Double one-sided cross-validation of local linear hazards

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1. Additional simulation results

Here we include additional details about the simulation studies described in the paper in Section 7.

Figure 1 shows the five simulated hazard functions. Table 1 shows the simulation results in the case of adding left truncation (case 2).

Table 2 reports the percentage of times where the cross-validation score:

$$\widehat{Q}_K(b) = n^{-1} \left\{ \sum_{i=1}^{n} \int_0^T \left[\widehat{\alpha}_{b,K}(s)\right]^2 Y_i(s) w(s) ds - 2 \sum_{i=1}^{n} \int_0^T \widehat{\alpha}_{b,K}^{[i]}(s) w(s) dN_i(s) \right\},$$

for the kernel K (cross-validation) or for the one-sided kernels K_L and K_R (Do-validation), have more than one minima on the considered grid of bandwidths. For simplicity we only show the results for two of the five simulated models. The non-presented models show a similar behaviour. The small number of cases where the Do-validation criterion runs into having several local minima is an indicator for the stability of Do-validation compared to cross-validation.

Table 1. Simulation results for datasets with left-truncation. Measure m_1 in columns 3–6 is the empirical MISE for each bandwidth estimate (multiplied by 100 for models 1 to 4 and by 1000 for model 5). The last column shows the relative error Rel_err that compares Do-validation with standard cross-validation. Measures m_2 and m_3 are the average and the standard deviation of the differences $\widehat{b} - \widehat{b}_{ISE}$, respectively.

	Criteria	ISE	$MISE$	CV	DO	Rel_err
Model 1, n =100	m_1	2.556	2.944	4.590	3.539	2.07
	m_2		0.024	0.141	0.201	
	m_3		0.274	0.620	0.580	
n =1000	m_1	0.340	0.398	0.563	0.429	2.51
	m_2		0.000	-0.032	-0.024	
	m_3		0.117	0.179	0.136	
n =10000	m_1	0.055	0.061	0.077	0.063	2.75
	m_2		-0.009	-0.019	-0.010	
	m_3		0.054	0.082	0.060	
Model 2, n =100	m_1	2.871	3.456	5.459	3.912	2.50
	m_2		0.010	0.004	0.022	
	m_3		0.152	0.258	0.197	
n =1000	m_1	0.471	0.542	0.742	0.591	2.25
	m_2		-0.005	-0.017	-0.001	
	m_3		0.068	0.107	0.085	
n =10000	m_1	0.082	0.089	0.108	0.092	2.66
	m_2		0.001	-0.009	-0.001	
	m_3		0.031	0.050	0.036	
Model 3, n =100	m_1	5.199	5.749	7.980	7.388	1.27
	m_2		-0.035	0.174	0.278	
	m_3		0.227	0.567	0.585	
n =1000	m_1	0.909	0.965	1.171	1.033	2.12
	m_2		-0.003	-0.006	0.001	
	m_3		0.045	0.080	0.064	
n =10000	m_1	0.184	0.190	0.210	0.197	2.00
	m_2		-0.003	-0.002	0.008	
	m_3		0.020	0.035	0.026	
Model 4, n =100	m_1	3.907	4.147	6.591	5.141	2.18
	m_2		0.566	0.004	0.123	
	m_3		0.531	0.943	0.881	
n =1000	m_1	0.596	0.653	0.920	0.745	2.17
	m_2		0.104	0.124	0.147	
	m_3		0.327	0.672	0.603	
n =10000	m_1	0.168	0.182	0.214	0.196	1.64
	m_2		-0.041	-0.003	0.076	
	m_3		0.146	0.214	0.172	
Model 5, n =50000	m_1	0.063	0.070	0.080	0.073	1.71
	m_2		0.079	0.274	-0.273	
	m_3		1.413	2.088	1.687	
n =75000	m_1	0.047	0.051	0.057	0.053	1.70
	m_2		-0.165	0.278	-0.196	
	m_3		1.183	1.779	1.431	
n =100000	m_1	0.036	0.039	0.043	0.041	1.51
	m_2		0.145	0.219	-0.182	
	m_3		1.095	1.638	1.352	

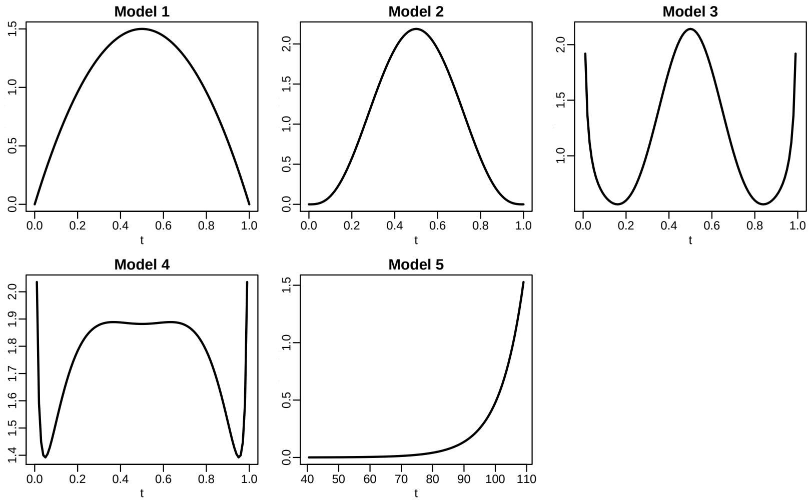

Fig. 1. The simulated models: the true hazard functions. Model 5 is a parametric specification for the mortality dataset in Iceland presented in the paper in Section 6.

Table 2. Percentage of cases where the cross-validation and the Do-validation scores have multiple local minima.

	Without left-truncation		With left-truncation	
	CV	*DO*	*CV*	*DO*
Model 2, n =100	17.2	1.2	21.2	2.4
n =1000	7.2	0.0	12.4	0.0
n =10000	5.6	0.0	8.4	0.0
Model 5, n =50000	18.0	2.4	2.0	0.0
n =75000	11.2	1.6	2.8	0.0
n =100000	12.0	1.2	3.6	0.0