0,\ldots,p, \\
X^\mu(\tau=0,\sigma)\,|B\rangle &= 0 \quad \mu = p+1,\ldots,9, \\
(\psi^\mu \pm i\tilde{\psi}^\mu)\,|B\rangle &= 0 \quad \mu = 0,\ldots,p, \\
(\psi^\mu \mp i\tilde{\psi}^\mu)\,|B\rangle &= 0 \quad \mu = p+1,\ldots,9,
\end{aligned} \tag{9}$$

or in terms of modes,

$$\begin{aligned}
-\alpha_n^\mu\,|B\rangle &= \tilde{\alpha}_{-n}^\mu\,|B\rangle\,, \quad \mu = 0,\ldots,p, \\
\alpha_n^\mu\,|B\rangle &= \tilde{\alpha}_{-n}^\mu\,|B\rangle\,, \quad \mu = p+1,\ldots,9, \\
\psi_r^\mu\,|B\rangle &= \mp i\tilde{\psi}_{-r}^\mu\,|B\rangle\,, \quad \mu = 0,\ldots,p, \\
\psi_r^\mu\,|B\rangle &= \pm i\tilde{\psi}_{-r}^\mu\,|B\rangle\,, \quad \mu = p+1,\ldots,9.
\end{aligned} \tag{10}$$

In the above r is half integer in the NS$\otimes$NS sector and integer in the R$\otimes$R sector.

The relevant boundary conditions for a crosscap are [20, 19]

$$\begin{aligned}
\frac{\partial}{\partial\tau}X^\mu(\sigma+\pi,\tau=0) &= -\frac{\partial}{\partial\tau}X^\mu(\sigma,\tau=0) \\
X^\mu(\sigma+\pi,\tau=0) &= X^\mu(\sigma,\tau=0).
\end{aligned} \tag{11}$$

In constructing Op-planes one compactifies $9-p$ directions on circles and performs T-duality in these directions [15, 16]. The boundary conditions in the compact directions then read

$$X^\mu(\sigma+\pi,\tau=0) = -X^\mu(\sigma,\tau=0). \tag{12}$$

This fixes the transverse position of the Orientifold plane at $x^\mu = 0$. The effect of a crosscap state on modes is given by equation (10) but with an extra $(-1)^n$ factor inserted.

The boundary and crosscap state are constructed as eigenvectors and as such do not have a fixed normalisation. However, one may normalise the crosscap relative to the boundary state by looking at the factorisation of cylinder and Möbius strip diagrams in the open string channel [19, 20]. We denote by η_{10} the normalisation of a crosscap with all Neumann boundary conditions relative to the boundary state of a D9-brane.

We shall adopt here the fermionic zero mode conventions of [28] for the Dp-brane boundary state,

$$|B_\psi\rangle^{(0)} = \mathcal{M}_{AB}\,|A\rangle\,|\tilde{B}\rangle\,, \tag{13}$$

where

$$\mathcal{M} = \frac{T_p}{32}(C\Gamma^0\ldots\Gamma^p)\left(1 \pm i\Gamma^{11}\right). \tag{14}$$

Here C is the charge conjugation matrix and $A, B, \ldots$ are 32-dimensional indices for spinors in 10 dimensions. The fermionic zero modes are represented by the following action

$$d_0^\mu\,|A\rangle\,|\tilde{B}\rangle = \frac{1}{\sqrt{2}}(\Gamma^\mu)^A{}_C(1)^B{}_D\,|C\rangle\,|\tilde{D}\rangle \tag{15}$$

The right moving fermions' representation can be obtained from the boundary state's effect on the fermionic modes.

The $C^{(p)}$ state with momentum k is placed at $\tau = \infty$ and is given by

$$\langle C^{(p)}, k \,| = \frac{1}{p!} C_{\mu_1 \ldots \mu_p} \Gamma^{\mu_1 \ldots \mu_p}_{A\tilde{B}} \, \langle A \,| \otimes \langle \tilde{B} \,| \otimes \langle k \,| \,. \tag{16}$$

With the above conventions

$$\langle C^{(p)}, k \,|B\rangle = \frac{T_p}{p!} \varepsilon^{\mu_1 \ldots \mu_{p+1}} C^{(p)}_{\mu_1 \ldots \mu_{p+1}}, \tag{17}$$

in agreement with (1).

3 The four form amplitudes

In this section we discuss the trR^2 WZ term in the action. The first part shows in some detail how the D-brane calculation is performed while the second part of the section obtains the O-plane amplitude in a straightforward generalisation of the D-brane calculation.

3.1 The D5-brane amplitude

We are interested in scattering two gravitons and a R⊗R two form, $C^{(2)}$, in the presence of a D5-brane in order to verify the D-brane worldvolume term

$$\frac{T_5}{24(4\pi)^2} \int_{\mathcal{B}_5} C^{(2)} \mathrm{tr} R \wedge R, \tag{18}$$

in the low energy effective action. We do this by choosing a worldsheet time coordinate τ and a periodic space coordinate σ. Furthermore, we fix the position of the $C^{(2)}$ vertex operator and that of the D5-brane. In effect we need to compute

$$\begin{aligned} A_2 &= \langle\, C^{(2)}, k^0 \,|\, V_g(k^1) \Delta V_g(k^2) \Delta \,|B\rangle \\ &= (4\pi)^{-2} \int_{|z|\geq 1} \frac{d^2 z}{|z|^2} \int_{|y|\geq 1} \frac{d^2 y}{|y|^2} \langle\, C^{(2)}, k^0 \,|\, T\left\{ V_g\left(k^1, y, \bar{y}\right) V_g\left(k^2, z, \bar{z}\right)\right\} |B\rangle \,, \end{aligned} \tag{19}$$

where V_g is a graviton vertex operator in an appropriate picture, Δ is a closed string propagator and $T\{\ldots\}$ indicates time ordering. In this calculation all momenta shall be restricted to the worldvolume directions, and we denote by M_2 the matrix element above.

On the disc the superghost number anomaly is -2. Thus we can work with the gravitons in the $(0,0)$ picture and the R⊗R two form in the $\left(-\frac{1}{2}, -\frac{3}{2}\right)$ picture. Alternately, the R⊗R two form can be in the $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ picture with one of the gravitons in the $(-1,0)$ picture while the other in the $(0,0)$ picture.

Turning to the ghost factor, BRST invariance of the boundary state fixes the ghost zero mode piece of $|B\rangle$ to be $\frac{1}{2}\left(c_0 + \tilde{c}_0\right) |{\downarrow\downarrow}\rangle$. The propagator next to the boundary has to have a ghost piece $\frac{1}{2}(b_0 + \tilde{b}_0)$ since the cylinder has only one Teichmüller parameter. The other propagator will contain the usual $\oint b(z) \oint \tilde{b}(\bar{z})$ factor while the vertices will have factors of $c\tilde{c}$ in order to make them BRST invariant. Thus the state $\langle\, C^{(2)}, k^0 \,|$ will have a $c_{-1}\tilde{c}_{-1}$ ghost piece. Combining these we see that the ghost factor is

$$\langle \Omega |\, c_{-1}\tilde{c}_{-1} c(z) \tilde{c}(\bar{z}) \,|{\downarrow\downarrow}\rangle = \langle \Omega |\, c_{-1}\tilde{c}_{-1} c(z) \tilde{c}(\bar{z}) c_1 \tilde{c}_1 \,|\Omega\rangle = 1. \tag{20}$$

At this point we note that the ghost piece will be the same for the 4 graviton scattering discussed in the next section. In fact the ghost piece for any tree level process in the cylinder channel will simply be equal to 1.

The fermionic zero modes play an important role in the calculation. The boundary state contribution amounts to a factor of $\Gamma^0 \ldots \Gamma^5$. Furthermore, the right-moving spin field $\tilde{S}^\alpha$ gets converted into a left-moving spin field by the boundary state, resulting in a trace over the Clifford algebra. We will have to saturate the $\Gamma^0 \ldots \Gamma^5$ factor by picking out terms with precisely 6 fermionic zero modes in them. Since we are dealing with massless, physical states we have[4] $k^2 = k \cdot \zeta = 0$, we can only select precisely two zero modes from each graviton vertex, one contracted into ζ and the other into k.[5] The $C^{(2)}$ state contributes a further two gamma matrices. Thus the fermion zero modes contribute a factor

$$W = \frac{32}{4} \varepsilon^{\mu_1 \ldots \mu_6} C^{(2)}_{\mu_1 \mu_2} \zeta^1_{\mu_3} k^1_{\mu_4} \zeta^2_{\mu_5} k^2_{\mu_6}, \tag{21}$$

where 32 and 4 come from the gamma matrix trace and from the representation of the fermionic zero modes as in equation (15), respectively.

As a result of the above discussion we obtain modified graviton vertices in the $(0,0)$ and $(-1,0)$ pictures, which subject to the above zero mode factor are

$$\begin{aligned}
V^M_{(-1,0)}(k,\zeta,z,\bar{z}) &= -\frac{\zeta_\mu}{2}(\psi^\mu \pm i\tilde{\psi}^\mu) e^{ikX} = -\frac{\zeta_\mu}{2} \Psi^\mu e^{ikX}, \\
V^M_{(0,0)}(k,\zeta,z,\bar{z}) &= \left[-\zeta_\mu (\partial X^\mu + \frac{k_\nu}{2} \psi^\nu \psi^\mu) + \zeta_\mu (\bar{\partial} \tilde{X}^\mu + \frac{k_\nu}{2} \tilde{\psi}^\nu \tilde{\psi}^\mu) + \frac{\pm i}{2} (\zeta_\mu \psi^\mu k_\nu \tilde{\psi}^\nu - k_\mu \psi^\mu \zeta_\nu \tilde{\psi}^\nu) \right] e^{ikX} \\
&= -\zeta_\mu (\partial^* \mathcal{X}^\mu + \frac{k_\nu}{2} \Psi^\nu \Psi^\mu) e^{ikX} = P(k,\zeta,z,\bar{z}) e^{ikX}.
\end{aligned} \tag{22}$$

where the prefactor $P(k,\zeta,z,\bar{z})$ denotes the term in square brackets in the above equation. Here $\Psi^\mu(z,\bar{z}) = \psi^\mu(z) \pm i\tilde{\psi}^\mu(\bar{z})$, $\mathcal{X}^\mu(z,\bar{z}) = X^\mu(z) + \tilde{X}^\mu(\bar{z})$ and $\partial^* = \partial - \bar{\partial}$. The fact that we can split the amplitude in this way makes the computation substantially easier as we are now dealing with half as many multiplicative factors. We define

$$\begin{aligned}
a &= \frac{z}{y-z}, \quad \bar{a} = \frac{\bar{z}}{\bar{y}-\bar{z}}, \quad A = \bar{a} - a = \frac{\mathrm{Im}(z\bar{y})}{|z-y|^2}, \\
b &= \frac{1}{\bar{y}z - 1}, \quad \bar{b} = \frac{1}{y\bar{z}-1}, \quad B = \bar{b} - b = \frac{\mathrm{Im}(z\bar{y})}{|z\bar{y}-1|^2}.
\end{aligned} \tag{23}$$

The matrix element in equation (19) can then be written as

$$M_2 = \frac{T_5 W}{4} \langle C^{(2)}, k^0 \,|\, P_1 e^{ik^1 X} P_2 e^{ik^2 X} \,| B \rangle = \frac{T_5 W}{4} \langle C^{(2)}, k^0 \,|\, (P_1 + A_1)(P_2 + A_2) \,| B \rangle \, \mathcal{E}, \tag{24}$$

where $P_1 = P(k^1, \zeta^1, y, \bar{y})$, $\quad P_2 = P(k^2, \zeta^2, z, \bar{z})$ and

$$A_1 = \frac{\zeta^1 \cdot k^2}{2}(A - B), \quad A_2 = -\frac{\zeta^2 \cdot k^1}{2}(A-B), \quad \mathcal{E} = e^{\left[ik^1 \cdot X^-, ik^2 \cdot X^*\right]}. \tag{25}$$

X^- is the annihilation part of X while X^* is X with all annihilation operators converted to creation operators using the boundary state.

The prefactor P_2 may now be bounced off the boundary state converting it into P_2^* which contains only creation operators. The relevant prefactor commutator is

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[4] A symmetric traceless polarisation tensor $\zeta_{\mu\nu}$ can be expressed as $\zeta_{\mu\nu} = \sum_{i=1}^n a_i \zeta^i_\mu \zeta^i_\nu$ with a_i constant. Without loss of generality we take $\zeta_{\mu\nu} = \zeta_\mu \zeta_\nu$.

[5] This follows from the fact that $\zeta{\cdot}\Gamma \zeta{\cdot}\Gamma = k{\cdot}\Gamma k{\cdot}\Gamma = 0$.

$$[P_1, P_2^*] = H_{12} + G_{12}, \tag{26}$$

where G_{12} and H_{12} are given by

$$\begin{aligned} G_{12} &= -\frac{1}{4}(A+B)k^1_{[\sigma}\zeta^1_{\mu]}k^{2[\mu}\zeta^2_{\rho]}\Psi^\rho\Psi^\sigma, & (27)\\ H_{12} &= \zeta^1\cdot\zeta^2\left(\frac{zy}{(z-y)^2}+\frac{z\bar{y}}{(z\bar{y}-1)^2}+c.c.\right)+\frac{1}{8}\zeta^1_{[\mu}k^1_{\nu]}\zeta^{2[\nu}k^{2\mu]}(A+B)^2, & (28)\end{aligned}$$

where $\zeta^1_{[\mu}k^1_{\nu]} = \zeta^1_\mu k^1_\nu - \zeta^1_\nu k^1_\mu$. Since G_{12} is normal ordered it will not contribute to the two graviton scattering amplitude. $\mathcal{E}$ is given by

$$(y-z)^{k^1\cdot k^2/4}\left(y-\frac{1}{\bar{z}}\right)^{k^1\cdot k^2/4} y^{-k^1\cdot k^2/4}\times c.c. \tag{29}$$

The total matrix element is

$$M_2 = \frac{T_5 W}{8}\left\{H_{12}-\frac{1}{4}\zeta^1\cdot k^2\zeta^2\cdot k^1(A-B)^2\right\}|y-z|^{k^1\cdot k^2/2}|y-\frac{1}{\bar{z}}|^{k^1\cdot k^2/2}|y|^{-k^1\cdot k^2/2}. \tag{30}$$

The amplitude as it stands contains second order poles and thus would appear to factorise on tachyonic excitations. These undesirable poles are expected when using vertex operators in certain pictures [31].[6] It can be shown that they combine into terms which contain first order poles and total derivatives. Performing this operation the amplitude under consideration becomes

$$A_2 = \frac{T_5 W}{128\pi^2}\int_{|z|\geq 1}\frac{d^2z}{|z|^{2+k^1\cdot k^2/2}}\int_{|y|\geq 1}\frac{d^2y}{|y|^{2+k^1\cdot k^2/2}}\frac{(k^1\cdot\zeta^2 k^2\cdot\zeta^1 - k^1\cdot k^2\zeta^1\cdot\zeta^2)\mathrm{Im}^2(z\bar{y})}{|z-y|^{2-k^1\cdot k^2/2}|z\bar{y}-1|^{2-k^1\cdot k^2/2}}. \tag{31}$$

Since we are concerned with scattering in the world-volume of D-branes the transverse momenta vanish. In that case momentum conservation implies that $k^i\cdot k^j = 0$ and so $\mathcal{E} = 1$. The region of integration can be divided into two sub-regions - $|y| < |z|$ and $|z| < |y|$. In each of these regions one may expand a, b and their complex conjugates in a power series. This gives

$$\begin{aligned} &\int_{|z|\geq 1}\frac{d^2z}{|z|^2}\int_{|y|\geq 1}\frac{d^2y}{|y|^2}AB = 2\int_{|z|\geq 1}\frac{d^2z}{|z|^2}\int_{|z|\geq|y|\geq 1}\frac{d^2y}{|y|^2}AB\\ &= 2\int_{r=1}^{\infty}\frac{drd\theta_1}{r}\int_{\rho=1}^{\infty}\frac{d\rho d\theta_2}{\rho}\sum_{m,n=1}^{\infty}\left(\frac{\rho}{r}\right)^m\left(e^{i(\theta_1-\theta_2)m}-e^{i(\theta_2-\theta_1)m}\right)\left(\frac{1}{r\rho}\right)^n\left(e^{i(\theta_2-\theta_1)n}-e^{i(\theta_1-\theta_2)n}\right), \quad (32)\end{aligned}$$

where $y = \rho e^{i\theta_2}$ and $z = re^{i\theta_1}$. Integrating out the angular variables one obtains

$$4(2\pi)^2\int_{r=1}^{\infty}\int_{\rho=1}^{r}\sum_{n=1}^{\infty}r^{-2n}\frac{d\rho}{\rho}\frac{dr}{r} = 4(2\pi)^2\sum_{n=1}^{\infty}\int_{r=1}^{\infty}r^{-2n-1}\ln r dr. \tag{33}$$

Since

$$\int_{r=1}^{\infty}r^{-2n-1}\ln r dr = \frac{1}{4n^2}, \tag{34}$$

[6]For a recent addition to this reasoning in the context of D-branes see [32].

the integral in equation (32) becomes

$$4\pi^2 \sum_{n=1}^{\infty} \frac{1}{n^2} = 4\pi^4 \left| B_2 \right| = \frac{2\pi^4}{3}, \tag{35}$$

where B_n is the n-th Bernoulli number. We may then write down the final amplitude

$$A_2 = \frac{T_5 \pi^2}{192} \varepsilon^{\mu_1 \dots \mu_6} C^{(2)}_{\mu_1 \mu_2} k^1_{\mu_3} \zeta^1_{\mu_4} k^2_{\mu_5} \zeta^2_{\mu_6} (\zeta^1 \cdot k^2 \zeta^2 \cdot k^1 - k^1 \cdot k^2 \zeta^1 \cdot \zeta^2), \tag{36}$$

We observe that the evaluation of the integral differs by a factor of 2 from the one in the work of Craps and Roose [23].[7]

3.2 The O9-plane amplitude

In this subsection we consider scattering $C^{(6)}$ with two gravitons on a O9-plane. The crosscap state which represents the O9-plane has a zero mode contribution of the form $\Gamma^0 \dots \Gamma^9$, while the $C^{(6)}$ state also has six gamma matrices. The fermionic zero mode contribution to the crosscap amplitude is then

$$W = \frac{32}{4} \varepsilon^{\mu_1 \dots \mu_{10}} C^{(6)}_{\mu_1 \dots \mu_6} k^1_{\mu_7} \zeta^1_{\mu_8} k^2_{\mu_9} \zeta^2_{\mu_{10}}. \tag{37}$$

The crosscap state is much like the boundary state except that it contains an extra $(-1)^n$ in the exponential terms of the coherent states describing the boundary state. This modifies B to

$$B' = \frac{-1}{1 + y\bar{z}} - \frac{-1}{1 + z\bar{y}}, \tag{38}$$

while A remains unchanged. Thus the final integration now includes a factor of $(-1)^n$. It is easy to sum the appropriate series using

$$\sum_{k=1}^{\infty} (-1)^k \frac{1}{k^{2n}} = \frac{(1 - 2^{2n-1})\pi^{2n}}{(2n)!} |B_{2n}|. \tag{39}$$

The amplitude for scattering a $C^{(6)}$ and two gravitons off a boundary can be obtained as a generalisation of the calculation in the previous section. It follows from equation (39) that the crosscap amplitude is $-\frac{1}{2}\eta_{10}$ times the boundary amplitude.

4 The eight-form amplitudes

In this section the eight-form gravitational couplings of D-branes and O-plane states are studied. We first discuss the D9-brane amplitude and then, as before, by a simple modification obtain the O9-plane amplitude. This will allow us to explicitly check gravitational anomaly cancellation in the ten dimensional Type I theory.

4.1 The D9-brane amplitude

In order to confirm the ten-dimensional term we calculate the amplitude for four gravitons and one $C^{(2)}$ on a D9-brane. This is given by

$$A_4 = (4\pi)^{-4} \int_{|z_4| \geq 1} \frac{d^2 z_4}{|z_4|^2} \int_{|z_3| \geq 1} \frac{d^2 z_3}{|z_3|^2} \int_{|z_2| \geq 1} \frac{d^2 z_2}{|z_2|^2} \int_{|z_1| \geq 1} \frac{d^2 z_1}{|z_1|^2} M_4, \tag{40}$$

[7] This may account for the discrepancy that these authors reported between the amplitude and the low energy effective action.

where

$$M_4 = \langle C^{(2)}, k^0 \,|\, T\left\{V_g\left(k^1, z_1, \bar{z}_1\right) V_g\left(k^2, z_2, \bar{z}_2\right) V_g\left(k^3, z_3, \bar{z}_3\right) V_g\left(k^4, z_4, \bar{z}_4\right)\right\} |B\rangle \,. \tag{41}$$

As in the D5-brane case the R⊗R two form induces a trace over the fermionic zero modes of the boundary state and the vertex operators. This contributes a factor

$$W = \frac{32}{16}\varepsilon^{\mu_1\ldots\mu_{10}} C^{(2)}_{\mu_1\mu_2}\zeta^1_{\mu_3}k^1_{\mu_4}\ldots\zeta^4_{\mu_9}k^4_{\mu_{10}}, \tag{42}$$

and allows for the definition of modified graviton vertex operators as before. Defining

$$\begin{aligned} a_{ij} &= \frac{z_j}{z_i - z_j}, \quad b_{ij} = \frac{1}{\bar{z}_i z_j - 1}, \\ A_{ij} &= \frac{\mathrm{Im}(z_i\bar{z}_j)}{|z_i - z_j|^2}, \quad B_{ij} = \frac{\mathrm{Im}(z_i\bar{z}_j)}{|z_i\bar{z}_j - 1|^2}. \end{aligned} \tag{43}$$

the matrix element may be written as

$$\begin{aligned} M_4 &= \frac{T_9 W}{16}\,\langle C^{(2)}, k^0 \,|\, P_1 e^{ik^1X} P_2 e^{ik^2X} P_3 e^{ik^3X} P_4 e^{ik^4X}\,|B\rangle \\ &= \frac{T_9 W}{64}\mathcal{E}\,\langle C^{(2)}, k^0 \,|\, (P_1 + A_1)(P_2 + A_2)(P_3 + A_3)(P_4 + A_4)\,|B\rangle \,, \end{aligned} \tag{44}$$

where the P_i are defined as previously and

$$A_i = \sum_{j\neq i} \frac{\zeta^i \cdot k^j}{2}(A_{ij} - B_{ij}). \tag{45}$$

$\mathcal{E}$ is given by

$$\mathcal{E} = \prod_{\substack{i=1 \\ j>i}}^{4} (z_i - z_j)^{k^i\cdot k^j/4}(z_i - \frac{1}{\bar{z}_j})^{k^i\cdot k^j/4} z_i^{-k^i\cdot k^j/4} \times c.c. \tag{46}$$

The computation of the matrix element uses the commutator $\left[P_i, P_j^*\right]$ obtained previously, where now the G_{ij} operator has a non-trivial effect. The overall matrix element is

$$\begin{aligned} M_4 = &\frac{T_9 W}{256}\left[\mathrm{tr}(R^1R^2)\mathrm{tr}(R^3R^4)A_{12}B_{12}A_{34}B_{34} + \mathrm{tr}(R^1R^3)\mathrm{tr}(R^2R^4)A_{13}B_{13}A_{24}B_{24} + \mathrm{tr}(R^2R^3)\mathrm{tr}(R^1R^4)A_{23}B_{23}A_{14}B_{14}\right]\mathcal{E} \\ &+\frac{T_9 W}{512}\mathrm{tr}(R^1R^2R^3R^4)\left[(A_{14}B_{12} + A_{12}B_{14})(A_{23}A_{34} + B_{34}B_{23}) + (A_{23}B_{34} + A_{34}B_{23})(A_{12}A_{14} + B_{12}B_{14})\right]\mathcal{E} \\ &+\frac{T_9 W}{512}\mathrm{tr}(R^1R^3R^2R^4)\left[(A_{14}B_{13} + A_{13}B_{14})(A_{23}A_{24} + B_{24}B_{23}) + (A_{23}B_{24} + A_{24}B_{23})(A_{13}A_{14} + B_{13}B_{14})\right]\mathcal{E} \\ &+\frac{T_9 W}{512}\mathrm{tr}(R^1R^3R^4R^2)\left[(A_{12}B_{13} + A_{13}B_{12})(A_{34}A_{24} + B_{24}B_{34}) + (A_{34}B_{24} + A_{24}B_{34})(A_{13}A_{12} + B_{13}B_{12})\right]\mathcal{E}, \end{aligned} \tag{47}$$

which we integrate over the measure in equation (40). Here the traces are expressed in terms of the momenta and polarisations of the gravitons (see equation (54) below).

The integral of the first term (the $\mathrm{tr}(R^2)^2$ term) in (47) splits into a product of two integrals encountered in the previous section. The integration gives a factor of $(2\pi^4/3)^2 = 4\pi^8/9$. Now consider the $\mathrm{tr}(R^1R^2R^3R^4)$ term. This contains two kinds of integrals: ones involving three A's and one B and ones with three B's and one A. It is easy to see by manipulating the indices on the z_i, that each of the four AAAB terms contributes the same amount. Similarly all the BBBA integrals are the same. Let us then integrate first, say, $A_{12}B_{14}B_{23}B_{34}$. As in the previous section we split the region of integration into two parts namely $|z_1| > |z_2|$ and $|z_1| < |z_2|$. We then expand in a series of z_2/z_1 or z_1/z_2. After integrating out the angular coordinates θ_i, where $z_i = x_i e^{i\theta_i}$, the two regions again combine to give

$$\begin{aligned}
&\prod_{i=1}^{4}\int_{|z_i|\geq 1}\frac{d^2z_i}{|z_i|^2}A_{12}B_{14}B_{23}B_{34} = 64\pi^4\int_{x_2,x_3,x_4=1}^{\infty}\int_{x_1=1}^{x_2}\sum_{n=1}^{\infty}(x_2x_3x_4)^{-2n}\,\frac{dx_4dx_3dx_1dx_2}{x_4x_3x_1x_2}\\
&= 16\pi^4\sum_{n=1}^{\infty}\frac{1}{n^2}\int_{x_2=1}^{\infty}x_2^{-2n-1}\ln x_2 dx_2 = 4\pi^4\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{2\pi^8}{45}.
\end{aligned} \tag{48}$$

The AAAB integrals are a bit more involved as now one has 8 regions, but again they can be integrated fairly easily to give for each AAAB integral $14\pi^8/45$. The final amplitude then becomes

$$\begin{aligned}
A_4 =\ & \frac{T_9\pi^4}{73728}\varepsilon^{\mu_1\ldots\mu_{10}}C^{(2)}_{\mu_1\mu_2}k^1_{\mu_3}\zeta^1_{\mu_4}\ldots k^4_{\mu_9}\zeta^4_{\mu_{10}}\zeta^1_{[\mu_1}k^1_{\mu_2]}\\
&\times\Big[\zeta^{2[\mu_2}k^{2\mu_1]}\zeta^3_{[\mu_3}k^3_{\mu_4]}\zeta^{4[\mu_4}k^{4\mu_3]} + \zeta^{3[\mu_2}k^{3\mu_1]}\zeta^2_{[\mu_3}k^2_{\mu_4]}\zeta^{4[\mu_4}k^{4\mu_3]} + \zeta^{4[\mu_2}k^{4\mu_1]}\zeta^2_{[\mu_3}k^2_{\mu_4]}\zeta^{3[\mu_4}k^{3\mu_3]}\Big]\\
&+\frac{T_9\pi^4}{46080}\varepsilon^{\mu_1\ldots\mu_{10}}C^{(2)}_{\mu_1\mu_2}k^1_{\mu_3}\zeta^1_{\mu_4}\ldots k^4_{\mu_9}\zeta^4_{\mu_{10}}\zeta^1_{[\mu_1}k^1_{\mu_2]}\\
&\times\Big[\zeta^{2[\mu_2}k^{2\mu_3]}\zeta^3_{[\mu_3}k^3_{\mu_4]}\zeta^{4[\mu_4}k^{4\mu_1]} + \zeta^{3[\mu_3}k^{2\mu_3]}\zeta^2_{[\mu_3}k^2_{\mu_4]}\zeta^{4[\mu_4}k^{4\mu_1]} + \zeta^{3[\mu_2}k^{3\mu_3]}\zeta^4_{[\mu_3}k^4_{\mu_4]}\zeta^{2[\mu_4}k^{2\mu_1]}\Big].
\end{aligned} \tag{49}$$

As we will see below the symmetry factors of the two eight form terms are the same, thus we may compare their ratio

$$\frac{\mathrm{tr}R^4}{(\mathrm{tr}R^2)^2} = \frac{8}{5}, \tag{50}$$

which is precisely the one expected for the expansion of $\sqrt{\hat{A}}$ in equation (2).

4.2 The O9-plane amplitude

As in the case of the $\mathrm{tr}R^2$ coupling we now calculate the amplitude involving the R⊗R two-form, four gravitons and a crosscap. In effect we are calculating the contribution of a process on a non-orientable worldsheet to the Green-Schwarz gravitational anomaly cancelling term. Again there are only minor changes to the amplitude given in equation (47). Namely the B_{ij} become

$$B'_{ij} = \frac{-1}{1+z_i\bar{z}_j} - \frac{-1}{1+z_j\bar{z}_i}, \tag{51}$$

while the A_{ij} remain fixed. Equations (33)-(35) and (48) show that the coefficients of $(\mathrm{tr}R^2)^2$ and $\mathrm{tr}R^4$ in the boundary state calculation are proportional to $\left(\sum\frac{1}{n^2}\right)^2$ and $\sum\frac{1}{n^4}$, respectively. The crosscap, as discussed above introduces a factor of $(-1)^n$ into these sums. Using equation (39) we have

$$\left(\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\right)^2 = \frac{1}{4}\left(\sum_{n=1}^{\infty}\frac{1}{n^2}\right)^2,$$
$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n^4} = -\frac{7}{8}\sum_{n=1}^{\infty}\frac{1}{n^4}. \tag{52}$$

Thus, subject to the crosscap normalisation, the crosscap $(\mathrm{tr}R^2)^2$ and $\mathrm{tr}R^4$ terms are $\frac{\eta_{10}}{4}$ and $-\frac{7\eta_{10}}{8}$ times the D-brane couplings, respectively. Since the symmetry factors of the two eight form terms are the same we compare their ratio

$$\frac{\mathrm{tr}R^4}{(\mathrm{tr}R^2)^2} = -\frac{28}{5}, \tag{53}$$

which is precisely the one expected for the expansion of $\sqrt{L(R/4)}$ in equation (4).

5 Comparison with the D-brane action

In order to confirm the WZ D-brane action in equation (2) it is necessary to write it in a form comparable with the amplitude calculations of the previous sections. The linearised Ricci two-form in terms of graviton polarisation and momentum is given by

$$R_{\mu\nu}{}^{ab} = (\zeta_\mu k_\nu - k_\mu\zeta_\nu)(\zeta^a k^b - k^a\zeta^b), \tag{54}$$

where a, b are tangent bundle indices.

The symmetry factors of the $\mathrm{tr}R^2$ term is 2 while both the $\mathrm{tr}R^4$ and $(\mathrm{tr}R^2)^2$ terms have a symmetry factor of 8. The D-brane WZ action then gives the following amplitudes

$$\begin{aligned}
&\frac{T_5}{48(4\pi)^2}\varepsilon^{\mu_1\ldots\mu_6}C^{(2)}_{\mu_1\mu_2}\zeta^1_{\mu_3}k^1_{\mu_4}\zeta^2_{\mu_5}k^2_{\mu_6}(\zeta^1\!\cdot k^2\zeta^2\!\cdot k^1 - \zeta^1\!\cdot\zeta^2 k^1\!\cdot k^2)\\
&+\frac{T_9}{4608(4\pi)^4}\varepsilon^{\mu_1\ldots\mu_{10}}C^{(2)}_{\mu_1\mu_2}\zeta^1_{\mu_3}k^1_{\mu_4}\ldots\zeta^4_{\mu_9}k^4_{\mu_{10}}\zeta^1_{[\mu_1}k^1_{\mu_2]}\\
&\times\left[\zeta^{2[\mu_2}k^{2\mu_1]}\zeta^3_{[\mu_3}k^3_{\mu_4]}\zeta^{4[\mu_4}k^{4\mu_3]} + \zeta^{3[\mu_2}k^{3\mu_1]}\zeta^2_{[\mu_3}k^2_{\mu_4]}\zeta^{4[\mu_4}k^{4\mu_3]} + \zeta^{4[\mu_2}k^{4\mu_1]}\zeta^2_{[\mu_3}k^2_{\mu_4]}\zeta^{3[\mu_4}k^{3\mu_3]}\right]\\
&+\frac{T_9}{2880(4\pi)^4}\varepsilon^{\mu_1\mu_2\ldots\mu_9\mu_{10}}C^{(2)}_{\mu_1\mu_2}\zeta^1_{\mu_3}k^1_{\mu_4}\ldots\zeta^4_{\mu_9}k^4_{\mu_{10}}\zeta^1_{[\mu_1}k^1_{\mu_2]}\\
&\times\left[\zeta^{2[\mu_2}k^{2\mu_3]}\zeta^3_{[\mu_3}k^3_{\mu_4]}\zeta^{4[\mu_4}k^{4\mu_1]} + \zeta^{3[\mu_2}k^{3\mu_3]}\zeta^2_{[\mu_3}k^2_{\mu_4]}\zeta^{4[\mu_4}k^{4\mu_1]} + \zeta^{3[\mu_2}k^{3\mu_3]}\zeta^4_{[\mu_3}k^4_{\mu_4]}\zeta^{2[\mu_4}k^{2\mu_1]}\right].
\end{aligned} \tag{55}$$

Let us now compare this with the amplitudes calculated in the previous sections. While the exact normalisation of graviton vertex operators can be obtained from first principles, this seems unnecessarily convoluted. Instead we will show that the terms calculated in the amplitudes above are in the correct ratios to each other.

The graviton normalisation (say v) is obtained by comparing coefficients in equations (36) and (55) giving $v = (2.\pi^2)^{-1}$. Now the $\mathrm{tr}(R^2)^2$ coefficient in the amplitude is

$$\frac{\pi^4 v^4 T_9}{73728} = \frac{T_9}{4608(4\pi)^4}, \tag{56}$$

which corresponds to precisely the result expected in equation (55). Similarly one can confirm the $\mathrm{tr}(R^4)$ term's coefficient.

6 The O-Plane action and Green-Schwarz anomaly cancellation

In the course of this calculation it became clear that the gravitational couplings of orientifold planes were somewhat different from the ones postulated in [21].

Type I theory in 10 dimensions has a tadpole cancellation due to its gauge group being SO(32) [19, 20] giving $\eta_{10} = -32$. In lower dimensions the number of O-planes doubles, thus $\eta_d = -2^{d-5}$. Since in sections 3 and 4 we expressed the O-plane couplings as fractions of the D-brane couplings it is now straightforward to confirm the couplings in equation (4). Explicitly the four form coupling of the O9-plane was $-\eta_{10}/2 = 16$ times that of the D9-brane, while the eight-form $\mathrm{tr}R^4$ and $(\mathrm{tr}R^2)^2$ couplings are $\eta_{10}/4 = -8$ and $-7\eta_{10}/8 = -28$, respectively times the D9-brane coupling. These are in exact agreement with equation (4). Gravitational couplings of other O-planes can be similarly deduced and are summarised in (3).

Finally one can confirm the Green-Schwarz gravitational anomaly cancelling terms [24] of Type I theory. These are

$$S_{GS} = T_9 \int \frac{2}{(4\pi)^2} C^{(6)}{}_\wedge \mathrm{tr}R^2 + \frac{1}{12(4\pi)^4} C^{(2)}{}_\wedge \left[\frac{1}{4}\mathrm{tr}(R^2)^2 + \mathrm{tr}R^4\right] . \tag{57}$$

It is a crucial consistency check that the gravitational couplings of 32 D9-branes and an O9-plane reproduce the Green-Schwarz terms. In fact again exact agreement is found.

Acknowledgements

I would like to thank Michael Green for introducing me to this problem and for his continued support throughout its completion. I would also like to acknowledge the many useful discussions with Matthias Gaberdiel and Marco Barrozo.

Added note concerning O-planes and the Hirzebruch polynomial

Following the interpretation of (4) in terms of the square root of the Hirzebruch polynomial [33] it is very plausable that the O-plane WZ action can be determined from a generalisation of the anomaly inflow argument that was used in [10] to determine the D-brane WZ term. For example, two O7-planes intersect in a manner that gives rise to a chiral 5+1 dimensional $N = 1$ supersymmetric theory in the intersection domain. The spectrum includes a second rank antisymmetric self-dual tensor which has an anomaly obtained by descent from the eight form piece of the Hirzebruch polynomial.

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