DERIVATION OF NON-STATIONARY STOCHASTIC PROCESSES COMPATIBLE WITH SEISMIC RESPONSE/DESIGN SPECTRA

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In this paper the problem of deriving non-stationary stochastic processes defined by a parametric evolutionary power spectrum (EPS) compatible with a given (target) design spectrum is addressed. An inverse stochastic dynamics problem is formulated and solved in a least-square sense to determine the requisite EPS. This involves the incorporation of a "peak factor" which is used to relate statistically the target spectrum to the EPS. Special attention is focused on deriving design spectrum compatible processes of specific "effective duration" as commonly defined in the field of earthquake engineering. Specifically, the design spectrum of the Chinese GB 50011 aseismic code is considered as a paradigm of a target spectrum. Comprehensive Monte Carlo analyses are undertaken to numerically estimate GB 50011-compatible median peak factor spectra, given in a polynomial form. These spectra are associated with the first passage problem for linear oscillators excited by uniformly modulated colored non-stationary processes of various durations. The derived peak factor spectra used in conjunction with the herein adopted stochastic formulation yield an excellent level of agreement between the GB 50011 spectrum and the ensemble average response spectra of simulated EPS-compatible accelerograms of different effective durations. Additional numerical results pertaining to the design spectra of the European EC8 code and the GB 50011 code are included to show how the behavior of the target spectrum in the range of long periods affects the choice of the assumed spectral form of the EPS. It is envisioned that the herein derived stochastic processes can be used to facilitate the aseismic design of structures regulated by contemporary code provisions in a Monte Carlo-based or random vibration-based context of analysis.

Keywords: non-stationary process, design spectrum compatible accelerograms, inverse problem, Monte Carlo simulation, peak factors.

1 Introduction

Contemporary code provisions regulating the aseismic design of structured facilities represent the seismic action via smooth response/design elastic and inelastic spectra (e.g. CEN 2004). Such spectra facilitate the design of ordinary structures significantly by means of linear response spectrum-based kinds of analyses (e.g. Clough and Penzien 1993). However, the aseismic design of critical structures calls for additional nonlinear time-history analyses to be performed within a Monte Carlo-based framework. This requires representing the seismic severity by ensembles of seismic accelerograms whose response spectra are compatible (i.e. in a close agreement) with a prescribed design spectrum.

Such design spectrum compatible accelerograms can be obtained either by scaling properly selected field recorded ground motions or by synthesizing time-histories belonging to appropriately defined stochastic processes compatible with a given design spectrum (e.g. Giaralis and Spanos (2009a) and references therein). Obviously, design spectrum compatible stochastic processes can also be used as input to perform random vibrations-based analyses during the preliminary design stages of structures regulated by specific aseismic codes to circumvent tedious Monte Carlo analyses (e.g. Giaralis and Spanos 2010).

This paper considers a stochastic formulation originally proposed by Spanos and Vargas Loli (1985) to address the issue of deriving a non-stationary stochastic process compatible with a given (target) design spectrum. Specifically, the considered formulation relies on an analytical expression to relate a parametrically defined evolutionary power spectrum (EPS) with the target design spectrum via the concept of a "peak factor" in a statistical manner (see also Vanmarcke 1976). The sought EPS characterizes a non-stationary stochastic process in the frequency domain whose spectral content and time-evolving intensity are controlled by the assumed parametric form of the EPS. The unknown parameters defining the EPS are determined by solving an inverse stochastic dynamics problem involving a system of non-linear equations. In this respect, base-line corrected realizations compatible with the thus derived EPS are construed as (artificial) seismic accelerograms (Giaralis and Spanos 2009). The statistical nature and quality of the achieved compatibility of ensembles of these accelerograms with the target spectrum is governed by the assumed value for the peak factor in the considered formulation. The peak factor is closely associated with the first passage problem of the response of stochastically excited linear single-degree-of-freedom (SDOF) oscillators (e.g. Vanmarcke 1976). It depends on the stochastic input excitation and on the natural frequency and ratio of critical damping of the assumed oscillators. However, for the case of non-stationary input processes, no convenient analytical formula for the peak factor is available to facilitate the solution of the herein considered inverse problem (see also Giaralis and Spanos (2009b) and references therein).

In this respect, Giaralis and Spanos (2009a) utilized the aforementioned formulation adopting a constant value for the peak factor in tandem with a wavelet-based spectral matching scheme to derive artificial accelerograms compatible with the design spectrum of the European (EC8) aseismic code provisions (CEN 2004). Moreover, Spanos et al. (2009) extended further the above integrated wavelet-based approach to obtain accelerograms satisfying the compatibility criteria with the design spectrum of the GB 50011 aseismic code effective in China (GB 50011 2001). In particular, frequency-dependent peak factors (peak factor spectra) have been incorporated in the herein adopted formulation to generate ensembles of accelerograms whose average response spectrum achieved enhanced agreement with the GB 50011 design spectrum. These peak factor spectra have been derived numerically by pertinent Monte Carlo analysis considering uniformly modulated Kanai-Tajimi (K-T) (Kanai 1957) EPSs compatible with the GB 50011 spectrum. More recently, Giaralis and Spanos (2009b) conducted comprehensive Monte Carlo simulations to examine the statistics of peak factors for uniformly modulated Clough-Penzien (C-P) EPSs (Clough and Penzien 1993) compatible with the EC8 design spectrum. Median peak factor spectra compatible with the EC8 design spectrum for various damping ratios and for all soil conditions prescribed in the EC8 regulations were reported. These spectra were used in conjunction with the previously described stochastic formulation to derive EPSs compatible with the EC8 design spectrum. It was shown that the thus derived EPSs achieved a significantly improved level of compatibility with the EC8 spectrum compared to the one accomplished by assuming a constant peak factor.

In view of the previous research work on the topic, this study focuses on clarifying certain aspects of practical merit regarding the selection of the assumed parametric form of the EPS required in the adopted stochastic formulation. In particular, the considered formulation is further extended to derive design spectrum compatible non-stationary processes of a specific prescribed "effective duration" as is commonly defined in the field of earthquake engineering (Trifunac and Brady, 1975). For this purpose, the GB 50011 is set as the target spectrum. Appropriate Monte Carlo analysis is undertaken to derive GB 50011 median peak factors for input K-T spectra of various effective durations. These peak factors are then used to derive GB 50011 compatible EPSs of different predefined durations. This is an important practical consideration since the duration of the strong ground motion is known to be a parameter influencing the response of seismically excited structures (e.g. Iervolino et al. 2006).

Furthermore, pertinent numerical results associated with the GB 50011 and the EC8 design spectra are provided to elucidate the fact that the choice of a valid parametric phenomenological model governing the frequency content of the EPS hinges on the behavior of the target design spectrum in the range of long periods.

2 Mathematical Background

2.1 *Parametric form of the evolutionary power spectrum (EPS)*

Let the acceleration trace of the strong ground motion due to an earthquake be modeled by a uniformly modulated non-stationary stochastic process $u_g(t)$. That is,

$$u_g(t) = A(t)\, y(t)\,, \qquad (1)$$

where $A(t)$ is a deterministic envelop function dependent on time t and $y(t)$ is a zero-mean stationary stochastic process. For sufficiently "slowly-varying" envelop functions, the process $u_g(t)$ can be reliably represented in the domain of frequencies ω by a two-sided evolutionary power spectrum (EPS) $G(t,\omega)$ given by the expression (Priestley 1965)

$$G(t,\omega) = \left|A(t)\right|^2 Y(\omega), \left|\omega\right| \le \omega_b\,, \qquad (2)$$

where ω_b is the highest frequency contained in the $u_g(t)$ process, and $Y(\omega)$ is the power spectrum of the stationary process $y(t)$.

Herein, the envelop function given by the equation (Bogdanoff et al. 1961)

$$A(t) = Cte^{-bt/2} \qquad (3)$$

is adopted to account for the time-varying intensity observed in typical field recorded accelerograms pertaining to historic seismic events. In the above equation the parameter C controls the intensity of the ground acceleration process. Furthermore, the parameter b is related to the effective duration T_{eff} defined as (Trifunac and Brady 1975)

$$T_{eff} = t_{95} - t_{05}\,, \qquad (4)$$

by means of the following system of non-linear equations (Spanos et al. 2009)

$$\begin{cases} \left(b^2 t_{95}^2 + 2bt_{95} + 2\right) e^{-bt_{95}} = 0.1 \\ \left(b^2 t_{05}^2 + 2bt_{05} + 2\right) e^{-bt_{05}} = 1.9 \end{cases}. \qquad (5)$$

In Eqs. (4) and (5) t_{05} and t_{95} denote the time instants at which the 5% and the 95% of the total energy of the acceleration process has been released, respectively.

For the purposes of this study the Kanai-Tajimi (K-T) spectrum given by the equation (Kanai 1957)

$$Y_{KT}(\omega)=\frac{1+4\zeta_g^2\left(\frac{\omega}{\omega_g}\right)^2}{\left(1-\left(\frac{\omega}{\omega_g}\right)^2\right)^2+4\zeta_g^2\left(\frac{\omega}{\omega_g}\right)^2}, \quad (6)$$

and the Clough-Penzien (C-P) spectrum given by the equation (Clough and Penzien 1993)

$$Y_{CP}(\omega)=Y_{KT}(\omega)\times \frac{\left(\frac{\omega}{\omega_f}\right)^4}{\left(1-\left(\frac{\omega}{\omega_f}\right)^2\right)^2+4\zeta_f^2\left(\frac{\omega}{\omega_f}\right)^2}, \quad (7)$$

are considered in conjunction with Eq. (2). These phenomenological models account for the influence of the surface soil deposits on the frequency content of the propagating seismic waves via the "stiffness" (ω_g) and "damping" (ζ_g) parameters. The C-P spectrum incorporates an additional high-pass filter whose cut-off frequency and "steepness" are determined by the parameters ω_f and ζ_f. This filter suppresses the low frequencies allowed by the K-T spectrum: a quite desirable property to realistically capture the frequency content exhibited by field recorded strong ground motions (e.g. Giaralis and Spanos, 2009a). Further comments on the importance of selecting appropriately the spectral form of $Y(\omega)$ appearing in Eq. (2) for the purposes of this study are included in section 3.2 in light of pertinent numerical results.

2.2 *Formulation and solution of the inverse stochastic dynamics problem*

It is possible to relate an EPS $G(t,\omega)$ to a given elastic relative displacement (deformation) response/design spectrum $S_d(\omega_n, \zeta)$ via the concept of the "peak factor" r (e.g. Vanmarcke, 1976) by relying on the equation (e.g. Spanos and Vargas Loli, 1985; Giaralis and Spanos, 2009b)

$$S_d(\omega_n,\zeta)=r(\omega_n,\zeta,G,p)\times \max_t\left\{\sigma(t,\omega_n,\zeta,G)\right\}. \quad (8)$$

In the above equation, σ denotes the standard deviation of the time-evolving amplitude of the response process $x(t)$ of a linear quiescent single-degree-of-freedom (SDOF) oscillator base-excited by the stochastic process $u_g(t)$ (see also Spanos and Lutes, 1980). The governing equation of motion of the above system reads as

$$\begin{aligned}\ddot{x}(t)+2\zeta\omega_n\dot{x}(t)+\omega_n^2x(t)&=-u_g(t)\\ x(0)=\dot{x}(0)&=0\end{aligned}, \quad (9)$$

where ζ and ω_n are the oscillator's ratio of critical viscous damping and natural frequency, respectively, while a dot over a symbol denotes time differentiation. Furthermore, the peak factor r is the scalar by which one needs to multiply the peak standard deviation of the response amplitude attained at some time instant t^* (assumed to be equal to the peak standard deviation of the response process $x(t)$) to reach a certain response level S_d with probability p.

An approximate point-wise solution of Eq. (8) can be obtained by minimizing the error (Spanos and Vargas Loli, 1985; Giaralis and Spanos, 2009a)

$$e=\sum_{j=1}^{2N}\left(S_j-q_j\right)^2, \quad (10)$$

in which

$$S_j = \begin{cases} S_d^2\left(\omega_j,\zeta\right), j=1,...,N \\ 0 \quad , j=N+1,...,2N \end{cases}, \quad (11)$$

$$q_j = \begin{cases} \dfrac{r^2\pi C^2 t_j^{*2} e^{\left(-bt_j^*\right)} Y\left(\omega_j\right)}{2\zeta\omega_j^3}, j=1,...,N \\ \gamma_{j-N}^2\left(2t_{j-N}^* - bt_{j-N}^{*2}\right) - \\ 2\gamma_{j-N}\left(1-bt_{j-N}^*\right) - 2b + \\ 4\zeta\omega_{j-N} e^{\left(-\gamma_{j-N}t_{j-N}^*\right)}, j=N+1,...,2N \end{cases}, \quad (12)$$

and

$$\gamma_j = 2\zeta\omega_j - b, \quad (13)$$

at a certain set of N natural frequencies $\{\omega_j\}$; j= 1,…,N. In Eq. (12) the symbol t_j* denotes the time instant at which the variance σ corresponding to the linear SDOF oscillator with natural frequency ω_j is maximized. In all of the ensuing numerical results, a Levenberg-Marquardt algorithm with line search (see e.g. Nocedal and Wright 1999) is used to solve the over-determined nonlinear least-square fit optimization problem of Eq. (10). The unknowns to be determined are the t_j* time instants and the parameters involved in the definition of the EPS form as detailed in the previous section. In this context, it is pointed out that the parameter b can be treated as an unknown "free" parameter to be determined by the optimization algorithm (e.g. Giaralis and Spanos, 2009; Spanos et al., 2009). Alternatively, it can be held fixed at a predefined value corresponding to a specific effective duration. In this way, the optimization algorithm is "forced" to yield an EPS corresponding to a non-stationary process of specific duration. Additional comments along with numerical results on this issue are included in the following section.

Upon determination of the parameters defining the EPS, one can employ any random field simulation technique to generate samples of the underlying non-stationary process compatible with the EPS (e.g. Spanos and Zeldin, 1998). Further, these samples need to be base-line corrected to yield physically realizable (artificial) accelerograms (e.g. Giaralis and Spanos 2009). To this end a reasonable "compatibility criterion" between the pursued EPS and the target design spectrum S_d is to require the average response spectrum of a sufficiently large ensemble of EPS compatible accelerograms to lie close to the S_d. This criterion implies adopting peak factors r for p= 0.5 in Eq. (8). In the subsequent numerical applications peak factors dependent on the natural frequency ω_n and on the input EPS G, for ζ=5% and for p= 0.5 are utilized. These are derived from Monte Carlo simulations as has been proposed in Giaralis and Spanos (2009b) and in Spanos et al. (2009) to by-pass the fact that no reliable analytical expression for r exists.

3 Numerical Applications

3.1 *Derivation of design spectrum compatible EPS of specific duration*

In this section the applicability of the herein adopted formulation to define design spectrum compatible non-stationary processes of a prescribed effective duration (T_{eff}) as defined by Trifunac and Brady (1975) is assessed. This is accomplished by utilizing the one-to-one relation established by Eqs. (4) and (5) between the b parameter appearing in Eq. (3) and T_{eff}. The design spectrum prescribed in the current aseismic code provisions (GB 50011 2001) effective in China for 5% ratio of critical damping is used as a paradigm of a target design spectrum (Eq. (A.1) of the Appendix). The K-T spectral form of Eq. (6) is assumed in solving the inverse stochastic dynamics problem as detailed in section 2.2 (see also Spanos et al. 2009).

For the purposes of this study, Monte Carlo analyses to estimate median peak factor spectra for different effective durations are performed following Giaralis and Spanos (2009b). This is a necessary step to achieve enhanced agreement between the target design spectrum and the average response spectrum of populations of EPS compatible accelerograms. To this aim, K-T evolutionary power spectra (EPSs) compatible with the GB 50011, for three different values of the *b* parameter, and for all the 14 values of the characteristic period T_g prescribed by GB 50011 (see Eq. (A.1) of the Appendix) are considered. These spectra have been derived by assuming a constant value for the peak factor and by treating the *b* parameter as a constant in solving the minimization problem of Eq. (10). The *b* parameter has been taken equal to $0.30s^{-1}$, $0.40s^{-1}$, and $0.50s^{-1}$ corresponding to effective durations of approximately 18s, 14s, and 11s, respectively. For each of the obtained K-T EPSs a suite of 10000 spectrum-compatible non-stationary artificial accelerograms is generated using the auto/cross-correlation (ACM), auto-regressive-moving-average (ARMA) filtering technique for random field simulation (see e.g. Spanos and Zeldin 1998, Giaralis and Spanos 2009a). Further, these accelerograms are base-line adjusted by performing zero-phase acausal high-pass filtering using a fourth-order Butterworth filter with cut-off frequency 0.10Hz (see also Boore 2005, Giaralis and Spanos 2009a). Next, each suite of the thus generated and processed accelerograms is input to a series of 200 linear SDOF oscillators with natural periods $T_n = 2\pi/\omega_n$ ranging from 0.02sec to 6sec. The damping ratio of these oscillators is taken equal to 5%. For each oscillator considered the response ensembles ($x^{(k)}(t)$; k=1,2,…,10000) corresponding to the aforementioned suites of accelerograms are calculated via numerical integration of Eq. (9). Finally, populations of peak factors ($r^{(k)}$; k=1,2,…,10000) are computed from the above ensembles as the ratio of the population of peak responses over the maximum standard deviation of the response ensemble (Giaralis and Spanos 2009b). That is,

$$r\left(T_n,\zeta_n,G\right)=\frac{\max\limits_t\left\{\left|x\left(t,T_n,\zeta_n,G\right)\right|\right\}}{\max\limits_t\left\{\sqrt{E\left\{x^2\left(t,T_n,\zeta_n,G\right)\right\}}\right\}},\quad(14)$$

in which $E\{\cdot\}$ is the mathematical expectation operator. Note that the obtained peak factor populations are independent of the peak ground acceleration and the peak factor assumed in the derivation of the considered K-T EPSs.

In Figure 1 the median of the peak factor populations calculated by Eq. (14) for each suite of input accelerograms are plotted versus the natural period T_n of the considered oscillators (median peak factor spectra). It is observed that the median peak factor spectra corresponding to a certain value of the *b* parameter (i.e. to a certain effective duration) are in a close agreement. This suggests that the effective duration of the input non-stationary processes has a non-negligible influence on these spectra. Nevertheless, it is clear that the variation of the spectral content of the considered processes reflecting the different shapes of the GB 50011 design spectrum dependent on the T_g (see Eq. (A.1) of the Appendix), has a minor effect on the median peak factor spectrum. Interestingly, similar trends regarding the influence of the critical damping ratio on median peak factor spectra have been also observed by Giaralis and Spanos (2009b). In this regard, it is reasonable to consider the average of these median spectra for each value of the *b* parameter considered. Moreover, by fitting a polynomial curve to the above averaged median peak factor spectra an analytical expression to approximate the numerically derived median peak factors is reached. The eigth-order polynomials plotted in Figure 1 and expressed by the equation

$$r(T)=\sum_{j=0}^{7} p_j T^j \text{ , } 0.02s \le T \le 6\text{s} \quad (15)$$

approximate reasonably well the averaged median peak factor spectra. The coefficients p_j of these polynomials are reported in Table 1.

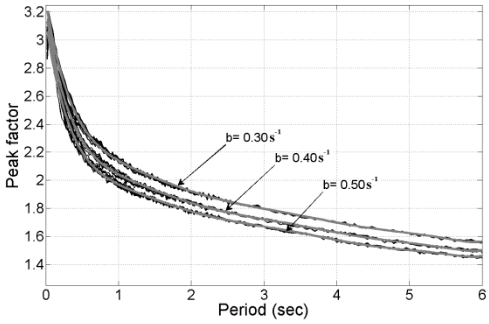

Figure 1. Median peak factor spectra compatible with the GB 50011 design spectrum for all values of T_g (thin black lines) and fitted average peak factor spectra given by Eq. (15) (thick gray lines), for various values of the *b* parameter.

Table 1. Polynomial coefficients of the fitted average peak factor spectra of Eq. (15).

Coefficient	b=0.30s^{-1} ($T_{eff}\approx$ 18s)	b= 0.40s^{-1} ($T_{eff}\approx$14s)	b=0.50s^{-1} ($T_{eff}\approx$11s)
p_0	3.2452	3.1711	3.1012
p_1	-2.8625	-3.0196	-3.1086
p_2	3.1451	3.4074	3.5941
p_3	-1.9687	-2.1573	-2.3085
p_4	0.7003	0.7724	0.8361
p_5	-0.1404	-0.1556	-0.1702
p_6	0.0148	0.0164	0.0181
p_7	-0.0006	-0.0007	-0.0008

Table 2 includes K-T EPSs compatible with the GB 50011 spectrum for α_{max}= 1.20g and T_g=0.70s corresponding to two different effective durations. These spectra have been derived by solving the optimization problem of Eq. (10) treating the *b* parameter as a constant and using the fitted average median peak factor spectra shown in Figure 1 for *b*= 0.30s^{-1} and *b*= 0.50s^{-1}, as appropriate. As expected, the value of the *C* parameter related to the amplitude of the adopted envelop function (Eq. (3)) is significantly larger for the EPS corresponding to the *b*= 0.50s^{-1} due to the reduced duration of the underlying non-stationary process compared to the *b*=0.30s^{-1} case. However, the ζ_g and ω_g parameters associated with the spectral content of the two non-stationary processes considered do not change significantly. In Figure 2 average pseudo-acceleration response spectra for ensembles of 500 baseline-corrected artificial accelerograms compatible with the EPSs of Table 2 are plotted along with the GB 50011 target design spectrum. These average response spectra lie close to each other indicating that the two non-stationary processes defined by means of the EPSs of Table 2 of different durations are consistent in terms of peak response accelerations. To further illustrate this point, arbitrarily chosen individual realizations compatible with the aforementioned EPSs are shown in Figure 3. The fact that they belong to non-stationary processes of different durations is evident from these time-traces. The pseudo-acceleration response spectra of these accelerograms are included in Figure 2.

Table 2. K-T evolutionary power spectra compatible with the GB 50011-2001 design spectrum (T_g=0.70s; α_{max}= 1.20g) for different predefined effective durations.

EPS Parameter (units)	(b=0.30s^{-1}) $T_{eff}\approx$ 18s	(b=0.50s^{-1}) $T_{eff}\approx$ 11s
C(cm/s$^{2.5}$)	11.33	21.11
b(1/s)	0.30	0.50
ζ_g	0.83	0.88
ω_g(rad/s)	7.89	7.57

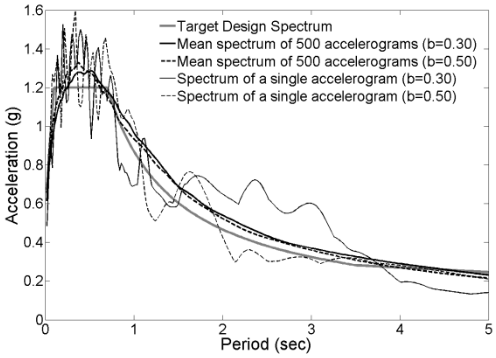

Figure 2. Artificial accelerograms of different effective durations compatible with the K-T evolutionary power spectra of Table 2.

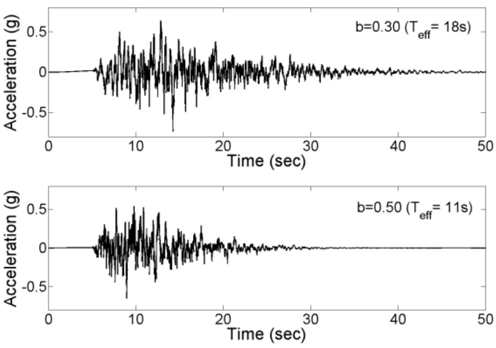

Figure 3. Accelerograms of different effective durations compatible with the K-T evolutionary power spectra of Table 2.

It is further noted that both the average response spectrum curves included in Figure 2 are in a close agreement with the target design spectrum. Similar results, not included here for brevity, have been obtained for other GB 50011 shapes and values of effective duration. In this regard, it can be argued that the EPSs derived by solving the minimization problem of Eq. (10) in conjunction with appropriately derived peak factor spectra can be used for structural aseismic design scenarios mandating the consideration of strong ground motions of specific duration in the context of Monte Carlo and/or random vibration analyses (e.g. Roberts and Spanos 2003).

3.2 *Selection of the spectral form of the assumed evolutionary power spectrum*

The previous section discussed the proper selection of the *b* parameter involved in the definition of the envelop function *A(t)* in Eq. (2) to obtain design spectrum compatible processes of specific duration. In this section the issue of choosing appropriately the parametric form of the stationary power spectrum *Y(ω)* (Eq. (2)) which controls the frequency content of the sought processes is addressed. This choice depends on the behaviour of the target design spectra in the range of long periods (Giaralis and Spanos 2009a, Spanos et al. 2009). To further clarify this point, the design spectra prescribed by the Chinese GB 50011 code (Eq. (A.1) of the Appendix), and the European EC8 code (Eq. (A.2) of the Appendix) for 5% ratio of critical damping, are considered. Specifically, the GB 50011 spectrum for α_{max}= 1.20g and T_g= 0.40g, and the EC8 spectrum for α_g= 0.25g and soil conditions B are taken as representative examples. These two target spectra are plotted in Figure 4 and 5 in terms of pseudo-acceleration and relative displacement, respectively. It is seen that in the range of long periods (in the particular case herein considered for T>2s), the GB 50011 relative displacement spectrum increases monotonically. This is due to the fact that the GB 50011 poses over-conservative (high) demands in terms of structural strength for flexible systems by prescribing a slow (linear) rate of decay to the last segment of the pseudo-acceleration design spectrum. This provision is associated with the fact that flexible structures require significantly more elaborate detailing during construction to reach the same level of structural performance when exposed to the seismic hazard as compared with stiffer structures in a performance-based design framework. The aforementioned attributes of the analytical expression of the GB 50011 spectrum can only be accommodated by adopting a spectral form *Y(ω)* rich in low frequencies in deriving compatible EPSs as described in section 2.2. The K-T spectrum of Eq. (6) defines such a spectral form and has been successfully used in the previous section and in Spanos et al. (2009) as a mathematical instrument to accommodate the GB 50011 spectrum within the context of the herein adopted formulation. It is noted, in passing, that the low-frequency content allowed by the K-T spectrum is regarded as "spurious" as it does not reflect what is observed in field recorded accelerograms. However, it appears that this content is crucial in rendering the solution of the optimization problem of Eq. (10) numerically feasible in dealing with the GB 50011 design spectrum (Spanos et al. 2009). Clearly, it is not a coincidence that the GB 50011 design spectrum does not comply with the theory of structural dynamics suggesting that the maximum deformation of very flexible seismically-excited SDOF oscillators is equal to the peak ground displacement.

Nevertheless, the design spectrum of the EC8 is characterized by a behavior in the range of long periods which seems to capture better the physics of the underlying structural dynamics problem (e.g. Faccioli et al. 2004). The EC8 pseudo-acceleration spectrum drops at an exponential rate for T>2s in such a manner so that the corresponding relative displacement spectrum attains a constant value for very flexible oscillators. This allows for utilizing phenomenological models to represent more realistically the low-frequency content of the strong ground motion than the K-T spectrum, such as the C-P spectrum given by Eq. (7) (Giaralis and Spanos 2009a).

In this regard pertinent numerical results are further included to support the aforementioned comments and to illustrate the applicability of the adopted methodology to derive EPSs compatible with different target spectra. In particular, presents a K-T EPS and a C-P EPS compatible with the GB 50011 and the EC8 spectra of Figures 4 and 5, respectively. These EPSs have been derived by solving the optimization problem of Eq. (10). The quality of the point-wise matching achieved is depicted via the dots included in Figures 4 and 5. In obtaining these EPSs the parameter b has been treated as a "free" unknown parameter and the median peak factor spectra provided by Spanos et al. (2009) for the GB 50011 and by Giaralis and Spanos (2009b) for the EC8 have been utilized. These peak factor spectra are shown in Figure 6; they were derived via Monte Carlo simulations similar to those discussed in the previous section.

Table 3. Evolutionary power spectra compatible with the target spectra of Figure 5.

Parameter (units)	C-P (EC8) (soil B;α_g=0.25g)	K-T (GB 50011) (T_g=0.40g;α_{max}=1.20g)
C(cm/s$^{2.5}$)	10.16	17.37
b(1/s)	0.54	0.50
ζ_g	0.65	0.72
ω_g(rad/s)	12.76	15.67
ζ_f	0.85	-
ω_f(rad/s)	2.15	-

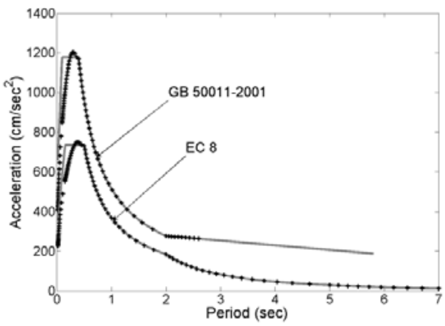

Figure 4. Target pseudo-acceleration design spectra and point-wise least square matching for the EPSs spectral forms considered in Table 3.

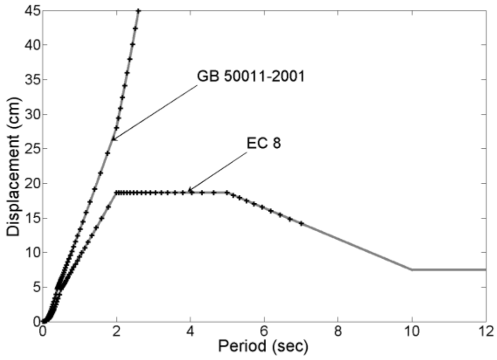

Figure 5. Target relative displacement design spectra and point-wise least square matching for the spectral forms considered in Table 3.

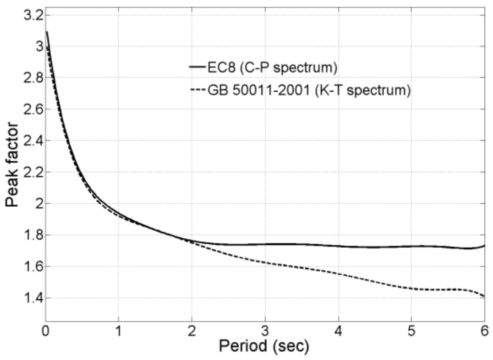

Figure 6. Median peak factor spectra for ζ= 5% pertaining to the GB 50011-2001 (Spanos et al. 2009) and to the EC8 (Giaralis and Spanos 2009b).

As it has been previously reported in Spanos et al. (2009) if point-wise matching is pursued to include values of ω_j appearing in Eqs. (11)~(13) corresponding to periods beyond $6.5T_g$ in the GB 50011 case the optimization algorithm fails to converge to an acceptable solution. This is due to the aforementioned behavior of the GB 50011 relative displacement spectra which does not converge to a constant value. However, in the EC8 case, the assumed C-P spectral form is able to trace the target spectrum to much higher natural periods. Notably, these differences in the frequency content of the assumed spectral forms necessitated by the need to accommodate the different design spectra influence the shape of the corresponding median peak factor spectra, as well. This is seen in Figure 6 where the median peak factor spectra corresponding to C-P and K-T spectral forms coincide for periods up to 2s. For longer periods the EC8 compatible peak factors attain a constant value, while the GB 50011 peak factor spectrum is monotonically decreasing.

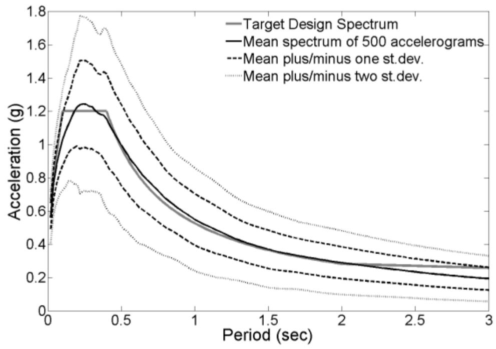

Figure 7. Statistics of pseudo-acceleration response spectra of an ensemble of 500 simulated accelerograms compatible with the GB 50011-2001 design spectrum (T_g=0.40sec; α_{max}= 1.20g).

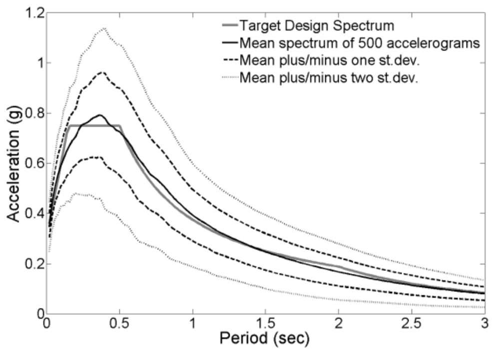

Figure 8. Statistics of pseudo-acceleration response spectra of an ensemble of 500 simulated accelerograms compatible with the EC8 design spectrum (soil B; α_g= 0.25g).

In Figures 7 and 8 average response spectra of 500 accelerograms compatible with the K-T and the C-P EPSs of Table 3, respectively, are compared with the target spectra. In both cases, the use of frequency-dependent median peak factor spectra in deriving the design spectrum compatible EPSs have achieved enhanced agreement between the average response spectrum and the target spectrum for T<2s. For periods longer than 2s, the average response spectra in the K-T case lies below the target spectrum, which is not the case for the C-P spectrum. This result confirms numerically that the GB 50011 code poses unrealistically high demands on flexible structures.

4 Concluding remarks

The issue of deriving evolutionary power spectra (EPSs) of the separable kind defined parametrically by a time-dependent envelop function and a stationary power spectrum compatible with a given response/design (target) spectrum has been addressed. These EPSs characterize uniformly modulated non-stationary processes whose samples are treated as accelerograms representing strong ground motions consistent with the target spectrum. A formulation of an inverse stochastic dynamics problem to relate such an EPS to the target spectrum on a probabilistic basis has been adopted involving the consideration of a "peak factor" (e.g. Giaralis and Spanos 2009a). This inverse problem is solved in an approximate point-wise least-squared sense by considering frequency-dependent peak factors (peak factor spectra). These spectra have been obtained by pertinent comprehensive Monte Carlo-based analyses to satisfy an appropriate compatibility criterion between the EPS and the target spectrum (e.g. Giaralis and Spanos 2009b). That is, the average response spectrum of a sufficiently large population of EPS compatible accelerograms is in a close agreement with the target spectrum.

Special attention has been directed to elucidate certain issues of practical importance requiring the appropriate selection of the parametric form of the EPS involved in the herein adopted formulation. Specifically, numerical results pertaining to the design spectrum of the Chinese GB 50011 aseismic code have been furnished to point out the fact that the adopted formulation is capable of deriving design spectrum compatible EPSs characterized by a prescribed "effective duration". This has been accomplished by assigning appropriate values to the parameter controlling the width of the envelop function used in the definition of the EPSs. Further, Monte Carlo simulations have been carried out to numerically derive polynomial expressions of median peak factor spectra for various effective durations consistent with the GB 50011 design spectrum. These spectra have been incorporated in solving the considered inverse problem to yield non-stationary processes of different effective durations of enhanced compatibility with the GB 50011 spectrum. It is noted that the effective duration is regarded as a critical parameter in the field of earthquake engineering influencing the destructive potential of the strong ground motion (see e.g. Iervolino et al. 2005). Therefore, the thus derived EPSs can significantly facilitate Monte Carlo-based or random vibration-based (e.g. Roberts and Spanos 2003) analyses in structural design scenarios where accounting for the effective duration is deemed essential.

Furthermore, it has been argued that caution must be exercised in choosing an appropriate parametric spectral form assumed by the considered EPS. Particularly, this choice depends on the behavior of the target design spectrum in the range of long periods. This point has been confirmed by comparing numerical results pertaining to the design spectra of the Chinese GB 50011 code and of the European EC8 code. It has been pointed out that the Kanai-Tajimi spectral form is required to accommodate the monotonically increasing relative displacement spectral ordinates of the GB 50011 spectrum. Further, capturing the convergent-to-a-constant-value trend of the relative displacement EC8 spectrum necessitates the incorporation of a high-pass filter to suppress the low frequencies of the EPS spectral form. A reasonable candidate for this purpose is the Clough-Penzien spectrum. Finally, the influence of the spectral content of the assumed input EPS to the shape of numerically derived median peak factor spectra has also been clarified.

Appendix A. Design Spectra of the Chinese GB 50011 and the European EC8 codes

The elastic relative displacement design spectrum for oscillators with ζ= 5% and natural period T, is defined in the current aseismic code provisions effective in China (GB 50011, 2001) by the expression

$$S_d = \left(\frac{T}{2\pi}\right)^2 \times a_{\max} \times \begin{cases} (0.45+5.5T) & , \quad 0 \le T \le 0.1 \\ 1 & , \quad 0.1 \le T \le T_g \\ \left(\dfrac{T_g}{T}\right)^{0.9} & , \quad T_g \le T \le 5T_g \\ \left(0.2^{0.9} - \dfrac{T-5T_g}{50}\right) & , 5T_g \le T \le 6 \end{cases} . \qquad \text{(A.1)}$$

In the above equation α_{max} denotes the maximum spectral ordinate in terms of the pseudo-acceleration, and T_g is the "characteristic period" which differentiates the shape of the design spectrum to account for various soil conditions and intensity levels as defined by the GB 50011. T_g can take on 14 different values ranging from 0.25s to 0.95s (see also Spanos et al. 2009).

The elastic relative displacement design spectrum for oscillators with ζ= 5% and natural period T, is defined by the current European aseismic code (EC8) by the expression (CEN, 2004)

$$S_d = \left(\frac{1}{2\pi}\right)^2 \times a_g S \times \begin{cases} T^2\left[1+\dfrac{1.5T}{T_B}\right] & ,0 \le T \le T_B \\ 2.5T^2 & ,T_B \le T \le T_C \\ 2.5T_C T & ,T_C \le T \le T_D \\ 2.5T_C T_D & ,T_D \le T \le T_E \\ T_C T_D\left[2.5 - \dfrac{1.5(T-T_E)}{T_F-T_E}\right] & ,T_E \le T \le T_F \\ T_C T_D & ,T \ge T_F \end{cases} . \qquad \text{(A.2)}$$

In the last equation α_g is the peak ground acceleration, S is an amplification factor dependent on the soil conditions, and T_B, T_C, T_D, $T_{E,}$ and T_F are the corner periods defining the various branches of the design spectrum also dependent on the soil conditions. The EC8 prescribes five different soil conditions to capture the influence of the surface soil layers.

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