CES Technology and Business Cycle Fluctuations*

Cristiano Cantore
University of Surrey

Paul Levine
University of Surrey

Joseph Pearlman
City University London

Bo Yang
Xi'an Jiaotong - Liverpool University
University of Surrey

September 1, 2015

Abstract

We contribute to an emerging literature that brings the constant elasticity of substitution (CES) specification of the production function into the analysis of business cycle fluctuations. Using US data, we estimate by Bayesian-Maximum-Likelihood methods a standard medium-sized DSGE model with a CES rather than Cobb-Douglas (CD) technology. We estimate a elasticity of substitution between capital and labour well below unity at 0.15-0.18. In a marginal likelihood race CES decisively beats the CD production and this is matched by its ability to fit the data better in terms of second moments. We show that this result is mainly driven by the implied fluctuations of factor shares under the CES specification. The CES model performance is further improved when the estimation is carried out under an imperfect information assumption. Hence the main message for DSGE models is that we should dismiss once and for all the use of CD for business cycle analysis.

JEL Classification: C11, C52, D24, E32.
Keywords: CES production function, DSGE model, Bayesion estimation, Imperfect information

*An earlier version of this paper was presented at the first MONFISPOL Conference, Nov 4 – 5, 2010, London Metropolitan University. Thanks are due to participants and the discussant Peter McAdam in particular, for valuable comments. We also thank the Editor Christopher Otrok and one anonymous referee for valuable comments. We acknowledge financial support from the EU Framework Programme 7 and from the ESRC, project no. RES-062-23-2451. The corresponding author is Cristiano Cantore, email c.cantore@surrey.ac.uk.

1 Introduction

This paper extends a standard medium-sized DSGE model developed by Christiano *et al.* (2005) and Smets and Wouters (2007) to allow for a richer and more data coherent specification of the production side of the economy. The idea is to enrich what has become the workhorse DSGE model by relaxing the usual Cobb-Douglas production assumption in favour of a more general CES function. This then allows for cyclical variations in factor shares, the estimation of the capital/labour elasticity of substitution and biased technical change.

The CES production function has been used extensively in many area of economics since the middle of the previous century (Solow (1956) and Arrow *et al.* (1961)). Thanks to La Grandville (1989), who introduced the concept of *normalization*, it has been extensively used in growth theory.[1] Factor substitution and the bias in technical change feature an important role in many other areas of economics[2] but, until recently have been largely disregarded in business cycle analysis. On the empirical side León-Ledesma *et al.* (2010) show that normalization improves empirical identification.[3]

The concepts of biased technical change and imperfect factor substitutability between factors of production have been introduced in business cycle analysis by Cantore *et al.* (2014b). They show that the introduction of a normalized CES production function into an otherwise standard Real Business Cycle (RBC) and/or New Keynesian (NK) DSGE model significantly changes the response of hours worked to a technology shock in both settings and that such response might change as well within each model depending on the parameters related to the production process. They also show how the introduction of biased technical change and imperfect substitutability allow movements in factor shares which appear to fluctuate at business cycle frequencies in the data but are theoretically

[1] La Grandville (1989) showed that it was possible to obtain a perpetual growth in income per-capita, even without any technical progress. See Cantore and Levine (2012) for a recent perspective on normalization.

[2] The value of the substitution elasticity has been linked to differences in international factor returns and convergence (e.g., Klump and Preissler (2000), Mankiw (1995)); movements in income shares (Blanchard (1997), Caballero and Hammour (1998), Jones (2003)); the effectiveness of employment creation policies (Rowthorn (1999)), etc. The nature of technical change, on the other hand, matters for characterizing the welfare consequences of new technologies (Marquetti (2003)); labour-market inequality and skills premia (Acemoglu (2002)); the evolution of factor income shares (Kennedy (1964), Acemoglu (2003)) etc.

[3] They show that using a normalized approach permits to overcome the 'impossibility theorem' stated by Diamond *et al.* (1978) and simultaneously identify the elasticity of substitution and biased technical change.

constant under the Cobb-Douglas specification.[4] Indeed there is mounting evidence in the literature[5] that whilst constant factor shares might be a good approximation for growth models where the time span considered is very long, at business cycle frequencies those shares are not constant. This is clearly shown in Figure 1 for the US data used to estimate our model.

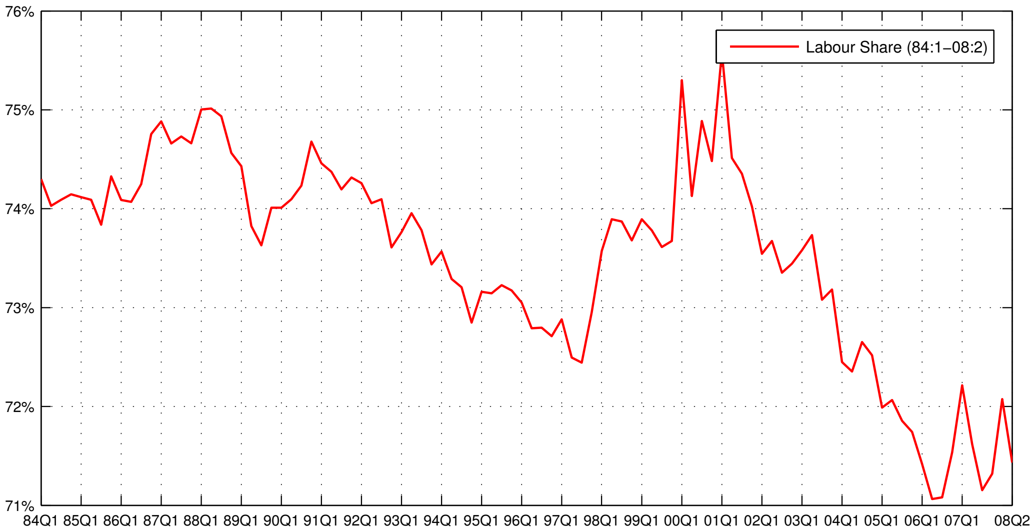

Figure 1: US Labour Share (Source: U.S. Bureau of labour Statistics)

There is also increasing interest in reconciling the apparent long run constancy of factor shares with the short-run fluctuations. León-Ledesma and Satchi (2011) approach is based on the choice of technologies by firms in terms of their capital intensity and results in a new class of production functions that produces short-run capital-labour complementarity but yields a long-run unit elasticity of substitution.[6] Koh and Santaeulalia-Llopis (2014) use a non-constant elasticity of substitution production function allowing the elasticity to vary over time in the short run while keeping the Cobb-Douglas assumption in the long run. They estimate this elasticity in a perfectly competitive setting and find that the elasticity is countercyclical and shocks are on average biased towards labour.[7]

[4] Furthermore Cantore *et al.* (2012) test empirically the model(s) developed by Cantore *et al.* (2014b) using rolling-windows Bayesian techniques in order to check if the documented time-varying relation between hours worked, productivity and output (see Fernald (2007) and Galí and Gambetti (2009) among others) can be explained using the threshold rule.

[5] See for example Blanchard (1997), Jones (2003, 2005), McAdam and Willman (2013) and Ríos-Rull and Santaeulália-Llopis (2010).

[6] See also León-Ledesma and Satchi (2015) for an approach based on the concept of a technology frontier.

[7] They show how their model is able to solve four labour market puzzles that had been previously

In the DSGE literature however, most models continue to use the Cobb-Douglas assumption even if the empirical evidence provided through the years has ruled out the possibility of unitary elasticity of substitution (see among others Antràs (2004), Klump *et al.* (2007), Chirinko (2008) and León-Ledesma *et al.* (2010)). In this paper we show that the introduction of a CES production function in a medium-scale DSGE model in the spirit of Christiano *et al.* (2005) and Smets and Wouters (2007) makes it possible to exploit the movements of factor shares we observe in the data to improve significantly the performance of the model. To the best of our knowledge, we are the first to compare the empirical implications of CD and CES production functions in a medium scale DSGE context using Bayesian-Maximum-Likelihood comparison.[8]

The main results of our paper are first: in terms of model posterior probabilities, impulse responses, second moments and autocorrelations, the assumption of a CES technology significantly improves the model fit. Second, this finding is robust to the information assumption assumed for private agents in the model. Indeed allowing the latter to have the same (imperfect) information as the econometrician (namely the data) further improves the fit compared with the standard assumption that they have perfect information of all state variables including the shock processes. Third, using US data, we estimate by Bayesian-Maximum-Likelihood (BML) the elasticity of substitution between capital and labour to be 0.15-0.18, a value broadly in line with the literature using other methods of estimation.[9]

solved only in isolation and with the introduction of frictions and/or non-competitive settings. The four puzzles being: i) The low (or even negative) correlation between real wage and output; ii) the vanishing procyclicality of labour productivity (see Galí and van Rens (2014)); iii) the negative correlations of labour share and output and its components including its overshooting response of labour share to productivity; iv) the negative hours-productivity correlation.

[8]Choi and Ríos-Rull (2009) and Koh and Santaeulalia-Llopis (2014) also present a comparison of CD vs CES. Choi and Ríos-Rull (2009) use a calibrated version of their model with CES with a specific value of the elasticity of substitution (0.75), that turns out to be quite high compared to recent estimates, and show that the implied responses of the economy to a productivity shock are very similar, apart from the labour share response that shows a mild overshooting. Koh and Santaeulalia-Llopis (2014) instead compare NCES (non-constant elasticity of substitution) with constant CES and CD production. Our comparison here differs from theirs on three dimensions. First they perform a non-linear estimation of their model while here we are estimating a first order approximation of a DSGE model using perturbation methods. For this reason we are able to compute posteriors probabilities. Second we are not allowing the elasticity of substitution between capital and labour to vary over time. Third we are comparing the production functions in a medium scale DSGE model with nominal and real frictions allowing for a larger set of shocks while their comparison of their model against non-competitive settings is done conditional only to productivity shocks.

[9]See, for example, Table 2 in Rowthorn (1999), Chirinko (2008), León-Ledesma *et al.* (2010) and the survey by Klump *et al.* (2012).

As we show later in the paper the first result is mainly driven by the implied fluctuations of factor shares under the CES specification. Although the main focus of the paper is not to match labour share moments and business cycle behaviour, this paper is related to a fast growing literature on the cyclical behaviour of the labour share which we briefly review below in order to put our model and results in context.

Although cyclical properties of factor shares have been understudied for many years, scholars have reached a wide consensus regarding two business cycle characteristics of the labour share. Labour share is countercyclical (but with a low correlation) on impact and it overshoots following productivity innovations.[10] This overshooting behaviour has been presented by Ríos-Rull and Santaeulália-Llopis (2010) who find that, conditional on productivity shocks, the labour share in U.S. is negatively correlated with contemporaneous output but positively with lagged output (5 quarters or more).

To explain the contercyclicality some authors have focused on market rigidities that prevent prices and volumes to adjust perfectly and rapidly to shocks. Gomme and Greenwood (1995) and Boldrin and Horvath (1995) introduce labour and insurance contracts into the market which enable risk sharing between workers and firms.[11]

Choi and Ríos-Rull (2009) introduce a RBC model with search and matching frictions and a non-competitive labour market, where wages and hours are not determined by their marginal products but by a bargaining process. Wages then exceed the marginal product of labour, while employment remains fixed due to the search frictions. This leads to a sluggish, countercyclical behaviour of the share.[12] Bentolila and Saint-Paul (2003) argue that changes in the product market markups, occurring due to imperfect competition, may cause a cyclical movement of the labour share as they fluctuate over the business cycle. A procyclical markup in the product market may cause countercyclical

[10]Hansen and Prescott (2005) discuss macroeconomic dynamics in the light of countercyclical labour shares in the US between 1954 and 1993. The European Commission (2007) showed detailed business cycle behaviour of the labour share for European countries confirming countercyclicality for all countries except Germany. Choi and Ríos-Rull (2009) and Ríos-Rull and Santaeulália-Llopis (2010) find that the labour share in the short run is relatively volatile, countercyclical, highly persistent, lagging output and overshoots the initial loss after a positive technology shock.

[11]Similar to the idea of introducing sticky wages into a model through implicit contracts by Boldrin and Horvath (1995) and Gomme and Greenwood (1995) suggests that only a subset of workers and firms bargain over wages at each point in time so that wages cannot be adjusted immediately after an increase in productivity. After such an increase prices and productivity go up while there is only a sluggish adjustment of aggregates wages.

[12]Also Justiniano and Michelacci (2012) discuss the behaviour of the labour share in a search and matching context.

shifts in the labour share. Shao and Silos (2014) introduce costly entry of firms in a model with frictional labour markets and find that the key to replicate factor share dynamics is to have a weak correlation between the real interest rate and output. Colciago and Rossi (2015) show that Nash bargaining delivers a countercyclical labour share while countercyclical mark-ups deliver the overshooting response. What all these papers have in common is that there are frictions or adjustments in the short run that create endogenous fluctuations in the labour share while maintaining the CD assumption in production.[13] Koh and Santaeulalia-Llopis (2014) instead present a competitive market framework, using a flexible production function (non-constant CES in the short run and CD in the long run) and are able to replicate and match the cyclical behaviour of the labour share and various labour market variables. From this literature it emerges that we are able to match second moments of the labour share in different settings. We have a wide set of different competing models explaining the same behaviour in terms of second moments. Here in this paper we provide an additional one which does not require an extensive modification of the workhorse medium scale DSGE used extensively in the literature and by policy-makers. We follow the conclusion of Choi and Ríos-Rull (2009) and Cantore *et al.* (2014b) that, in order to match the cyclical behaviour of the labour share, we should focus on a more general specification of the production side of the economy, rather than on non-competitive factor markets.

Finally there is also a growing literature looking at the long-run behaviour of the labour share in the U.S. pointing towards an observed decline in its trend since the 70s (see Piketty and Zucman (2014) and Neiman and Karabarbounis (2014) amongst others.) This implies that, in order to match empirical phenomena, the macroeconomic literature should move away from linear and stationary models, as in the one presented here, and develop models of the U.S. economy featuring a non-stationary labour share. However there remains a substantial degree of uncertainty and disagreement in the literature regarding the long-run decline in the labour share. Muck *et al.* (2015) show that the consequences of the ambiguity in the empirical definition of the labour share do not matter much for the short run, but do affect its long-run trend. Their preferred measure, amongst the various examined, does show a decline from the 70s but also a substantial increase before that.

[13] Apart from the last section in Choi and Ríos-Rull (2009).

For this reason they suggest that we should focus not only on the decline in recent years but extend the sample in order to explain this observed hump shaped response. Bridgman (2014) focuses on the differences between gross and net labour share. He shows that once we take into account items that do not add to capital, depreciation and production taxes net labour share is within historical averages and does not show a significant decline as we observe in gross measures. Finally Koh *et al.* (2015) show that labour share computed in an economy with only structures and equipment capital is indeed stationary, which actually is, the case of the economy considered here.

The rest of the paper is organised as follows. Section 2 summarise the model with particular attention paid to the normalization of the CES production function. Section 3 sets out the BML estimation of the model. Section 4 examines the ability of the models to capture the main characteristics of the actual data as described by second moments, discusses identification and sensitivity issues and presents the models implied response of labour share conditional on productivity shocks. Section 5 concludes the paper.[14]

2 Summary of the Model

Here we summarise the Smets and Wouters (2007) augmented model (SW) with a wholesale and a retail sector, Calvo prices and wages, CES production function, adjustment costs of investment and variable capital utilization.[15] Figure 2 illustrates the model structure. The novel feature is the introduction of a CES production function in the wholesale sector, instead of the usual Cobb-Douglas form. This generalization then allows for the identification of both labour-augmenting and capital-augmenting technology shocks. As in Smets and Wouters (2007) we use a household utility function compatible with a balance growth path in the steady state, but we adopt a more standard functional form used in the RBC literature. However we do not adopt Kimball aggregators for final output and composite labour.[16] Again as in their paper we introduce a monopolistic trade-union that

[14]The online Appendix available here presents the full derivation of the model, the estimation result together with figures for prior and posterior distribution of the parameters for the baseline model estimated with asymmetric information and identification and sensitivity analysis details for the labour market and input elasticity of substitution parameters.

[15]For the full model derivation and equilibrium conditions see the Section A in the online Appendix.

[16]The motivation for generalizing Dixit-Stiglitz aggregators is to bring estimates of price and wage contract lengths into line with micro-econometric evidence. In fact our estimates for US data are compatible with the simpler Dixit-Stiglitz formulation.

allows households to supply homogeneous labour. Then as long as preference shocks are symmetric, households are identical in equilibrium and the complete market assumption is no longer required for aggregation. The supply-side of the economy consists of competitive retail sector producing final output and a monopolistically competitive wholesale sector producing differentiated goods using the usual inputs of capital and work effort. Households consume a bundle of differentiated commodities, supply labour and capital to the production sector, save and own the monopolistically competitive firms in the goods sector. Capital producers provide the capital inputs into the wholesale sector.[17]

The sequencing of decisions is as follows:[18]

1. Each household supplies homogeneous labour at a price $W_{h,t}$ to a trade-union. Households choose their consumption, savings and labour supply given aggregate consumption (determining external habit). In equilibrium all household decisions are identical.

2. Capital producing firms convert final output into new capital which is sold on to intermediate firms.[19]

3. A monopolistic trade-union differentiates the labour and sells type $N_t(j)$ at a price $W_t(j)$ to a labour packer in a sequence of Calvo staggered wage contracts. In equilibrium all households make identical choices of total consumption, savings, investment and labour supply.

4. The competitive labour packer forms a composite labour service according to a constant returns CES technology $N_t = \left(\int_0^1 N_t(j)^{(\zeta-1)/\zeta} dj\right)^{\zeta/(\zeta-1)}$ and sells onto the

[17]There are other differences with Smets and Wouters (2007): (i) Our price and wage mark-up shocks follow an AR(1) process instead of the ARMA process chosen by SW; (ii) in SW the government spending shock is assumed to follow an autoregressive process which is also affected by the productivity shock; (iii) we have a preference shock instead of the risk-premium shock. Chari *et al.* (2009) criticise the risk premium shock arguing that it has little interpretation and is unlikely to be invariant to monetary policy. We prefer our somewhat simpler set-up and we expect none of the differences to affect the main focus of the paper which is on the comparison between CD and CES production functions.

[18]Sequencing matters for the monopolistic trade-unions and intermediate firms who anticipate and exploit the downward-sloping demand for labour and goods respectively. Different set-ups with identical equilibria are common in the literature. Monopolistic prices can be transferred to the retail sector. When it comes to introducing financial frictions, for example, as in Gertler and Karadi (2011) the introduction of separate capital producers as in our set-up is convenient, but not essential in the SW model without such frictions.

[19]Alternatively one can assume households rent capital to firms. Introducing capital producers gives a more flexible set-up suitable for the introduction of a banking sector and financial firms as in Gertler and Karadi (2011).

intermediate firm.

5. Each intermediate monopolistic firm f using composite labour and capital rented from capital producers to produce a differentiated intermediate good which is sold onto the final goods firm at price $P_t(f)$ in a sequence of Calvo staggered price contracts.

6. Competitive final goods firms use a continuum of intermediate goods according to another constant returns CES technology to produce aggregate output $Y_t = \left(\int_0^1 Y_t(f)^{(\mu-1)/\mu} df\right)^{\mu/(\mu-1)}$.

We then solve the model by backward induction starting with the production of final goods as we show in Section A of the online Appendix. Below we discuss in details the CES production and its *normalization*, the choice of utility function and list the exogenous shocks of the model.

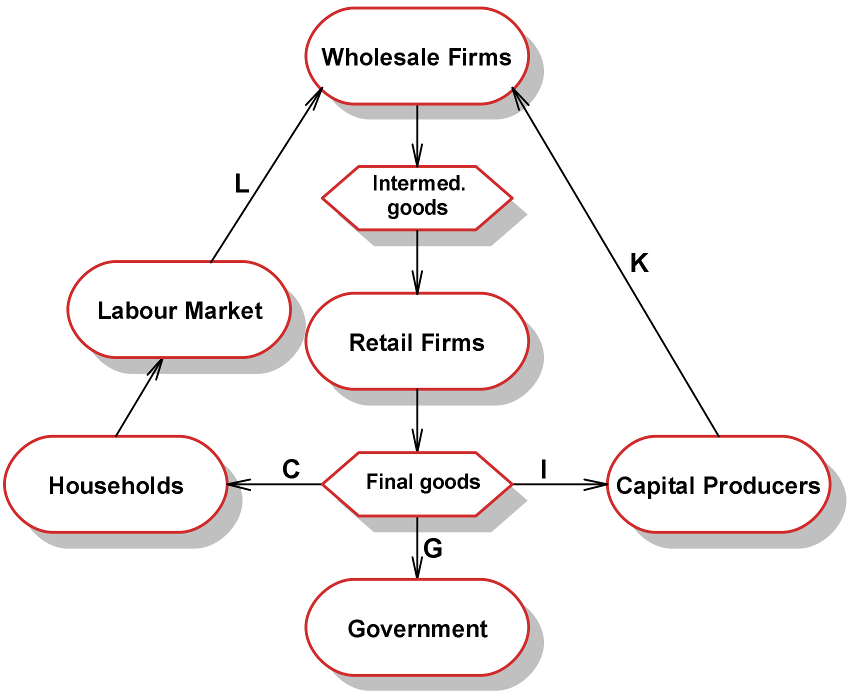

Figure 2: Model Structure

2.1 The Normalized CES Production Function

The production function is assumed to be CES as in Cantore *et al.* (2014b) which nests Cobb-Douglas as a special case and admits the possibility of neutral and non-neutral technical change. Here we adopt the 're- parametrization' procedure described in Cantore and Levine (2012) in order to normalize the CES production function:

$$\begin{aligned} Y_t &= \left[\alpha_k (ZK_t U_t K_t)^{\psi} + \alpha_n (ZN_t N_t)^{\psi}\right]^{\frac{1}{\psi}} - F\,;\ \psi \neq 0\ \&\ \alpha_k + \alpha_N \neq 1 \\ &= (ZK_t U_t K_t)^{\alpha_k} (ZN_t N_t)^{\alpha_n} - F\,;\ \psi \to 0\ \&\ \alpha_k + \alpha_N = 1 \end{aligned} \quad (1)$$

where Y_t, K_t, N_t and U_t are wholesale output, capital and labour inputs and variable capital utilisation respectively at time t and ψ is the substitution parameter and α_k and α_n are sometimes referred as distribution parameters. The terms ZK_t and ZN_t capture capital-augmenting and labour-augmenting technical progress respectively.[20] Calling σ the elasticity of substitution between capital and labour,[21] with $\sigma \in (0, +\infty)$ and $\psi = \frac{\sigma-1}{\sigma}$ then $\psi \in (-\infty, 1)$. When $\sigma = 0 \Rightarrow \psi = -\infty$ we have the Leontief case, whilst when $\sigma = 1 \Rightarrow \psi = 0$ (1) collapses to the usual Cobb-Douglas case.

From the outset a comment on dimensions would be useful. Technology parameters are not measures of efficiency as they depend on the units of output and inputs (i.e., are not dimensionless[22]) and the problem of normalization arises because unless $\psi \to 0$, α_n and α_k in (1) are not shares and in fact are not dimensionless.

[20] We are in the case of Hicks neutrality if $ZK_t = ZN_t > 0$, Solow neutrality if $ZK_t > 0$ and $ZN_t = 0$ and Harrod neutrality in the case of $ZK_t = 0$ and $ZN_t > 0$.

[21] The elasticity of substitution for the case of perfect competition, where all the product is used to remunerate factor of productions, is defined as the elasticity of the capital/labour ratio with respect to the wage/capital rental ratio. Then calling W the wage and $R + \delta$ the rental rate of capital we can define the elasticity as follows:

$$\sigma = \frac{d\frac{K}{L}\frac{L}{K}}{d\frac{W}{R+\delta}\frac{R+\delta}{W}}.$$

See La Grandville (2009) for a more detailed discussion.

[22] For example for the Cobb-Douglas production function in the steady state, $Y = K^{\alpha}(AN)^{1-\alpha}$, by dimensional homogeneity, the dimensions of A are (output per period)$^{\frac{1}{1-\alpha}}$ / ((person hours per period)$\times$ (machine hours per period)$^{\frac{\alpha}{1-\alpha}}$). For some this poses a fundamental problem with the notion of a production function - see Barnett (2004). Units can be chosen so that when $N = 1$ and $K = 1$, then $Y = 1$ implying $A = 1$. For the equilibrium to be independent of the choice of units, it follows that it must be independent of the steady state value A. This is readily demonstrated in what follows.

Marginal products of labour and capital are respectively

$$F_{N,t} = \frac{Y_t}{N_t}\left[\frac{\alpha_n(ZN_tN_t)^{\psi}}{\alpha_k ZK_tU_tK_t^{\psi}+\alpha_n(ZN_tN_t)^{\psi}}\right] = \alpha_n ZN_t^{\psi}\left(\frac{Y_t}{N_t}\right)^{1-\psi} \tag{2}$$

$$F_{K,t} = \frac{Y_t}{K_t}\left[\frac{\alpha_k ZK_tU_tK_t^{\psi}}{\alpha_k ZK_tU_tK_t^{\psi}+\alpha_n(ZN_tN_t)^{\psi}}\right] = \alpha_k(U_tZK_t)^{\psi}\left(\frac{Y_t}{K_t}\right)^{1-\psi} \tag{3}$$

The equilibrium of real variables depends on parameters defining the RBC core of the model ϱ, σ_c, δ, ψ, α_k and α_n, and those defining the NK features. Of the former, ϱ, ψ and σ_c are dimensionless, δ depends on the unit of time, but unless $\psi = 0$ and the technology is Cobb-Douglas, α_k and α_n depend on the units chosen for factor inputs, namely machine units per period and labour units per period. To see this rewrite the wholesale firm's foc[23] in terms of factor shares

$$\frac{W_tN_t}{P_t^WY_t} = \alpha_n ZN_t^{\psi}\left(\frac{Y_t}{N_t}\right)^{-\psi} \tag{4}$$

$$\frac{(R_t-1+\delta)K_t}{P_t^WY_t} = \alpha_k(U_tZK_t)^{\psi}\left(\frac{Y_t}{K_t}\right)^{-\psi} \tag{5}$$

where $P_t^W \equiv MC_tP_t$ is the price of wholesale output, W_t is the nominal wage and R_t is the real interest rate. Then we have

$$\frac{W_tN_t}{(R_t+\delta)} = \frac{\alpha_n}{\alpha_k}\left(\frac{ZK_tU_tK_t}{ZN_tN_t}\right)^{-\psi} \tag{6}$$

Thus α_n (α_k) can be interpreted as the share of labour (capital) iff $\psi = 0$ and the production function is Cobb-Douglas. Otherwise the dimensions of α_k and α_n depend on those for $\left(\frac{ZK_tU_tK_t}{ZN_tN_t}\right)^{\psi}$ which could be for example, (effective machine hours per effective person hours)$^{\psi}$. In our aggregate production functions we choose to avoid specifying unit of capital, labour and output.[24] It is impossible to interpret and therefore to calibrate or estimate these 'share' parameters.

There are two ways to resolve this problem; 're-parameterize' the dimensional parameters α_k and α_n so that they are expressed in terms of dimensionless ones, with all

[23]Equations A.6 and A.7 in the Online Appendix.

[24]Unlike the physical sciences where particular units are explicitly chosen so dimension-dependent parameters pose no problems. For example the fundamental constants such as the speed of light is expressed in terms of metres per second; Newtons constant of gravitation has units cubic metres per (kilogram $\times$ second^2) etc.

parameters to be estimated or calibrated (see Cantore and Levine (2012)), or 'normalize' the production function in terms of deviations from a steady state. We consider these in turn.

2.1.1 Re-parametrization of α_n and α_k

On the balanced-growth path (BGP) consumption, output, investment, capital stock, the real wage and government spending are growing at a common growth rate g driven by exogenous labour-augmenting technical change $\overline{ZN}_{t+1} = (1+g)\overline{ZN}_t$, but labour input N is constant.[25] As is well-known a BGP requires either Cobb-Douglas technology or that technical change must be driven solely by the labour-augmenting variety (see, for example, Jones (2005)). Then $ZK_t = ZK$ must also be constant along the BGP.

On the BGP let capital share and wage shares in the wholesale sector be α and $1-\alpha$ respectively. Then using (4) and (5) we obtain our *re-parameterization* of α_n and α_k:

$$\alpha_k = \alpha \left(\frac{\bar{Y}_t}{ZK\bar{U}_t\bar{K}_t} \right)^{\psi} \tag{7}$$

$$\alpha_n = (1-\alpha) \left(\frac{\bar{Y}_t}{\overline{ZN}_t N} \right)^{\psi} \tag{8}$$

Note that $\alpha_k = \alpha$ and $\alpha_n = 1-\alpha$ at $\psi = 0$, the Cobb-Douglas case.[26] This completes the stationarized equilibrium defined in terms of *dimensionless* RBC core parameters ϱ, σ_c, ψ,α and δ which depends on the unit of time, plus NK parameters. In (7) and (8) dimensional parameters are expressed in terms of other endogenous variables Y^W, N and K which themselves are functions of $\theta \equiv [\sigma, \psi, \alpha, \delta, \cdots]$. Therefore $\alpha_n = \alpha_n(\theta)$, and $\alpha_k = \alpha_k(\theta)$ which expresses why we refer to this procedure as reparameterization.

There is one more normalization issue: the *choice of units* at some point say $t = 0$ on the steady state BGP. We use for simplicity $\bar{Y}_0 = \overline{ZN}_0 = ZK = 1$[27] but, as it is straightforward to show that having expressed the model in terms of dimensionless parameters re-parameterization makes the steady state ratios of the endogenous variables of the model independent of this choice.

[25] If output, consumption etc are defined in per capita terms then N can be considers as the proportion of the available time at work and is therefore both stationary and dimensionless.

[26] And as argued before if $\alpha \in (0,1)$ $\alpha_k + \alpha_n = 1$ iff $\psi = 0$.

[27] By assuming $\bar{Y}_0 = 1$ we implicitely assume $\bar{Y}_0^W = \frac{\bar{Y}_0}{1-c}$.

2.1.2 The Production Function in Deviation Form

This simply bypasses the need to retain α_k and α_n and writes the dynamic production function in deviation form about its steady state as

$$\frac{Y_t}{\bar{Y}_t} = \left[\frac{\alpha_k(ZK_tU_tK_t)^{\psi} + \alpha_n(ZN_tN_t)^{\psi}}{\alpha_k(ZK\bar{U}_t\bar{K}_t)^{\psi} + \alpha_n(\overline{ZN}_tN)^{\psi}}\right]^{\frac{1}{\psi}} = \left[\frac{\alpha_k\left(\frac{ZK_tU_tK_t}{ZK\bar{U}_t\bar{K}_t}\right)^{\psi}}{\alpha_k + \alpha_n\left(\frac{\overline{ZN}_tN}{ZK\bar{U}_t\bar{K}_t}\right)^{\psi}} + \frac{\alpha_n\left(\frac{ZN_tN_t}{\overline{ZN}_tN_t}\right)^{\psi}}{\alpha_k\left(\frac{ZK\bar{U}_t\bar{K}_t}{\overline{ZN}_tN}\right)^{\psi} + \alpha_n}\right]^{\frac{1}{\psi}}$$

Then from (7) and (8) we can write this simply as

$$\frac{Y_t}{\bar{Y}_t} = \left[(1-\alpha)\left(\frac{ZK_tU_tK_t}{ZK\bar{U}_t\bar{K}_t}\right)^{\psi} + \alpha\left(\frac{ZN_tN_t}{\overline{ZN}_tN}\right)^{\psi}\right]^{\frac{1}{\psi}} \tag{9}$$

as in Cantore *et al.* (2014b). The steady-state normalization now consists of $\overline{ZN}_0 = \bar{Y}_0 = ZK = 1$[28] and is characterized entirely by fixed shares of consumption, investment and government spending and by labour supply as a proportion of available time, all dimensionless quantities apart from the unit of time.

Using either of these two approaches, as showed by Cantore and Levine (2012), the steady state ratios of the endogenous variables and the dynamics of the model are not affected by the starting values of output and the two source of shocks ($\bar{Y}_0, \overline{ZN}_0$, ZK) which only represent choice of units. Crucially, this implies also that changing σ does not change our steady state ratios and factor shares, impulse response functions are directly comparable, and parameter values are consistent with their economic interpretation.

2.2 Utility Function

The household utility function is of a Cobb-Douglas-CRRA form chosen to be compatible with a balanced-growth steady state and allowing for external habit-formation:

$$\begin{aligned}
U_t &= \frac{eB_t((C_t - \chi C_{t-1})^{(1-\varrho)}(1-N_t)^{\varrho})^{1-\sigma_c} - 1}{1-\sigma_c} && (10)\\
U_{C,t} &= eB_t(1-\varrho)(C_t - \chi C_{t-1})^{(1-\varrho)(1-\sigma_c)-1}((1-N_t)^{\varrho(1-\sigma_c)}) && (11)\\
U_{N,t} &= -eB_t\varrho(C_t - \chi C_{t-1})^{(1-\varrho)(1-\sigma_c)}(1-N_t)^{\varrho(1-\sigma_c)-1} && (12)
\end{aligned}$$

[28] Which is almost identical to the one used in Cantore *et al.* (2014b) although they normalize as well hours worked to 1 using the accounting identity $\bar{Y} = (\bar{R}+\delta)\bar{K} + \bar{W}\bar{N}$.

where C_t is aggregate consumption, χ is the habit formation parameter, σ_c measures relative risk aversion, ϱ is the inverse of the Frisch elasticity of labour supply and eB_t is a preference shock.

2.3 Shock Processes

As we show in the Online Appendix the model has a total of seven AR(1) exogenous shocks[29] plus the iid shock to the monetary policy rule. The seven autoregressive shocks are in order: capital-augmenting, labour-augmenting, investment specific, preference, government spending, price and wage mark-up.

3 Estimation

We estimate the linearized version of the model around zero steady state inflation by Bayesian methods using DYNARE. We use the same observable set as in Smets and Wouters (2007) in first difference at quarterly frequency but extend the sample length to the second quarter of 2008, before the outbreak of the 2008 crisis. Namely, these observable variables are the log differences of real GDP, real consumption, real investment and real wage, log hours worked, the log difference of the GDP deflator and the federal funds rate. As in Smets and Wouters (2007), hours worked are derived from the index of average hours for the non-farm business sector and we divide hourly compensation from the same sector by the GDP price deflator to obtain the real wage. All series are seasonally adjusted. All data are taken from the FRED Database available through the Federal Reserve Bank of St.Louis and the US Bureau of Labour Statistics. The sample period is 1984:1-2008:2. The corresponding measurement equations for the 7 observables are:[30]

[29] Expressed in log deviation forms: $\log X_t - \log X = \rho_X(\log X_{t-1} - \log X) + \epsilon_{X,t}$. Where ρ_X and ε_X will be the autoregressive parameter and the standard deviation of shock X respectively.

[30] Where I_t, Π_t and $R_{n,t}$ are investment, inflation and nominal interest rate in the model.

$$
\begin{bmatrix} D(\log GDP_t) * 100 \\ D(\log CONS_t) * 100 \\ D(\log INV_t) * 100 \\ D(\log WAGE_t) * 100 \\ \log(GDPDEF_t/GDPDEF_{t-1}) * 100 \\ FEDFUNDS_t/4 \\ HOURS_t \end{bmatrix} = \begin{bmatrix} \log\left(\frac{Y_t}{\overline{Y}_t}\right) - \log\left(\frac{Y_{t-1}}{\overline{Y}_{t-1}}\right) + \text{ctrend} \\ \log\left(\frac{C_t}{\overline{C}_t}\right) - \log\left(\frac{C_{t-1}}{\overline{C}_{t-1}}\right) + \text{ctrend} \\ \log\left(\frac{I_t}{\overline{I}_t}\right) - \log\left(\frac{I_{t-1}}{\overline{I}_{t-1}}\right) + \text{ctrend} \\ \log\left(\frac{W_t/P_t}{\overline{W_t/P_t}}\right) - \log\left(\frac{W_{t-1}/P_{t-1}}{\overline{W_{t-1}/P_{t-1}}}\right) + \text{ctrend} \\ \log\left(\frac{\Pi_t}{\overline{\Pi}}\right) + \text{conspie} \\ \log\left(\frac{R_{n,t}}{\overline{R_n}}\right) + \text{consrn} \\ \log\left(\frac{N_t}{\overline{N}}\right) + \text{conslab} \end{bmatrix}
$$

The first four potentially non-stationary observables (output Y_t, consumption C_t, investment I_t and the real wage W_t/P_t) are taken in first differences, while gross inflation Π_t, the gross nominal interest rate $R_{n,t}$, and hours worked N_t, are used in levels. We introduce a common trend in the first-differenced variables and a specific one for inflation, nominal interest rate and hours worked.

3.1 Bayesian Methodology

Bayesian estimation entails obtaining the posterior distribution of the model's parameters, say θ, conditional on the data. Using the Bayes' theorem, the posterior distribution is obtained as:

$$
p(\theta|Y^T) = \frac{L(Y^T|\theta)p(\theta)}{\int L(Y^T|\theta)p(\theta)d\theta} \tag{13}
$$

where $p(\theta)$ denotes the prior density of the parameter vector θ, $L(Y^T|\theta)$ is the likelihood of the sample Y^T with T observations (evaluated with the Kalman filter) and $\int L(Y^T|\theta)p(\theta)d\theta$ is the marginal likelihood. Since there is no closed form analytical expression for the posterior, this must be simulated.

One of the main advantages of adopting a Bayesian approach is that it facilitates a formal comparison of different models through their posterior marginal likelihoods, computed using the Geweke (1999) modified harmonic-mean estimator. For a given model $m_i \in M$ and common data set, the marginal likelihood is obtained by integrating out vector θ,

$$L\left(Y^T|m_i\right) = \int_\Theta L\left(Y^T|\theta, m_i\right) p\left(\theta|m_i\right) d\theta \tag{14}$$

where $p_i\left(\theta|m_i\right)$ is the prior density for model m_i, and $L\left(Y^T|m_i\right)$ is the data density for model m_i given parameter vector θ. To compare models (say, m_i and m_j) we calculate the posterior odds ratio which is the ratio of their posterior model probabilities (or Bayes Factor when the prior odds ratio, $\frac{p(m_i)}{p(m_j)}$, is set to unity):

$$PO_{i,j} = \frac{p(m_i|Y^T)}{p(m_j|Y^T)} = \frac{L(Y^T|m_i)p(m_i)}{L(Y^T|m_j)p(m_j)} \tag{15}$$

$$BF_{i,j} = \frac{L(Y^T|m_i)}{L(Y^T|m_j)} = \frac{\exp(LL(Y^T|m_i))}{\exp(LL(Y^T|m_j))} \tag{16}$$

in terms of the log-likelihood. Components (15) and (16) provide a framework for comparing alternative and potentially misspecified models based on their marginal likelihood. Such comparisons are important in the assessment of rival models, as the model which attains the highest odds outperforms its rivals and is therefore favoured. Given Bayes factors, we can easily compute the model probabilities $p_1, p_2, \cdots, p_n$ for n models. Since $\sum_{i=1}^n p_i = 1$ we have that $\frac{1}{p_1} = \sum_{i=2}^n BF_{i,1}$, from which p_1 is obtained. Then $p_i = p_1 BF(i,1)$ gives the remaining model probabilities.

3.2 Likelihood Comparison of Models

We compare four different model specifications in order to see if the introduction of factor substitutability and/or the biased technical change improves the fit of the estimation. In the first row of Table 1 we present the likelihood density of the model with the CD production function where only the labour-augmenting technology shock is present. In the second row we introduce the CES and calibrate the elasticity of substitution to 0.4 (CES0), following the literature as in Cantore *et al.* (2014b) and Klump *et al.* (2012), and introduce the capital-augmenting shock whilst in rows 3 (CES1) and 4 (CES2) we estimate σ in a model with and without the latter shock. Strictly speaking a meaningful likelihood comparison that provides information about σ is only possible between row 1 and 3 (where we can compare like for like without adding a further exogenous shock).

Table 1 reveals that Models with the CES production function clearly outperforms its

CD counterpart with a posterior probability of 100%. This suggests that incorporating a CES production function offers substantial improvements in terms of the model fitness to the data in the US economy. The differences in log marginal likelihood are substantial. For example, the log marginal likelihood difference between the first two specifications is 12.47 corresponding to a posterior Bayes Factor of 2.6041e+05. As suggested by Kass and Raftery (1995), the posterior Bayes Factor needs to be at least $e^3 \approx 20$ for there to be a positive evidence favouring one model over the other.

Model	σ	Technology shocks	Log data density	Difference with CD
CD	1	ZN	-544.60	0
CES0	calibrated=0.4	ZK & ZN	-532.13	12.47
CES1	estimated=0.15	ZN	-528.50	16.10
CES2	estimated=0.15	ZK & ZN	-528.31	16.29

Table 1: Marginal Likelihood Comparison Between CD and CES Specifications

3.3 Estimation under the Standard Information Assumptions

In this section we make the standard information assumption in solving rational expectations models that economic agents have perfect information about the realizations of current shocks and other relevant macroeconomic variables, alongside their knowledge of the model, parameter values and the policy rule, whereas the econometrician uses only observable data. In effect the private sector has more information than the econometrician, so we refer to this case as *asymmetric information* (AI).

The joint posterior distribution of the estimated parameters is obtained in two steps. First, the posterior mode and the Hessian matrix are obtained via standard numerical optimization routines. The Hessian matrix is then used in the Metropolis-Hastings (MH) algorithm to generate a sample from the posterior distribution. Two parallel chains are used in the Monte-Carlo Markov Chain Metropolis-Hastings (MCMC-MH) algorithm. Thus, 250,000 random draws (though the first 30% 'burn-in' observations are discarded) from the posterior density are obtained via the MCMC-MH algorithm, with the variance-covariance matrix of the perturbation term in the algorithm being adjusted in order to obtain reasonable acceptance rates (between 20%-30%).

Estimation results from posteriors maximization are presented in Section D of the online Appendix. We used the same priors as Smets and Wouters (2007) for common parameters whereas we used a loose prior for the elasticity of substitution between capital and labour in order to see if the data are informative about the value of this parameter. A few structural parameters are kept fixed or calibrated in accordance with the usual practice in the literature of choosing them to target steady state values of some observed variables (see Table 2).[31]

Calibrated parameter	Symbol	Value
Discount factor	β	0.99
Depreciation rate	δ	0.025
Growth rate	g	$\frac{\delta}{4}$
Substitution elasticity of goods	ζ	7
Substitution elasticity of labour	μ	7
Variable capital utilization	γ_1	$\frac{1}{\beta} + \delta - 1$
Fixed cost	$\frac{F}{Y}$	$1 - MC = \frac{1}{\zeta} = 0.1429$
Preference parameter	ϱ	$\frac{1-N}{1+N(c_y(1-\chi)/\alpha-1)}$
Implied steady state relationship		
Government expenditure-output ratio	g_y	0.2
Investment-output ratio	i_y	$\frac{(1-\alpha)\delta}{(1/\beta-1+\delta)}$
Consumption-output ratio	c_y	$1 - g_y - i_y$

Table 2: Calibrated Parameters

First we focus on the posterior estimates obtained using the most general CES model, CES2. As shown in Tables D.1 and D.2 in the online Appendix, the point estimates under the CES assumption are tight and plausible. In particular, focusing on the parameters characterizing the degree of price stickiness and the existence of real rigidities, we find that the price indexation parameters are estimated to be smaller than assumed in the prior distribution (in line with those reported by Smets and Wouters (2007)). The estimates of the indexation parameters γ_p and γ_w imply that inflation is intrinsically not very persistent in the CES model specifications. The posterior mean estimates for the Calvo parameters, ξ_p and ξ_w, imply an average price contract duration of around 2.31 quarters and an average wage contract duration of around 1.84 quarters, respectively. These results are in general consistent with the findings from empirical works on the DSGE modelling in the US economy. It is interesting to note that the risk-aversion parameter (σ_c) is estimated to be less than assumed in the prior distribution, indicating that the inter-temporal elasticity

[31]F is chosen to eliminate monopolistic profits in equilibrium. ϱ is chosen to target the proportion of worked hours available, N, an estimated parameter.

of substitution (proportional to $1/\sigma_c$) is estimated to be about 0.86 in the US, which is plausible as suggested in much of RBC literature. As expected, the policy rule estimates imply a fairly strong response (α_π) to expected inflation by the US Fed Reserve and the degree of interest rate smoothing (α_r) is fairly strong.

Figure E.1 in the online Appendix plots the prior and posterior distributions for the above CES model. The location and the shape of the posterior distributions are largely independent of the priors we have selected since priors are broadly less informative. Most of the posterior distributions are roughly symmetric implying that the mean and median coincide. According to Figure E.1, there is little information in the data for some parameters where prior and posterior overlap.[32] Perhaps the most notable finding comes from the estimation of the parameter σ - our key parameter in the CES setting. As a result of assuming a very diffuse prior with large standard deviation, we find that the data is very informative about this parameter (as clearly shown in the figure, curves do not overlap each other and are very different) and the point estimate of σ in Table D.1 is close to the plausible values. This further provides strong evidence to support the empirical importance of the CES assumption.

We now turn to the comparisons between parameter estimates under CD and CES specifications. Parameter posteriors that are quantitatively different[33] from the estimation using a Cobb-Douglas specification are underlined in Tables D.1 and D.2.

Starting with the parameters related to the exogenous shocks (Table D.1) we notice that the estimated standard deviations of the newly introduced capital-augmenting technology shock is very small but, probably because of its introduction, the standard deviation of the investment specific shock reduces significantly (from 4.16 in the CD specification to 3.06 in the CES case).[34] We also notice that the estimated standard deviations of the mark-up shocks are lower under the CES specification and the standard deviation of the preference shock is slightly higher. The autoregressive parameters of the exogenous shocks are not affected significantly by the CES choice.

Posterior estimation of the investment adjustment costs parameter (ϕ^X) reduces by 0.75 when we estimate the model under CES showing once again how introducing factor-

[32] In particular parameters ρ_{ZK}, σ_c, γ_p, γ_w and α are weakly identified.

[33] Difference in posteriors up to 0.05 were not considered quantitatively relevant here.

[34] The two shocks are clearly related and it is very likely that when ZK is absent ZI is capturing "capital-biased" technological progress.

biased technical change affects significantly the estimation of 'investment-related' parameters. The parameters of the utility function also appear to be affected by the choice of the production function (σ_c reduces by 0.94 and χ reduces by around 0.21). Regarding the parameters associated with sticky prices and wages both the probabilities of no price-adjustment (ξ_p) and no wage-adjustment (ξ_w) change significantly, decreasing from 0.77 to 0.57 and 0.60 to 0.46, respectively. Monetary policy weights (except the weight on inflation which increases slightly), real and nominal trends estimations are not affected by the introduction of factor substitutability and biased technical change.

3.4 Estimation under Symmetric Imperfect Information

In our estimation procedure up to now we have followed the standard practice of assuming agents forming rational expectations have perfect information of all states, include shock processes, whereas the econometrician only observes the data variables. This we refer to as the *asymmetric information* (AI) assumption. Based on the methodology developed in Levine *et al.* (2012), we now relax this extreme information assumptions and instead assume that both private agents and the econometrician have the *same imperfect information* (II) and both use the Kalman Filter, the former to form expectations of unobserved state variables and the latter to calculate the likelihood in the Bayesian estimation. Although perfect information on idiosyncratic shocks may be available to economic agents, it is implausible to assume that they have full information on economy-wide shocks. Furthermore, as shown in Levine *et al.* (2012), as long as the number of shocks exceed the number of observables (as is the case in our set-up), the solutions under II and AI differ and the latter provides an alternative source of endogenous persistence that offers the potential for capturing the dynamics observed in the data. We therefore empirically address these alternative information assumptions to assess whether parameter estimates are consistent across them and which of these choices leads to a better model fit.

We provide the estimation results from posteriors maximization for Model CES2, the 'best' model in Table 1, under II. The following table provides a formal Bayesian comparison between CES2 under AI and II respectively.[35]

[35] To complete the comparison of CD and CES under either AI or II we have also compared the two production function assumption under II. Similar results to those under AI were obtained and are not reported.

Model	σ	Technology shocks	Log data density	Difference with CD
CES2AI	estimated=0.15	ZK & ZN	-528.31	16.29
CES2II	estimated=0.18	ZK & ZN	-524.59	20.01

Table 3: Marginal Likelihood Comparison Between AI and II

It is clear from Table 3 that the assumption of imperfect information leads to a better fit as implied by the marginal likelihoods. Recalling Kass and Raftery (1995), a Bayes factor of 10-100 or a log data density range of [2.30, 4.61] is 'strong to very strong evidence'. For our Model CES2, there is 'strong' evidence in favour of the II assumption. The differences in log data density or the posterior odds ratio are substantive when comparing models assuming both CES and II with the model with CD. For example, the log marginal likelihood difference between Model CES2 under II and Model CD is 20.01. In order to choose the former over Model CD, we need a prior probability over Model CD 4.9004e+08($= e^{20.01}$) times larger than our prior probability over Model CES2 under II and this factor is decisive.

Parameter	Prior mean	Post. mean CES II (AI case)	5% CES (II)	95% CES (II)	Prior	Prior s.d.
ρ_{ZN}	0.5	0.9470 (0.9443)	0.9059	0.9932	beta	0.2
ρ_{ZK}	0.5	0.4980 (0.4441)	0.1887	0.8325	beta	0.2
ρ_G	0.5	0.9613 (0.9631)	0.9418	0.9782	beta	0.2
ρ_{ZI}	0.5	0.7961 (0.7429)	0.6817	0.9117	beta	0.2
ρ_P	0.5	0.9672 (0.9744)	0.9399	0.9955	beta	0.2
ρ_W	0.5	0.9750 (0.9656)	0.9565	0.9938	beta	0.2
ρ_B	0.5	0.8728 (0.9311)	0.7977	0.9503	beta	0.2
ε_{ZN}	0.1	0.6905 (0.6833)	0.5944	0.7793	invg	2.0
ε_{ZK}	0.1	0.0804 (0.0744)	0.0240	0.1492	invg	2.0
ε_G	0.5	1.9947 (1.9904)	1.7650	2.2374	invg	2.0
ε_{ZI}	0.1	2.8668 (3.0647)	1.4762	4.2147	invg	2.0
ε_P	0.1	0.3771 (0.3756)	0.3093	0.4370	invg	2.0
ε_W	0.1	0.9781 (0.9482)	0.8328	1.1282	invg	2.0
ε_M	0.1	0.1604 (0.1579)	0.1371	0.1832	invg	2.0
ε_B	0.1	1.0863 (1.4997)	0.7681	1.3876	invg	2.0

Table 4: Posterior Results for the Exogenous Shocks (II vs AI)

We now turn to the comparisons between parameter estimates under AI and II (Tables 4 and 5). Overall, the parameter estimates are plausible and reasonably robust across information specifications, despite the fact that the II alternative leads to a better model fit based on the corresponding posterior marginal likelihood. It is interesting to note that assuming II for the private sector reinforces the evidence that the *ZK* and *ZI* shocks are related when CES is introduced. We notice that in the II case the estimated standard

Parameter	Prior mean	Post. mean CES II (AI case)	5% CES (II)	95% CES (II)	Prior	Prior s.d.
σ	1	0.1788 (0.1542)	0.0735	0.2841	gamma	1
N	0.4	0.5343 (0.5136)	0.4184	0.6758	beta	0.1
ϕ	0.5	0.7959 (0.7860)	0.7052	0.8838	beta	0.15
ϕ^X	2	1.7097 (1.9210)	0.5106	2.6885	norm	1.5
σ_c	1.5	1.2609 (1.1571)	0.6997	1.8070	norm	0.375
χ	0.7	0.3324 (0.3445)	0.2111	0.4341	beta	0.1
ξ_w	0.5	0.4384 (0.4577)	0.3373	0.5373	beta	0.1
ξ_p	0.5	0.5634 (0.5677)	0.4788	0.6538	beta	0.1
γ_w	0.5	0.4578 (0.4489)	0.2123	0.7005	beta	0.15
γ_p	0.5	0.3656 (0.3512)	0.1434	0.5742	beta	0.15
α	0.3	0.3489 (0.3553)	0.2629	0.4379	norm	0.05
α_π	1.5	2.3968 (2.3771)	2.1130	2.6680	norm	0.25
α_r	0.75	0.7959 (0.7911)	0.7598	0.8336	beta	0.1
α_y	0.125	0.0592 (0.0667)	0.0174	0.0974	norm	0.05
$conspie$	0.625	0.5462 (0.5732)	0.4783	0.6118	gamma	0.1
$ctrend$	0.4	0.4609 (0.4975)	0.4110	0.5113	norm	0.1

Table 5: Posteriors Results for Model Parameters (II vs AI)

deviations of the capital-augmenting technology shock (ZK) is slightly larger and as a result the standard deviation of the investment specific shock (ZI) further reduces (from 3.06 under AI to 2.87 in the II case). This again confirms our finding early that when ZK is absent ZI is capturing "capital-biased" technological progress and the degree of which depends on whether the shocks are observed or not by the private sector. The other significant change in estimates from AI to II is from the investment adjustment costs parameter (ϕ^X) and this shows how assuming II helps provide evidence that introducing factor-biased technical change affects significantly the estimation of 'investment-related' parameters. Our model comparison analysis contains one important result suggesting that a combination of incorporating CES and with information set II offers a decisive improvement in terms of the model fit, dominates the standard CD model with a very large log-likelihood difference of around 20.

4 Model Validation

After having shown the model estimates and the assessment of relative model fit to its other rivals with different restrictions, we use them to investigate a number of key macroeconomic issues in the US. The model favoured in the space of competing models may still be poor (potentially misspecified) in capturing the important dynamics in the data. To further evaluate the absolute performance of one particular model against data, it is necessary to compare the model's implied characteristics with those of the actual data. In this section, we address the following questions: (i) can the models capture the underlying characteristics of the actual data? (ii) Are the parameters, especially the labour market and production ones, identified? Are the results robust to calibration and estimation assumptions? (iii) are the models able to reproduce the overshooting property of the labour share conditional to productivity innovations?

4.1 Standard Moment Criteria

Summary statistics such as first and second moments have been standard as means of validating models in the literature on DSGE models, especially in the RBC tradition. As the Bayes factors (or posterior model odds) are used to assess the relative fit amongst a number of competing models, the question of comparing the moments is whether the models correctly predict population moments, such as the variables' volatility or their correlation, i.e. to assess the absolute fit of a model to macroeconomic data.

To assess the contributions of assuming different specifications of production function in our estimated models, we compute some selected second moments and present the results in this section. Table 6 presents the (unconditional) second moments implied by the above estimations and compares with those in the actual data. In terms of the standard deviations, all models generate relatively high volatility (standard deviations) compared to the actual data (except for the interest rate and the CD production assumes constant labour share). Overall, the estimated models are able to reproduce broadly acceptable volatility for the main variables of the DSGE model and all model variants can successful replicate the stylized fact in the business cycle research that investment is more volatile than output whereas consumption is less volatile. In line with the Bayesian model comparison, the models with CES technology fit the data better in terms of implied volatility, getting closer to the data in this dimension (we highlight the 'best' model (performance) in bold). Note that all our CES models clearly outperform the CD model in capturing the volatilities of all variables (except for hours) and CES2 with II does extremely well at matching the investment and real wage volatilities in the data. Furthermore by not imposing a constant labour share as in the CD model CES2 with II generates the standard deviation very close to the data but performs badly for hours. As suggested by the likelihood comparison, the differences in generating the moments between the CES specification with only the shock ZN and the CES with both ZK and ZN shocks are qualitatively very small.

	Standard Deviation							
	Output	Consumption	Investment	Wage	Inflation	Interest rate	Hours	Labour share
Data	0.58	0.53	1.74	0.66	0.24	0.61	2.47	2.07
	(0.50,0.69)	(0.46,0.62)	(1.54,2.01)	(0.57,0.82)	(0.21,0.27)	(0.55,0.70)	(2.09,2.94)	(1.81,2.37)
CD	0.78(0.05)	0.69(0.05)	1.87(0.20)	0.99(0.09)	0.43(0.07)	0.44(0.07)	**2.84**(0.84)	0
CES1	0.69(0.05)	0.66(0.03)	1.82(0.16)	0.73(0.05)	0.43(0.08)	0.50(0.08)	5.76(1.47)	3.83(0.84)
CES2	0.69(0.05)	0.66(0.03)	1.81(0.16)	0.73(0.05)	0.43(0.08)	0.50(0.09)	5.79(1.48)	3.85(0.89)
CES2II	**0.69**(0.05)	**0.66**(0.03)	**1.80**(0.16)	**0.72**(0.05)	**0.36**(0.08)	**0.50**(0.09)	4.09(1.48)	**2.59**(0.89)
	Cross-correlation with Output							
Data	1.00	0.61	0.64	-0.11	-0.12	0.22	-0.25	-0.05
	(-)	(0.46,0.74)	(0.47,0.75)	(-0.40,0.10)	(-0.31,0.10)	(0.02,0.39)	(-0.48,-0.00)	(-0.26,0.16)
CD	1.00	0.76(0.06)	0.57(0.06)	0.59(0.09)	**-0.11**(0.09)	-0.37(0.05)	0.11(0.02)	-
CES1	1.00	0.43(0.07)	0.63(0.04)	0.13(0.11)	-0.08(0.06)	-0.11(0.05)	0.03(0.02)	-0.23(0.04)
CES2	1.00	0.43(0.07)	0.63(0.04)	**0.13**(0.11)	-0.07(0.07)	**-0.11**(0.05)	**0.03**(0.02)	**-0.23**(0.04)
CES2II	1.00	**0.47**(0.07)	**0.63**(0.04)	0.15(0.11)	-0.06(0.07)	-0.15(0.05)	0.06(0.02)	-0.28(0.04)
	Autocorrelation (Order=1)							
Data	0.28	0.17	0.56	0.17	0.54	0.96	0.93	0.90
	(0.19,0.37)	(0.07,0.27)	(0.46,0.66)	(0.07,0.26)	(0.45,0.64)	(0.87,1.00)	(0.84,1.00)	(0.81,1.00)
CD	0.37(0.05)	0.42(0.06)	**0.58**(0.06)	0.50(0.05)	0.69(0.05)	0.89(0.02)	0.93(0.03)	-
CES1	0.29(0.04)	0.32(0.05)	0.61(0.04)	0.38(0.05)	0.79(0.07)	0.94(0.02)	0.99(0.01)	0.99(0.01)
CES2	0.29(0.04)	**0.31**(0.05)	0.61(0.04)	0.37(0.05)	0.79(0.08)	**0.94**(0.02)	0.99(0.01)	0.99(0.01)
CES2II	**0.28**(0.04)	0.33(0.05)	0.63(0.04)	**0.36**(0.05)	**0.66**(0.08)	0.91(0.02)	0.98(0.01)	**0.98**(0.01)

Table 6: Second Moments of the Model Variants (at the Posterior Means)†

Note. † Numbers in bold represent the closest fit to the data.
For the empirical moments computed from the dataset the bootstrapped 95% confidence bounds based on the sample estimates are presented in parentheses; For the moments generated by the estimated models the standard errors based on the model simulation are shown in parentheses. We sample randomly 1000 of the retained parameter draws from the posterior distribution of the estimated models. For each sample the models are solved and simulated to give the finite sampling distribution of the second moments in which we construct the standard errors.

Table 6 also reports the cross-correlations of the seven observable variables plus labour share vis-a-vis output. All models perform successfully in generating the positive contemporaneous correlations of consumption and investment observed in the data. All CES models fit the output-investment correlation with the data very well. The highlighted numbers in this category together with the evidence above show that the feature of CES in the model is particularly important in characterizing the investment dynamics. However, as evidence from the implied volatilities confirms, the main shortcoming of all the models, including the preferred one, is the difficulty at replicating the cross-correlations of output with hours and the real wage, and mimicking the volatility observed in the hours data. This is not a very surprising result because, for DSGE models with even more frictions and distortions in the labour market, those moments are usually difficult to match. One promising way of matching those correlations is given by Koh and Santaeulalia-Llopis (2014). They allow the elasticity of substitution between capital and labour to be time varying and find that its countercyclicality helps in matching the cross correlations of wage and labour productivity with output. All models also fail to predict the positive correlation between output and interest rate and CES models have problems in replicating the negative contemporaneous cross-relation between inflation and output. This is consistent with the work of Smets and Wouters (2003) as they find that the implied cross-correlations with the interest rate and inflation are not fully satisfactory. Nevertheless, the results in general show that, in the models where the CES specification is present, cross-correlations of endogenous variables are generally closer to those in the actual data.[36] It is the empirical relevance of the CES feature that helps to explain the better overall fit as found in the likelihood race.

To summarise, overall BML based methods suggest that the ability of the model's second moments to fit those of the data generally match the outcome of the likelihood race. The CES model assuming the II set delivers a better fit to the actual data for most of the second moment features in Table 6. However, as noted above, the differences in the second moments of the two competing CES variants are very small.

4.2 Autocorrelation Functions

We have so far considered autocorrelation only up to order 1. To further illustrate how the estimated models capture the data statistics and persistence in particular, we now plot the autocorrelations up to order 10 of the actual data and those of the endogenous variables generated by the model variants in Figure 3.

The CES models all stay within the 95% uncertainty bands and II CES2 model performs a little better than its AI counterpart. The inflation autocorrelations generated by CD model lie outside the 95% uncertainty bands. Of particular interest is that, when assuming CES production, the implied autocorrelograms fit very well the observed au-

[36] Looking at the standard errors in Table 6, the CES2II specification is generating moments lying within or closer to the 95% confidence intervals of their data counterparts. The CD model performs poorly and indeed has cross-correlations of wage and interest rate *vis-a-vis* output lying well outside the 95% confidence intervals.

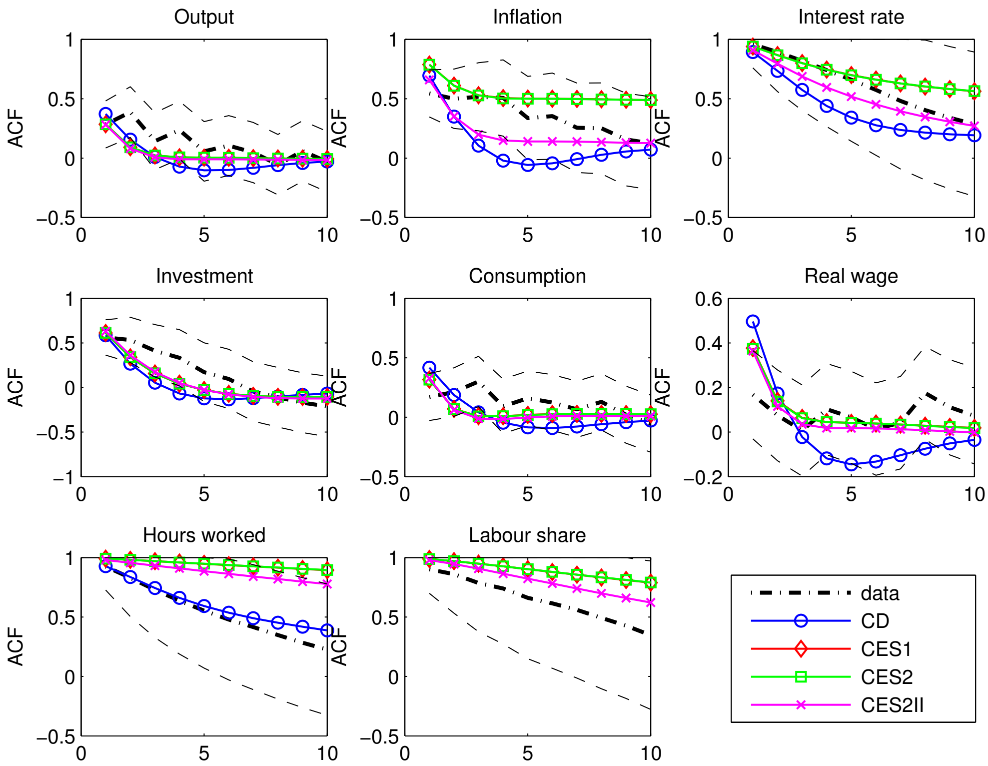

Figure 3: Autocorrelations of Observables in the Data and in the Estimated Models✠

Note. ✠ The approximate 95% confidence bands are constructed using the large-lag standard errors (See Anderson (1976)).

tocorrelation of inflation, interest rate, investment and real wage, whilst the CD model generates much less sluggishness and is less able to match the autocorrelation of inflation, interest rate and real wage observed in the data from the second lag onwards. Overall we find that, with nominal price stickiness in the models and highly correlated estimated price markup shocks, inflation persistence can be captured closely in DSGE models when CES production is assumed.

When it comes to output, all models perform well in matching the observed output persistence. However, the hours is more autocorrelated in all models than in the data, but now the model with the CD feature gets much closer to the data for higher order autocorrelations. All models match reasonably well the autocorrelations of investment and consumption. To summarise, the results for higher order autocorrelations for the most part show that the DSGE models under the more general CES production function are better at capturing the main features of the US data, strengthening the argument that the assumption of CES helps to improve the model fit to data.

4.3 Identification and Sensitivity Analysis

In order to verify the robustness of the results discussed above, we conduct a series of experiments and checks on the variants of our CES model. In this section we address the following two questions: (a) How well identified are those parameters related to the labour market? (b) How sensitive are the results to the value of the capital/labour elasticity of substitution? We focus then on: the labour elasticity of substitution that enters in the wage mark-up μ, wage indexation γ_w, the Calvo coefficient for wage contracts ξ_w and the elasticity of substitution, σ. Initially, we carry out a formal check on the inherent identifiability of the models structure by running a series of tests described in Iskrev (2010a, 2010b). The procedure in Iskrev (2010b) reveals whether there are identification problems, stemming from the Jacobian matrix of the mapping from the structure parameters to the moments of the observation. We carry out the local identification check for the whole set of the estimated parameters in the model at a chosen central tendency measure at the defined priors. For the 'global' check we conduct the Monte Carlo exploration in the entire prior space by drawing 10000 sets of parameters to evaluate the Jacobian matrix. These tests reveal that the Jacobian matrix is of full rank everywhere in the space of the prior range so all these parameters, including our key elasticity of substitution parameter for the CES production σ, are well-identified within a theoretically admissible range defined by the prior distribution.

We then examine more carefully the identification of γ_w, ξ_w, μ and σ at the means of the prior and posterior distributions and using the prior uncertainty. To completely rule out a flat likelihood and account for the possibility of weak identification we quantify the identification strength and check collinearity between the effects of different parameters on the likelihood (following the sensitivity measure in Iskrev (2010a)). If there exists an exact linear dependence between a pair and among all possible combinations their effects on the moments are not distinct and the violation of this condition must indicate a flat likelihood and lack of identification. Table F.1 in the online Appendix reports the sensitivity measure and collinearity results. Note that all four parameters are sensitive in affecting the likelihood through their effects on the moments of the observed variables. ξ_w, in the prior space or estimated, has the strongest effects and μ is the weakest on the moments among them. The collinearity results suggest, that there is no linear dependence between the columns of the Jacobian matrix (non-identification) across the prior points and the estimated parameters in our various CES models.

Alternatively, model CES2 is then tested by using diffuse priors for μ, γ_w and ξ_w, based on the normal distribution with one standard deviation for μ and the open unit interval with $[0.229, 0.733]$ at the 95% probability level for γ_w and ξ_w, reflecting our uncertainty about the values of these parameters. We find that the posteriors are relatively insensitive to the change.[37]

As an additional check on our results, we experiment with different priors for σ and different parameterizations for γ_w and ξ_w, and we re-estimate the preferred model CES2II. As already noted, our estimate of σ is almost at the lower bound of the previous available estimates in the literature[38] so this part of the exercise focuses on checking whether our results with CES are sensitive to an alternative set of parameterization and/or higher degree of σ. Our complete findings are available upon request, and the remainder of this section restricts itself to key results.

Model CES2II is re-estimated imposing the Smets and Wouters (2007) estimates of the wage dynamics parameters γ_w and ξ_w. We find that the posteriors are sensitive to this change and the log marginal likelihood deteriorates significantly (-537.63), suggesting that it fails to compete with all previous models. In Section 3.2 we have already compared a model with σ calibrated to 0.4 and found that the model performance is in fact worse than the cases when σ is estimated. In this section we carry out a further test by re-estimating the model under II, using this calibration, combined with the Smets and Wouters (2007) estimates. It shows that calibrating the model with higher degrees of σ leads to a worse model fit.

It is also interesting to know whether the dynamics of the variables are altered and the moments of which variables are most strongly affected by the parameter changes. To find out, we compute model-generated unconditional moments which are reported in Table F.3 in the online Appendix. Both models using the alternative parameterization show an

[37]The full set of results are in Table F.2 of the online Appendix.
[38]See Klump *et al.* (2012) for a survey.

improvement in matching the standard deviation of hours worked at the expense of other moments – the ability of matching almost all other moments in this dimension is distorted, generating much volatility than the data. The performance is even worse when $\sigma = 0.4$ is imposed and in this case the volatility of labour share is under-estimated, lying well below the lower bound of the confidence bands.[39] Overall, the results we show in this section suggest that the parameter identification, estimation precision of the NK labour market and the estimated CES elasticity in our CES model should not be a reason for concern given the data.

4.4 Productivity and the Labour Share

In this section we present the performance of the model(s) focusing on the conditional correlation of the labour share to productivity. We want to check if our model is able to replicate the overshooting behaviour of the labour share following productivity shocks. As showed by Ríos-Rull and Santaeulália-Llopis (2010), following productivity shocks, the labour share in U.S. is negatively correlated with contemporaneous output but positively with lagged output (after 5 quarters or more). First we show that all of our models with the CES specification can reproduce an impulse response of the labour share to the labour augmenting shock that is negative on impact and turns positive afterwards.[40] Figure 4 shows the impulse response of the labour share in the four models conditional on a ZN shock simulated at the posterior mean.[41] Even if this proves that, in principle, our CES models can match the overshooting behaviour of the labour share there are two of points to raise here. First Ríos-Rull and Santaeulália-Llopis (2010) and Koh and Santaeulalia-Llopis (2014) have shown, in their SVAR analysis, that the change in sign of the impulse response of the labour share tends to happen in between quarter 4 and quarter 10.[42]

[39] For completeness, we also check the sensitivity in the conditional moments by comparing the estimated impulse responses. The results are not reported here.

[40] This property of the CES production function was shown already by Choi and Ríos-Rull (2009) and Cantore *et al.* (2014b). In the ECB working paper version of Cantore *et al.* (2014b), as opposed to the published version that has a unit root in the labour augmenting shock, it is evident how both RBC and NK models with CES can reproduce such impulse response of the labour share conditional on a labour augmenting shock. This has to do with the sensitivity in the response of hours to the elasticity of substitution showed by Cantore *et al.* (2014a).

[41] Results are qualitatively similar using the posterior mode and median and are available under request.

[42] Ríos-Rull and Santaeulália-Llopis (2010) use a bivariate SVAR using productivity and labour share and they find a change in sign at quarter 4 while Koh and Santaeulalia-Llopis (2014) use a trivariate SVAR adding also output and they find a change in sign few quarters later although this is never significant at 95% level but it is significant at 90% after 10 quarters approximately.

Here the change in sign is happening after 20 quarters and it is significant at 95% after 25 quarters. Second our ZN shock might not be the same identified by Ríos-Rull and Santaeulália-Llopis (2010) and Koh and Santaeulalia-Llopis (2014) in their SVAR.

Hence we proceed with an exercise similar to the one in Koh and Santaeulalia-Llopis (2014) in order to determine how well our model matches the IRFs of labour share and output to productivity shocks from the estimated VAR documented in their paper. Here we draw upon their *comparability principle* together with the result that the RE solution to any linearized DSGE model can be expressed as in infinite VAR in a subset of observable variables and approximated by a finite VAR (see Fernandez-Villaverde *et al.* (2007) and Levine *et al.* (2012)). Then we can make a meaningful comparison between our model and the data counterparts by applying the same identification strategy of productivity shocks. In Figure 4 we showed the response of labour share to a labour augmenting shock while in Koh and Santaeulalia-Llopis (2014) they show the empirical response to a productivity residual Z_t defined as[43]

$$Z_t = Y_t - (1 - LS_t)K_t - LS_tN_t \tag{17}$$

where Y_t, LS_t, K_t and N_t correspond to the model generated output, labour share, capital and hours. Here we follow their analysis and we use the same definition of productivity residual as in (17) that enable us to compare the results across models with different identification of productivity. We construct the same productivity measure from simulated data from all four models presented here and we then use this measure together with the model-generated data for labour share and output to estimates the same trivariate VAR as in Koh and Santaeulalia-Llopis (2014) using the same Cholesky factorization.[44]

Results are shown in Figure 5 and show clearly that the simple introduction of the CES in an otherwise standard, medium scale DSGE model is able to reproduce qualitatively the shape of the response of productivity, labour share and output to a productivity innovation in line with the findings of Ríos-Rull and Santaeulália-Llopis (2010) and Koh

[43]They define it in growth rates in order to account for the fact that the data are non-stationary while in our models growth is absent and hence they produce stationary data.

[44]We simulate each model producing 10000 observation for the model(s) implied labour share, ouput, investment and hours. As in Koh and Santaeulalia-Llopis (2014) capital is recovered following the perpetual inventory model which means that we use model-generated data for capital that abstract from investment specific technical change and variable capital utilisation. Note that here, in contrast with Koh and Santaeulalia-Llopis (2014), the depreciation rate is constant by definition. We then discard the first 1000 observation and use the rest to estimate the trivariate SVAR. We then present the median responses and the 95% confidence intervals.

and Santaeulalia-Llopis (2014). Indeed the result does not change much with respect to the inclusion, or not, of capital augmenting technical change and with respect to the information assumption used in the estimation. A closer look at the quantitative implication of this exercise and at the shape of the impulse response of the labour share and output shows how the timing of the overshoot differs across the models. The CES1, with only labour augmenting technical change, produces an overshooting, on average, after 10 quarters, in line with the results in Koh and Santaeulalia-Llopis (2014). The other two models, with both ZN and ZK shocks and estimated using alternative information assumptions, produce an overshooting response after 20 quarters, hence much later than the empirical evidence suggests.

This analysis then suggests that the inclusion of a CES production function in an otherwise standard DSGE model with nominal and real frictions is able to reproduce at least qualitatively the overshooting response of the labour share to productivity innovation. However future research[45] is needed to quantify how much closer to the data this model can get in comparison to other models in the literature that were able to produce such a response (see Ríos-Rull and Santaeulália-Llopis (2010), Koh and Santaeulalia-Llopis (2014) and Colciago and Rossi (2015)) and if this property comes at the expenses of distorting the responses and second moment of other labour market variables as discussed by Koh and Santaeulalia-Llopis (2014).[46]

5 Conclusions

This paper contributes to the rapidly rising literature that brings the CES specification of the production function into the analysis of business cycle fluctuations. Whilst other papers including Choi and Ríos-Rull (2009), Cantore *et al.* (2012), Cantore *et al.* (2014b) and Koh and Santaeulalia-Llopis (2014) have explored this issue in different contexts, this is the first one to confirm decisively, in a log-linear Bayesian framework, the importance of CES rather than CD production functions. In a marginal likelihood race our estimated best CES model (estimated under imperfect information) with an elasticity well below

[45] One promising avenue would be to estimate a DSGE-VAR in the spirit of Del Negro and Schorfheide (2004).

[46] Here, for example, we have noted that our CES models are not able to match the volatility of hours and the cross correlation of real wage with output.

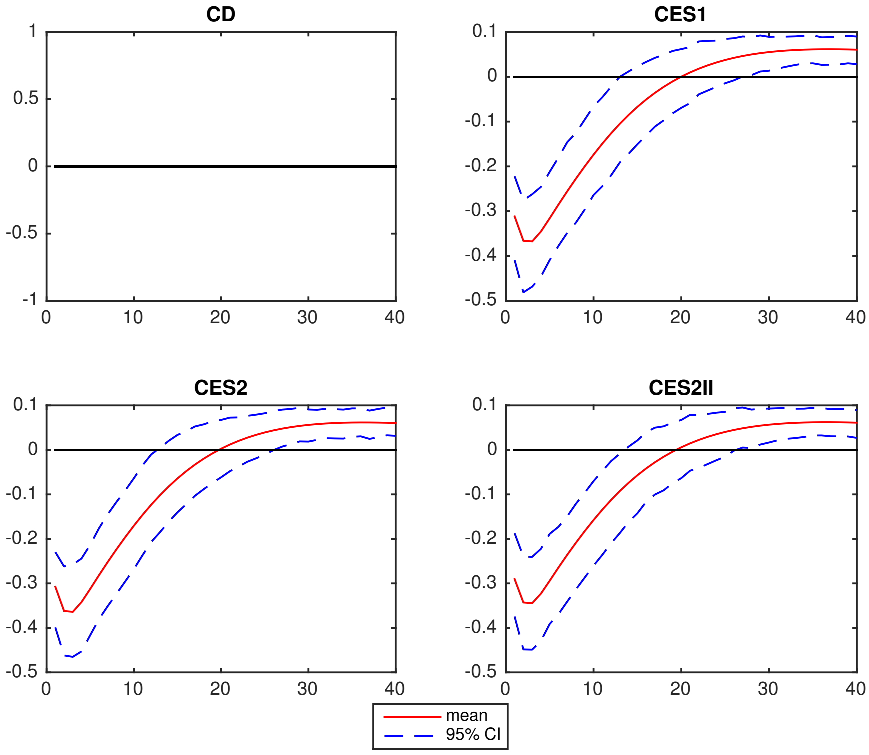

Figure 4: Labour Share IRFs to a Labour Augmenting Shock Across Models

unity at 0.18 beats the CD production function by a substantial log-likelihood of over 20. Assuming equal prior model probabilities, this implies that posterior model probabilities are 4.8517e+08:1 in favour of the CES.[47] The principal reason for this result is that movements of factor shares with the CES specification help substantially to fit the data. The marginal likelihood improvement is matched by the ability of the CES model to get closer to the data in terms of second moments, especially the volatilities of output, consumption and the real wage, and the autocorrelation functions for inflation and the nominal interest rate. We also showed that the CES specification, independently of the informational assumption made in the estimation, is able to reproduce, at least qualitatively, the overshooting response of the labour share to productivity innovations. The main message then for DSGE models is that we should dismiss once and for all the use of CD for business cycle analysis.

[47] But this does not necessarily imply that the less likely model is not useful for forecasting. See Geweke and Amisano (2011) for the use of an 'prediction pool' consisting of an optimal choice of the log of the linear combination of the predictive densities over the two models.

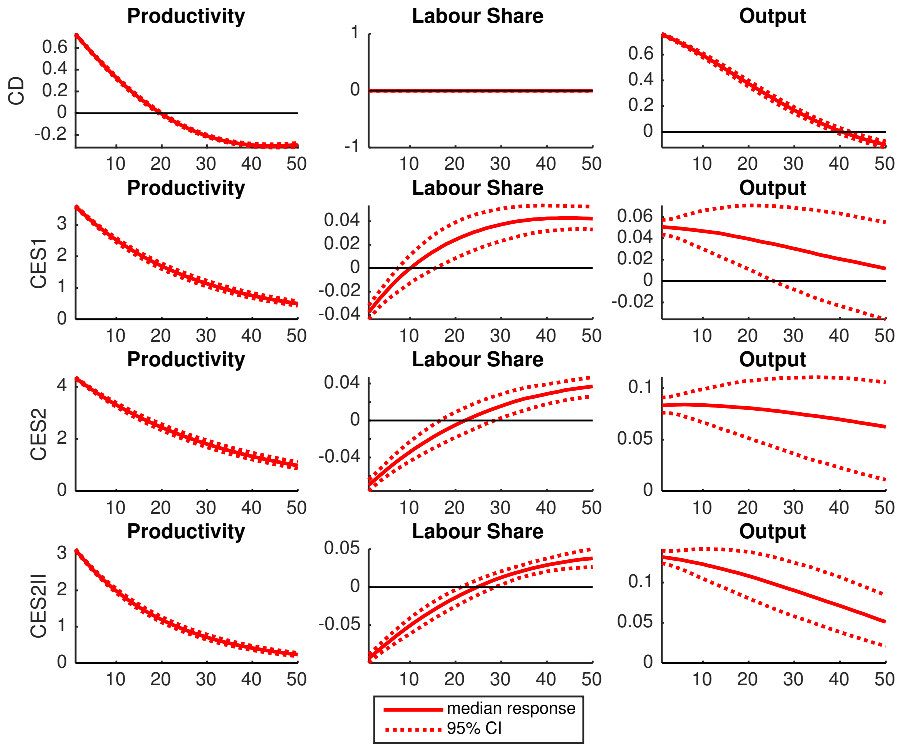

Figure 5: SVAR Labour Share Implied IRFs to a Productivity Innovation using Model Generated Data

Despite these positive results, one area where the CES model remains a concern, in terms of model misspecification, is in the second moments involving wages and hours. For example both CD and CES models fail to reproduce the negative correlation between output and hours; furthermore the CES model produces far too much persistence in hours.[48] This shortcoming is important because as pointed out by Rowthorn (1999), a low capital-labour substitutability is crucial for understanding unemployment persistence. This suggests that future research should also look more closely at the labour market and introduce search-match frictions and unemployment alongside CES production.[49]

A further feature of the paper is to pose the question of whether the superior fit of CES over CD production functions is robust to the information assumptions and whether imperfect information (II) can further improve the model fit using the CES specification.

[48]However, Koh and Santaeulalia-Llopis (2014) present a non-linear framework with a non-constant elasticity of substitution production function and in which a time-varying countercyclical elasticity of substitution helps to better match the second moments of labour market variables.

[49]See Cantore *et al.* (2014a) using a simpler calibrated model.

Indeed we find this is the case. Our model comparison analysis suggests that a combination of incorporating CES and with information set II offers a decisive improvement in terms of the model fit, dominating the standard CD model with a very large log marginal likelihood difference. When using II we also find that the poor performance from the CES models in terms of capturing hours volatility can be improved.[50]

Finally, our CES specification allows us also to introduce a capital-augmenting shock alongside the labour-augmenting variety. However we find this does not bring about an improvement in the model fit and the contribution of the a capital-augmenting shock in the variance decomposition is small. We have noted the well-known result that a BGP requires either CD technology, or that technical change must be driven solely by the labour-augmenting variety. This raises an obstacle to the prospect of unifying business cycle analysis with long-term endogenous growth based on CES technology. One possible way forward is to follow León-Ledesma and Satchi (2011) and Koh and Santaeulalia-Llopis (2014); they independently provide, in two different settings, models in which the CES prevails in the short-run but converges to CD in the long run, thus allowing a capital-augmenting technical change contribution to long-run growth. Hence we believe further research on this topic should incorporate this flexible-type of production function into the SW-type model of this paper and to assess its empirical performance.

[50] In the Working Paper version we also fit various DSGE-VAR models to the same data used to estimate the DSGE models and compare the DSGE impulse responses with those derived from the VARs as an alternative way of validating the model performance that follows Del Negro and Schorfheide (2004) and Del Negro *et al.* (2007). The Working Paper also investigates the contribution of each of the structural shocks to the forecast error variance of the observable variables in the models. In the medium to long run the supply shocks and the exogenous spending shock together dominate, but the contribution of government spending shock to output variability is small, the wage mark-up shock explaining a bigger part of the long-run variations in output. In contrast, the monetary policy shock and preference shock have little impact on for output variability, regardless of forecast horizon.

References

Acemoglu, D. (2002). Technical Change, Inequality and the Labor Market. *Journal of Economic Literature*, **40**(1), 7–72.

Acemoglu, D. (2003). Labor- and capital-augmenting technical change. *Journal of the European Economic Association*, **1**, 1–37.

Anderson, O. (1976). *Time Series Analysis and Forecasting: the Box-Jenkins Approach*. Butterworths, London.

Antràs, P. (2004). Is the US Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution. *Contributions to Macroeconomics*, **4**, Article 4.

Arrow, K. J., Chenery, H. B., Minhas, B. S., and Solow, R. M. (1961). Capital-labor substitution and economic efficiency. *Review of Economics and Statistics*, **43**(3), 225–250.

Barnett, W. (2004). Dimensions and cconomics: some problems. *Quarterly Journal of Austrian Economics*, **7**(1), 95–104.

Bentolila, S. and Saint-Paul, G. (2003). Explaining Movements in the Labor Share. *The B.E. Journal of Macroeconomics*, **3**(1), 1–33.

Blanchard, O. J. (1997). The Medium Run. *Brookings Papers on Economic Activity*, **2**, 89–158.

Boldrin, M. and Horvath, M. (1995). Labor Contracts and Business Cycles. *Journal of Political Economy*, **103**(5), 972–1004.

Bridgman, B. (2014). Is Labor's Loss Capital's Gain? Gross versus Net Labor Shares. BEA Working Papers 0114, Bureau of Economic Analysis.

Caballero, R. J. and Hammour, M. L. (1998). Jobless growth: Appropriablity, factor substitution and unemployment. *Carnegie-Rochester Conference Proceedings*, **48**, 51–94.

Cantore, C. and Levine, P. (2012). Getting normalization right: Dealing with 'dimensional constants' in macroeconomics. *Journal of Economic Dynamics and Control*, **36**(12), 1931–1949.

Cantore, C., Ferroni, F., and León-Ledesma, M. A. (2012). The dynamics of hours worked and technology. Banco de España Working Papers 1238, Banco de España.

Cantore, C., Levine, P., and Melina, G. (2014a). A fiscal stimulus and jobless recovery. *The Scandinavian Journal of Economics*. Forthcoming.

Cantore, C., León-Ledesma, M., McAdam, P., and Willman, A. (2014b). Shocking stuff: Technology, hours, and factor substitution. *Journal of the European Economic Association*, **12**(1), 108 – 128.

Chari, V. V., Kehoe, P. J., and McGrattan, E. R. (2009). New Keynesian models: Not yet useful for policy analysis. *American Economic Journal: Macroeconomics*, **1**(1), 242–66.

Chirinko, R. S. (2008). Sigma: The Long and Short of It. *Journal of Macroeconomics*, **30**(2), 671–686.

Choi, S. and Ríos-Rull, J.-V. (2009). Understanding the Dynamics of the Labor Share: the Role of non-Competitive Factor Prices. *Annales d'Economie et de Statistique*, (95-96), 251–277.

Christiano, L., Eichenbaum, M., and Evans, C. (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, **113**, 1–45.

Colciago, A. and Rossi, L. (2015). Firm Entry, Endogenous Markups and the Dynamics of the Labor Share of Income. *Macroeconomic Dynamics*, **19**(6).

Del Negro, M. and Schorfheide, F. (2004). Priors from general equilibrium models for VARs. *International Economic Review*, **45**, 643–673.

Del Negro, M., Schorfheide, F., Smets, F., and Wouters, R. (2007). On the fit of New Keynesian models. *Journal of Business and Economic Statistics*, **25**(2), 123–162.

Diamond, P. A., McFadden, D., and Rodriguez, M. (1978). Measurement of the elasticity of substitution and bias of technical change. In M. Fuss and D. McFadden, editors, *Production Economics, Vol. 2*, pages 125–147. Amsterdam and North Holland.

European Commission (2007). *The Labour Income Share in the European Union*, chapter 5, Employment in Europe 2007. European Commission.

Fernald, J. G. (2007). Trend breaks, long-run restrictions, and contractionary technology improvements. *Journal of Monetary Economics*, **54**(8), 2467–2485.

Fernandez-Villaverde, J., Rubio-Ramirez, J., Sargent, T., and Watson, M. W. (2007). ABC (and Ds) of Understanding VARs. *American Economic Review*, **97**(3), 1021–1026.

Galí, J. and Gambetti, L. (2009). On the sources of the great moderation. *American Economic Journal: Macroeconomics*, **1**(1), 26–57.

Galí, J. and van Rens, T. (2014). The Vanishing Procyclicality of Labor Productivity. Cepr discussion papers, C.E.P.R. Discussion Papers.

Gertler, M. and Karadi, P. (2011). A model of unconventional monetary policy. *Journal of Monetary Economics*, **58**(1), 17–34.

Geweke, J. (1999). Using simulation methods for Bayesian econometric models: inference, development, and communication. *Econometric Reviews*, **18**, 1–127.

Geweke, J. and Amisano, G. (2011). Optimal prediction pools. *Journal of Econometrics*, **164**, 130141.

Gomme, P. and Greenwood, J. (1995). On the cyclical allocation of risk. *Journal of Economic Dynamics and Control*, **19**(1-2), 91–124.

Hansen, G. D. and Prescott, E. C. (2005). Capacity constraints, asymmetries, and the business cycle. *Review of Economic Dynamics*, **8**(4), 850–865.

Iskrev, N. (2010a). Evaluating the strenght of identification in dsge models. an a priori approach. unpublished manuscript.

Iskrev, N. (2010b). Local identification in dsge models. *Journal of Monetary Economics*, **57**, 189–202.

Jones, C. I. (2003). Growth, capital shares, and a new perspective on production functions. mimeo, Stanford University.

Jones, C. I. (2005). The shape of production functions and the direction of technical change. *Quarterly Journal of Economics*, **120**(2), 517–549.

Justiniano, A. and Michelacci, C. (2012). The Cyclical Behavior of Equilibrium Unemployment and Vacancies in the United States and Europe. *NBER International Seminar on Macroeconomics*, **8**(1), 169 – 235.

Kass, R. E. and Raftery, A. E. (1995). Bayes Factors. *Journal of the American Statistical Association*, **90**, 773–795.

Kennedy, C. (1964). Induced bias in innovation and the theory of distribution. *Economic Journal*, **74**, 541–547.

Klump, R. and Preissler, H. (2000). CES production functions and economic growth. *Scandinavian Journal of Economics*, **102**, 41–56.

Klump, R., McAdam, P., and Willman, A. (2007). Factor Substitution and Factor Augmenting Technical Progress in the US. *Review of Economics and Statistics*, **89**(1), 183–92.

Klump, R., McAdam, P., and Willman, A. (2012). The normalized CES production function: Theory and empirics. *Journal of Economic Surveys*, **26**(5), 769–799.

Koh, D. and Santaeulalia-Llopis, R. (2014). The shape of the aggregate production function over the business cycle and its implications for the labor market. mimeo.

Koh, D., Santaeulalia-Llopis, R., and Zheng, Y. (2015). Labor share decline and the capitalization of intellectual property products. mimeo.

La Grandville, O. de. (1989). In Quest of the Slutzky Diamond. *American Economic Review*, **79**, 468–481.

La Grandville, O. de. (2009). *Economic Growth: A Unified Approach*. Cambridge University Press.

León-Ledesma, M. A. and Satchi, M. (2011). The Choice of CES Production Techniques and Balanced Growth. Studies in Economics 1113, School of Economics, University of Kent.

León-Ledesma, M. A. and Satchi, M. (2015). Appropriate Technology and the Labour Share. Studies in Economics 1505, School of Economics, University of Kent.

León-Ledesma, M. A., McAdam, P., and Willman, A. (2010). Identifying the elasticity of substitution with biased technical change. *American Economic Review*, **100**(4), 1330–57.

Levine, P., Pearlman, J., Perendia, G., and Yang, B. (2012). Endogenous persistence in an estimated DSGE Model under imperfect information. *Economic Journal*, **122**(565), 1287 – 1312.

Mankiw, N. G. (1995). The growth of nations. *Brookings Papers on Economic Activity*, **1**, 275–310.

Marquetti, A. A. (2003). Analyzing historical and regional patterns of technical change from a classical-marxian perspective. *Journal of Economic Behavior and Organization*, **52**, 191–200.

McAdam, P. and Willman, A. (2013). Medium run redux. *Macroeconomic Dynamics*, **17**(04), 695–727.

Muck, J., McAdam, P., and Growiec, J. (2015). Will the true labor share stand up? Working Paper Series 1806, European Central Bank.

Neiman, B. and Karabarbounis, L. (2014). The Global Decline of the Labor Share. *The Quarterly Journal of Economics*, **129**(1), 61–103.

Piketty, T. and Zucman, G. (2014). Capital is Back: Wealth-Income Ratios in Rich Countries 1700?2010. *The Quarterly Journal of Economics*, **129**(3), 1255–1310.

Ríos-Rull, J.-V. and Santaeulália-Llopis, R. (2010). Redistributive shocks and productivity shocks. *Journal of Monetary Economics*, **57**(8), 931–948.

Rowthorn, R. (1999). Unemployment, wage bargaining and capital-labour substitution. *Cambridge Journal of Economics*, **23**, 413–425.

Shao, E. and Silos, P. (2014). Accounting For The Cyclical Dynamics Of Income Shares. *Economic Inquiry*, **52**(2), 778–795.

Smets, F. and Wouters, R. (2003). An estimated Stochastic Dynamic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association*, **1(5)**, 1123–1175.

Smets, F. and Wouters, R. (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach. *American Economic Review*, **97**(3), 586–606.

Solow, R. M. (1956). A contribution to the theory of economic growth. *Quarterly Journal of Economics*, **70**, 65–94.