A Panel Data Test for Poverty Traps

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Abstract

This paper develops a threshold panel data nonlinearity test for poverty traps. The new testing strategy extends the work on nonlinearity tests for panel data by considering threshold nonlinearities in the fixed-effects components. Monte Carlo simulations are conducted to evaluate the finite-sample performance of these tests. his test is applied to the relationship between GDP per capita and capital stock per capita. Our application to a panel of countries for the period 1973-2007 uncovers the presence of two regimes determined by the level of capital stock per capita. The conclusions from our test also support the existence of a poverty trap determined by a capital stock per capita level at the 11% quantile of its pooled worldwide distribution.

Keywords: nonlinearity tests, panel data, poverty traps, threshold models.

JEL classification: C33, C12, C13, O1

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1 Introduction

Poverty has a tendency to persist over time. This raises the suspicion that underdevelopment is a state of equilibrium and that there are forces at work that tend to restore the equilibrium every time there are small improvements in living conditions. Moreover, this gives the idea of a vicious circle of poverty as a "constellation of forces tending to act and react upon one another in such a way as to keep a poor country in a state of poverty" (Nurkse, 1953, p.4). A poverty trap arises when poor individuals or countries are faced with two distinct equilibria, one below the poverty line and one above it. Individuals or countries with a sufficiently low income or asset endowments are trapped in the poor-equilibrium, and small improvements are not enough to escape the forces bringing them back to this level. This idea was first applied in the early development economics literature. For instance, Rosenstein-Rodan (1943) "big-push" theory, where countries needed a big enough inflow of capital to break the vicious cycle of poverty, implicitly assumes a dual-equilibrium process. Moreover, the theory of different "convergence clubs" (Baumol, 1986; DeLong, 1988; Quah, 1993, 1996, 1997) turns fundamentally on the existence of an exclusionary mechanism that keeps members of one group or club facing a lower level equilibrium from moving to another group or club with a higher level equilibrium.

To motivate the existence of a poverty trap we follow Barro and Sala-i-Martin (2004, ch.1) exposition of a Solow-Swan type model with a generic country that has access to a traditional and a modern technology. This simple model produces a poverty trap if nonlinearities exist in the production function describing the relationship between GDP per capita and capital stock per capita.[1] We, then, develop a poverty trap test by means of a nonlinearity test of the production function, where the nonlinearity may be present either in the capital stock (capital elasticity), in the technology parameter, or in both. The latter two require panel data and country-specific fixed-effects. This new testing strategy extends the work of Hansen (1999) on nonlinearity tests for panel data[2] by also considering the possibility of a threshold effect on the fixed-effects component,

[1]While our work is framed within the dual-equilibrium paradigm, the existence of poverty traps is also contested on the empirical growth literature. For instance, Jones and Olken (2008) claim that "almost all countries in the world have experienced rapid growth lasting a decade or longer", and that "economic growth can be easily reversed, often leaving countries no better off than they were prior to the expansion" (p.584).

[2]Threshold nonlinearity tests for panel data have been recently applied by Nautz and Scharff (2012) for the case of inflation and relative price variability.

a procedure that was recently suggested by Bick (2010), who proposes a threshold regime switching model for studying inflation and economic growth.

The closest contributions to our application are Durlauf and Johnson (1995) and Bloom, Canning, and Sevilla (2003). These authors however do not implement formal tests to determine the presence of nonlinearities in the production function or make use of a panel of observations. That is, they consider multiple regimes in the production function and test for this using a cross-section of countries. We argue that the panel data set-up is more appropriate because of potential model misspecification. If there are idiosyncratic differences across countries and regimes, inference made on cross-sectional only models may be misleading. Including additional control variables to capture countries' idiosyncratic characteristics or initial conditions (such as education) does not solve the potential misspecification problem, because which variables should be included is an empirical problem that will depend on the available data and nature of the countries under study. A more pragmatic alternative, which we follow, is to consider differences in the country-specific intercept parameters, i.e. fixed-effects. These fixed-effects can be interpreted as differences in the countries' technology parameters for the Solow-Swan type production function regression or differences in country-specific variables, e.g. natural resources, governance, human capital, for the regression equation in growth models. For instance, Graham and Temple (2006) find that multiple equilibria is associated with differences in aggregate total factor productivity (TFP). It is also reasonable to assume that these fixed-effects are themselves functions of the capital stock, as in Romer (1986) and Azariadis and Drazen (1990), or depend on the initial conditions of the endogenous variables in the presence of historical self-reinforcement (Mookherjee and Ray, 2001). Therefore, we believe that a model that allows for threshold effects on the country-specific fixed-effects is an important contribution to the empirical growth literature.

Our empirical analysis is concerned with a panel data containing 138 countries and covering 35 years, from 1973 to 2007. There is strong statistical evidence to reject the linear model due to threshold effects. The interpretation of this nonlinearity depends on the testing method and the threshold estimate. Thus, whereas the standard F-test developed in Hansen (1999) reports a threshold at the 41% quantile of the pooled worldwide distribution of capital stock per capita, our nonlinear method allowing for a change in both fixed effects and capital per-capita yield a threshold at 11%. These findings suggest changes in technology parameters and other nonobservable factors that depend on the amount of capital. The difference between our method and that of Hansen (1999) might therefore be attributed to model misspecification. A more restricted model only allowing for changes in the fixed effects but taking capital elasticity as constant across regimes also rejects linearity. In this case, the capital threshold is 46% of the overall distribution of capital. These results give evidence of the importance of considering the possibility of changes in all the parameters of growth regression equations.

The article is structured as follows. Section 2 discusses the extensive literature on growth models dealing with poverty traps and multiple equilibria with special attention to empirical papers proposing statistical tests for detecting threshold models. Section 3 sets the theoretical foundations for this paper and introduces different tests for the hypotheses of multiple equilibria derived from the existence of more than a single production function. Section 4 presents a Monte Carlo study to assess in finite samples the performance of these tests in terms of size and power. These nonlinear tests are implemented in Section 5 to analyze empirically the existence of poverty traps and/or multiple equilibria in the relationship between income and capital per capita. Section 6 concludes. Tables and figures are gathered in an appendix.

2 Literature Review

In economic growth theory, neoclassical models like Solow (1956) or Diamond (1965) assume that countries with similar characteristics and with access to similar technologies may show temporary differences in output growth levels, but these will disappear in the long-run steady state equilibrium. Furthermore, if the fundamental factors can be assumed to be the same across countries their income levels and growth rates should be distributed around a single expected value. The empirical evidence, however, contradicts this claim. While some countries manage to sustain high growth rates of per capita income others stagnate in low growth traps exhibiting low levels of economic development.

There is an extensive literature on models able to generate poverty traps and multiple equilibria. One mechanism that has been extensively analyzed is increasing returns to scale. This mechanism implies the existence of a threshold value such that once it is exceeded, increasing returns to scale can make investment and capital accumulation more productive, and lead to a self-sustaining high equilibrium. Azariadis and Drazen (1990) focus on technological externalities that permit returns to scale to rise rapidly whenever economic and social variables exceed certain thresholds. Among the types of externalities considered by these authors are spillovers from the stocks of physical and human capital. Durlauf and Johnson (1995) explore this possibility and develop a statistical model for describing nonlinearities in cross country growth rates due to returns to scale in stock of capital. Bloom, Canning, and Sevilla (2003) discuss endogenous fertility (see also Barro and Becker (1989), Becker, Murphy, and Tamura (1990)) as another mechanism for the generation of poverty traps. In this model improvements on health or the accumulation of knowledge beyond some critical point increase the return to human capital to lead parents to produce fewer children but invest more in each child. Other possible mechanisms are, for example, the existence of imperfect credit markets, lack of credibility of legal entities and of trustable political institutions. Azariadis (1996), and more recently Azariadis and Stachurski (2005), present an excellent survey of theoretical models with the potential to produce poverty traps.

Surprisingly, there are very few empirical studies assessing the validity of each of the above theories. Some exceptions are Bloom, Canning, and Sevilla (2003) that allow for the existence of two regimes in GDP per capita. These authors propose a Markov regime switching model. The income per capita variable changes between regimes with probabilities $p(x)$ and $1 - p(x)$, respectively, where x is a set of exogenous characteristics. The nature of this statistical model, however, does not permit to identify the theoretical model generating the nonlinearity. In a related context, Durlauf and Johnson (1995) apply nonlinear regression tree methods to test the existence of convergence clubs among countries. These authors model the threshold nonlinearity by grouping countries with similar characteristics and running different growth regression equations for each club. This seminal contribution finds strong evidence of different rates of convergence for developed and developing economies. Other influential empirical studies are Lee, Pesharan, and Smith (1997) that consider international per capita output and study their growth for 102 countries using panel data; Quah (1996, 1997) investigates the existence of twin peaks in the empirical distribution of national per capita income, Bianchi (1997) and Paap and van Dijk (1998) introduce mixtures of distributions to describe the divergence in growth among countries.

3 Econometric tests for two regimes in the production function

This section introduces threshold nonlinearity tests in a panel data context. The null hypothesis corresponds to a linear model and the alternative to a nonlinear model defined by a capital per capita threshold variable that can affect not only the relationship between income and capital per capita but also other unobserved components explaining income per capita. These factors are reflected in the fixed-effects. First, we describe the theoretical foundations on the existence of poverty traps, the corresponding cross-sectional econometric tests and their limitations. Next, we consider the extension to panel data.

3.1 Cross-sectional econometric tests

To motivate the existence of a poverty trap we follow Barro and Sala-i-Martin (2004, ch.1) exposition of a Solow-Swan type model with a generic country that has access to a traditional (A) and a modern (B) technology. Each technology is represented by a Cobb-Douglas production function with two factors of production (labor and capital) and constant returns to scale.[3] In per capita terms, these production functions are $Y_A = A\ K^{\alpha}$ and $Y_B = B\ K^{\beta} - c$, where Y is a measure of per capita national income or GDP, K the per capita capital stock and $\alpha, \beta \in (0,1)$ and $B > A > 0$ are technology parameters. It is assumed that in order to exploit the better technology the country has to pay a setup cost, that is given in per capita terms by $c > 0$. The envelope production function takes the form $Y = max\{A\ K^{\alpha}; B\ K^{\beta} - c\}$. Different from Barro and Sala-i-Martin (2004, ch.1) we allow for the possibility of $\alpha \neq \beta$, that is, distinct capital elasticities, and for simplicity, assume that $\alpha < \beta$ to ensure non-reversibility. After simple algebra, the log-linearized envelope production

[3] We use a very stylized model. Different production functions can be used here controlling for education levels (in a Mincerian sense) or other observable variables. In the following subsection, we argue that these variables should be captured by the country-specific fixed effect.

function can be expressed as the combination of the two above functions:

$$y = \begin{cases} a + \alpha k, & \text{if } k \leq \tilde{k}, \\ b + \beta k + \ln\left(1 - \frac{c}{BK^{\beta}}\right), & \text{if } k > \tilde{k}, \end{cases} \tag{1}$$

where for notational convenience we use $x = lnX$. Thus, $\tilde{k} = ln\tilde{K}$ is the solution to $A\ K^{\alpha} = B\ K^{\beta} - c$ that represents the threshold where the country is better off by adopting the modern technology. Of course if $\alpha = \beta$, then $\tilde{K} = [c/(B - A)]^{1/\alpha}$.

The poverty trap hypothesis arises because the countries' production functions can be modeled as (1). This dual production function does not imply the existence of a poverty trap. The latter depends on the existence of different steady state solutions of the growth model. Assuming, as in the Solow-Swan model, that the population growth rate ($n > 0$), the savings rate ($s \in (0, 1)$) and the depreciation rate ($\delta \in (0, 1)$) are exogenous, multiple equilibria arise if there are at least two steady states with capital stock per capita $K^* < \tilde{K} < K^{**}$ such that $A\alpha {K^{*}}^{\alpha-1} = \frac{n+\delta}{s} = B\beta {K^{**}}^{\beta-1} - c/K^{**}$. We focus on tests for nonlinearity of the log production function because this is one of the most important cases where the poverty trap can be motivated.

In statistical terms, the null hypothesis of a single production function is

$$y_i = d + \gamma k_i + \varepsilon_i, \tag{2}$$

with $i = 1, 2, ..., N$, a cross-sectional sample of N countries, ε_i a random variable describing the component of the production function not explained by per capita capital, and satisfying $E[\varepsilon_i] = 0$ and $E[\varepsilon_i \varepsilon_j] = 0$ for $i \neq j$, $i, j > 0$, and d an exogenous parameter usually identified with technology factors.

The alternative competing model is a simplification of model (1) given by a threshold model with two regimes for each country:

$$y_i = \begin{cases} a + \alpha k_i + \varepsilon_i, & \text{if } k_i \leq \tilde{k}, \\ b + \beta k_i + \varepsilon_i, & \text{if } k_i > \tilde{k}, \end{cases} \tag{3}$$

To obtain this model we assume that $\ln\left(1 - \frac{c_i}{B\tilde{K}_i^{\beta}}\right) \approx 0$ for every country in the study. That is, the cost of switching to a better technology is negligible compared to the per-capita output level obtained in the new regime with a capital of $\tilde{K}$. In addition, model (3) also assumes that the two output regimes are determined by the same threshold value of log capital per-capita, $\tilde{k}$, for all countries. We believe that this assumption is not very restrictive given that the model is in per-capita terms. It seems natural to think that the levels of capital per-capita that induce a high-productivity regime are similar across countries. As we discussed previously, for each economy the threshold $\tilde{K}_i$ is the solution to $A_i\ K_i^{\alpha} = B_i\ K_i^{\beta} - c_i$. Now, using simple algebra and taking logs we obtain the following expression

$$\tilde{k}_i = \frac{b_i - a_i}{\alpha - \beta} + \frac{1}{\alpha - \beta} \ln\left(1 - \frac{c_i}{B\tilde{K}_i^{\beta}}\right),$$

which under the previous assumption simplifies to

$$\tilde{k}_i = \frac{b_i - a_i}{\alpha - \beta}. \tag{4}$$

Hence, using the same threshold $\tilde{k}_i = \tilde{k}$ for every country implies that $b_i - a_i$ also takes the same value across countries.[4] That is, assuming that all the economies switch to a new regime when they achieve the same levels of per-capita capital implies a constraint on the amount of heterogeneity between regimes and countries. This assumption will be further discussed in the next section because it is also required to avoid the incidental parameter problem.

After simple algebra it can be shown that this model admits the following representation:

$$y_i = x_i(\tilde{k})'\rho + \varepsilon_i, \tag{5}$$

with $x_i(k) = (1, I(k_i > k), k_i, k_i I(k_i > k))'$, $\rho = (\rho_0, \rho_1, \rho_2, \rho_3)'$ where $\rho_0 = a$, $\rho_1 = b - a$, $\rho_2 = \alpha$ and $\rho_3 = \beta - \alpha$; $I(\cdot)$ is an indicator function that takes a value of 1 if the event is true and zero otherwise.

The hypothesis of interest is linearity of the model, $H_0 : \rho_1 = \rho_3 = 0$ against a threshold effect

[4] We thank an anonymous referee for pointing this out.

in either the intercept or slope.[5]

3.2 Panel data econometric tests

A crucial point to note is that the technology parameters, d, a and b, are assumed to be the same across countries. To the best of our knowledge all empirical tests making allowance for multiple regimes in the production function and based on a cross-section of countries implicitly assume this, see for example Durlauf and Johnson (1995) or Bloom, Canning, and Sevilla (2003). There are two main advantages obtained from relaxing this assumption and making allowance for different technologies across countries. First, the econometric models are more flexible for describing the pattern of each economy. Second, it is the issue of model misspecification. If in fact, there are idiosyncratic differences across countries and regimes, inference made on the above model may be misleading. The question now is whether there is a unique intercept parameter or there are different intercepts for different countries. One alternative explored in the literature, as in Durlauf and Johnson (1995) or Bloom, Canning, and Sevilla (2003), is to include additional control variables to capture countries' idiosyncratic characteristics or initial conditions. However, this does not solve the potential misspecification problem, because which variables should be included is an empirical problem that will depend on the available data and nature of the countries under study. A more pragmatic alternative, which we follow, is to consider differences in the intercept parameters as fixed-effects. These fixed-effects can be interpreted as differences in the countries' technology parameters ($d_i \neq d_j, i \neq j$) for the Solow-Swan type production function regression or differences in country-specific variables, e.g. natural resources, governance or human capital. For instance, Graham and Temple (2006) find that multiple equilibria is associated with differences in aggregate TFP. It is also reasonable to assume that d, a and b are themselves functions of the capital stock, as in Romer (1986) and Azariadis and Drazen (1990), or depend on the initial conditions of the endogenous

[5]The same methodology can be applied to study the existence of convergence clubs in growth rates. The linear model is

$$\Delta y_{it} = \alpha_0 + \alpha_1 y_{i0} + \varepsilon_i$$

with $\Delta y_{it} = y_{it} - y_{i0}$, and y_{it} and y_{i0} denoting terminal and initial per capita income of country i, respectively. The alternative model is

$$\Delta y_{it} = \begin{cases} \alpha_{01} + \alpha_{11} y_{i0} + \varepsilon_i, & \text{if } y_{i0} \leq c, \\ \alpha_{02} + \alpha_{12} y_{io} + \varepsilon_i, & \text{it } y_{i0} > c. \end{cases}$$

The rate of convergence for each "club" is characterized by the parameters α_{11} and α_{12}.

variables in the presence of historical self-reinforcement (Mookherjee and Ray, 2001).

Consider now a panel data of N countries for T periods (indexed by $t = 1, 2, ..., T$). In order to consider the different effects in a single model we consider the following model:

$$y_{it} = d_i + \gamma k_{it} + \varepsilon_{it}, \tag{6}$$

that represents a unique production function that only differs across countries in a country-specific technology parameter, which for convenience is denoted d_i. The competing model that extends (3) by allowing for nonlinearities in this parameter is

$$y_{it} = \begin{cases} a_i + \alpha k_{it} + \varepsilon_{it}, & \text{if } k_{it} \leq \tilde{k}, \\ b_i + \beta k_{it} + \varepsilon_{it}, & \text{if } k_{it} > \tilde{k}. \end{cases} \tag{7}$$

Model (7) allows the fixed-effects to vary with the evolution of the country and its transition from an undeveloped to a developed state (and, at least in theory, the other way round). Note from the previous section that the assumption of a unique threshold $\tilde{k}$ across countries limits the amount of heterogeneity in the model. We formalize this now and use $\rho_1 = b_i - a_i$ to denote the difference in the fixed-effects between regimes. Interestingly, the assumption is crucial to solve the incidental parameter problem. That is, in principle the estimation of the country-specific effects a_i and b_i is problematic as the number of observations N increases, and with it the number of parameters to be estimated. This phenomenon also implies the inconsistency of the structural parameters, which are the parameters measuring the dependence between GDP per capita and capital stock per capita. Fortunately, the availability of a panel data and the constraint $b_i - a_i = \rho_1$ for every i, makes possible to disentangle the uniqueness of the equilibrium outcome in the presence of country-specific differences in other variables from the presence of multiple regimes in the production function. To test for these two hypotheses, we introduce a more convoluted F-type test to account for the regime-dependent fixed-effects. This framework extends Hansen (1999) that studies nonlinearity tests robust to the presence of fixed-effects that are constant across regimes.

The demeaned specification of the linear model with fixed-effects as in (6) is

$$y_{it}^{*} = \gamma k_{it}^{*} + \varepsilon_{0,it}^{*}, \tag{8}$$

with * being the within demeaning transformation, i.e $f_{it}^{*} = f_{it} - \overline{f}_{i}$, with $\overline{f}$ denoting the sample mean for each country i for $t = 1, \ldots, T$, and $\varepsilon_{0,it}^{*}$ the demeaned error term of the model under the null hypothesis. The specification corresponding to the alternative hypothesis, described in equation (7), can be written as,

$$\begin{aligned} y_{it}^{*} &= (b_i - a_i) I^{*}(k_{it} > \tilde{k}) + \alpha k_{it}^{*} + (\beta - \alpha)\left(k_{it} I(k_{it} > \tilde{k})\right)^{*} + \varepsilon_{it}^{*} \\ &= \rho_1 I^{*}(k_{it} > \tilde{k}) + \rho_2 k_{it}^{*} + \rho_3 \left(k_{it} I(k_{it} > \tilde{k})\right)^{*} + \varepsilon_{it}^{*}, \end{aligned} \tag{9}$$

assuming that $b_i - a_i = \rho_1$ and where $\rho_2 = \alpha$, $\rho_3 = \beta - \alpha$. The hypothesis of no threshold effect in the accumulation of capital is

$$Test\ a: \ H_0^a : \rho_1 = \rho_3 = 0 \text{ vs. } H_1^a : \rho_1 \neq 0 \ or \ \rho_3 \neq 0.$$

A similar approach was developed by Hansen (1999). However, this author imposed no changes in the country-specific fixed-effects for the different regimes and evaluated only differences in the slope (i.e. $a_i = b_i \ \forall i$ in our setting). This corresponds to the following alternative model regression equation

$$y_{it}^{*} = \rho_2 k_{it}^{*} + \rho_3 \left(k_{it} I(k_{it} > \tilde{k})\right)^{*} + \varepsilon_{it}^{*}. \tag{10}$$

The implied null hypothesis in Hansen (1999) is

$$Test\ b: \ H_0^b : \rho_3 = 0 \text{ vs. } H_1^b : \rho_3 \neq 0,$$

that assumes $\rho_1 = 0$.

Finally, a test for the nonlinearities produced by Barro and Sala-i-Martin model considers

$$y^*_{it} \;=\; \rho_1 I^*(k_{it} > \tilde{k}) + \rho_2 k^*_{it} + \varepsilon^*_{it}. \tag{11}$$

This model takes the capital elasticity to be constant across regimes. The implied null hypothesis is

$$Test\ c: \; H^c_0 : \rho_1 = 0 \text{ vs. } H^c_1 : \rho_1 \neq 0$$

assuming that $\rho_3 = 0$.

In the next sections, we compare estimates and testing procedures for H^a_0, H^b_0 and H^c_0.

4 A Nonlinear Test for Poverty Traps

For any given $\widetilde{k}$, the coefficients of all models can be estimated by ordinary least squares (OLS). This involves estimating a set of coefficients for the null and alternative models. For each case, define the vector of residuals $\hat{e}_0$, $\hat{e}^a_1$, $\hat{e}^b_1$ and $\hat{e}^c_1$. Following Chan (1993) and Hansen (1999) the estimation of the threshold parameter is done by minimization of the concentrated sum of squared residuals of each alternative model:

$$\widehat{S}^j(k) = \hat{e}^j_1(k)'\hat{e}^j_1(k), \tag{12}$$

for $j = a, b, c$ indexing the different alternative models. Moreover, define

$$\hat{\tilde{k}}^j = \underset{k \in \kappa}{\arg\min}\, \widehat{S}^j(k), \tag{13}$$

as the threshold estimates for each model, with $j = a, b, c$. The consistency of the threshold parameter $\tilde{k}$ follows from standard arguments in threshold models for time series (see Chan, 1993; Gonzalo and Pitarakis, 2002) and its application to panel data (see Hansen, 1999).

Nonlinearity tests have the difficulty that under the null hypothesis of linearity $\widetilde{k}$ cannot be identified (see Hansen, 1996, for a general discussion). This problem was first studied by Andrews

(1993), Andrews and Ploberger (1994) and in the context of threshold regression models in time series by Hansen (1997) using F-tests. Hansen (1999) extends the use of F-tests for threshold models to a panel data context. Wald tests can be derived similarly using the score function of the demeaned process, see Hansen (1996).

The test statistics to discriminate between (8) and its alternative hypotheses are similar in spirit to Hansen (1999). We consider statistics based on the stochastic process

$$F^j(\hat{\hat{k}}^j) = N(T-1)\left(\frac{\widehat{S}_0 - \widehat{S}_j(\hat{\hat{k}}^j)}{\widehat{S}_j(\hat{\hat{k}}^j)}\right), \tag{14}$$

for $j = a, b, c$, where $\widehat{S}_0 = \hat{e}_0'\hat{e}_0$ is the concentrated sum of squares under the null hypothesis. These processes converge weakly to a nonlinear function of a Gaussian process with covariance kernel that depends on moments of the sample, and thus critical values cannot be tabulated. Following Davies (1977, 1987) and Andrews and Ploberger (1994) the test statistics that we propose are the supremum, average and exponential average, that is,

$$\begin{aligned} supF^j &= \sup_{k\in\Gamma} F^j(k), \\ aveF^j &= \operatorname*{average}_{k\in\Gamma} F^j(k), \\ expaveF^j &= \exp \operatorname*{ave}_{k\in\Gamma} F^j(k), \end{aligned}$$

for $j = a, b, c$. Andrews and Ploberger (1994) show that the exponential average test is optimal in terms of power in very general frameworks. On the other hand, the supremum test has the advantage of providing very valuable information about the location of the rejection, and hence of the threshold value. We approximate the liming distribution of these statistics using bootstrap.

The bootstrap algorithm to approximate the p-value of the different supremum, average and exponential average tests is the same of Hansen's (1999) approach. The difference in p-values between our method and Hansen's panel data nonlinearity test is due to the models estimated under the alternative hypothesis. Whereas in Hansen's approach $\widehat{S}(\hat{\hat{k}})$ is computed from the residuals of model (10), in the methodology introduced in this paper we construct the statistic with the residuals of model (9). The rest of bootstrap procedure is analogous; the algorithm is as follows:

Algorithm:

- Generate a grid of $M = 1, \dots, m$ different k values, with $k \in \kappa$, let $\Gamma = (k_1, \dots, k_m)$.
- Generate a sequence of NT observations $\{\nu_{it}^{(h)}\}_{i=1,t=1}^{N,T}$ indexed by h with $h = 1, \dots, H$, from a $N(0,1)$ distribution.
- Regress $\nu_{it}^{(h)}$ on k_{it}^*, obtain the residuals for the null hypothesis model $e_{0,it}^{(h)} = \nu_{it}^{(h)} - \widehat{\gamma}^{(h)} k_{it}^*$, and compute $\widehat{S}_0^{(h)}$.
- Similarly, obtain the residuals for each alternative hypothesis model for fixed $k \in \kappa$, and obtain the sum of squared residuals: $\widehat{S}^{j(h)}(k)$, $j = a, b, c$, as in eq. (12).
- Construct $F^{j(h)}(k)$, $k \in \kappa$, $j = a, b, c$, as in eq. (14).
- Compute $supF^{j(h)} = \sup_{k \in \Gamma} F^{j(h)}(k)$, $aveF^{j(h)} = \underset{k \in \Gamma}{ave} F^{j(h)}(k)$, $expaveF^{j(h)} = \underset{k \in \Gamma}{expave} F^{j(h)}(k)$, $j = a, b, c$, for each $h = 1, \dots, H$.
- For the supremum, average or exponential average cases this procedure gives a random sample of H simulated observations. The empirical p-value is computed as the percentage of these artificial observations which exceed the actual test statistic,

$$\widehat{p}^H = \frac{1}{H} \sum_{h=1}^{H} I(statF^{j(h)} \geq statF^j), j = a, b, c,$$

  where $statF$ is either $supF$, $aveF$ or $expaveF$. Hansen (1996) shows that this empirical p-value converges in probability to the true asymptotic p-value, in this case under the null hypothesis.

5 Monte Carlo Experiments

The Monte Carlo simulation experiments in this section examine the finite-sample performance of the nonlinearity tests proposed in the previous section. The test statistics analyzed are the supremum, average and exponential average methods discussed above.

We consider a null data generating process, which corresponds to a linear model with fixed-effects:

$$(i) \quad y_{it} = a_i + \beta k_{it} + \varepsilon_{it},$$

with $a_i \sim N(1,1)$, and $k_{it} \sim N(0,1)$ and $\varepsilon_{it} \sim N(0,1)$ mutually independent random variables. Alternative DGPs are

(ii) $y_{it} = a_i + \beta k_{it} + \gamma k_{it} I(k_{it} > \widetilde{k}) + \varepsilon_{it}$,

(iii) $y_{it} = a_i + \rho_1 I(k_{it} > \widetilde{k}) + \beta k_{it} + \varepsilon_{it}$,

(iv) $y_{it} = a_i + \rho_1 I(k_{it} > \widetilde{k}) + \beta k_{it} + \gamma k_{it} I(k_{it} > \widetilde{k}) + \varepsilon_{it}$,

with $\rho_1 = 1.5$; this implies that $b_i = a_i + \rho_1$. Throughout the experiments $\beta = 1$, $\gamma = 0.5$ and $\widetilde{k} = 0$. The domain of the threshold parameter is the space $\kappa = \{k \in \mathbb{R}, \text{s.t.} F(k) \in [0.05, 0.95]\}$ with $F(\cdot)$ the distribution function of the random variable k. We consider several panel sizes. First we evaluate the tests performance under very small panel sizes ($N = 10, T = 5; N = 20, T = 10$). Then we consider similar panel sizes to those used in the application ($N = 100, T = 10; N = 100, T = 20; N = 250, T = 10; N = 250, T = 10$). We use $H = 200$ bootstrap internal simulations to get the p-value and we use 500 Monte Carlo simulations to compute the empirical size and power. Tables 1, 2 and 3 report empirical estimates of size and power evaluated at 5% and 1% significance level for the supremum, average and exponential average test statistics, respectively.

[INSERT TABLES 1,2,3 ABOUT HERE]

These simulations are consistent with the findings of Hansen (1996) and Andrews and Ploberger (1994). Whereas the supremum test overestimates the size of the test (see rows starting with (i), null DGP), the average and exponential average methods provide very reliable estimates for rather small sample sizes. In every case, for fixed N, increasing the T dimension of the panel produces more accurate empirical sizes.

The power of the tests yields interesting results. As expected, Hansen (1999) test, $Test\ b$ (test for threshold effects in the slope only), has the highest power for DGP (ii), while $Test\ c$ (test for threshold effects in the fixed-effects only) shows the highest power for DGP (*iii*). By construction Hansen (1999) test should not detect nonlinearities on capital produced by changes in the fixed-effects, as in DGP (*iii*). However, rejection rates are rather high for this test and model. In a similar vein, $Test\ c$, should not detect nonlinearities in the slope, because this test only looks at changes in the fixed effects. However, rejection rates are high for DGP (*ii*). This determines that these tests are not able to discriminate the source of the threshold effect, even though in this case, the fixed-effects are uncorrelated with the k variable. The reason is that the threshold effect implicitly produces a correlation between the fixed-effects and the covariate.

The joint F-test, $Test\ a$ (test for threshold effects in the slope and fixed-effects), has (correctly) the highest power for detecting non-linearities in both slope and fixed-effects, DGP (*iv*). Moreover, it shows good power for DGPs (*ii*) and (*iii*). Therefore, this tests can be used when the nature of the nonlinearity is unknown.

6 Detection of Poverty Traps: An Empirical Study

The application of this study consists on determining statistically whether the relationship between per capita gross domestic product (GDP) and capital stock per capita is linear or exhibits discontinuities due to the accumulation of capital. Two different approaches can be followed. First, we could opt for a panel with a large time-series dimension but a short cross-country variation. Second, a shorter time-series dimension with a larger cross-country variability. Each approach has its own merits and disadvantages. The former corresponds to long-term historical analysis where the overall process of industrialization can be evaluated. However, it relies mostly on developed countries data, for which long historical statistics are available. The second corresponds to the empirical growth literature (conditional convergence, multiple equilibria). Most studies on this approach start in 1960 or 1970, where standardized cross-country statistics started to be systematically collected. Following Durlauf and Johnson (1995) and Bloom, Canning, and Sevilla (2003), which we believe are the closest studies to ours, we use the second approach.

In order to do this we create a panel data set using World Development Indicators. Our simple model of poverty traps contains the country's GDP per capita and capital stock per capita. While the former can be found in any cross-country panel database, the latter is difficult to obtain. To our knowledge the largest panel data covering both developed and developing countries was constructed by Crego, Larson, Butzer, and Mundlak (2000). Those authors provide a comprehensive study to estimate aggregate capital stocks in a systematic way and they use the perpetual inventory method to obtain fixed capital estimates for 63 countries between 1948 and 1992 resulting in 1323 observations. We follow their methodology and construct our own capital stock estimates using data from gross capital formation, using the perpetual inventory method with a yearly depreciation of 7% and obtain estimates for the period 1973 to 2007. Our final balanced panel data contains 138 countries and 35 years (1973 to 2007) resulting in 4830 observations.[6] We apply Levin, Lin, and Chu (2002) tests for unit roots in panel data. In all cases we reject the null hypothesis of a unit root.[7] Therefore, we can apply the asymptotics proposed in this paper which are the same as those in Hansen (1999).

The data is fitted to the four processes detailed in the preceding section. These processes are (i) a linear model with fixed-effects, (ii) Hansen's fixed-effects model (10), (iii) model (9) making allowance for regime switching in the fixed-effects and capital elasticity, and (iv) model (11), making allowance for regime-switching only in the fixed-effects component. The estimates of the relevant parameters are in Table 4. The latter two processes are also the building blocks for the corresponding nonlinearity tests discussed above.

All testing methods reject the linear model against the nonlinear alternative. These results are

[6] The sample contains Afghanistan, Albania, Algeria, Angola, Antigua and Barbuda, Argentina, Australia, Austria, Bangladesh, Barbados, Belgium, Belize, Benin, Bhutan, Bolivia, Botswana, Brazil, Bulgaria, Burkina Faso, Burundi, Cambodia, Cameroon, Canada, Cape Verde, Central African Republic, Chad, Chile, Colombia, Comoros, Dem. Rep. Congo, Costa Rica, Cuba, Cyprus, Denmark, Djibouti, Dominica, Dominican Republic, Ecuador, El Salvador, Equatorial Guinea, Ethiopia, Fiji, Finland, France, Gabon, The Gambia, Germany, Ghana, Greece, Grenada, Guatemala, Guinea, Guinea-Bissau, Guyana, Haiti, Honduras, Hungary, Iceland, India, Indonesia, Ireland, Israel, Italy, Jamaica, Japan, Jordan, Kenya, Kiribati, Kuwait, Lebanon, Lesotho, Liberia, Libya, Luxembourg, Madagascar, Malawi, Malaysia, Maldives, Mali, Malta, Mauritania, Mauritius, Mexico, Fed. Sts. Micronesia, Mongolia, Morocco, Mozambique, Namibia, Nepal, Netherlands, New Zealand, Nicaragua, Niger, Norway, Oman, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Poland, Portugal, Puerto Rico, Qatar, Romania, Rwanda, Saudi Arabia, Senegal, Seychelles, Sierra Leone, Singapore, Solomon Islands, Somalia, South Africa, Spain, Sri Lanka, St. Lucia, Sudan, Suriname, Swaziland, Sweden, Switzerland, Tanzania, Thailand, Togo, Tonga, Tunisia, Turkey, Uganda, United Arab Emirates, United Kingdom, United States, Uruguay, Vanuatu, Vietnam, Zambia and Zimbabwe.

[7] For GDP per capita, it has a Levin, Lin, and Chu (2002) pooled Dickey-Fuller test coefficient of -0.04685 without lags and -0.05258 augmented one lag. For capital stock per capita, it has a Levin, Lin, and Chu (2002) pooled Dickey-Fuller test coefficient of -0.02272 without lags and -0.05623 augmented one lag. In all cases the null hypothesis is rejected with a p-value<0.01.

in line with Durlauf and Johnson (1995) and Bloom, Canning, and Sevilla (2003), where multiplicity of equilibria appeared. In our case, this corresponds to a nonlinear production function. Moreover, these methods allow us to compute explicit threshold candidates: $K = 1,744,881$, $K = 448,553$ and $K = 1,960,935$ for models (ii), (iii) and (iv), respectively. This corresponds to 41%, 11% and 46% quantiles of the pooled worldwide distribution of capital stock per capita, respectively. Interestingly, the threshold effect is observed not only in the capital variable but also in the fixed-effects component supporting the theory of changes in technology parameters and other non-observable factors that depend on the amount of capital. Note that the difference between the estimated thresholds is substantial, and potential model misspecification may play a significant role in explaining this. In particular, model (iii) implies that much of the nonlinear effect may affect the country-specific characteristics (i.e. fixed-effects), and that the threshold to achieve this *better* technology is lower than otherwise predicted. Although the intercept of process (iii) is negative the model also shows the incremental effect on per-capita income of exceeding a capital threshold. This fact highlights the nonlinear technology effects for higher values of capital leading to higher values of TFP.

Finally note that the dependence reflected in model (iv) might be, however, due to the misspecification of the nonlinearity in the capital per-capita variable. A similar conclusion can be obtained from the results observed from Hansen's specification in model (ii), aimed to only detect nonlinearities in the capital elasticity term. From these results, it seems sensible to consider model (iii) as the most accurate representation of the true relationship between the variables. It is also remarkable the differences in threshold estimates across models.

[INSERT TABLES 4, 5 ABOUT HERE]

To round off the empirical application Table 5 reports the same specifications allowing now for year effects. This is done by considering a dummy variable for each year. The results reflect the presence of a statistically significant effect for some of the years in the sample. It is interesting to note that this effect is produced in clusters. Thus, for the first years of the study (1973-1977) and the nineties (1992-1995 and 1998) we observe a marginal negative effect in per-capita income. For

1980-1981 and 2004-2006 the effect is positive reflecting a recovery of the worldwide economy after periods of turbulence. In fact, for the latest period of the study this effect is between 5 and 10 times the effect observed in the rest of the sample. This statistical phenomenon illustrates the failure of simple models of economic growth to describe the rapid growth in per-capita income during the last years (although before the 2008 worldwide crisis). For the remaining years the effect of the year is not statistically significant. Finally note that the qualitative implications of the four econometric models and hypothesis tests remain unchanged.

7 Conclusion

We developed a new econometric test to evaluate the existence of poverty traps and multiple equilibria. The test makes use of the panel data structure with country-specific fixed-effects and tests for threshold nonlinearities on the model's slope and fixed-effects. The empirical results confirm the existence of a nonlinear model in modeling GDP per capita and capital stock per capita (in logs).

It would be interesting to apply this methodology at the microeconomic level by studying households income dynamics (see for instance Galor and Zeira, 1993; Antman and McKenzie, 2007; Imai, Gaiha, and Kang, 2012). This would provide further support to the theory of poverty traps at the individual or family level. Our methodology would be especially useful for panel data surveys where family specific characteristics can be controlled for.

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TABLES

Table 1 - Monte Carlo simulations - Supremum

SUP	N=20, T=5		N=20, T=10		N=100, T=10		N=100, T=20		N=250, T=10		N=250, T=20	
α	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01
$Slope\&fixed-effects$ $H_0^a:\rho_1=\rho_3=0$ vs. $H_1^a:\rho_1\neq 0$ or $\rho_3\neq 0$												
(i)	0.148	0.036	0.104	0.026	0.092	0.020	0.054	0.016	0.114	0.032	0.078	0.018
(ii)	0.408	0.202	0.466	0.230	0.980	0.900	1.000	1.000	1.000	1.000	1.000	1.000
(iii)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(iv)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$Fixed-effects$ $only$ $H_0^c:\rho_1=0$ vs. $H_1^b:\rho_1\neq 0$, assuming $\rho_3=0$												
(i)	0.134	0.034	0.088	0.016	0.074	0.026	0.052	0.014	0.110	0.024	0.084	0.012
(ii)	0.336	0.154	0.348	0.166	0.93	0.778	1.000	0.998	1.000	0.998	1.000	1.000
(iii)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(iv)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$Slope$ $only$ $H_0^b:\rho_3=0$ vs. $H_1^b:\rho_3\neq 0$, assuming $\rho_1=0$ (Hansen, 1999)												
(i)	0.102	0.024	0.092	0.020	0.076	0.014	0.050	0.010	0.074	0.024	0.064	0.020
(ii)	0.454	0.226	0.586	0.368	0.992	0.952	1.000	1.000	1.000	1.000	1.000	1.000
(iii)	0.606	0.306	0.980	0.796	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(iv)	0.544	0.230	0.966	0.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Notes: Simulations based on 500 Monte Carlo experiments.

Table 2 - Monte Carlo simulations - Average

AVE	N=20, T=5		N=20, T=10		N=100, T=10		N=100, T=20		N=250, T=10		N=250, T=20	
α	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01
$Slope\&fixed-effects$ $H_0^a:\rho_1=\rho_3=0$ vs. $H_1^a:\rho_1\neq 0$ or $\rho_3\neq 0$												
(i)	0.106	0.018	0.084	0.022	0.088	0.014	0.050	0.016	0.074	0.020	0.066	0.018
(ii)	0.478	0.248	0.602	0.382	0.994	0.958	1.000	1.000	1.000	1.000	1.000	1.000
(iii)	0.968	0.796	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(iv)	0.952	0.802	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$Fixed-effects$ $only$ $H_0^c:\rho_1=0$ vs. $H_1^b:\rho_1\neq 0$, assuming $\rho_3=0$												
(i)	0.134	0.034	0.084	0.022	0.074	0.018	0.040	0.012	0.102	0.028	0.080	0.020
(ii)	0.336	0.154	0.394	0.18	0.964	0.870	1.000	1.000	1.000	0.998	1.000	1.000
(iii)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(iv)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$Slope$ $only$ $H_0^b:\rho_3=0$ vs. $H_1^b:\rho_3\neq 0$, assuming $\rho_1=0$ (Hansen, 1999)												
(i)	0.086	0.014	0.076	0.022	0.066	0.012	0.046	0.018	0.062	0.020	0.056	0.014
(ii)	0.474	0.244	0.614	0.386	0.992	0.972	1.000	1.000	1.000	1.000	1.000	1.000
(iii)	0.318	0.042	0.946	0.522	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(iv)	0.338	0.080	0.952	0.420	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Notes: Simulations based on 500 Monte Carlo experiments.

Table 3 - Monte Carlo simulations - Exponential Average

EXPAVE	N=20, T=5		N=20, T=10		N=100, T=10		N=100, T=20		N=250, T=10		N=250, T=20	
α	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01
$Slope\&fixed-effects\ H_0^a : \rho_1 = \rho_3 = 0$ vs. $H_1^a : \rho_1 \neq 0\ or\ \rho_3 \neq 0$												
(i)	0.130	0.036	0.106	0.014	0.092	0.012	0.050	0.014	0.094	0.030	0.070	0.014
(ii)	0.440	0.232	0.550	0.322	0.992	0.938	1.000	1.000	1.000	1.000	1.000	1.000
(iii)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(iv)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$Fixed-effects\ only\ H_0^c : \rho_1 = 0$ vs. $H_1^b : \rho_1 \neq 0$, assuming $\rho_3 = 0$												
(i)	0.134	0.030	0.084	0.022	0.072	0.022	0.040	0.014	0.102	0.028	0.072	0.018
(ii)	0.350	0.168	0.394	0.180	0.948	0.826	1.000	1.000	1.000	0.998	1.000	1.000
(iii)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(iv)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$Slope\ only\ H_0^b : \rho_3 = 0$ vs. $H_1^b : \rho_3 \neq 0$, assuming $\rho_1 = 0$ (Hansen, 1999)												
(i)	0.088	0.020	0.086	0.028	0.072	0.012	0.042	0.018	0.064	0.018	0.058	0.018
(ii)	0.482	0.246	0.624	0.394	0.992	0.974	1.000	1.000	1.000	1.000	1.000	1.000
(iii)	0.546	0.262	0.988	0.808	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(iv)	0.496	0.190	0.98	0.754	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Notes: Simulations based on 500 Monte Carlo experiments.

Table 4 - Empirical growth panel data model

Threshold in		slope	slope&fixed-effects	fixed-effects
ρ_1			-3.285	0.207
			(0.122)	(0.011)
ρ_2	0.689	0.646	0.562	0.651
	(0.004)	(0.005)	(0.007)	(0.004)
ρ_3		0.014	0.252	
		(0.001)	(0.009)	
Threshold (K per capita, US$)		1,744,881	448,553	1,960,935
Fraction of observations below		0.41	0.11	0.46
p value of F_{ave} test		0.00	0.00	0.00
p value of F_{sup} test		0.00	0.00	0.00
p value of F_{expave} test		0.00	0.00	0.00
N		99		
T		35(1973-2007)		
NT		3465		

Notes: All regression coefficient estimates are statistically significant with p-value$<$ 0.01

Table 5 - Empirical growth panel data model with dummies for time effects

Threshold in		slope	slope&fixed-effects	fixed-effects
ρ_1			-3.310	0.210
			(0.123)	(0.011)
ρ_2	0.658	0.623	0.528	0.624
	(0.009)	(0.008)	(0.010)	(0.008)
ρ_3		0.014	0.256	
		(0.001)	(0.009)	
Threshold (K per capita, US$)		1,744,881	405,342	1,960,935
Fraction of observations below		0.41	0.10	0.46
p value of F_{ave} test		0.00	0.00	0.00
p value of F_{sup} test		0.00	0.00	0.00
p value of F_{expave} test		0.00	0.00	0.00
N		99		
T		35(1973-2007)		
NT		3465		

Notes: All regression coefficient estimates are statistically significant with p-value$<$ 0.01