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New Method for Non-Paraxial Beam Propagation

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***Abstract*- A new method for solving the wave equation is presented, which is non-paraxial and can be applied to wide-angle beam propagation. It shows very good stability characteristics in the sense that relatively larger step-sizes can be taken. An implementation using the collocation method is presented, in which only simple matrix multiplications are involved and no numerical matrix diagonalization or inversion is needed. The method is, hence, faster and is also highly accurate.**

OCIS Codes:130.2790, 230.7370, 350.5500

I. Introduction

Recently several schemes have been suggested for wide-angle and bi-directional beam propagation through guided-wave devices.[1-13] In general, this non-paraxial propagation would involve solving directly the wave equation, which contains a second order partial derivative with z (the general direction of propagation) as against the first order partial derivative in the paraxial wave equation. All the methods for non-paraxial beam propagation discussed in the literature approach this problem iteratively, in which a numerical effort equivalent to solving the paraxial equation several times is involved. The actual number of iterations depends on the desired accuracy and the obliquity of the beam. Many of these methods neglect the backward propagating components and solve the one-way wave equation; but even methods that deal with bi-directional propagation employ special techniques either to suppress or to model evanescent modes, which are a source of instability in these methods.[8-10] In all these methods, the

square root of the propagation operator involved in the wave equation is approximated in various ways. One of the approximations used is based on the Padé approximants.[1-11] We have recently shown that a direct numerical solution (DNS) of the scalar wave equation gives very good accuracy and is also numerically efficient.[14] The method is non-paraxial and hence, is applicable to wide-angle as well as to bi-directional propagation. We used the collocation method[15-17] to formulate our equations. In this paper, we present a new method to solve the non-paraxial wave equation using symmetrized splitting of the operators. Examples show that this method is more tolerant to larger step sizes than other methods including the DNS.[14]

II. Split-step non-paraxial (SSNP) method

For simplicity, we shall confine our discussions in this paper to two-dimensional wave propagation for which the scalar wave equation is given by

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} + k_0^2 n^2(x,z)\,\psi(x,z) = 0\,, \tag{1}$$

where $\psi(x,z)$ represents one of the Cartesian components of the electric field (generally referred to as the scalar field) and $n^2(x,z)$ defines the refractive index distribution of the medium. The time dependence of the field has been assumed to be $\exp(i\omega t)$ and $k_0 = \omega / c$ is the free space wave number.

Equation (1) can be rewritten as

$$\frac{\partial \mathbf{\Phi}}{\partial z} = \mathbf{H}(z)\,\mathbf{\Phi}(z)\,, \tag{2}$$

where

$$\mathbf{\Phi}(z)=\begin{bmatrix}\psi\\ \frac{\partial\psi}{\partial z}\end{bmatrix},\qquad \mathbf{H}(z)=\begin{bmatrix}0 & 1\\ -\nabla_t^2-k_0^2n^2 & 0\end{bmatrix}. \tag{3}$$

The operator **H** can be written as a sum of two operators, one representing the propagation through a uniform medium of index, say n_r, and the other representing the effect of the index variation of the guiding structure; thus,

$$\begin{aligned}\mathbf{H}(z)&=\mathbf{H}_1+\mathbf{H}_2(z)\\ &=\begin{bmatrix}0 & 1\\ -\nabla_t^2-k_0^2n_r^2 & 0\end{bmatrix}+\begin{bmatrix}0 & 0\\ k_0^2(n_r^2-n^2) & 0\end{bmatrix}\end{aligned} \tag{4}$$

A formal solution of Eq. (2), after using the symmetrized splitting of summation of operators as in Eq. (4), can be written as

$$\mathbf{\Phi}(z+\Delta z)=\mathbf{P}\,\mathbf{Q}(z)\,\mathbf{P}\,\mathbf{\Phi}(z)+O\left((\Delta z)^3\right) \tag{5}$$

$$\mathbf{P}=\mathrm{e}^{\frac{1}{2}\mathbf{H}_1\Delta z},\quad \mathbf{Q}(z)=\mathrm{e}^{\mathbf{H}_2\Delta z}\ . \tag{6}$$

The operator **P** represents propagation in the uniform medium n_r over a distance of $\Delta z/2$, and hence, can be evaluated using any method like the collocation, finite-difference or FFT methods. The operator $\mathbf{Q}(z)$ can also be easily evaluated due to the specific form of the matrix and it can be easily seen that

$$\mathbf{Q}(z)=\begin{bmatrix}1 & 0\\ k_0^2(n_r^2-n^2)\Delta z & 1\end{bmatrix}, \tag{7}$$

since,

$$(\mathbf{H}_2)^m=\mathbf{0},\qquad m\geq 2 \tag{8}$$

due to the special form of the matrix $\mathbf{H}_2$. It may be noted that for lossless propagation the matrix **P** would be Hermitian, while the matrix **Q** always has a determinant value equal to unity.

The method given above can be implemented with any of the numerical methods employed to solve the wave equation, *e.g.*, the FFT-BPM, FD-BPM or the collocation method. In this paper, we discuss the implementation using the collocation method, and the implementation using the FD-BPM will be discussed elsewhere.

III. Implementation in the Collocation method

We have implemented the SSNP formalism in the collocation method, in which the wave equation is converted to a matrix ordinary differential equation using the representation of the field $\psi(x,z)$ as a linear combination of a set of orthogonal basis functions, $\phi_n(x)$:

$$\psi(x,z) = \sum_{n=1}^{N} c_n(z)\,\phi_n(x) \tag{9}$$

where $c_n(z)$ are the expansion coefficients, n is the order of the basis functions and N is the number of basis functions used in the expansion. The choice of $\phi_n(x)$ depends on the boundary conditions and the symmetry of the guiding structure. The coefficients of expansion, $c_n(z)$, are unknown and represent the variation of the field with z. In the collocation method,[15-17] these coefficients are effectively obtained by requiring that the differential equation, Eq. (1), be satisfied *exactly* by the expansion, Eq. (9), at N collocation points $x_j, j = 1,2,....,N$, which are chosen such that these are the zeroes of $\phi_{N+1}(x)$. Thus,

using this condition and with some algebraic manipulations,[15-17] one converts the wave equation, Eq. (1), into a matrix ordinary differential equation:

$$\frac{\mathrm{d}^2\boldsymbol{\Psi}}{\mathrm{d}z^2} + \left[\mathbf{S}_0 + k_0^2 n_r^2 \mathbf{I} + \mathbf{R}(z)\right] \boldsymbol{\Psi}(z) = 0 \tag{10}$$

with

$$\boldsymbol{\Psi}(z) = \begin{bmatrix} \psi(x_1, z) \\ \psi(x_2, z) \\ \vdots \\ \psi(x_N, z) \end{bmatrix}, \qquad \mathbf{R}(z) = k_0^2 \begin{bmatrix} \Delta n^2(x_1, z) & 0 & . & 0 \\ 0 & \Delta n^2(x_2, z) & . & . \\ . & . & . & 0 \\ 0 & . & 0 & \Delta n^2(x_N, z) \end{bmatrix}, \tag{11}$$

where $\Delta n^2(x_m, z) = n^2(x_m, z) - n_r^2$, $m = 1, 2, \cdots, N$, and $\mathbf{S}_0$ is a constant known matrix defined by the basis functions.[15-17] We refer to Eq. (10) as the *collocation equation*. In deriving this equation from the wave equation, Eq. (1), no approximation has been made except that N is finite and Eq. (10) is exactly equivalent to Eq. (1) as $N \to \infty$. Thus, the accuracy of the collocation method improves indefinitely as N increases. The collocation equation is a matrix ordinary differential equation and can be solved as an initial value problem using any standard method such as the Runge-Kutta method as we have done in the DNS.[14] In this paper, we solve this equation using the SSNP discussed in Sec. II.

We have chosen here a set of sinusoidal functions as the basis functions[16,17] and following the procedure outlined in Sec. II, we obtain the formal solution of Eq. (10) as in Eq. (5), with the operators **P** and **Q**, and the field function $\boldsymbol{\Phi}$ now being block matrices:

$$\boldsymbol{\Phi}(z) = \begin{bmatrix} \boldsymbol{\psi} \\ \dfrac{\mathrm{d}\boldsymbol{\psi}}{\mathrm{d}z} \end{bmatrix}, \qquad \mathbf{P} = \exp\left\{ \frac{\Delta z}{2} \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -(\mathbf{S}_0 + k_0^2 n_r^2 \mathbf{I}) & \mathbf{0} \end{bmatrix} \right\}, \qquad \mathbf{Q}(z) = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{R}(z) & \mathbf{I} \end{bmatrix}, \tag{12}$$

where **I** and **0** the unit and null matrices, respectively. The operator **P** represents propagation in uniform medium of index n_r over a distance of $\Delta z/2$ and can be easily obtained as a constant square matrix using the basis functions and their properties.[17] It has to be evaluated only once. Each propagation step thus requires 12 multiplications of an $N \times N$ square matrix with a column matrix except at the first and the last steps where 8 such multiplications are additionally required. We would like to emphasize that using the sinusoidal basis functions in the collocation method here has an advantage, since no FFT, matrix inversion or matrix diagonalization need be done for propagation through uniform medium and all matrices involved are obtained analytically; the details are presented in the Appendix.

IV. Numerical Results

We consider a number of examples to show the effectiveness of the method. In the first example, we consider the propagation of the fundamental mode through a tilted graded-index waveguide,[5] with index profile given by $n^2(x) = n_s^2 + 2n_s\Delta n \,\mathrm{sech}^2(2x/w)$, n_s=2.1455, Δn=0.003, w=5 μm and λ=1.3 μm. The computation was done with 530 collocation points and the width of the numerical window was about 185 μm. As a measure of accuracy, we computed an error (*ERR*), which includes the effects of both the dissipation in power as well as the loss of shape of the propagating mode:

$$ERR = 1 - \frac{\left| \int \psi_{exact} \psi_{cal}\, dx \right|^2}{\int |\psi_{inp}|^2 dx \cdot \int |\psi_{exact}|^2 dx} \tag{13}$$

where ψ_{inp}, ψ_{calc} and ψ_{exact} are the input, the propagated and the exact fields, respectively.[18]

The first result for a straight waveguide, which we have plotted in Fig. 1, shows the performance of the method in respect of stability of the method for relatively larger values of Δz. The direct numerical solution (DNS) based on the Runge-Kutta solution of the collocation equation[14] becomes unstable for $\Delta z > 0.1\mu$m, whereas the SSNP method remains stable even for 1 μm. To the best of our

knowledge, a step size equal to or larger than 1 μm for non-paraxial propagation has not been reported earlier. Even with such a large step, an accuracy better than 0.001 in propagation over a distance of 1000 μm is significantly better than those reported in the literature. In Fig.2, we have plotted the error in propagation (*ERR*) as a function of the tilt angle. The figure shows that the SSNP method gives accuracy of the order of 10^{-4} even with a step size of 0.25 μm , which is much better than those obtained by Shibayama *et al.*[5] To illustrate the point, let us consider the error for a tilt angle of 50° . The error in the best results reported by Shibayama *et al.*[5] for the 3-step GD scheme is about 0.04 with Δz =0.05 μm , whereas in our method the error is less than 0.001 with Δz =0.25 μm . This would thus mean much faster and accurate propagation. Of course, one gets better accuracy with the DNS as the single step error in the Runge-Kutta method (used in the DNS) is $O\left((\Delta z)^5\right)$ as against $O\left((\Delta z)^3\right)$ in the SSNP method, but then the computation effort is significantly reduced with the latter method.

We next consider propagation of the TE_1 mode in step index waveguides. Fig. 3 shows a plot of *ERR* as a function of the of propagation steps for the step index waveguide[6] with n_{co} =1.002, n_{clad} =1.000, λ =1.0 μm , w =15.092 μm . Even with a step size as large as 0.4 μm , the propagation is extremely stable and highly accurate while DNS becomes unstable for this step size. Fig. 4 shows performance with variation in tilt angle of the waveguide. We can see that the present method and DNS[14] curves are very close, except for SSNP method with step size 0.4 μm at 0^{o} where error is higher. However, the error value even with 0.4 μm step size is better than that reported by Yamauchi *et al.*[6] at 50^{o}. The SSNP method gives better accuracy with twice the step size used by Yamauchi *et al.*[6]; in fact, only 500 computation points are required as against 1800 by Yamauchi *et al.*[6]

Figures 5 and 6 show performance of the method for the TE_1 mode in the benchmark waveguide[19] with n_{co} =3.3, n_{clad} =3.17, λ =1.55 μm , w =8.8 μm . As the refractive index change from core to cladding is very large here, only small step sizes can be taken, yet the SSNP method is stable for a step

size of 0.2 μm , as shown in Fig. 5. In fact, the performance at large tilt angles with 0.2 μm is quite close to that for the DNS,[14] as shown in Fig. 6. We may note that oscillatory behaviour in the error curves becomes more pronounced for the step-index waveguide with larger index jump at the core-cladding interface (compare Figs. 3 and 5). This may be attributed to the fact that any discretization would approximate the index-step by an interpolating curve between two successive sample points around the step. Expectedly, this oscillation becomes larger as Δz increases from 0.01 μm to 0.2 μm (although the logscale deceptively shows nearly equal oscillations).

The final example is that of the propagation of the TE_{10} mode in the benchmark waveguide[19] described above and we have obtained the power remaining in the guide after propagation of 100 μm at a tilt angle of 20^o. Table 1 compares the SSNP method with other methods. It is quite obvious from the table that with fewer points, the SSNP method shows higher accuracy. The method is faster than the DNS,[14] taking only about half the time. It is also much easier to implement.

An important parameter to choose is the reference refractive index, n_r . Although, in principle, its value can be arbitrarily chosen, the value in general affects the accuracy. However, as Fig. 7 shows, the accuracy is largely insensitive to the choice of n_r .

We would like to add that the SSNP method and the DNS method are also not very sensitive to perturbations in the value of the initial field or its derivative. We have carried out preliminary investigations by adding and subtracting a small error (10^{-3}) alternately in the initial field and its derivative at successive sample points. The error in the overlap integral was 1.9×10^{-2} as against 2.6×10^{-5} for propagation of the TE_1 mode at 0^o for 100 μm with a propagation step size of 0.1 μm , in the benchmark waveguide[19] where n_{co} =3.3, n_{clad} =3.17, λ =1.55 μm , w =8.8 μm . Thus, the propagation remains stable.

V. Conclusions

We have presented a new method to solve the non-paraxial wave equation based on a symmetrized splitting of the operator. We have implemented this method with the collocation method. We have also included comparison with reported results of other methods. The method shows better stability with relatively larger step sizes being possible. The method involves only simple multiplication of matrices and no numerical diagonalization or inversion of any matrix is needed. It is therefore much faster and easier to implement, and is more efficient than other methods.

Appendix: **Evaluation of** $\exp(\mathbf{H_1}\Delta z)$

This amounts to a solution of the collocation equation, Eq. (10), without the $\mathbf{R}(z)$ term, i.e., propagation in a medium of uniform refractive index, n_r over a distance of Δz. That is, solution of the equation:

$$\frac{\mathrm{d}^2\boldsymbol{\Psi}}{\mathrm{d}z^2} + \mathbf{S}\ \boldsymbol{\Psi}(z) = 0 \tag{A1}$$

where $\mathbf{S} = \mathbf{S}_0 + k_0^2 n_r^2 \mathbf{I}$ is a constant matrix. Equation (A1) can also be written as

$$\frac{\partial \boldsymbol{\Phi}}{\partial z} = \mathbf{H}_1\, \boldsymbol{\Phi}(z) \tag{A2}$$

where $\boldsymbol{\Phi}(z)$ is defined in Eq. (12), and

$$\mathbf{H}_1 = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{S} & \mathbf{0} \end{bmatrix}$$

is a constant matrix and has to be evaluated just once. A formal solution Eq. (A2) can be written:

$$\boldsymbol{\Phi}(z + \Delta z) = \mathrm{e}^{\mathbf{H}_1 \Delta z} \boldsymbol{\Phi}(z). \tag{A3}$$

The evaluation of $\exp(\mathbf{H_1}\Delta z)$ can be done by diagonalization of $\mathbf{H}_1$; however, $\mathbf{H}_1$ is a $2N \times 2N$ non-symmetric matrix and its diagonalization may involve complex matrix algebra and hence, present some difficulties. We present here a much simpler and analytical method to evaluate $\exp(\mathbf{H_1}\Delta z)$.

Since Eq. (A1) represents propagation in a uniform medium, the propagation can be obtained by eigenvalue decomposition method. Thus, the solution of Eq. (A1) over a single step can be written as

$$\boldsymbol{\Psi}(z+\Delta z)=\cos(\sqrt{\mathbf{S}}\Delta z)\boldsymbol{\Psi}(z)+\frac{1}{\sqrt{\mathbf{S}}}\sin(\sqrt{\mathbf{S}}\Delta z)\boldsymbol{\Psi}'(z) \tag{A4}$$

$$\boldsymbol{\Psi}'(z+\Delta z)=-\sqrt{\mathbf{S}}\sin(\sqrt{\mathbf{S}}\Delta z)\boldsymbol{\Psi}(z)+\cos(\sqrt{\mathbf{S}}\Delta z)\boldsymbol{\Psi}'(z)\,. \tag{A5}$$

Using this solution in Eq. (A3) gives

$$e^{\mathbf{H}_1\Delta z}=\begin{pmatrix}\cos(\sqrt{\mathbf{S}}\Delta z) & \frac{1}{\sqrt{\mathbf{S}}}\sin(\sqrt{\mathbf{S}}\Delta z)\\ -\sqrt{\mathbf{S}}\sin(\sqrt{\mathbf{S}}\Delta z) & \cos(\sqrt{\mathbf{S}}\Delta z)\end{pmatrix} \tag{A6}$$

In order to evaluate the functions of the matrices involved in Eq. (A6), we use the diagonalization procedure. Thus, let $\mathbf{S}=\mathbf{V}\boldsymbol{\Lambda}\mathbf{V}^{-1}$ where $\mathbf{V}$ and $\boldsymbol{\Lambda}$ are the eigenvectors and eigenvalues of $\mathbf{S}$, respectively. Then, we have

$$\sqrt{\mathbf{S}}\Delta z=\mathbf{V}\,(\sqrt{\boldsymbol{\Lambda}}\Delta z)\,\mathbf{V}^{-1}\qquad\text{with}\quad\sqrt{\boldsymbol{\Lambda}}=diag.(\sqrt{\Lambda_i})\,, \tag{A7}$$

and :

$$\cos(\sqrt{\mathbf{S}}\Delta z)=\mathbf{V}\cos(\sqrt{\boldsymbol{\Lambda}}\Delta z)\,\mathbf{V}^{-1} \tag{A8}$$

$$\sin(\sqrt{\mathbf{S}}\Delta z)=\mathbf{V}\sin(\sqrt{\boldsymbol{\Lambda}}\Delta z)\,\mathbf{V}^{-1} \tag{A9}$$

Thus

$$e^{\mathbf{H}_1\Delta z}=\begin{pmatrix}\mathbf{V} & 0\\ 0 & \mathbf{V}\end{pmatrix}\begin{bmatrix}\cos(\sqrt{\boldsymbol{\Lambda}}\Delta z) & \frac{1}{\sqrt{\boldsymbol{\Lambda}}}\sin(\sqrt{\boldsymbol{\Lambda}}\Delta z)\\ -\sqrt{\boldsymbol{\Lambda}}\sin(\sqrt{\boldsymbol{\Lambda}}\Delta z) & \cos(\sqrt{\boldsymbol{\Lambda}}\Delta z)\end{bmatrix}\begin{pmatrix}\mathbf{V}^{-1} & 0\\ 0 & \mathbf{V}^{-1}\end{pmatrix}. \tag{A10}$$

The operator **P** in Eq. (12) is thus given by Eq. (A10) with Δz replaced by $\Delta z/2$.

In the case of sinusoidal basis functions in the collocation method,[16] the form of $\mathbf{S}$ is such that the eigenvalues decomposition required as per Eq. (A7) is simply done analytically. In this case, we choose the basis functions as

$$\begin{aligned} \phi_n(x) &= \cos(\nu_n x) \qquad \text{for } n = 1,3,5,\cdots N-1 \\ &= \sin(\nu_n x) \qquad \text{for } n = 2,4,6,\cdots N \end{aligned} \tag{A11}$$

where $\nu_n = n\pi / 2L$, with computation window being from –*L* to *L*. The collocation points are at

$$x_j = \left(\frac{2j}{N+1} - 1\right) L, \qquad j = 1,2,3\cdots, N\,. \tag{A12}$$

The matrix $\mathbf{S}$ in this case is then given by[16,17]

$$\mathbf{S} = \mathbf{A}\,\mathbf{G}\,\mathbf{A}^{-1} + k_o^2 n_r^2 \mathbf{I} = \mathbf{A}(\mathbf{G} + k_o^2 n_r^2 \mathbf{I})\mathbf{A}^{-1} \tag{A11}$$

where **A** is a constant square matrix with elements as $A_{ij} = \phi_j(x_i)$ and the matrix **G** is given by

$$\mathbf{G} = diag.(-\nu_1^2 \ -\nu_2^2 \ -\nu_3^2 \cdots -\nu_N^2)\,. \tag{A12}$$

Thus, we have

$$\mathbf{V} = \mathbf{A} \qquad \text{and} \qquad \Lambda_i = k_o^2 n_r^2 - \nu_i^2 \tag{A13}$$

Further, it can be shown that

$$\mathbf{V}^{-1} = \mathbf{A}^{-1} = \left(\frac{2}{N+1}\right)\mathbf{A}^T \tag{A14}$$

Thus, no matrix eigenvalue equation need be solved. With these values of $\mathbf{V}$ and $\mathbf{\Lambda}$, one obtains from Eq. (A10):

$$e^{\mathbf{H}_1\Delta z} = \left(\frac{2}{N+1}\right)\begin{pmatrix}\mathbf{A} & 0\\ 0 & \mathbf{A}\end{pmatrix}\begin{bmatrix} c_1 & 0 & \cdots & 0 & s_1 & 0 & \cdots & 0 \\ 0 & c_2 & \cdots & 0 & 0 & s_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & c_N & 0 & 0 & \cdots & s_N \\ \tilde{s}_1 & 0 & \cdots & 0 & c_1 & 0 & \cdots & 0 \\ 0 & \tilde{s}_2 & \cdots & 0 & 0 & c_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \tilde{s}_N & 0 & 0 & \cdots & c_N \end{bmatrix}\begin{pmatrix}\mathbf{A}^T & 0\\ 0 & \mathbf{A}^T\end{pmatrix} \tag{A15}$$

where

$$c_i = \cos(\sqrt{\Lambda_i}\Delta z), \qquad s_i = \frac{1}{\sqrt{\Lambda_i}}\sin(\sqrt{\Lambda_i}\Delta z) \quad \text{and} \quad \tilde{s}_i = -\sqrt{\Lambda_i}\,\sin(\sqrt{\Lambda_i}\Delta z).$$

In cases, where Λ_i is imaginary [see Eq. (A13)], the quantities c_i, s_i and $\tilde{s}_i$ remain real and sine and cosine functions are evaluated through the corresponding hyperbolic functions.

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Table I: Comparison of error/power loss in propagation to 100 μm in the benchmark[19] step index waveguide for TE_{10} modes using different methods.

Method	N_z	N_x	Power in waveguide at 20^o
SSNP	1000	800	~0.96
DNS[14]	1000	800	~0.90
AMIGO[19]	1429	1311	~0.95
FD2BPM[19]	1000	2048	~0.95
FTBPM[19]	1000	256	~0.55
LETI-FD[19]	200	1024	~0.15

Figure Captions

Fig. 1 Error in propagation as a function of the number of propagation steps with Δz for the graded index waveguide[5].

Fig. 2 Error in propagation with the tilt angle of the graded-index waveguide[5] for propagation up to 100 μm .

Fig. 3 Error in propagation as a function of the number of propagation steps with Δz for the step index waveguide.[6]

Fig. 4 Error in propagation with the tilt angle of the step-index waveguide[6] for propagation up to 100 μm .

Fig. 5 Error in propagation as a function of the number of propagation steps with Δz for the benchmark step-index waveguide[19].

Fig. 6 Error in propagation with the tilt angle of the benchmark step-index waveguide[19] for propagation up to 100 μm .

Fig. 7 Error in propagation with the reference refractive index for the benchmark step-index waveguide[19] for propagation up to 100 μm with step size 0.1 μm at 40^{o}.

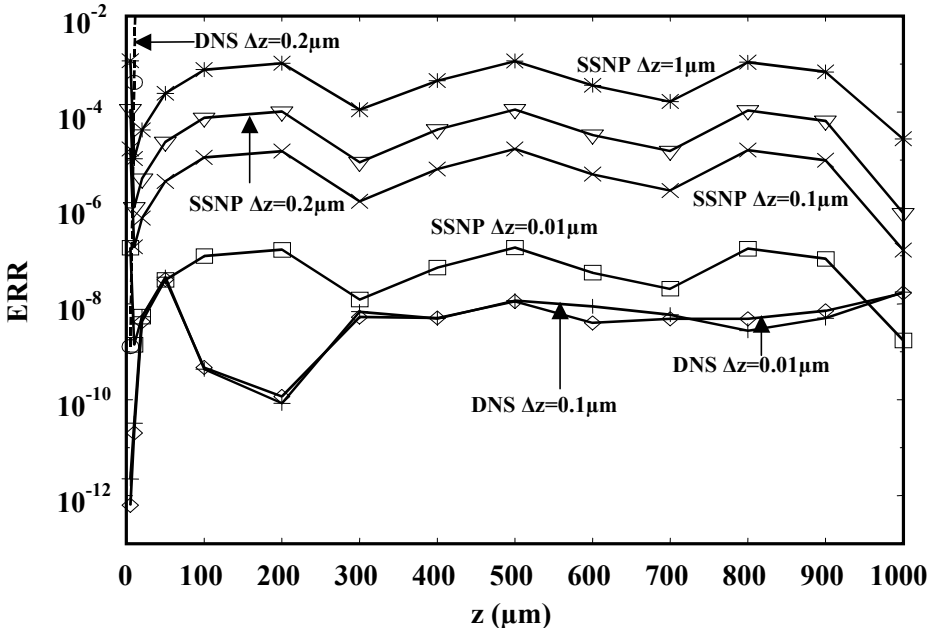
DNS Δz=0.2μm
SSNP Δz=1μm
SSNP Δz=0.2μm
SSNP Δz=0.1μm
SSNP Δz=0.01μm
DNS Δz=0.01μm
DNS Δz=0.1μm
ERR
z (μm)
10-2
10-4
10-6
10-8
10-10
10-12
0
100
200
300
400
500
600
700
800
900
1000

Fig. 1

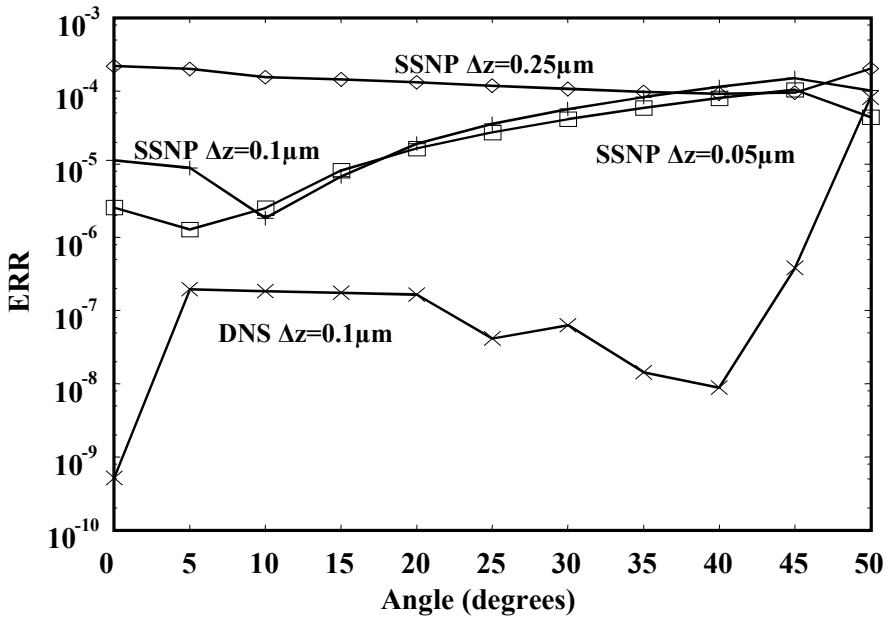
SSNP Δz=0.25µm
SSNP Δz=0.1µm
SSNP Δz=0.05µm
DNS Δz=0.1µm
ERR
Angle (degrees)
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
10^-10
0
5
10
15
20
25
30
35
40
45
50

Fig. 2

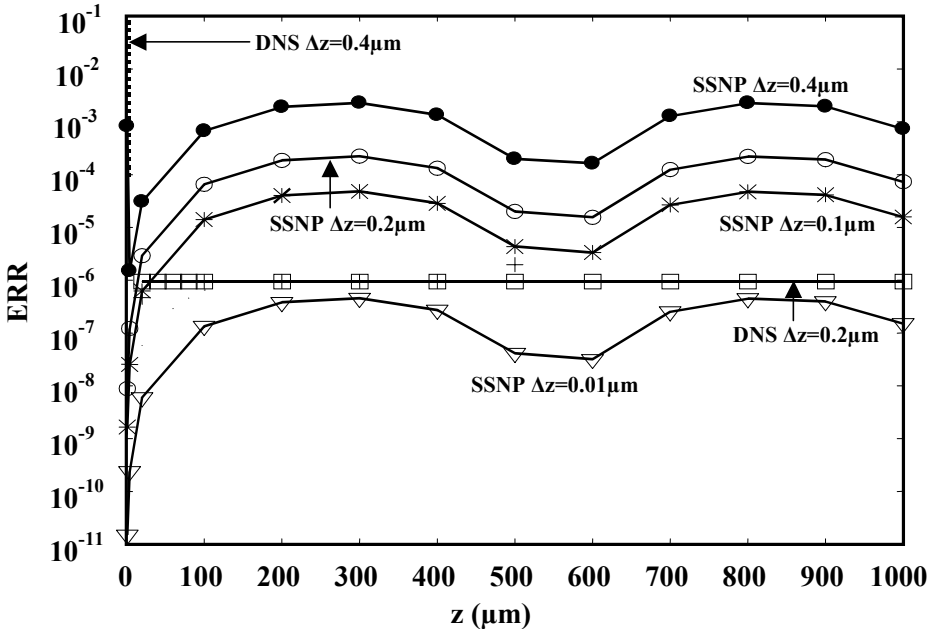
DNS Δz=0.4μm
SSNP Δz=0.4μm
SSNP Δz=0.2μm
SSNP Δz=0.1μm
DNS Δz=0.2μm
SSNP Δz=0.01μm
ERR
z (μm)
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
10^-10
10^-11
0
100
200
300
400
500
600
700
800
900
1000

Fig. 3

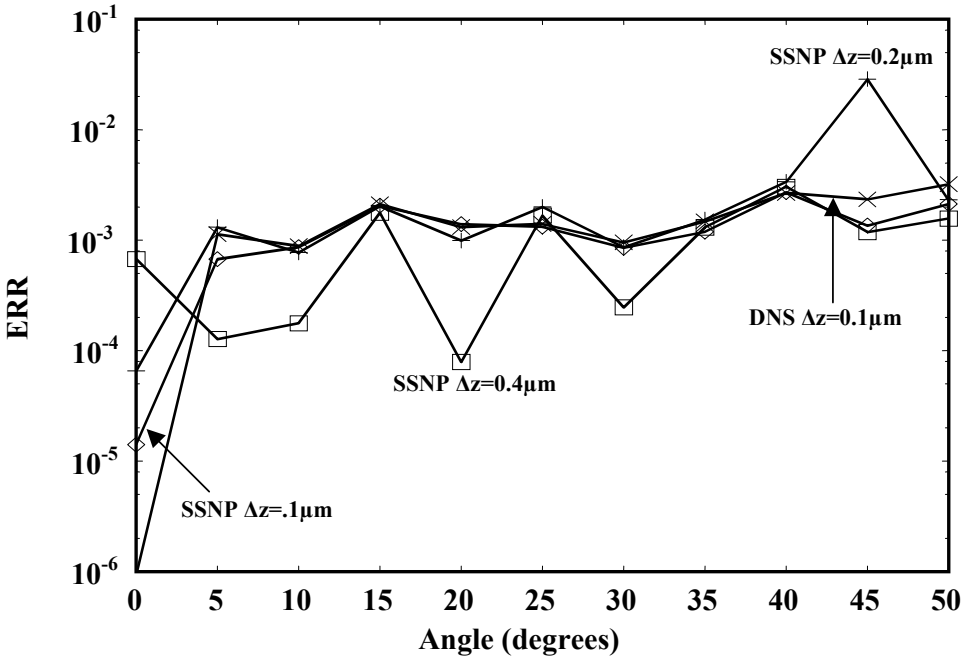
ERR
Angle (degrees)
SSNP Δz=0.2μm
DNS Δz=0.1μm
SSNP Δz=0.4μm
SSNP Δz=.1μm

Fig. 4

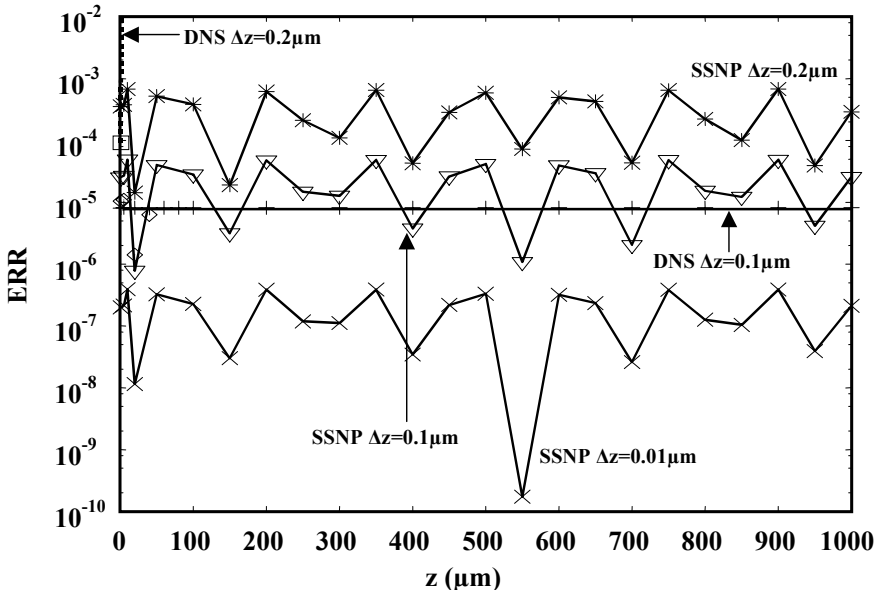
DNS Δz=0.2μm
SSNP Δz=0.2μm
DNS Δz=0.1μm
SSNP Δz=0.1μm
SSNP Δz=0.01μm
ERR
z (μm)
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
10^-10
0
100
200
300
400
500
600
700
800
900
1000

Fig. 5

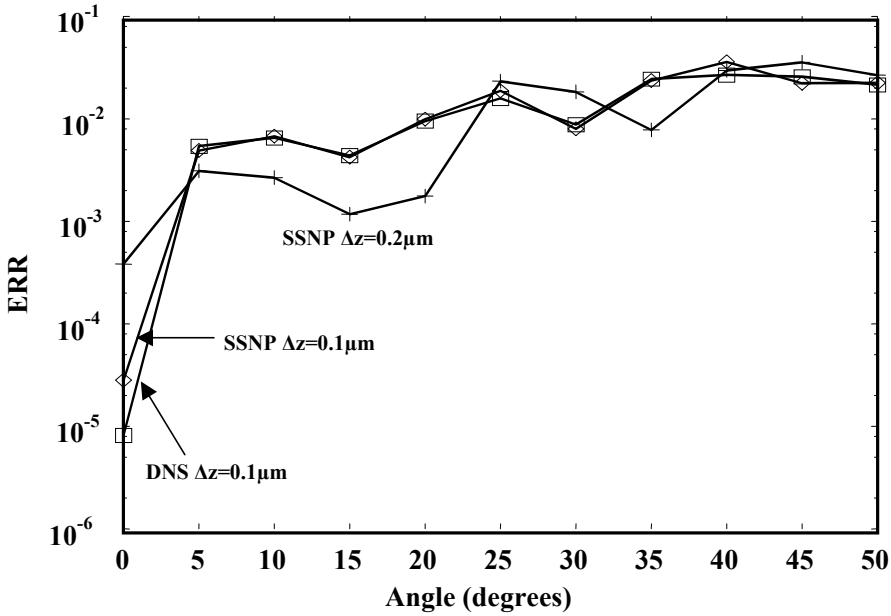

Fig. 6

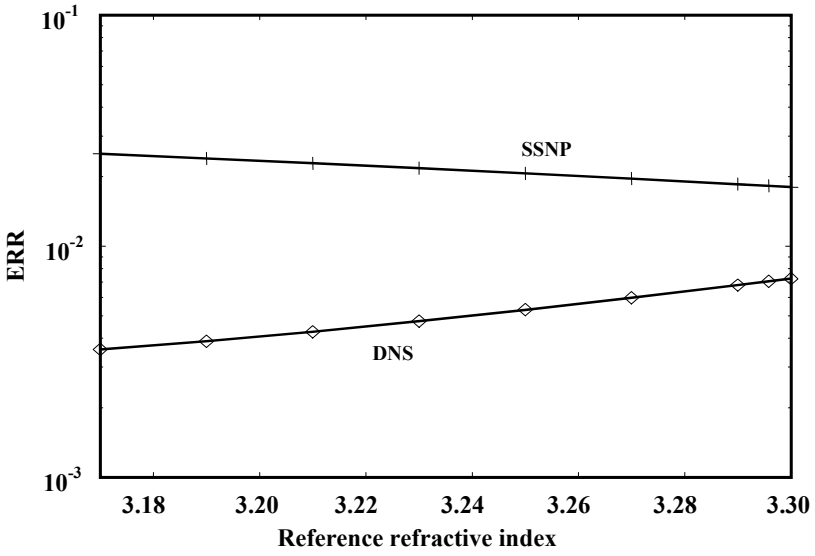
10^-1
10^-2
10^-3
ERR
SSNP
DNS
3.18
3.20
3.22
3.24
3.26
3.28
3.30
Reference refractive index

Fig. 7