An Examination of the Optimal Timing Strategy for a Slow Trader Investing in a High Frequency Trading Technology

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July 6, 2015

Abstract

This paper examines, using a real options approach, the optimal time for financial market investors to adopt a high frequency trading (HFT) technology. When the level of fast trading in the market is high, investors should wait longer before adopting when the cost of the technology is high, and vice versa. However, when the market is highly fragmented, they should invest early (wait longer) when the cost of doing so is high (low).

Furthermore, the equilibrium level of investment prescribed by the model exceeds the socially optimal level, and investors should wait longer (invest earlier) than the optimal time when the technology is relatively cheap (expensive) in order to be more socially optimal.

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Keywords: real options, high frequency trading, fragmented markets, welfare.

JEL Classification Numbers: C61, G1, G20, L1.

1 Introduction

This paper uses a real options approach to determine and analyse the optimal time for financial market investors (referred to as "traders" hereafter) to invest in a high frequency trading (HFT) technology. HFT is a type of algorithmic trading that uses sophisticated computer algorithms to implement vast amounts of trades in extremely small time intervals. The analysis is motivated by the fact that over the last decade, the state of financial markets has changed considerably. In the first instance, markets have become highly fragmented. There are now more than 50 trading venues for U.S. equities - 13 registered exchanges and 44 so called Alternative Trading Systems (see Biais et al. [1] and O'Hara and Ye [14]). Hence, traders must search across many markets for quotes and doing so can be costly as it may delay full execution of their orders.

In response to the increase in market fragmentation, so called HFT technologies have been developed to reduce the associated costs borne by traders. For example, traders can buy colocation rights (the placement of their computers next to the exchange's servers) which gives them fast access to the exchange's data feed, they can invest in smart routers which can instantaneously compare quotes across all trading venues and then allocate their orders accordingly, or they can invest in high-speed connections to the exchanges via fiber optic cables or microwave signals. Proprietary trading desks, hedge funds, and so called pure-play HFT outlets are investing large sums of money into such technologies in an effort to outpace the competition. Indeed, according to Hoffmann [9], recent estimates suggest that HFTs are now responsible for more than 50% of trading in U.S. equities.

In a recent paper, O'Hara [13] details the many ways in which market microstructure has changed over the past decade and calls for a new approach to research in this area which "reflects the new realities of the high frequency world". Nevertheless, there has been a growth in the literature on HFT in recent years, but much of this literature is empirical. On the whole, the consensus has been that HFT improves liquidity (Hendershott et al. [8] and Hasbrouck and Saar [6]), is highly profitable (Menkveld [11]), and improves price discovery (Hendershott and Riordan [7] and Brogaard et al. [2]).

The theoretical literature in this area is scant, but Biais et al. [1] develop a model of HFT in a Glosten and Milgrom [4] type framework, which is the most closely related model to the one in this paper. In particular, I incorporate the HFT model of Biais et al. [1] into a real options framework such that the option takes the shape of an exchange option where one payoff flow is exchanged for another (see Smets [15] or Thijssen [16] for examples of "exchange-type" real options models). This enables the optimal time for traders to invest in a HFT, as a function of the level of HFT activity already accounted for in the market as well as the degree of market fragmentation, to be determined analytically. The analytical solution takes the form of a threshold policy: invest if the payoff flow from adopting exceeds the threshold level, otherwise refrain from doing so.

Since the seminal contribution by Myers [12], it has become quite standard practice in the corporate finance literature to view most irreversible (at least partially so) investment projects as real options. The main feature of real options analysis is the recognition that the investor has the option to postpone investing until some future date. Hence, the investor has some flexibility over his investment decision which is not accounted for in classical Marshallian investment evaluation. This flexibility has economic value (typically referred to as the option value of waiting) owing to the irreversibility, and because the payoff from investing is uncertain. Investing in the HFT technology is both an irreversible and an uncertain investment, and can be entered into at any time. Therefore, a real options approach is appropriate for evaluating this particular investment decision. To the best of my knowledge, the approach has not been previously applied in a HFT context, nor has the HFT issue been considered from a timing perspective.

The novel contribution of the paper, from the real options perspective, is the application of the methodology to the "new" market microstructure environment of fragmented markets and HFTs. Hence, I restrict the examination of the optimal policy to these two characteristics: When the cost of investing is low, the threshold decreases in the level of HFT in the market and increases in the level of market fragmentation. This is driven by the option value of waiting which dominates the present value effect. However, when the cost of investing is high, the present value effect dominates and the threshold increases in the level of HFT and decreases in the degree of market fragmentation.

Finally, from a welfare perspective, adhering to the optimal policy prescribed by the model yields an equilibrium level of HFT which always exceeds that level which is socially optimal. Biais et al. [1] obtain a similar result in their paper and analyse possible policy responses to this problem. For example, they provide an analysis of the effects a Pigovian tax would have if imposed on HFTs. The socially optimal level of HFT would be reached if the tax imposed is equal to the externalities generated by HFTs. Their analysis can be implemented in the same way to this model by simply raising the total cost of investing and the result would be the same. However, the model is solved from an optimal timing perspective and this generates another response: in order to be socially optimal, traders should wait longer before investing in the HFT technology than the real options approach suggests when the cost of investing is relatively low, and vice versa when this cost is high.

The remainder of this paper is organised as follows. The model is set up and solved in the next section. Section 3 analyses the implications of the model for investment timing and welfare, and Section 4 concludes. All figures are placed in the appendix.

2 The Model

2.1 The Trading Environment

Consider a risk-neutral market trader contemplating investment in a HFT technology. Investing in the technology incurs a sunk cost $I > 0$. Before investing, the trader is a regular (slow) trader who trades in fragmented markets where fast high frequency traders (HFTs) also trade. Hence, he is exposed to the impact such traders have on the likelihood of his orders getting executed at favourable prices. Once he invests, however, he becomes one of the HFTs.

I assume a similar market and trading environment to that of Biais et al. [1]. In particular, I capture a fragmented market where slow traders compete with HFTs by assuming there is a size-one continuum of trading venues distributed on a circle and indexed clockwise from 0 to 1. At each point in time dt, only a fraction $\lambda < 1$ of the trading venues are "liquid". In the context of this model, a trading venue is liquid if it allows the trader to fully execute his order at his specified order price. At time t, the liquid venues are located on the circle in an interval of size λ, starting at a venue v_t such that v_t is uniformly distributed on the circle and i.i.d. across periods. Moreover, at any point in time there are a continuum of traders in the market, some of which are HFTs and some of which are slow. In order to execute his desired trade, a trader must find a trading venue that is liquid. When a trader enters the market, he is uncertain about which venues are liquid since he does not know v_t.

HFTs have extremely fast connection speeds to the market and can observe all venues instantaneously (Biais et al. [1]). Thus, they find a liquid one with certainty upon arrival to the market. However, a slow trader must search for liquid trading venues and finding one can take time. I assume that at each point in time, a slow trader can only send orders to one trading venue on the circle. His choice of venue is random and uniformly drawn from the unit circle. Therefore, with probability λ the venue is liquid and the slow trader can execute his order. If it is not liquid, the trader cannot trade at t and with probability $(1-\phi)$ he will exit the market and not trade again. However, with probability ϕ, he waits and tries to trade again at the next time point $t + dt$.

I denote by α (respectively $1-\alpha$) the mass of fast (resp. slow) institutions in the market at time t. Intuitively, and indeed is the case in the model of Biais et al. [1], an increase in the level of fast trading α reduces the expected gains of both fast and slow institutions. To capture this, I let the profit flow dynamics from trading, $(X_t)_{t\geq 0}$, for both a fast and a slow trader follow an arithmetic Brownian motion of the form

$$dX_t = \kappa(\alpha) \left[\mu dt + \sigma dW_t\right], \tag{1}$$

such that $(W_t)_{t\geq 0}$ is a standard Weiner process, and $\kappa(\alpha) \in (0,1)$ (for $\alpha \in (0,1)$), $\kappa'(\alpha) < 0$ for all $X_t \geq 0$ and $\kappa'(\alpha) > 0$ for all $X_t < 0$.

A fast trader is one who has already invested in the HFT technology and, thus, all his orders are filled with certainty at the time of submission. Hence, I assume that he earns a profit stream of $(X_t)_{t\geq 0}$ with certainty.

A slow trader, however, must search for quotes and finding a liquid venue can take time. For some arbitrary time $t \geq 0$, he finds a liquid venue with probability $\lambda\left((1-\lambda)\phi\right)^t$. Thus, he obtains a profit of $\lambda\left((1-\lambda)\phi\right)^t X_t$ in each period t.

If the current state of the profit flow process is x and the slow trader, who discounts profits at the rate $r > 0$, decides to adopt the HFT at the stopping time τ, then via the strong Markov property of diffusions, the value of this policy to the trader, denoted by $V(x)$, is given by

$$
\begin{aligned}
V(x) =& E_x\left[\int_0^\tau \lambda\left((1-\lambda)\phi\right)^t e^{-rt} X_t dt + \int_\tau^\infty e^{-rt} X_t dt - e^{-r\tau} I\right] \\
=& E_x\left[\int_0^\infty \lambda\left((1-\lambda)\phi\right)^t e^{-rt} X_t dt\right] + E_x\left[e^{-r\tau} F(X_\tau)\right], \\
=& \frac{\lambda\kappa(\alpha)}{r - \ln((1-\lambda)\phi)}\left(x + \frac{\mu}{r - \ln((1-\lambda)\phi)}\right) + E_x\left[e^{-r\tau} F(X_\tau)\right]
\end{aligned} \tag{2}
$$

where

$$
\begin{aligned}
F(X_\tau) =& E_{X_\tau}\left[\int_0^\infty \left(1 - \lambda\left((1-\lambda)\phi\right)^t\right) e^{-rt} X_t dt\right] - I \\
=& \frac{\kappa(\alpha)}{r}\left(X_\tau + \frac{\mu}{r}\right) - \frac{\lambda\kappa(\alpha)}{r - \ln((1-\lambda)\phi)}\left(X_\tau + \frac{\mu}{r - \ln((1-\lambda)\phi)}\right) - I
\end{aligned} \tag{3}
$$

is the trader's payoff function from adoption, E_y denotes the expectation operator under the family of probability measures $(P_y)_{y\in(-\infty,\infty)}$, and I assume that $F(X_\tau) > 0$.

The problem for the trader is to find a value function $V^*(x)$ and a stopping time τ^* such that the following optimal stopping problem is solved:

$$V^*(x) := E_x\left[e^{-r\tau^*}F(X_{\tau^*})\right] = \sup_{\tau\in\mathcal{T}} E_x\left[e^{-r\tau}F(X_\tau)\right], \tag{4}$$

for $\mathcal{T}$ the set of stopping times.

I show in the next subsection that there is a unique trigger X^* such that adoption of the HFT is optimal as soon as X^* is hit from below.

2.2 The Model Solution

Theorem 1. *Let $f : \mathbb{R} \to \mathbb{R}$ be defined by*

$$f(x) = e^{-\beta_1 x}F(x)$$

where β_1 is the positive (real) root of the quadratic equation

$$\mathcal{Q}(\beta) \equiv \frac{1}{2}\left(\kappa(\alpha)\right)^2\sigma^2\beta^2 + \kappa(\alpha)\mu\beta - r = 0. \tag{5}$$

If f attains a unique maximum at X^*, then the optimal stopping problem (4) is solved by

$$V^*(x) = \begin{cases} e^{-\beta_1(X^*-x)}F(X^*) & \text{if } x < X^* \\ F(x) & \text{if } x \geq X^*. \end{cases} \tag{6}$$

Furthermore, the optimal stopping time is given by

$$\tau^* := \inf\{t \geq 0 | X_t \geq X^*\}. \tag{7}$$

Proof. Before I prove the theorem, I show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ has a maximiser which is unique.

$$f'(x) = 0 \Longleftrightarrow F'(x) = \beta_1 F(x)$$

and x is a maximiser iff $F''(x) < \beta_1 F'(x)$. But, from (3), $F''(x) = 0$ and $F'(x) > 0$. Therefore, any solution to $f'(x) = 0$ is a maximiser since $\beta_1 > 0$ also. Furthermore, since $F(x)$ is linear in x, the solution to $f'(x) = 0$ has to be unique.

I next prove the result of the theorem using the conditions outlined in Thijssen [17]. According to that paper, the conditions to be checked are as follows:

1. The value function must dominate the payoff function; i.e., $V^* > F$,

2. The value function is a smooth function; i.e., $V^* \in C^1$,

3. $$\mathcal{L}_X V^* - rV^* = 0$$

   for all $x < X^*$. $\mathcal{L}_X \in C^2$ is the partial differential operator defined by

   $$\mathcal{L}_X g(x) = \lim_{t \downarrow 0} \frac{E[e^{-rt} g(X_t)] - g(x)}{t}$$

   where $g \in C^2$.

4. V^* is superharmonic; i.e.,

   $$\mathcal{L}_X V^* - rV^* \leq 0$$

   for all $x \geq X^*$,

5. the family $\{V^*(X_\tau)|\tau \leq \tau_C\}$ is uniformly integrable with respect to $(P_x)_{x\in(-\infty,\infty)}$, where $\tau_C = \inf\{t \geq 0|X_t \notin (-\infty, X^*)\}$.

The conditions are verified as follows:

1. Let X_{NPV} be such that $F(X_{NPV}) = 0$. Since $F'(x) > 0$ and $F(X^*) > 0$ by assumption, it must be the case that $X^* > X_{NPV}$. For all $x \in (-\infty, X_{NPV}]$, $F(x) < 0$ and, thus, $V^*(x) > F(x)$.

   For all $x \in [X^*, \infty)$, $V^*(x) = F(x)$.

   Finally, for all $x \in (X_{NPV}, X^*)$, suppose $V^* < F$. Then it must be the case that

$$e^{-\beta_1(X^*-x)}F(X^*) < F(x) \Longleftrightarrow e^{-\beta_1 X^*}F(X^*) < e^{-\beta_1 x}F(x).$$

   However, this condition cannot hold if X^* is the unique maximiser of $f(x)$. Therefore, it must be the case that $V^* > F$ on (X_{NPV}, X^*) also. This proves that the value function dominates the payoff function.

2. It is clear that V^* is continuous. Moreover, V^* is differentiable at X^* since

$$\begin{aligned}\lim_{x\uparrow X^*} V^*(x) &= \lim_{x\uparrow X^*} \beta_1 e^{-\beta_1(X^*-x)}F(X^*) \\ &= \lim_{x\uparrow X^*} \beta_1 e^{-\beta_1(X^*-x)}\frac{F'(X^*)}{\beta_1} \text{ [from } f'(X^*) = 0] \\ &= F'(X^*).\end{aligned} \tag{8}$$

3. Let $\widetilde{X}_t := e^{-rt}X_t$. For some $g \in C^2$, the infinitesimal generator of $(\widetilde{X}_t)_{t\geq 0}$

is equal to the partial differential operator

$$\begin{aligned}\mathcal{L}_{\widetilde{X}}g(x) =& \lim_{t\downarrow 0}\frac{E[e^{-rt}g(X_t)]-g(x)}{t} = \mathcal{L}_X g(x) - rg(x)\\ =&\frac{1}{2}(\kappa(\alpha))^2\sigma^2\frac{\partial^2 g(x)}{\partial x^2} + \kappa(\alpha)\mu\frac{\partial g(x)}{\partial x} - rg(x).\end{aligned}$$

For all $x \leq X^*$,

$$\frac{\partial V^*(x)}{\partial x} = \beta_1 e^{-\beta_1(X^*-x)}F(X^*) \text{ and } \frac{\partial^2 V^*(x)}{\partial x^2} = \beta_1^2 e^{-\beta_1(X^*-x)}F(X^*),$$

then it follows from (5) that

$$\begin{aligned}\mathcal{L}_{\widetilde{X}}V^*(x) =& \frac{1}{2}(\kappa(\alpha))^2\sigma^2\frac{\partial^2 V^*(x)}{\partial x^2} + \kappa(\alpha)\mu\frac{\partial V^*(x)}{\partial x} - rV^*(x)\\ =&\left(\frac{1}{2}(\kappa(\alpha))^2\sigma^2\beta_1^2 + \kappa(\alpha)\mu\beta_1 - r\right)e^{-\beta_1(X^*-x)}F(X^*) \qquad (9)\\ =&0\end{aligned}$$

as required.

4. For all $x \geq X^*$

$$\begin{aligned}\mathcal{L}_X V^* - rV^* =& \mathcal{L}_X F - rF\\ =&\frac{1}{2}(\kappa(\alpha))^2\sigma^2 F''(x) + \kappa(\alpha)\mu F'(x) - rF(x) \qquad (10)\\ <&\left(\kappa(\alpha)\mu - \frac{r}{\beta_1}\right)F'(x)\end{aligned}$$

since $F''(x) = 0$, and $f'(x) < 0$ and $F(x) > 0$ for $x \geq X^*$. However, it is easy to check that $(\kappa(\alpha)\mu - r/\beta_1) < 0$ and, since $F'(x) > 0$, it must therefore be the case that $\mathcal{L}_X V^* - rV^* < 0$ for $x \geq X^*$. Hence, V^* is superharmonic.

5. The Borel function $g : [0, \infty) \to [0, \infty)$, defined by $g(y) = y^2$ is a

uniform integrability test function since g is increasing and convex in y and $\lim_{y\to\infty} g(y)/y = \infty$ (see Thijssen [17]). Then the family $\mathcal{Y} := \{Y_t \leq Y_{\tau_C}\}$ is uniformly integrable with respect to $(P_y)_{y\in(-\infty,\infty)}$ if $\sup_{Y_t\in\mathcal{Y}} E[g(|Y_t|)] < \infty$. Applying this to our function gives,

$$\begin{aligned}\sup_{t\leq\tau_C}\{\int g(|V^*(x)|)dP_x\} =& \sup_{x\leq X(\tau_C)} E[(V^*(x))^2] \\ =& \sup_{x\leq X(\tau_C)} E[e^{-2\beta_1(X^*-x)}](F(X^*))^2] \\ \leq& (F(X^*))^2 < \infty\end{aligned} \tag{11}$$

since $X^* > x$ for all $t \leq \tau_C$.

■

From the theorem, we get that X^* is given by

$$X^* = \frac{1}{\beta_1} - \Psi(r,\mu,\lambda,\phi) + \Phi(r,\lambda,\phi,\alpha)I, \tag{12}$$

where

$$\Psi(r,\mu,\lambda,\phi) = \frac{\mu}{((1-\lambda)r - \ln((1-\lambda)\phi))}\left(\frac{(r-\ln((1-\lambda)\phi))^2 - \lambda r^2}{r(r-\ln((1-\lambda)\phi))}\right)$$

and

$$\Phi(r,\lambda,\phi,\alpha) = \frac{r\,(r-\ln((1-\lambda)\phi))}{\kappa(\alpha)\,((1-\lambda)r - \ln((1-\lambda)\phi))}.$$

3 Model Implications

3.1 Investment Timing

Proposition 1. *The more high frequency traders there are in the market, the sooner a slow trader will invest in the HFT technology when the cost of investing is low, but the longer he will wait when this cost is high.*

To see this, consider the following equation:

$$\frac{\partial X^*}{\partial \alpha} < 0 \Longleftrightarrow \Phi(r, \lambda, \phi, \alpha) I < \frac{1}{\beta_1} \tag{13}$$

since $X^* > 0 \Longrightarrow \kappa'(\alpha) < 0$.

Proposition 1 says that an increase in the level of fast trading will lead other slow traders to be more forthcoming with adopting the HFT technology when the cost of doing so is relatively low. This result arises from the option effect of waiting. When the cost of investing in the technology is relatively low, the option effect dominates the present value effect (cf. equation (13)). When α rises, the value of waiting decreases since a relatively low investment cost implies a high opportunity cost to waiting. Therefore, the slow traders will be more forthcoming with investing. This result is depicted in Figure 1.

However, when the cost of investing is high, the present value effect dominates the option effect and the condition given by (13) does not hold. A high α reduces the present value from investing (arising from the assumption that high frequency traders reduce gains from trade) implying that the opportunity cost of waiting is low and, hence, traders will be more reluctant to invest (see Figure 2).

To obtain an explanation for the observed results, I examine the relative value of being fast. The relative value of being fast, denoted by $\Delta(\alpha)$, is the amount by which the expected trading profits for high frequency traders, denoted by $V^f(\alpha)$, exceeds the expected trading profits for slow traders, $V^s(\alpha)$:

$$
\begin{aligned}
\Delta(\alpha) =& V^f(\alpha) - V^s(\alpha) \\
=& E_x\left[\int_0^\infty \left(1 - \lambda\left((1-\lambda)\phi\right)^t\right) e^{-rt} X_t dt\right] \\
& - E_x\left[\int_0^\infty \lambda\left((1-\lambda)\phi\right)^t e^{-rt} X_t dt\right] \\
=& \frac{\kappa(\alpha)}{r}\left(x + \frac{\mu}{r}\right) - \frac{2\lambda\kappa(\alpha)}{r - \ln((1-\lambda)\phi)}\left(x + \frac{\mu}{r - \ln((1-\lambda)\phi)}\right).
\end{aligned} \tag{14}
$$

Since $\kappa'(\alpha) > 0$ for all $x < 0$ and $\kappa'(\alpha) < 0$ otherwise, the relative value of being fast decreases in α if and only if

$$
\frac{\partial \Delta(\alpha)}{\partial \kappa(\alpha)} = \begin{cases} \text{Positive} & \text{if } x \geq 0 \\ \text{Negative} & \text{if } x < 0. \end{cases} \tag{15}
$$

Indeed, from Figures 3 and 4, we see this is the case.

Therefore, an increase in the level of high frequency trading reduces the relative value of being fast. When the cost of investing is low, the fall in the value of waiting overshadows the effect of the fall in relative value of being fast and traders are eager to avail of the low cost by adopting the technology early. On the other hand, when this cost is high, the relative value of being fast provides less compensation to the trader for incurring the high cost and, moreover, this dominates the effect of a fall in the value of waiting. Therefore, as we observe in Figure 2, the higher the level of fast trading, the more reluctant is the trader to adopt the technology when the cost of doing so is high.

It is not possible to obtain knife-edge comparative static results with respect to the market fragmentation parameter λ. However, Figures 5 and 6 imply that X^* is convex in λ when the cost of investment is low, and concave in λ when the cost is high.

A high value of λ implies a low degree of market fragmentation and a relatively easier trading environment for a slow trader to find liquid venues. Furthermore, the expected profitability of fast institutions increases in the degree of market fragmentation,[1] and so too does the relative value of being fast (this follows from equation (14)).

Accordingly, when the cost of investing in the technology is low, the option effect on the optimal time to invest dominates the present value effect and an increase in the level of market fragmentation will make the trader wait longer before investing (see Figure 5) even though his expected profit from doing so is higher. On the other hand, when the cost of investing is high, a higher level of fragmentation actually speeds up investment. In this instance, the present value effect dominates and, since the relative value of being fast is higher owing to the more fragmented environment, this effect more than compensates the trader for the high cost of investing implying he is willing to invest sooner and incur the high cost. A possible explanation for this, as suggested by Biais et al. [1] for a different but related effect in their model, is that the trader anticipates that if it remains slow while others are fast, he will obtain very low profits as the market gets more and more fragmented. Hence, the relative value of being fast becomes more valuable and the sooner he will invest.

Furthermore, the convexity of the low cost effect implies that the more fragmented the market becomes, the lesser is the positive impact on the value of waiting since the marginal effect of λ on the optimal time to invest is positive.

[1] Since $F(x)$, given by (3), decreases in λ.

The concavity of the high cost case, however, implies that the present value effect is strengthened as the market becomes increasingly fragmented.

While I depict the impact of α and λ for both high and low costs of investment, the relatively high cost results are more pertinent because in practice, trading firms invest massive amounts of money in high frequency technology. For example, Laughlin et al. [10] estimate that achieving a 3-millisecond decrease in communication time between Chicago and New York markets costs in the range of \$500 million.

Finally, as shown in Theorem 1, the net present value threshold X_{NPV} asserts that investment in the high frequency technology should take place earlier than the real options threshold suggests. This is one of the most widely known and cited results in real options analysis and is typically driven by the fact that the standard net present value theory of investment does not account for uncertainty over future payoffs nor the fact that the investments are, at least partially, irreversible (see, for example Dixit and Pindyck [3]). It also assumes that investing is a now-or-never decision with no value of waiting. The driving force in this model is the same as in standard models because, since $F(X_{NPV}) = 0$, where $F(x)$ is given by (3), it is easily verified that

$$X^* - X_{NPV} = \frac{1}{\beta_1},$$

and any term involving β_1 pertains to the option effect, or the value of waiting.

In the next subsection, I examine the welfare implications of adhering to the optimal policy prescribed by the model.

3.2 Welfare Implications of the Optimal Investment Time

So far, the level of fast trading in the market α has been exogeneous. However, in order to analyse the implications arising from the model for social welfare, it is necessary to endogenise α. In order to do so, I assume in the customary way (see, for example, Grossman and Stiglitz [5], Hoffmann [9] and Biais et al. [1]) that all traders are born slow but have the opportunity to become fast at an exogeneously determined cost $I > 0$. Then, an interior equilibrium requires that traders are indifferent about being fast or slow; in other words, that the relative value of being fast equals the cost of investing. Therefore, the equilibrium level of fast trading α^* is the value of α that solves

$$\Delta(\alpha^*) = I.$$

Utilitarian welfare is equal to

$$W(\alpha) = \alpha \left(V^f(\alpha) - I\right) + (1-\alpha)V^s(\alpha)$$

and the socially optimal level of fast trading, denoted by α^*_{SO}, is the value of α that maximises $W(\alpha)$. Thus α^*_{SO} solves

$$\Delta(\alpha^*_{SO}) - I + \alpha^*_{SO}\frac{\partial V^f(\alpha)}{\partial \alpha}|_{\alpha=\alpha^*_{SO}} + (1-\alpha^*_{S0})\frac{\partial V^s(\alpha)}{\partial \alpha}|_{\alpha=\alpha^*_{SO}} = 0. \tag{16}$$

From Figure 7, it is clear that the equilibrium level of fast trading α^*, always exceeds the socially optimal level no matter how fragmented the market. This is because the greater the level of high frequency trading in the market, the lower the expected trading profit for slow traders and therefore, high frequency traders exert a negative externality on the slow traders which they do not internalise when making their investment decisions.

A similar result emerges from the models of Biais et al. [1] and Hoffmann [9]. Since high frequency traders are a source of the negative externality, both papers analyse possible policy responses to this problem. Hoffmann [9] discusses the implications of imposing restrictions such as minimum resting times for limit orders and undifferentiated limits to message traffic, while Biais et al. [1] discuss the implications of imposing a ban on high frequency traders, as well as providing an analysis of the effects a pigovian tax would have if imposed on high frequency traders.

However, the optimal timing model in this paper provides a relatively more straightforward response: in order to be socially optimal, traders should wait longer before investing in the HFT technology than the real options approach suggests when the cost of investing is relatively low, and vice versa when this cost is high. This is because X^* decreases (increases) in α when the cost of investing is low (high) owing to the option (present value) effect (cf. Proposition 1).

4 Conclusion

In recent years, the state of market microstructure has changed considerably. There are many ways in which these changes have come about (see O'Hara [13] for a detailed description), but one of the biggest changes is that markets have become highly fragmented. When markets are fragmented, traders must search across many markets for venues which will execute their orders at their specified prices. This can result in delayed or partial execution which is costly. In response to the increase in market fragmentation, there has been a demand for speed by traders, and various types of expensive technologies have been developed. Such technologies enable traders to compare all trading venue instantaneously or obtain a glimpse of the true state of the market before everyone else.

In this paper I derive a model, using techniques from real options analysis, which provides insights into the optimal time traders should invest in high frequency trading technologies. The model prescribes waiting longer when the cost of the technology is very high and the level of high frequency trading activity in the market is also high. On the other hand, when the cost is relatively low, traders should adopt quickly when the level of fast trading is high. Moreover, in a highly fragmented market environment, traders should invest in the technology early when the cost of doing so is high, and vice versa when this cost is low.

Finally, I show that the equilibrium level of investment prescribed by the model is excessive from a social welfare perspective, and that in order to be more socially optimal, traders should wait longer before investing in the HFT technology than the model suggests when the cost of investing is relatively low, and vice versa when this cost is high.

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Appendix

A Figures

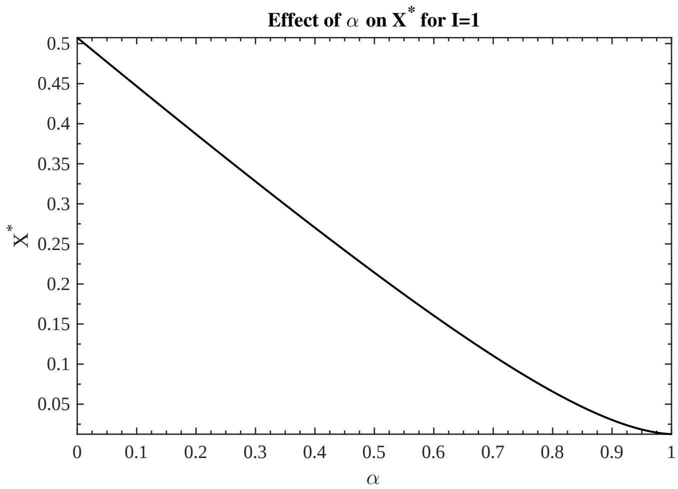

Figure 1: Parameter values: $I = 1$, $\phi = 0.5$, $\lambda = 0.5$, $\kappa(\alpha) := 1 - 0.75\alpha$, $\sigma = 0.2$, $r = 0.05$, $\mu = 0.02$.

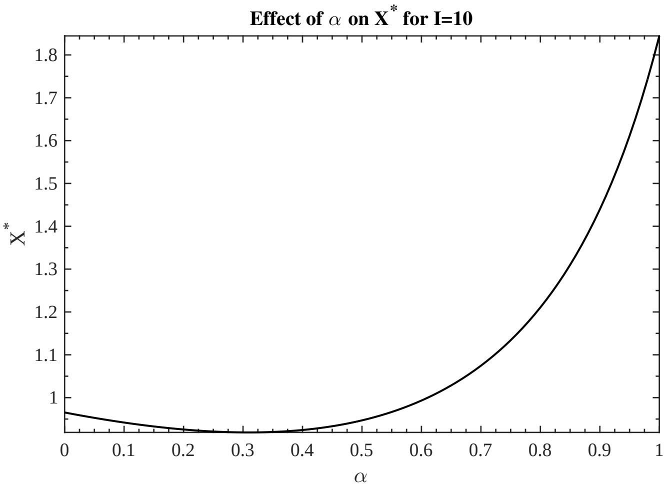

Figure 2: Parameter values: $I = 10$, $\phi = 0.5$, $\lambda = 0.5$, $\kappa(\alpha) := 1 - 0.75\alpha$, $\sigma = 0.2$, $r = 0.05$, $\mu = 0.02$.

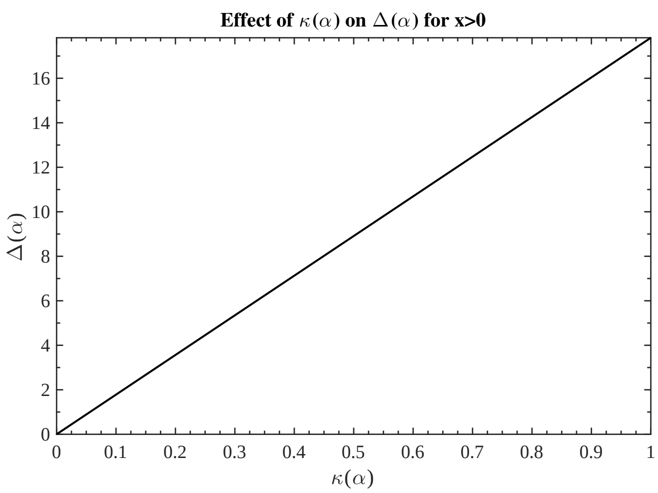

Figure 3: Parameter values: $x = 0.5$, $\phi = 0.5$, $\lambda = 0.5$, $\sigma = 0.2$, $r = 0.05$, $\mu = 0.02$.

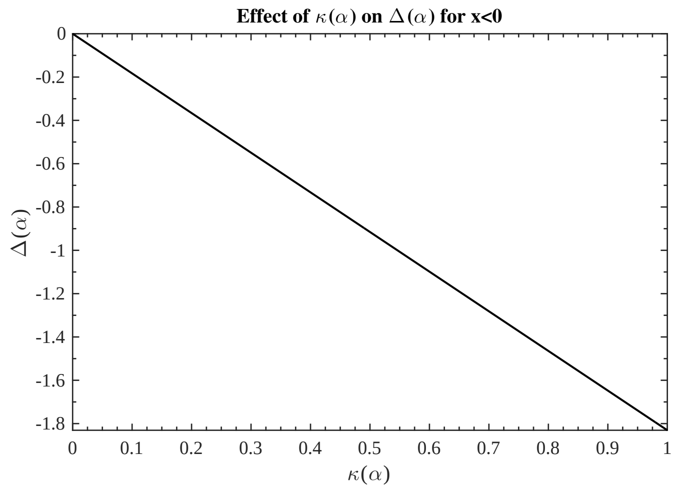

Figure 4: Parameter values: $x = -0.5$, $\phi = 0.5$, $\lambda = 0.5$, $\sigma = 0.2$, $r = 0.05$, $\mu = 0.02$.

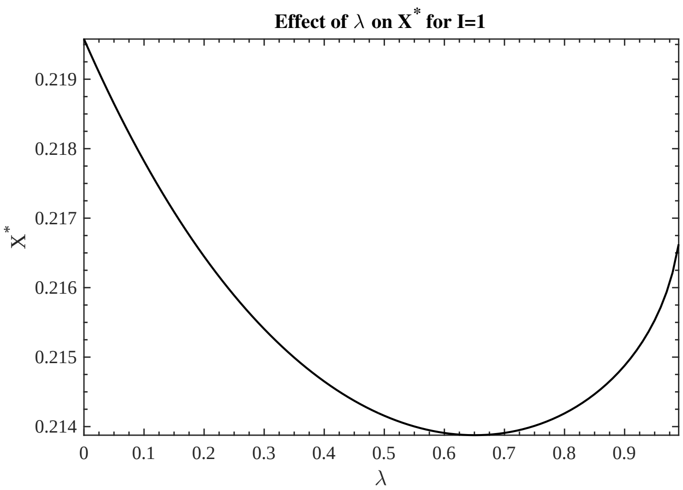

Figure 5: Parameter values: $I = 1$, $\phi = 0.5$, $\alpha = 0.5$, $\kappa(\alpha) := 1 - 0.75\alpha$, $\sigma = 0.2$, $r = 0.05$, $\mu = 0.02$.

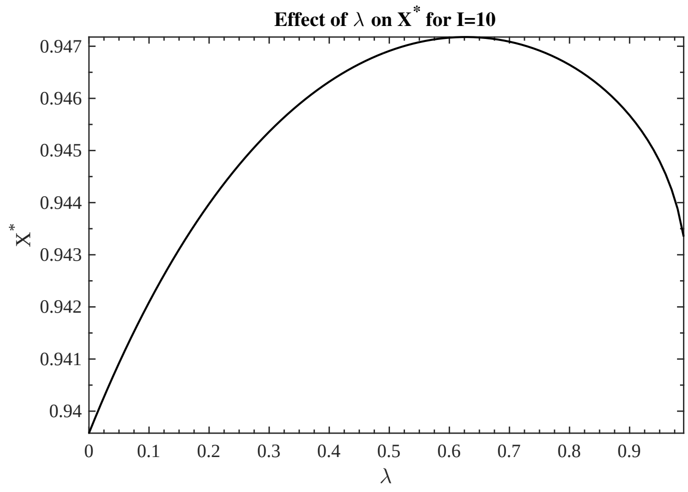

Figure 6: Parameter values: $I = 10$, $\phi = 0.5$, $\alpha = 0.5$, $\kappa(\alpha) := 1 - 0.75\alpha$, $\sigma = 0.2$, $r = 0.05$, $\mu = 0.02$.

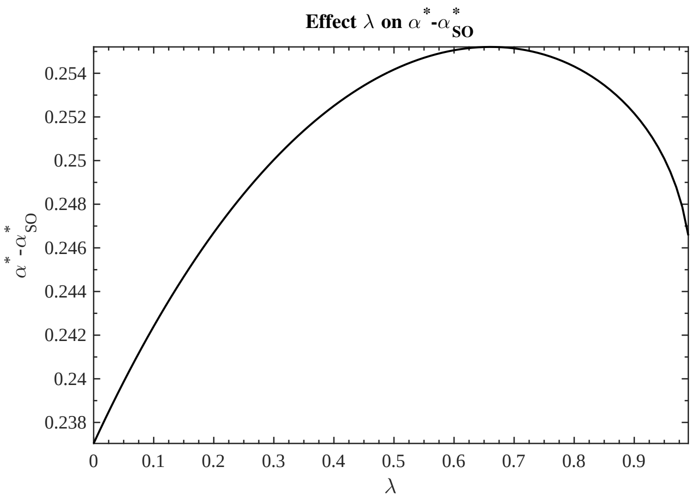

Figure 7: The parameter values are $I = 10$, $x = 0.5$ $\phi = 0.5$, $\kappa(\alpha) := 1-0.75\alpha$, $\sigma = 0.2$, $r = 0.05$, $\mu = 0.02$.