A stochastic approach for deriving effective linear properties of bilinear hysteretic systems subject to design spectrum compatible strong ground motions

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ABSTRACT: A novel statistical linearization based approach is proposed to derive effective linear properties (ELPs), namely damping ratio and natural frequency, for bilinear hysteretic oscillators subject to seismic excitations specified by an elastic response/design spectrum. First, an efficient numerical scheme is adopted to derive a power spectrum, satisfying a certain statistical criterion, which is compatible with the considered seismic spectrum. Next, the thus derived power spectrum is used in conjunction with a frequency domain higher-order statistical linearization formulation to substitute a bilinear hysteretic oscillator by a third order linear system. This is done by minimizing an appropriate error function in the least square sense. Then, this third-order linear system is reduced to a second order linear oscillator characterized by a set of ELPs by enforcing equality of certain response statistics of the two linear systems. The ELPs are utilized to estimate the peak response of the considered hysteretic oscillator in the context of linear response spectrum-based dynamic analysis. In this manner, the need for numerical integration of the nonlinear equation of motion is circumvented. Numerical results pertaining to the European EC8 elastic response spectrum are presented to demonstrate the applicability and usefulness of the proposed approach. These results are supported by Monte Carlo analyses involving an ensemble of 250 non-stationary artificial EC8 spectrum compatible accelerograms. The proposed approach can hopefully be an effective tool in the preliminary aseismic design stages of yielding structures following either a force-based or a displacement-based methodology.

KEY WORDS: Effective linear properties; Design spectrum; Power spectrum; bilinear hysteretic; Stationary stochastic process.

1 INTRODUCTION

Aseismic code provisions define seismic severity via elastic design spectra associated with the peak response of linear viscously damped single-degree-of-freedom (SDOF) oscillators exposed to a "design" strong ground motion. However, ordinary structures are designed to behave inelastically (i.e. to suffer structural damage) for the "design earthquake". To account for this nonlinear/hysteretic behavior within a response spectrum-based analysis framework, inelastic design spectra of reduced coordinates by a strength reduction factor R are usually prescribed by regulatory agencies. These spectra provide the peak response of hysteretic SDOF oscillators with T_n natural period of small oscillations. The development of inelastic spectra relies either on a straightforward computation of the peak inelastic deformation or on R-μ-T_n relations, where μ is the ductility ratio. In both cases, comprehensive Monte Carlo analyses involving numerical integration of the nonlinear equations governing the motion of the hysteretic oscillators exposed to ensembles of field recorded seismic accelerograms are required (e.g. [1],[2]). Alternatively, approximate linearization techniques can be used for this purpose (e.g. [3]-[5]).

These techniques approximate the peak inelastic response by considering the peak response of an equivalent linear system (ELS) characterized by effective linear properties (ELPs), that is, damping ratio and natural frequency. Though results for various hysteretic constitutive laws are available the simple bilinear hysteretic law is the most extensively considered in such studies and the most commonly assumed in the everyday practice of earthquake resistant design of yielding structures. Most of the existing studies in the literature assume deterministic harmonic input to derive ELPs. This is done by averaging various quantities of interest over one cycle of the hysteretic response ([4],[5]). Herein, a recently proposed by the first two authors ([6],[7]) statistical linearization based approach is adopted and further extended to derive ELPs from bilinear hysteretic oscillators associated with any given elastic response spectrum. Notably, this is achieved without resorting to computationally demanding integration of the underlying nonlinear equation of motion.

The adopted linearization approach comprises two steps. First, a stationary stochastic process of finite duration is derived via a computationally efficient numerical scheme to achieve compatibility with a given design spectrum in a statistical sense. This process is defined in the frequency domain by means of a non-parametric power spectrum. Next, the thus derived power spectrum is treated as the input spectrum to perform statistical linearization [8]. In this manner, an equivalent linear system (ELS) is defined whose properties depend both on the nonlinear system and on the considered design spectrum.

In the original work of Giaralis and Spanos [7] an early statistical linearization formulation [9] assuming Gaussian narrow-band response of the considered nonlinear system has been used to derive a second order ELS corresponding to a linear SDOF oscillator. It relied on stochastic averaging over one period of oscillation. Herein, an efficient frequency-domain statistical linearization solution is formulated. It replaces the bilinear hysteretic system by a third order linear system [8],[10]. This statistical linearization formulation is based on less restrictive assumptions than the one adopted in [7] allowing for the treatment of bilinear hysteretic oscillators exhibiting strong nonlinear behavior (see also [11] and [12]). However, the thus derived third order ELS does not correspond to any particular physical system and cannot be readily related to a response/design spectrum pertaining to the peak response of linear SDOF oscillators. To this end, an additional step is introduced which considers an effective second order linear system. This system is obtained by enforcing equality of its displacement and velocity response variances with those of the third order ELS. The reduced-order effective linear system corresponds to a SDOF linear oscillator characterized by an effective damping ratio and an effective natural frequency. These effective linear properties are then used in conjunction with design spectra defined for various damping ratios to estimate the peak response of the underlying bilinear hysteretic oscillator.

Numerical data pertaining to various bilinear hysteretic oscillators exposed to the elastic response/design spectrum prescribed by the European aseismic code provisions (EC8) [13] are provided to demonstrate the effectiveness and applicability of the proposed approach. Furthermore, Monte Carlo based analyses involving an ensemble of 250 non-stationary artificial EC8 spectrum compatible accelerograms are also included. They pertain to R-μ-T_n relationships for both the considered hysteretic oscillators and the corresponding effective linear oscillators supporting the usefulness of the approach.

2 THEORETICAL BACKGROUND

2.1 Derivation of design spectrum compatible finite duration stationary stochastic processes

Consider a one-sided power spectrum $G(\omega)$ representing in the domain of frequencies ω a stationary zero-mean stochastic process $g(t)$ of finite duration T_s. This spectrum can be related to a given (target) response/design pseudo-acceleration seismic spectrum $S_\alpha(T,\zeta)$, with $T=2\pi/\omega_j$ being the natural period of oscillation, via the concept of a "peak factor" η_j by relying on the equation (e.g. [14])

$$S_a\left(\frac{2\pi}{\omega_j},\zeta\right)=\eta_j\omega_j^2\sqrt{\int_0^\infty\frac{G(\omega)}{\left(\omega^2-\omega_j^2\right)^2+\left(2\zeta\omega\omega_j\right)^2}d\omega}. \quad (1)$$

In the above equation, ω_j and ζ denote the natural frequency and ratio of critical damping, respectively, of a linear single-degree-of-freedom (SDOF) base excited by the process $g(t)$. In determining the peak factor η_j appearing in Eq. (1) the following approximate semi-empirical expression is herein adopted [14]

$$\eta_j=\sqrt{2\ln\left\{2v_j\left[1-\exp\left(-q_j^{1.2}\sqrt{\pi\ln\left(2v_j\right)}\right)\right]\right\}}, \quad (2)$$

with

$$v_j=-\frac{T_s}{2\pi\ln(0.5)}\omega_j, \quad (3)$$

and

$$q_j=\sqrt{1-\frac{1}{1-\zeta_j^2}\left(1-\frac{2}{\pi}\tan^{-1}\frac{\zeta_j}{\sqrt{1-\zeta_j^2}}\right)^2}, \quad (4)$$

where

$$\zeta_j=\frac{\zeta}{1-\exp\left(-2\zeta\omega_jT_s\right)}. \quad (5)$$

Equations (2) to (5) allow for calculating reliably the median peak factor of a linear oscillator with properties ω_j and ζ subject to clipped white noise input of duration T_s. In this regard, Eq. (1) establishes the following criterion: considering an ensemble of realizations of the process $g(t)$, half of the population of their response spectra will lie below S_a (i.e. S_a is the median response spectrum) [7],[14].

Given a target spectrum S_a, an estimate of the power spectrum $G(\omega)$ conforming with the aforementioned criterion can be evaluated recursively at a specific set of M equally spaced natural frequencies $\omega_k= \omega_0+ (j\text{-}0.5)\Delta\omega$; $j= 1,2,\ldots,M$ using the equation [15]

$$G\left[\omega_k\right]=\begin{cases}\dfrac{4\zeta_k}{\omega_k\pi-4\zeta_k\omega_{k-1}}\left(\dfrac{S_\alpha^2\left(2\pi/\omega_k,\zeta\right)}{\eta_k^2}-\Delta\omega\sum_{i=1}^{k-1}G\left[\omega_i\right]\right);\ \omega_k>\omega_0 \\ 0\ ;\ 0\le\omega_k\le\omega_0\end{cases}. \quad (6)$$

In Eq. (6), ω_0 is the lowest frequency for which Eq. (2) is defined (see also [7]). An approximation of the pseudo-acceleration response spectrum $A[2\pi/\omega_k,\zeta]$ corresponding to the discrete power spectrum $G[\omega_k]$ obtained by Eq. (6) can be determined by using Eqs. (1) to (5). For this purpose, the integral appearing in Eq. (1) can be efficiently evaluated numerically using the spectral moment formulae due to Pfaffinger [16] (see also [17]). Note that, in general, $A[2\pi/\omega_k,\zeta]$ may not lie as close as desired to the target spectrum S_α for all the considered ω_k natural frequencies. In this respect, $G[\omega_k]$ can be further modified iteratively to improve the point-wise matching of the response spectrum $A[2\pi/\omega k,\zeta]$ with the target spectrum. This can be done by means of the following equation written at the N-th iteration (e.g. [18])

$$G^{(N+1)}\left[\omega_j\right]=G^{(N)}\left[\omega_j\right]\left(\frac{S_a\left[2\pi/\omega_j,\zeta\right]}{A^{(N)}\left[2\pi/\omega_j,\zeta\right]}\right)^2. \quad (7)$$

Next, the design spectrum compatible power spectrum $G[\omega_j]$ obtained from Eqs. (6) or (7) is used in conjunction with the method of statistical linearization to determine effective natural frequency and damping parameters associated with bilinear hysteretic oscillators.

2.2 A frequency-domain statistical linearization solution for bilinear hysteretic systems

Consider a unit-mass quiescent bilinear hysteretic SDOF system with ratio of critical viscous damping ζ, base-excited by a stationary acceleration process $g(t)$. The process is defined in the frequency domain by the one-sided power spectrum $G(\omega)$. The motion of the considered system is governed by the system of differential equations [19]

$$\ddot{x}(t)+2\zeta\omega_n\dot{x}(t)+a\omega_n^2x(t)+(1-a)\,\omega_n^2z(t)=-\frac{g(t)}{x_y}, \quad (8)$$

and

$$\dot{z}=\dot{x}\left[1-U(\dot{x})U(z-1)-U(-\dot{x})U(-z-1)\right]. \quad (9)$$

In this equation x_y is the yielding displacement, x is the relative response displacement process normalized by x_y, α is the post-yield to pre-yield stiffness ("rigidity") ratio (e.g. the value α=0 corresponds to a perfectly elasto-plastic oscillator), and $\omega_n=(f_y/x_y)^{1/2}$ is the pre-yielding natural frequency with f_y being the yielding strength. Furthermore, z is an additional "state" related to the response of the system by the differential Eq. (9) which mathematically captures the bilinear perfectly elasto-plastic hysteretic behavior. In the previous equations and hereafter the dot over a symbol denotes differentiation with respect to time and $U(\cdot)$ is the Heaviside step function. A graph of the restoring force of the bilinear hysteretic oscillator defined by Eqs. (8) and (9) for zero damping ratio is shown in Figure 1. Also included are certain response quantities of practical interest in the earthquake resistant design of structures (i.e. the strength reduction factor R and the ductility ratio μ).

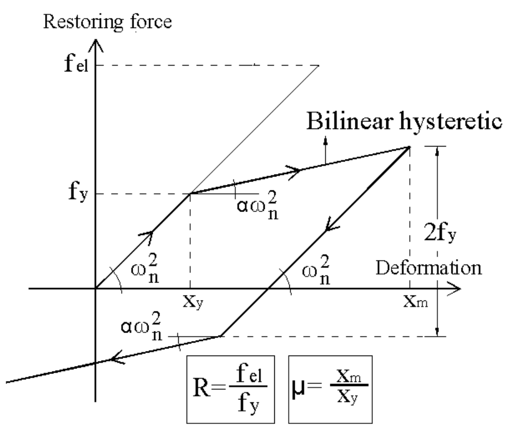

Figure 1. Bilinear hysteretic restoring force and definitions of the strength reduction factor R and ductility μ.

Note that Eq. (8) is a linear differential equation containing three states ($x, \dot{x}$, and z) related by a non-linear equation (Eq. (9)). In this junction, the statistical linearization procedure for non-linear multi-degree-of-freedom systems described in [11] and based on the work of Kazakov [20] is applied to replace Eq. (9) by the following linear differential equation (see also [8] and [12])

$$\dot{z}(t)+c_{eq}\dot{x}(t)+k_{eq}z(t)=0. \quad (10)$$

In this equation, the equivalent linear coefficients c_{eq} and k_{eq} are determined by requiring minimization of the mean square error in replacing Eq. (9) by Eq. (10). By approximating the processes $\dot{x}$ and z as jointly Gaussian the following expressions for determining c_{eq} and k_{eq} are derived [8]

$$c_{eq}=2\varepsilon\left(\frac{\pi}{2\alpha\beta}\right)^{1/2}\times \int_{\sqrt{\beta}}^{\infty}\exp\left[-(1+\rho)u^2\right]\left[1+erf(\rho u)\right]du-1, \quad (11)$$

and

$$k_{eq}=\frac{2\varepsilon\exp\left(\mu^2-\beta\right)}{\alpha}\times \left[\frac{1}{2}\exp\left(-\mu^2\right)+\rho\left(\frac{\pi}{2}\right)^{1/2}\left(1+\exp\mu\right)\right], \quad (12)$$

where

$$\varepsilon=\frac{1}{2\pi\sqrt{\Delta}};\ a=\frac{E\{z^2\}}{2\Delta};\ \beta=\frac{E\{\dot{x}^2\}}{2\Delta};\ \mu=\frac{\gamma}{\sqrt{\alpha}}, \quad (13)$$

and

$$\gamma=\frac{E\{\dot{x}z\}}{2\Delta};\ \Delta=E\{\dot{x}^2\}E\{z^2\}-\left(E\{\dot{x}z\}\right)^2. \quad (14)$$

In Eq. (12) $erf(\cdot)$ is the standard error function and in Eqs. (13) and (14), and, henceforth, $E\{\cdot\}$ denotes the mathematical expectation operator. From the above expressions it is seen that c_{eq} and k_{eq} depend on the variance of the processes $\dot{x}$ and z, and their cross-variance. To this end, a frequency domain formulation relying on the spectral input/output relations for linear systems and the Wiener-Khinchin theorem can be devised to calculate these response moments. Specifically, [10]

$$E\{\dot{x}^2\}=\int_0^{\infty}\omega^2\left|\frac{i\omega+k_{eq}}{\sum_{k=0}^{3}(i\omega)^kA_k}\right|^2\frac{G(\omega)}{x_y^2}d\omega, \quad (15)$$

and

$$E\{z^2\}=\int_0^{\infty}\left|\frac{-i\omega c_{eq}}{\sum_{k=0}^{3}(i\omega)^kA_k}\right|^2\frac{G(\omega)}{x_y^2}d\omega, \quad (16)$$

in which $i=\sqrt{-1}$ and

$$\begin{aligned}A_0&=a\omega_n^2k_{eq}\\A_1&=a\omega_n^2+2\zeta\omega_nk_{eq}-(1-a)\,\omega_n^2c_{eq}\\A_2&=k_{eq}+2\zeta\omega_n\\A_3&=1\end{aligned}, \quad (17)$$

while the cross-variance term can be computed by the equation

$$E\{\dot{x}z\}=-\frac{k_{eq}}{c_{eq}}E\{z^2\}. \quad (18)$$

Equations (11), (12), (15), (16), and (18) form a system of non-linear equations with five unknowns, namely, c_{eq}, k_{eq}, $E\{\dot{x}^2\}$, $E\{z^2\}$, and $E\{\dot{x}z\}$. This system can be readily written as a standard minimization problem and solved numerically by any qualified optimization routine. In all of the ensuing numerical examples an optimization algorithm built-in MATLAB® using a trust region dog-leg search method is used for this purpose (see also [21]).

Upon determination of the c_{eq}, k_{eq} parameters, an "equivalent" linear third order system is established governed by the differential Eqs. (8) and (10). Various researchers have shown both theoretically and through numerical experimentation that this particular higher-than-a-second-order linear system captures the response statistics of various hysteretic systems exhibiting strong nonlinear behavior in an acceptable manner (see e.g. [8],[12] and [22]). However, this third order linear system cannot be readily related to a response/design spectrum pertaining to the peak response of linear SDOF oscillators. To this end, in the next section an approach to reduce the system order is introduced by relying on a specific statistical criterion.

2.3 Derivation of effective linear properties from the 3rd order equivalent linear system

Let y be the normalized by x_y deformation of an "auxiliary" linear SDOF oscillator of critical viscous damping ζ_{eff} and natural frequency ω_{eff} base excited by the stationary acceleration process $g(t)$. The governing equation of motion of this auxiliary system reads as

$$\ddot{y}(t)+2\zeta_{eff}\omega_{eff}\dot{y}(t)+\omega_{eff}^2 y(t)=-g(t)/x_y, \qquad (19)$$

and zero initial conditions apply. For the purposes of this work, it is sought to relate the above second order linear system to the third order ELS derived by means of statistical linearization to replace bilinear hysteretic systems excited by the process $g(t)$ as detailed in the previous section. This can be accomplished by enforcing equality of the variances of the processes x and y, that is,

$$E\{x^2\}=\int_0^\infty \frac{G(\omega)/x_y^2}{\left(\omega^2-\omega_{eff}^2\right)^2+\left(2\zeta_{eff}\omega_{eff}\omega\right)^2}d\omega, \qquad (20)$$

and of the variances of the processes $\dot{x}$ and $\dot{y}$, that is,

$$E\{\dot{x}^2\}=\int_0^\infty \frac{\omega^2 G(\omega)/x_y^2}{\left(\omega^2-\omega_{eff}^2\right)^2+\left(2\zeta_{eff}\omega_{eff}\omega\right)^2}d\omega. \qquad (21)$$

The variance appearing in the lhs of Eq. (20) can be determined by the expression

$$E\{x^2\}=\int_0^\infty \left|\frac{i\omega+k_{eq}}{\sum_{k=0}^{3}(i\omega)^k A_k}\right|^2 \frac{G(\omega)}{x_y^2}d\omega. \qquad (22)$$

Further, the variance appearing in the lhs of Eq. (21) is a known quantity determined upon solving the nonlinear system of equations considered in the statistical linearization solution of the previous section. In this regard, Eqs. (20) and (21) define a system of nonlinear equations which can be solved for the unknown effective linear properties ζ_{eff} and ω_{eff} of the second order linear system corresponding to a linear SDOF oscillator. To this aim, the same optimization algorithm used to obtain the statistical linearization solution is employed to solve the above two-by-two system of non-linear equations in obtaining the numerical results presented in the next section. Conveniently, there exist spectral moment formulae to numerically evaluate the integrals in Eqs. (20) and (21) for the discrete power spectra derived as detailed in section 2.1 in a computationally efficient manner [16],[17].

2.4 Estimation of the peak deformation of bilinear hysteretic systems from the effective linear properties

Following the approach established in [6] and [7], the response/design spectrum compatible power spectra obtained by Eqs. (6) or (7) is used as the input spectrum to derive effective SDOF linear oscillators corresponding to bilinear hysteretic systems via the preceding statistical linearization based methodology. This is accomplished by solving sequentially one five-by-five system and one two-by-two system of non-linear equations derived in sections 2.2 and 2.3, respectively. In this context, for any bilinear hysteretic oscillator characterized by an initial stiffness T_n, a yielding displacement x_y and a rigidity ratio α exposed to the seismic hazard represented by a specific response/design spectrum $S_\alpha(T,\zeta)$, the herein proposed methodology yields a set of effective linear properties ζ_{eff} and ω_{eff}. These effective linear properties are explicitly associated with both the pre-specified response/design spectrum and the considered non-linear system. In this regard, a reasonable estimate of the peak deformation of bilinear hysteretic oscillators can be determined by the expression

$$\max_t\{|x(t)|\}\approx \frac{S_a\left(2\pi/\omega_{eff},\zeta_{eff}\right)}{\omega_{eff}^2}. \qquad (23)$$

That is, using a family of response spectra defined for various damping ratios. In this manner, the need for numerically integrating the non-linear Eqs. (8) and (9) of motion for an ensemble of seismic accelerograms compatible with the considered response/design spectrum is by-passed. Obviously, the reliability of the estimated peak value will depend on the severity of the induced non-linear response and will lie within the well-quantified approximation induced by the application of the statistical linearization method (see e.g. [8]).

Note that in case dependable response spectra are not available for damping ratios other than ζ an estimate of the peak non-linear response can be achieved by the equation relying on the concept of the peak factor and the fact that the

$$\max_t\{|x(t)|\}\approx \eta_{eff}\int_0^\infty \frac{G(\omega)/x_y^2}{\left(\omega^2-\omega_{eff}^2\right)^2+\left(2\zeta_{eff}\omega_{eff}\omega\right)^2}d\omega, \qquad (24)$$

which relies on the concept of the peak factor and the fact that the response variance of x is determined upon performing statistical linearization. In the latter equation η_{eff} can be calculated from Eqs. (2) to (5) by setting $\zeta=\zeta_{eff}$.

3 NUMERICAL APPLICATION TO THE EC8 DESIGN SPECTRUM

The elastic response/design spectrum of the European aseismic code [13] is herein considered as a paradigm to assess the usefulness and applicability of the proposed methodology. Effective linear properties corresponding to various bilinear hysteretic oscillators are derived. Specifically, the EC8 (target) pseudo-acceleration response spectrum for peak ground acceleration 0.36g (g= 981cm/sec^2), ground type B and damping ratio ζ= 0.05 (gray thick line in Figure 2), is considered to represent the induced seismic action. The broken line of Figure 3 corresponds to a discrete power spectrum compatible with the EC8 target spectrum computed by means of Eq. (6) assuming T_s = 20s and $\Delta\omega$= 0.1rad/s. Furthermore, this power spectrum is modified by performing four iterations using Eq. (7). The obtained modified spectrum is also shown in Figure 3.

The pseudo-acceleration response spectra associated with the two power spectra of Figure 3 are plotted in Figure 2 and compared with the target spectrum. These response spectra have been determined analytically by Eqs. (1) to (5). It can be seen that the iteratively matched power spectrum achieves a close compatibility with the target spectrum (see also [7]). This is further confirmed by considering the median spectral ordinates of an ensemble of 2000 20s long stationary signals compatible with the iteratively modified spectrum (plotted as dots in Figure 2), which have been synthesized using a random simulation filtering technique based on an auto-regressive-moving-average filter (see for example [23]).

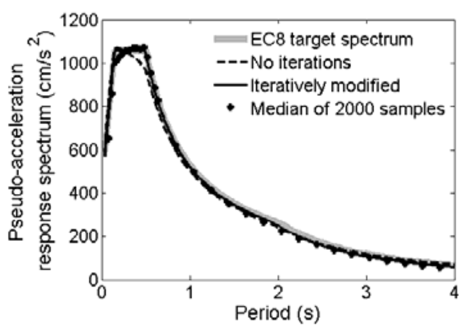

Figure 2. Target EC8 design spectrum, response spectra computed by Eqs. (1) to (5) pertaining to the power spectra of Figure 3, and median response spectrum of 2000 simulated signals compatible with the iteratively modified power spectrum of Figure 3.

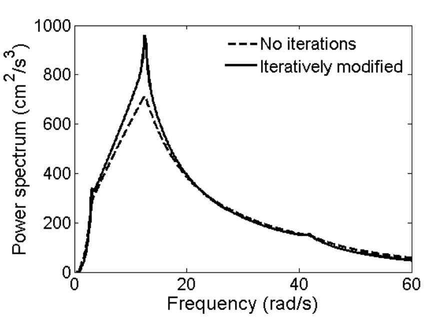

Figure 3. Power spectra obtained by Eq. (6) and by Eq. (7) (4 iterations) compatible with the EC8 spectrum of Figure 2.

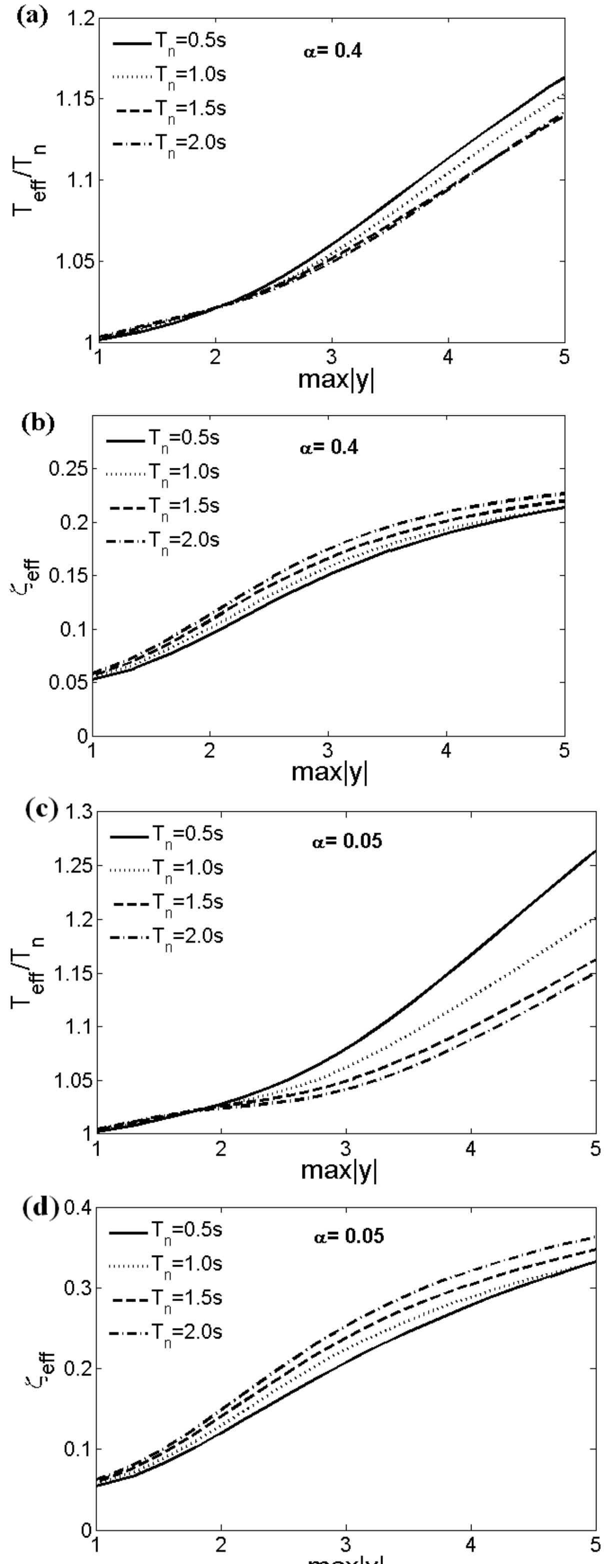

Figure 4. Effective linear properties (T_{eff} and ζ_{eff}) for various bilinear hysteretic oscillators with damping coefficient ζ=5% rigidity ratio α and pre-yield natural period T_n compatible with the EC8 spectrum of Figure 2.

Next, the iteratively modified power spectrum of Figure 3 is used to obtain effective linear properties (ELPs) $T_{eff}= 2\pi/\omega_{eff}$ and ζ_{eff} via the statistical linearization-based method detailed in sections 2.2 and 2.3 for various bilinear hysteretic oscillators. Shown in Figure 4 are ELPs corresponding to viscously damped bilinear oscillators with ζ=0.05, four different pre-yield stiffness values expressed by $T_n= 2\pi/\omega_n$, and with rigidity ratios α=0.4 (panels (a) and (b) of Figure 6) and α=0.05 (panels (c) and (d) of Figure 6). These properties are plotted against the "ductility" max|y| to quantify the severity of the nonlinear response. For the purposes of this study, max|y| is numerically evaluated as the average of the peak responses of the effective linear system of Eq. (19) excited by an ensemble of 250 artificial non-stationary accelerograms compatible with the target EC8 spectrum of Figure 2. These signals have been generated by a wavelet-based stochastic methodology recently proposed by Giaralis and Spanos [24]. Figure 5 includes the average, the maximum, the minimum and the standard deviation over the average of the spectral ordinates in terms of pseudo-acceleration of this ensemble of accelerograms as a function of the natural period. In this context, the range of max|y| is accomplished by varying the yielding displacement x_y of the considered nonlinear oscillators.

Note that max|y| may not coincide with the ductility demand μ as defined in Figure 1, because of the approximation involved in deriving the ELPs via the proposed statistical linearization based approach. Larger discrepancy is expected for bilinear oscillators forced to exhibit more severe nonlinear response (see also [7]). This point is illustrated in Figure 6 which includes R-μ-T_n relations based on averaged response time-histories obtained via numerical integration of the considered bilinear hysteretic oscillators (dots of various shapes) and of the corresponding effective linear oscillators (lines of various types) considering as input the ensemble of the seismic signals of Figure 5. Figure 6 further shows the standard deviation over the average of the obtained μ and max|y| to shed light on the statistical nature of the presented Monte Carlo analysis based numerical results. In all plots of Figure 6 the strength reduction factor R is computed as the ratio of the average value of the peak response of the corresponding linear oscillator of natural period T_n excited by the ensemble of the considered 250 design spectrum compatible accelerograms over the yielding force f_y (see also Figure 1).

Examining the data included in Figure 4, it is evident that the herein proposed approach yields results that are in agreement with engineering intuition. In general, the departure from the linear response, quantified by larger values of max|y|, yields "softer" effective linear systems characterized by longer natural periods. Furthermore, the effective damping ratio increases monotonically with max|y| to account for the increased energy dissipation through more severe plastic/ hysteretic behaviour of the corresponding nonlinear oscillators.

Finally, Figure 7 illustrates the manner in which the ELPs derived from the adopted design spectrum-based statistical linearization procedure as those reported in Figure 4 can be used to approximate the peak values of certain response quantities of the associated nonlinear systems by using the EC8 design spectrum. This is done for various levels of viscous damping. In particular, consider a specific viscously damped bilinear hysteretic oscillator with damping ratio ζ= 5% and pre-yield natural period T_n exposed to the EC8 elastic design spectrum (vertical broken lines). One can focus, following the horizontal arrows, to a vertical solid line corresponding to an effective linear system characterized by T_{eff} and ζ_{eff} obtained by the statistical linearization based methodology herein adopted and "read" the related spectral ordinate. In this manner, an estimate of the peak response of the considered structural system is achieved without the need to have available suites of spectrum compatible accelerograms and to numerically integrate the governing nonlinear equation of motion.

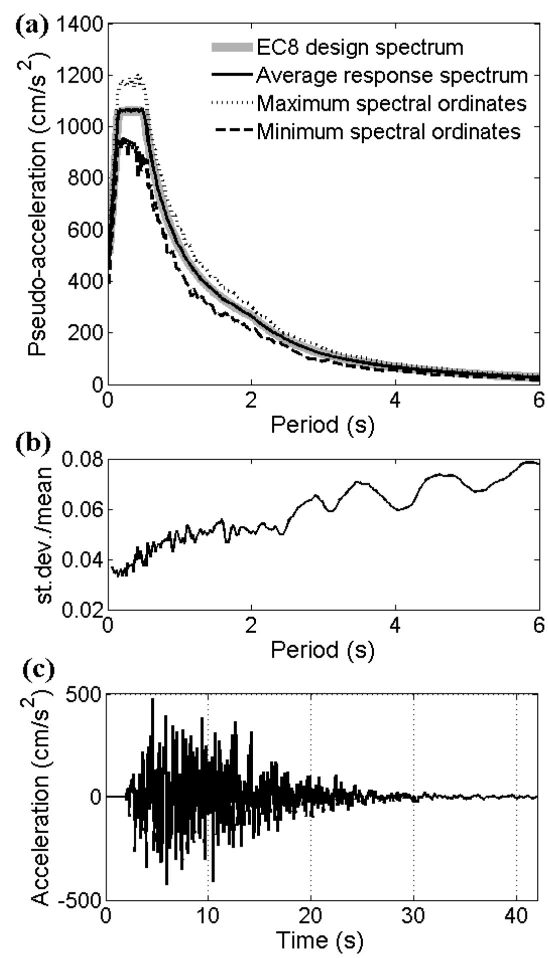

Figure 5. Statistics of response spectra of an ensemble of 250 artificial non-stationary accelerograms compatible with the EC8 response spectrum of Figure 2 (panels (a) and (b)), and time-history of an arbitrarily chosen signal belonging to the considered ensemble (panel (c)).

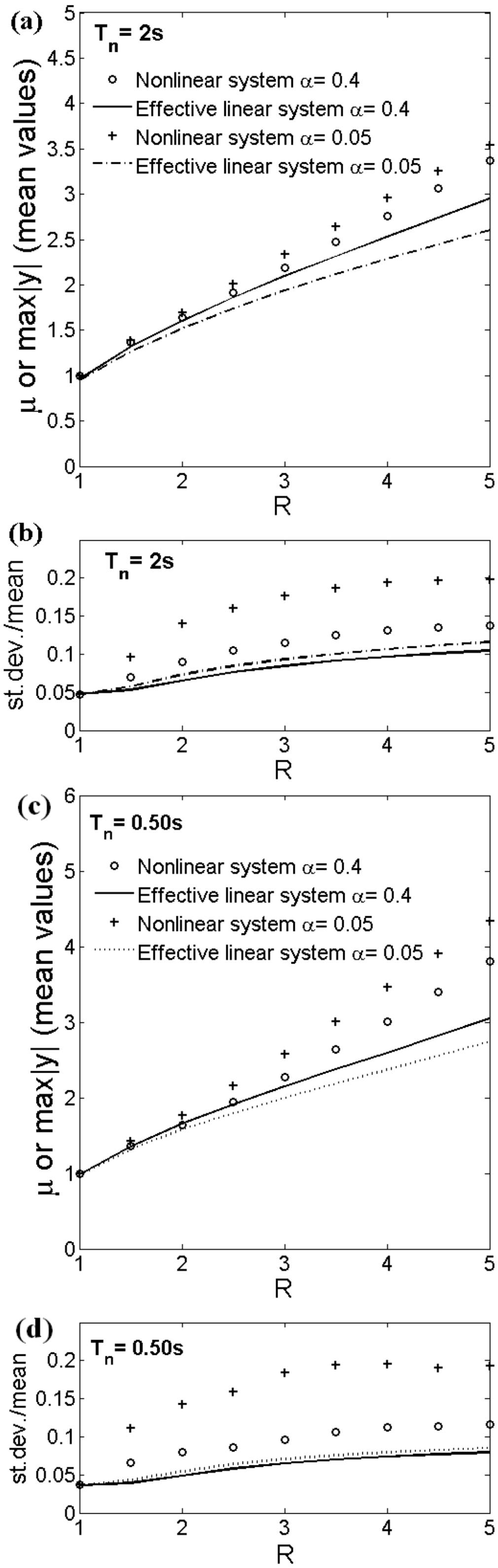

Figure 6. Mean (panels (a) and (c)) and standard deviation over mean (panels (b) and (d)) values of the peak deformation normalized by the yielding displacement x_y of various bilinear hysteretic oscillators and of the corresponding effective linear systems excited by the suite of accelerograms of Figure 5.

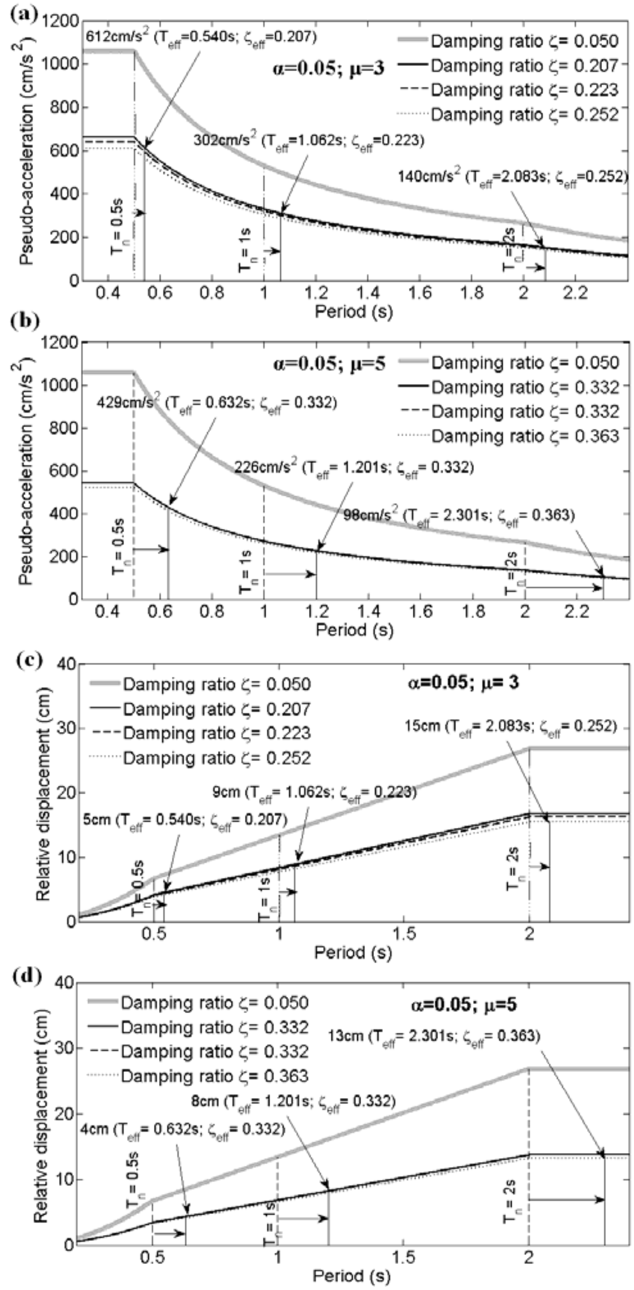

Figure 7. Peak response estimation in terms of pseudo-acceleration (panels (a) and (b)) and of deformation (panels (c) and (d)) of various viscously damped bilinear hysteretic oscillators using their corresponding effective linear properties and the EC8 elastic response spectrum.

4 CONCLUDING REMARKS

A novel statistical linearization based approach has been proposed to derive effective linear properties (damping ratio and natural frequency) (ELPs) corresponding to effective linear single-degree-of-freedom oscillators (ELSs) from viscously damped bilinear hysteretic oscillators excited by strong ground motions defined by a given response/design spectrum. Those ELPs have been determined by solving successively two systems of nonlinear equations (one five-by-five and one two-by-two) which can be efficiently done using any qualified optimization algorithm. The first system of equations provides a frequency domain statistical linearization solution which replaces a bilinear oscillator with a third order linear system. The second system of equations is associated with a system reduction step to reduce the third order to a second order linear system. Both solutions involve the consideration of a numerically derived power spectrum compatible with the given design spectrum, in a probabilistic sense. An efficient recursive formula has been adopted to determine the required power spectrum.

The thus derived ELPs have been used to obtain reliable estimates of the peak response of strongly nonlinear bilinear hysteretic systems without integrating the nonlinear equations of motion. This point has been ascertained by pertinent numerical results associated with the EC8 elastic response/design spectrum. Specifically, EC8 compatible ELPs corresponding to various viscously damped bilinear oscillators of different rigidity ratios and pre-yield stiffness have been obtained. Furthermore, strength reduction-ductility-natural period relationships have been derived for the considered hysteretic systems and for the associated ELSs within a Monte Carlo based analyses pertaining to an ensemble of 250 EC8 design spectrum compatible non-stationary time-histories obtained via a wavelet-based stochastic approach. Finally, the derived ELPs have been utilized to estimate the peak responses of the bilinear hysteretic oscillators by using the elastic EC8 response spectrum for various damping ratios.

It is expected that the proposed approach can be further used to facilitate various aseismic design procedures relying on the definition of ELSs such as the direct displacement based method [25]. For this purpose, perhaps a semi-empirical correction maybe incorporated to enhance the reliability of the estimated peak deformations. Such a corrective factor can be calibrated by means of comprehensive Monte Carlo analyses.

ACKNOWLEDGMENTS

The partial support through the “Pump Priming Fund” granted by City University London is gratefully acknowledged by the first author.

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