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Application of Agent Based Modeling to Insurance Cycles

A thesis submitted for the degree of Doctor in Philosophy

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Acknowledgments

Declaration

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Abstract

Traditional models of analyzing the general insurance market often focus on the behavior of a single insurer in a competitive market. They assume that the major players in this market are homogeneous and have a common goal to achieve a same long-term business objective, such as solving profit (or utility) maximization. Therefore these individual players in the traditional models can be implemented as a single representative economic agent with full rationality to solve the utility optimization. To investigate insurance pricing (or underwriting) cycles, the existing literature attempts to model various isolated aspects of the market, keeping other factors exogenous. We find that a multi-agent system describing an insurance market affords a helpful understanding of the dynamic interactions of individual agents that is a complementary to the traditional models. Such agent-based models (ABM) try to capture the complexity of the real world. Thus, economic agents are heterogeneous and follow different behavioral rules depending on their current unique competitive situations or comparative advantages relating to, for example, their existing market shares, distribution channels, information processes and product differentiations. The real-world continually-evolving environment leads agents to follow common rules of thumb to implement their business strategies, rather than completely be utility-maximizer with perfect foresight in an idealized world. The agents are adaptively learning from their local competition over time. In fact, the insurance cycles are the results of these dynamic interactions of agents in such complex system.

Chapter 1

Introduction

At the 2008 UK Actuarial Profession Risk and Investment Conference, the agent-based modeling (ABM) working party (consisting of Jon Palin, Nick Silver, Andrew Slater and Andrew Smith) presented a paper on "Complexity Economics: Application and relevance to actuarial work". The paper introduced the concepts of ABM to actuaries and outlined how they might be developed further in the future. The paper was well received and the general consensus was that this would be a promising area for future development.

Lloyd's Franchise Performance Director, Rolf Tolle, recently stated that "mitigating the insurance cycle is the 'biggest challenge' facing managing agents in the next few years." All industries experience cycles of growth and decline, 'boom and bust'. These cycles are particularly important in the insurance and re-insurance industry as they are especially unpredictable, due to a combination of uncertain market competition and insurers' pricing behavior.

The purpose of this PhD research is to use ABM to model and hence understand cyclicality in the insurance industry from a newly developed bottom-up approach. This behavioral simulation could be a powerful complementary tool to traditional top-down analyses. It aims to analyze the macro-dynamics of a competitive market via a simulation of the individual interactive micro-behavior of heterogeneous agents. In the case of insurance cycle, we explain that the market cycle is a result of heterogeneous insurers' responses to the dynamic changes of market uncertainty.

The so-called "insurance cycle" or "underwriting cycle" has the following process: A 'soft' period in the cycle is a period in which premiums are low, capital base is high and competition is high. Premiums continue to fall as insurers offer cover at low rates. Established businesses are forced to compete for risk losing business in the long term. As a result of this unsustainable development over time, less stable companies are driven out of the market which decreases competition, whilst larger companies' capitals are reduced; hence premiums rise rapidly. The market hardens and underwriters are less likely to take on risks due to the risk of becoming insolvent. The lack of competition and high rates once again makes the market profitable, thereby attracting more companies to join the market whilst existing businesses begin lowering rates again to compete. This causes market saturation and insurance cycles continue. Such endogenous cyclical processes are mainly caused by market uncertainty and insurers' responses to the changes in competition, although it is often triggered by a major claim burst such as follows a catastrophic event.

Insurance markets are generally competitive, but they are characterized by imperfections and inefficiencies that sometimes prevent the markets from reaching competitive equilibrium. Such market failures come from informational asymmetries, contract frauds, regulatory interventions, capital constraints, etc (Cummins and Dionne, 2008). Analyses of market dynamics can be focused on different perspectives of players in the system; either customers (the buyer) or insurers (the seller) as economic agents. However, the limitations of traditional insurance market models have been recognized recently, such as: (1) They ignore the heterogeneity of individual players; (2) They assume that economic players have full rationality and the power to foresee the long term future; (3) The environment is static in a way that makes it possible to achieve profit optimization in some states; (4) The market and agents are separately considered rather than simulating them together to see the emerging result.

Daykin et al. (1994) argue that insurance market cycle is hard to be analyzed by any existing individual explanation alone and that insurers can not be considered in isolation. This is a dynamic phenomenon that involves many interactions among different explanations and individual agents. A single company's action cannot represent the dynamics of a whole market, because different companies in the market have different motivations and abilities to response the changing environment. As Taylor (2009) also argues, the existing literature contains numerous studies of particular isolated aspects of the market. The emphasis of his approach to insurance market modeling is less on the detail of any single aspect but more on

the integration of all market dynamics into a single model. He addresses the fact that an insurer's pricing decision not only needs to balance risk and return from its own perspective, but also must compare its price with competitors in a competitive market. To improve these limitations of traditional insurance market model, the approach of ABM simulation provides a possible solution.

Proponents of ABM view the economy as a complex adaptive system: the structure of the markets, the interplay between agents and time lags are the cause of much of the complexity and interesting behavior we see in the real world. Much of science (including economics) is reductionist; an attempt to reduce the world to its basic elements. However, the interaction of these elements causes behavior (often described as emergence), which cannot be predicted by studying the elements themselves. The agents of a complex system, often by following simple rules, form a system that behaves qualitatively differently from the individual agents themselves. A good example of this sort of system is an ant colony. The colony as a whole can perform complex tasks, such as defending the colony from an aggressor by way of individual ants processing simple pieces of information (Beinhocker, 2007). ABM attempts to capture this and typically has the following features (Palin et al., 2008):

1. Heterogeneous agents: a finite number of heterogeneous agents follow different rules that reflect common real-world behaviors; these rules can be very simple (for example, if more customers come to buy an insurance from a particular insurer, then this insurer increases its

price) to highly complicated (for example, this insurer may wait for a while in order to increase market share to destroy its competitors). Neo-classical economic models, which tend to assume there is only a single representative agent or an infinite number of homogeneous agents. They simplify these different behavioral rules as a unified rule that is based on perfect rationality.

2. Adaptation: agents have limited information to make their decisions in a complex real-world system that is different from a theoretical-world with perfect rationality. They have to keep learning from their own mistakes and others who have influences on them, in order to fit or adapt to the system. Therefore, these behavioral rules are evolving and adapting the dynamics of system over time. Following the example of an insurer above, if the rule of waiting is better than increasing price suddenly, then more agents will adapt this rule. However, when too many insurers use this rule, the market becomes too competitive and decreases the profitability, more insurers will change their rules again.

3. Local interactions: agents in the real-world environment often interact and influence each other locally, this also contrasts with neo-classical economic models that assume any decision of an individual agent can affect all other agents in the system. This creates both heterogeneity and agents' adaptive nature, since agents have different limited information sets that only contain the situations of their local environment. For example, one rule may fit well in one local environment but may not suit other places. A large insurer may in-

crease price rapidly when many relatively small insurers around it, but it should not change price suddenly when other large insurers are competing it in the same business segment.

4. Feedback loops and externalities: the dynamic process with adaptive nature can have a positive or negative feedback between the individuals and the system, which often moves the system away from an rational economic equilibrium. For example, an insurer increases its price after it saw the market average price is increasing, but this insurer's action actually contributes the overall market price to increase further. Likewise, insurers often reduce their individual prices too much to follow the market price reduction. Again, this contrasts with neo-classical economic models, which assume that systems always return to equilibrium shortly. They assume that insurers are rational (or have a full information set) enough to ignore the misleading information in the market. While there is a feedback loop between an individual insurer and the whole system, there is also a loop of externality between the individual insurer and other insurers. This means that a single decision of an individual insurer affects the whole system, and the system affects other insurers. Then, the other insurers' reactions feedback to the system and affect the original insurer again.

We apply ABM to build an artificial system that captures several key features of the non-life insurance market. For example, insurers have different short-term objectives at particular times. A big insurer has a large capital and a well-diversified risk portfolio. Therefore, a large insurer prefers

a stable market price, since its price is close to the fundamental value of unit risk which stabilizes its market share. On the other hand, a small insurer is keen to diversify its risk portfolio through different forms of price competition, so its short-term objective is to increase market share rather than stabilize its market price. Insurers' objectives and pricing decisions are dependent on both their current market position and the level of competition. The interactions of these different multi-agents are often difficult to be analyzed by traditional models. The above five unique features of ABM assist us to understand the dynamic nature of the insurance market. Our focus is on the market cycles.

We implement our ABM of insurance market in two steps. First step is to build a base case model of our insurance market. We locate insurance companies equally in an one-dimensional (1-D) circular street, where customers are uniformly distributed along the street. Insurers cannot move their locations but have a competitive advantage to attract customers who are closer to them than other insurers. Insurers price their contracts, based on their local demand situations. In fact, this local situation is also affected by the pricing decisions of an insurer's direct competitors. Second step is to build some extensions that are based on the first base case model. We introduce a two-dimensional (2-D) planar space where insurers have an option to move their locations. We call this dynamic movements as non-price strategical competition, because insurers not only compete with price but also change their locations in order to attract more customers. We compare the simulation results from these two steps to analyze the impact of non-price strategical movement on market cycles and individual

insurers' performance. We conclude that, although there is no significant impact on overall market performance whether insurers can or cannot move in the system, the impacts on individual insurers are significantly different.

Based on the simulation results of the base case model that are validated by the real-world data from the UK insurance sectors, we confirm our hypothesis of explaining insurance cycles endogenously. We argue that the non-life insurance market has a monopolistically competitive nature with differentiated products, despite the contracts are similar. This is because non-price product characteristics or attributes do matter that are modeled by the insurers' individual fixed locations, they generate a differentiation of the insurance products of different insurers. When we are close to the extremes in the cycle (either at the bottom of a soft phase or the top of a hard phase), the market becomes more monopolistic and local, prices are less dispersed. The role of non-price product characteristics in customers' purchasing decision becomes relatively more significant since prices are very similar globally over the market, customers react to the product differentiation and become more localized in terms of their purchase. When we are in the middle parts of the cycle, the market is more competitive and diffuse, prices are more dispersed. Non-price product characteristics are less relevant to customers' purchasing decision since prices are very variable globally over the market, customers react primarily to prices, and will buy insurance from cheaper insurers. A more detailed and comprehensive description of this hypothesis is explained in Chapter 4.

In the extensions of the base case model, our insurance market captures

one key behavioral rule (or bias) of individual companies (or economic decision makers in general). Insurers are more likely to compare themselves with competitors who have similar business strategies and/or sizes. We call these competitors as "neighbors" of an insurer, because they compete with the insurer directly in the local market. Insurers often define their profit targets as a percentage of average market loss ratio in the business segments where they focus on. Also, insurers compare their annual performances in their business reports with a few selected competitors who have similar market positions. Financial analysts in the equity market (or rating agencies) also value the stocks and risks of insurers by comparing a group of companies with similar business strategies and sizes of market share, this leads the managers of insurance companies to keep following each other (Doherty and Phillips, 2002). As Banerjee (1992) argues that decision makers are influenced in their decision making by what others around them are doing. Ariely et al. (2010) and Simonsohn and Ariely (2008) experimentally examine and find evidence that decision makers use a behavioral rule to value a product by looking at other people's pricing decisions, even when this valuation method is not applicable. This real-world observation, so-called "keeping up with the Joneses", is supported by empirical research in Behavioral Economics (Gali, 1994). For example, a consumer's total utility not only positively depends on his own consumption, but also negatively links to his peers' consumptions. Economists often find, for the same amount of consumption, a person feels relatively happier when he lives with poor people who consume less than this person in comparison to a rich group. This simple behavior often creates a risk of "herding", since people often try to follow the action of their peer group.

This is not good for an economic agent since it causes over-consumption or over-pricing of an asset. However, it is human nature that has a huge impact on real-world systemic risks of financial markets. We try to model and understand this behavior in our insurance market.

The structure of this PhD thesis as follows: Chapter 2 provides the introduction of insurance cycle, its evidence and real-world observations, recent developments in the literature. Chapter 3 gives the general introduction about the Agent-based Modeling, includes its distinctive features and economic applications. We introduce and explain our first base case model of insurance market in Chapter 4. In Chapter 5, we extend this base case with further analyses, such as extending 1-D circular street to 2-D planar space, introducing agents' movements on the map of product attributes and taking capital constraint into accounts of insurers' pricing decision. Finally, Chapter 6 concludes our research and provides directions for future research.

Chapter 2

Insurance Cycles

The persistent fluctuations of insurance premium in the Non-Life Insurance market significantly affect the overall business performance of insurance companies. In the real world, an insurance cycle can have a significant impact on the stability of Non-Life Insurance companies. Trufin et al. (2009) study the increase in ruin probabilities when the premium income of insurance company is subject to insurance cycles. Both their analytical and numerical results support this conclusion.

Some useful applications of understanding insurance cycle are in the field of individual business risk management (Simmons and Cross, 1986; Line et al., 2003) and in preventing systemic risks in the whole industry (Gron, 1994; Maher, 2006). From a business risk management perspective, Lloyd's Franchise Performance Director Tolle (2007) said that "mitigating the insurance cycles was the 'biggest challenge' facing managing agents in the next few years." From a regulatory perspective, as Meyers (2007) mentioned, "... insurance cycle contributes an artificial volatility to under-

writing results that lies outside the statistical realm of insurance risk. For internal model development under Solvency II, underwriting cycles must be analyzed, because the additional volatility could produce a higher capital requirement..."

Jones and Ren (2006) present a model for analyzing the impact of insurance cycles on an insurer's surplus. It includes a strategy parameter that indicates how an insurer responds to the cycles and allows one to analyze ruin probabilities under the different strategies that mix between maintaining market share and conserving capital (Boor, 2004; Felisky and Goodall, 2007; Wright, 2008). However, their model is static and assumes that the insurer does not change strategy with the evolution of the insurance cycle. They emphasize that a dynamic model will be necessary to analyze the ruin probability if one assumes that strategies do change over the cycles. A similar suggestion of linking the insurance cycles and ruin theory is made by Daykin et al. (1994).

Kaufmann et al. (2001) and DArch et al. (1997) apply exogenously-induced cycles into their "dynamic financial analysis" (DFA) models. They used a homogeneous Markov chain model in discrete time to assign one of the three different states of market competition to each line of business for each projection year (i.e. from weak to average competition, and finally to strong competition). Both of them report the growing interest in DFA models in the Non-Life Insurance industry[1]. DArch et al. (1998) argue that

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[1]There are some DFA software products for Non-Life Insurance companies available in the market. Each of them relies on its unique DFA methodology, since DFA combines many economic and mathematical concepts and methods which are almost impossible

the enhancement of the public-access DFA model is an on-going process and that one important required improvement is to continue developing the insurance cycle module.

Industrial practitioners are keen to understand this industry-specific behavior, but academics involved with macro-economic business cycles seem to have little interest in this particular sector. This imbalance between industry and academia has two main reasons. The first reason, as Harrington and Nichaus (2003) discovered, is that historical insurance cycles do not appear to be strongly related to the general business cycles of the overall economic activity. Grace and Hotchkiss (1995) also found those external unanticipated economic shocks in the general economy to have little effect on insurance cycles. The second main reason is caused by the traditionally conservative features of the insurance market. As Mooney (1986) states, insurance is a necessity good and the demand for insurance products is relatively stable. Therefore, investment activity in the insurance market is relatively passive when compared to other financial markets, such as the stock market, the derivative market, etc. The process of innovation in both products and technologies has always been slower in the insurance market than other markets (Daykin et al., 1994).

However, many recent developments in the financial market are changing the traditional insurance market. The growing activities of insurance companies are playing increasingly significant roles in the overall stability

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to identify. Our ABM application to cycles will be an advantage if added into the DFA models in the near future.

of the financial market. Barrieu and Albertini (2009) edit a volume about Insurance-Linked Securities (ILS) that is written mainly by practitioners, which gives an excellent overview of the challenging field of modern insurance. As Krutov (2010) describes, ILS and certain reinsurance instruments provide the way of investing in insurance directly by many different parties. Securitization of insurance risk has also become an important tool for risk and capital management that can be utilized by insurance companies alongside the more traditional approaches. Meanwhile, Shelp and Ehrbar (2009) and Spencer (2010) describe the huge influence in the broad financial market that was brought by the 'boom and bust' history of the insurance giant AIG. During the past five years of global financial crisis, many stories were revealed by insiders that clearly blurred the natural boundary between the insurance market and the rest of the financial market. For example, both Lewis (2010) and Sorkin (2009) reveal the growing connection of hedge funds with insurance companies, which are bridged by trading many well-known structured finance products.

This expanding interconnection between the insurance market and the rest of the financial market demonstrates the importance of understanding the cycles. The correlation between insurance cycles and general business cycles will become significant in line with these recent developments. Risk managers and regulators in the insurance industry will benefit from a complete understanding of insurance cycles in order to perform enterprise risk management or to prevent systemic risks (Ingram et al., 2012).

2.1 Non-Life Insurance Market

Non-Life Insurance is commonly known as General Insurance in the UK. It typically comprises any insurance that is not related to the insured individual's death or other events, such as terminal or critical illness (these cases are known as: Life-Health Insurance). It is also called Property-Casualty or Property-Liability Insurance in the US. It accounts for around one quarter (24.8%) of the total gross written premiums in the UK insurance market, nearly 3.5% of the total UK GDP. It is estimated that an annual average of £1,000 of Non-Life Insurance premiums are spent by each person in the UK (Data Source: CEA 2012).

In terms of product classes, Non-Life Insurance is broadly divided into three main areas in the UK: personal lines, commercial lines and the London market[2]. In terms of business classes, Non-Life Insurance can generally be split into six main classes (the individual market shares are shown in the pie chart of Figure 2.1): Motor, General Liability, Legal Expenses, Property, MAT (i.e. Marine, Aviation and Transport) and other non-life items (CEA, 2012).

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[2]The London market consists of a large number of companies (more than 300) which operate in the City of London, together with a well-known Lloyd's insurance syndicate. It includes specialist reinsurance companies, as well as the reinsurance subsidiaries of direct writers. It mainly focuses on large international risks that include marine and aviation, large property risk, liability and reinsurance classes.
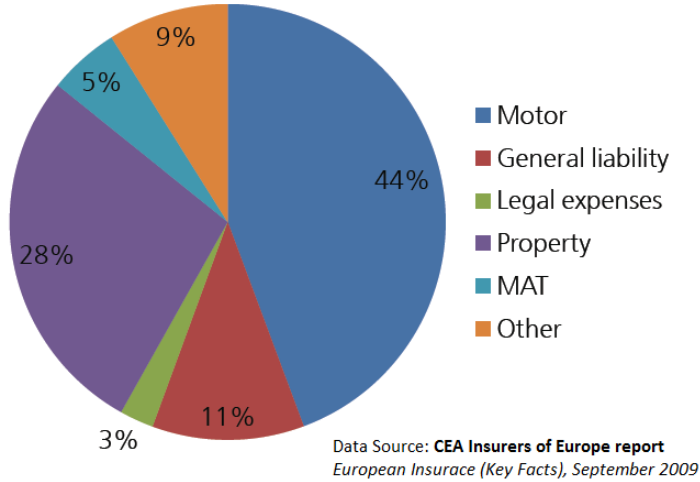
Figure 2.1: European P&C insurance premiums by product (2012)

The Non-Life Insurance market is traditionally very competitive, despite there being some obvious existing barriers to entry. Government regulation attempts to ensure a continued operation of the market by imposing minimum capital requirements and solvency standards on individual companies. Insurers also require some sophisticated technologies and expertise in this specialized market.

In spite of these barriers, there are several causative factors behind the competitive nature. Firstly, capital committed to support insurance companies is subject to the usual rules of supply and demand: when there is an attractive profitability, then there will be enough new capital to create new competitors into the market (Booth et al., 2005). Informational efficiency that has been improved by advanced IT developments is one of the drivers for competition in the non-life insurance market. Secondly, the idea of insurance is to diversify risks through many different kinds of pooling arrangements. One of the important means of risk diversification is to do

global business. The UK market may be of interest to other international insurance companies or other global financial entities (Skipper and Kwon, 2007). Finally, insurers have not been able to patent, copyright or franchise their products, because it is relatively hard (i.e. compares to other goods market, but not impossible) to develop differentiated products and niche markets. Even though it is a specialized market, the new entrants can easily follow the prices of the existing companies without initially investing huge amounts of capital (Harrington and Nichaus, 2003).

As an example, considering the UK insurance market in the fifteen years from 1997 to 2012, there were about 1,000 insurance companies (both Life and Non-Life) operating in the market. Most of them were national insurance companies (around 50% in terms of company number and 80% in terms of market share in total premiums). Some were other European and non-EU insurance companies' branches in the UK market. The remainders were subsidiaries fully owned by other financial entities (see Figure 2.2).

The largest five Non-Life insurers in the UK had less than 40% market share in 2000, and the next five largest companies accounted for less than 10% of the market share. The 15 largest companies occupied less than 50% of the overall Non-Life UK market in 2000, although there was a slight increase in market share for the big 15 companies since 2008 (see Figure 2.3).
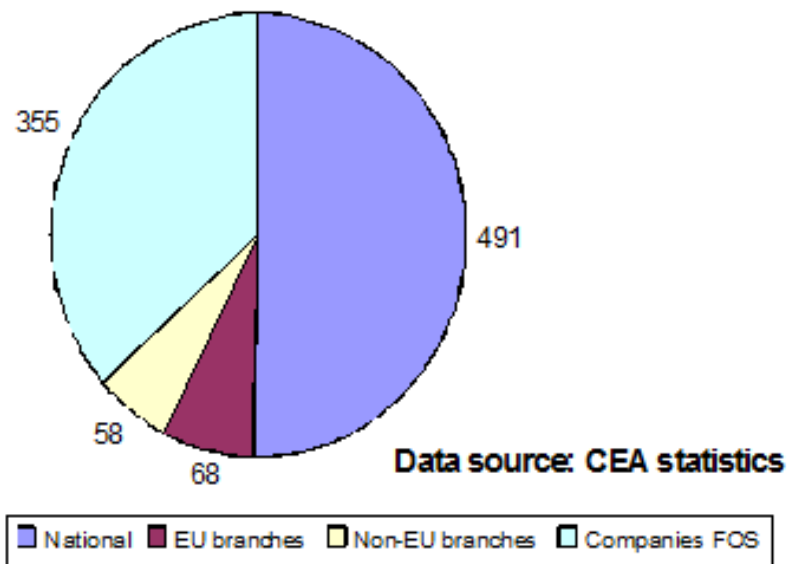
Figure 2.2: Numbers of UK insurers by types of entity (CEA, 2012)
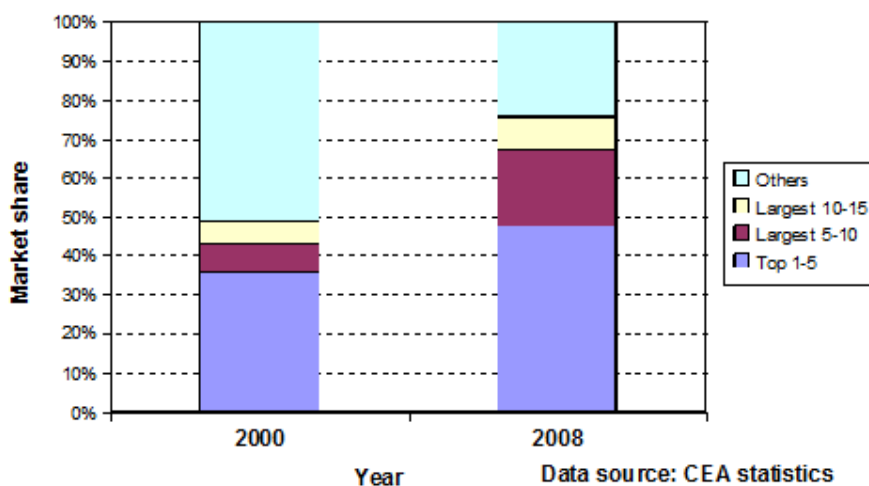
Figure 2.3: Market share of the largest insurers in the UK (CEA, 2012)

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2.2 Cycles: Evidence and Observation

An insurance cycle is a repeated process whereby rates, premiums and profits alternately rise and fall over time, rather than grow smoothly. It often occurs in the Non-Life Insurance sector, where the insured risks are relatively harder to measure and the potential claims are more difficult to predict than in the Life-Health insurance sector (Lyons et al., 1996).

We refer to a soft market when the curve of the price is downward, and a hard market when the price is moving up (see Figure 2.4 for the cycle of general market loss ratio in the UK as an simple illustration, more detailed real market data validations will be discussed in Chapter 4). In a competitive market, the usual cyclical behavior starts with a hard market when higher profit attracts new entrants. The increased competition between new and existing firms pushes the individual companies to lower premium or relax terms and conditions in order to maintain or to seize the market share. The mixed strategies of excess capacity and competitive price-cutting and terms, lead to weakening insurance companies improperly operating their business (Stewart, 1980; Smith, 1981).
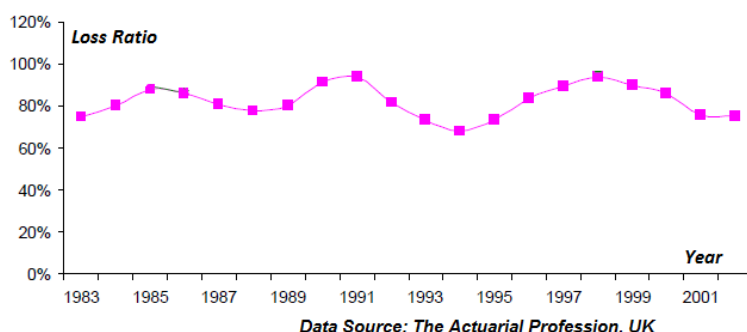
Figure 2.4: An illustration of insurance cycles

Due to the variability associated with the business, these unsustainable levels of low premium rates may persist for some time and might even show a profit prior to the expected future claims occurring. In the long term, when those claims occur along with some unexpected large shocks, there will be large amounts of losses as a result of this mis-pricing. Inevitably, some insurers will become insolvent and they will be forced to leave the market by regulation or by investors withdrawing their finances. The market then moves from profit to loss in a soft-market. However, normally there are some well-established and well-capitalized insurers who will be able to recover these losses from many other different sources of finance, such as by the transfer of capital from other departments within the group. Therefore, they may recover quickly from this poor performance environment. Eventually, they will become profitable since less competition provides the condition for increasing premium rates. This new profitability will soon attract new entrants, and so the cycle continues (Booth et al., 2005).

During the 20th century in the US, many researchers observed the existence of about six to eight-year insurance cycles in the Non-Life Insurance market as a whole, and many different lengths of cycles in the individual business lines (Venezian, 1985; Simmons and Cross, 1986; Doherty and Kang, 1988; Grace and Hotchkiss, 1995). Cummins and Outreville (1987) find that the cycle length is between six and eight years in the six out of 13 developed countries in their sample, and that cycles are presented in automobile insurance profits in all six countries tested, with an average cycle length of 7.1 years.

Lamm-Tennant and Weiss (1997) explore further and find the presence of insurance cycles at an international level. Their research extends the sample of Cummins and Outreville (1987) and updates some data periods. They also expand the scope by examining results in five additional lines of business and cycles in the average loss ratio in addition to the overall underwriting profit ratio. Fenn and Vencappa (2005) test the determinants of cycles in the UK motor insurance market. Their results confirm the existence of a second-order auto-regressive structure to economic loss ratios, which generates a cyclical response in underwriting profits with peaks and troughs around every eight years. Chen et al. (1999) investigate the presence of insurance cycles in Asia. They analyze the insurance markets in Japan, Malaysia, Singapore, South Korea, and Taiwan. Their model supports the existence of the cycles in Asia, the insurance cycles being found in at least one line of all the five Asian countries tested.

Booth et al. (2005) reproduce a table from (Roth, 1984), which shows the combined ratio for US stock property-casualty companies. A combined ratio of greater than 100 represents an underwriting loss: the ratio represents the losses and underwriting expenses dividend by total premiums. It is clear from their figures that there is a cyclical pattern. Daykin et al. (1994) provide some empirical observations of actual behavior of Non-Life Insurance in various countries, such as from the whole markets of both the UK and the USA. They also collect corresponding data from Finland, both the whole Finnish market and six of the largest non-life insurers in Finland. The observations exhibit common features of more or less irregular cycles

several years in length. Harrington and Nichaus (2003) argue that many cycle observers believe and some evidence suggests that the insurance cycle causes actual premiums to fluctuate in a cyclical fashion around fair premium levels.

2.3 Time Series Analysis of Insurance Cycles

To test and determine insurance cycles based on empirical real market data, Weiss (2007) provides and reviews a summary of some time series analyses from early literature. Cointegration analysis can be used to determine whether insurance market performance are related to other economic factors, such as general demand of the economy, interest rates, investment opportunities, etc. If other variables have cyclical behavior, then insurance cycle follows a similar pattern. Autoregression analysis can be applied to test the cycles and the length of the autocorrelation process.

Based on the cointegration analysis of the relationship between aggregate market underwriting margins and interest rates (i.e. 90-day Treasury bill rates) between 1930 and 1989 in the US, Haley (1993) presents evidence that it has a negatively related and cointegrated relationship. However, Haley (1995) expands the cointegration analysis to 17 individual lines of insurance business, he finds that these individual lines are not necessarily cointegrated with the risk-free interest rate. Choi et al. (2002) support the negative and cointegraing relationship between aggregate market per-

formance and interest rate. Grace and Hotchkiss (1995) conduct a similar analysis, which not only support Haley (1993), but also find a positive cointegrating relationship between the combined ratio and other economic factors: GDP (represents a proxy for demand of insurance) and the consumer price index. As Weiss (2007) emphasizes that these findings of relationship between insurance market performance and key economic variables are important because they link the insurance cycle with the general macroeconomic business cycle.

From the empirical US data on major lines of non-life insurance, Venezian (1985) recognizes the underwriting profits can be modeled by a second-order auto-regression process. The analysis shows that cycles appear in several lines of insurance business, but the lengths of the cycles can vary and the phases of cycles among different lines do not necessarily coincide. Similarly, Cummins and Outreville (1987) provide an international analysis of insurance cycle model, as an AR(2) process:

$$\Pi_t = \alpha_0 + \alpha_1 \Pi_{t-1} + \alpha_2 \Pi_{t-2} + \epsilon_t \tag{2.1}$$

where Π_t can be many different market performance variables in period t, such as the average price, market premium, loss ratio, combined ratio or inverse loss ratio, market profits, etc. ϵ_t is an i.i.d error term with $E(\epsilon_t) = 0$ and $Var(\epsilon_t) = \sigma_\epsilon^2$. In this $AR(2)$ process, cycles occur if the coefficients on the lagged terms $\alpha_1 > 0$, and $-1 < \alpha_2 < 0$, and $\alpha_1^2 + 4\alpha_2 < 0$, that is, if complex roots exist (Trufin et al., 2009). As Trufin et al. (2009) suggest that such ARMA analysis can be misleading if the underlying time series

are not stationary. Therefore, it is necessary to check the stationarity before fitting any ARMA model.

The length of cycles is then equal to:

$$\text{Cycle Length} = \frac{2\pi}{arccos(\alpha_1/2\sqrt{-\alpha_2})} \tag{2.2}$$

For example, in the case of Winter (1991), he estimates the process of loss ratio for US property-liability insurance between 1948 and 1988 as: $\Pi_t = 0.42 + 1.02\Pi_{t-1} - 0.48\Pi_{t-2}$, the length of cycle is a period of 8.6 years.

More recently, Wang et al. (2010) develop a regime-switching model for calibrating and simulating insurance cycles. They argue this model is better than an autoregressive approach since it captures the asymmetrical features of the downward versus upward cycle paths. Based on statistical tests on the real market data from SNL Financial, the National Association of Insurance Commissioners and A.M.Best's Aggregates and Average (1976-2010), they observe the following asymmetrical statistical behavior: (1) it takes more years for prices to go down than to go up; (2) Price going up tends to be larger than going down; (3) The volatilities are different in the hardening phases vs. softening phases of a cycle path.

2.4   Existing Explanations and Challenges

The US "Liability Insurance Crisis" of the mid to late 1980s motivated many industrial practitioners to start thinking about the existence of in-

surance cycles. In this period the aggregate Non-Life Insurance premiums increased by over 70% per annum in 1985 and 1986, and limitations on the availability of insurance were widely reported (Fenn and Vencappa, 2005). Both causes and mechanisms of the cycles are studied extensively in the insurance literature, particularly in this period of the 1980s (Stewart et al., 1991).

The major industry-specific inducing factors of the cycles are discussed in (Daykin et al., 1994). They include these factors: time lag effect, claim process delays, inflation uncertainty, fluctuations in investment return, changes in demand, large unexpected external shocks in claim losses, and market competitive activities. Weiss (2007) provides a good summary and a more detailed discussion is given by Derien (2008). Much of the literature analyses the explanations of the insurance cycles from individual or combinations of these cycle-inducing factors.

The theoretical debates of insurance cycle mainly focus on either fundamental or technical aspects of the pricing process of insurance premiums. The fundamental (economic) view of Non-Life Insurance products' prices reflects intrinsic value of risk and insurance (also referred as the price of risk). For example, the changes in interest rate affect the investment return of the assets which leads to change future premium. The second part of pricing decision comes from technical elements such as time delays in financial reporting, i.e. those elements do not reflect the true value of the product, but will inevitably affect the decision making. The common approaches of the pricing techniques can be found in most Risk Management

& Insurance or Actuarial Science textbooks (Hart et al., 2007). It can be mainly classified as: cost-based approach, supply meets demand approach, price negotiation approach, incentive mechanisms approach and financial option model-based pricing method (Madsen and Pedersen, 2002; Madsen et al., 2005).

Major existing explanations are summarized and reviewed as follows:

Market competition cause structure fluctuations: This idea comes from the growing interest in economics of industrial organization (Porter, 1980). It suggests that the cycles are caused by periods of "destructive competition" followed by oligopolistic coordination to cut back supply in order to increase prices before the next round of competition arrives (Stewart, 1980, 1985; Stewart et al., 1991). The main drawback of this argument comes from the difficulty of implementing a complete mathematics-based model that applies dynamic interactions of the complex market behavior. Dutang et al. (2012) and Dutang (2012) apply a game-theoretic approach to analyze insurance market cycles. They consider a discrete framework and insurers are subject to competition as each seek market share and profits based on defined objective function and constraint function. From their initial results of a simple case which only includes three insurers, they conclude the competition modeled through Nash equilibrium alone cannot explain the presence of insurance cycles. Recently, increasing attention on Complexity Economics and the necessity of building dynamic economics models has promoted the development of interconnected dynamical models (Winter, 1994).

Irrational forecasting errors due to "naive" actuarial forecasting (also known as extrapolation hypothesis): This idea focuses on the premium pricing stage and assumes that future expenses are relatively stable when compared to the premiums. Venezian (1985) argues that those "naive" pricing procedures are unable to reflect the changes of expectations about future claim costs. Cummins and Outreville (1987) argue that if under rational expectation theory, the prices of the insurance products reflect all information (about future costs) that is currently available. Venezian demonstrates how such estimation errors can lead to cycles. The drawback is that his hypothesis assumes an unrealistic degree of irrationality on the part of insurers who fail to learn from the experience of past forecasting errors (Feldblum, 2000; Fenn and Vencappa, 2005).

Time delays due to adjustment costs and reporting lags (also known as accounting cycles: rational-expectations/institutional-intervention hypothesis): This idea suggests that the pricing behavior of insurers is consistent with both assumptions of the insurance market being competitive and insurers holding rational expectations to update their pricing information. However, it focuses on the inevitable time delay in getting this updated information (Balzer and Benjamin, 1980; Cummins and Outreville, 1987; Berger, 1988; Dagg, 1995; Booth et al., 2005). This theory predicts a long-run relationship between premiums and the expected value of claims that is consistent with rational expectation theory, subject to second order serial correlation of the short run deviations from this long run relationship. However, an unanswered question of this explanation

is why there is no obvious cyclical behavior in the Life-Health Insurance market, despite the time delays that also occur in this Life-Health sector (Haberman, 1992; Zimbidis and Haberman, 1993).

Capital constraints due to the imbalance between internal and external finances (also known as "cash flow underwriting" cycles): It states that financial capital cannot be adjusted instantaneously when insurers need them. There are costs in getting external funding to adjust premium level in response to a change of expected claim loss. This induces a degree of persistence to a change of new pricing information (Winter, 1988; Gron, 1990, 1992, 1994). This explanation also states that if the investment return in the external financial market is more than the internal earning, then insurers would do better to invest in external opportunities rather than hold capital (Doherty and Kang, 1988; Doherty and Garven, 1995). It links cycles with the change of interest rates. However, this explanation has lost favor recently, since the insurance cycles have lost no force despite the stability of interest rates at the beginning of the 2000s (Feldblum, 2000).

Micro-foundations of firm-level factors (also known as "Financial quality hypothesis" and "Option pricing hypothesis"): It incorporates default risk *endogenously* by viewing premium pricing as being analogous to the pricing of risky corporate debts (Cummins, 1991). Similar idea of comparing insurance products to financial assets can be found in two discussion papers (Madsen and Pedersen, 2002; Madsen et al., 2005). They indicate that one way to view the buying of insurance is to

consider it as buying a call option on losses. It refers to the fact that the insured parties care about the insolvency risk of the insurer (Harrington and Danzon, 1994). The buyers of the insurance product demand financial quality of their contracts. It predicts a positive (or negative, depend on short- or long-run) relationship between surplus and underwriting profits. When firms have lower insolvency risks, they can set higher premiums for the same covers (Doherty and Garven, 1986; Cummins and Danzon, 1997). Elements such as interest rate and investment return affect the solvency level of the insurer and will have an influence on premium.

Behavioral models of underwriters' "mass psychology": This idea interprets cycles as being dependent on the psychology of the underwriters (Kunreuther, 1989; Kunreuther et al., 2013), such behavioral biases in insurance decision as status quo or reference level bias (Tversky and Kahneman, 1991), availability bias (Kahneman and Tversky, 1973) and more generally prospect theory (Kahneman and Tversky, 1979). It argues that insurers are over-confident about their private information. This bias leads to mis-adjustment of their own prices, since the price is more related to the subjective judgment rather than fundamental value. It also argues that insurers become optimistic and compete strenuously for new business during profit years and feel pessimistic when the soft market arrives (Ligon and Thistle, 2007). Broader discussions of behavioral elements such as fears, ambitions, individual level motivations etc, related to insurers can be found in the article (Fitzpatrick, 2004). However, the fundamental problem of this is the assumption of a uniform psychology among underwriters (Feldblum, 2000): over-confidence may cancel out under-confidence if it is

assumed that insurers are heterogeneous in terms of psychology.

As Fenn and Vencappa (2005) state, "although many researchers try to explain cycles from individual factor, separating these hypotheses empirically has proved difficult." They find that the insurance cycle is a multi-factorial phenomenon and each of the existing explanations mentioned above is capable of explaining some parts of the cycles in a complementary way. Some of them discover the prime triggers of the motion. For example, high market competition forces insurers to act aggressively. Others recognize those factors that increase the amplitude of the cycle. For instance, both the extrapolation hypothesis and the institutional-intervention hypothesis realize the difficulties of updating new information to reflect the true value of a product in the real world.

More likely, there is a feedback process that combines these different factors. As an example, insurance cycles can have an impact on insurers' pricing decisions, while the outcomes of these pricing decisions will feed into the cycles (Boor, 2004). Most importantly, all of these competing theories of insurance cycle require exogenous factors to elicit cycles, such as "destructive competition" from external forces, interest rates, errors of actuarial methods, delays in information process, external shocks affect capitals, rating agency reviews insurers' financial quality, etc.

Chapter 3

Agent-Based Modeling

There are some existing explanations of insurance cycle that partially explain the existence of the cycle from different aspects, but a full picture is still not clear (Daykin et al., 1994). A complete and strict explanation of the cycle is needed in order to see the full picture, although this would add considerable difficulties to the problem and make any model largely intractable.

The complexity of these intractable difficulties is often ignored by the traditional "reductionism" approach of conventional economic models. Some of these models assume a single representative rational agent, while others assume there is no dynamic interaction among the agents in a system. They often ignore the "Fallacy of Composition", which states that the whole is *not* equal to the sum of its parts (Ehrentreich, 2007). In a dynamically evolutionary system, the relationship between the parts and the whole is non-linear, and such complexity as a whole is hardly explained by analyzing any one individual part.

In his speech at the International Conference on Complex Systems in 2000, the Nobel Prize winner Kenneth Arrow stated that until the 1980s the '*sea of truth*' in economics laid in simplicity, whereas since then it has become recognized that '*the sea of truth lies in complexity*'. In response to Arrow's statement, a newly developed approach of conducting research, so-called "Agent-Based Modeling (ABM)", sheds light on solving intractable problems in such complex systems (Pyka and Fagiolo, 2005).

Nevertheless, these existing "Ceteris Paribus" based individual explanations of insurance cycles provide useful knowledge for us to implement a more integrated model to understand the cycles. From them, we know that insurance cycles have occurred in the past and that it is difficult to predict them in the future. We recognize that it will become a dangerous task if the risk management model designers do not understand the triggers of the motion for the cycles. It is necessary to emphasize that the goal of our research is neither to replace any existing explanation of the cycles, nor to add another new individual explanation onto the existing list of these different explanations.

In fact, our research project aims to achieve the following goals: (1) to recognize the competitive Non-Life Insurance market as a complex system; (2) to understand the dynamic interactions among the heterogeneous agents in the market; (3) to apply the different features from these existing explanations of cycles in order to model the agents and the market activities; (4) to integrate these different and individual and independent

explanations into one complete and dynamic ABM model; (5) to build our own model of an artificial insurance market based on behavioral simulations; and finally (6), to give a dynamic picture of understanding the insurance cycles.

The reason for us using ABM to achieve our goal is because: The competitive Non-Life Insurance market is a complex system. Agent-based models attempt to capture the emergent complex dynamics of real-world systems and typically have the following features that are consistent with our research (Palin et al., 2008):

1. Heterogeneous agents: a finite number of heterogeneous agents follow different rules that reflect common real-world behaviors; these rules can be very simple (for example, if more customers come to buy an insurance from a particular insurer, then this insurer increases its price) to highly complicated (for example, this insurer may wait for a while in order to increase market share to destroy its competitors). This process of repeated interaction among these agents over time endlessly changes both their initial states and progressive patterns. This dynamic interaction leads individuals to become heterogeneous, even if their initial states were similar (Tesfatsion, 2006). Neo-classical economic models, which tend to assume there is only a single representative agent or an infinite number of homogeneous agents. They simplify these different behavioral rules as a unified rule that is based on perfect rationality.

2. Adaptation: agents have limited information to make their decisions

in a complex real-world system that is different from a theoretical-world with perfect rationality. They have to keep learning from their own mistakes and others who have influences on them, in order to fit or adapt to the system (Pyka and Fagiolo, 2005). Therefore, these behavioral rules are evolving and adapting the dynamics of system over time. Following the example of an insurer above, if the rule of waiting is better than increasing price suddenly, then more agents will adapt this rule. However, when too many insurers use this rule, the market becomes too competitive and decreases the profitability, more insurers will change their rules again.

3. Local interactions: agents in the real-world environment often interact and influence each other locally, this also contrasts with neo-classical economic models that assume any decision of an individual agent can affect all other agents in the system. This creates both heterogeneity and agents' adaptive nature, since agents have different limited information sets that only contain the situations of their local environment. For example, one rule may fit well in one local environment but may not suit other places. A large insurer may increase price rapidly when many relatively small insurers around it, but it should not change price suddenly when other large insurers are competing it in the same business segment.

4. Feedback loops and externalities: the dynamic process with adaptive nature can have a positive or negative feedback between the individuals and the system, which often moves the system away from an rational economic equilibrium. For example, an insurer increases

its price after it saw the market average price is increasing, but this insurer's action actually contributes the overall market price to increase further. Likewise, insurers often reduce their individual prices too much to follow the market price reduction. Again, this contrasts with neo-classical economic models, which assume that systems always return to equilibrium shortly. They assume that insurers are rational (or have a full information set) enough to ignore the misleading information in the market. While there is a feedback loop between an individual insurer and the whole system, there is also a loop of externality between the individual insurer and other insurers. This means that a single decision of an individual insurer affects the whole system, and the system affects other insurers. Then, the other insurers' reactions feedback to the system and affect the original insurer again.

Figure 3.1 illustrates the dynamic process of a complex adaptive system with agents. ABM simulations are now becoming popular to analyze some problems in the field of behavioral economics, due to the above main unique features (LeBaron, 2005).
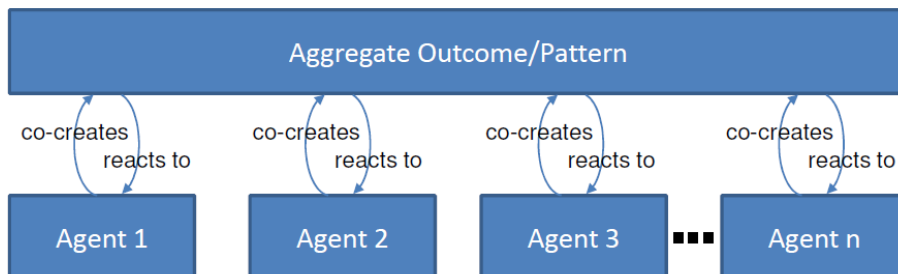
Figure 3.1: An illustration of a complex adaptive system

In Economics, as Tesfatsion (2000, 2003, 2006) conclude that, "... decentralized market economies are complex adaptive systems.". In this kind of system, the main difference with traditional economic analysis is that randomness and determinism are both relevant to the overall behavior. Such systems exist on the edge of chaos; they may exhibit almost regular behavior, but then change dramatically and unpredictably in time and/or space as a result of small changes in conditions (Vicsek, 2002). In order to deal with this complexity, an individual needs to be adaptive rather than perfectly rational as traditional theory assumed[1]. Axelrod (1997) also argues that the adaptation may be at the individual level through learning, or it may be at the population level through differential survival and reproduction of the more successful individuals. Either way, the consequences of adaptive processes are often very hard to capture by traditional analysis when there are many interacting agents following rules that have non-linear effects.

In order to solve the complexity, ABM-based computer simulations will provide a useful way of doing research. Daykin et al. (1994) state that, "a complete and strict simultaneous treatment of several insurers taking

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[1]We often see 'adaptive' in relation to 'rational'; for example, rational expectation theories were developed in response to perceived flaws in theories based on adaptive expectation (Mankiw, 2009). Under adaptive expectations, expectations of future value of an economic variable are based on past value. However, rational expectations assume individuals are able to take all available information to make decisions. Therefore, whether agents are adaptive or fully rational, both of them are trying to maximize their goals. The distinction is the way of dealing with different information sets or the ability of foreseeing the future. Bruun (2005) suggests that "Besides being a complex dynamic system..., the [competitive] economy is also adaptive." She argues that even people are rational, although they would be better off if they were just adaptive since the world is too complex to be rational. As a complex system is non-linear and often has no equilibrium, rational agents would be better to use rule of thumb to adapt/respond to the dynamic system in order to achieve a Nash-equilibrium.

decisions independently adds considerably to the dimensions of the problem, making it largely intractable ..." However, they emphasize that the interactions among the agents in the competitive market are too important to ignore and it is worthwhile adopting even very approximate approaches to explore the possible impact. Nowadays, the computer is more powerful than it was in the early 1990s and provides a great opportunity to further explore the advantage of the simulation.

In this section, both recent developments and future prospects of the ABM are introduced. It is necessary to know the distinctive features of the ABM when comparing it to the traditional economic models. The rest of this section focuses on some ABM applications in Economics.

3.1 Introduction of ABM: Development and Prospect

The complex adaptive systems bring many difficult questions for traditional economic analysis, such as (1) how to model individual behavior, (2) how to understand the learning process of an individual, and (3) how to build a quantitative model in order to implement the complex interactions among these agents. Some of these problems can be explained separately based on different fields of science. However, ABM in Economics (a.k.a Agent-Based Computational Economics) provides a way to link these standalone science via a blend of concepts and tools from individual research, studies within areas, such as Evolutionary Economics, Cognitive Science and Computer

Science (Tesfatsion, 2000). (See Figure 3.2.)
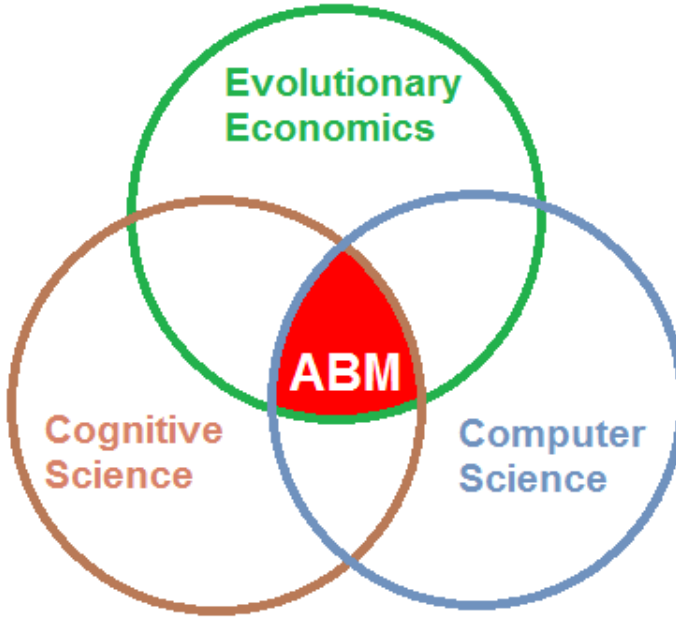
Figure 3.2: An illustration of agent based models

Evolutionary Economics stresses complex inter-dependencies, competition and selections through the actions of diverse agents from experience and interactions in the market. The properties of emergence from evolution states the large-scale effects from some locally interacting small and simple agents (Pyka and Fagiolo, 2005). Evolutionary economists are surprised to see how small innovations or technology breakthroughs are able to change the big world. The area of Evolutionary Economics started with Schumpeter (1934, 1942), and has been recognized by mainstream economists since the studies by Hayek (1948). Schelling (1969, 1978, 1980, 1986) recognize that simple behavior can have a big impact on the macro-environment. Numerous studies on evolutionary economics are reviewed by Nelson (1995) and Nelson and Winter (2002).

Cognitive Science teaches us how the same information or data will have different effects on different people. It also explains how different meanings can be taken by people with different experiences. An individual's cognitive ability forms their rules of thumb and their skills of pattern recognition. Ross (2005) and Sutherland (2007) confirm that personal experience matters when discussing an individual's way of thinking. There are several different learning algorithms that have been modeled by artificial intelligence research (Tesfatsion, 2007): (a) "Reactive Reinforcement Learning" is to update an agent's future behavior automatically in response to successive rewards attained through previous actions taken. The well-known "Tit for Tat" strategy is one of good examples to illustrate this learning idea (Axelrod, 1990). (b) "Belief-based Learning" is to update an agent's future behavior automatically in response to the agent's measurement of his different beliefs. (c) "Anticipatory Learning" is a forward looking approach, it is to update an agent's future behavior automatically in response to future rewards. (d) "Evolutionary Learning" is to improve an agent's future behavior automatically by combining some most relatively successful strategies into one new strategy. (e) "Connectionist Learning" is to improve an agent's future behavior automatically by connecting all possible information that the agent obtained in the past. The agent calculates the weighted average of these information. This impressive learning ability can be achieved by artificial neural networks on the computer. As Tesfatsion (2007) states, it is a decentralised information processing paradigm inspired by biological nervous systems, like the human brain.

Fast developments in computer technology have benefited researchers in terms of them being able to take the opportunities and advantages of simulation techniques. The dynamic interactions can be easily implemented by using simple rules. The learning process can be represented by using genetic algorithms or neural networks on the machine. Many new developments in machine learning will definitely further improve the ability and results of simulation (LeBaron et al., 1999). Artificial Intelligence ('machines that think' or problem solving mechanisms) first appeared as a significant disciplinary division during the 1950s. Both the move toward a networked computing environment and the development of object-oriented programming techniques bring many advantages for scientific research in Economics (O'Sullivan and Haklay, 2000).

In fact, ABM is the kind of simulation where computer technology is used with a combination of both Evolutionary Economics and Cognitive Science. Axelrod (1997) states that, "to appreciate the value of simulation as a research methodology, it pays to think of it as a new way of conducting scientific research[2]." Simulation is a way of doing research through experiments. The ABM simulation tools can be used for testing the impact of

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[2]Traditional, there are two ways of doing research. The first way is through 'Induction', the discovery of patterns in the empirical data. The other way is based on 'Deduction', which involves specifying a set of axioms and proving consequences that can be derived from those simplified assumptions. In contrast to both traditional methods, simulation starts with a set of explicit assumptions that is similar to deduction but it does not prove theorems. Instead, a simulation is keen to generate data that can be analyzed inductively. The simulated data comes from a rigorously specified set of rules rather than directly measured from the real world. Therefore, simulation has the ability to control the experiment, which offers the researchers a tool for scenario testing. Axelrod (1997) summaries that, "While induction can be used to find patterns in data, and deduction can be used to find consequences of assumptions, simulation modeling can be used as an aid intuition."

new policies, potential strategies or decisions in the dynamic market.

3.2 Distinctive Features of ABM

In a public lecture at LSE (2010), both Professor Geoffrey Hodgson and Paul Ormerod criticise mainstream economic analysis. Hodgson (1988, 1999a,b, 2001, 2004) argue that the early static analysis of economics is not sufficient for modern economics. However, it is also too unrealistic to do economics by relying on rational expectation. Ormerod (1997, 1999, 2006) provide some ideas about social network and complex systems in order to understand the 'rule of thumb' of human beings. Both Hodgson and Ormerod recommend that the approach of ABM can be used to fill the current gap in the economic analysis. Some distinctive unique features of ABM when comparing to the traditional economic analysis are summarized as follows:

Bottom-up rather than top-down: Most traditional models use top-down analysis to explain economic problems. Such way of building a model simplifies problems initially but, this simplicity has drawbacks as it foregoes some factors that may become important at later stages. It assumes all individuals are the same as a single representative agent and they are managed by the market. The market allocates the resource and improves efficiency for the economy, but ABM has a different view about the role of the market. It argues that the market does not have the ability to oversee individuals, since ABM recognizes that the autonomous interacting heterogeneous agents are too complex to look after (Tesfatsion and Judd, 2006).

Instead, ABM studies the self-organizing capabilities of decentralized entities to react in the market. Therefore, ABM focuses on individuals rather than a single representative agent. It is a bottom-up approach.

Dynamic rather than static model: Tesfatsion (2006) introduces a common procedure of implementing an ABM application. Once initial model conditions are set, all subsequent events in these agent-based models are initiated and driven by agent-agent and agent-environment interactions. No further outside interventions by the modeler are permitted. The model is dynamic in terms of the interactions among the heterogeneous agents and there are feedback loops between the market and the agents. As Arthur (2000, 2005) notice, this complex system involves out-of-equilibrium dynamics. The ABM approach emphasizes the dynamic process rather than concentrating on a solution to achieve possible equilibrium.

Consistency with other models, rather than being standalone: Many recently developed standalone models or economics theories can partially overcome some problems of early traditional economics analysis, such as DSGE modeling, game theory, and behavioral economics (Camerer and Lowenstein, 2003). ABM provides an opportunity to work consistently with them in order to achieve a full picture to explain economic problems. ABM has the feature of producing data by simulation rather than entirely relying on historical data. ABM is able to analyze the non-equilibrium states. The agents in the ABM follow rules of thumb which is similar to the idea from behavioral economics. As LeBaron (2006) states, ABM may

be a critical tool for implementing the behavioral models.

Computer simulation-based research with learning capacity: Johnson (2001) notices that the ABM builders are often those economists who have some computer programming skills. It is essential for ABM researchers to understand some kinds of object-oriented programming language such as Java or Objective-C. ABM is different from the usual simulations in economic analysis. It features some techniques such as neural network and genetic algorithms. The agents not only have the ability to recognize the patterns, but also learn from these patterns to improve themselves.

3.3 ABM Applications in Economics and Insurance

Tesfatsion (2010) maintains a website which is fully dedicated to Agent-based Computational Economics. There are growing numbers of ABM applications which link to explain early unsolved economic questions and behavior. It is impossible to cover details of all new developments here, but the following few ABM applications are worth mentioning. They are briefly reviewed for the purpose of our research.

The idea of using ABM for investigating a wide range of social structures can be found in the Sugarscape model (Epstein and Axtell, 1996; Epstein, 1999; Beinhocker, 2007). More specifically, its use in explaining the exis-

tence of segregation in human choices is based on the theory of Schelling (1969, 1986). Models such as "Tit-for-tat" prove that simplest strategies can beat more sophisticated ones (Axelrod, 1990). The literature demonstrates how useful ABM can be in analyzing complex ideas through basic simple rules.

There are many more specific applications of ABM applied to explain common economic problems. Tesfatsion (1999) uses ABM to analyze an evolutionary labor market with adaptive search ability. Banal-Estaol and Ruprez-Micola (2009) use ABM simulation and "Reactive Reinforcement Learning" to study how the diversification of electricity generation portfolio influences wholesale price. Farmer (1999, 2001) introduce the ABM application to make real investment strategies from an academic's point of view to a practical investment model. Similar ABM applications in the investment field are used by Lettau (1997) and Chakrabarti and Roll (1999). Lettau studies an example of mutual fund flows to analyze the portfolio decisions of boundedly rational agents in a financial market, where the learning process in the application is modeled via genetic algorithm. Chakrabarti and Roll build an application to analyze some herding behavior in the financial world.

In the field of macroeconomics, Arifovic (1996) and Arifovic and Gencay (2000) study statistical properties of the time series of the exchange rate data generated in the environment where agents update their savings and portfolio decisions using the genetic algorithm. Moreover, the ABM model of Arifovic and Masson (1999) tries to explain the currency crisis. Allen

and Carroll (2001) apply ABM to agents who are learning to choose consumption. The well-known artificial stock market ABM model was built by researchers in the Santa Fe Institute (SFI) in 1980-1990s (Arthur et al., 1996; Ehrentreich, 2002, 2006). The so-called "second generation" SFI model was updated by LeBaron (1999, 2001, 2005). They try to explain some stylized facts in the stock market. Similarly, Ladley and Schenk-Hopp (2009) find that many stylized facts of limit-order markets are not dependent on individual strategic behavior. They can be simply obtained from the interaction of the market mechanism and non-strategic zero-intelligence agents.

In terms of analyzing economic cycles, the traditional cobweb theory (also known as: Pork or Cattle Cycles models) describes cyclical supply and demand in a competitive market, where firms are price takers and the amount of their outputs must be decided before the market prices are observed (Kaldor, 1934). The market price will decrease when supply is more than demand for common goods, and vice versa. In fact, firms' decisions on outputs are based on their expectations about the price in the next period, but the actual price in the next period also depends on firms' current decisions on outputs. So there are feedback loops between the decisions of outputs and the market prices and the key to understanding such market dynamics is the nature of firms' expectations (Nerlove, 1958).

Economists face some complex problems, such as dynamic interactions among heterogeneous agents, feedback loops, non-equilibrium, etc. In this situation of complexity, ABM provides useful tools to improve the tradi-

tional economic models. It does not reject the traditional findings from the cobweb model; it adds extra contributions to the economic theory by improving the economic agents' expectation formations. For example, Arifovic (1994) uses a genetic algorithm (GA, i.e. an ABM tool) to update firms' decision rules on next-period productions and sales. The GA tool is implemented in an ABM simulated cobweb market and its several distinctive features have the capacity to better model the learning behavior (expectation formations) of firms in the real world, e.g. using genetic operators such as *reproduction*, *crossover*, *mutation* and *election* to select more suitable rules or pricing strategies in the real world (Holland and Miller, 1991). Their results show that the ABM simulations with the GA tool can better capture several features of the experimental behavior of human subjects than other traditional models (Nerlove, 1958; DeCanio, 1979). Chen and Yeh (1996) generalize Arifovic's work by applying genetic programming (GP) learning in their ABM simulations.

Recently, the approach of ABM simulation and Complexity Economics in general are becoming popular in the field of Actuarial Science and Insurance Economics. Parodi (2012a,b, 2009) introduce computational intelligence (or machine learning) and its applications can be useful to improve existing methods in general insurance. Taylor (2009) constructs a simple dynamic model with artificial agents (i.e. insurers) to compete locally by comparing direct competitors with similar firm sizes. Although Taylor does not specifically call this model as ABM simulation, there are many similar features of ABM approach and it aims to capture the real world stylized facts in non-life insurance market.

Mills (2010) summarizes many interesting examples from complexity science in order to invite actuaries to join this research field and apply useful and insightful techniques to improve existing problems in the field of insurance. Ingram et al. (2012) apply the theory of plural rationality (i.e. it suggests individuals moving in and out of different solidarities in different parts of their lives, rather than having an unchanged rational or irrational behavior in their whole life) and present a simple ABM model that agents move among four forms of social solidarity. This dynamic movements among different solidarities and the interactions of different agents with different associated nature in each solidarity can suggest some stylized facts of insurance market.

3.4 ABM Model Evaluation: Verification and Validation

Midgley et al. (2007) state that both verification and validation of ABMs are difficult[3] because the heterogeneity of the (computational) economic agents and the possibility of different emergent macro-behaviors come from the dynamic interactions among these micro-agents. Fagiolo et al. (2007) describe the problem of finding an appropriate method for conducting empirical validation as the Achilles' heel of the ABM approach to economic modeling.

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[3]Verification is to determine that one is 'solving the equations right' or implementing the ABM simulations correctly, while validation is to evaluate that one has 'solved the right equation' or set the right Agent-based Model (Boehm, 1981). Both of them are two necessary steps in common procedures of the model assurance.

Nevertheless, some common methods of model verification and validation can be summarized and applied in our model evaluation in the later Chapters as follows:

Face validity (Verification): By using diagrams to graphically represent the simulated data, we can see whether the preliminary results are (or looks like) reasonable. Some practitioners in the general insurance market may give us advice. For example in our model, if more numbers of insurers are competing in the market that increases the competition, then the average market price should be lower. Otherwise, there must be some errors in the implementation. It is a subjective measure that is based on users' limited opinion and knowledge about the real world (Banks et al., 2004).

Turing test (Verification): This technique may be applied as well. We send both simulated outputs and empirical data to some pricing actuaries, let them distinguish whether one data is from simulation or not. We collect and weight their results to make our decision, since people have different subjective view. If there are some artificially deterministic patterns in the simulated outputs, then the stochastic processes in the model must have been implemented incorrectly, such as claims distribution, random shocks. Bharathy and Silverman (2010) suggest that a Turing's test could improve the strengths of face validation when validating Agent-based social systems.

Internal validity and tracing (Verification): In contrast with external validity (e.g.: Face validity and Turing's test) that compares

model/simulation results with real-world system, the internal validity tests the correctness of the internal logic (Mehrens and Lehman, 1991). For example, if we change the claim distribution from Pareto to other more risky distribution, the simulated average prices should be higher in order to reflect the risk-based pricing process. Otherwise, there must be some logic errors in the model. However, from the experience of our model, many unexpected results that initially seems to be no logic, but they actually are emerged from complex interactions.

Sensitivity analysis (Validation): This is an important step in a model that contains many parameters (e.g. ABMs), it shows the effect of the different parameters and their values. We change or adjust the input values and the internal parameters such as agents' number, then we determine the consequence upon its output. This procedure assists us to eliminate some unnecessary parameters in order to avoid some problems due to over-parameterisation. However, from our model experience, some parameters may not be significant individually, but a combination of them may have big impacts on the result. Therefore, the judgments are based on economic theories (Leamer, 1985).

Predictive validation with backtesting (Validation): This technique is used to compare the model's prediction with actual system behavior. We apply some of existing historical data to calibrate our model first, we then use the remaining data to determine if the model behaves as the system supposes to. In our case, we have the data set of motor insurance market in different countries for various periods, so

we can use the first half of all years (or one of countries) for calibration purpose and next half of periods (or another country) for validation. We finally check whether our simulated results are consistent with the real data from the UK ABI and US A.M.Best.

Statistical validation (Validation): Finally, common statistical tests can be conducted here, they can help us to determine if the model's behavior has an acceptable range of accuracy. It significantly increases the credibility of the model. In the initial stage, the particular interests are the distributions of claim and loss ratio, together with their moments and other common statistics. Graphical tests are useful for us to spot any big difference. Some simple methods help us to check if our two samples (simulated data from the model and observed data from the real market) are considered belonging to the same distribution, such methods as Quantile-Quantile plot and Box plot, etc. For the purpose of statistical significance tests, we should perform some goodness-of-fit tests, such as the Chi-Square test and Kolmogorov-Smirnov test. The Chi-Square tests whether outcome frequencies follow a specified distribution and the Kolmogorov-Smirnov tests whether two samples are drawn from identical distributions. We can also test the autocorrelation of the time series data. Particular testing procedures will be discussed in the Chapter 4 of this thesis, and Klugman et al. (2012) provide a more detailed explanation.

Some principles should be applied to our model specification and evaluation: (1) both the nature and the goal of our ABM analysis should be

understood clearly, whether a qualitative representation of real world results or quantitative; (2) any specification of our ABM should be based on economic theories as closely as possible, such as from behavioral economic theory; (3) sensitivity analysis should be applied to some combinations of parameters and to select the most important parameters; (4) estimation on some parameters that have observable data in the real market, while making reasonable assumptions for those parameters without obtainable data.

3.5 Examples of existing ABM

We present a couple of ABM examples with the most commonly used 2-dimensional environment and summarize some of their key features. Both of these existing models can be implemented by different programming languages, but we use the NetLogo platform (Wilensky, 1999) as an example to illustrate the basic elements[4]. Each presentation follows four steps: *model objective, implementation, agent rules, applications.*

3.5.1 Forest-fire model

Model objective: As Mills (2010) states that the Forest-fire Model has a 2-D environment on a lattice. It aims to simulate and understand the spread of a fire through a forest. It shows, as the density of trees becomes high, even a small initial fire will have a devastating consequence. More importantly, the relationship between density and burned area is not linear, there is a

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[4]NetLogo is a multi-agent programmable modeling environment. See http://ccl.northwestern.edu/netlogo/

critical point (or tipping point) where results a dramatic change.

Implementation: As shown in Figure 3.3 that is implemented by NetLogo (Wilensky, 1997), there are also three main parts: left side PART 1 is the initial model setup that controls the density of forest in the 2-D environment. The middle PART 2 is the simulation environment where green color represents the tree and dark area is the empty cell (unwooded area). Right side PART 3 shows the simulation results. The trees are the agents, so they are fixed in the environment and they can be in three states: with or without being on fire, or become burned. Both trees and fires are randomly generated in the environment. Based on the results in PART 3, when density is below 50 percent, there is a very low probability of having a systemic risk (i.e. the percent of burned area is small). However, when the density reaches 60 percent, it changes the percent of burned area a lot (i.e. increases from 1.7% to 78.4%).
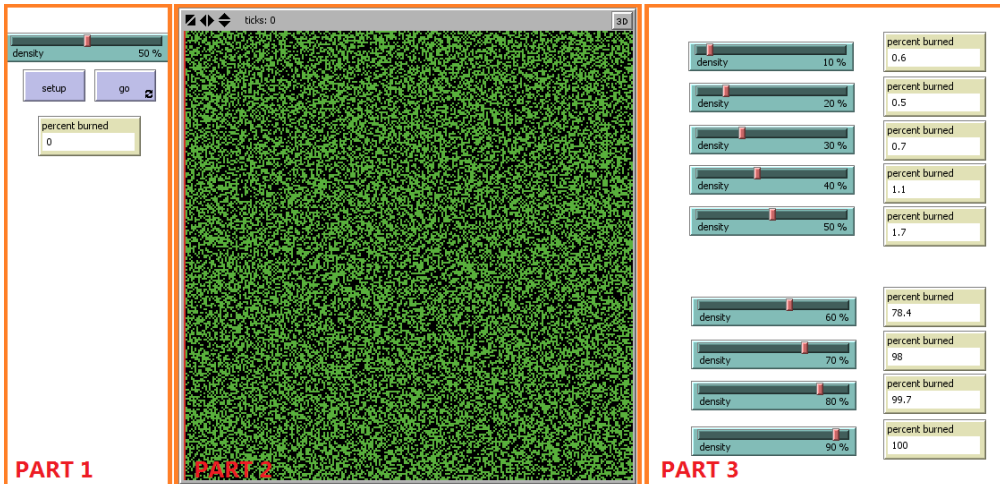
Figure 3.3: Forest fire model

Agent rules: The behavior rule in this model is simple but it still captures the key feature of a complex system, i.e. the presence of a non-linear threshold that affects dynamic results completely. The basic model assumes that fire only spreads to neighboring trees in four directions: north, east, south and west. If a burning tree has no neighbor, then the fire will be stopped after the tree is burned. However, if we apply this simple model to more complex real-world problems, then other behavior rules can also be tested in different situations, such as adding the impact of wind speed, the form of topography, etc.

Real-world applications: Although this 2-D model looks simply, it can be used in many situations to understand the complexity of non-linear relationship and systemic risk management, such as contagion in financial markets, domino effect in social network and demand surge in insurance risk management, etc.

3.5.2 Sugarscape model

Model Objective: Sugarscape Model aims to simulate and analyze some complex social problems (Epstein and Axtell, 1996), such as population growth, wealth distribution, etc. The basic model setup is simple, but it can be extended to many different social situations.

Implementation: As shown in Figure 3.4, there are three main parts in the NetLogo platform screen. Left side PART 1 includes the initial model assumptions and parameter values. Right side PART 3 produces the out-

comes of simulation. The middle PART 2 illustrates the dynamics of agents' interactions in a 2-D environment: the red dots are agents that are randomly distributed on the 2-D grid environment. Grid cells contain sugar that the agents aim to occupy and collect in order to grow agents' wealth. Cells are colored according to their sugar amounts, the more yellow colored cells contain more sugar. Once an agent collects sugar in one cell, the cell requires some time to produce new sugar. Agents interact and compete each other in such complex system.

Agent rules: There can be many different agents' behavioral rules, depending on a particular real-world situation that researchers aim to analyze. For example, the rules of agent movement can manage the following agents' behavior: speed of movement, direction, vision, etc.

Real-world applications: Different agents' behavioral rules can explain many different social problems. For example, if the speed of movement is too quick but agents are myopic with short vision, then this results aggressive competition among agents to move toward a same area in order to compete sugar (i.e. a herding behavior). The results of this herding behavior are similar to the recent crashes in stock and credit markets.
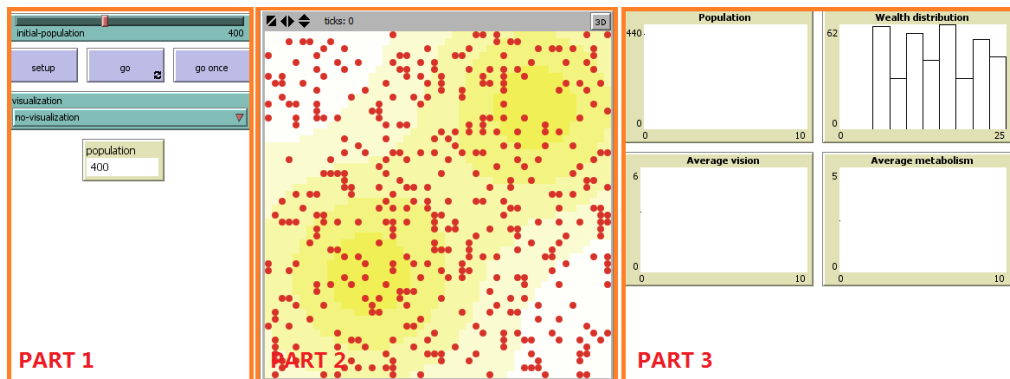
Figure 3.4: Sugarscape model

Chapter 4

An ABM of the Non-life Insurance Market

4.1 Introduction

The major existing (and competing) theories of insurance cycle are reviewed in Chapter 2, together with their main drawbacks and limitations of explaining this market dynamics of non-life insurance. Chapter 3 introduces the approach of Agent-based modeling (ABM) as a newly developed research tool that provides a helpful and complementary way to improve our understanding of cyclic market aggregate behavior, since ABM mainly focuses on understanding the basic individual agent's behavioral patterns (e.g. insurers' rules of thumb in the real-world environment to dealing with different market situations) and interacting them in an artificial system to achieve many complex aggregate stylized facts that are appeared in the real world. This chapter applies ABM to explain the cyclic behavior of insurance premium and market combined ratio in the non-life insurance market.

This chapter is organized as follows. Section 2 reviews and summarizes the distinguishing features of our ABM insurance market that aims to address the limitations and challenges of existing traditional models, in comparison to the previous studies of underwriting cycle. Section 3 presents our base case model in detail along with addressing main assumptions and simplifications. Section 4 explains the real market data from the UK insurance sectors. Section 5 discusses the estimations of model parameter values. Simulation results are evaluated and illustrated in Section 6, followed by the key procedures of verification and validation for our model in line with real market data in Section 7. Then, we discuss the results and the further sensitivity analysis of key parameters in Sections 8. Finally, Section 9 concludes this chapter.

4.2 Literature Review

4.2.1 Monopolistic competition

Existing insurance cycle literature recognizes the competitive nature of this market (Winter, 1994). Most models assume perfect competition and require assumptions such as: (1) Insurance products are homogeneous: insurance contracts have similar terms and conditions among different insurers; (2) Every insurer is a price taker: no individual insurer has a significant market power to decide the market price, but the price is decided by the aggregation of all insurers; (3) Zero transaction costs and perfect information of exchanging goods or services between buyers and sellers: insured

customers pay premium regularly and insurers compensate their customers when claims are incurred, so the process is straightforward. In this competitive market, price reflects the average cost of risks (i.e. it should be stable on average if the market is large enough to diversify individual risks, but sometimes it may have random fluctuations around the mean because of the uncertain nature of this business), and therefore the market should not exhibit patterns such as cycles, in theory(Cummins and Dionne, 2008).

However, several sources of market imperfection cause cyclic behavior. For example, Cummins and Outreville (1987) argue the time delay of processing past information cause auto-correlation in price, since it takes time to get useful information from raw claim data; Harrington and Danzon (2000) discuss how the practical limitations of liability law and insurance regulation affect the competitive price, since government has a responsibility to control the price within a certain range therefore maintaining a stable market; Danzon (1983) finds that rating bureaus in U.S. property-liability insurance market facilitate entry and competition by allowing small insurers to compete without the substantial information or loss experience available to larger ones; this leads to the cyclic dynamics of excessive capacity and insolvency in the industry.

In contrast with the assumption of "perfect competition" in the traditional insurance market literature, we suggest that non-life insurance market is characterized by monopolistic competition (Davies and Cline, 2005). Our argument in favor of the "monopolistic competition" assumption and against the above three main structural characteristics of perfect competi-

tion are:

1. Although there are a large number of insurers, and products (i.e. insurance policies and contracts) are generally similar, they are differentiated in terms of non-price characteristics or product attributes. Schlesinger and Schulenburg (1991) emphasize that, whereas the insurance contract might be roughly similar from insurer to insurer, the insurance product is a service and will be variable. More specifically, such different services (Wells and Stafford, 1995) include: insurers' reputation and branding, their perceived reliability and trustworthiness, also the relationships between insurers and customers (Crosby and Stephens, 1987), ease of policy cancellation, online websites, access to information and communication, protection and safety of personal data, marketing and advertising methods, sales promotional activity, convenience, out-of-hours phone service and complaint handling (Doerpinghaus, 1991), etc.

2. The market is therefore segmented with slightly different products targeted at different customers' preferences over these characteristics. Monopolistic competition means that insurers face highly, but not perfectly, price-elastic demand. Although no single insurer has significant power to affect the market price, each of them has local power to influence the price in its particular market segment, and in neighboring segments. Insurers maximize profits by setting prices competitively and also through non-price competition. They can identify ex-post whether other insurers make higher profits in other areas of the market (i.e. different groups of customers) and will

strategically target these more profitable areas.

3. Although it is straightforward to exchange services in insurance business once the contract is completed, it requires investments and costs for both buyers and sellers to start a contract and collect mutually trusted information. Insurers invest in developing skills so as to better underwrite the risks of their existing customers and attract potential new customers, while customers expend both money and time to understand the benefits of their contracts and find suitable insurers (Schlesinger and Schulenburg, 1993). This process of searching and matching entails transactional costs, which means that the insurance market is not perfectly competitive.

Our recognition of monopolistic competition suggests that each individual insurer has local power to affect the price in its targeted customer groups, although this localized impact is not enough to influence the whole market. There are two important questions related to this: (1) Since an insurer has the power to affect the local price in its market segment, instead of being a pure price-taker, as suggested under perfect competition, the insurer's price decision and strategic behavior now become significant to its profitability. How to model these kinds of individual behavior? (2) The market is loosely segmented with heterogeneous insurers and customers, so how does one explain how the collection of these individual parts affects the overall market? The following subsections discuss these issues.

4.2.2   Behavioral economics

Behavioral Economics purports to provide a more realistic approach to real-world behavior and to individual agents' decision-making processes compared to early, classical economic models with rational agents (Simon, 1987). It argues that economic agents may not achieve full rationality in reality (i.e. they exhibit bounded rationality) and these agents are not necessary to purely seek the goal of profit and standalone utility maximization since heuristics may influence decision instead of being strict logic (Camerer and Lowenstein, 2003). For example, the findings from Prospect Theory (Kahneman and Tversky, 1979) are very relevant to pricing decisions in the insurance market. Recently, Blake et al. (2013) apply some of these findings to investigate optimal investment strategies in defined contribution pension plans under loss aversion. One reason for the use of simple heuristics may be that collective experience in organizations teaches managers that some heuristics can deliver near-optimal or robust performance. For example, Conlisk (2003) shows that adaptive heuristics for stopping searches, based on an aspiration target which involves an average of past performance, can approximate optimal stopping rules. Owadally et al. (2013) also show that a simple rule based on deviations from a target can lead to robust contribution rules for a savings and investment plan. Baumol and Quandt (1964) investigate rules of thumb which managers may use to achieve "optimally imperfect" pricing decisions in practical settings.

Kunreuther et al. (1993) find that insurers attach greater importance to losses than to gains of equivalent magnitude, they are willing to give up

more gains in order to avoid equivalent losses (i.e. loss aversion). As a result, they update their prices more aggressively when they reach an insolvency level. Meanwhile, insurers and customers often overweight small probabilities (e.g. few occurrences but large severity) and underweight large probabilities (e.g. more frequency but small severity) of events. This is often referred to as non-linear probability weighting or availability bias.

Kunreuther et al. (1993) also recognize that insurers often use a reference level to set prices rather than depending purely on updated new information about the risk of its portfolio (i.e. reference dependence or status quo bias). This reference level may take the form of the past price, or the average market performance, or the direct competitors' decisions, or the average capital level, etc. This is consistent with the interdependent (benchmark) decision-making behavior that is proposed by Gali (1994), who argues that the happiness of an economic agent not only depends on his/her own consumption but also on his comparison with other agents' levels of consumption (i.e. so-called "keeping up with Joneses").

Kunreuther et al. (2013) use some real-world examples (so-called "anomalies") to explain the impact of behavioral decision processes on both supply and demand in the insurance market. They argue that, in the real world, neither insurers' sale of insurance, nor customers' purchase of insurance, occurs purely for the purpose of risk transfer as rational decision-making would dictate. Both parties may have other insurance-related goals that are not consistent with being rational agents. As Kunreuther et al. (2013) mentioned that insurers' goals are influenced by their different stakehold-

ers. Likewise, customers may also have four main goal categories that may influence their purchases: investment goals, satisfying legal or other official requirements, compensating their personal feeling of worry or regret, and meeting social cognitive norms.

Kunreuther (1989) analyses the role of actuaries and underwriters' behavior in the insurance product design and pricing decision. He finds that insurers are often not willing to offer protection against some uncommon risks, even though new opportunities with low overall supply and low competition could be profitable. The empirical evidence from their investigations of actuaries and underwriters suggests that there are ambiguities associated with these risks which affect the judgment of actuaries and underwriters, rather than being based on an rational profit-maximization analysis.

Stone (1973) also observes that underwriters' pricing behavior often follows a safety-first approach in order to avoid insolvency risk rather than maximizing the expected profits of their risk portfolio. Doherty and Phillips (2002) analyze the interconnection between changing A.M. Best's rating standards and build-up of capital by U.S. property-liability insurers. They show that the pressure for insurers to maintain their existing rate levels provides a plausible explanation of the dramatic build-up of capital in the industry when the credit rating agency increases the capital requirements during the 1990's, rather than being considered as an rational profit-maximization decision. Insurers' apparent inertia to switch their rating methods also seems to be irrational.

Grace et al. (2004) find that insurers stick with rating guidelines based on recent loss history, rather than using more analytical and scientific CAT models when they estimate future premiums after suffering a major catastrophic event. As a result, after a major disaster, insurers often increase future premiums excessively or reduce the supply of catastrophe coverage drastically.

Ligon and Thistle (2007) suggest that such boundedly rational behavior by market participants can help to explain the underwriting cycle. They develop a model where insurers are overconfident about the private information which they hold on expected losses. The insurers also overreact to new information which becomes available to them. These insurers often overestimate the precision of their own abilities when deciding correct prices, which may lead to cyclic deviation of market price from the true expected risk.

Similarly, Boyer and Outreville (2011) present a behavioral pricing model that focuses on underwriter sentiment. They illustrate that cycles can be purely driven by a biased perception of the random process that generates future losses. Fear and greed on financial markets can preclude rational behavior. Fitzpatrick (2004) describes how fear can contribute to the cycle formation dynamics. For example, insurance companies may create powerful short-term incentives (e.g. bonus and promotion based on short term performance) for underwriters to sell as many policies as possible at irrationally unprofitable price levels under normal circumstances. Such a culture then generates 'fear' when huge losses are realized in more extreme

circumstances.

Also, Ingram et al. (2012) discuss the role of "surprise" in pricing decision when agents are moving from one social state to another in order to understand the human-side dynamics of the insurance cycle based on the theory of Plural Rationalities (or so-called "cultural theory of risk") (Douglas and Wildavsky, 1982, p. 174). They argue that individual agents often move in and out of many different personal situations when making decisions, they may make rational decisions in some particular situations while being irrational in other circumstances. The rotation of these different states may create cycles.

Agent-based Modeling (ABM) provides a way to model such boundedly rational behavior by individuals (LeBaron, 2000). It makes it possible to implement simple behavioral rules followed by insurers. Because insurers do not exhibit pure profit-maximizing behavior, these rules allow us to represent commonly observed real-world behavior such as non-linear reaction to gain and loss by insurers (i.e. Prospect Theory), and performance comparison against competitors or benchmarks (i.e. interdependent decision-making behavior).

4.2.3 Economic location models

Two characteristics of monopolistically competitive markets are important in our model structure: (1) sellers have a degree of control over price locally, although no one can dominate the market globally; (2) customers

perceive that there are non-price differences among different sellers' products or services, which depend on customers' individual product preference. The main difference between monopolistic and perfectly competitive markets is that the products are heterogeneous in monopolistic competition, because strategic non-price competition between insurers leads to product differentiation.

We use economic location models as our main tool to analyze the dynamics of insurance markets (Hotelling, 1929; Salop, 1979; D'Aspremont et al., 1979; Economides, 1986b). These models use "location" and "distance" as a way to define and capture product differentiation. The models can be based on geographic location with physical distance as in the original model of Hotelling (1929), but generally distance represents variation in customers' preferences in a product characteristic space (Salop, 1979). For example, a customer rents a DVD from the nearest shop because it costs less on transportation and saves time (i.e. geographic location in a physical distance). A customer could also buy a particular bottle of perfume from many options with similar prices at one counter because she has a unique preference and it has a closest preference distance between the customer and the perfume (i.e. customers' preference distance). Although physical distance is easy to measure and observe, the distance between customers' preferences is abstract. It may be possible to calibrate the distance between customers' preferences using customers' product experience (Riordan, 1986).

As mentioned in Schlesinger and Schulenburg (1991), insurance policies

are also different in terms of non-price characteristics about the insurance product, e.g. provision of online or telephone service, product bundling, etc. Nowadays the physical locations of direct insurance companies and intermediate brokers are less relevant to the majority of customers' purchasing decisions because of the development of advanced information and communication technologies (Petersen and Rajan, 2002). So the location space is more "a preference space for product characteristics" (Degryse and Ongena, 2005). A customer is at a particular location because he prefers a particular set of characteristics about the insurance policy. His location is his own most preferred product specification. He might have a utility function in terms of price as well as in terms of this particular product specification, where the distances between the customer's personal unique location and the particular characteristics of potential products can be seen as extra costs (e.g. the cost of searching a product that matches customer's preference or the cost of switching from existing products to a better option) (Berger et al., 1989). Location or the distance between suppliers and purchasers may also represent the cost of acquiring the necessary information to build mutual trust (Schlesinger and Schulenburg, 1993).

Location, or distance differentiation, in our model represents and combines different sources of costs which affect customers' insurance purchasing decision, and which are not related to the actual market price of the product. Such "non-price costs" include the physical transportation cost (Regan and Tennyson, 2000), the information searching cost (Dahlby and West, 1986; Posey and Yavas, 1995; Posey and Tennyson, 1998), and the customer switching cost (Schlesinger and Schulenburg, 1993). Both phys-

ical transportation and information searching activity are much cheaper nowadays than in the past, but they are still relevant to insurers' selection of distribution channels and methods of selling their new products (e.g. whether through traditional approaches of direct human contact, or online websites without human intervention). Customer switching cost arises because of customer inertia and customer loyalty, and is consistent with findings from Behavioral Economics. For example, Liebman and Zeckhauser (2008) argue that, in reality, customer inertia and confusion lead to customers sticking with overpriced and poorly designed policies, and this explains the anomalies of consumers' irrational purchasing behavior as outlined in (Kunreuther et al., 2013, p. 40).

4.2.4   Emergent behavior in complex systems

Behavioral Economics attempts to understand the individual behavior of an economic agent with bounded rationality. Economic Location Models capture the main characteristics of a monopolistically competitive system. One of our main contributions is to combine these two elements into one agent-based model. As explained in Chapter 3 where Agent-based Modeling was introduced, the key benefit and advantage of ABM is the ability to simulate the interaction between heterogeneous agents, in an endogenous fashion and in a complex system (monopolistic completion based on an economic location model). This can generate emergent and complex behavior which is hard to be explained by using traditional economic models but which appears commonly in the real world.

The bottom-up approach of agent-based modeling can explain many stylized facts in the real world. For example, Froot et al. (1992) disagree with the standard models of informed speculation, which suggest that rational traders learn by acquiring private information, unavailable to other investors, when they hold assets over a long horizon. Instead, they argue that speculators' behavior is of a short-term nature and speculators rely on learning from other traders' decisions, which explains herding phenomena in real financial markets. Similarly, Lux (1995) uses a self-organizing process to understand herd behavior in speculative market and the emergent results of bubbles and crashes.

Arthur et al. (1996) suggest an ABM of asset pricing based on heterogeneous agents who continually adapt their individual expectations to the current market, but this market is endogenously created by the aggregation of these individual expectations. It has a recursive nature and a feedback loop, since an individual agent's expectation is formed on the basis of its anticipation of other agents' expectations. Thus, individual beliefs or expectations become endogenous to the market in the system, and constantly compete within an ecology of other agents' beliefs, so that agents co-evolve over time.

Returning to our insurance market and underwriting cycle literature, Lai and Witt (1992) and Lai et al. (2000) attempt to analyze cycles from an endogenous view. However, if one uses the definition of endogenous expectation of Arthur et al. (1996), their models are not completely endogenous. Although the model of Lai et al. (2000) suggests that changes in both

insurers' and customers' expectations about the future environment may cause cycles and crises, the sources of these expectation changes are still based on exogenous variables rather than endogenous factors (e.g. other insurers' expectations). An example of exogenous variables in their model is interest rates, which may be affected by credit markets as well as regulatory or political risk. Our model tries to offer a more complete view of endogenous market dynamics, one of the outcomes being the emergence of insurance cycles.

4.3 Model Specification

4.3.1 Outline and model assumptions

In our ABM of the insurance market, the market process is similar to Taylor (2009). The insurance market is simplified and contains insurers and customers only. It operates in discrete time, the unit of time is arbitrary but may be thought of as the minimum period during which an insurer is able to reassess and respond to its market circumstances (e.g. one year).

As illustrated in Figure 4.1, insurers interact in each time period $(t, t+1)$ according to the following steps: (1) At time t, each insurer offers its own unique market competitive price to all customers. (2) Every customer calculates the total cost of purchasing insurance from each insurer. This includes both the price of insurance from a particular insurer and the distance-related costs between the customer and this insurer. The distance-related costs are explained in the subsection of Economic Location Models

above, which states that customers incur a smaller cost when the characteristics of a particular insurer's products are closer to matching customers' personal preference or product specification than other competitors. (3) Based on the estimated total costs, customers select the lowest option, and the whole market is balanced between supply and demand. Insurers collect premiums after being selected by their customers at time t. (4) Customers' total claims are randomly generated and paid by insurers at time $t+1$. (5) Insurers update their underwriting results after paying the claims to their customers. (6) Market performance is updated at the end of each time period. This includes combined (or loss) ratios, premiums and profits, both for the whole industry and individual insurers. (7) Based on newly updated market performance, insurers decide what their competitive price will be in the next time period.

The market process then recommences at the beginning of the next period. Beyond some basic assumptions[1] which are also made by both Taylor (2009) and Maynard (2012), we also assume that the market is monopolistically competitive, rather than perfectly competitive. It is compulsory for each customer to purchase an insurance policy in each time period. The next several subsections discuss each step in greater detail.

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[1] Common market simplifications, such as: Zero claim inflation; Short tail business; No tax and administration expenses; No agency cost and complicated moral hazard problems; No external capital market, reinsurance, brokers, intermediary agent; No international operation, multinational currency, exchange rate; No other complex investment vehicle, financial product, hedging instrument, derivatives, legal issues, accounting and reporting policy, etc.
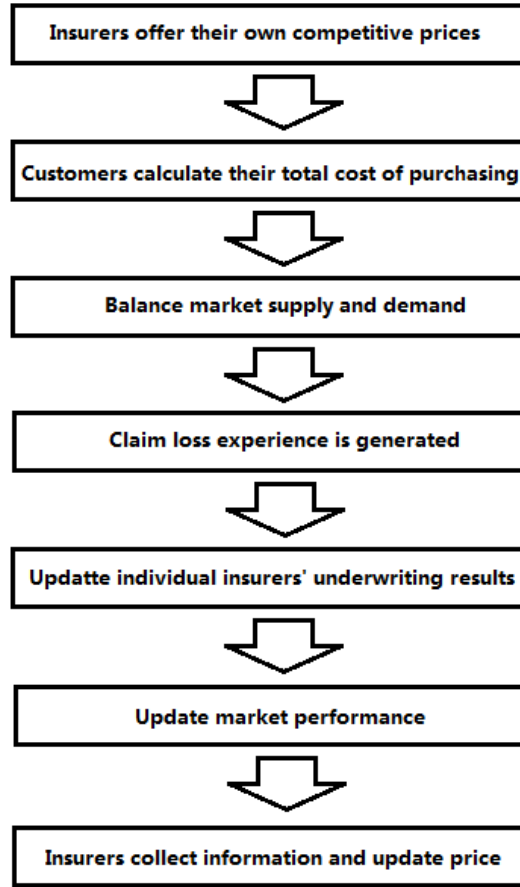
Figure 4.1: Market process of Non-life insurance market

4.3.2 Insurer's market competitive price

(1) Neoclassical Price Theory

It is a standard result in neoclassical price theory that a firm's profits are maximized when marginal cost (MC) equals marginal revenue. This occurs when the firm charges the price P^* given by

$$P^* = MC \times (1 + \epsilon^{-1})^{-1} \tag{4.1}$$

where ϵ is the price-elasticity of demand. See for example Hirschey and Pappas (1996, p. 637) or Petersen and Lewis (1999, p. 429).

Equation (4.1) is an example of cost-plus or mark-up pricing. Price is formulated as a function of cost which is marked-up through the term $(1 + \epsilon^{-1})^{-1}$. If a market is competitive and its demand elasticity is high, then the price-cost margin $(P^* - MC)/MC$ and mark-up P^*/MC will be lower. In a perfectly competitive market, with perfect elasticity of demand, there is no mark-up and firms charge a price equal to the marginal cost and make no economic profit.

(2) Cost-plus or Mark-up Pricing

Management accounting studies report that cost-plus pricing is very prevalent among firms in a wide range of industries. For example, Drury and Tayles (2006) report that 60% of 112 firms surveyed in the U.K. use it in some forms to set prices. Many firms practise an approximate form of cost-plus or mark-up pricing. Typically, they use a rule of thumb rather than a precise calculation of marginal cost and elasticity of demand. For example, the marginal cost MC is Equation (4.1) is replaced by an average cost and the elasticity of demand is estimated based on recent sales data. See Lucas (2003) and Petersen and Lewis (1999, p. 427).

Although cost-plus pricing through a rule of thumb is sometimes criticized as simplistic, it is also regarded as a practice which enables managers to determine optimal prices efficiently in a practical setting. See for example Petersen and Lewis (1999, p. 429), Hirschey and Pappas (1996, p. 640),

Lavoie (2001) and Lee (1999, p. 201). It is expensive and difficult to collect information on demand schedules, marginal costs and marginal revenues, so cost-plus pricing represents a pragmatic proxy for marginal analysis.

Turning specifically to the insurance industry, there appears to be no published work on marginal cost and revenue analysis by insurers. It appears that insurers have only limited information about demand schedules and uncertain knowledge of consumers. Indeed, Warthen and Sommer (1996) refer to the elasticity of demand for individual insurers as a variable that cannot be estimated. Monopolistic competition means that firms are interdependent and will adapt their strategies in response to the market environment and to each other, so the demand function faced by any one firm is highly variable and not easy to measure. A form of cost-plus pricing based on Equation (4.1) is therefore more practical than a comprehensive marginal analysis of revenue and cost. Indeed, Booth et al. (2005, p. 404) suggest that the insurance premium calculation consists of a two-stage process, the first being a costing exercise with actuarial input and the second being a pricing stage which involves a commercial adjustment to the cost.

(3) Pricing by Insurers

We assume in our model that insurers price their policies using a modified form of Equation (4.1). First, in the absence of information on incremental cost and revenue, insurers use the average cost of a policy. Long-run average cost and marginal cost are often not too different (Petersen and Lewis, 1999, p. 429). Much of the cost is variable rather than fixed (Feldblum, 2001). In the case of insurance business, the expected loss on a contract

is the variable cost, and this is the key element in premium considera-
tion. Second, the average cost is based on established actuarial premium
principles (Kaas et al., 2008). Detailed premium principles and proper-
ties are included in the Appendix. In Equation (4.1), MC is therefore
replaced by $\widetilde{P}_{it} + \alpha F_{it}$, where $\widetilde{P}_{it}$ is the pure premium based on the ex-
pected claim cost faced by insurer i at time t, and αF_{it} is a risk loading or
safety loading ($\alpha > 0$ is a loading factor and F_{it} is the standard deviation
of claims experienced by insurer i at time t). This is consistent with the
Standard Deviation Premium Principle, which states that a risk-averse in-
surer requires a loading to compensate for the uncertainty of its risk-taking
business, with risk being measured using the standard deviation. Third,
we define the log-mark-up m_{it} employed by insurer i at time t through the
following equation:

$$P_{it} = \left(\widetilde{P}_{it} + \alpha F_{it} \right) e^{m_{it}} \tag{4.2}$$

Since the exponential function has the following expansion based on the
Taylor Approximation:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... \quad \forall x$$

so if x is small enough, the approximation is:

$$e^x = 1 + x$$

Therefore, Equation (4.1) can be approximated to:

$$
\begin{aligned}
P^* &= MC \times (1 + \epsilon^{-1})^{-1} \\
&= MC \times (e^{-\epsilon^{-1}}) \\
&= \left(\widetilde{P_{it}} + \alpha F_{it} \right) e^{m_{it}}
\end{aligned}
$$

Where $m_{it} = -\epsilon_{it}^{-1}$ for insurer i at time t. To evaluate the optimal mark-up, the management of an insurance company must estimate price-elasticity of demand ϵ_{it}. This varies (a) at different price points, (b) in different segments of the market where customers have different product preferences, (c) at different stages of the underwriting cycle. Insurers cannot therefore estimate their price-elasticity of demand precisely (Warthen and Sommer, 1996). Note that individual insurers will experience different price sensitivity in different segments of the market and at different times. This is due to the characteristics of monopolistic competition (Davies and Cline, 2005). This suggests that each individual insurer locally has the power to affect the price in its targeted customer groups and market segments, but any one of these local-level impacts is not enough to influence the whole market. In the case of motor insurance, the market demand as a whole is relatively inelastic relative to price, but the local demand is very elastic for an individual insurer (Feldblum, 2001).

Following the general example given by Hirschey and Pappas (1996, p. 639), we assume that insurer i calculates a crude elasticity of demand at

time t using an arc-elasticity over the previous two periods:

$$\widehat{\epsilon}_{it} = \frac{(Q_{i,t-1} - Q_{i,t-2})/(Q_{i,t-1} + Q_{i,t-2})}{(P_{i,t-1} - P_{i,t-2})/(P_{i,t-1} + P_{i,t-2})} \tag{4.3}$$

where $Q_{i,t}$ denotes the number of insurance policies sold by insurer i at time t and $P_{i,t}$ denotes the price of these policies. (The above approximates the point elasticity of demand $\epsilon = \frac{\partial Q/Q}{\partial P/P}$ where Q is quantity demanded and P is price.)

Comparing Equations (4.1) and (4.2), and using Equation (4.3), a first-order approximation suggests a crude estimate of the mark-up as follows:

$$\widehat{m}_{it} \approx -\frac{1}{\widehat{\epsilon}_{it}} = -\frac{(P_{i,t-1} - P_{i,t-2})/(Q_{i,t-1} + Q_{i,t-2})}{(Q_{i,t-1} - Q_{i,t-2})/(P_{i,t-1} + P_{i,t-2})} \tag{4.4}$$

This approximation is based on $-\widehat{\epsilon}_{it}^{-1}$ being typically small in a highly competitive market where individual insurers face a gently sloping, and nearly horizontal, demand curve. Finally, insurer i estimates the optimal mark-up by updating its previous estimate using a weighted average[2]:

$$m_{it} = \beta \widehat{m}_{it} + (1 - \beta)m_{i,t-1} \tag{4.5}$$

where $0 < \beta \leq 1$. It is noteworthy that exponential weighted moving averages such as in Equation (4.5) occur in macroeconomic models of the business cycle with variable mark-ups and sticky prices (Rotemberg and

---

[2]In the (rare) circumstances where $Q_{i,t-1} = Q_{i,t-2}$, which would lead to an infinite value of $\widehat{m}_{it}$ in Equation (4.4), one may calculate the arc-elasticity using values at time $t - 3$, but this would lead to outdated estimates based on different market conditions and possibly distant price points. Instead, we assume that the insurer does not update its estimate in Equation (4.5) and sets $\widehat{m}_{it} = m_{i,t-1}$.

Woodford, 1999).

4.3.3 Customer's total cost of purchasing an insurance

(1) Nature of Customers

In line with the simple insurance market models of Taylor (1986, 1987, 2009), we use the following specification for customers. Each customer selects the policy at the beginning of each time period. The terms and conditions of one policy are the same for every customer. These include: a fixed contract period, identical inception and renewal times (e.g. at the beginning of every year), two parties only (i.e. the customer and insurer without intermediaries such as brokers involved), etc. Therefore, all procedures that customers need to perform in our simulation can be summarized as follows: (1) At the beginning of each time period, customers search and select one insurance policy according to their selection rule (i.e. the lowest available total cost); (2) Customers immediately pay the premium to their insurers at the beginning of each time period; (3) Their risks are randomly generated by some common claim distributions; (4) Their claims are paid by the selected insurers at the end of each time period, at which point customers start to search and select again for the next period after insurers update their prices.

We make some further assumptions about customers. Customers have no connection with each other (Independent); The risks of customers are identically distributed (Identical); Customers care about the cost of pur-

chasing insurance in the next period only, and disregard future periods (Myopic behavior); Customers only use insurance to exchange the risk over a fixed period and there is no other motivation or negotiation process between sellers and buyers (Passive); There is no moral hazard or fraud (Honest behavior); Customers take all opportunities to collect information, so they know all prices in the market (Diligent).

(2) Location and Distance

A combination of Economic Location models and ABM simulations provides a good way to understand the dynamics of monopolistic competition. Examples of Economic Location models include: two competing shops in a linear street of fixed length (Hotelling, 1929), several competing firms in a circle city (Salop, 1979), and hybrid variations. In our base case analysis, we use a one-dimensional circle city that is similar to Salop (1979) as illustrated in Figure 4.2. The circular city model is also used by Ladley (2013) in his inter-bank lending model to define relationships between banks and households. It assumes a large number of consumers who are identical (except for their locations) and who are uniformly distributed along the larger circle in Figure 4.2. Insurers (shown as smaller circles on Figure 4.2) are also evenly located on the larger circular space. We also assume that neither customers nor insurers change their locations, Therefore, our analysis focuses on price competition, and location plays a role of segmenting the whole market into local competition (Salop, 1979). We define Δ_{ij} as the shortest distance[3] between insurer i and customer j along the circumfer-

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[3]When implementing the distance on MATLAB, we map the circle city to a fixed line with 1 unit length and the shortest distance Δ_{ij} between an insurer i at location

ence.
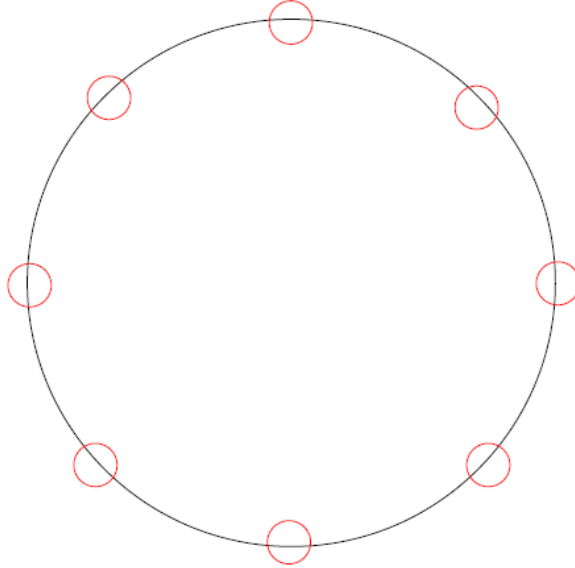
Figure 4.2: Economic location model: a circle city

(3) Customer's Total Cost Function

We simplify our customers' decision rules by assuming that they minimize a total cost function. A more elaborate decision rule would involve a utility function, such as the one used by Schlesinger and Schulenburg (1991). Customer j has the following total cost function of purchasing an insurance from an insurer i at time t of period $(t, t+1)$:

$$TC_{ij,t} = P_{it} + \gamma \Delta_{ij} \tag{4.7}$$

A and one customer j at location B is:

$$\Delta_{ij} = min\left(abs(A - B), abs(1 - A + B), abs(1 + A - B)\right) \tag{4.6}$$

where $abs(.)$ is an absolute value MATLAB function.

where γ is monetary cost per unit of distance. This is similar to utility loss per unit of distance in Schlesinger and Schulenburg (1991), signifying a reduction in utility from having to buy a policy which does not satisfy the consumer's ideal product characteristics. The product-characteristics distance Δ_{ij} affects the ability of insurer i to attract the customer j. A small Δ_{ij} leads to have a low cost of purchasing insurance from insurer i for the customer j at time t of each period. This is similar to the inter-bank lending model of Ladley (2013) where depositors select a bank to place their funds. He argues that the distance between a bank and a potential depositor affects the bank's attractiveness to the depositor. Ladley (2013) uses a linear distance to model transaction costs, although he also tests alternative functions, and finds that these generate few qualitative differences.

4.3.4 Balancing market supply and demand

Every customer ranks all insurers from the lowest total cost to the highest. A customer chooses the insurer with the lowest total cost, from the customer's point of view, unless this insurer has reached full capacity, in which case the customer chooses an insurer with the next lower total cost. An insurer reaches full capacity if it uses all of its existing capital to support its insurance business. We assume that the claims of customers are i.i.d, therefore insurers select the potential customers by fulfilling their individual capacity. Once they reach their full capacities, they stop taking other customers. An insurer's total capacity in each time period depends on its existing level of capital and the required solvency ratio, and it de-

fines the maximum total gross premiums that an insurer can take in each time period. To avoid insurers' selection biases, we distribute customers randomly in the system and insurers select interested customers randomly until they reach their full capacity. For example, if an insurer can only take up to 10 customers, but there are 20 customers waiting in a queue that is represented by 20 cells in a column, the insurer will select the first ten cells. Since the 20 customers were randomly allocated to the cells in the column, the insurer actually selects the 10 customers randomly (i.e.: avoided selection bias).

4.3.5 Claim loss experience

Following Taylor (2009), we generate a simple claim loss experience that is stochastic. This loss experience may include catastrophe (CAT) events to understand the impact of such events on market dynamics (Taylor, 2009), but we focus on the normal loss situations in our base case analysis.

Let

- $\mathbb{L}_{(M \times T)}$: a random loss matrix that contains the total loss of each of M individual customers in T time periods[4].

- $\mathbb{B}_{(M \times T)}$: a random claim frequency matrix that contains non-CAT claim count for each of M individual customers in T time periods, where each element of the matrix follows an independent Bernoulli distribution. This simplification means that each customer can be in

---

[4]Losses for each customer in past periods, i.e. before time 0, are also simulated, so that insurers have some claims data on which to base their prices as from time 0.

only one of two possible states in each time period: either at least one claim is made or there is no claim at all. Customers may have several claims at different stages during one time period, but insurers may sum up these claims into one cell for this particular period.

- $\mathbb{G}_{(M \times T)}$: a random claim severity matrix that contains non-CAT claim amount for each of M individual customers in T time periods, where each element of the matrix follows a Gamma distribution with two parameters defining its mean μ_G and coefficient of variation CoV_G. Therefore the shape parameter of the Gamma distribution is $1/(CoV_G^2)$ and the scale parameter is $\mu_G(CoV_G^2)$.

Then the claim loss experience for all time periods is:

$$\mathbb{L}_{(M \times T)} = \mathbb{B}_{(M \times T)} \circ \mathbb{G}_{(M \times T)} \tag{4.8}$$

where $(B \circ G)_{i,j} = (B)_{i,j} \times (G)_{i,j}$ is the Hadamard product that takes two matrices of the same dimensions, and produces another matrix where each element (i, j) is the product of elements (i, j) of the original two matrices. Each cell in $\mathbb{L}_{(M \times T)}$ represents the total loss for an individual customer by a particular time.

4.3.6 Individual underwriting results

In line with the equation for defining underwriting result of an insurance company (Daykin et al., 1994, p. 327), an individual insurer's underwriting result is traditionally measured as the excess of earned premiums in the year over incurred claims and expenses. However, we assume zero expense

in our model and also ignore the interest rate (investment return) in a single period. Therefore, our underwriting result for insurer i at time t is:

$$\pi_{it} = GWP_{it} - Claim_{it} \tag{4.9}$$

where:

- π_{it}: Underwriting profit or loss;

- GWP_{it}: Gross written premium (which is '$Total Exposure \times Insurer Price$');

- $Claim_{it}$: Total claim payments (which is the sum of '$Customer Selection \times Individual Loss$');

Furthermore, following the balance sheet results (Taylor, 2009) and the basic equation (Daykin et al., 1994, p. 327), we have the following accumulation process for capital K_{it}, insurer $i \in \mathcal{I}$ has capital $K_{i,t+1}$ at the end of an underwriting period $(t, t+1)$ as:

$$K_{i,t+1} = K_{it} + \pi_{it} \tag{4.10}$$

4.3.7 Overall market performance

We measure the aggregate market performance in a ratio form. The most commonly used ratios are:

$$\text{Loss ratio} = \frac{\sum_{i=1}^{I} Claim_{it}}{\sum_{i=1}^{I} GWP_{it}} \tag{4.11}$$

and

$$\text{Combined ratio} = \frac{\sum_{i=1}^{I} Claim_{it} + \sum_{i=1}^{I} Expense_{it}}{\sum_{i=1}^{I} GWP_{it}} \tag{4.12}$$

Since we ignore both expense and investment return in our model, the above two ratios are identical in our simulation. However, we need to take extra care when we use real market data to calibrate and validate our model, since these real market performance ratios contain both expense and investment loadings.

4.3.8   Re-pricing

At the end of each time period, insurers collect information from the market to update their prices. They use such information as the mean and standard deviation of their own total portfolio loss to update their safety loading in Equation (4.2), the quantity of insurance sold to update their mark-up adjustment in Equation (4.5).

The *theoretical pure risk premium* is:

$$P_0 = \lambda\mu \tag{4.13}$$

Where λ is the mean of claim frequency and μ is the average claim amount. In reality, insurers estimate the pure premium from their own experience and the market average based on Credibility Theory (Kaas et al., 2008, p. 203-227). We employ a simple example of the *practical pure risk premium*:

$$\widetilde{P}_{it} = z\bar{X}_{it} + (1-z)\lambda'\mu' \tag{4.14}$$

where

$$\bar{X}_{it} = wX_{i,t-1} + (1-w)\bar{X}_{i,t-1} \tag{4.15}$$

In the above:

- λ: expected claim frequency per unit risk exposure;

- μ: expected claim size or amount;

- λ': market collected average claim frequency per unit risk exposure;

- μ': market collected average claim size or amount;

- z: credibility factor of insurer experience, $z \in [0, 1]$;

- $\bar{X}_{it}$: insurer i's past weighted average claims experience at time t;

- w: weight on the loss experience of the previous period, $w \in [0, 1]$;

- $X_{i,t-1}$: insurer i's average realized claim at time $(t-1)$.

The credibility factor z explains the credibility or acceptability of an insurer's own experience. It depends on the size of the insurer's portfolio in theory. The weight w determines the importance of the experience of the previous period against the past claim history, given that more recent data are normally more relevant to the decision of current risk premium.

In our case, both individual insurers' own claim experience and market average claim data are generated from the simulations, therefore we can calculate the prices or estimate the risks based on these generated data. However, in reality, as Nielsen et al. (2012a, p. 2-3) suggest when data is scarce or unreliable, insurers need to use different approaches to include their own prior knowledge into their estimations. Prior knowledge can come in many forms (or guises). One of the most important sources is

external data that are related to the insurer's business, such as market average data, or data about their main competitors (Nielsen et al., 2012b; Guillen et al., 2008a; Gustafsson et al., 2007). Other prior knowledge could be on parametric shapes that have shown themselves to be relevant and helpful in related studies, or even nonparametric shapes taken from sources other than data at hand (Buch-Kromann et al., 2007). In particular, when insurers try to quantify the operational risk of their business, they have to acquire as much prior knowledge as possible and then adjust this with their own real observations (Bolance et al., 2013). Furthermore, suggestions from industry experts are also very important (Clemen and Winkler, 1999).

4.4 Data

We use actual data from the UK non-life insurance market to calibrate and validate our model. The data is collected from the Association of British Insurers (ABI) database between year 1983 and 2011. This includes loss ratios and combined loss ratios, on an annual basis, for UK motor insurance, UK property insurance, and all of UK non-life insurance business.

Figure 4.3 shows both UK motor insurance market loss ratios and combined ratios from 1983 to 2011. The two series of ratios follow a similar cyclic pattern, since the main difference between them is the expense, which is stable over time as a proportion of premium (i.e. about 20-25 percent).
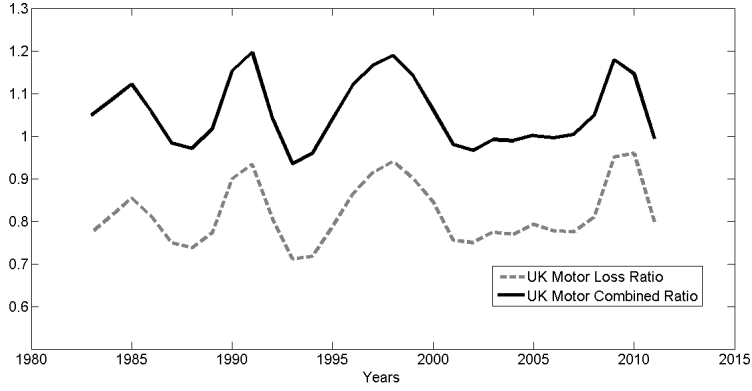
Figure 4.3: Actual market data: UK motor sector ratios (Source: ABI)

These market performance ratios are calculated as follows:

$$RealMarketLossRatio \ = \ \frac{Claims}{Premiums}$$

$$= \ \frac{ActualClaimCost}{ExpectedClaimCost + ExpenseLoading}$$

and

$$RealMarketCombinedRatio \ = \ \frac{Claims + Expense}{Premiums}$$

$$= \ \frac{ActualClaimCost + ActualExpense}{ExpectedClaimCost + ExpenseLoading}$$

Table 4.1 shows basic statistics for the UK motor insurance market. We find that: (1) the mean of loss ratios is below one (i.e. 0.8190) because the premium includes both the expected claims to match the actual claims and expenses loadings to cover normal business operations. (2) the mean of combined ratios is above one (i.e. 1.0550) because insurers can earn

investment return on the premium which they collect in advance of claim payments. Therefore it is possible for insurers to have the sum of expected claim and expected expense to be less than the sum of actual claim and actual expense, since the investment return may cover the losses in their underwriting results that is not included in the combined ratios.

Table 4.1: Basic statistics of UK motor sector (Year: 1983-2011)

	Mean	StandardDeviation	CoefficientOfVariation
Loss Ratio	0.8190	0.0728	0.0889
Combined Ratio	1.0550	0.0787	0.0746

We ignore both expenses and investment incomes in our model simulation. Therefore, our simulated loss ratios are the same as simulated combined ratios which is close to one, rather than being below one (as actual UK motor market loss ratio) or above one (as actual UK motor market combined ratio). This is because without taking consideration of expense and investment loadings, the expected loss (i.e. premium) should be close to the actual loss (i.e. claim). As Daykin et al. (1994) mention, both the combined ratio and the loss ratio should drift around one since the insurance market is competitive overall. If it is below one, it will attract more competition in the long run. If it is above one, most insurers will become insolvent in the long run. For this reason, when we calibrate or validate our model, we focus on the Standard Deviation and Coefficient of Variation of the actual market data, and keep the mean of our simulated ratios close to one.

4.5 Parameter Estimation

4.5.1 Summary of parameters

This subsection summarizes parameters in our base case model and their initial values appear in Table 4.2.

- N: Number of insurers;

- M: Number of customers;

- T: Time periods of a single simulation;

- α: Risk (or safety) loading factor in the price Equation (4.2);

- β: Weight of current market mark-up in the overall mark-up Equation (4.5);

- γ: Weight of location element in the customer's total cost function (4.7);

- b: Bernoulli distribution parameter in Equation (4.8);

- μ_G: Mean of Gamma distribution in Equation (4.8);

- CoV_G: Coefficient of variation of Gamma distribution in Equation (4.8);

- w: Weight of past claim experience in the claim estimation (4.15);

- z: Credibility of own experience in the pure risk premium function (4.14);

- K_0: Initial capital for every insurer in the capital accumulation Equation (4.10).

Table 4.2: Initial values of base case parameters

Parameter	Initial Value	Explanation
N	20	Balance simulation time and real-world representation
M	1000	Balance simulation time and real-world representation
T	1000	Balance simulation time and stability of results
α	0.001	Estimation and calibration from real-market data
β	0.3	Estimation and calibration from real-market data
γ	0.05	Estimation and calibration from real-market data
b	1	Every customer has claim but different amount
μ_G	100	Keep the average amount of random claims to be 100
CoV_G	0.1	Keep variance low to focus analysis on price
w	0.2	Use a rule of thumb (20/80) to simplify this process
z	0.2	Use a rule of thumb (20/80) to simplify this process
K_0	100000	Avoid insolvency during simulations

4.5.2 Initialization of parameter values

The parameters α, β and γ in Table 4.2 are the key parameters in our model, and their estimation is discussed in the next section. Here we briefly discuss how we chose the remaining parameters, whose values we found not too significantly affect our simulation results and conclusions.

We follow three steps:

Step 1 Reasonableness check: We firstly test different values of each parameter in a reasonable range, as observed from the real-world insurance market, e.g.: $N \in [2, 100]$, $M \in [100, 1000000]$, $T \in [50, 10000]$, and $K_0 > 0$. We continually narrow the range until we reach a level where there is no material difference in the simulation results; we then try to balance the simulation time and real-world representation of these values. This step is applied to set the following parameters in Table 4.2: N, M, T, and K_0.

Step 2 Justification from existing literature: Some parameters may not play a significant role in our main ABM analysis but necessary for the simulation to run, we try to assume initial values that are close to existing literature. For example, claim distributions can be generated by many different models, our implementation is in line with Taylor (2009) since this paper has a similar market structure and aims to understand a similar research question.

Step 3 Real-world rules of thumb: The initial values of other less important parameters are assumed by applying some real-world practical rules of thumb, such as market practitioners' feedback, industry experts' opinion, etc. For example, we use some common rules such as 20/80 (or 50/50) on weight w and credibility z of current claim experience in insurers' price estimation. Furthermore, an initial sensitivity analysis shows that the changes of these parameters have no significant impact on our key simulation results. This is because claim volatility is low since our focus is on insurers' pricing dynamics and our aim was to keep claim stable. Therefore, the stable claim samples lead insurers' claim experience to be similar to overall market experience over time, which means that the weight w and credibility z do not have a sizable effect on the model results.

4.5.3 Estimation of key parameters

The three key parameters in our model are α, β and γ. They are similar to the "base case dynamical parameters" of Taylor (2009). We calibrate them based on real market data in this subsection, then we perform further

sensitivity tests to determine how they affect insurance cycles. They are:

- Safety (Risk) Loading factor α in Equation (4.2): This governs the mean of loss ratio. We set α to normalize mean loss ratio at 100%. In theory as mentioned in Daykin et al. (1994), the market loss ratio should drift around one since insurance market is overall competitive. If it is below one, it will attract more competition in the long run. If it is above one, most insurers will become insolvent in the long run.

- Mark-up Weight β in Equation (4.5): It manages the insurer's reaction to current market environment. We perform a brute-force grid search with refinement to equate standard deviation and lag-1 auto-correlation with real data.

- Location Weight γ in the customer's total cost function (4.7): It controls the role of an insurer's non-price competitive advantage in a customer's purchase decision. We also apply a brute-force grid search of a combination of location and mark-up weights with refinement equate standard deviation and lag-1 auto-correlation with real data.

More details appear below.

1. Parameter estimation for the key parameters α, β and γ proceeds by a version of the method of moments. Because our data is autocorrelated, we would like to solve for α, β, γ by equating the following statistics calculated from the simulated data to the corresponding statistics from the UK sample data: (1) $\mu_{Sim} = \mu_{Real}$; (2) $\sigma_{Sim} = \sigma_{Real}$; and (3) $\rho_{Sim} = \rho_{Real}$, where μ_{Sim}, σ_{Sim} and ρ_{Sim} represent the mean, standard deviation and lag-1 autocorrelation from the

simulated sample path and, whereas μ_{Real}, σ_{Real} and ρ_{Real} represent the respective statistics from the sample time series data.

2. In theory, we could use a grid search procedure whereby we try a large number of combinations of values for (α, β, γ), carry out a large number of years simulation, calculate the mean μ_{Sim}, standard deviation σ_{Sim} and lag-1 autocorrelation ρ_{Sim}, minimize a weighted penalty function such as $abs(\mu_{Sim} - \mu_{Real}) + w_1 \times abs(\sigma_{Sim} - \sigma_{Real}) + w_2 \times abs(\rho_{Sim} - \rho_{Real})$, and then refine the grid around the penalty-minimizing triplet (α, β, γ), and start the grid search again, etc.

3. In practice, we observe from our initial simulations that α controls the mean, but has little effect on the higher moments of the simulated time series. We can also see that this is true by construction in our model if we assume that the mark-up (or elasticity of demand) is serially independent of aggregate claims (from Equations (4.2), (4.4) and (4.5)). So we can choose α to satisfy condition (1) $\mu_{Sim} = \mu_{Real}$ above (if μ_{Real} is a reasonable mean loss ratio). However, both real market loss and combined ratios contain real world elements of expense and investment returns as explained in Section 4.4, and therefore they are slightly different from our simulated results since we ignore these elements. As a result, a more reasonable mean loss ratio is close to one given that the motor insurance market is competitive (Daykin et al., 1994). If it is below one, it will attract more competition in the long run. If it is above one, most insurers will become insolvent in the long run. Since Table 4.1 shows similar values of standard deviation for both real-world loss ratio and combined ratio, we can

chose to target any one of these two ratios as long as we target the mean of our simulated ratios to be close to one.

4. We then use a grid search procedure with refinement to estimate β and γ: we have found that there are several pairs of values of (β and γ) which satisfy condition (2) $\sigma_{Sim} = \sigma_{Real}$. It gives a surface when we plot standard deviation of simulated data vs. β and γ, and there is a line of values of (β and γ) corresponding to the sample standard deviation (We have found that we can always raise the standard deviation of claims process to achieve this.) Likewise, there is no unique pair of values of (β and γ) which satisfies condition (3) $\rho_{Sim} = \rho_{Real}$. One possible strategy is to choose (β and γ) which minimizes $w_1 \|\sigma_{Sim} - \sigma_{Real}\| + w_2 \|\rho_{Sim} - \rho_{Real}\|$, but the choice of w_1 and w_2 is then arbitrary. Instead, we choose the pair of values (β and γ) which satisfies condition (2) and we minimize $\|\rho_{Sim} - \rho_{Real}\|$, i.e. this gives us the closest approximation of condition (3). The valid key parameter combinations are summarized in Figure 4.4.

5. As a result, the initial values for the three key parameters are estimated by a brute-force grid search procedure with refinement, equating standard deviation and lag-1 auto-correlation with real data. The estimated parameter values for α, β and γ are given in Table 4.2.

Standard Deviation					
Beta	0	0.3	0.5	0.7	1
Gamma					
0	0.0038	0.0038	0.0038	0.0038	0.0038
0.03	0.0032	0.0714	0.3233	0.3919	0.5543
0.05	0.0032	0.0735	0.1758	0.2390	0.8876
0.07	0.0032	0.0781	0.1584	0.2110	0.2793
0.1	0.0032	0.0980	0.1682	0.1975	0.2587

Rho1					
Beta	0	0.3	0.5	0.7	1
Gamma					
0	0.0178	0.0178	0.0178	0.0178	0.0178
0.03	-0.0208	0.289	0.2088	0.061	-0.109
0.05	-0.0208	0.3914	0.1217	-0.0295	-0.0178
0.07	-0.0208	0.403	0.2128	0.0313	-0.1015
0.1	-0.0208	0.5474	0.0865	-0.0444	-0.0144

Figure 4.4: Key Parameters' Estimations: statistics (standard deviation and lag-1 autocorrelation) for loss ratio under different parameter combinations

4.6 Simulation Results

In this section, we show the main results of our model and illustrate the dynamics of the motor insurance market. The detailed discussions are included in the next several subsections, but a key summary follows:

- Claim experience and market premium: We generate a sample of non-CAT claims for each customer at each time period, so the overall market losses are stable over time. This simplification helps us to capture the dynamic cyclic behavior of price and loss ratio that are due to insurers' interactions, rather than being driven by major market losses.

- Loss ratio: Casual inspection reveals that there are cyclic patters over

both short and long periods in the market loss ratio, as well as in the loss ratios of individual insurers. However, individual insurers' results are more volatile than the overall market and different insurers have different cycle periods. This may reflect local market competition involves in the process.

- Mark-up pricing adjustment dynamics: The mark-up adjustment has a direct impact on the market competitive price of an insurer, it is worth looking at its pattern over time. Not surprisingly, both short and long term average results of current estimated mark-up such as in Equation (4.5) exhibit cyclic patterns.

- Cyclic market behavior: The autocorrelation function and partial autocorrelation function suggest that market combined ratios, premiums and profits over time follow an AR(2) process.

Simulated data collection: we follow a few steps to test and select our simulated data. (1) We use different claim samples that are generated from same distribution parameters to run different lengths of simulations (e.g. test 100, 500, 1000, 2500, and 5000 time periods). (2) We compare the average results of key variables (e.g.: market premium, loss ratio and profit, etc) from each simulation with same time periods, together with their statistics. (3) We find that a time length of 500 periods and more gives us stable results under different claim samples, since our claim distribution has a low coefficient of variation and therefore stable claim samples. This means that we can use a single simulation (e.g. a particular claim sample time series) over (at least) 500 time periods to analyze our model. (4) We chose 1000 time periods for our experiment, since it balances both

the simulation time and result stability. (5) We also discard the first 100 periods to avoid transient results at initialization, which means that all statistical analyses later are performed based on the 900 periods after this first 100 periods.

4.6.1   Claim experience and market premium

Figure 4.5 shows one simulation over 500 time periods of claim experience, insurers' offered prices, and premiums paid, all averaged over the market. According to rational expectations theory, insurers' prices should follow the claim pattern over time in response to new information about risk in a fully competitive market (Cummins and Outreville, 1987). However, in monopolistic competition, the interaction of insurers plays a key role in the determination of the market competitive price. Therefore, it is not surprising that prices in our simulation behave differently from claims, as may be seen in Figure 4.5. The thick dark line is the market average claim experience, which is stable over time. The thin dark line is the market average premium, which is cyclic and more volatile than claim. The dotted line is the average of all insurers' offered prices, which is higher than market average premium since customers prefer and select the lowest possible prices. This figure only provides an initial snapshot about the prices and claims in our simulation over time. More detailed results will be discussed later when we analyze the cyclic patterns of simulated loss ratios. There appears to be cycles in between 6 and 8 time periods, therefore we will show some results with a shorter length of timespan.
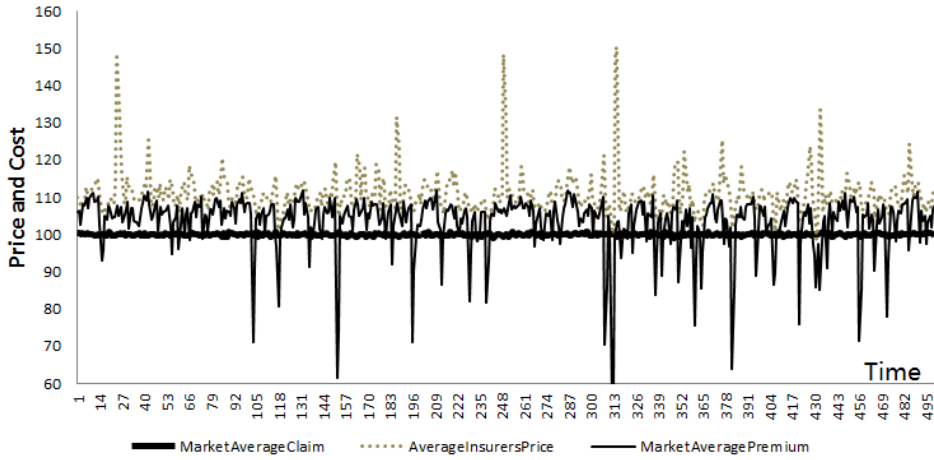
Figure 4.5: Claim experience: comparison of claim and premium 1

Figure 4.6 shows the histograms of market average losses (i.e. the left diagram) and market average premiums (i.e. the right diagram) in one simulation over 1000 time periods. It is apparent that they are distributed differently. Average claims in the market are approximately normally distributed over 1000 time periods as expected by the Central Limit Theorem, while the market average premiums are clearly non-normally distributed: it is easy for insurers to reduce prices to attract customers, but it is difficult to increase prices and retain customers to cover their poor claim experiences. Therefore, the distribution of market average premiums over 1000 time periods is negatively skewed.
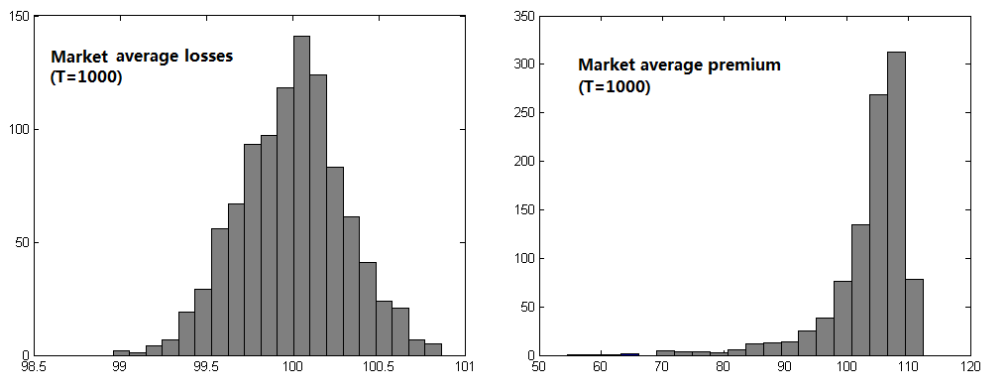
Figure 4.6: Claim experience: comparison of claim and premium 2

4.6.2 Loss ratio

Our simulated loss ratio is a ratio of claims paid to premiums collected at each time period (see Equation (4.11)), so it should follow a similar pattern to premiums assuming non-CAT claims. Figure 4.7 shows the market loss ratio, which depicts the performance of the overall market, over the first 50 time periods. A casual observation of the chart seems to indicate a cyclic pattern.
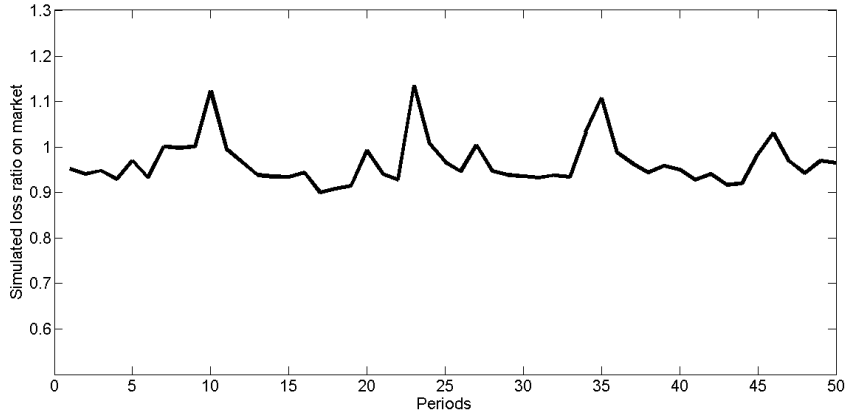
Figure 4.7: Loss ratio: market performance over 50 periods

Figure 4.8 shows the Autocorrelation (AcF: left diagram) and Partial autocorrelation (PAcF: right diagram) plots of the simulated market loss ratios over 1000 time periods. It is strongly suggestive of an AR(2) process. The AcF plot illustrates the correlations are diminishing and changing directions, they become insignificant after a few time lags. The PAcF plot shows both first and second lags are significant autocorrelated. An AR(2) process of loss ratio is consistent with existing insurance cycle literature (Cummins and Outreville, 1987).
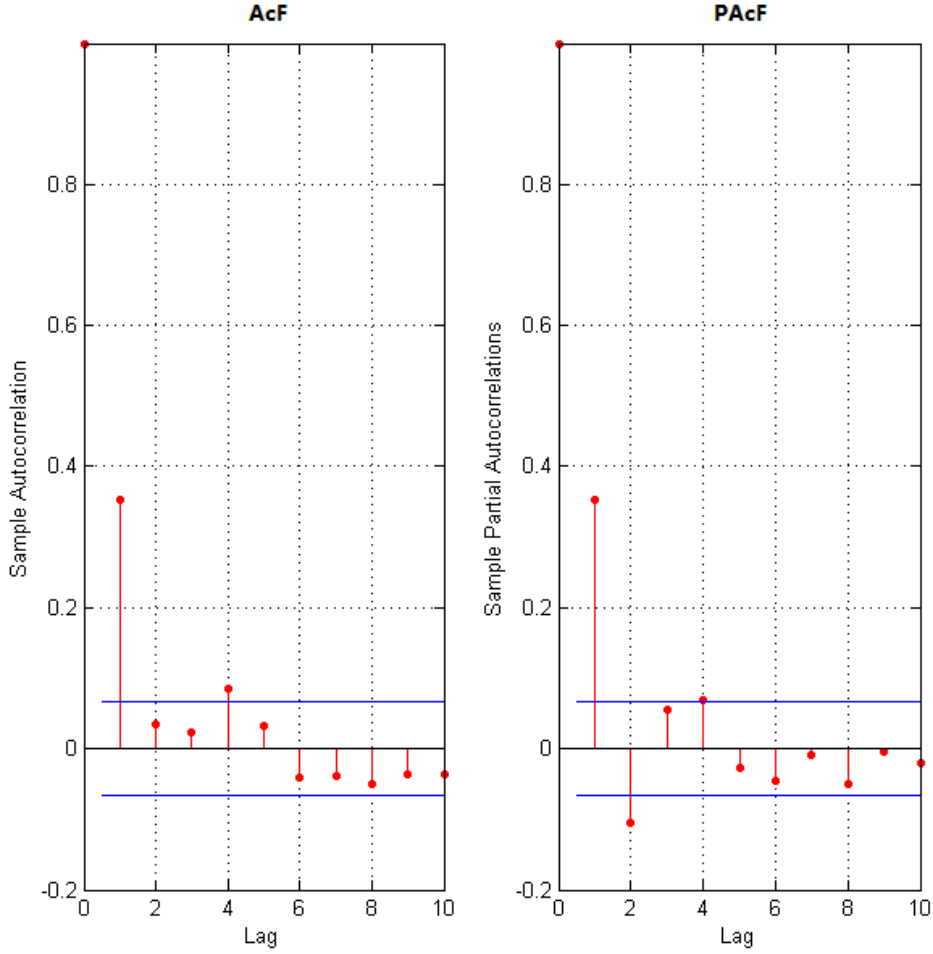
Figure 4.8: Loss ratio: AcF and PAcF correlograms

The market loss ratio is a combination of individual insurers' performance in the whole market, therefore we expect that individual insurers also experience cyclic performance. In Figure 4.9, we randomly select an individual insurer and show its loss ratio over 50 time periods. A cyclic pattern is immediately obvious. One might expect that an individual insurer's performance is more volatile than the market average performance.
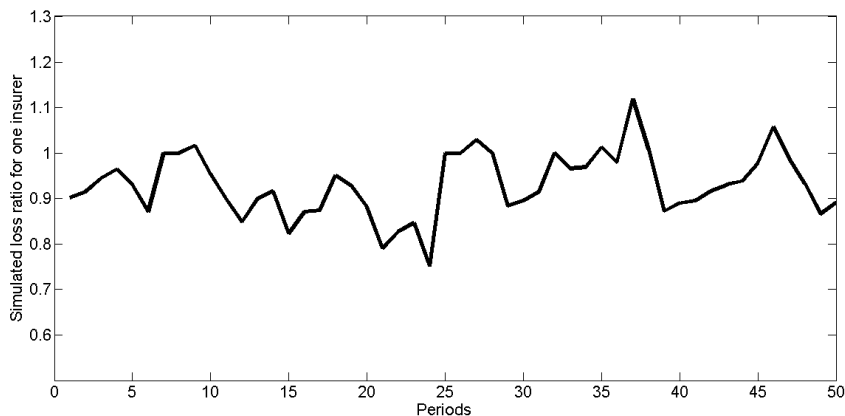
Figure 4.9: Loss ratio: an individual insurer over 50 periods

4.6.3 Mark-up pricing dynamics

Insurers' individual mark-up adjustments m_{it} in Equation (4.2) reflect the current situations of local market competition and insurers' individual elasticity of demand. When local demand is elastic, insurers are unable to increase their prices since customers will change their product selection quickly to other cheaper options. Therefore, a more competitive local environment will force insurers to adjust their mark-up more cautiously until the local competition becomes less intensive. As a result, although we cannot directly measure the local market competition intensity, the change and pattern of mark-up adjustments provide useful information about the changes in local market conditions and its impact on the price and insurers' performance.

Figure 4.10 shows the market average of all insurers' mark-up adjustments

over 50 time periods. Two conclusions may be drawn. (1) As discussed when Equation (4.4) was introduced, the values of mark-up are typically small in a highly competitive market where individual insurers face a gently sloping, and nearly horizontal, demand curve. (2) There is also a cyclic pattern in the values of mark-up over time, which reflects the cyclic nature of market competition.
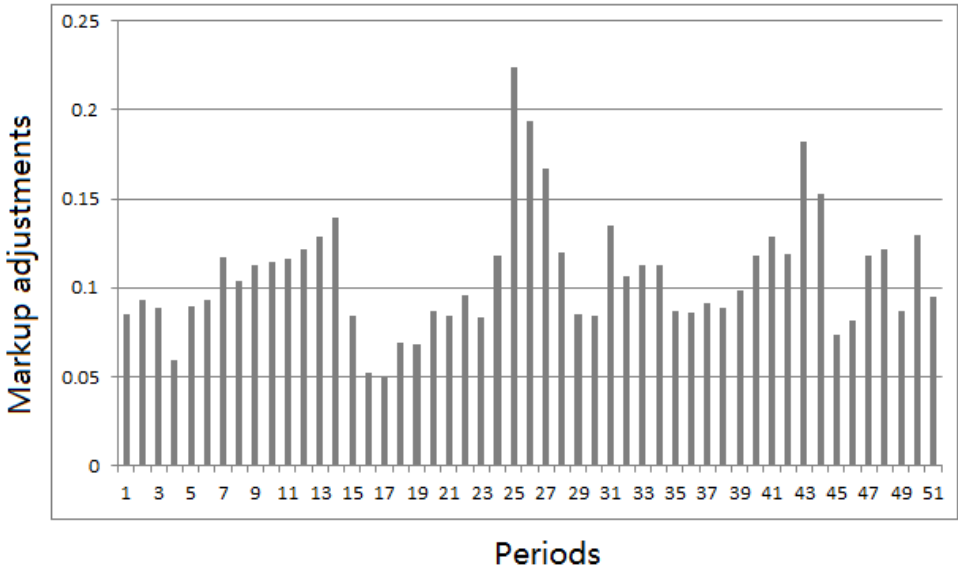
Figure 4.10: Mark-up adjustment: market average over 50 periods

Figure 4.11 shows the mark-up adjustment for a randomly chosen individual insurer, over 50 time periods. Similar to the comparison of market average and individual insurers' loss ratio as above, it also exhibits a cyclic pattern and a more volatile result than a market average. On rare occasions, individual markup adjustments appear to be negative. Negative numbers mean that the relationship between price and quantity is positive, i.e. price of policies and quantity of policies sold sometimes increase

at the same time. This appears counterintuitive, but it arises because the purchasing decision of customers not only depends on price, but also depends on location (i.e. local competitions among neighbors). When location dominates in the total cost function, the relationship between price and quantity of an insurer may be positive. For example, if an insurer increases its price at the same time as neighboring insurers are increasing their prices, then it is possible for the insurer to sell more policies if its price increase is less than the price increase of its neighbors.
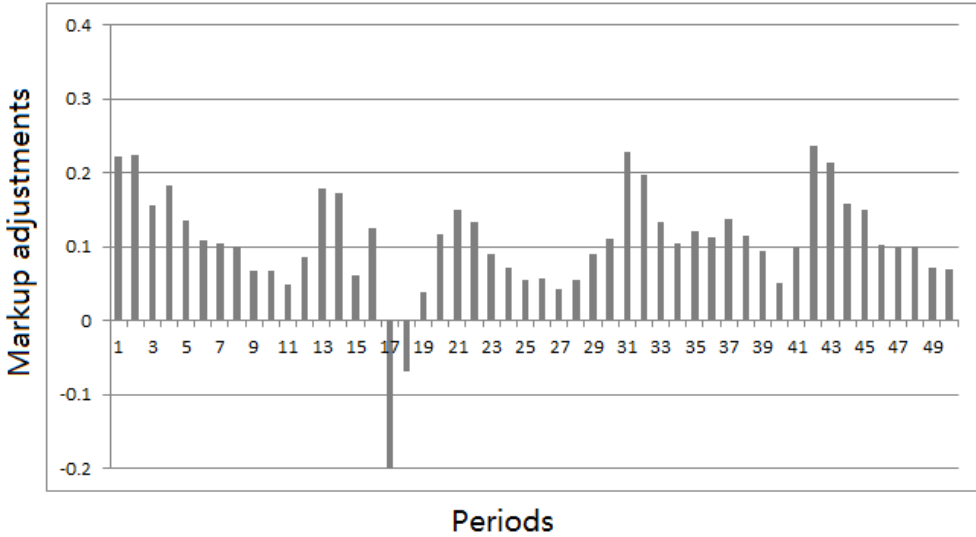
Figure 4.11: Mark-up adjustment: an invidual insurer over 50 periods

4.6.4 Cyclic behavior of market premium

One useful way to look at a cycle is by showing the correlograms of autocorrelation function (AcF) and partial autocorrelation function (PAcF). Figure 4.12 shows that the market premium has a similar AcF and PAcF as the loss ratio shown in Figure 4.7. Correlations are diminishing as lag

increases in the AcF plot; they change signs, and become insignificant after a few time lags. The PAcF plot shows that both first and second lags are significantly autocorrelated.
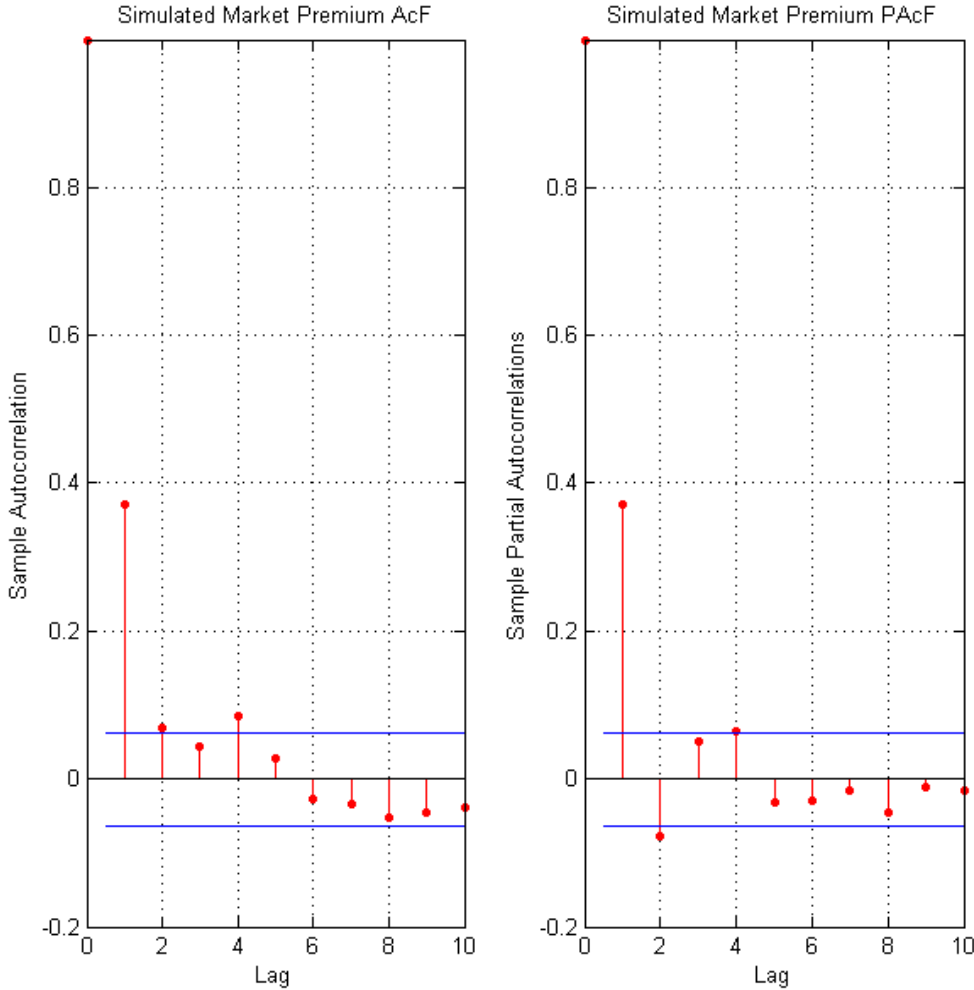
Figure 4.12: Market premium: AcF and PAcF correlograms

4.7 Verification and Validation

4.7.1 Basic model verification

Claim parameter

We test different combinations of parameters for claim distributions. There are nine different cases, as laid out in Figure 4.3. The key conditions for the generated claims are: (1) We try to set the mean of simulated claims equal to 100, since this keeps our insurers' price around 100. (2) We try to use the claim distribution and combine other model parameters so that the mean of simulated loss ratios is close to one, and the standard deviation and coefficient of variation are close to the statistics of our sample of UK insurance market data. Therefore, Scenario 9 as highlighted in Table 4.3 is the most suitable option.

	Claim count Binomial		Claim amount GAMMA			Simulated Claim		Simulated Loss Ratio			
	n	p	Mean	Stdev	CoV	Mean	ClaimStdev	Mean	Variance	Stdev	CoV
Case1	1	0.01	10000.00	1000.00	0.1	100	1000	0.943	0.1214	0.3484	0.3694
Case2	1	0.1	1000.00	100.00	0.1	100	302	0.967	0.0213	0.1459	0.1510
Case3	1	0.2	500.00	50.00	0.1	100	201	0.968	0.0111	0.1054	0.1089
Case4	1	0.3	333.33	33.33	0.1	100	154	0.968	0.0097	0.0985	0.1017
Case5	1	0.5	200.00	20.00	0.1	100	101	0.971	0.0078	0.0883	0.0909
Case6	1	0.8	125.00	12.50	0.1	100	51	0.976	0.0074	0.0860	0.0881
Case7	1	1	100.00	30.00	0.3	100	30	0.9793	0.007	0.0837	0.0854
Case8	1	1	100.00	20.00	0.2	100	20	0.9805	0.0067	0.0819	0.0835
Case9	1	1	100.00	10.00	0.1	100	10	0.988	0.0057	0.0755	0.0764

Table 4.3: Model verification: claim parameter illustration 1

To verify the implementation of our model, we should expect that a more volatile risk claim portfolio should have more volatile loss ratios since insurers have less certainty about claim experience and therefore price is relative unstable. At the same time, such high volatility should require a higher risk loading to compensate for the extra uncertainty, therefore

it leads to a lower loss ratio (i.e. the ratio of claim over premium). As illustrated in Figure 4.13, our results do verify this. When the standard deviation of simulated claims decreases from Case 1 to Case 9 as shown in Table 4.3, the mean of simulated loss ratio increases and the standard deviation decreases in Figure 4.13.
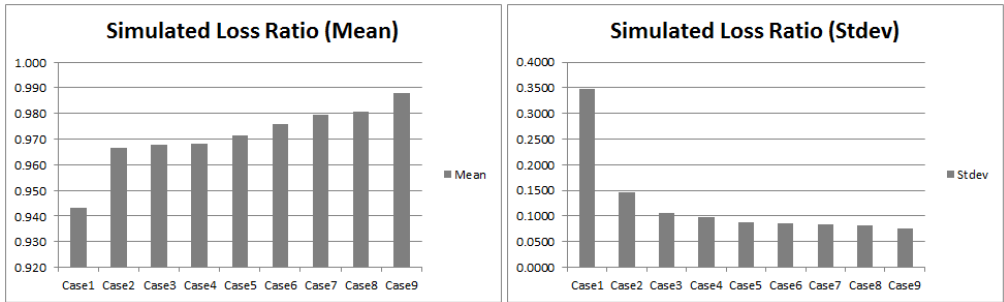
Figure 4.13: Model verification: claim parameter illustration 2

Figure 4.14 shows the coefficient of variation (ratio of standard deviation to mean) of the loss ratio. This increases as the volatility of the simulated claim increases, consistent with Figure 4.13.
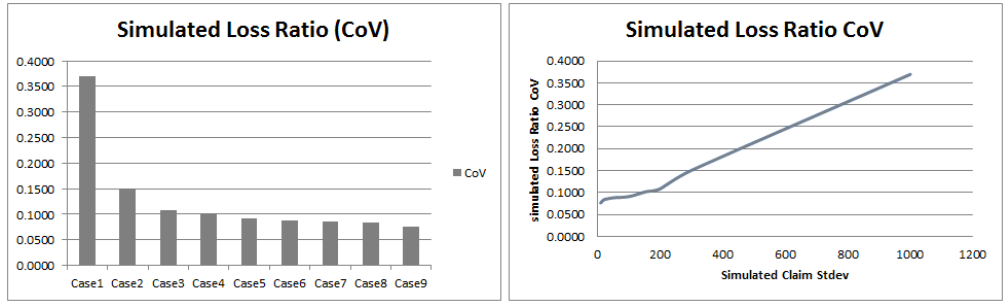
Figure 4.14: Model verification: claim parameter illustration 3

Testing varying market concentration

We should expect that an increase in the number of insurers in a fixed market will bring more competition, and this should therefore lead to lower premiums, and an increase in the combined ratio in the insurance market. Meanwhile, higher competition will force insurers to react to the environment more quickly and more aggressively than in a less competitive situation, therefore the volatility of insurers' performance and market combined ratio is also increased. The results from our model are indeed consistent with this, as shown in Figure 4.15.
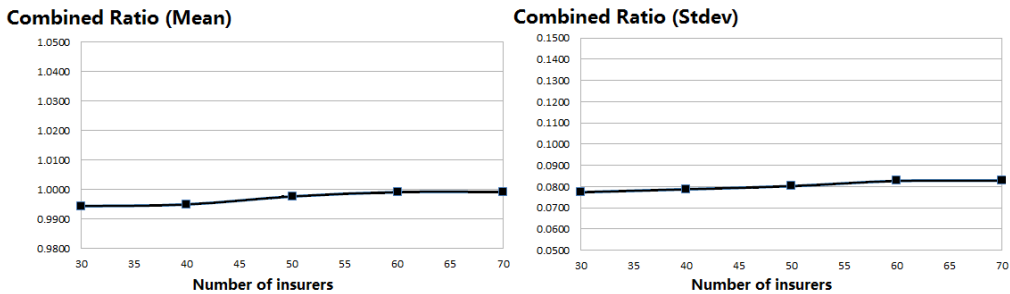
Figure 4.15: Model verification: number of insurers

Likewise, we should also expect that an increase in the number of customers in a fixed market will bring less competition (i.e. opposite to the increase in the number of insurers). Therefore, this should lead to higher premiums and the combined ratio will fall. Simultaneously, lower competition will allow insurers to react to the environment more slowly and less vigorously than in a more competitive situation. The volatility of insurers' performance and market combined ratio are expected to decrease. As illustrated in Figure 4.16, our model results verify this suggestion.

Combined Ratio (Mean)

Combined Ratio (Stdev)
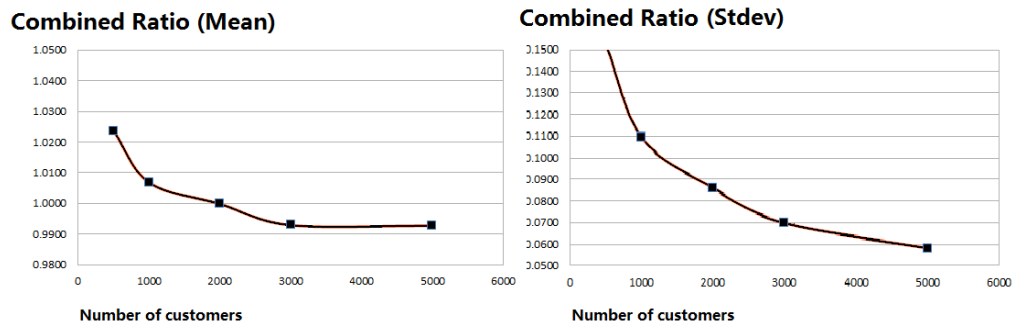
Figure 4.16: Model verification: number of customers

Distribution of firms' sizes

Axtell (1999) suggests that one way to verify an ABM market is to check that the distribution of firms' sizes follows a power law, i.e. a small percentage of firms in the market have the most market share. For example, the real UK non-life market also has such a distribution as shown in Table 4.4, around 90 percent of market share is occupied by the 10 largest insurers.

Table 4.4: Model verification: actual market firm size distribution

Business Class	Market share 2010	Market share 2011
Personal line top 5 insurers	63.7%	63.0%
Personal line top 10 insurers	85.3%	82.9%
Personal line top 20 insurers	98.5%	97.0%
Commercial line top 5 insurers	67.8%	65.7%
Commercial line top 10 insurers	90.4%	87.1%
Commercial line top 20 insurers	98.8%	97.2%
Whole industry top 5 insurers	66%	64%
Whole industry top 10 insurers	88%	85%
Whole industry top 20 insurers	99%	97%

In fact, if we simulate our model many times over a fixed time horizon

and collect all insurers' final capital accumulations in each simulation, the distribution of insurers' sizes also appears to decline very fast, in accordance with a power law. This is depicted in Figure 4.17, together with a log-log plot of a power law distribution.
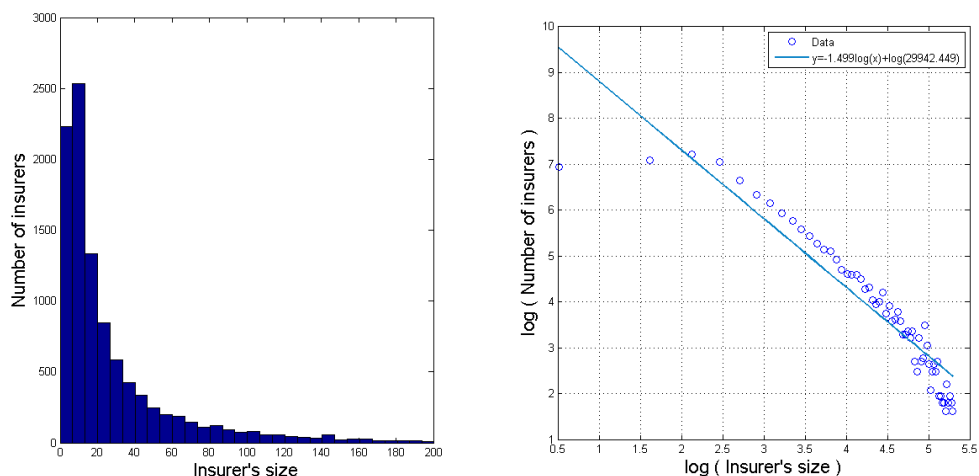
Figure 4.17: Model verification: simulated firm size distribution

4.7.2   Model validation

We use real-world data from ABI UK database to validate our model. The data period is from 1983 to 2011 (a total of 29 years of annual combined ratios). We collect three samples of UK data: (1) UK motor insurance annual combined ratios (labeled as UK Motor); (2) UK property insurance annual combined ratios (labeled as UK Property); and (3) Global results of insurers who are based in the UK (labeled as UK World). A snapshot of these actual UK market data samples are displayed in the bar plots of Figure 4.18.
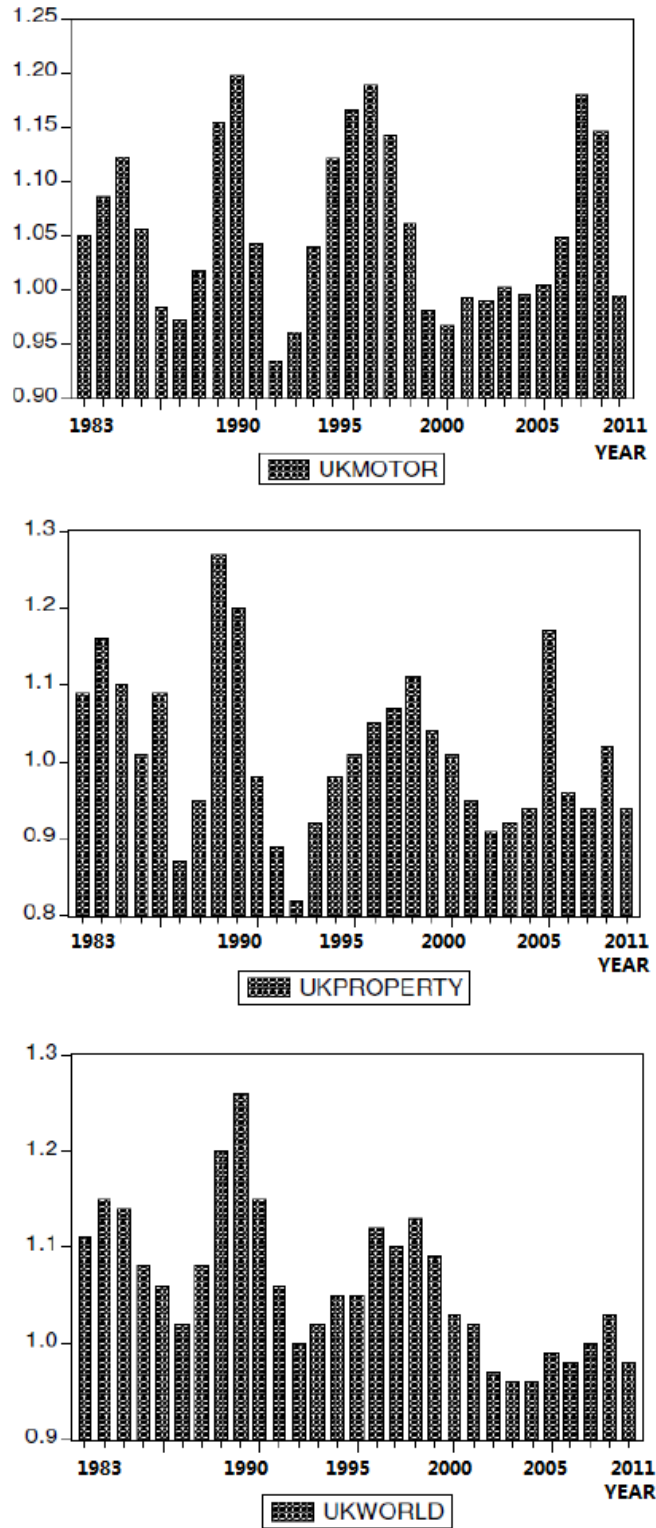
Figure 4.18: Actual UK market data (annual combined ratios 1983-2011)

To validate our model, we randomly simulate two samples of combined ratios from our model: (1) the first is 29 periods long time interval and is randomly cut from a simulation over a total of 1000 periods in order to compare with real world 29 years data; (2) the second is 900 periods long (we generate data over 1000 periods but discard the first 100 to avoid transient results at initialization). A snapshot of these simulated data samples are displayed in the bar plots of Figure 4.19.
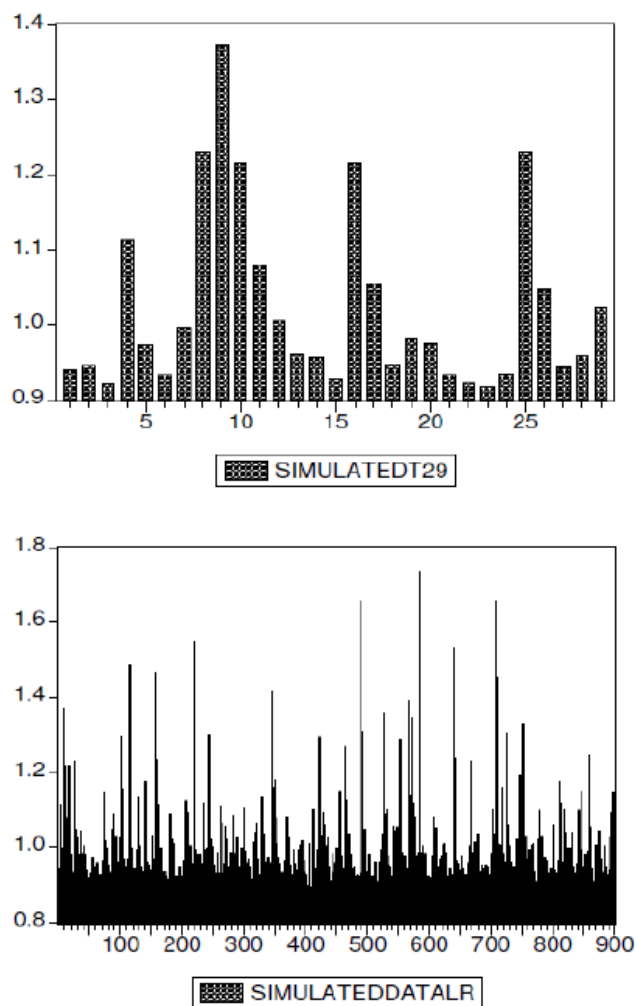
Figure 4.19: Model validation: simulated combined (or loss) ratios

Testing stationarity (ADF and PP tests)

We test the stationarity of the data series in the three sample datasets as well as in the two simulated datasets. The results are included in Table 4.5, where the simulated sample of 29 years is labeled as Simulated T29 and the simulated sample of 900 years is labeled as Simulated T900. All results based on the augmented DickeyFuller test (ADF) reject the null hypothesis of a unit root at the 90% level, most of them are at 95% level. Therefore, there is no statistical evidence to suggest that the data is non-stationary. More detailed statistics are included in the Appendix.

	ADF Statistic	ADF Critical Value	PP Statistic	PP Critical Value	Decision	Result
UK motor	-6.075116	1% => -3.6959	-2.610687	10% => -2.6242	ADF reject null hypothesis at 1%	Stationary (ADF)
UK property	-4.138845	1% => -3.6959	-3.331301	5% => -2.9705	Both tests reject null hypothesis at 5%	Stationary (ADF)
UK world	-2.904013	10% => -2.6265	-1.927247	10% => -2.6242	ADF reject null hypothesis at 10%	Stationary (ADF)
Simulated T29	-3.504021	5% => -2.9750	-3.142629	5% => -2.9705	Both tests reject null hypothesis at 5%	Stationary
Simulated T900	-11.65701	1% => -3.4403	-19.42269	1% => -3.4403	Both tests reject null hypothesis at 1%	Stationary

Table 4.5: Model validation: testing stationarity of data series

Identification of AR Process

The autocorrelations and partial autocorrelations in the sample datasets and in the simulated datasets indicate that an AR(2) process might be a good fit. The correlogram for the UK Motor sample data is shown in Table 4.6. The partial autocorrelations are significant at lags 1 and 2, but not at higher lags, which is suggestive of an AR(2) process.

```
Date: 08/06/13   Time: 22:36
Sample: 1 29                          Correlogram of UKMOTOR
Included observations: 29

    Autocorrelation    Partial Correlation     AC     PAC    Q-Stat   Prob

         |  ▇▇▇      |      |  ▇▇▇   |     1   0.566   0.566  10.285  0.001
         | ▇|        |      ▇▇▇|     |     2  -0.121  -0.649  10.771  0.005
       ▇▇|           |      | ▇|     |     3  -0.452   0.073  17.821  0.000
       ▇▇|           |      |▇ |     |     4  -0.408  -0.238  23.813  0.000
        ▇|           |      | ▇|     |     5  -0.155   0.064  24.708  0.000
         | |         |      | ▇|     |     6   0.045  -0.212  24.786  0.000
         | |         |      | ▇|     |     7   0.049  -0.104  24.886  0.001
         | |         |      |▇ |     |     8  -0.060  -0.184  25.038  0.002
         |▇|         |      | |       |    9  -0.110  -0.046  25.586  0.002
         |▇|         |      |▇ |     |    10  -0.083  -0.223  25.912  0.004
         | |         |      | |       |   11   0.014   0.026  25.922  0.007
         | ▇|        |      |▇ |     |    12   0.128  -0.107  26.792  0.008
         | ▇|        |      | |       |   13   0.165   0.014  28.329  0.008
         | ▇|        |      |▇ |     |    14   0.088  -0.152  28.788  0.011
         | ▇|        |      |▇ |     |    15  -0.075  -0.145  29.150  0.015
```

Table 4.6: UK motor correlogram: ACF and PACF

We compare different Autoregressive models and confirm AR(2) is the best process to model the data series, based on the Akaike information criterion (AIC). The value of AIC provides a measure of the relative quality of a statistical model based on a given set of data. It deals with the trade-off between the goodness-of-fit of the selected model and its complexity. Given a set of candidate models for the data, the preferred model is the one with the minimum AIC value. A summary of AIC results is included in Table 4.7. Disregarding the marginally more negative AIC value for AR(1) compared to AR(2) for the 29-year simulated data, an AR(2) model appears to be the best fit for all the actual and simulated datasets. More detailed statistics regarding the AR(2) model (e.g. Autocorrelation Function ACF and Partial Autocorrelation Function PACF, correlograms) are included in the Appendix.

Table 4.7: Model validation: AIC results of AR process

Data Sample	AR(1)	AR(2)	AR(3)
UK Motor	-2.5002	-3.2670	-3.2376
UK Property	-1.6851	-1.7530	-1.6475
UK World	-3.0910	-3.2885	-3.1694
Simulated T29	-1.4917	-1.4594	-1.3691
Simulated T900	-2.0103	-2.0162	-2.0157

Diagnostic check on residuals

Uncorrelatedness check (Residual Correlogram and Durbin-Watson statistic): After fitting an AR(2) process, we check for correlation in the residuals. Table 4.8 shows that there is no significant autocorrelation in the residuals in the case of the UK Motor data. The correlograms for the remaining four sample and simulated datasets may be found in the Appendix and also confirm uncorrelatedness of the residuals. The Durbin-Watson (DW) statistics from the AR(2) regression for all five datasets are collected in Table 4.9, where they are compared with the lower (d_L) and upper (d_U) critical values at 1% significance points from the DW significance tables.

Date: 08/07/13 Time: 09:20			Correlogram of Residuals
Sample: 3 29			
Included observations: 27			
Q-statistic probabilities adjusted for 2 ARMA term(s)			

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.177	0.177	0.9478	
		2	-0.175	-0.213	1.9052	
		3	0.034	0.118	1.9425	0.163
		4	-0.018	-0.097	1.9536	0.377
		5	-0.103	-0.052	2.3279	0.507
		6	-0.114	-0.115	2.8165	0.589
		7	-0.129	-0.119	3.4708	0.628
		8	-0.047	-0.033	3.5637	0.735
		9	0.071	0.045	3.7827	0.804
		10	0.006	-0.040	3.7846	0.876
		11	0.030	0.047	3.8294	0.922
		12	-0.020	-0.099	3.8509	0.954

Table 4.8: UK motor residual testing: uncorrelatedness check

Table 4.9: Testing residuals (uncorrelatedness): Durbin-Watson statistics

Data Sample	Durbin-Watson statistics	1% Significance d_L	1% Significance d_U
UK Motor	1.5654	1.053	1.332
UK Property	2.0430	1.053	1.332
UK World	1.8542	1.053	1.332
Simulated T29	2.0347	1.053	1.332
Simulated T900	1.9900	1.653	1.693

The results from Table 4.9 indicate that all Durbin-Watson statistics (DW) are greater than the upper critical values (d_U) at 1% significance point. Therefore, there is no statistical evidence that the error terms are positively autocorrelated. Likewise, there is no negative autorcorrelation, since $(4 - DW) > d_U$ for all AR(2) regressions either.

Evaluating goodness-of-fit

(1) Kolmogorov-Smirnov (KS) test: we use a two-sample K-S test to check the goodness-of-fit of our ABM to the insurance market data. We simulate our ABM for a period of 900 years generating loss ratios with the same mean and standard deviation as in the UK Motor sample data. The results are graphically represented in Figure 4.20. The top two histograms show the distributions of actual Motor Insurance market combined (or loss) ratios (i.e. left diagram) and our simulated data (i.e. right diagram). Both of them are right-skewed compare to the normal distribution and most of loss ratios are distributed around one, as suggested by Daykin et al. (1994). The bottom two diagrams are the QQ-plot between actual and simulated

data (i.e. left diagram) and their empirical CDF (i.e. right diagram). Although there are some large outliers in the long series of simulated data, the simulated data from the ABM are a close fit to the actual market data.
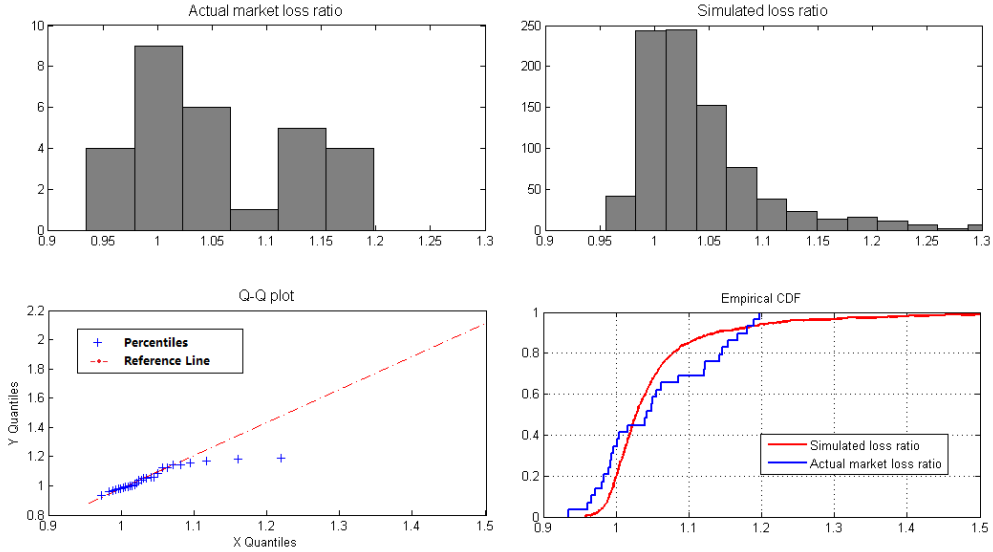
Figure 4.20: Validation: goodness-of-fit 1 (UK motor data)

We use a two-sample Kolmogorov-Smirnov (K-S) test to compare the distribution of the simulated loss ratios with the distribution of the actual market loss ratios. (The alternative Pearson Chi-Square test could also be used, but it is less powerful for small sample sizes, so we prefer the K-S test.) The K-S test cannot reject the null hypothesis that the simulated loss ratios and the actual market loss ratios come from the same continuous probability distribution (i.e. the p-value is 0.1997). This confirms that our ABM has a close fit to the actual insurance market, and provides powerful validation for our model.

We also test the *annual changes* in the loss ratio. It is useful to consider this as the changes reflect insurers' reactions to market competition. Histograms for the changes in the loss ratio are shown in the top two panels in Figure 4.21. The bottom two panels in Figure 4.21 exhibit the Q-Q plot and empirical CDFs. Visual inspection seems to suggest that our model fits the data reasonably closely. This is indeed confirmed by a two-sample Kolmogorov-Smirnov (K-S) test (i.e. the p-value is 0.1363) on the changes in the loss ratios in the data, compared to the changes in the simulated loss ratios.
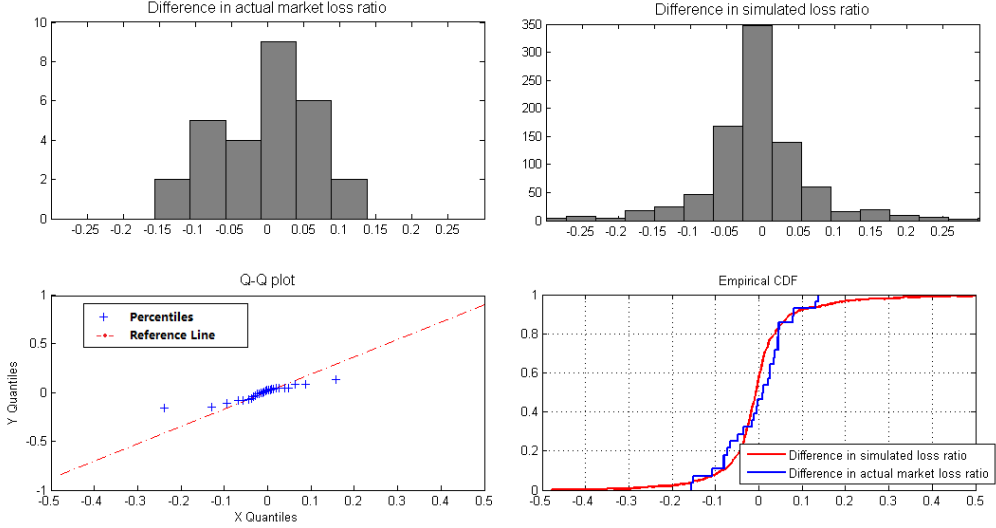
Figure 4.21: Validation: goodness-of-fit 2 (UK motor data)

(2) Chow test (for model evaluation): The so-called Chow test is a common application of the F-test to test for the presence of a structural break and to perform model evaluation (Greene, 2003, p. 130-139). In our case, we have two data sets: one is collected from a real-world insurance market and the other is generated from our ABM simulation. The real-

world data comes from a complex system that we do not know and that appears to be an AR(2) process. The simulated data is collected from our ABM insurance market model and another AR(2) process has been fitted to this data. Therefore, we want to compare these two AR(2) processes in order to evaluate the goodness-of-fit of our model.

We randomly select 29 continuous data points from the 900-year large simulated data sample, then fit an AR(2) process to each of three sets of data: (1) the actual market data which we labeled as "UK Motor" earlier, (2) the selected simulated data, (3) a concatenation of these two datasets. We collect the residuals from these regressions, and calculate the sum of squared residuals in each case. Let S_1, S_2 and S_3 denote the sum of squared residuals when an AR(2) is fitted to datasets (1), (2) and (3) respectively. Table 4.10 summarizes the relevant calculated values. We then use the Chow test to test whether there is a significant difference between the AR(2) processes fitted to the two different datasets (1) and (2) above.

Table 4.10: Chow test: UK motor insurance data

AR(2) regression	Sum of squared residuals	Data points
UK Motor actual data	0.048112	29
Simulated data	0.345649	29
A combined data	0.450799	58

The Chow test statistic is:

$$
\begin{aligned}
F_{(k,N_1+N_2-2k)} &= \frac{[S_3 - (S_1 + S_2)]/(k)}{(S_1 + S_2)/(N_1 + N_2 - 2k)} \\
&= \frac{[0.450799 - (0.048112 + 0.345649)]/(3)}{(0.048112 + 0.345649)/(29 + 29 - 6)} \\
&= 2.51
\end{aligned}
$$

In the above, N_1 and N_2 are the number of observations in each data groups and k is the total number of parameters (i.e. $k = 3$ in an AR(2) process). The test statistic follows the F distribution with k and $N_1 + N_2 - 2k$ degrees of freedom. The critical value at 5% level with degrees of freedom (3, 52) is about 2.8. Hence, we cannot reject the null hypothesis that the AR(2) process fitted to the actual market loss ratio is the same as the AR(2) process fitted to the simulated loss ratio from our model. This again provides strong validation that our agent-based model is a close fit to the actual UK motor insurance market.

4.8 Discussion of Results

Our results in Section 4.6 at page 97 show that cycles emerge endogenously in our agent-based model of the motor insurance market. Furthermore, the validation procedures in Section 4.7 at page 108 confirm that our model has a good fit to the insurance market data described in Section 4.4 at page 88. In this section, we discuss these results further and ask whether our agent-based model can provide insights into the workings of the insurance market.

Firstly, it is worth stating that the AR(2) time series fitted to the actual UK Motor insurance loss ratios in Table 4.11 imply an insurance cycle period of 7 years (i.e. $\alpha_1 = 1.08729$, $\alpha_2 = -0.828939$). An AR(2) process $\Pi_t = \alpha_0 + \alpha_1 \Pi_{t-1} + \alpha_2 \Pi_{t-2} + \epsilon_t$, where Π_t is the simulated market loss ratios at time t. ϵ_t is an i.i.d error term with $E(\epsilon_t) = 0$ and $Var(\epsilon_t) = \sigma_\epsilon^2$. In this $AR(2)$ process, cycles occur if the coefficients on the lagged terms $\alpha_1 > 0$, and $-1 < \alpha_2 < 0$, and $\alpha_1^2 + 4\alpha_2 < 0$, that is, if complex roots exist

(Trufin et al., 2009). The length of cycles is then equal to:

$$\text{Cycle Length} = \frac{2\pi}{arccos(\alpha_1/2\sqrt{-\alpha_2})}$$

Based on the coefficients in Table 4.11, it meets the conditions of the coefficients on the lagged terms α_1 and α_2 of forming cycles. This compares favorably with the cycle period of 8 years (i.e. $\alpha_1 = 0.440515$, $\alpha_2 = -0.095599$), as estimated in our agent-based model, using AR(2) process fitted to the simulated 900 years in Table 4.12.

```
Dependent Variable: UKMOTOR
Method: Least Squares
Date: 08/06/13   Time: 16:52
Sample(adjusted): 3 29
Included observations: 27 after adjusting endpoints
Convergence achieved after 3 iterations
```

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.048922	0.011685	89.76331	0.0000
AR(1)	1.087290	0.139579	7.789802	0.0000
AR(2)	-0.828939	0.143072	-5.793859	0.0000

R-squared	0.721064	Mean dependent var		1.054085
Adjusted R-squared	0.697820	S.D. dependent var		0.081449
S.E. of regression	0.044773	Akaike info criterion		-3.269969
Sum squared resid	0.048112	Schwarz criterion		-3.125987
Log likelihood	47.14458	F-statistic		31.02066
Durbin-Watson stat	1.565367	Prob(F-statistic)		0.000000

| Inverted AR Roots | .54+.73i | .54 -.73i | | |

Table 4.11: UK Motor AR2 regression

```
Dependent Variable: SIMULATEDDATALR
Method: Least Squares
Date: 08/06/13   Time: 23:49
Sample(adjusted): 3 901
Included observations: 899 after adjusting endpoints
Convergence achieved after 2 iterations
```

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.975174	0.004488	217.2831	0.0000
AR(1)	0.440515	0.033263	13.24329	0.0000
AR(2)	-0.095599	0.033267	-2.873690	0.0042

R-squared	0.169282	Mean dependent var		0.975194
Adjusted R-squared	0.167427	S.D. dependent var		0.096610
S.E. of regression	0.088152	Akaike info criterion		-2.016167
Sum squared resid	6.962683	Schwarz criterion		-2.000145
Log likelihood	909.2670	F-statistic		91.29221
Durbin-Watson stat	1.989955	Prob(F-statistic)		0.000000

Inverted AR Roots	.22+.22i	.22 -.22i		

Table 4.12: Simulated 900 years AR2 regression

Although AR(2) processes are commonly fitted to loss ratios in insurance cycle analysis (Venezian, 1985; Cummins and Outreville, 1987; Winter, 1988; Trufin et al., 2009), it is also worth stating here that the Akaike Information Criterion (AIC) suggests that an ARMA(1,1) model may be a better fit to the UK Motor combined ratio data. AIC values for different time series models are given in Table 4.13. Nevertheless, the correlograms and partial correlograms in Table 4.6 and in the Appendix suggest that a pure AR model is also acceptable, and we proceed with such an AR model, as much of the literature on insurance cycles does.

Table 4.13: ARMA model selection: UK motor insurance data

ARMA model	Akaike Information Criterion (AIC)
ARMA(1,1)	**-3.771342**
ARMA(1,2)	-3.68369
MA(1)	-3.652445
MA(2)	-3.638675
ARMA(2,2)	-3.417043
ARMA(2,1)	-3.330944
MA(3)	-3.291313
AR(2)	**-3.269969**
AR(3)	-3.237646
AR(1)	-2.500187

Secondly, it is worth highlighting the presence of cycles in our agent-based model. Unlike in other models, these cycles appear endogenously in our ABM. As we discussed in Chapter 2, there are many competing explanations for insurance cycles: cycles in interest rates, entry and exit of insurers into the market, irrational expectations through actuarial forecasting methods, regulatory lags, shocks to capital held by capital-constrained insurers, etc. In all of these models, an exogenous factor is required to initiate and drive underwriting cycles. No such exogenous variable appears in our ABM, and yet cycles are present.

This is a remarkable result which deserves further comment. A possible explanation for this is that our agent-based model captures a number of essential features of insurance markets, which may be missing from other

models. The insurance market is not perfectly competitive, but is monopolistically competitive with differentiated products. Price competition drives the market, but customers' non-price preferences over product characteristics and insurers' attributes, as well as behavioral responses by both insurers and customers, act to dampen and turn prices, thereby generating cycles.

In our model, the elasticity of demand appears to vary both over time as well as over the product characteristic space. Insurers estimate a possible mark-up over the actuarially determined premium, when pricing their policies. This mark-up is based on the demand elasticity which they are estimating: see Equation (4.3). Figure 4.22 shows the crude mark-up estimated by 4 randomly chosen insurers over 30 years. (They calculate this mark-up in accordance with Equation (4.4).) The mark-up clearly varies over time and indeed appears to be cyclic.
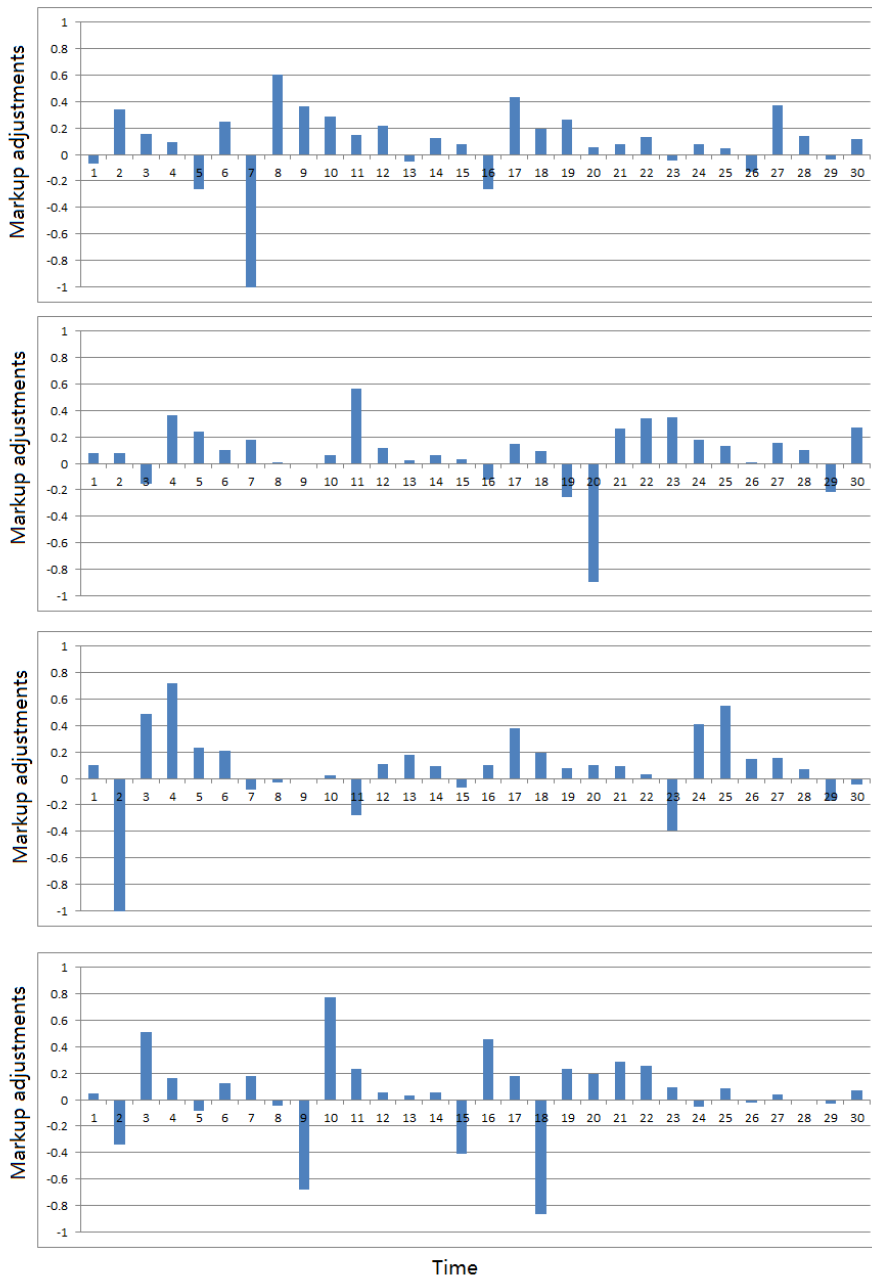
Figure 4.22: Mark-up over 30 years of 4 insurers

In Figure 4.23, we show the variation over 50 years of the average distance, in the product characteristic space, between an insurer and its customers. Some customers are switching from insurer to insurer over time,

but one might anticipate that the group of customers which is 'close' to an insurer (i.e. which prefers this insurer and its product over competitors) would remain relatively stable over time. However, the variation in Figure 4.23 shows that the market appears to become more segmented and 'local', and then less so, over time. In other words, the market goes through phases: there are times, presumably of relatively higher prices, when price dictates customer choice, and there are other times, presumably when prices are lower, when non-price preferences are of greater importance in customer decisions. The response of insurers in terms of pricing, and of customers in terms of product selection, then drives the market and appears to engender cycles.
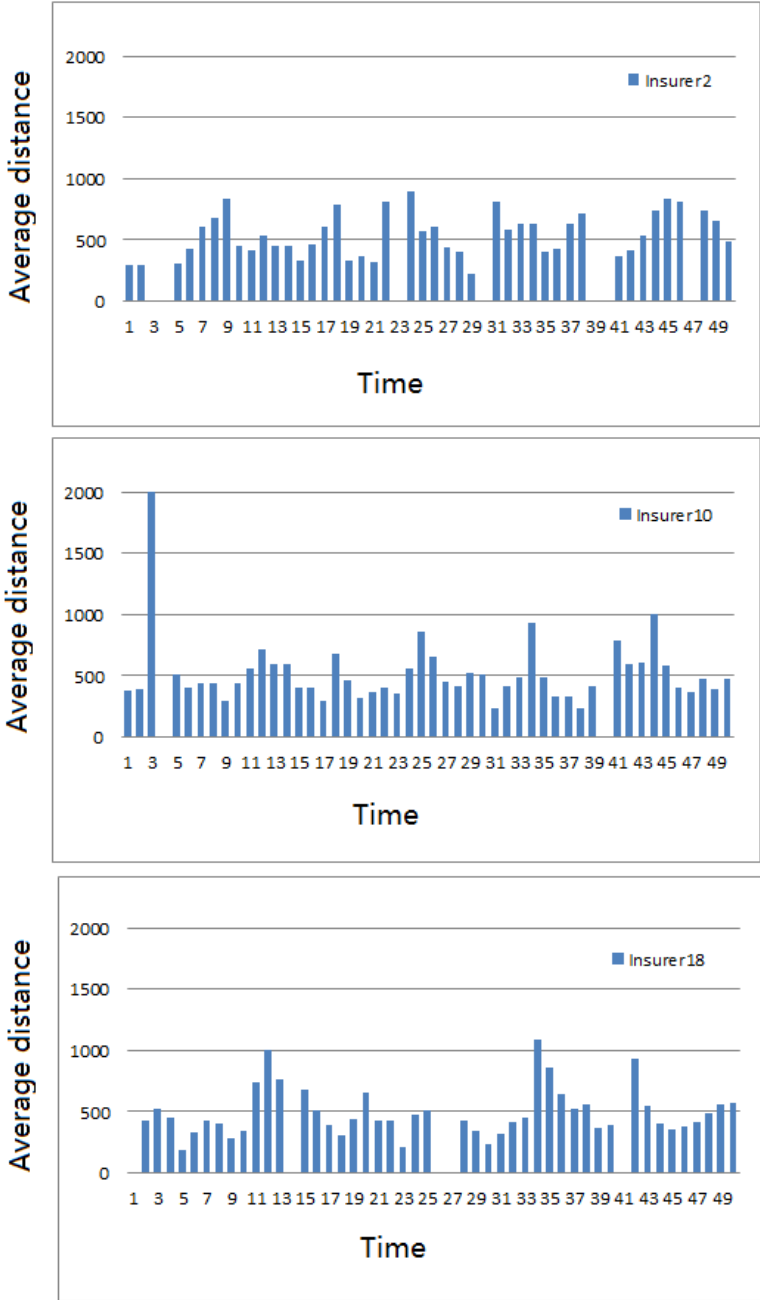
Figure 4.23: Average distance over 50 years

It is also interesting to examine the effect of the two parameters which may be under the control of insurers and regulators: the safety loading factor α and the mark-up adjustment factor β. The safety loading factor

α, which appears in Equation (4.2), represents insurer's attitude to risk. One might expect that, if insurers raise their safety loading factor, they will demand a higher premium, and this will reduce and stabilize the loss ratio in the overall market. Figure 4.24 does indeed show that both the mean and the standard deviation of the loss ratio in our agent-based model initially fall as α increases. It also shows that the mean and standard deviation of loss ratio start to increase as α increases beyond a certain value. This would appear to be an instance of the "winner's curse" phenomenon (Doherty and Posey, 1997). If insurers are too sensitive to risk and apply a high safety loading, customers will be more likely to switch to other insurers as they search for lower prices. Insurers which are under-pricing, possibly because of initial favorable claims experience or a low customer base, then acquire more customers, but subsequently suffer large losses, contributing eventually to an increase in the level and volatility of the loss ratio for the insurance market as a whole.
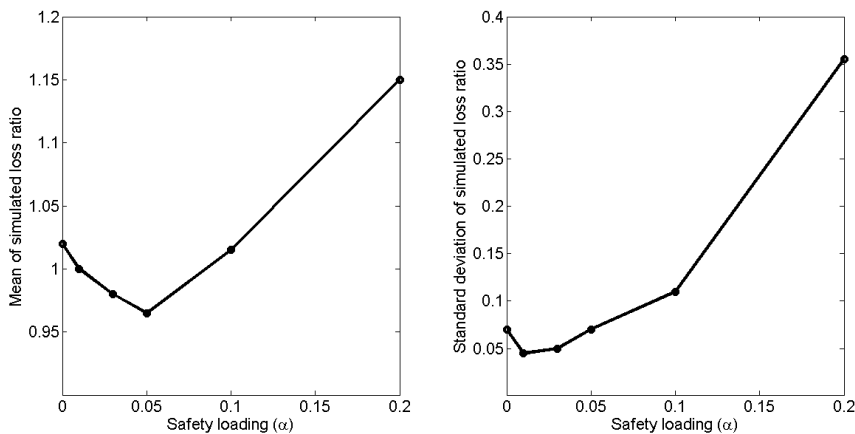
Figure 4.24: Sensitivity analysis: safety loading

The mark-up adjustment factor β, which appears in Equation (4.5), governs how much weight insurers place on past experience when estimating the mark-up which they can apply to the actuarial premium when they price their products[5] . The larger β is, the less reliant insurers are on the past. Figure 4.25 shows that loss ratios are less autocorrelated (particularly at lag 1) as β increases. Thus, as β increases, large loss ratios are less likely to be followed by large loss ratios. This is a sensible result because faster reaction by individual insurers to local market competition may help the market as a whole avoid entrenched underwriting cycles. Technological innovations in insurance product design and sales - such as insurance aggregators in personal lines, telematics usage-based insurance contracts employing 'big data' machine learning - may enable insurers to learn about their market and react to competition much faster, and thus price their products in a more accurate and timely way. This may reduce the risk of large losses from deep troughs in cycles in insurance markets.

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[5]The location (product preference) weight γ: Both parameters α and β represent an insurer's attitude towards business risk and market competition, γ reflects a customer's non-price product preference in the purchasing decision. It represents the stickiness or inertia of a customer to a particular insurer or product. If parameter γ is small, this means customers care the price P_{it} of insurance more than the insurance contract meets their individual product specification or personal preference when γ is large. The larger the γ, the more importance of an insurer's targeted product specification to this particular targeted customer segment, the less role of price in this group of customers' purchasing decision. We consider the special case when $\gamma = 0$, then "Location" (or product differentiation) plays no role in the competition, the market becomes from monopolistic to perfect competition. Then, customers purchase the insurance with the lowest price in each time period, they switch from their existing insurers to other cheaper options in every time step because there is no any other cost to influence their switching decisions. The market dynamics are similar to the phenomenon of so-called "winner's curse" (Doherty and Posey, 1997): those mistakenly underpriced insurance products attract all customers in every business period.
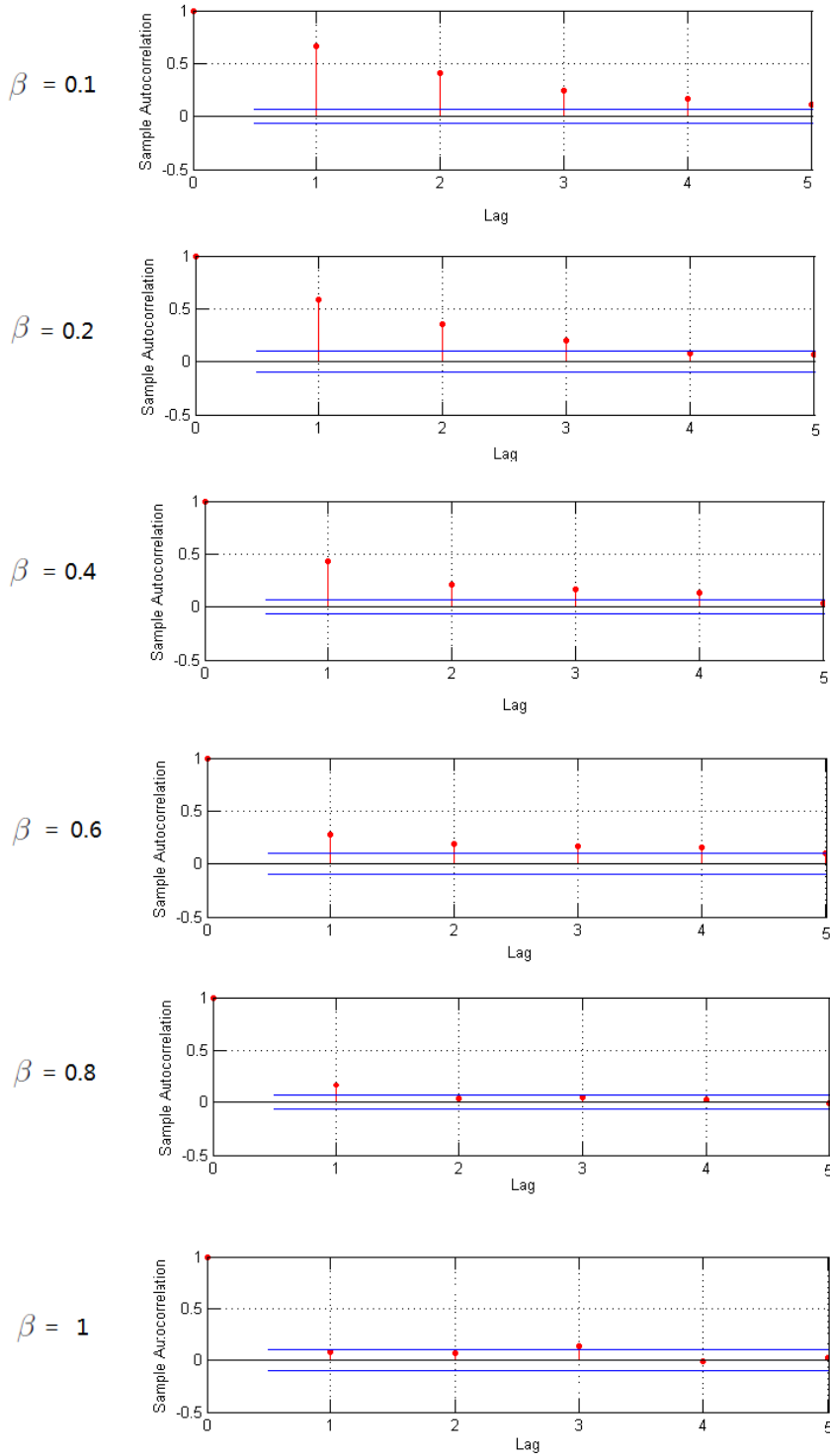
Figure 4.25: Sensitivity analysis: weight of mark-up adjustment

4.9 Conclusion

In this chapter, we presented an agent-based model (ABM) of a non-life insurance market, with the aim of understanding the dynamics which may generate insurance cycles. Unlike other models and theories of insurance cycles, our model had no exogenous factor driving cycles, and yet we found that cycles in loss ratios emerge. While we could not dismiss the role of exogenous factors such as capital shocks and interest rate fluctuations in promoting and maintaining insurance cycles, our ABM showed that cycles can arise endogenously if insurance markets are modeled as competitive, but not perfectly competitive. In particular, we made use of an economic location model to capture product differentiation and non-price preferences. In our model, insurers competed against each other and priced their products by calculating a premium based on past experience, and then adjusting through cost-plus or mark-up pricing. This provided a realistic description of actuarial and insurance practice. We calibrated and validated our agent-based model using UK motor insurance data. Simple time series analysis showed that cycles were present in the simulated insurance loss ratios from our ABM, and that these cycles were comparable to those in the actual data. Our ABM showed that the monopolistically competitive nature of non-life insurance markets, with their differentiated products and with boundedly rational behavior of insurers and customers, may inherently create cycles, without requiring external factors.

Chapter 5

Extensions to the ABM Base Case

5.1 Introduction

In the previous chapters, we separately introduced agent-based modeling and the phenomenon of insurance cycles. Chapter 4 presented an agent-based model of the motor insurance market, which was calibrated on actual UK data, and we found that cycles emerged without exogenous factors. In this chapter, we extend the model of Chapter 4 in three ways. First, we consider the impact of capital constraint on the insurer's pricing decision. We include an adjustment to the pricing function which takes into account the capital of the insurer. Second, we recognize that insurers can adapt to customers' preferences and change their strategies, and therefore move on the product characteristic space. Third, we allow for a larger-dimensional product characteristic space, rather than the one-dimensional circular space used heretofore.

The simulation follows the same market process of the base case in Chapter 4. It still contains insurers and customers only. It also operates in discrete time. Insurers still interact in each time period $(t, t+1)$ according to the steps detailed below and illustrated in Figure 5.1: (1) At time t, each insurer offers its own unique market competitive price to all customers. (2) Given all insurers' prices, every customer calculates the total cost of purchasing insurance from each insurer. (3) Based on the estimated total costs, customers select the lowest option until supply and demand are balanced in the market. Insurers collect all premiums after being selected by their potential customers at time t. (4) Customers' total claims are randomly generated and paid by insurers at time $t+1$. (5) Insurers update their underwriting results after paying the claims to their customers at time $t+1$. (6) The market performance is updated at the end of each time period. This includes both the whole industry and individual insurers' loss ratios, premiums and profits. (7) All extensions in this Chapter 5 are implemented in this final step. In Chapter 4 base case, insurers decide their next period competitive prices in this step to begin a new round of competition at $t+1$. However, in this chapter, we update the insurer's price function using a capital-based adjustment. Insurers not only update their price, but also change their location. They compete not just on price but also on product characteristics and other attributes.

The structure of this chapter is as follows: Section 2 summarizes and explains the key elements that we try to target in the model extensions, along with the purposes for us to understand them. They are: (1) the role of

capital in an insurer's pricing decision (i.e. price function); (2) the role of an insurer's non-price competition in our dynamic system; (3) the impact of different insurers' behavior rules on both the market performance as a whole and different individual insurers' performance. Section 3 focuses on the role of an insurer's capital in its price function. The model structure is the same as the base case in Chapter 4, but we adapt the insurer's pricing equation by adding an adjustment that includes an insurer's existing capital level. In Section 4, we model our insurers as moving agents, so they can move their positions on the 1-D space of Circle City to compete with each other. These movements represent non-price competition, since insurers aim to compete by moving toward customers, seeking to satisfy their preferences and attract more customers from competitors. These movements can be modeled through insurers' behavioral rules and associated parameters. Finally, in Section 5, we generalize our 1-D Circle City to 2-D Planar Space for several reasons. It performs a robustness check for the early 1-D versions, since the results of explaining insurance cycle from this 2-D version are similar and consistent with the early 1-D versions. It also increases the flexibility of moving agents, so it generalizes our model in line with the real market competition. We can think about this 2-D space as a strategy map, so insurers can use this kind of ABM simulations to monitor and plan their business strategies in the real world.
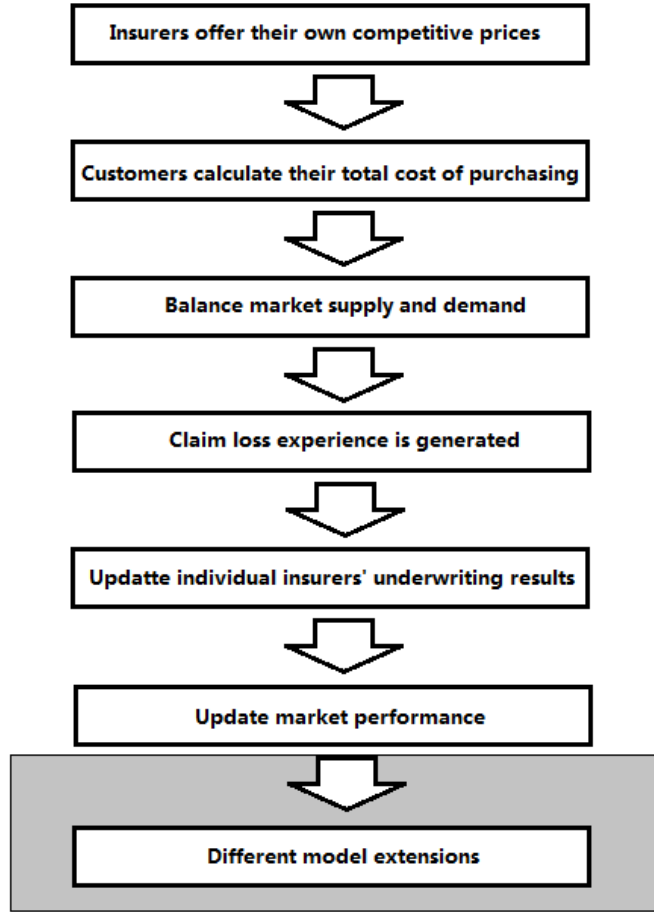
Figure 5.1: Market process: extended versions

5.2 Key Elements in our Model Extensions

5.2.1 The role of capital in pricing decision

In the previous Chapter 4, we simplify the role of capital in an insurer's pricing decision by artificially setting the individual capitals of insurers high enough to avoid the possibility of insolvency, for several reasons: (1) we aim to focus our initial analysis on the basic interactions of insurers in the base case, such as their reactions to the market demand and claim experience; (2) we believe that capital is one of contributing factors of am-

plifying insurance cycle, but it is not the original source of cycle motion; (3) if we model capital in consistent with the real world, we have to artificially assume other elements as well, such as dividend distribution (e.g. the level of dividend payout ratio, etc) or entry-exit activities. We think that it would be better for us to keep our insurance market system closed without being complicated by capital outflow (e.g. dividend or insolvent insurers' exit) and inflow (e.g. government and investors' capital injections or new entrants).

Obviously, as addressed in Chapter 2 literature review on insurance cycle, an insurer's capital plays an important role of affecting its pricing decision. There are three different hypotheses related to this discussion and had been tested on real market data by researchers. First, the capacity constraint hypothesis states that internal capitals are cheaper than external capitals, therefore insurers are willing to reduce price when they have more capitals. This hypothesis predicts a negative relationship between insurer's capacity and underwriting margin. Gron (1994) provides an empirical test by examining this relationship. Based on the results of testing data on four insurance business lines, they generally support it and unanticipated decreases in capacity cause higher profitability and price. Second, the financial quality hypothesis extends the capacity constraint model by allowing insurer's default risk to be endogenous. It predicts a negative relationship between insurer's capacity and underwriting margin in the short run, but positive in the long run. Finally, the option pricing hypothesis as a contingent claim analysis is different from both hypotheses above, which states the insurance policy is similar to risky debt. It

predicts a positive relationship in both short and long run. More detailed discussions and related literature review on these capital related hypotheses are included in the following section, when we apply a capital-based adjustment in our price equation.

5.2.2 Non-price competition in an dynamic system

The customer's total cost function (i.e. $TC_{ij,t} = P_{it} + \gamma \Delta_{ij}$) appears in Chapter 4, where Δ_{ij} represents the element of non-price competition. In Chapter 4, we assume that the positions of insurers are fixed in the "product attribute circle city" over time, so insurers cannot change their non-price strategies (although they have some non-price competitive advantages to attract the customers who are close to their locations). The purpose of this simplification is that we believe non-price competition matter in the overall market dynamics (e.g. competition on product characteristics or attribute to meet customers' preferences) and they generate a differentiation of the insurance products of different insurers, particularly in a monopolistically competitive insurance market. However, we understand that the aggregate insurance market cycles are mainly dominated by the insurers' pricing decisions. As our cycle hypothesis suggests that "price competition drives the aggregate cycle, but non-price competition acts to dampen and helps to turn the cycle when top and bottom are reached". Now in this Chapter 5, we relax this assumption, so insurers can move their locations after determining the best position for them to move. We analyze this non-price competition to confirm whether our initial hypothesis is true or not. Furthermore, non-price competition may not have a direct impact

on aggregate market performance, but it does affect individual insurers'
performance and their individual cycles directly.

It is easy to define price competition, which states that firms are compet-
ing on the basis of offering a lower price. However, it is useful to specify
the definition of non-price strategic competition in our model. Common
non-price competition is based on product differentiation and distribu-
tion method, which is the process of distinguishing a product to make it
more attractive to a particular target customer. Stigler (1968) lists some
non-price variables when firms offer similar products, such as advertising,
durability of product, investment advice or warranties of free repairs. In
the financial services industry, the nature of a product is often similar. So
the non-price competition is focused on information and knowledge about
the customers. Insurance companies often compete through improving
the mutual knowledge between themselves and potential customers. They
select different channels to distribute their products and advertise their
brands, thus increasing the customers' knowledge about the firms. Mean-
while, the insurers collect customer data from different sources and use
sophisticated IT systems to analyze this information, thus increasing the
insurers' knowledge about the customers. In our model, we model all of
these non-price strategies by using one variable. The key idea of this im-
plementation is similar to traditional economic non-price location models,
in that we assume the non-price strategy incurs extra cost, but it gains a
specific competitive advantage to the firm.

5.2.3 Model agent's behavior rules

Basically, any agent's decision can be modeled by defining some behavior rules in ABM simulations. Fascinatingly, although these rules look like very simple and observes them easily from the real market, the interactions of these basic rules can produce some very complex systemic results, such as insurance cycle in our case. We have already modeled the price equation in Chapter 4, now there are a few behavior rules to manage the insurers' non-price strategic movements. The pricing behavior is consistent with the traditional risk averse utility and supply-demand market competition, which means that more uncertain risks should be charged at a higher price and the more competitive market should offer a lower price, and vice versa. However, the rule of non-price strategic movement is new to the traditional cycle models that are based on a perfectly competitive market, although it has become more popular in the area of ABM research and in the field of Behavioral Economics. The major advantage of ABM simulation is to understand the impacts of these (non-traditional) behavior rules in the real world.

An insurer's belief about a competitor's current action and future strategy plays a key role in deciding its own action in the next period. We model this belief by using two different behavior rules to understand which one of them or a combination of them is significant for analyzing the real-world insurance market, such as insurance cycle.

Rule case 1: Before an insurer tests all available options of changing its (non-price) business strategy at the start of next period, the insurer

assumes that all other insurers will remain in their existing business strategies and will maintain their current prices in the next period. The insurer will select a business strategy in the next period that should earn a maximum expected profit, assuming others will not change their current positions.

Rule case 2: An insurer assumes that all other insurers will remain in their existing (non-price) business strategies and will maintain their current prices in the next period, but this insurer will compare its current profit with others who are selected to be its direct competitors (i.e. those insurers who have similar size/capital or business strategy of this insurer). The insurer then selects the business strategy in next period that is closer to the business strategy of the top performing player in this peer group.

Intuitively, these two cases reflect two different beliefs of insurers about the creation of profit in next period. The first case tells us that the insurer considers its own profit and selects an available business strategy to react to the current environment (assuming others do not change in the next period). The optimal business strategy can be summarized as avoiding higher competition to earn more profit by focusing their specialized market segments over time, or by moving away from other competitors. The second case tells a different story, since the insurer also considers other competitors' profits. It changes its own strategy towards the top performing competitor among its peer group. As a result, there is a herding behavior. Insurers believe that a better strategy has a better performance, so they aim to move toward the top performing player. As Abel (1990) mentioned

that agents are "keeping up with the Joneses".

In reality, there may be some interactions between these two behavior rules; sometimes one rule is more significant than the other one (LeBaron et al., 1999). In the insurance market, companies need to earn more profits, but they also need to compare their performance with other direct competitors. Particularly, in an uncertain environment, when the absolute performance is difficult to be defined, the relative comparison among a peer group becomes more important. This dynamic behavior of herding may affect insurance market cycles (Feldblum, 2000).

5.3   Extension 1: Capital-based Adjustment in Price

5.3.1   Roles of capital in price

Choi et al. (2002) compare a few alternative models of insurance pricing as theories of the underwriting cycle. They discuss the relationship between an insurer's capital (i.e. surplus) and its price in both short and long run. They define "long-run" as an equilibrium relationship between capital and price, and "short-run" as for the temporary impact of capital on the dynamic of price. Although different hypotheses have different predictions about this relationship in different lengths of period, it is clear that the level of capital plays an important role of affecting an insurer's price. Here, we review three major hypotheses that provide different conclusions about this relationship. It is useful for us to compare these theories, before adding

a capital-based adjustment into our insurers' price equation in this model extension.

Capacity constraint hypothesis

Over the past two decades, capacity constraint hypothesis is the most popular theory of understanding insurance cycle and has been discussed by many researchers (Winter, 1988, 1991, 1994; Gron, 1990, 1992, 1994; Doherty and Garven, 1995). Rather than assuming capital market is perfect or assuming an insurer's capital adjusts quickly to the market environment as suggested in Cummins and Outreville (1987), it assumes that capital market is imperfect and insurers take both time and cost to adjust their capitals in the short term. Choi et al. (2002) provide empirical evidence to support that the slow adjustment of capital implies the persistence of cycles, also see (Niehaus and Terry, 1993; Haley, 1993, 1995; Grace and Hotchkiss, 1995; Fung et al., 1998). Therefore, this hypothesis focuses on short-run price determination, since capital can be adjusted fully in the long term to match the market demand. For example, a negative shock to the overall capital of insurance market reduces market supply, so the market price has to be increased to balance the existing demand in the short run. As existing insurers enjoy a higher price than before, the capital accumulates by earning more profits over time, so the market returns to the long-run equilibrium. As a result, it predicts that capital and price have a negative relationship in the short run, but no relationship in the long run.

Financial quality hypothesis

An extension to the capacity constraint hypothesis, Cummins and Danzon (1997) further develop a model of price determination that includes the negative impact of default risk on demand for insurance. The model suggests that price may increase or decrease following a loss shock that reduces an insurer's capital, depending on factors such as the effect of the shock on the price elasticity of demand. Since demand for insurance links to the financial quality of an insurer, the price is also affected by the insurer's quality. Similar arguments are in both Harrington and Danzon (1994) and Cagle and Harrington (1995). While capacity constraint hypothesis analyses the impact of capital on overall market supply (i.e. a high market capital means a large supply, so this situation reduces market price if short term demand of insurance is fixed, vice versa), financial quality hypothesis focuses more on the downside risk and the impact of insolvency risk on the market price. It argues that capital not only affects supply of insurance in the short run, but also the total demand of policyholders. For the same example of a negative shock to the capital, insurers now have a resistance to increase the market price, since customers reduce their demands when the overall default risk of insurers is increased. If the effect of supply change is larger than demand change, then it predicts a similar relationship to capacity constraint hypothesis. However, in the long run, as Choi et al. (2002) suggest an insurer with a higher level of capital is able to charge a higher price, since customers' willingness to pay is higher. As a result, it predicts that capital and price have a negative relationship in the short run (i.e. same as capacity constraint hypothesis), but a positive relationship in

the long run (i.e. different from capacity constraint hypothesis).

Option pricing hypothesis

The option pricing hypothesis is based on the contingent claim analysis from the traditional financial pricing theory, which states an insurance policy is similar to a risky debt (Cummins, 1991). The idea is that insurers have the option to default as same to a call option on the assets of the company. On the other side, policyholders hold a short position in a put option on the same assets of the company with an exercise price equal to aggregate losses. Different from other pricing hypotheses, option pricing hypothesis argues that the policyholders actually bear the risk of default and should be compensated by this risk taking activity. According to the traditional option theory, the value of the bankruptcy has a negative relationship with a firm's capital in both short and long run. Therefore, if the capital of an insurer is low and its default risk is high, then customers require a low price to compensate their risk sharing with this insurer. The empirical test of Sommer (1996) on the cross-sectional data supports option pricing hypothesis. As a result, it predicts that capital and price have a positive relationship in the short run (i.e. different from both capacity constraint hypothesis and financial quality hypothesis), and a positive relationship as well in the long run (i.e. same as financial quality hypothesis).

Summary of relationships between capital and price

All three different pricing hypotheses link the insurer's capital to its pricing decision, but they have different predictions in either short or long run situations. Table 5.1 provides a conclusion. Both capacity constraint and

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financial quality theories suggest a negative relationship between capital and price in the short run: when an insurer's capital increases, its price decreases. Both financial quality and option pricing theories suggest a positive relationship between capital and price in the long run, but option pricing theory also expects a positive relationship in the short run.

Table 5.1: Summary of three pricing models: capital vs price

Hypothesis	Short-run relationship	Long-run relationship
Capacity constraint	Negative	None
Financial quality	Negative	Positive
Option pricing	Positive	Positive

5.3.2 Model extension: capital in price equation

As explained in the previous subsection, the relationship between the level of capital and its role on sitting price is complicated based on different pricing hypotheses (see Table 5.1). However, the prediction of capacity constraint hypothesis has the most supporters in both theoretical insurance cycle papers and empirical data tests (Choi et al., 2002). Therefore, we apply the short-run negative relationship between capital and price to extend our model, by adding an extra capital-based adjustment (i.e. $e^{s_{it}}$) in our early price equation in Chapter 4.

The exended price equation is:

$$P'_{it} = P_{it} \, e^{s_{it}} \tag{5.1}$$

$$= \left((\widetilde{P}_{it} + \alpha F_{it}) e^{m_{it}} \right) e^{s_{it}} \tag{5.2}$$

Where P'_{it} is the extended price in this Chapter 5 and P_{it} is the early Chapter 4 price Equation (4.2). In line with the implementation of Taylor (2009), we define the estimated capital-based adjustment of price equation as follows:

$$s_{it} = -\frac{K_{it} - K_0}{K_0} \tag{5.3}$$

Where K_{it} is the existing capital of an insurer i at time t and K_0 is the benchmark capital. In the case of Taylor (2009), the benchmark is the steady state solvency (i.e. a long term ideal situation of an insurer's solvency level in a perfect competitive market). As Taylor argues, an insurer's premium should increase (decrease) as its current solvency ratio (i.e. SolvencyRatio = Capital/Exposure, it is denoted by S_{it}) falls below (rises above) the steady state solvency (i.e. it is denoted by S_0), as implemented by this equation (Taylor, 2009):

$$P_{it} = P_0 \; e^{-k(S_{it}-S_0)} \tag{5.4}$$

Where P_0 is the steady state premium per exposure unit (i.e. theoretical pure risk premium) that is sufficient to pay future claims in the long run and k is a premium-to-solvency sensitivity parameter. In our model extension, we use an insurer's initial capital level at the beginning of simulation as our benchmark capital, since we ignore insolvency risk and entry-exit issues in our model by starting with a high level of initial capital. Our objective is to minimize the exogenous factors and focus on the endogenous interactions of insurers within this closed system. The ideas behind this implementation are: (1) Insurers care about their capital situations and af-

fect their pricing decisions (Choi et al., 2002); (2) When their capitals are below the benchmark level, they start to increase their prices as suggested by capacity constraint hypothesis (Niehaus and Terry, 1993; Haley, 1993); (3) The level of price increase or decrease depends on the gap between existing capital and the benchmark, and also the weight of this gap that is relative to the benchmark capital (i.e. a relative measure). As Gron and Winton (2001) suggest that this relative level of supply disruption affects insurers' prices. (4) The exponential function is consistent with Taylor (2009), which explains that the bigger gap below the benchmark the more nervous for an insurer to increase its price, vice versa.

Nevertheless, one of main benefits from implementing an ABM system is providing us an experimental field to test different hypotheses. For this purpose, we can test the three hypotheses in Table 5.1 and see which results are closest to the real data. In order to achieve that, we introduce a new parameter κ which controls the role of capital level in price function, as follows:

$$P'_{it} = P_{it}\, e^{\kappa(-s_{it})} \tag{5.5}$$

where

$$s_{it} = -\frac{K_{it} - K_0}{K_0} \tag{5.6}$$

that means: If $\kappa < 0$, there is a negative relationship between capital level and price (i.e. Capacity Constraint Theory and Financial Quality Theory: a more safe company is willing to charge less price). If $\kappa = 0$, there is no short-term relationship between capital and price (i.e. that is the original price function in Chapter 4). If $\kappa > 0$, there is a positive relationship (i.e.

Option Pricing Theory: a more safe company is able to charge more). The simulation results are included in the following Table 5.2:

KAPPA	MeanLR	S.D.LR	Rho(1)	Rho(2)	
-1	0.9974	0.0860	0.4625	-0.1954	
-0.5	0.9951	0.0922	0.4593	-0.1843	
-0.3	0.9934	0.1030	0.4436	-0.1813	
-0.1	0.9765	0.0894	0.4058	-0.1615	
0	0.9729	0.0735	0.3914	-0.1066	Benchmark
0.1	0.9664	0.0927	0.3931	-0.0889	
0.3	0.9577	0.1005	0.4671	-0.0671	
0.5	0.8556	0.1212	0.5454	0.1967	
1	0.6058	0.3428	0.7648	0.2011	

Table 5.2: Model extension: three versions of capital relation in price equation (MeanLR and S.D.LR are the mean and standard deviation of loss ratios; Rho(1) and Rho(2) are the partial autocorrelation of first and second time lags of loss ratios.)

Based on the statistical results in Table 5.2, we understand: (1) When $\kappa < 0$ or is close to $\kappa = 0$, there are cyclic patterns of loss ratios over time. Therefore, there are cycles in loss ratio time series that are similar to real-world situations, i.e. $Rho(1) > 0$ and $Rho(2) < 0$. The reason for this is: since the relationship between capital and price is negative, insurers increase price when the capital is low that will increase overall profits. A high overall profit will build up capitals, which will decrease prices. In fact, the loss ratios continually shift their directions around the mean, which form cycles. (2) When $\kappa > 0$ and is far away from $\kappa = 0$, there is no cycle in loss ratio time series, i.e. both $Rho(1) > 0$ and $Rho(2) > 0$. This is because: when capital is high (low), insurers keep increasing (decreasing) price and earn more (less) profit that will further build up (reduce) capital.

5.3.3 Simulation results

We follow the market process in Figure 5.1 to run this first model extension and use the same parameter values as in Chapter 4 base case model. Therefore, the only difference between this extension and the base case of Chapter 4 is the extended price Equation (5.1). We compare the autocorrelation function (AcF) and partial autocorrelation function (PAcF) of the market loss ratios from both models. Figure 5.2 shows the results of the base case in Chapter 4. Based on both AcF (left) and PAcF (right), the market loss ratios follow an AR(2) process over time since both first and second lags are significantly opposite to each other, while other lags are statistically insignificant.
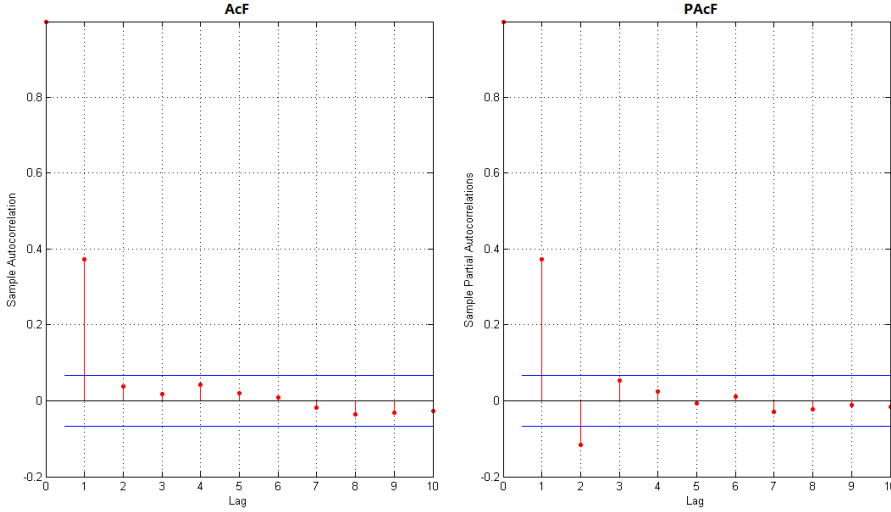

Figure 5.2: Market loss ratio: AcF and PAcF of the base case model

Figure 5.3 shows the results of the model extension 1. A similar AR(2) process of market loss ratios is also appeared. However, if comparing with Figure 5.2, the values of both first and second lag coefficients become wider from 0.38 to 0.43 in the first lag and from -0.12 to -0.2 in the second lag.
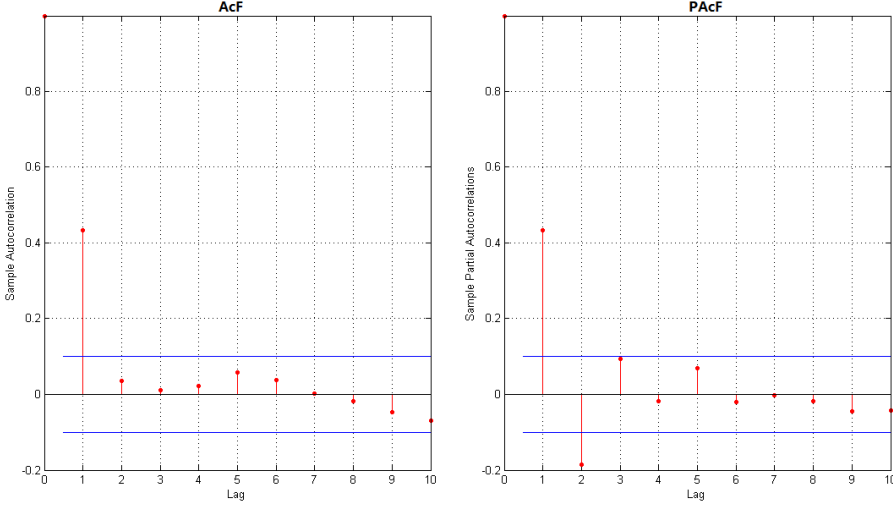
Figure 5.3: Market loss ratio: AcF and PAcF of the extended model 1

According to the AR(2) process:

$$\Pi_t = \alpha_0 + \alpha_1 \Pi_{t-1} + \alpha_2 \Pi_{t-2} + \epsilon_t$$

where Π_t is the simulated market loss ratios. ϵ_t is an i.i.d error term with $E(\epsilon_t) = 0$ and $Var(\epsilon_t) = \sigma_\epsilon^2$. In this $AR(2)$ process, cycles occur if the coefficients on the lagged terms $\alpha_1 > 0$, and $-1 < \alpha_2 < 0$, and $\alpha_1^2 + 4\alpha_2 < 0$, that is, if complex roots exist (Trufin et al., 2009). The length of cycles is then equal to:

$$\text{Cycle Length} = \frac{2\pi}{arccos(\alpha_1/2\sqrt{-\alpha_2})}$$

Based on the results of our simulations, both of the above two cases meet the conditions of the coefficients on the lagged terms α_1 and α_2 of forming cycles. Also, based on the calculation of cycle length and the estimated values of α_1 and α_2, the base case model in Chapter 4 has a cycle with a period of 6.34 years and the extended model has a period of 5.87 years. This

result is sensible, because when insurers add the capital-based adjustment into their pricing decisions and the relationship between capital and price is negative in the short run, insurers will start to reduce (increase) price when they perform well (badly). This will reduce the length of cycle between a hard market and a soft market.

5.3.4 Conclusion

Based on the simulation results of this model extension 1 and its comparison with the base case in Chapter 4, we conclude that the change of an insurer's capital has an impact on the cyclic pattern of market loss ratios over time. It supports our initial endogenous cycle hypothesis, which suggests that "capital constraint" is an external contributing factor of amplifying cycles (i.e. it expands the AR(2) process, both the first and second lag coefficients become wider). The key endogenous source of cycle is the interaction of insurers' competition in a monopolistically competitive market.

5.4 Extension 2: 1-D Circle City with Moving Agents

5.4.1 Roles of strategical movement in non-price competition

In Chapter 4, we used an economic location model to capture some key features of monopolistic competition, but we assumed that agents were

fixed in product characteristic space. The locations on this space capture the differentiation of insurers' products, from the point of view of customers. In this section, we allow insurers (agents) to move in this space. This means that insurers can change their business strategies, alter their products or the way that they are perceived by customers, and thus move to more profitable market segments. In short, insurers can compete using non-price strategies, as well as on price.

Therefore, firms compete through both price and non-price strategies. Price strategy is related to the nature of product (i.e. the expected risk of customers and the level of demand) and non-price business strategy is represented by the distance between firms and potential customers. Firms attract customers from their competitors by a combination of offering a lower price and being located closer to them in order to achieve profit maximization. In a simple example, two DVD rental shops compete with each other on a straight street. We assume that they can charge different prices for identical products and move their shop locations without incurring any cost. Customers buy/rent DVDs from the shop with the lowest total cost of the DVDs and their travel expenses. Hotelling (1929) shows that the market reaches stability in competition, when both shops locate at the middle point of the street and offer an identical price. He assumes that both firms are identical and customers are independent, identical and uniformly distributed on the street. This simple case can be extended to more complex situations. Brekke et al. (2006) apply the Hotelling model to analyze the quality and location of choices under price regulation. Economides (1986a) analyses firms' decisions of minimal and maximal product

differentiation in Hotelling's duopolistic competition. He also expands the original Hotelling model from one-dimensional products to two dimensions (Economides, 1986b), increases the number of firms to more than two competitors (Economides, 1993) and updates the firms' actions as a sequential decision making process (Economides et al., 2002).

In this section, we analyze the impacts of insurers' location movements on both market aggregate results and insurers' individual performance. Before discussing the simulation results, we explain how we model this dynamic movement.

5.4.2 Non-price business strategy modeling

We illustrate the insurance market by using the diagram in Figure 5.4. This is an example for illustration purposes, the actual simulation may change the number of insurers and customers, etc. We define the small red bubbles are individual insurers (e.g. from insurer A to H), and the large black circle is the circular street on which many customers are uniformly distributed. Insurers' locations on the street define their product characteristics, hence differentiating them from each other. Customers' locations reflect their individual non-price product preferences. In Chapter 4, neither insurers nor customers can move on the space over time, their locations are fixed. The insurer's location defines the level of attractiveness to customers. Insurers only make one strategical decision that is the competitive price for the next period.
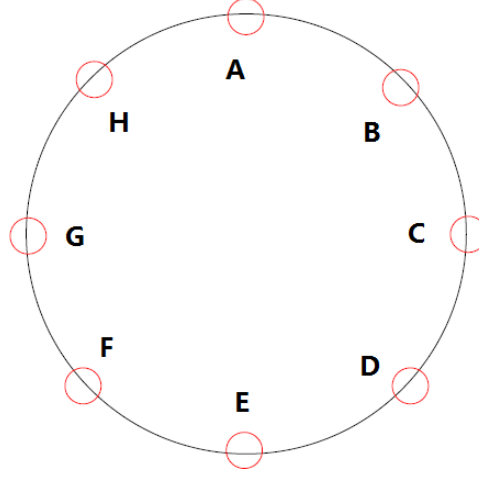
Figure 5.4: Model dynamics: a circle city

Now, in this model extension 2, insurers have two strategical decisions to make: a pricing decision, and a non-price business strategy decision. They move toward new locations where they expect to earn more profits, as in the example of DVD rental shops. We use three behavioral parameters to manage insurers' movement on the space of product characteristics, as summarized in Table 5.3. We explain these parameters in the following three subsections.

Table 5.3: Behavioral parameters in model extension

Parameter	Range of values	Explanation
θ_1	$[0, 1]$	Speed of an agent's movement
θ_2	$[0, 1]$	Scope of an agent's comparison
θ_3	$[0, 1]$	Proportion of insurers who move

Speed of an insurer's movement

Insurers can only move along the circular street (i.e. the large black circle in Figure 5.4), where the customers are uniformly distributed and fixed on

the street. Therefore, the maximum range of movement is the circumference and the possible direction is either clockwise or anti-clockwise. We normalize the circumference to be of unit length. The speed parameter θ_1 controls insurers' moving speed by specifying by how much they can move along the circumference from one time period to the next. This behavioral parameter has two real-world applications: (1) It reflects the speed of strategical adjustment of insurers in the market to react the current competition. In different insurance sectors, insurers may exhibit different speeds of reaction to competition. We can test the impact of this variation in speed on the dynamics of the insurance cycle. For example, personal motor insurance is data-driven, whereas commercial Directors & Officer (D& O) liability insurance is mainly based on underwriting skill and experience. Therefore, the speed of strategical change is faster in motor sector than D&O. (2) We can also test some real-world scenarios where the speed of insurers' market reaction becomes faster over time. For example, product cover of motor insurance traditionally is updated slowly with a time delay to change the policy conditions, but it is becoming faster since newly developed technologies, such as comparison websites (insurance aggregators), are becoming popular with both insurers and customers. Customers can now design their own policies to suit their circumstances.

Scope of an insurer's comparison

The scope of an insurer's movement determines the proximity of its direct competitors. It controls how many neighbors there are against which an insurer might compare its performance. It then decides to move toward the most profitable location (i.e. customer group or market segment). If there

are eight insurers in the market, then an insurer selects a non-negative integer[1] number of competitors who have the nearest location from the insurer. In Figure 5.4, if insurer A only selects two direct competitors against which to compare its performance, then it should select insurers B and H. On the other hand, if A selects four competitors, then it should select insurers B, H, C, and G. After comparing its performance with that of its selected direct competitors, insurer A moves toward the direct competitor who earns the highest profit. This behavioral parameter reflects the following real-world behavior: (1) Although the whole insurance market is very competitive, most insurers in this complex system often compare their performance locally. For example, a large insurer only targets some competitors with a similar large size and a specialist firm only focuses on the competitors in its niche market. (2) With the assistance and contribution from different market participants (e.g. news agency, market regulator and industry association ABI), the scope of an insurer's comparison may change over time due to the increased speed of information flow in the market. This scope parameter can help us to test the impact of such changes in the real world.

Proportion of insurers who move

Parameter θ_3 is the proportion of insurers who can move on the circular street in Figure 5.4 and change their locations and thus their business strategy. If it is zero, then no insurer can move and the extended model returns to the base case of Chapter 4. If it is one, then all insurers are able

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[1]Use Int(x) function, that is calculated as rounding the number $\theta_2 \times 8 - 1$ to the next smaller integer, or be zero if it is less than one

to move in the system. We use it to analyze whether non-price strategical movement actually creates a benefit to insurers or not, and how much the impact of this parameter will be on the whole market performance. When insurers are able to move, they move toward the location of the direct competitor who has the highest profit. Therefore, the movement reflects so-called "Keeping or catching up with the Joneses" feature in Behavioral Economics. In other words, it leads to herding behavior if a group of insurers move together and toward the same direction. In fact, this parameter can help us to understand the impact of agents' herding activities. If a large percentage of insurers are herding toward a particular strategy, then it would be better for some insurers to stay in their individual market segments where there is less competition. Likewise, when most insurers stay fixed in their existing market segments, it would be better for some insurers to move and explore more profitable opportunities. This behavioral parameter can help us to analyze this kind of real-world dynamics, which is similar to Arthur's El Farol Bar problem (Arthur, 1994).

Local comparison

The above three behavioral parameters manage an insurer's strategical movement. An insurer's decision as to its next location depends on its comparison of performance with its direct competitors. Insurers carry out a local comparison as follows (we use insurer A in Figure 5.4 as an example):

1. At the beginning of each discrete time period, the simulation generates a $n \times n$ symmetric matrix, where n is the total number of insurers. Each cell in this matrix represents a distance (measured

along the circumference) between two insurers, but all cells in the main diagonal are zeros since the distance between an insurer and itself is zero.

2. In this distance matrix, the elements in the row corresponding to insurer A represent the distances between insurer A and all other insurers. The lowest distance is between insurer A and itself, i.e. $(A, A) = 0$. For example, an element (A, B) in the matrix is the distance between insurer A and insurer B, which is equal to another element (B, A). Based on this distance matrix, insurer A sorts other insurers by proximity, i.e. from the smallest distance to the largest one.

3. Depending on the scope of an insurer's movement (i.e. the behavioral parameter θ_2), insurer A selects an integer $(\theta_2 \times n - 1)$ number of other insurers from the lowest distance insurer to the $(\theta_2 \times n - 1)$th low distance insurer to to be its direct competitors. These insurers are the peer group against which the insurer compares itself. Other remaining insurers become irrelevant in the local comparison of insurer A.

4. After insurer A selects its group of direct competitors in each time period, it compares its own performance with this group of peers at the end of each time period. We use individual loss ratios instead of profits to measure the insurers' performance, since profit is an absolute value that can be affected by an insurer's size. There is also a $n \times n$ performance matrix in the simulation, which records all selected direct competitors' performance for each insurer.

5. Insurers can move along the circular street at the end of every time period, therefore both distance and performance matrices are updated. The peer group representing an insurer's direct competitors also changes in each time period, based on new information about insurers' locations.

The above procedure explains the steps in selecting an insurer's direct competitors with close locations (i.e. similar customer preference, market segment or product characteristics). This is a common activity in the real-world insurance market, since an insurer often compares its business performance with the competitors in the market segments that are similar to the insurer. Likewise, insurers often compare their performance with other insurers who have similar sizes, i.e. a large insurer is more interested to compare with other large insurers' performance than small insurers (Taylor, 2009). When selecting direct competitors with a similar size, an insurer follows a similar procedure to selecting direct competitors with a similar location.

In our model, we can control whether insurers select direct competitors based on the location or size alone, or both location and size. We have two "switch" parameters for strategy and size comparisons, which can only be either one or zero. When the "strategy comparison switch" is on (i.e. it becomes one) and "size comparison switch" is off (i.e. it becomes zero), each insurer will only select its direct competitors among the insurers with closest distance. Likewise, when the "strategy comparison switch" is off (i.e. it becomes zero) and "size comparison switch" is on (i.e. it becomes

one), each insurer will only select its direct competitors from insurers with similar sizes. If both the "strategy comparison switch" and "size comparison switch" are on, then each insurer will select half of its competitors based on size and the other half based on location (there be an overlap when these two criteria are used). In reality, insurers may weight these criteria when choosing a subset of competitors against which they compare themselves. This could be implemented and simulated, but we then face the difficulty of estimating a suitable weight. We have chosen an equal weighting for the sake of simplicity.

Location decision and strategical movement

After every insurer follows the above steps to select its own group of direct competitors, it identifies the highest profitable insurer among this peer group. The insurers start to move toward the location of the best performing competitor. This is implemented as follows:

1. Each insurer has access to all of its direct competitors' performances that are recorded in the $n \times n$ performance matrix. This means that an row A contains the loss ratios of all of A's direct competitors (loss ratios for unselected insurers are recorded as zero in this matrix).

2. There is an index matrix that defines the lowest loss ratio (i.e. except zeros in the performance matrix) in each row. This means that the insurer corresponding to this row will now move toward the location of this indexed competitor (i.e. the top performer in the peer group). For example, if the element (A, B) (i.e. the element in row A and column B) in the index matrix is equal to 1 and other elements in

row A are zeros, then insurer B is the top performer and insurer A will move toward the location of insurer B.

3. The index matrix maps the location of the selected best performing competitor for each insurer. For example, if insurer A selects its direct competitors as insurers B, H, C and G in Figure 5.4 and the competitor G has the best performance, then insurer A will move toward the location of competitor G in an anti-clockwise movement.

4. The movement is dependent on the speed parameter[2] θ_1. Similarly, if insurer H has an identical behavior rule, then it is more likely to move toward competitor G as well. This is because insurer H is close to insurer A and they have a similar peer group of direct competitors. Therefore, this kind of dynamics has the nature of herding. However, if some insurers are unable to move (i.e. parameter θ_3 manages the proportion of insurers who can move in the system), the market dynamics is affected by this group of fixed insurers. Both local comparison and herding behavior cause some problems of overheating the competition, which means insurers compete too high in some market segments but too low in other segments.

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[2]Ideally, the speed parameter should be implemented as a maximum speed of an agent's movement, i.e. agents can move at any speed up to this maximum and the exact speed can be dependent on a more complicated decision rule. However, for simplicity, we assume a fixed speed, it means that agents move by θ_1 at each time step. We try to keep θ_1 small enough to avoid excessively large jumps on the space. This implementation is similar to many 2D ABM models as agents move from one cell to a neighboring cell.

5.4.3 Simulation results

When running the simulations of this model extension 2, we use the same parameter values from the base case in Chapter 4, except these behavioral parameters as discussed in this section. We test ten different scenarios (i.e. ten different combinations of three behavioral parameters) of this extended model, they are:

Scenario 1: (parameter values $\theta_1 = \theta_2 = \theta_3 = 0$) This is a special case where we fix the locations of all agents in the system, therefore it is also the base case of Chapter 4.

Scenario 2: (parameter values $\theta_1 = 0.01$ $\theta_2 = 0.2$ $\theta_3 = 0.2$) We use relatively small values for the behavioral parameters, this means that agents move relatively slow and they compare their performance with a small peer group of direct competitors, and only a few of them can move their locations in the system. In this Scenario 2, it defines that 20% of agents in the system can move and 80% cannot (i.e. $\theta_3 = 0.2$), and they only compare their performance with 20% of the insurers who have similar size and location (i.e. $\theta_2 = 0.2$), and finally they move 1% of the total distance (i.e. the circumference of the circular street in Figure 5.4) in each time period (i.e. $\theta_1 = 0.01$).

Scenario 3: (parameter values $\theta_1 = 0.01$ $\theta_2 = 0.2$ $\theta_3 = 0.5$) We increase the proportion of insurers who can move their locations from 20% to 50% (i.e. $\theta_3 = 0.5$) and keep other parameters unchanged as in Scenario 2, therefore more insurers are able to change their non-price strategies.

Scenario 4: (parameter values $\theta_1 = 0.01$ $\theta_2 = 0.2$ $\theta_3 = 0.8$) We increase the proportion of insurers who can move their locations from 50% to 80% (i.e. $\theta_3 = 0.8$) and keep other parameters unchanged as in Scenario 2, therefore more insurers are able to change their non-price strategies.

Scenario 5: (parameter values $\theta_1 = 0.01$ $\theta_2 = 0.2$ $\theta_3 = 1$) We further increase the proportion of insurers who can move their locations from 80% to 100% (i.e. $\theta_3 = 1$) and keep other parameters unchanged as in Scenario 2, therefore more insurers are able to change their non-price strategies.

Scenario 6: (parameter values $\theta_1 = 0.01$ $\theta_2 = 0.2$ $\theta_3 = 0.5$) We use this Scenario 6 to compare with Scenario 3, we let insurers to compare their performance with the selected direct competitors *who have similar location but not similar size*, instead of comparing direct competitors who have both similar size and location in Scenario 3.

Scenario 7: (parameter values $\theta_1 = 0.01$ $\theta_2 = 0.2$ $\theta_3 = 0.5$) We use this Scenario 7 to compare with Scenario 3, we let insurers to compare their performance with the selected direct competitors *who have similar size but not similar location*, instead of comparing direct competitors who have both similar size and location in Scenario 3.

Scenario 8: (parameter values $\theta_1 = 0.01$ $\theta_2 = 0.8$ $\theta_3 = 0.5$) We increase the insurers' scope of comparing their performance from $\theta_2 = 0.2$ in Scenario 3 to $\theta_2 = 0.8$ and keep other parameters unchanged as in Scenario 3.

Scenario 9: (parameter values $\theta_1 = 0.05$ $\theta_2 = 0.2$ $\theta_3 = 0.5$) We increase the speed of agent's movement from $\theta_1 = 0.01$ in Scenario 3 to $\theta_1 = 0.05$ and keep other parameters unchanged as in Scenario 3.

Scenario 10: (parameter values $\theta_1 = 0.05$ $\theta_2 = 0.8$ $\theta_3 = 0.5$) We increase both the insurers' scope of comparing their performance (from $\theta_2 = 0.2$ in Scenario 3 to $\theta_2 = 0.8$) and the speed of agent's movement (from $\theta_1 = 0.01$ to $\theta_1 = 0.05$) and keep the proportion of insurers who can move unchanged as in Scenario 3.

Based on these ten different scenarios, we aim to understand the impacts of these behavioral parameters on both the overall market dynamics and individual insurers' performance. These different scenarios help us to answer several questions: (1) What is the impact on insurance market cycle when (more) insurers are able to move their non-price strategies? (2) What is the impact on insurers' performance when insurers move faster? (3) What is the impact when insurers compare a larger peer group to make their decisions of movement? (4) What are the market results when insurers select their peer groups based purely on firm size, or location, or both?

Market performance

Table 5.4 includes the market performance (i.e. the mean and standard deviation of market loss ratios) of the above ten different scenarios: we use "Slow/Fast" to represent two different speed parameter values $\theta_1 = 0.01/0.05$ and "Small/Large" to represent two different scope parameter values $\theta_2 = 0.2/0.8$, also "MeanLR" and "StdevLR" are the mean and standard deviations of simulated loss ratios over 1000 time periods. All

simulations are based on a same claim sample and other parameters remain unchanged as in the base case of Chapter 4.

Table 5.4: Scenario testing: extension 2 (market performance)

Scenarios	Speed	Scope	% MovingAgents	MeanLR (Market)	StdevLR (Market)
Scenario 1	Fixed	Fixed	0%	0.97	0.08
Scenario 2	Slow	Small	20%	0.99	0.09
Scenario 3	Slow	Small	50%	1.00	0.13
Scenario 4	Slow	Small	80%	1.05	0.09
Scenario 5	Slow	Small	100%	1.08	0.06
Scenario 6	Slow	Small	50%	1.02	0.13
Scenario 7	Slow	Small	50%	1.01	0.13
Scenario 8	Slow	Large	50%	1.01	0.10
Scenario 9	Fast	Small	50%	1.03	0.09
Scenario 10	Fast	Large	50%	1.03	0.10

The summary of these results is as follows:

- Comparing Scenario 2 with Scenario 1 that allows insurers to move their non-price strategies to compete in the system, the overall market performance becomes worse: the mean of market loss ratio increases from 0.97 to 0.99 and the standard deviation increases from 0.08 to 0.09. This is sensible, because the market becomes more competitive if the insurers are able to move their non-price strategies.

- Comparing Scenarios 1,2,3,4,5 when the proportion of moving agents increases, the market performance becomes worse and worse: the mean of market loss ratio increases. However, the standard deviation increases initially before the proportion of moving agents reaches to a half, then decreases from 50% to 100%. This pattern tells us that the volatility of market performance is affected by the proportion of moving insurers in the market, but the relationship is not linear.

- Comparing Scenario 3 with both Scenario 6 and Scenario 7, where the differences are the insurers in Scenario 3 select their direct competitors based on both size and location while Scenario 6 only looks at location and Scenario 7 only looks at size. The standard deviation of loss ratios is similar, but Scenario 3 has a slightly better average market performance than both Scenario 6 and Scenario 7. This suggests that it is better for insurers to compare the competitors who have both similar size and location when deciding to move their locations.

- Comparing Scenario 3 with Scenario 8, insurers in Scenario 8 select more direct competitors into their peer groups when comparing their performance than in Scenario 3. As a result, this brings more competition into some locally profitable market segments. It changes the profitability of these market segments and leads a high average market loss ratio, due to the pressure from high competition. However, a large value of "scope parameter" in Scenario 8 also provides more flexibility for insurers to move their next period strategies, which creates more opportunities to earn higher profits at the same time, therefore the actual impact on market performance is hard to be defined.

- Comparing Scenario 3 with both Scenario 9 and Scenario 10, insurers' speed of moving their locations are faster in both Scenario 9 and Scenario 10 than in Scenario 3. This leads to have a more competitive market and produce a lower market performance on average (i.e. the mean of loss ratios increases from 1.00 in Scenario 3 to 1.03.).

Individual insurers' performance

After testing the market performance, we divide insurers into two groups: one group contains all insurers who are able to move (i.e. we call it as "moving group") and the other group includes those insurers cannot move (i.e. "fixed group"). We look at the average performance of these two groups and the summary is in Table 5.5. On average, the insurers in the fixed group perform better than those in the group with moving insurers. This is because moving agents often follow a herding behavior and compete with each other aggressively. Although some of the fixed insurers individually perform badly in some periods when their local competition is high as the moving agents invade their local market segments, other fixed insurers who are far away from the areas of intense competition perform better. Therefore, the standard deviation of the loss ratio of the fixed group is higher than the moving group.

Table 5.5: Scenario testing: extension 2 (individual performance)

Scenarios	MeanLR (Fixed)	StdevLR (Fixed)	MeanLR (Moving)	StdevLR (Moving)
Scenario 1	0.9706	0.0799	n/a	n/a
Scenario 2	0.9576	0.0918	1.0084	0.0521
Scenario 3	0.9736	0.0847	0.9997	0.0575
Scenario 4	1.0140	0.0664	1.0216	0.0444
Scenario 5	n/a	n/a	1.0241	0.0465
Scenario 6	0.9628	0.0860	1.0213	0.0365
Scenario 7	0.9641	0.0862	1.0000	0.0407
Scenario 8	0.9576	0.0949	1.0099	0.0569
Scenario 9	0.9985	0.0695	1.0156	0.0544
Scenario 10	0.9710	0.0942	1.0217	0.0467

Herding behavior and insurance cycle over time

We also look at individual insurers' performance over time, to understand the impact of herding behavior on the market dynamics (e.g. insurance cycle) in the system.

Firstly (testing the speed of insurers' movements), we compare Scenario 3 with Scenario 9, both cases have 50% of insurers move their locations (i.e. $\theta_3 = 0.5$) and all of these insurers select a small number of direct competitors to compare their performance (i.e. $\theta_2 = 0.2$). The only difference is the speed of insurers' movements (i.e. slow $\theta_1 = 0.01$ in Scenario 3 and fast $\theta_1 = 0.05$ in Scenario 9). Therefore, we compare the impact of insurers' speed on the herding dynamics and market cycles.

Figure 5.5 illustrates the movements of 20 insurers in the circle city. The x-axis represents the time period and the y-axis defines the locations of 20 insurers over time who are initially distributed equally on the circular street. There are 10 insurers start to move their locations at the beginning of simulation based on the behavioral parameters (i.e. we call them as "moving agents") and the rest of insurers stay at their original positions over time (i.e. "fixed agents"). The insurers who are able to move their locations will always move, and those stay at one location will always fix at that location. This implementation may represent two types of insurers in the real-world insurance market: one group of insurers always focus on their special business strategies (i.e. niche market), while another group of insurers change their strategies according to the profitability in different

market segments. The moving insurers move toward the best performing competitors in their individual peer groups. The competition in local market segments are changed by the movements of insurers, so insurers move from less profitable market segments to other segments with higher potential profits. Some moving insurers change their directions before they reach the most profitable area if the speed of movement is slow. Following the movements of insurers in Figure 5.5 over time, the whole market is divided into two main parts at the end of simulation.
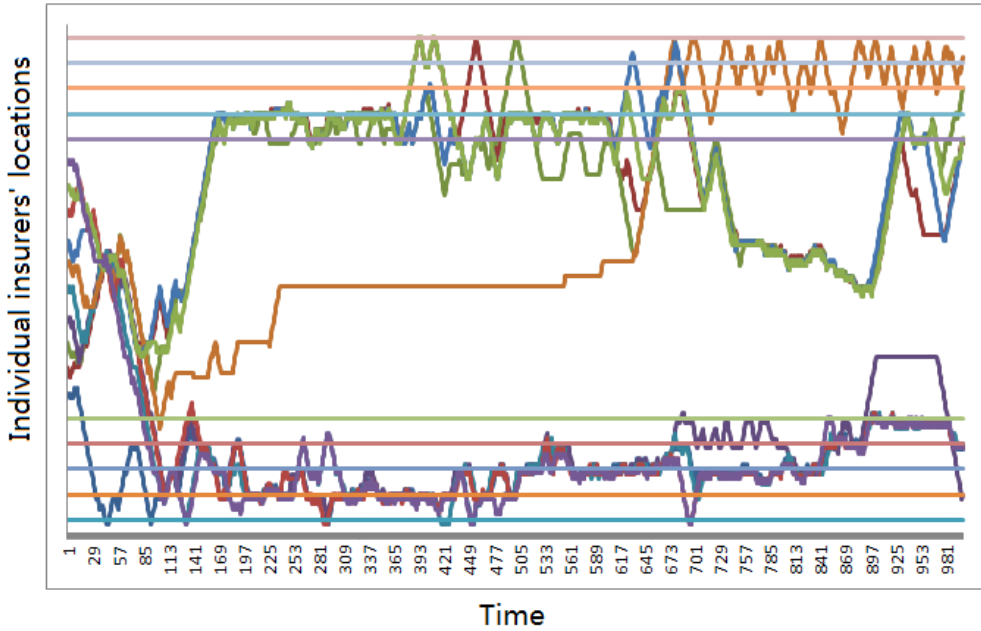
Figure 5.5: Scenario 3 Insurers' location movements

Figure 5.6 shows the average market loss ratios for the two groups of insurers (i.e. a group of 10 insurers who are able to move and another group of 10 insurers are fixed) over four time intervals. The x-axis represents four time intervals and the y-axis is the mean of loss ratios. Each time

interval includes 250 time periods of loss ratios (e.g. the periods from 1 to 250 years are included in the first time interval on the x-axis). From the figure, we have two conclusions: (1) Fixed insurers have better performance than moving insurers over time, so the herding behavior is bad for moving insurers in this system on average. (2) Insurers' performance change over time: moving agents initially improve their performance by adjusting and moving their strategies, so they earn some profits from some fixed insurers. However, once the moving insurers herd together and start to compete with themselves, their performance become worse and worse.
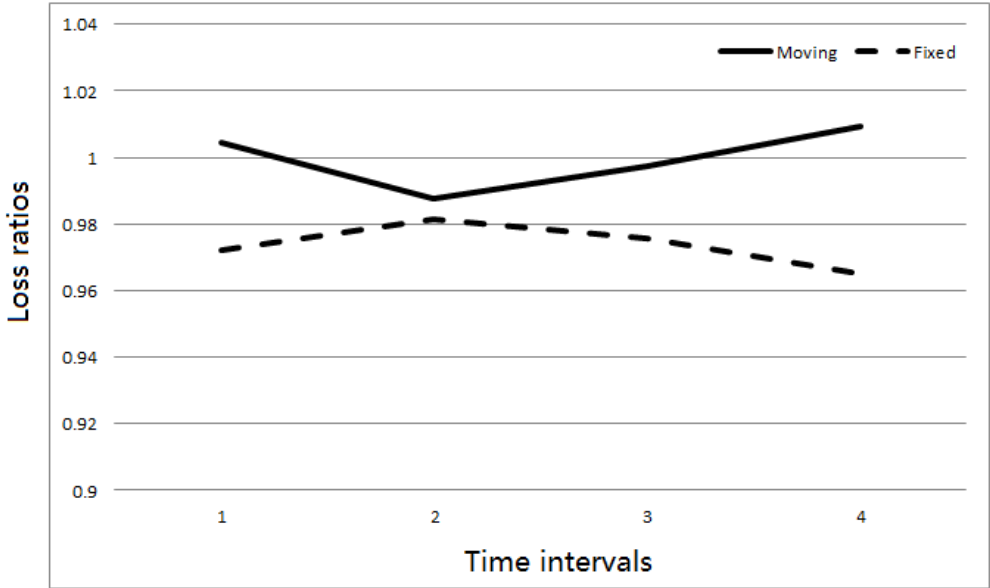
Figure 5.6: Scenario 3 Average performance of two insurer groups

Figure 5.7 of Scenario 9 shows a similar diagram of location movement in Figure 5.5 of Scenario 3. The difference is that insurers herd together faster and closer in Scenario 9 than in Scenario 3, because the speed of insurers' movements are increased in Scenario 9. Moving agents shift be-

tween the top and bottom market segments, this generates individual profit cycles in these two local markets. In each market segment, average profits increase when moving insurers leave, vice versa.
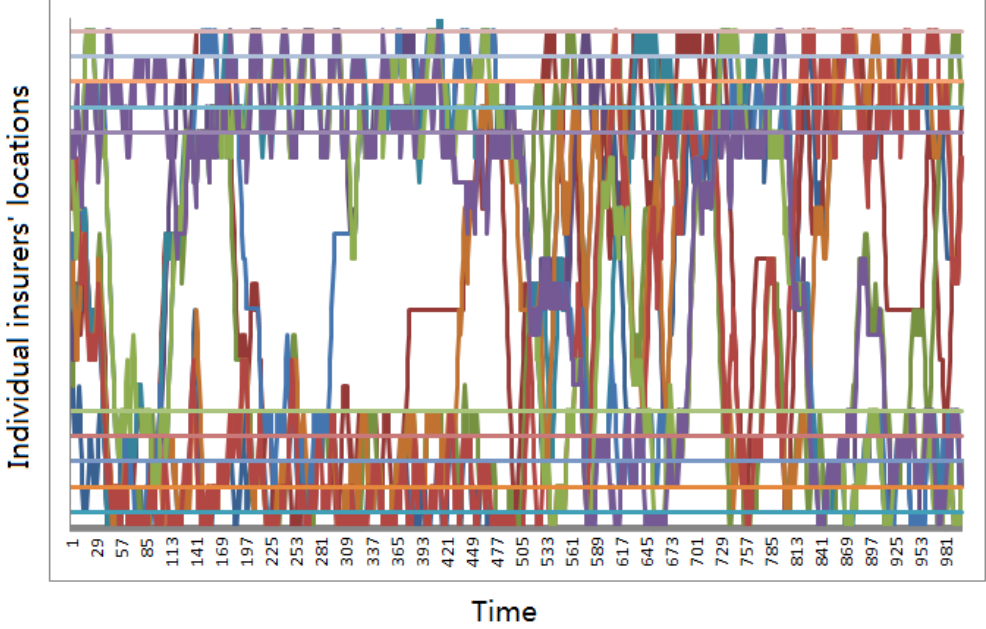

Figure 5.7: Scenario 9 Insurers' location movement

Figure 5.8 shows the average performance of two groups of different insurers (i.e. fixed and moving) over 4 time intervals. There are cyclic patterns in both insurer groups. When moving agents enter the fixed insurers' market segments, the average performance of fixed insurers becomes worse due to a higher competition. The individual performance cycle in each group is due to the movements of moving agents.
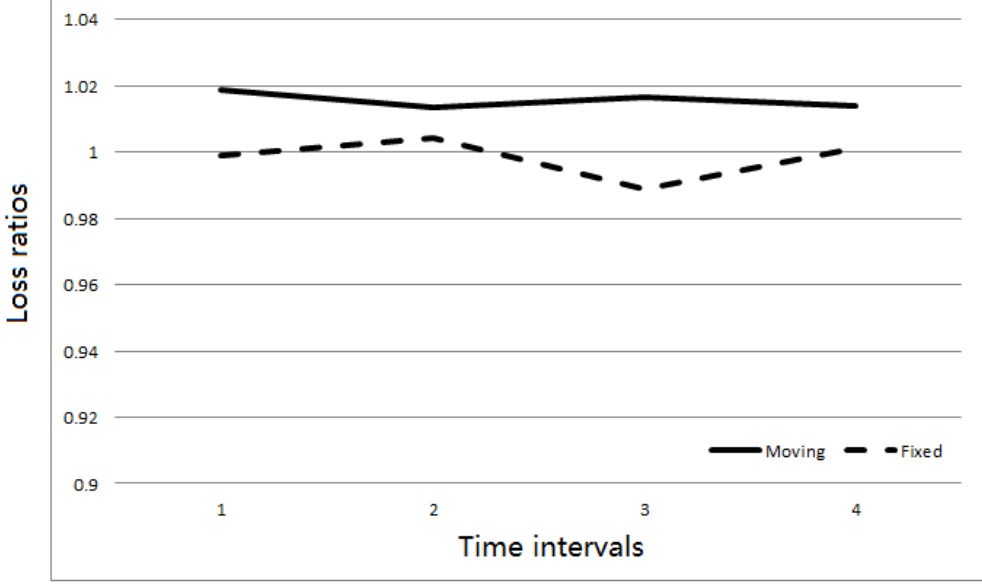
Figure 5.8: Scenario 9 Average performance of two insurer groups

Secondly (testing the scope of insurers' comparison), we compare Scenario 3 with Scenario 8, both cases have 50% of insurers move their locations (i.e. $\theta_3 = 0.5$) and all of them move their locations with a slow speed (i.e. $\theta_1 = 0.01$). The only difference is the scope of insurers' local comparison (i.e. $\theta_2 = 0.2$ in Scenario 3 and $\theta_2 = 0.8$ in Scenario 8) that means moving insurers in Scenario 3 select 20% of other insurers to compare their performance and move toward the best performing competitors, but they select 80% of other insurers in Scenario 8. Therefore, we compare the impact of insurers' scope on the herding dynamics and market cycles.

Figure 5.9 is the location movement of Scenario 8. It is similar to the above Figure 5.5 of Scenario 3 and Figure 5.7 of Scenario 9. Comparing to Figure 5.5 of Scenario 3, the scope of insurers' local comparison leads

insurers to observe the best performing competitor more globally in the system, so moving agents are more likely to herd together and compete in same local market segments. All of these moving agents move between profitable market segments, but the profitable market segments become less profitable after they move in.
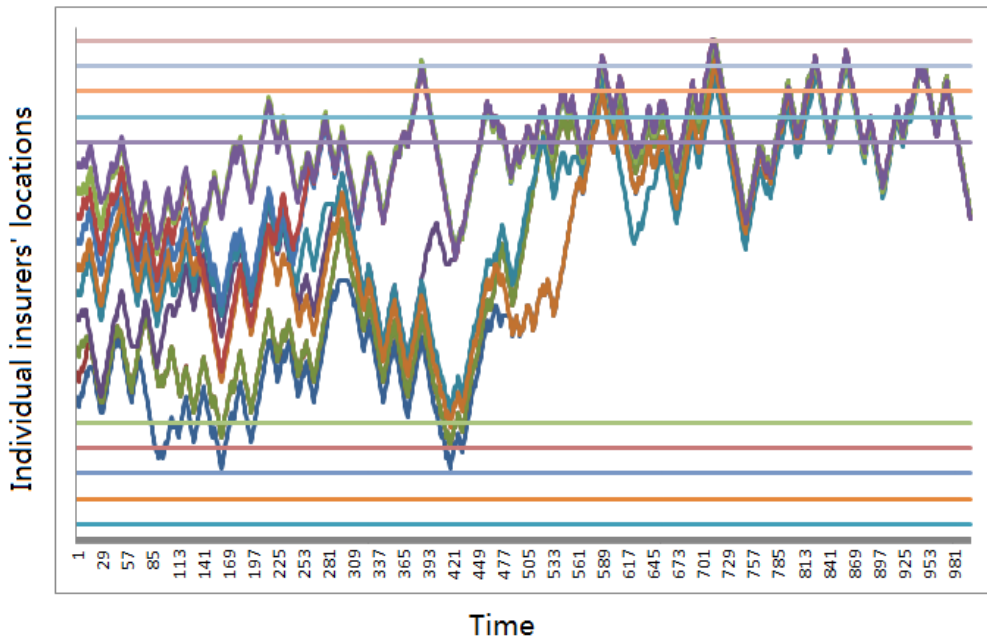

Figure 5.9: Scenario 8 Insurers' location movement

Figure 5.10 shows a cyclic pattern of performance in the group of fixed insurers. This is due to the dynamics of "move in and move out" of moving insurers. The performance of moving agents become worse over time, since they herd together closer over time and compete with themselves more aggressively.
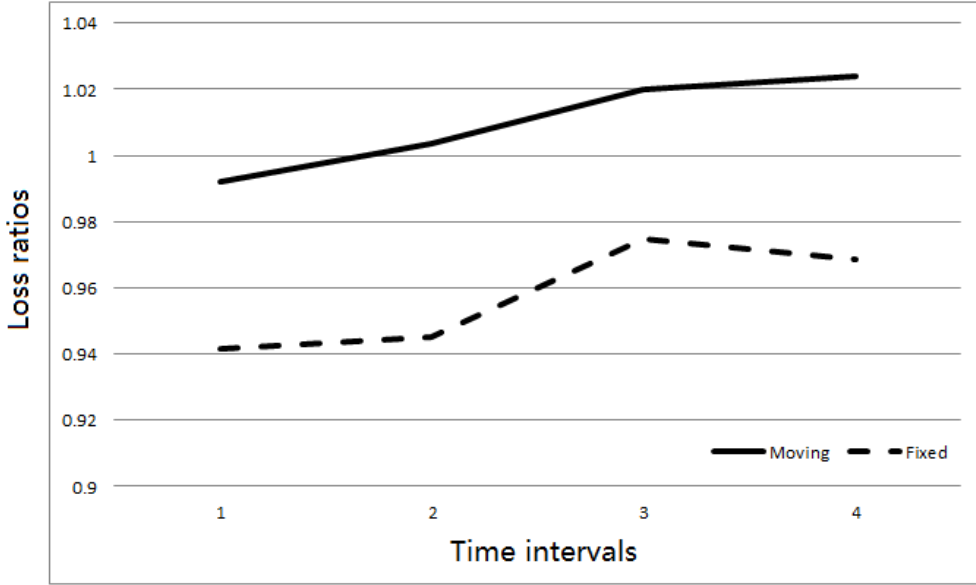
Figure 5.10: Scenario 8 Average performance of two insurer groups

Finally (testing the proportion of insurers who can move), we compare Scenario 2, Scenario 3, Scenario 4 and Scenario 5, all of these four scenarios have moving insurers who move slowly and compare a small number of direct competitors to decide their next movements. However, the number of moving agents in the system increases from 4 to 20. We look at the impact of this change on the insurers' performance and market dynamics. Figure 5.11 illustrates the location movements for all scenarios: Scenario 2 is the top-left diagram, Scenario 3 is top-right, Scenario 4 is bottom-left and Scenario 5 is the bottom-right diagram. In consistent with traditional location model (Hotelling, 1929), the long term equilibrium is for moving agents to herd together, particularly in Scenario 5 when all insurers can move.

Figure 5.12 includes the individual performance of two insurer groups

Figure 5.11: Scenarios 2,3,4,5 Insurers' location movement
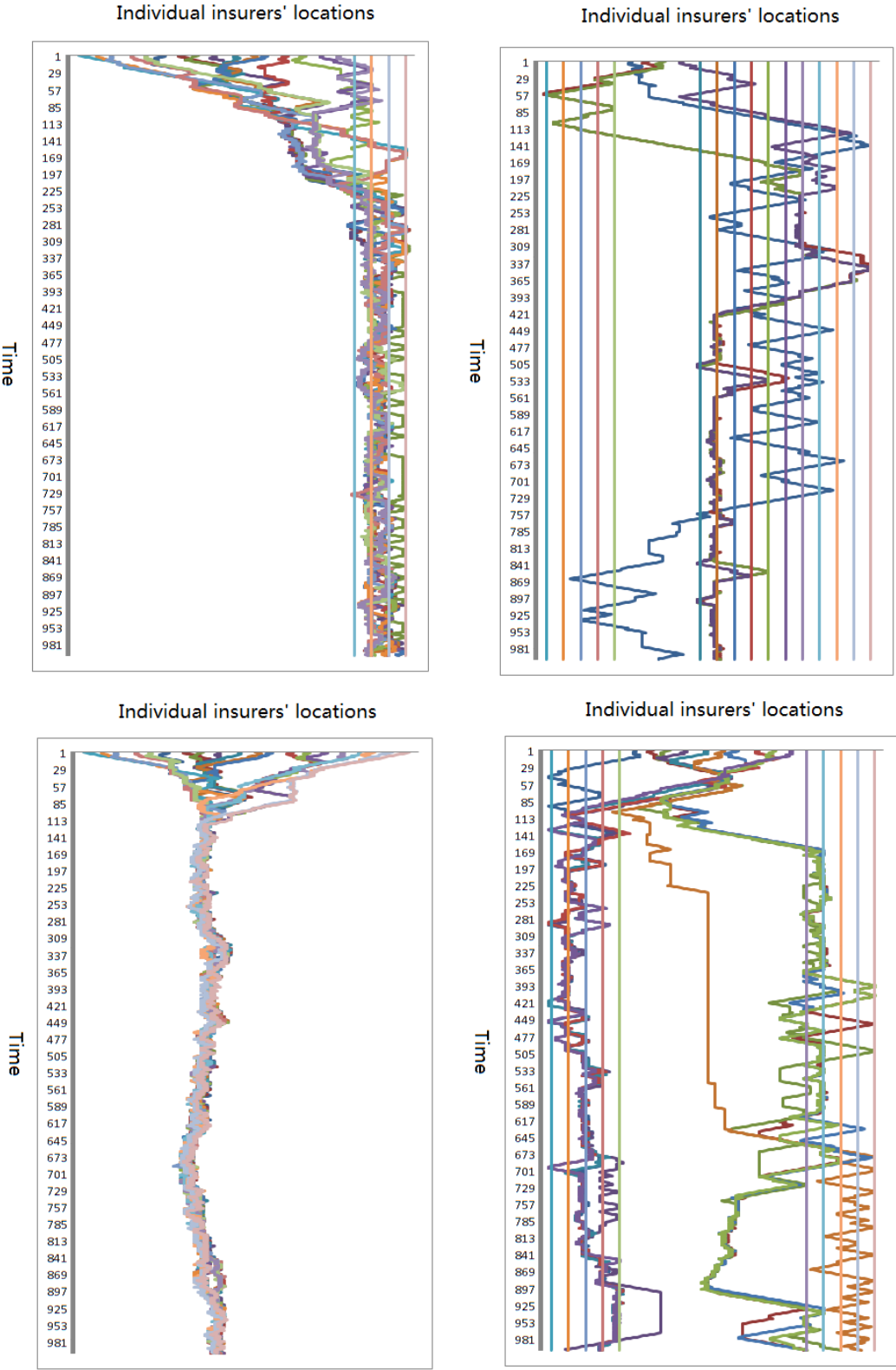
(i.e. fixed and moving groups) for each corresponding case. There are a few interesting conclusions: (1) Herding behavior is bad for moving agents, since the performance is worse than fixed agents on average. However, as discussed in the traditional location model (Hotelling, 1929), the best strategy for moving agents is to keep together based on game theoretical analysis and the system achieves a Nash equilibrium. (2) The market competition is increased by increasing the proportion of moving insurers. Although this is good to the customers as the price of insurance is decreased due to the competitive nature, some insurers suffer large losses in the competition. This is because they follow similar non-price strategies, so the only way to compete is to reduce their prices (i.e. less on product differentiation). More detailed simulation results of cycles in the individual insurers' local markets are included in Appendix A.5.

5.4.4   Conclusion

In this model extension 2, we take the benefits of ABM that aim to target insurers' individual behavior from a bottom-up approach to understand the market dynamics. We are not only able to understand the overall insurance market cycle that is based on the base case analysis of Chapter 4, but also understand the individual insurers' performance in their local market segments and the interactions between agents with different behavior rules. Individual local market segments often have cyclic performance patterns as well, since the moving agents continually adapt their business strategies and move between less profitable segments to more profitable ones. The herding behavior creates high competition among themselves in their local
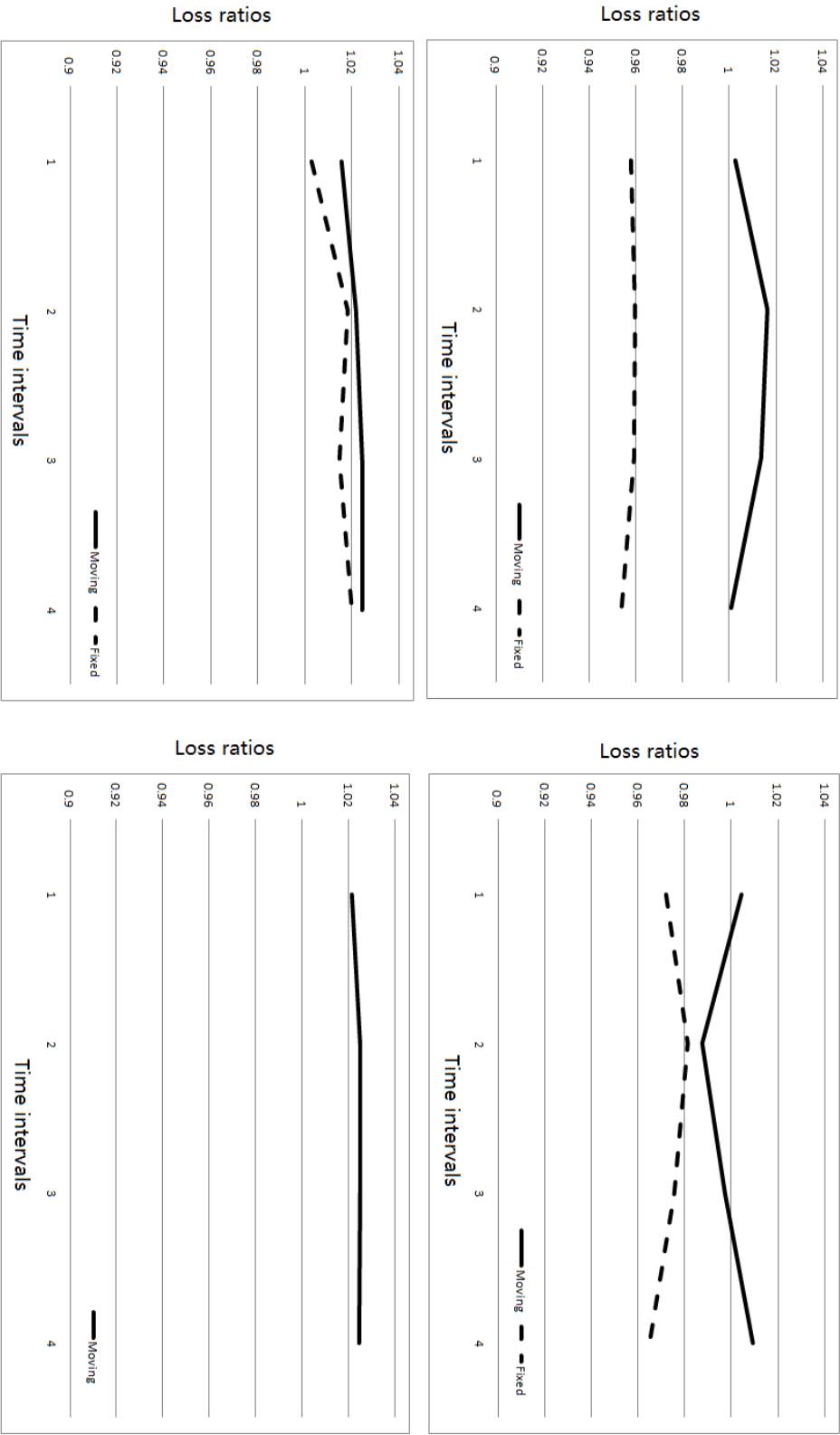
Figure 5.12: Scenarios 2,3,4,5 Average performance of two insurer groups

market segments. This kind of cyclic behavior in different market segments is consistent with literature in insurance cycle. Feldblum (2001) discusses that the shift of insurers' strategical focuses may cause cycles within the market. He finds that the pattern of the cycle differs by line of insurance business. Berger et al. (1999) analyze the merit of strategic focus in the insurance market. They find that some specialists are worth to fix their strategies in particular niche markets (i.e. this suggestion is similar to the fixed agents in our model). Gron and Winton (2001) find different lines of insurance business have different dynamic patterns of cycle. They suggest that this may be due to the capital flow between these different lines within the insurance market over time. Chen et al. (2007) compare the difference between nonspecialized strategy insurers with specialized ones. They conclude that nonspecialists are more efficient for some types of insurers, whereas specialists are more efficient for other types. The type of insurers depends on: whether insurers are large or small, in personal line or commercial line of business, cost efficient or not, in a high or low competitive market segments, etc.

5.5   Extension 3: 2-D Planar Space

5.5.1   A more generalized system

In this model extension 3, we provide a more generalized version of our ABM of insurance market. Although an 1-D circle city is enough for us to better understand the insurance cycle than traditional models, a 2-D framework is more helpful for the users of ABM to visualize the whole dy-

namics of the market. It is a common implementation in the field of ABM research, as we introduced some existing 2-D ABM examples in Chapter 3, such as Forest-fire model (Mills, 2010) and Sugarscape model (Palin et al., 2008). In this section, we summarize the main updated elements of our ABM in this extension. This is followed by a comparison of the simulation results on both aggregate market and individual insurers' cycles from this 2-D extension with the early three 1-D versions.

From 1-D circle city to 2-D planar space

Figure 5.13 illustrates how we extend the original 1-D circle city (i.e. left diagram) to a 2-D planar space (i.e. right diagram). In the left diagram, insurers are equidistantly located and customers are uniformly distributed along the circular street. Now, in the right diagram, both insurers (i.e. red dots) and customers (i.e. blue bubbles) are randomly distributed in the system. A key difference between these two different dimensional spaces is the corner or border effect when agents measure the distances among themselves and customers. The original 1-D circle city model is introduced by Salop (1979), in order to avoid the corner effect of Hotelling's linear street model (Hotelling, 1929). In our case, the system is closed by the four boundaries and each agent's location is defined by a coordinate in the xy-plane, therefore their distances are also measured by Euclidean distance. The reason for us to build this 2-D space with corner or border effect is the following: if we think this 2-D planar space as a strategy map of insurers that defines customers' product preference in motor insurance sector. The x-axis defines the product preference of customers with different ages from young to old, while the y-axis defines the preference of

customers with different market value vehicles from low to high. Therefore, any single blue bubble in the system defines a product preference of one customer at a particular age with a specific car. Insurers want to target different customer groups, as defined by the red dots. Insurers' movements in the space represent their strategical decisions to target different market segments. As an example of moving horizontally from left to right, this means that insurers target from young customers to old age groups. We think, in reality, young customers (e.g. 18 years old) have very different insurance product preference to old customers (e.g. 70 years old), but they are more similar to the age group who are close to them (e.g. age between 20 and 30 years old.) Therefore, for insurers who are changing their strategical targets, it is more likely for them to move smoothly between different market segments with similar preferences (i.e. They are moving from 18 to 20-30s years old market segments first, then moving towards 70 years old.). In this case, once insurers reach a particular corner or border, they will not cross the corner or border. [3]

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[3]Nevertheless, it is also possible to implement the system without corner or border effect, which can be represented by a torus or sphere. As Garcia (2005, p. 390) describes that "In networked ABMs, environments (or systems) are frequently modeled where agents of a similar type are spatially connected to neighbors in a torus (donut-shaped). In a two-dimensional space, the agent space looks like a lattice, but in a three-dimensional space, agents are on the edge of a lattice interact with their neighbors on the other side of the lattice, forming a donut-shape."
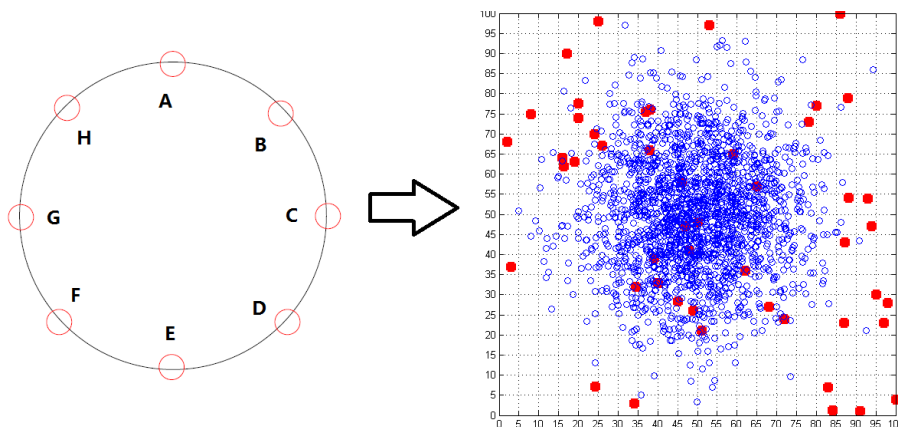
Figure 5.13: A more generalized system

Distribution of customers

Figure 5.14 provides a more realistic setup that models a real-world situation regarding customers' preferences on the 2-D map of product characteristics. Instead of distributing customers' preferences uniformly on the circular street that represents different product characteristics, the customers can be distributed using some density functions which are more representative of real-world situations. In reality, some market segments (i.e. product characteristics) have more customers (e.g. mainstream products) than other segments that only focus on some niche businesses. For simplicity, as an example in Figure 5.20, we assume that the customers are normally distributed on the two dimension in the product preference space. This would not be unreasonable, if we define the 2-D space as the motor insurance market that are segmented by the market value of the insured cars (e.g. x-axis) and customers' age (e.g. y-axis), since both data sets can be collected from ABI database.
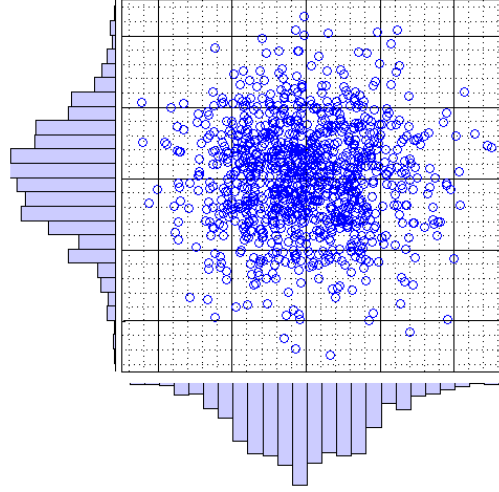
Figure 5.14: Distribution of customers

Insurers' behavior

Figure 5.15 illustrates the behavior of insurers in this 2-D space. As we explained in the 1-D circle city, insurers only compare their direct competitors (whom we call "neighbor") who have close distances between each other along the circular street and similar sizes in the system, and insurers only move their locations along the street in the model extension 2. In this 2-D model extension 3, insurers have more dimensions to select their direct competitors, but they still select direct competitors according to distance and size. The distance between two insurers is calculated by the Euclidean distance in a 2D plane, but it can be also implemented as the 'Manhattan' distance without changing the model results (Ladley and Rockey, 2010). If insurers are allowed to move, they will also have more flexibility of movement in terms of how they change their non-price business strategies in this 2-D space. In the Figure 5.15, we can use the size of bubbles to represent different insurers' capital levels. This helps the users of our ABM to

visualize the process of capital accumulation. The center of these bubbles defines the location of insurers, and therefore the distance is a straight-line between two centers that is measured by Euclidean Distance of two X-Y coordinates on the 2D space. For example, in the bottom left corner, there are three insurers I_1, I_2 and I_3 who are comparing each other. The lines between their centers are the distances and the sizes of the bubbles are their capital levels.
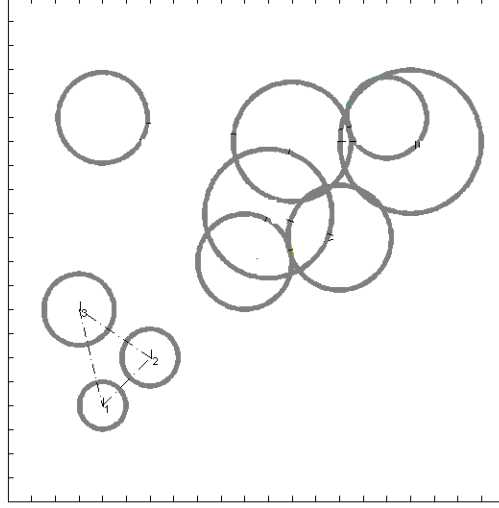

Figure 5.15: Insurers in the 2-D space

5.5.2   Simulation results

We use the same initial parameter values of the base case in Chapter 4 to run all simulations of this model extension 3. We follow three stages to compare the results from this more generalized 2-D extension with the results of early three different 1-D cases of our ABM. The three stages are: (1) Comparing with the base case model in Chapter 4, which ignores the role of capital in an insurer's price function. (2) Comparing with the model

extension 1, which includes the role of capital in insurer's price equation
and the positive short-term relationship between capital and price. (3)
Comparing with the model extension 2, that allows insurers to move their
locations and tests individual insurers' performance. Based on this three-
stage comparison of this 2-D version and the early 1-D versions, we find
that the results of early 1-D versions are robust and consistent with this
2-D extension.

Comparing results with Chapter 4 base case

Figure 5.16 shows the autocorrelation (AcF: left diagram) and partial au-
tocorrelation (PAcF: right diagram) of market loss ratios, based on the
same initial parameter values in Chapter 4 base case and the framework
of this extended 2-D system. Insurers have the same price function as in
Chapter 4 base case model and they are unable to move their locations in
the 2-D space. Comparing Figure 5.16 with Figure 4.8 of Chapter 4 base
case, it is also strongly suggestive of an AR(2) process and the coefficients
of the first two lags are similar to Chapter 4 base case. Therefore, we find
that the results of Chapter 4 base case are robust and appear to hold in a
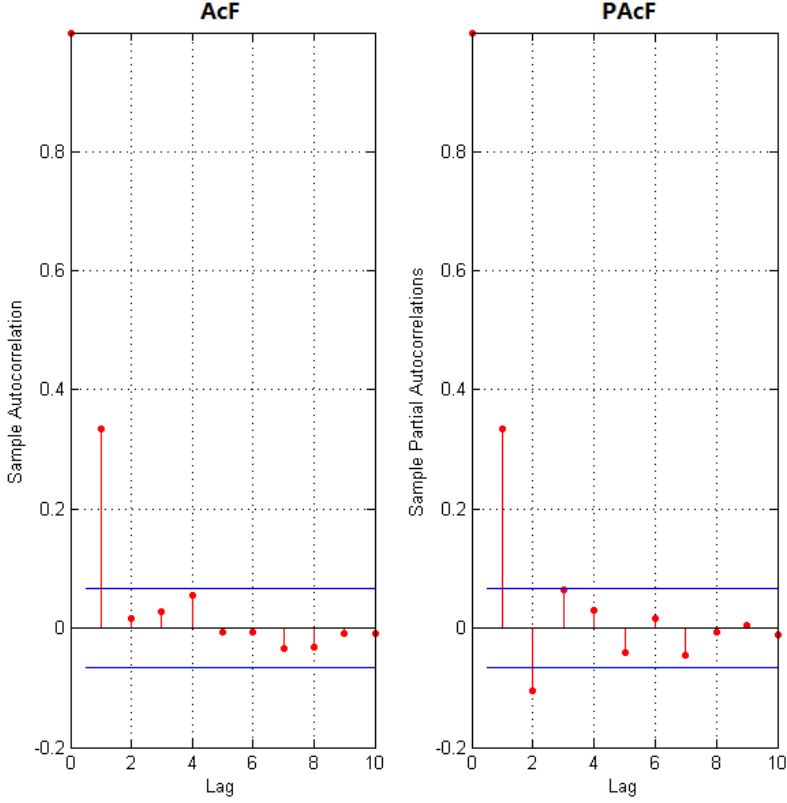more generalized setting.

Figure 5.16: Loss ratio: AcF and PAcF (comparison with Chapter 4 base case)

Comparing results with Chapter 5 extension 1

Figure 5.17 also shows the autocorrelation (AcF: left diagram) and partial autocorrelation (PAcF: right diagram) of market loss ratios, based on the same initial parameter values in Chapter 4 base case and the framework of this extended 2-D system. However, we update the insurer's price function that is in line with the model extension 1, which includes the role of capital level in the price function. Figure 5.17 provides a similar conclusion of model extension 1. Comparing Figure 5.17 with Figure 5.16, the autocorrelations at both lags 1 and 2 are amplified in 5.17 which agrees with the

conclusion in model extension 1. The level of capital has a contributing impact on the cyclic pattern of market loss ratios over time, but the main endogenous source of cycle is the interaction of insurers' competition in a monopolistically competitive market.
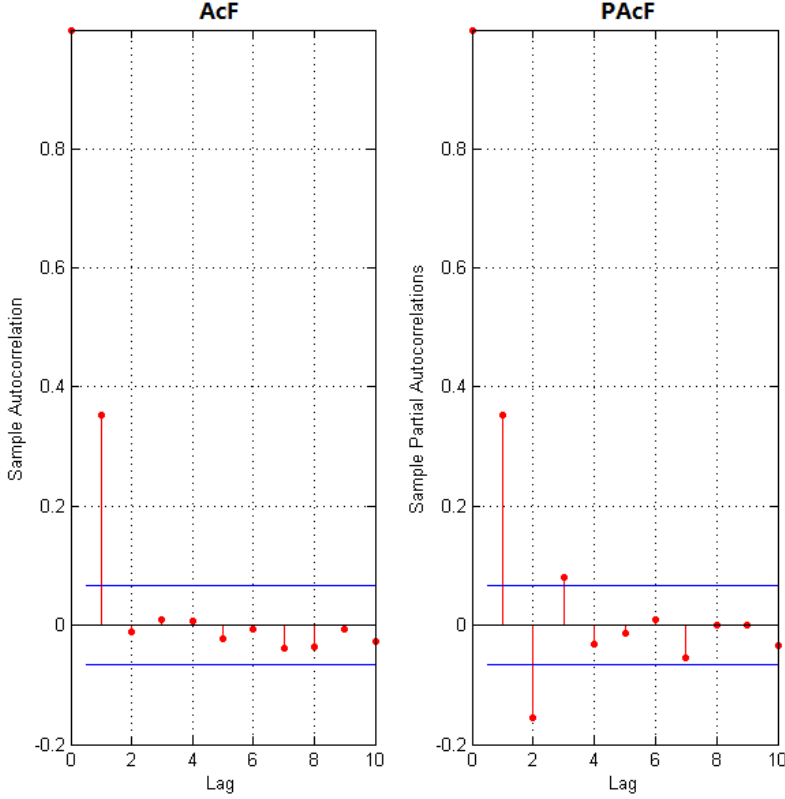

Figure 5.17: Loss ratio: AcF and PAcF (comparison with Chapter 5 extension 1)

Comparing results with Chapter 5 extension 2

We also allow insurers to move in this 2-D space, based on the same procedures of model extension 2. We test the impacts of three key behavioral parameters θ_1, θ_2, and θ_3, as summarized in the early Table 5.3 when we discussed the model extension 2.

Table 5.6 contains the mean and standard deviation of market loss ratios of all ten scenarios that are suggested in the model extension 2.

Table 5.6: Scenario testing: extension 3 (market performance)

Scenarios	Speed θ_1	Scope θ_2	%ofMovingAgents θ_3	MeanLR (Market)	StdevLR (Market)
Scenario 1	0	0	0%	0.90	0.13
Scenario 2	0.01	0.2	20%	0.93	0.18
Scenario 3	0.01	0.2	50%	0.96	0.25
Scenario 4	0.01	0.2	80%	1.02	0.30
Scenario 5	0.01	0.2	100%	1.06	0.10
Scenario 6	0.01	0.2	50%	0.92	0.05
Scenario 7	0.01	0.2	50%	0.93	0.10
Scenario 8	0.01	0.8	50%	0.95	0.12
Scenario 9	0.05	0.2	50%	0.97	0.29
Scenario 10	0.05	0.8	50%	0.98	0.30

Both mean and standard deviation have similar patterns as the conclusions of extension 2. To compare these results in Table 5.6 with extension 2 in Table 5.4, we found the following similarities and differences:

- Both extensions show that if insurers are able to move their non-price strategies, the market becomes more competitive and has less profit. Both extensions also show when both moving and fixed agents are more mixed in the system, the overall market profit decreases and risk (volatility) increases on average. Also, when the speed of insurers' movement is increased that leads to have a more competitive environment as in extension 2, it produces a lower and more volatile market performance. Similarly, due to the same conclusion in extension 2 when comparing scenario 3 with scenario 8, the impact of insurers' scope parameter on market performance is inconclusive.

- However, when comparing Scenario 3 with both Scenario 6 and Scenario 7, where the differences are the insurers in Scenario 3 select their direct competitors based on both size and location while Scenario 6 only looks at location and Scenario 7 only looks at size. The results are different from the model extension 2, both Scenario 6 and Scenario 7 have better performance than Scenario 3. This may suggest that insurer is better off to limit their comparison benchmarks in a more complex system, otherwise they are too volatile to change their non-price business strategies.

Table 5.7 contains the mean of loss ratios for two different insurer groups: a group of insurers who are able to move (i.e. moving agents) and the other group of insurers who cannot move (e.g. fixed agents). The results are consistent with the conclusion of model extension 2: On average, the insurers in the fixed group perform better than those in the group of moving insurers. This is because moving agents often follow a herding behavior and compete aggressively with each other.

Table 5.7: Scenario testing: extension 3 (individual performance)

Scenarios	MeanLR(Fixed)	MeanLR(Moving)
Scenario 1	0.9005	n/a
Scenario 2	0.9263	0.9430
Scenario 3	0.9351	0.9781
Scenario 4	0.9302	1.0403
Scenario 5	n/a	1.0607
Scenario 6	0.9058	0.9530
Scenario 7	0.9005	0.9634
Scenario 8	0.8936	1.0048
Scenario 9	0.9291	1.0030
Scenario 10	0.9408	1.0034

5.5.3   Conclusion

Based on the above simulation results of the 2-D extension and their comparisons with early three different 1-D cases, we find that the conclusions of this more generalized 2-D version are similar to the early 1-D versions. Therefore, our early cycle explanation is also robust in more generalized and realistic situations. The benefit of this 2-D model extension not only performs robustness testing on our cycle explanation, but also provides a useful visualization tool to analyze insurers' non-price strategical movement. This kind of modeling strategical movement is common in the field of ABM as introduced in Chapter 3, but we do not go further in this thesis since it is beyond our main objective of understanding insurance cycle.

5.6   Conclusion

In this chapter, we extend our model by following three steps.

Firstly, we include the level of capital into insurers' price function, therefore insurers' competitive prices depend on their existing capital levels. We analyze the impact of this extension on the market loss ratios and the pattern of insurance cycle. We find that the simulation results support our suggestion that capital constraint plays a role of amplifying cycle and it is one of contributing cycle factors.

Secondly, we allow insurers to move their non-price business strategies on the circular street in order to understand the individual performance of insurers with different behavior rules. We find that herding behavior is bad

for insurers on average, and local market segments appear different cycles over time that mainly depend on the actions of moving insurers. When moving insurers move toward a profitable market segment, this herding behavior creates a high competition in this particular segment and destroys the profitability. In fact, cycles shift between different market segments over time, which are consistent with empirical observations in different lines of insurance business.

Finally, we generalize the 1-D circle city model into a 2-D planar space that is a common type of environment in ABM research. We compare this more generalized 2-D model extension with the early three different cases of 1-D version: (1) The base case model in Chapter 4; (2) The model extension 1, which includes the role of capital in insurers' price function; (3) The model extension 2, which models the insurers' non-price strategical movement. The results of comparison show that, in terms of modeling and explaining insurance market cycle, both the 2-D generalized model and early 1-D versions provide similar conclusions. Therefore, we find that our early 1-D analyses of insurance cycle are robust.

Chapter 6

Conclusions

6.1 Summary

The aim of this thesis is to understand the insurance cycle endogenously. We believe that **_non-life insurance market is monopolistically competitive. Price competition drives the market, but non-price preferences, product differentiation, and behavioral responses act to dampen and turn prices, thereby generating cycles._**

Chapter 2 provides the background of market cycles in the non-life insurance market. Traditional approaches of analyzing the cycles are focused on time series analyses, such as using cointegration analysis, or modeling the market performance over time as a second-order autoregression process, or developing a regime-switching model to analyze performance between soft and hard markets. Most of existing theories of insurance cycle attempt to explain cycles through discovering exogenous factors, such as entry-exit of insurers, unexpected regulatory changes, capital flow in and out from

the market, cycle in interest rate, etc. We try to understand the cycles endogenously and focus on the interactions of insurers within this system. Agent-based modeling provides a possible way for us to model the interactions of insurers who have different behavior rules.

Chapter 3 introduces the agent-based modeling (ABM), its distinctive features and applications in insurance economics. We also discuss some possible methods of performing model evaluation and show a couple of simple ABM examples. The key reason of using ABM is because: it attempts to capture the emergent complex dynamics of real-world systems, like cycles in insurance market. Different from existing cycle literature, ABM recognizes that insurers are heterogeneous and they have different behavior rules to decide their actions. The rationality of insurers is limited by the information they have, so they have to learn and adapt from their current local market environment. They interact and compete with each other locally, which affect the local market environment at the same time.

Chapter 4 explains the base case of our ABM of non-life insurance market. We recognize that the insurance market is monopolistic competition, rather than being a perfect competition as traditional literature assumed. Monopolistic competition suggests that each insurer has local power to affect the price in its targeted customer groups. This is because customers have different product preferences, so insurers can use product differentiations to focus on particular market segments. We apply traditional economic location models to capture customers' product preferences and map them to a product attribute space (i.e. a circular street in Chapter 4). We also use

some recent findings from Behavioral Economics to model insurers' behavior of dealing with uncertainty, competition and product differentiation. By interacting both insurers and customers in this system, we are able to generate market performance cycles that match the real market data. We focus our analysis on insurer's pricing behavior, rather than allowing insurers to change their business focuses on the product attribute space. This is because we believe price competition play the key role of creating market cycles, while non-price strategical movements on the product attribute space have a small impact on the market dynamics. We use actual market data samples from the UK insurance sectors to calibrate and validate our simulated results.

Chapter 5 extends the base case model of Chapter 4 in three different ways. Firstly, by adding a capital-based adjustment in insurer's price equation and comparing the simulation results with the base case, we suggest that "capital constraint" is an external contributing factor of amplifying insurance cycles, rather than an original source or force of cycle motion. The key endogenous source or force of cycle is the interaction of insurers' competition in a monopolistically competitive market with uncertain nature, as explained in the previous Chapter 4. Secondly, we allow insurers not only change their prices, but also change their locations on the product attribute space. Although this kind of non-price strategical movement plays an insignificant role of affecting aggregate market dynamics, it affects individual insurers' performance. Insurers with different behavior rules of strategical movement perform differently. Therefore, cycles in individual market segments are different from each other. They are also different from

the aggregate market cycle. Finally, we provide a more generalized version of our model, by expanding it from 1-D structure to 2-D, distributing both insurers and customers on the product attribute space more realistically, and allowing insurers to have more flexibility to compare their performance with direct competitors and updating their non-price strategies. Comparing this 2-D generalized model with the early three different versions of 1-D structure, the simulation results are similar to the early versions. Therefore, we argue that the conclusions from the early 1-D model are robust.

6.2 Directions for future research

The distinctive features of ABM and its potential benefits are explained in early Chapter 3. The unique focus of ABM simulation is on the dynamic interactions of agents' behaviors. In the insurance market, there are important innovations and developments recently which enable the collection of customers' behavioral patterns when purchasing insurance (e.g.: comparison website, insurance aggregators, etc) and the analysis of customer risk behaviors (e.g.: telematics in motor insurance, usage-based insurance, etc). As Guillen et al. (2008b) state, it is becoming more and more important for insurers to monitor customer loyalty in a more competitive market as customers are shifting their purchasing behavior from traditional broker-based distribution to internet-based automatic system (Brockett et al., 2008; Guillen et al., 2009). An ABM-based behavioral simulation can be a powerful tool to predict these market trends and to investigate the potential impacts on insurers with different strategies.

The simulation results from our ABM of insurance market not only explain the insurance cycle endogenously, but also potentially provide some real-world applications. First, our ABM may help insurers to test optimal pricing strategies of achieving individual profit maximization in different market environments. Second, it may suggest some other business strategies that identify and minimize the risk of underwriting cycles, such as avoiding herd behavior and performing fundamental analysis to price insurance.

Based on our 2-D framework in the model extension 3 of Chapter 5, insurance market practitioners and regulators are able to visualize the non-price strategies and define their future movements better. Many possible questions are worth to research further in the future. Here we briefly discuss the following:

Local market comparison and peer review

From an insurance market practitioner's view (e.g. CEOs of major insurance companies, actuaries, underwriters, etc), an updated performance measure (i.e. loss ratio, combined ratio, etc) is necessary to manage their business operations. However, most of companies compare themselves with other insurers who have a similar size or total written premiums. We believe it would be better for insurers to compare both the size and the strategical business focus. Our 2-D model can define an insurer's major competitors' strategical focuses on customers' preferences, therefore insurers can use our model to trace their major competitors or define new

potential competitors in their existing focused market segments. Regular peer review is a necessary procedure for an insurer to manage its business, but it is crucial to define a proper group of peers. Otherwise, it may push the business strategy toward a wrong direction. For example, while Zurich Insurance Group has a similar size with AIG five years ago, but they have different strategical focuses. AIG was moving toward non-traditional strategies of financial innovations that are more focused on insuring credit protection for the structured debt securities (e.g. credit defaults swaps CDSs on collateralized debt obligations CDOs). AIG earned huge profits at the beginning of this innovative process, but suffered a lot after 2008 global financial crisis. Meanwhile, Zurich Insurance Group did not compare its performance with AIG and managed to avoid this crisis. It is worth for us to research further on the behavior of insurers' local comparison and the process of peer review, so we can test it on our ABM.

Strategical planning and investment

It is easy to see that, from our model, if a local market segment appears a high profitability, then it attracts other insurers move toward this particular segment and increases competition in this area. This kind of strategical movement is rational from a profit maximization point of view, but it creates a herding behavior among these competitors. The main reason for the herding is because insurers ignore the feedback loops between individual decisions and aggregate market results (i.e. complexity). When a profitable sub-market attracts a lot of new competitors, this sub-market becomes less profitable and even makes loss since a high competition (due to excess sup-

ply) reduces the price. To understand this complexity, we can simulate our model and test different behavioral rules of insurers, such as testing the speed of insurer's movement, deciding whether follow the herding or not, etc. These kinds of strategical scenario testing based on our ABM helps insurers understand the complexity to make better business investment plan. In fact, insurance market practitioners often find insurance cycles appear differently in different sectors and periods, some insurers move their business focuses by following the cycles (i.e. from less profitable sectors to more profitable areas), but other specialists remain in their niche markets over time (i.e. no matter the status of cycle in this particular niche area). It is worth for us to research further on the behavior of insurers' strategical movement, whether based on technical analysis (i.e. following the market trend and cycle) or fundamental analysis (i.e. following an insurer's own expertise and its specific knowledge).

Merger and acquisition

Throughout our early analyses, we assume that insurers are individual agents and their actions are independently made by themselves. Insurers always compete with each other. However, in terms of the purpose for risk mitigation in a cyclic environment, it is worth for us to research further and test one possible way of mitigating the risk through mergers and acquisitions (M&A). It diversifies the portfolio of business classes into different market segments. The procedure of its implementation is to modify the utility functions of our agents, let some agents cooperate among each other and form an insurance group. They share their profits

in each time period, but their business strategies are focused on different market segments. They consider their overall group-level profits to decide their strategical movements. In this way, they may suffer losses in some market segments with high competition, but can gain more profits in other less competitive market segments. In reality, some insurance giants may have subsidiaries to expand new businesses or create product innovations in some new sectors, together with their main businesses in the traditional market segments. Our model provides a useful tool to test the performance in these different market circumstances. Elango et al. (2008) investigate the relationship between product diversification and insurer's performance using US data over the 1994 through 2002 time period. They find that the relationship is complex and nonlinear. Liebenberg and Sommer (2008) find that undiversified insurers consistently outperform diversified insurers. This means that insurers who are strategic focused specialists have competitive advantages.

Insurers' reputation and customers' recognition

Similar to our analysis, Laver (2005) use ABM to model the dynamics of political competition as a complex system without assuming the existence of long term equilibrium. Political parties or their party leaders in the model of Laver (2005) are similar to insurers in our model and voters are similar to our customers. Political parties change their strategies by moving on the 2-D strategical map to attract voters and re-adapt their policy positions, while voters continually review party support and switch parties to increase their expectations based on their individual preferences. Laver

(2005) explore different algorithms for party adaptation that are similar to our insurers' behavioral rules, including "Aggregator" which states these agents adapt party policy to the ideal policy positions of party supporters, "Hunter" which states the agents repeat policy moves that were rewards or otherwise make random moves, "Predator" which states the agents move party policy toward the policy position of the largest party, and "Sticker" who never change party policy. To compare with our ABM and the real insurance market, insurers have similar reputations and customers often recognize these differences of insurers. Some insurers are very focused in their specific business areas who never change their strategies, while others move toward other business strategies whether by following other insurers or purely seeking more profitable area alone. It is interesting to test the benefits of these differences in reputation and recognition. It is worth for us to model customers' behavior as well, such as their recognitions and expectations about insurers.

Appendices

Appendix A

A.1 Premium Principles and Properties

Some common premium principles are mentioned in the insurance literature, Kaas et al. (2008) list ten principles in Chapter 5 of their textbook. We select five principles that are relevant to our premium function. They provide the foundation for understanding our implementation of insurer's premium.

Five premium principles are:

1. Net premium principle: premium should be positively related to the expected risk at least.
$$P(X) = E(X)$$

2. Expected value principle: premium should take into account of the future discount of time value (here α is a discount rate).
$$P(X) = (1 + \alpha)E(X)$$

3. Variance principle: premium should take into account of the future uncertainty of the outcome (here α is a weight of risk loading).
$$P(X) = E(X) + \alpha Var(X)$$

4. Standard deviation principle: similar to variance principle, but may take into account positive or negative correlation (here α is a weight of risk adjustment).
$$P(X) = E(X) + \alpha \sigma(X)$$

5. Exponential principle: basically, premium depends on the expected risk and uncertainty of future losses and the risk tolerance of an insurer (here α is the constant risk averse coefficient for an insurer).
$$P(X) \approx E(X) + \frac{1}{2}\alpha Var(X)$$

The reason for discussing these five principles is that they are commonly used in real-world insurance practice. Other principles that we do not include here are not practical in the real world (e.g. Percentile principle, Maximal Loss principle, or Esscher principle), or some of them can be represented by one of the above mentioned principles.

For example, the Exponential principle is a special case of both so-called "Zero Utility Premium" and "Mean Value Principle".

Five desirable properties of premium principles:

1. Non-negative loading: A premium without a positive loading will lead to ruin with certainty. Therefore, the premium for a risk of random loss X should be larger than the mean of this loss:

$$P(X) \geq \mu(X)$$

2. No rip-off: The maximal loss premium is a boundary case. If loss is unbounded, the premium is infinite. Therefore, the premium should be less than the maximal possible loss:

$$P(X) \leq max(X)$$

3. Consistency: If we raise the claim by some fixed amount c, then the premium should also be higher by the same amount.

$$P(X + c) = P(X) + c$$

4. Additivity: Pooling independent risks does not affect the total premium needed. Therefore, for independent risks X and Y:

$$P(X + Y) = P(X) + P(Y)$$

5. Iterativity: The premium for loss X can be calculated in two steps. First, conditional calculation on every possible of event Y, then apply same premium principle to calculate the expected risk of Y.

$$P(X) = P(P(X|Y))$$

Kaas et al. (2008) summarize the properties of different premium principles in a table on page 121-122. Within the above five listed principles, only the exponential premium and the net premium principle satisfy all of these five properties. Obviously, the major difference between these two premium options is that the net premium principle assumes insurers are risk neutral and the exponential premium assumes insurers are risk averse. For these reasons, the implementation of risk premium in our model is based on the exponential premium principle that we discuss later. As Kaas et al. (2008) state, the drawback of exponential utility based premium is that the insurer's existing capital level plays no role in the risk premium function, because the risk averse coefficient is a constant. On the other hand, this is also a strong point since it is very convenient not to have to know insurers' current capital, which they argue that the capital is generally either random or simply not precisely known at each time period. Although we consider that an insurer's capital (size) affects its price decision, the role of capital is not to change an insurer's risk tolerance. In fact, capital in our model has two roles. First, it measures the capacity/supply of each insurer. Second, our insurers also use capital levels to define their direct competitors and change their non-price strategies (e.g. agent's local comparison rule).

The premium principles only offer advice for us to define the risk premium, which means it is the minimal rational premium that an insurer can charge in a perfect com-

petitive market. It links to the insurer's risk aversion and the ruin probability, therefore it intends to provide the lowest possible rational price. However, the real market is dynamic and competition is changing over time, so the market competitive price should also include other competitive adjustment factors or loadings.

A.2 Details of Testing Stationarity (ADF test)

Augmented Dickey-Fuller Unit Root Test on UKMOTOR

ADF Test Statistic	-6.075116	1% Critical Value*	-3.6959
		5% Critical Value	-2.9750
		10% Critical Value	-2.6265

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(UKMOTOR)
Method: Least Squares
Date: 08/06/13 Time: 22:45
Sample(adjusted): 3 29
Included observations: 27 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
UKMOTOR(-1)	-0.741648	0.122080	-6.075116	0.0000
D(UKMOTOR(-1))	0.828939	0.143072	5.793859	0.0000
C	0.777931	0.129150	6.023488	0.0000

R-squared	0.668900	Mean dependent var	-0.003367
Adjusted R-squared	0.641308	S.D. dependent var	0.074758
S.E. of regression	0.044773	Akaike info criterion	-3.269969
Sum squared resid	0.048112	Schwarz criterion	-3.125987
Log likelihood	47.14458	F-statistic	24.24281
Durbin-Watson stat	1.565367	Prob(F-statistic)	0.000002

Phillips-Perron Unit Root Test on UKMOTOR

PP Test Statistic	-2.610687	1% Critical Value*	-3.6852
		5% Critical Value	-2.9705
		10% Critical Value	-2.6242

*MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: 3	(Newey-West suggests: 3)	
Residual variance with no correction		0.004165
Residual variance with correction		0.004254

Phillips-Perron Test Equation
Dependent Variable: D(UKMOTOR)
Method: Least Squares
Date: 08/06/13 Time: 22:48
Sample(adjusted): 2 29
Included observations: 28 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
UKMOTOR(-1)	-0.421422	0.162605	-2.591697	0.0155
C	0.443541	0.172368	2.573222	0.0161

R-squared	0.205304	Mean dependent var	-0.001979
Adjusted R-squared	0.174738	S.D. dependent var	0.073727
S.E. of regression	0.066977	Akaike info criterion	-2.500187
Sum squared resid	0.116634	Schwarz criterion	-2.405030
Log likelihood	37.00262	F-statistic	6.716896
Durbin-Watson stat	1.075947	Prob(F-statistic)	0.015462

Table A.1: UK Motor: testing Stationarity

Augmented Dickey-Fuller Unit Root Test on UKPROPERTY

ADF Test Statistic	-4.138845	1% Critical Value*	-3.6959
		5% Critical Value	-2.9750
		10% Critical Value	-2.6265

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(UKPROPERTY)
Method: Least Squares
Date: 08/06/13   Time: 22:49
Sample(adjusted): 3 29
Included observations: 27 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
UKPROPERTY(-1)	-0.867859	0.209686	-4.138845	0.0004
D(UKPROPERTY(-1))	0.328146	0.186355	1.760867	0.0910
C	0.871490	0.213389	4.084037	0.0004

R-squared	0.422229	Mean dependent var	-0.008148
Adjusted R-squared	0.374082	S.D. dependent var	0.120832
S.E. of regression	0.095596	Akaike info criterion	-1.752934
Sum squared resid	0.219326	Schwarz criterion	-1.608952
Log likelihood	26.66460	F-statistic	8.769488
Durbin-Watson stat	2.043081	Prob(F-statistic)	0.001384

Phillips-Perron Unit Root Test on UKPROPERTY

PP Test Statistic	-3.331301	1% Critical Value*	-3.6852
		5% Critical Value	-2.9705
		10% Critical Value	-2.6242

*MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: 3	(Newey-West suggests: 3)	
Residual variance with no correction		0.009412
Residual variance with correction		0.007559

Phillips-Perron Test Equation
Dependent Variable: D(UKPROPERTY)
Method: Least Squares
Date: 08/06/13   Time: 22:50
Sample(adjusted): 2 29
Included observations: 28 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
UKPROPERTY(-1)	-0.631661	0.182087	-3.469004	0.0018
C	0.636004	0.185860	3.421957	0.0021

R-squared	0.316401	Mean dependent var	-0.005357
Adjusted R-squared	0.290109	S.D. dependent var	0.119489
S.E. of regression	0.100676	Akaike info criterion	-1.685077
Sum squared resid	0.263525	Schwarz criterion	-1.589920
Log likelihood	25.59108	F-statistic	12.03399
Durbin-Watson stat	1.746602	Prob(F-statistic)	0.001835

Table A.2: UK Property: testing Stationarity

Augmented Dickey-Fuller Unit Root Test on UKWORLD

ADF Test Statistic	-2.904013	1% Critical Value*	-3.6959
		5% Critical Value	-2.9750
		10% Critical Value	-2.6265

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(UKWORLD)
Method: Least Squares
Date: 08/06/13   Time: 22:51
Sample(adjusted): 3 29
Included observations: 27 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
UKWORLD(-1)	-0.357211	0.123006	-2.904013	0.0078
D(UKWORLD(-1))	0.478443	0.178144	2.685713	0.0129
C	0.374823	0.131210	2.856674	0.0087

R-squared	0.327490	Mean dependent var	-0.006296
Adjusted R-squared	0.271447	S.D. dependent var	0.051972
S.E. of regression	0.044361	Akaike info criterion	-3.288461
Sum squared resid	0.047230	Schwarz criterion	-3.144479
Log likelihood	47.39422	F-statistic	5.843595
Durbin-Watson stat	1.854197	Prob(F-statistic)	0.008558

Phillips-Perron Unit Root Test on UKWORLD

PP Test Statistic	-1.927247	1% Critical Value*	-3.6852
		5% Critical Value	-2.9705
		10% Critical Value	-2.6242

*MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: 3	(Newey-West suggests: 3)	
Residual variance with no correction		0.002307
Residual variance with correction		0.002777

Phillips-Perron Test Equation
Dependent Variable: D(UKWORLD)
Method: Least Squares
Date: 08/06/13   Time: 22:51
Sample(adjusted): 2 29
Included observations: 28 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
UKWORLD(-1)	-0.227041	0.129005	-1.759940	0.0902
C	0.237075	0.137667	1.722090	0.0969

R-squared	0.106449	Mean dependent var	-0.004643
Adjusted R-squared	0.072082	S.D. dependent var	0.051746
S.E. of regression	0.049846	Akaike info criterion	-3.091003
Sum squared resid	0.064601	Schwarz criterion	-2.995845
Log likelihood	45.27404	F-statistic	3.097388
Durbin-Watson stat	1.235518	Prob(F-statistic)	0.090180

Table A.3: UK World: testing Stationarity

Augmented Dickey-Fuller Unit Root Test on SIMULATEDT29

ADF Test Statistic	-3.504021	1% Critical Value*	-3.6959
		5% Critical Value	-2.9750
		10% Critical Value	-2.6265

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(SIMULATEDT29)
Method: Least Squares
Date: 08/08/13   Time: 08:37
Sample(adjusted): 3 29
Included observations: 27 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SIMULATEDT29(-1)	-0.724252	0.206692	-3.504021	0.0018
D(SIMULATEDT29(-1))	0.271038	0.195280	1.387942	0.1779
C	0.745797	0.213068	3.500284	0.0018

R-squared	0.343607	Mean dependent var	0.002883
Adjusted R-squared	0.288908	S.D. dependent var	0.131279
S.E. of regression	0.110703	Akaike info criterion	-1.459491
Sum squared resid	0.294124	Schwarz criterion	-1.315509
Log likelihood	22.70313	F-statistic	6.281731
Durbin-Watson stat	2.034720	Prob(F-statistic)	0.006397

Phillips-Perron Unit Root Test on SIMULATEDT29

PP Test Statistic	-3.142629	1% Critical Value*	-3.6852
		5% Critical Value	-2.9705
		10% Critical Value	-2.6242

*MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: 3	(Newey-West suggests: 3)	
Residual variance with no correction		0.011419
Residual variance with correction		0.010047

Phillips-Perron Test Equation
Dependent Variable: D(SIMULATEDT29)
Method: Least Squares
Date: 08/08/13   Time: 08:37
Sample(adjusted): 2 29
Included observations: 28 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SIMULATEDT29(-1)	-0.563987	0.174567	-3.230770	0.0033
C	0.579937	0.179802	3.225409	0.0034

R-squared	0.286457	Mean dependent var	0.002996
Adjusted R-squared	0.259013	S.D. dependent var	0.128827
S.E. of regression	0.110895	Akaike info criterion	-1.491719
Sum squared resid	0.319740	Schwarz criterion	-1.396561
Log likelihood	22.88406	F-statistic	10.43787
Durbin-Watson stat	1.773534	Prob(F-statistic)	0.003338

Table A.4: Simulated 29 years: testing Stationarity

Augmented Dickey-Fuller Unit Root Test on SIMULATEDDATALR

ADF Test Statistic	-11.65701	1% Critical Value*	-3.4403
		5% Critical Value	-2.8652
		10% Critical Value	-2.5687

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(SIMULATEDDATALR)
Method: Least Squares
Date: 08/06/13   Time: 23:46
Sample(adjusted): 6 901
Included observations: 896 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SIMULATEDDATALR(-	-0.597642	0.051269	-11.65701	0.0000
D(SIMULATEDDATAL	0.043551	0.049172	0.885686	0.3760
D(SIMULATEDDATAL	-0.070473	0.044302	-1.590742	0.1120
D(SIMULATEDDATAL	-0.038821	0.038331	-1.012786	0.3114
D(SIMULATEDDATAL	0.006227	0.033530	0.185713	0.8527
C	0.582766	0.050077	11.63749	0.0000

R-squared	0.307678	Mean dependent var	-4.29E-05
Adjusted R-squared	0.303789	S.D. dependent var	0.105525
S.E. of regression	0.088049	Akaike info criterion	-2.015165
Sum squared resid	6.899892	Schwarz criterion	-1.983036
Log likelihood	908.7938	F-statistic	79.10596
Durbin-Watson stat	2.000363	Prob(F-statistic)	0.000000

Phillips-Perron Unit Root Test on SIMULATEDDATALR

PP Test Statistic	-19.42269	1% Critical Value*	-3.4403
		5% Critical Value	-2.8652
		10% Critical Value	-2.5687

*MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: 6	(Newey-West suggests: 6)	
Residual variance with no correction		0.007808
Residual variance with correction		0.007519

Phillips-Perron Test Equation
Dependent Variable: D(SIMULATEDDATALR)
Method: Least Squares
Date: 08/06/13   Time: 23:47
Sample(adjusted): 2 901
Included observations: 900 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SIMULATEDDATALR(-	-0.597895	0.030554	-19.56817	0.0000
C	0.583042	0.029941	19.47288	0.0000

R-squared	0.298938	Mean dependent var	-4.89E-06
Adjusted R-squared	0.298157	S.D. dependent var	0.105592
S.E. of regression	0.088460	Akaike info criterion	-2.010303
Sum squared resid	7.027076	Schwarz criterion	-1.999631
Log likelihood	906.6361	F-statistic	382.9132
Durbin-Watson stat	1.922730	Prob(F-statistic)	0.000000

Table A.5: Simulated 900 years: testing Stationarity
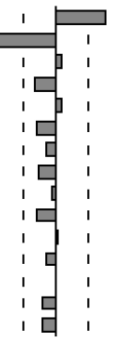
A.3 Details of AR(2) regression of data series

Table A.6: UK Motor ACF and PACF

Table A.7: UK Motor AR2 regression

```
Date: 08/06/13   Time: 22:37
Sample: 1 29                          Correlogram of UKPROPERTY
Included observations: 29

   Autocorrelation    Partial Correlation        AC      PAC    Q-Stat   Prob

                                          1    0.361   0.361   4.1890   0.041
                                          2   -0.157  -0.331   5.0101   0.082
                                          3   -0.244  -0.063   7.0628   0.070
                                          4   -0.326  -0.309  10.878    0.028
                                          5   -0.099   0.104  11.248    0.047
                                          6    0.109  -0.056  11.712    0.069
                                          7    0.156   0.076  12.708    0.080
                                          8    0.079  -0.091  12.976    0.113
                                          9    0.038   0.124  13.040    0.161
                                         10    0.019   0.003  13.057    0.221
                                         11   -0.037   0.036  13.124    0.285
                                         12   -0.076  -0.081  13.432    0.338
                                         13   -0.145  -0.100  14.618    0.332
                                         14   -0.088  -0.006  15.083    0.372
                                         15    0.047   0.015  15.222    0.436
```

Table A.8: UK Property ACF and PACF

```
Dependent Variable: UKPROPERTY
Method: Least Squares
Date: 08/06/13   Time: 22:10
Sample(adjusted): 3 29
Included observations: 27 after adjusting endpoints
Convergence achieved after 3 iterations
```

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.004184	0.021332	47.07303	0.0000
AR(1)	0.460287	0.188134	2.446588	0.0221
AR(2)	-0.328146	0.186355	-1.760867	0.0910

R-squared	0.221328	Mean dependent var	1.004444
Adjusted R-squared	0.156438	S.D. dependent var	0.104083
S.E. of regression	0.095596	Akaike info criterion	-1.752934
Sum squared resid	0.219326	Schwarz criterion	-1.608952
Log likelihood	26.66460	F-statistic	3.410849
Durbin-Watson stat	2.043081	Prob(F-statistic)	0.049689

| Inverted AR Roots	.23+.52i	.23 -.52i

Table A.9: UK Property AR2 regression

```
Date: 08/06/13   Time: 22:38           Correlogram of UKWORLD
Sample: 1 29
Included observations: 29

 Autocorrelation    Partial Correlation        AC    PAC   Q-Stat  Prob

                                        1   0.738  0.738  17.479  0.000
                                        2   0.351 -0.425  21.570  0.000
                                        3   0.028 -0.071  21.598  0.000
                                        4  -0.074  0.197  21.793  0.000
                                        5   0.036  0.196  21.841  0.001
                                        6   0.172 -0.050  22.992  0.001
                                        7   0.261  0.089  25.777  0.001
                                        8   0.208 -0.086  27.625  0.001
                                        9   0.131  0.139  28.394  0.001
                                       10   0.012 -0.174  28.401  0.002
                                       11  -0.078 -0.035  28.707  0.003
                                       12  -0.127 -0.080  29.565  0.003
                                       13  -0.172 -0.111  31.220  0.003
                                       14  -0.150  0.018  32.568  0.003
                                       15  -0.142 -0.108  33.860  0.004
```
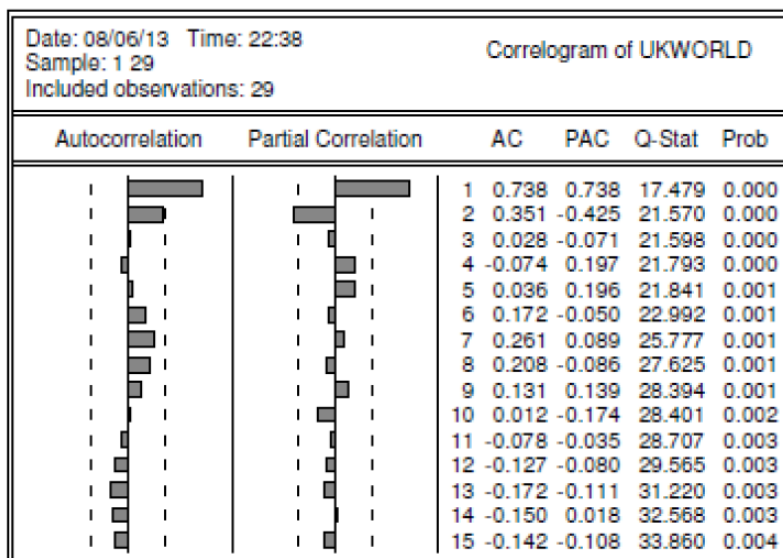
Table A.10: UK World ACF and PACF

```
Dependent Variable: UKWORLD
Method: Least Squares
Date: 08/06/13   Time: 22:26
Sample(adjusted): 3 29
Included observations: 27 after adjusting endpoints
Convergence achieved after 3 iterations
```

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.049305	0.024500	42.82880	0.0000
AR(1)	1.121233	0.178681	6.275050	0.0000
AR(2)	-0.478443	0.178144	-2.685713	0.0129

R-squared	0.675171	Mean dependent var	1.056667
Adjusted R-squared	0.648102	S.D. dependent var	0.074782
S.E. of regression	0.044361	Akaike info criterion	-3.288461
Sum squared resid	0.047230	Schwarz criterion	-3.144479
Log likelihood	47.39422	F-statistic	24.94251
Durbin-Watson stat	1.854197	Prob(F-statistic)	0.000001

Inverted AR Roots	.56+.41i	.56 - .41i

Table A.11: UK World AR2 regression

```
Date: 08/07/13   Time: 13:54          Correlogram of SIMULATEDT29
Sample: 1 29
Included observations: 29

  Autocorrelation    Partial Correlation      AC     PAC    Q-Stat   Prob

                                          1   0.436   0.436   6.1035   0.013
                                          2  -0.029  -0.271   6.1317   0.047
                                          3  -0.182  -0.064   7.2784   0.064
                                          4  -0.114   0.007   7.7420   0.101
                                          5  -0.081  -0.090   7.9866   0.157
                                          6  -0.092  -0.068   8.3155   0.216
                                          7  -0.030   0.033   8.3526   0.303
                                          8   0.029  -0.006   8.3895   0.396
                                          9   0.062   0.022   8.5609   0.479
```

Table A.12: Simulated 29 years ACF and PACF

```
Dependent Variable: SIMULATEDT29
Method: Least Squares
Date: 08/07/13   Time: 10:51
Sample(adjusted): 3 29
Included observations: 27 after adjusting endpoints
Convergence achieved after 3 iterations

  Variable      Coefficient    Std. Error    t-Statistic    Prob.

     C            1.029749      0.029440      34.97829      0.0000
   AR(1)          0.546786      0.195993       2.789832     0.0102
   AR(2)         -0.271038      0.195280      -1.387942     0.1779

R-squared            0.245593    Mean dependent var      1.028916
Adjusted R-squared   0.182726    S.D. dependent var      0.122455
S.E. of regression   0.110703    Akaike info criterion  -1.459491
Sum squared resid    0.294124    Schwarz criterion      -1.315509
Log likelihood      22.70313     F-statistic             3.906530
Durbin-Watson stat   2.034720    Prob(F-statistic)       0.033984

Inverted AR Roots     .27 -.44i      .27+.44i
```

Table A.13: Simulated 29 years AR2 regression

```
Date: 08/07/13   Time: 13:55
Sample: 1 901                    Correlogram of SIMULATEDDATALR
Included observations: 901

Autocorrelation    Partial Correlation       AC     PAC    Q-Stat   Prob

                                        1   0.402   0.402  146.11  0.000
                                        2   0.082  -0.095  152.15  0.000
                                        3   0.040   0.051  153.58  0.000
                                        4   0.064   0.042  157.24  0.000
                                        5   0.038  -0.006  158.58  0.000
                                        6  -0.005  -0.021  158.60  0.000
                                        7  -0.034  -0.028  159.64  0.000
                                        8  -0.023  -0.001  160.14  0.000
                                        9  -0.024  -0.019  160.64  0.000
```

Table A.14: Simulated 900 years ACF and PACF

Dependent Variable: SIMULATEDDATALR
Method: Least Squares
Date: 08/06/13   Time: 23:49
Sample(adjusted): 3 901
Included observations: 899 after adjusting endpoints
Convergence achieved after 2 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.975174	0.004488	217.2831	0.0000
AR(1)	0.440515	0.033263	13.24329	0.0000
AR(2)	-0.095599	0.033267	-2.873690	0.0042

R-squared	0.169282	Mean dependent var	0.975194
Adjusted R-squared	0.167427	S.D. dependent var	0.096610
S.E. of regression	0.088152	Akaike info criterion	-2.016167
Sum squared resid	6.962683	Schwarz criterion	-2.000145
Log likelihood	909.2670	F-statistic	91.29221
Durbin-Watson stat	1.989955	Prob(F-statistic)	0.000000

Inverted AR Roots	.22+.22i	.22 -.22i	

Table A.15: Simulated 900 years AR2 regression

A.4 Details of testing residuals of AR2 process

```
Date: 08/07/13   Time: 09:20
Sample: 3 29                              Correlogram of Residuals
Included observations: 27
Q-statistic probabilities adjusted for 2 ARMA term(s)
```

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.177	0.177	0.9478	
		2	-0.175	-0.213	1.9052	
		3	0.034	0.118	1.9425	0.163
		4	-0.018	-0.097	1.9536	0.377
		5	-0.103	-0.052	2.3279	0.507
		6	-0.114	-0.115	2.8165	0.589
		7	-0.129	-0.119	3.4708	0.628
		8	-0.047	-0.033	3.5637	0.735
		9	0.071	0.045	3.7827	0.804
		10	0.006	-0.040	3.7846	0.876
		11	0.030	0.047	3.8294	0.922
		12	-0.020	-0.099	3.8509	0.954

Table A.16: UK Motor residual testing: Uncorrelatedness

```
Date: 08/07/13   Time: 09:50
Sample: 3 29                              Correlogram of Residuals
Included observations: 27
Q-statistic probabilities adjusted for 2 ARMA term(s)
```

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.047	-0.047	0.0673	
		2	-0.122	-0.124	0.5314	
		3	0.020	0.007	0.5440	0.461
		4	-0.256	-0.275	2.7759	0.250
		5	-0.039	-0.068	2.8289	0.419
		6	-0.076	-0.173	3.0468	0.550
		7	-0.016	-0.055	3.0574	0.691
		8	0.036	-0.095	3.1122	0.795
		9	0.072	0.026	3.3385	0.852
		10	0.117	0.050	3.9688	0.860
		11	0.007	0.012	3.9712	0.913
		12	0.011	0.023	3.9772	0.948

Table A.17: UK Property residual testing: Uncorrelatedness

```
Date: 08/07/13   Time: 09:54                    Correlogram of Residuals
Sample: 3 29
Included observations: 27
Q-statistic probabilities adjusted for 2 ARMA term(s)

    Autocorrelation     Partial Correlation      AC     PAC    Q-Stat   Prob

                                          1   0.019   0.019   0.0105
                                          2   0.103   0.103   0.3457
                                          3  -0.044  -0.048   0.4089   0.523
                                          4  -0.042  -0.051   0.4681   0.791
                                          5   0.165   0.179   1.4348   0.697
                                          6  -0.102  -0.108   1.8256   0.768
                                          7   0.183   0.157   3.1402   0.678
                                          8  -0.072  -0.054   3.3536   0.763
                                          9   0.168   0.158   4.5797   0.711
                                         10   0.018  -0.013   4.5940   0.800
                                         11  -0.053  -0.038   4.7312   0.857
                                         12  -0.042  -0.104   4.8213   0.903
```

Table A.18: UK World residual testing: Uncorrelatedness
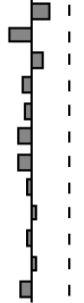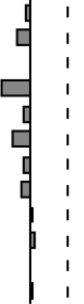
```
Date: 08/07/13   Time: 14:09                    Correlogram of Residuals
Sample: 3 29
Included observations: 27
Q-statistic probabilities adjusted for 2 ARMA term(s)

    Autocorrelation     Partial Correlation      AC     PAC    Q-Stat   Prob

                                          1  -0.031  -0.031   0.0282
                                          2   0.020   0.020   0.0413
                                          3  -0.060  -0.058   0.1571   0.692
                                          4  -0.061  -0.065   0.2846   0.867
                                          5  -0.034  -0.036   0.3260   0.955
                                          6  -0.079  -0.083   0.5590   0.968
                                          7   0.073   0.061   0.7652   0.979
                                          8   0.005   0.004   0.7661   0.993
                                          9   0.161   0.148   1.8874   0.966
                                         10  -0.162  -0.162   3.0929   0.928
                                         11  -0.181  -0.202   4.6973   0.860
                                         12   0.031   0.043   4.7463   0.907
```

Table A.19: Simulated 29 years residual testing: Uncorrelatedness
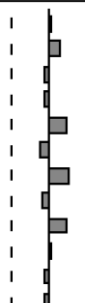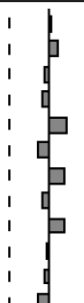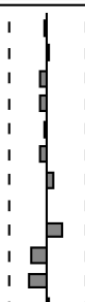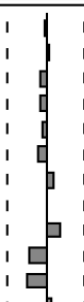
```
Date: 08/07/13   Time: 10:22
Sample: 3 901                          Correlogram of Residuals
Included observations: 899
Q-statistic probabilities adjusted for 2 ARMA term(s)

   Autocorrelation      Partial Correlation      AC       PAC    Q-Stat   Prob

                                              1   0.005    0.005   0.0201
                                              2  -0.016   -0.016   0.2463
                                              3   0.024    0.024   0.7614   0.383
                                              4   0.055    0.055   3.5298   0.171
                                              5   0.022    0.023   3.9851   0.263
                                              6  -0.005   -0.004   4.0102   0.405
                                              7  -0.029   -0.031   4.7672   0.445
                                              8  -0.006   -0.010   4.7985   0.570
                                              9   0.001   -0.002   4.7995   0.684
                                             10  -0.037   -0.036   6.0339   0.643
                                             11  -0.037   -0.033   7.3014   0.606
                                             12  -0.018   -0.016   7.5833   0.669
                                             13   0.007    0.008   7.6282   0.746
                                             14   0.044    0.048   9.3641   0.672
                                             15  -0.047   -0.042  11.412    0.576
                                             16   0.008    0.013  11.474    0.648
                                             17  -0.020   -0.027  11.854    0.690
                                             18   0.060    0.055  15.157    0.513
                                             19  -0.017   -0.017  15.408    0.566
                                             20  -0.032   -0.029  16.349    0.568
                                             21  -0.014   -0.016  16.536    0.621
                                             22  -0.005   -0.014  16.557    0.682
                                             23  -0.007   -0.007  16.606    0.735
                                             24   0.017    0.025  16.874    0.770
                                             25   0.029    0.035  17.651    0.776
                                             26   0.001    0.003  17.653    0.819
                                             27   0.058    0.055  20.732    0.707
                                             28  -0.008   -0.011  20.785    0.753
                                             29  -0.026   -0.024  21.430    0.766
                                             30  -0.017   -0.027  21.694    0.795
                                             31  -0.006   -0.013  21.726    0.831
                                             32   0.018    0.009  22.020    0.853
                                             33  -0.058   -0.048  25.126    0.762
                                             34   0.006    0.014  25.155    0.800
                                             35   0.009    0.015  25.237    0.831
                                             36   0.029    0.031  26.048    0.834
```

Table A.20: Simulated 900 years residual testing: Uncorrelatedness

A.5 Cycles in the individual insurers' local markets

If we look closer at the individual insurers in their local market segments in both a system with all fixed insurers (Case 1) and another system with 50% of insurers are moving agents (Case 2). From the autocorrelation function (AcF) and partial autocorrelation function (PAcF) in Figure A.1 (i.e.: a system when all insurers are fixed as Case 1) and Figure A.2 (i.e.: a system when 50% insurers are moving agents as Case 2), the cyclic patterns of overall market loss ratios are similar (i.e. an AR(2) process). However, if we look at the individual insurers' loss ratios in both systems, the individual insurers' local market performances are different. Figure A.3 and Figure A.4 of Case 1 show the partial autocorrelation function (PAcF) of all 20 fixed insurers in the system when all insurers are fixed. The individual insurers' loss ratios appear similar cycles (i.e. AR(2) process) to the overall market loss ratios in Figure A.1, since insurers are fixed in their local markets and maintain their local competitions over time. Then, we look at Figure A.5 of Case 2 that shows the 10 fixed insurers in the system when 50% of insurers are moving agents. Although most of them still have (weaker) cycles in their individual loss ratios, the cyclic patterns are different which are affected by the moving agents' herding behavior. Figure A.6 of Case 2 shows the PAcF of 10 moving insurers in this system. Since they are herding to each other and moving toward profitable local markets with feedback effects, their performances do not have cycles. This means, when they move into a profitable market segment, the profitability in this local market is reduced quickly by their herding behavior.
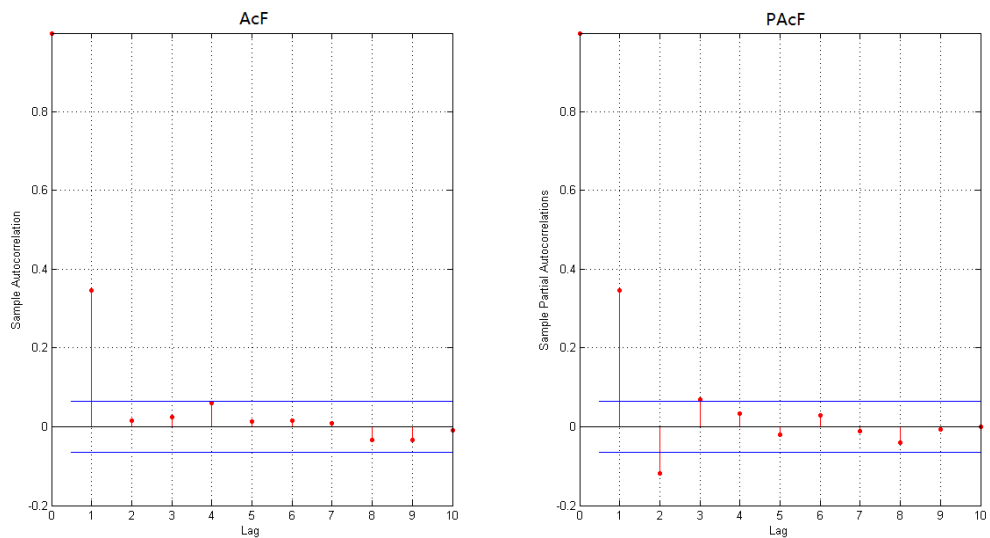
Figure A.1: (Case 1) Overall market loss ratio when all insurers are fixed
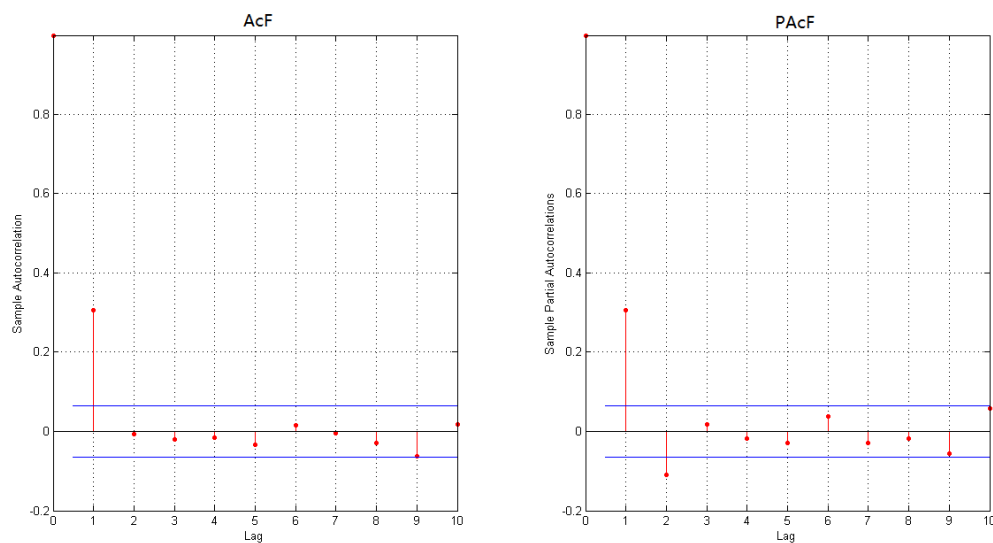
Figure A.2: (Case 2) Overall market loss ratio when 50% insurers are moving

Figure A.3: (Case 1) Individual insurers' loss ratios: the first half (All insurers are fixed)

Figure A.4: (Case 1) Individual insurers' loss ratios: the second half (All insurers are fixed)

Figure A.5: (Case 2) Individual insurers' loss ratios: 10 fixed insurers (in a system of 50% insurers are moving)

Figure A.6: (Case 2) Individual insurers' loss ratios: 10 moving insurers (in a system of 50% insurers are moving)

Bibliography

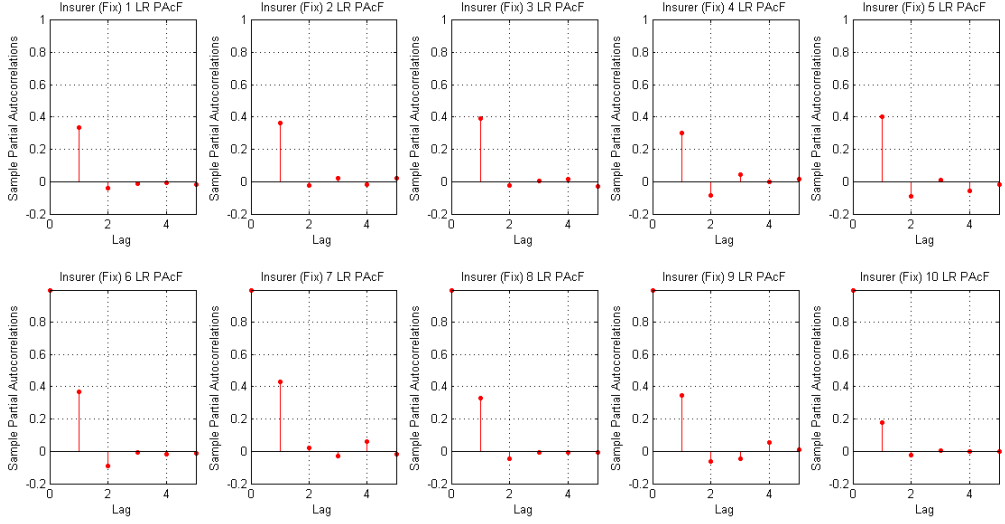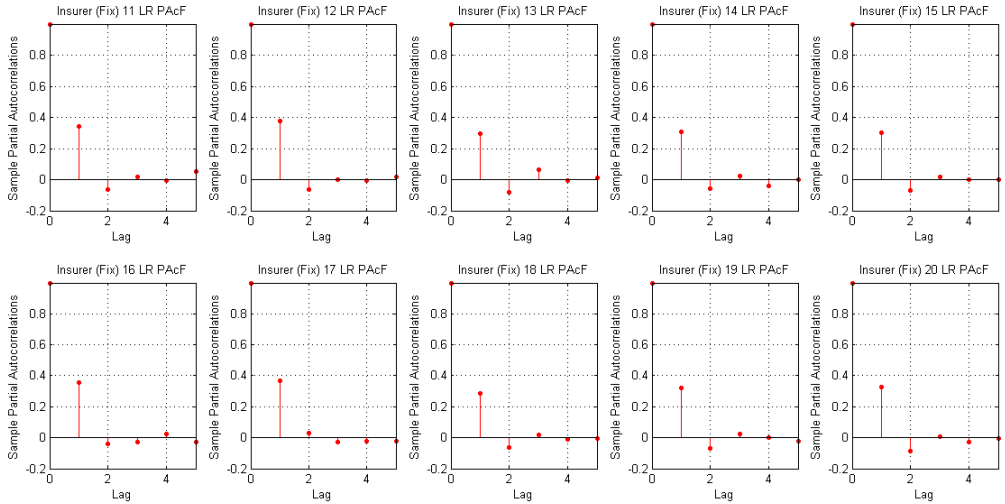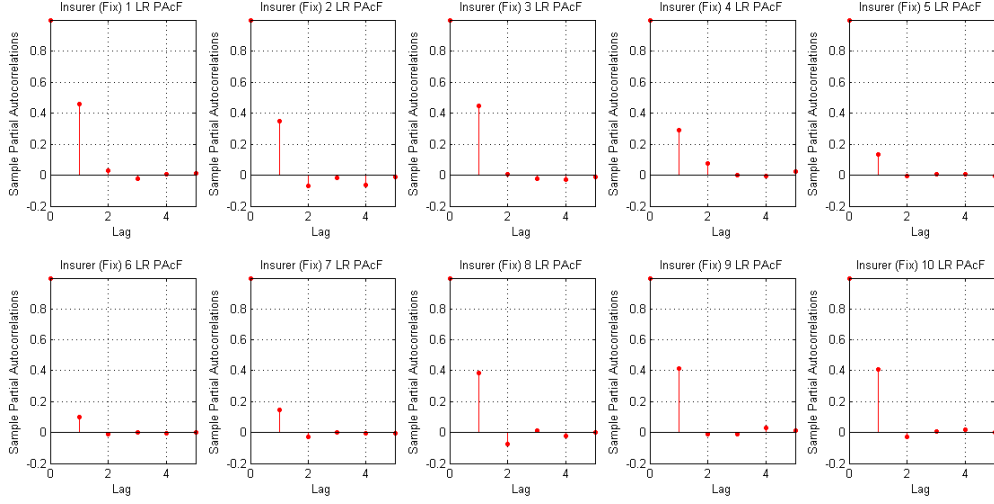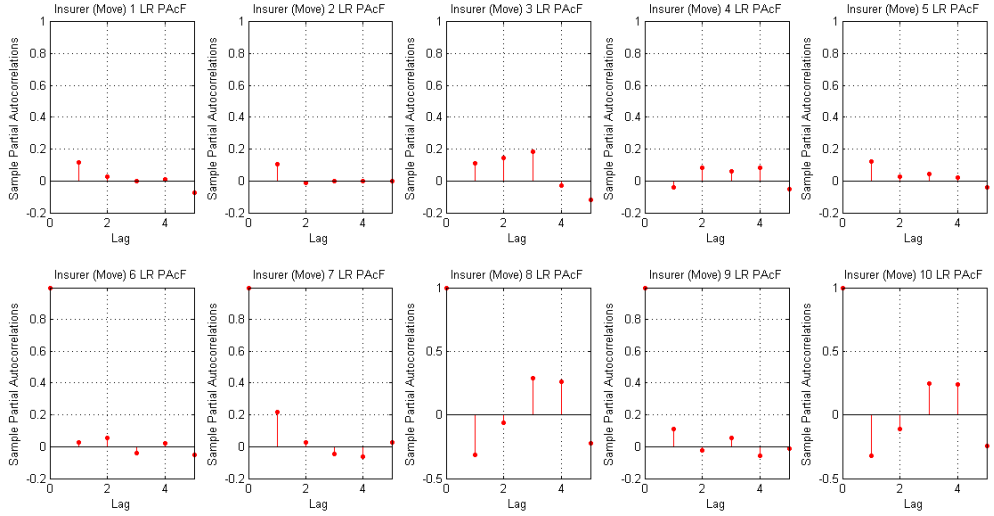Abel, A. B. (1990). Asset prices under habit formation and catching up with the joneses. *American Economic Review*, 80(2):38–42.

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