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# Supersymmetric Hidden Sectors for Heterotic Standard Models

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## Abstract

Within the context of the weakly coupled $E_8 \times E_8$ heterotic string, we study the hidden sector of heterotic standard model compactifications to four-dimensions. Specifically, we present a class of hidden sector vector bundles—composed of the direct sum of line bundles only—that, together with an effective bulk five-brane, renders the heterotic standard model entirely $N = 1$ supersymmetric. Two explicit hidden sectors are constructed and analyzed in this context; one with the gauge group $E_7 \times U(1)$ arising from a single line bundle and a second with an $SO(12) \times U(1) \times U(1)$ gauge group constructed from the direct sum of two line bundles. Each hidden sector bundle is shown to satisfy all requisite physical constraints within a finite region of the Kähler cone. We also clarify that the first Chern class of the line bundles need not be even in our context, as has often been imposed in the model building literature.

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# Contents

# 1   Introduction

The ten-dimensional theory of the massless modes of weakly coupled $E_8 \times E_8$ heterotic theory can arise in two ways. The first is directly from the $E_8 \times E_8$ heterotic superstring after decoupling the heavy string modes [1,2]. The second follows from compactifying eleven-dimensional $M$-theory on an $S^1/\mathbb{Z}_2$ orbifold in the limit of small radius [3–9]. Either way, the ten-dimensional effective action is a $N = 1$ supersymmetric theory with a metric, dilaton, two-form and $E_8 \times E_8$ gauge fields as the bosonic components. In addition, it can contain topological five-branes. This can be reduced to an $N = 1$ supersymmetric theory in four dimensions by appropriately compactifying on a

smooth Calabi-Yau threefold supporting gauge fields satisfying the hermitian Yang-Mills equations [10,11] with the five-branes wrapped on holomorphic curves [12]. The choice of the Calabi-Yau manifold and the gauge field background, that is, a slope-stable holomorphic vector bundle with vanishing slope, determines the low energy gauge group, spectrum, and coupling parameters [13–17].

In [18], it was shown that the low energy spectrum of a specific Calabi-Yau three-fold and holomorphic vector bundle with structure group $SU(4) \subset E_8$ is exactly that of the minimal supersymmetric standard model (MSSM) with three right-handed neutrinos and one pair of Higgs-Higgs conjugate chiral multiplets. There are no exotic or vector-like pairs of superfields. It was demonstrated in [19] that over a specific subspace of the Kähler cone, this vector bundle is slope-stable with vanishing slope and, hence, the low energy theory is $N = 1$ supersymmetric. For these reasons, this vacuum of the *observable* $E_8$ sector of the theory was called the heterotic standard model. To complete the vacuum, it is essential to present the explicit vector bundle for the second $E_8$ *hidden* sector. In previous work [20–22], it was expedient to choose this bundle to be trivial, requiring a five-brane sector with non-effective cohomology class to satisfy the anomaly constraint. This *anti-brane* allows one to raise the potential energy of the vacuum from negative to a small, positive cosmological constant [23], that is, the heterotic equivalent of the KKLT mechanism [24]. It was shown in [19] that, for vanishing five-brane class, a hidden sector $SU(n)$ bundle satisfying the Bogomolov bound [25] could exist in a subspace intersecting the region of stability of the observable sector vector bundle. The Bogomolov bound is a necessary condition for the hidden bundle to be $N = 1$ supersymmetric. At least on certain manifolds, satisfying this bound is also sufficient [26–28]. However, an explicit example was never constructed.

In a series of papers, both in weakly coupled [29–33] and strongly coupled [7,8,34] $E_8 \times E_8$ heterotic theory, the first non-trivial corrections—string one-loop and $M$-theory order $\kappa^{4/3}$ respectively—beyond tree level were constructed. These were presented for both the Fayet-Iliopoulos (FI) terms associated with anomalous $U(1)$ gauge factors and for the gauge threshold corrections. Furthermore, it was emphasized in [29–33] that gauge bundles with $U(n)$ structure groups, in addition to $SU(n)$ bundles, should be more closely scrutinized. Similarly, the appearance of line bundles on the stability wall boundaries of $SU(n)$ bundles was recognized in [34,35], and applied more widely to construct new realistic models in [36–38]. In this paper, within the context of the weakly coupled $E_8 \times E_8$ heterotic string, we will apply these ideas to construct a completely $N = 1$ supersymmetric hidden sector for the heterotic standard model. This hidden sector will have a non-trivial, but effective, five-brane class and a hidden sector vector bundle composed of a direct sum of line bundles. Together with the observable sector $SU(4)$ vector bundle, this provides an explicit $N = 1$ compactification vacuum for the heterotic standard model.

Specifically, we will do the following. In Section 2, we review the Calabi-Yau three-

fold and observable sector gauge bundle of the heterotic standard model introduced in [18, 19]. The generic form of the hidden sector bundle as a sum of line bundles is presented, as is an analysis of the explicit embedding of a line bundle into an $E_8$ vector bundle. Five-branes are also briefly discussed. Section 3 consists of an analysis of the four constraint conditions required for our compactification vacuum; namely, anomaly cancellation, slope-stability of the observable sector bundle, $N = 1$ supersymmetry of the hidden sector bundle and positivity of the threshold corrected gauge parameters. The explicit constraint equations for each of these conditions are given. Two specific examples are then presented in Section 4. The first is obtained using a single line bundle as the hidden sector bundle. Its exact embedding into the hidden $E_8$ is discussed, and it is shown to satisfy all required constraint conditions in a region of the Kähler cone. The associated low energy gauge group and spectrum of the hidden sector is presented. As a second example, we choose the hidden sector bundle to be a sum of two line bundles. Again, we elucidate its exact embedding into the hidden $E_8$, show that it satisfies all necessary constraints in a region of the Kähler cone and discuss the associated low energy gauge group. Finally, important technical issues regarding the embedding of line bundles into an $E_8$ vector bundle are discussed in Appendix A.

The analysis in this paper has a direct, but non-trivial, extension to strongly coupled heterotic $M$-theory [3,4,8,9,39–44]. This will be presented in a future publication.

# 2   The Compactification Vacuum

A four-dimensional, $N = 1$ supersymmetric effective theory of the weakly coupled $E_8 \times E_8$ heterotic string is obtained as follows. First, one compactifies ten-dimensional spacetime on an appropriate Calabi-Yau threefold, $X$. Second, it is necessary to construct two $E_8$ gauge bundles over the Calabi-Yau manifold. Two popular methods are to utilize a slope-stable holomorphic vector bundle with vanishing first Chern class or a line bundle. The latter is automatically slope-stable, but imposes the additional physical constraint that the FI-term must vanish. The associated gauge symmetries, spectrum and coupling parameters of the four-dimensional theory depend on the specific choice of the compactification. In this paper, we will examine a physically relevant subset of such vacua; namely, those with the Calabi-Yau threefold and observable sector vector bundle of the heterotic standard model [18, 19].

## 2.1   The Calabi-Yau Threefold

The Calabi-Yau manifold $X$ is chosen to be a torus-fibered threefold with fundamental group $\pi_1(X) = \mathbb{Z}_3 \times \mathbb{Z}_3$. Specifically, it is a fiber product of two rational elliptic $d\mathbb{P}_9$ surfaces, that is, a self-mirror Schoen threefold [43, 45–49] quotiented with respect to a freely acting $\mathbb{Z}_3 \times \mathbb{Z}_3$ isometry. Its Hodge data is $h^{1,1} = h^{1,2} = 3$ and, hence, there are three Kähler and three complex structure moduli. The complex structure moduli

will play no role in the present paper and we will ignore them. Relevant here is the degree-two Dolbeault cohomology group

$$H^{1,1}(X,\mathbb{C}) = \mathrm{span}_{\mathbb{C}}\{\omega_1,\ \omega_2,\ \omega_3\}, \tag{1}$$

where $\omega_i = \omega_{i a\bar{b}} dz^a d\bar{z}^{\bar{b}}$ are dimensionless harmonic $(1,1)$-forms on $X$ with the property

$$\omega_3 \wedge \omega_3 = 0, \quad \omega_1 \wedge \omega_3 = 3\omega_1 \wedge \omega_1, \quad \omega_2 \wedge \omega_3 = 3\omega_2 \wedge \omega_2. \tag{2}$$

Defining the intersection numbers as

$$d_{ijk} = \frac{1}{v}\int_X \omega_i \wedge \omega_j \wedge \omega_k, \quad i,j,k = 1,2,3 \tag{3}$$

where $v$ is a reference volume of dimension $(length)^6$, it follows from (2) that

$$d_{ijk} = \begin{pmatrix} \left(0,\frac{1}{3},0\right) & \left(\frac{1}{3},\frac{1}{3},1\right) & (0,1,0) \\ \left(\frac{1}{3},\frac{1}{3},1\right) & \left(\frac{1}{3},0,0\right) & (1,0,0) \\ (0,1,0) & (1,0,0) & (0,0,0) \end{pmatrix}. \tag{4}$$

The $\{ij\}$-th entry in the matrix corresponds to the triplet $(d_{\{ij\}k}|k=1,2,3)$.

Our analysis will require the Chern classes of the tangent bundle $TX$. Noting that the associated structure group is $SU(3) \subset SO(6)$, it follows that $\mathrm{rank}(TX) = 3$ and $c_1(TX) = 0$. Furthermore, the self-mirror property of this specific threefold implies $c_3(TX) = 0$. Finally, we find that

$$c_2(TX) = \frac{1}{v^{2/3}}\left(12\omega_1 \wedge \omega_1 + 12\omega_2 \wedge \omega_2\right). \tag{5}$$

We will use the fact that if one chooses the generators of $SU(3)$ to be hermitian, then the second Chern class of the tangent bundle can be written as

$$c_2(TX) = -\frac{1}{16\pi^2} \mathrm{tr}_{SO(6)} R \wedge R, \tag{6}$$

where $R$ is the Lie algebra valued curvature two-form.

Note that $H^{2,0} = H^{0,2} = 0$ on a Calabi-Yau threefold. It follows that $H^{1,1}(X,\mathbb{C}) = H^2(X,\mathbb{R})$ and, hence, $\omega_i$, $i = 1,2,3$ span the real vector space $H^2(X,\mathbb{R})$. Furthermore, it was shown in [50] that the curve Poincaré dual to each two-form $\omega_i$ is effective. Therefore, the Kähler cone is the positive octant

$$\mathcal{K} = H^2_+(X,\mathbb{R}) \subset H^2(X,\mathbb{R}). \tag{7}$$

The Kähler form, defined to be $\omega_{a\bar{b}} = ig_{a\bar{b}}$ where $g_{a\bar{b}}$ is the Calabi-Yau metric, can be any element of $\mathcal{K}$. That is, the Kähler form can be expanded as

$$\omega = a^i \omega_i, \quad a^i > 0, \quad i = 1,2,3. \tag{8}$$

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The real, positive coefficients $a^i$ are the three $(1,1)$ Kähler moduli of the Calabi-Yau threefold. Here, and through this paper, upper and lower $H^{1,1}$ indices are summed unless otherwise stated. The dimensionless volume modulus is defined by

$$V = \frac{1}{v} \int_X \sqrt{^6 g} \tag{9}$$

and, hence, the dimensionful Calabi-Yau volume is $vV$. Using the definition of the Kähler form and (3), $V$ can be written as

$$V = \frac{1}{6v} \int_X \omega \wedge \omega \wedge \omega = \frac{1}{6} d_{ijk} a^i a^j a^k. \tag{10}$$

It is useful to express the three $(1,1)$ moduli in terms of $V$ and two additional independent moduli. This can be accomplished by defining the scaled shape moduli

$$b^i = V^{-1/3} a^i, \quad i = 1, 2, 3. \tag{11}$$

It follows from (10) that they satisfy the constraint

$$d_{ijk} b^i b^j b^k = 6 \tag{12}$$

and, hence, represent only two degrees of freedom. Finally, note that all moduli defined thus far, that is, $a^i$, $V$ and $b^i$, are functions of the four coordinates $x^\mu$, $\mu = 0, \ldots, 3$ of Minkowski space $M_4$.

## 2.2 The Observable Sector Gauge Bundle

The $E_8 \times E_8$ vector bundle $V$ over $X$ is a direct sum of an *observable* sector bundle, $V^{(1)}$, whose structure group is embedded in the first $E_8$ factor, with a *hidden* sector bundle, $V^{(2)}$, with structure group in the second $E_8$. $V^{(1)}$ is chosen to be holomorphic with structure group $SU(4) \subset E_8$, thus breaking

$$E_8 \longrightarrow Spin(10). \tag{13}$$

Our analysis will require the Chern classes of $V^{(1)}$. Since the structure group is $SU(4)$, it follows immediately that $\operatorname{rank}(V^{(1)}) = 4$ and $c_1(V^{(1)}) = 0$. The heterotic standard model is constructed to have the observed three chiral families of quarks/leptons and, hence, $V^{(1)}$ must be chosen so that $c_3(V^{(1)}) = 3$. Finally, we find that

$$c_2(V^{(1)}) = \frac{1}{v^{2/3}} \Big( \omega_1 \wedge \omega_1 + 4\, \omega_2 \wedge \omega_2 + 4\, \omega_1 \wedge \omega_2 \Big). \tag{14}$$

Here, and below, it will be useful to note the following. Let $\mathcal{V}$ be an arbitrary vector bundle with structure group $\mathcal{G}$, and $\mathcal{F}^{\mathcal{V}}$ the associated Lie algebra valued two-form gauge field strength. If the generators of $\mathcal{G}$ are chosen to be hermitian, then

$$\frac{1}{8\pi^2} \operatorname{tr}_{\mathcal{G}} \mathcal{F}^{\mathcal{V}} \wedge \mathcal{F}^{\mathcal{V}} = \operatorname{ch}_2(\mathcal{V}) = \frac{1}{2} c_1(\mathcal{V}) \wedge c_1(\mathcal{V}) - c_2(\mathcal{V}), \tag{15}$$

where $ch_2(\mathcal{V})$ is the second Chern character of $\mathcal{V}$. Furthermore, we denote by $tr_{\mathcal{G}}$ the trace in the fundamental representation of the structure group $\mathcal{G}$ of the bundle. When applied to the vector bundle $V^{(1)}$ in the observable sector, it follows from $c_1(V^{(1)}) = 0$ that

$$c_2(V^{(1)}) = -\frac{1}{8\pi^2} \, tr_{SU(4)} \, F^{(1)} \wedge F^{(1)} = -\frac{1}{16\pi^2} \, tr_{E_8} \, F^{(1)} \wedge F^{(1)}, \qquad (16)$$

where $F^{(1)}$ is the gauge field strength for the visible sector bundle $V^{(1)}$ and $tr_{E_8}$ indicates the trace is over the fundamental $\underline{\mathbf{248}}$ representation of $E_8$. Note that the conventional normalization of the trace $tr_{E_8}$ includes a factor of $\frac{1}{30}$, the inverse of the dual Coxeter number of $E_8$. We have expressed $c_2(V^{(1)})$ in terms of $tr_{E_8}$ since the fundamental $SU(4)$ representation must be embedded into the adjoint representation of $E_8$ in the observable sector.

To preserve $N = 1$ supersymmmetry in four-dimensions, $V^{(1)}$ must be both slope-stable and have vanishing slope [51,52]. In the context of this paper, these constraints are most easily examined in the $d = 4$ effective theory and, hence, will be discussed below. Finally, when two *flat* Wilson lines are turned on, each generating a different $\mathbb{Z}_3$ factor of the $\mathbb{Z}_3 \times \mathbb{Z}_3$ holonomy of $X$, the observable gauge group is further broken to

$$Spin(10) \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}. \qquad (17)$$

## 2.3    The Hidden Sector Gauge Bundle

In the hidden sector, the vector bundle $V^{(2)}$ introduced in this paper will be constructed entirely as the sum of holomorphic line bundles. Let us briefly review the properties of such bundles on our specific geometry. Line bundles are classified by the divisors of $X$ and, hence, equivalently by the elements of the integral cohomology

$$H^2(X, \mathbb{Z}) = \left\{ a\omega_1 + b\omega_2 + c\omega_3 \mid a, b, c \in \mathbb{Z}, \ a + b = 0 \text{ mod } 3 \right\}. \qquad (18)$$

It is conventional to denote the line bundle associated with the element $a\omega_1 + b\omega_2 + c\omega_3$ of $H^2(X, \mathbb{Z})$ as

$$\mathcal{O}_X(a, b, c). \qquad (19)$$

Note that the $\omega_1$, $\omega_2$, $\omega_3$ are the natural basis of invariant integral forms on the *covering space*. In order to correspond to integral forms on the quotient Calabi-Yau manifold $X$, an element $a\omega_1 + b\omega_2 + c\omega_3$ has to satisfy the additional constraint $a + b = 0 \text{ mod } 3$ in order to be integral. This can also be seen from the intersection numbers (4), which are naively fractional. Only the intersection of classes satisfying $a + b = 0 \text{ mod } 3$ is integral. For the purposes of constructing a heterotic gauge bundle from a line bundle $\mathcal{O}_X(a, b, c)$, this is the only constraint required on the integers $a, b, c$. In particular, as explained in Appendix A, it is not necessary to impose that they be even for there to exist a spin structure on $V^{(2)}$. Although the auxiliary line bundle is not spin if $a$, $b$, $c$ are not even, the $E_8$ bundle is always spin.

We will choose the the hidden sector bundle to be

$$V^{(2)} = \bigoplus_{r=1}^{R} L_r, \qquad L_r = \mathcal{O}_X(\ell_r^1, \ell_r^2, \ell_r^3) \tag{20}$$

where

$$\ell_r^1 + \ell_r^2 = 0 \bmod 3, \quad r = 1, \ldots, R \tag{21}$$

for some positive integer $R$. The structure group is $U(1)^R$, where each $U(1)$ factor has a specific embedding into the hidden sector $E_8$ gauge group. It follows from the definition that $\mathrm{rank}(V^{(2)}) = R$ and that the first Chern class is

$$c_1(V^{(2)}) = \sum_{r=1}^{R} c_1(L_r), \quad c_1(L_r) = \frac{1}{v^{1/3}}\left(\ell_r^1 \omega_1 + \ell_r^2 \omega_2 + \ell_r^3 \omega_3\right). \tag{22}$$

Note that since $V^{(2)}$ is a sum of holomorphic line bundles, $c_2(V^{(2)}) = c_3(V^{(2)}) = 0$. However, the relevant quantity for the hidden sector vacuum is related to the second Chern character given in (15). Defining $F^{(2)}$ to be the gauge field strength for the hidden sector bundle $V^{(2)}$, this becomes

$$\frac{1}{8\pi^2} \mathrm{tr}_{U(1)^R} F^{(2)} \wedge F^{(2)} = \mathrm{ch}_2(V^{(2)}) = \sum_{r=1}^{R} \mathrm{ch}_2(L_r) = \sum_{r=1}^{R} \frac{1}{2} c_1(L_r) \wedge c_1(L_r) \tag{23}$$

since $c_2(L_r) = 0$. As in the observable sector, the $U(1)^R$ fundamental representation must be embedded into the adjoint representation of the hidden sector $E_8$. Hence, the physically relevant quantity is proportional to $\mathrm{tr}_{E_8}$ of $F^{(2)} \wedge F^{(2)}$, where we remind the reader that our normalization of the $E_8$ trace includes the $\frac{1}{30}$ as in (16). Specifically, the term of interest is

$$\frac{1}{16\pi^2} \mathrm{tr}_{E_8} F^{(2)} \wedge F^{(2)} = \sum_{r=1}^{R} \frac{2a_r}{8\pi^2} \mathrm{tr}_{U(1)} F_r^{(2)} \wedge F_r^{(2)} = \sum_{r=1}^{R} a_r \, c_1(L_r)^2 \tag{24}$$

with a group-theoretic factor

$$a_r = \frac{1}{4} \mathrm{tr}_{E_8} Q_r^2, \tag{25}$$

where $Q_r$ is the generator of the $r$-th $U(1)$ factor embedded into the **248** representation of the hidden sector $E_8$. Note that we have used (23) in going from the second to the third term in (24).

The definition of $a_r$ with the coefficient $\frac{1}{4}$ is, of course, a convention. However, it is justified by the following computation which we leave as an exercise for the reader to verify. Using the embedding $U(1) \subset SU(2) \subset E_7$ defined by eqns. (86) and (52) below, the normalized trace is given by $\mathrm{tr}_{E_8} Q^2 = \frac{1}{30} \cdot 60 \cdot 2$. Therefore, the minimal

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$U(1)$ embedding in $E_8$ leads to $a_{\min} = 1$, explaining the conventional normalization factor of $\frac{1}{4}$ in (25). In fact, by comparing the usual formula for the Chern character of a line bundle (23) with the $E_8$ characteristic class (24), one might have guessed that $a_r$ is half-integral. However, this is not true and $a_r$ is always integral. This is also crucially important for the contribution to the heterotic anomaly, which must be an integral cohomology class, to be well-defined. In Appendix A, we will discuss this in more detail.

## 2.4 Wrapped Five-Branes

In addition to the holomorphic vector bundles in the observable and hidden sectors, the compactification can also contain five-branes wrapped on two-cycles $\mathcal{C}_2^{(n)}$, $n = 1, \ldots, M$ in $X$. Cohomologically, each such five-brane is described by the $(2,2)$-form Poincaré dual to $\mathcal{C}_2^{(n)}$, which we denote by $W^{(n)}$. Note that to preserve $N = 1$ supersymmetry in the four-dimensional theory, these curves must be holomorphic and, hence, each $W^{(n)}$ be an effective class.

# 3 The Vacuum Constraint Conditions

In order for the Calabi-Yau threefold $X$, the observable and hidden sector vector bundles $V^{(1)}$, $V^{(2)}$ and the five-branes $W^{(n)}$ discussed above to form a consistent compactification, they must satisfy a set of physical constraints. These are the following.

## 3.1 Anomaly Cancellation

As discussed in [7–9, 51, 52], anomaly cancellation requires that

$$-\frac{1}{16\pi^2} \operatorname{tr}_{SO(6)} R \wedge R + \frac{1}{16\pi^2} \operatorname{tr}_{E_8} F^{(1)} \wedge F^{(1)} + \frac{1}{16\pi^2} \operatorname{tr}_{E_8} F^{(2)} \wedge F^{(2)} - \sum_{m=1}^{M} W^{(m)} = 0. \quad (26)$$

Using (6),(16) and (23),(24) the anomaly cancellation condition can be expressed as

$$c_2(TX) - c_2(V^{(1)}) + \sum_{r=1}^{R} a_r c_1(L_r) \wedge c_1(L_r) - W = 0, \quad (27)$$

where $W = \sum_{m=1}^{M} W^{(m)}$.

Condition (27) is expressed in terms of four-forms in $H^4(X, \mathbb{R})$. We find it easier to analyze its consequences by writing it in the dual homology space $H_2(X, \mathbb{R})$. In this case, the coefficient of the $i$-th vector in the basis dual to $(\omega_1, \omega_2, \omega_3)$ is given by

wedging each term in (27) with $\omega_i$ and integrating over $X$. Using (5),(14) and the intersection numbers (3),(4) gives

$$\frac{1}{v^{1/3}} \int_X \left( c_2(TX) - c_2(V^{(1)}) \right) \wedge \omega_{1,2,3} = \left( \tfrac{4}{3}, \tfrac{7}{3}, -4 \right). \tag{28}$$

Similarly, (3),(4) and (22) imply

$$\frac{1}{v^{1/3}} \int_X c_1(L_r) \wedge c_1(L_r) \wedge \omega_i = d_{ijk} \ell_r^j \ell_r^k, \quad i = 1, 2, 3. \tag{29}$$

Defining

$$W_i = \frac{1}{v^{1/3}} \int_X W \wedge \omega_i, \tag{30}$$

it follows that the anomaly condition (27) can be expressed as

$$W_i = \left( \tfrac{4}{3}, \tfrac{7}{3}, -4 \right)_i + \sum_{r=1}^{R} a_r d_{ijk} \ell_r^j \ell_r^k \geq 0, \quad i = 1, 2, 3. \tag{31}$$

The semi-positivity constraint on $W$ follows from the requirement that it be an effective class to preserve $N = 1$ supersymmetry.

## 3.2 Slope-Stability of the Observable Sector Bundle

As mentioned previously, to preserve $N = 1$ supersymmmetry in four-dimensions the holomorphic $SU(4)$ vector bundle $V^{(1)}$ associated with the observable $E_8$ gauge group must be both

- slope-stable (to admit a solution to the Hermitian Yang-Mills equation), and

- have vanishing slope (because there is no FI term for $SU(n)$ bundles).

Here, the slope of any bundle or sub-bundle $\mathcal{F}$ is defined as

$$\mu(\mathcal{F}) = \frac{1}{\text{rank}(\mathcal{F}) v^{2/3}} \int_X c_1(\mathcal{F}) \wedge \omega \wedge \omega, \tag{32}$$

where $\omega = a^i \omega_i$ is the Kähler form as in (8). The rank-4 bundle $V^{(1)}$ has vanishing slope since $c_1(V^{(1)}) = 0$. But, is it slope-stable? As shown in detail in [19, 23], one can identify a set of 7 "maximally destabilizing" line sub-bundles

$$\mathcal{O}_X(1, -1, -1), \ \mathcal{O}_X(-1, 1, -1), \ \mathcal{O}_X(-2, 2, 0), \ \mathcal{O}_X(2, -2, -1),$$
$$\mathcal{O}_X(2, -5, 1), \ \mathcal{O}_X(1, -4, 1), \ \mathcal{O}_X(-4, 1, 1). \tag{33}$$

It is a sufficient condition for stability to have all of their slopes be negative simultaneously. This singles out the subspace of the Kähler cone defined by the following 7 inequalities

$$-3(a^1 - a^2)(a^1 + a^2 + 6a^3) - 18a^1a^2 < 0$$
$$3(a^1 - a^2)(a^1 + a^2 + 6a^3) - 18a^1a^2 < 0$$
$$6(a^1 - a^2)(a^1 + a^2 + 6a^3) < 0$$
$$-6(a^1 - a^2)(a^1 + a^2 + 6a^3) - 18a^1a^2 < 0 \tag{34}$$
$$-3(5a^1 - 2a^2)(a^1 + a^2 + 6a^3) + 9a^1a^2 < 0$$
$$-3(4a^1 - a^2)(a^1 + a^2 + 6a^3) + 9a^1a^2 < 0$$
$$3(a^1 - 4a^2)(a^1 + a^2 + 6a^3) + 9a^1a^2 < 0.$$

These can be slightly simplified into the statement that $a^i, i = 1, 2, 3$ must satisfy at least one of the two sets of inequalities

$$
\left( a^1 < a^2 \leq \sqrt{\tfrac{5}{2}} a^1 \quad \text{and} \quad a^3 < \frac{-(a^1)^2 - 3a^1a^2 + (a^2)^2}{6a^1 - 6a^2} \right) \quad \text{or}
$$
$$
\left( \sqrt{\tfrac{5}{2}} a^1 < a^2 < 2a^1 \quad \text{and} \quad \frac{2(a^2)^2 - 5(a^1)^2}{30a^1 - 12a^2} < a^3 < \frac{-(a^1)^2 - 3a^1a^2 + (a^2)^2}{6a^1 - 6a^2} \right) \tag{35}
$$

The subspace $\mathcal{K}^s$ satisfying (34) is a full-dimensional subcone of the Kähler cone $\mathcal{K}$ defined in (7). It is a cone because the inequalities are homogeneous. In other words, only the angular part of the Kähler moduli are constrained but not the overall volume. Hence, it is best displayed as a two-dimensional "star map" as seen by an observer at the origin. This is shown in Figure 1. For Kähler moduli restricted to this subcone, the four-dimensional low energy theory in the observable sector is $N = 1$ supersymmetric.

## 3.3   $N = 1$ Supersymmetric Hidden Sector Bundle

In the heterotic standard model vacuum, the observable sector vector bundle $V^{(1)}$ has structure group $SU(4)$. Hence, it does not lead to an anomalous $U(1)$ gauge factor in the observable sector of the low energy theory. However, the hidden sector bundle $V^{(2)}$ introduced above consists entirely of a sum of line bundles and, therefore, has structure group $U(1)^R$. Each $U(1)$ factor leads to an anomalous $U(1)$ gauge group in the four-dimensional effective field theory and, hence, an associated $D$-term.

Let $L_r$ be any one of the sub-line bundles of $V^{(2)}$. Then, it was shown in [29] that the associated FI term is

$$
FI^{U(1)_r} \propto \mu(L_r) - \frac{g_s^2 l_s^4}{v^{2/3}} \int_X c_1(L_r) \wedge
$$
$$
\left( \sum_{s=1}^{R} a_s c_1(L_s) \wedge c_1(L_s) + \frac{1}{2} c_2(TX) - \sum_{m=1}^{M} (\tfrac{1}{2} + \lambda_m)^2 W^{(m)} \right), \tag{36}
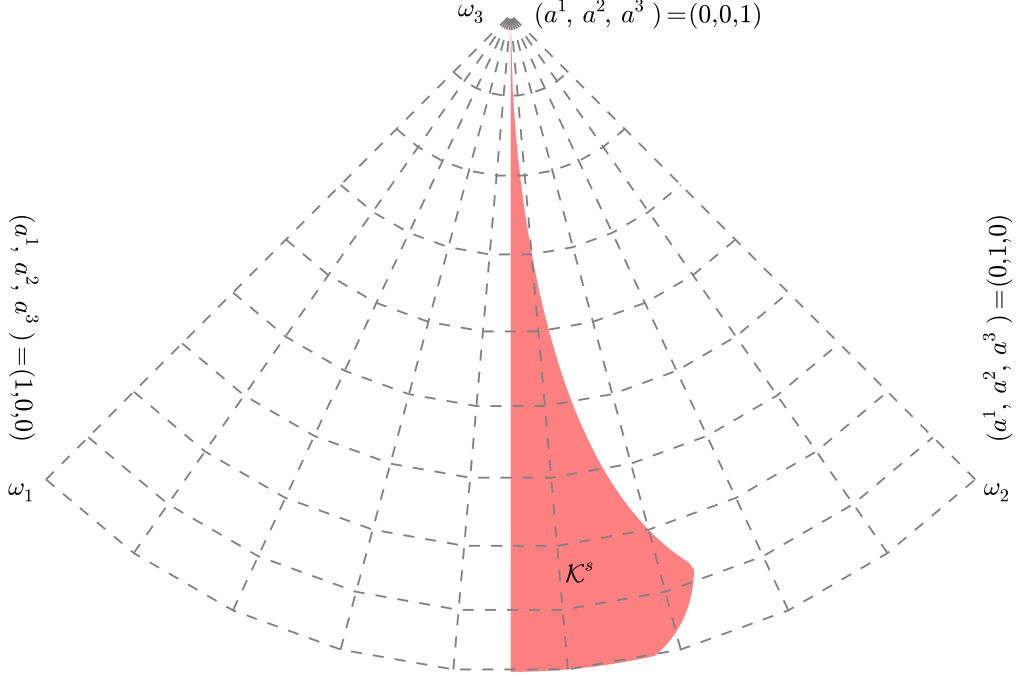$$

**Figure 1:** *Map projection of the unit sphere intersecting the Kähler cone, that is, the positive octant in $H^2(X, \mathbb{R}) \simeq \mathbb{R}^3$. The visible sector bundle $V^{(1)}$ is stable inside the red teardrop-shaped region $\mathcal{K}^s$. Every point in the projection represents a ray in the Kähler cone. For example, $(a^1, a^2, a^3) = (0, 1, 0)$ generates the ray in the $\omega_2$ direction.*

where $\mu(L_r)$ is given in (32) and

$$g_s = e^{\phi_{10}}, \quad l_s = 2\pi\sqrt{\alpha'} \tag{37}$$

are the string coupling and string length respectively. Furthermore, each $\lambda_n$ is a real modulus that, together with a self-dual two-form $\tilde{B}_n$, forms a tensor multiplet on the six-dimensional worldvolume of the $n$-th five-brane. The normalization of these moduli is chosen so that

$$\lambda_n \in \left[-\tfrac{1}{2}, \tfrac{1}{2}\right]. \tag{38}$$

The first term on the right-hand side, that is, the slope of $L_r$ defined in (32), is the tree level result.

The remaining terms are the string one-loop string corrections first presented in [29, 32, 53]. The general form of each $D$-term is as the sum of 1) the moduli dependent FI parameter (36) and 2) terms quadratic in the fields charged under the gauge symmetry weighted by their specific charge. In this paper, for simplicity, we will assume that all $U(1)^R$ charged zero-modes in the hidden sector have vanishing

expectation values. Then the hidden sector will be $N = 1$ supersymmetric if and only if the moduli-dependent FI parameter vanishes for each $L_r$. That is,

$$\int_X c_1(L_r) \wedge \omega \wedge \omega - g_s^2 l_s^4 \int_X c_1(L_r) \wedge$$

$$\left( \sum_{s=1}^R a_s c_1(L_s) \wedge c_1(L_s) + \frac{1}{2} c_2(TX) - \sum_{m=1}^M (\tfrac{1}{2} + \lambda_m)^2 W^{(m)} \right) = 0 \quad (39)$$

for $r = 1, \ldots, R$. Using (3), (4), (8), (22), (29), (30) and noting from (5) that

$$\frac{1}{v^{1/3}} \int_X \frac{1}{2} c_2(TX) \wedge \omega_i = (2, 2, 0)_i, \quad (40)$$

it follows that for each $L_r$ condition (39) can be written as

$$d_{ijk} l_r^i a^j a^k - \frac{g_s^2 l_s^4}{v^{2/3}} \left( d_{ijk} \ell_r^i \sum_{s=1}^R a_s \ell_s^j \ell_s^k + \ell_r^i (2, 2, 0)_i - \sum_{m=1}^M (\tfrac{1}{2} + \lambda_m)^2 \ell_r^i W_i^{(m)} \right) = 0 \quad (41)$$

where

$$V = \frac{1}{6} d_{ijk} a^i a^j a^k. \quad (42)$$

## 3.4   Gauge Threshold Corrections

The gauge couplings of the non-anomalous components of the $d = 4$ gauge group, in both the observable and hidden sectors, have been computed to the string one-loop level in [29–33]. Including five-branes, these are given by

$$\frac{4\pi}{g^{(1)2}} = \frac{1}{6v} \int_X \omega \wedge \omega \wedge \omega - \frac{g_s^2 l_s^4}{2v} \int_X \omega \wedge$$

$$\left( -c_2(V^{(1)}) + \frac{1}{2} c_2(TX) - \sum_{m=1}^M (\tfrac{1}{2} - \lambda_m)^2 W^{(m)} \right) \quad (43)$$

and

$$\frac{4\pi}{g^{(2)2}} = \frac{1}{6v} \int_X \omega \wedge \omega \wedge \omega - \frac{g_s^2 l_s^4}{2v} \int_X \omega \wedge$$

$$\left( \sum_{r=1}^R a_r c_1(L_r) \wedge c_1(L_r) + \frac{1}{2} c_2(TX) - \sum_{m=1}^M (\tfrac{1}{2} + \lambda_m)^2 W^{(m)} \right), \quad (44)$$

respectively. The first term on the right-hand side, that is, the volume $V$ defined in (10), is the tree level result. The remaining terms are the one-loop corrections first presented in [29].

Clearly, consistency of the $d = 4$ effective theory requires both $g^{(1)2}$ and $g^{(1)2}$ to be positive. It follows that the moduli of the four-dimensional theory are constrained to satisfy

$$\frac{1}{3}\int_X \omega \wedge \omega \wedge \omega - g_s^2 l_s^4 \int_X \omega \wedge \left( -c_2(V^{(1)}) \right.$$
$$\left. + \frac{1}{2}c_2(TX) - \sum_{m=1}^M (\tfrac{1}{2} - \lambda_m)^2 W^{(m)} \right) > 0 \tag{45}$$

and

$$\frac{1}{3}\int_X \omega \wedge \omega \wedge \omega - g_s^2 l_s^4 \int_X \omega \wedge \left( \sum_{r=1}^R a_r c_1(L_r) \wedge c_1(L_r) \right.$$
$$\left. + \frac{1}{2}c_2(TX) - \sum_{m=1}^M (\tfrac{1}{2} + \lambda_m)^2 W^{(m)} \right) > 0. \tag{46}$$

As in the previous subsections, one can use (3), (4), (8), (14), (29), (30) and (40) to rewrite these expressions as

$$d_{ijk}a^i a^j a^k - 3\frac{g_s^2 l_s^4}{v^{2/3}}\left( -(\tfrac{8}{3}a^1 + \tfrac{5}{3}a^2 + 4a^3) + 2(a^1 + a^2) - \sum_{m=1}^M (\tfrac{1}{2} - \lambda_m)^2 a^i\, W_i^{(m)} \right) > 0 \tag{47}$$

and

$$d_{ijk}a^i a^j a^k - 3\frac{g_s^2 l_s^4}{v^{2/3}}\left( d_{ijk}a^i \sum_{r=1}^R a_r \ell_r^j \ell_r^k + 2(a^1 + a^2) - \sum_{m=1}^M (\tfrac{1}{2} + \lambda_m)^2 a^i\, W_i^{(m)} \right) > 0 \tag{48}$$

respectively.

# 4 Specific Examples

## 4.1 Constraints for a Single Line Bundle

We now present an explicit $N = 1$ supersymmetric hidden sector for the weakly coupled heterotic standard model that satisfies all vacuum constraints. To do this, one must specify the number of line bundles $L_r$ and their exact embeddings into the hidden vector bundle $E_8$. Later in this section, we will consider the case of two

independent line bundles. However, for now we restrict ourselves to the simplest example consisting only of a single line bundle

$$V^{(2)} = L, \quad L = \mathcal{O}_X(\ell^1, \ell^2, \ell^3) \tag{49}$$

parametrized by integers

$$\ell^1, \ell^2, \ell^3 \in \mathbb{Z}, \quad \ell^1 + \ell^2 = 0 \text{ mod } 3. \tag{50}$$

Furthermore, the explicit embedding of $L$ into $E_8$ is chosen as follows. First, recall that

$$SU(2) \times E_7 \subset E_8 \tag{51}$$

is a maximal subgroup. With respect to $SU(2) \times E_7$, the $\underline{\mathbf{248}}$ representation of $E_8$ decomposes as

$$\underline{\mathbf{248}} \longrightarrow (\underline{\mathbf{1}}, \underline{\mathbf{133}}) \oplus (\underline{\mathbf{2}}, \underline{\mathbf{56}}) \oplus (\underline{\mathbf{3}}, \underline{\mathbf{1}}). \tag{52}$$

We embed the generator $Q$ of the U(1) structure group of $L$—more specifically, the generator of the $S(U(1) \times U(1))$ Abelian group of the induced rank two bundle $L \oplus L^*$ in $SU(2)$—so that under $SU(2) \to U(1)$ the two-dimensional $SU(2)$ representation decomposes as

$$\underline{\mathbf{2}} \longrightarrow 1 \oplus -1. \tag{53}$$

It follows that under $U(1) \times E_7$

$$\underline{\mathbf{248}} \longrightarrow (0, \underline{\mathbf{133}}) \oplus \Big((1, \underline{\mathbf{56}}) \oplus (-1, \underline{\mathbf{56}})\Big) \oplus \Big((2, \underline{\mathbf{1}}) \oplus (0, \underline{\mathbf{1}}) \oplus (-2, \underline{\mathbf{1}})\Big). \tag{54}$$

The generator $Q$ of this embedding of the line bundle can be read off from expression (54). Inserting this into (25), we find that

$$a = 1. \tag{55}$$

Having presented the hidden sector vector bundle, one must specify the number of five-branes. For simplicity, we assume that there is only one five-brane in this example. It then follows from (31), (41), (47) and (48) that the constraints for this

explicit example are given by

$$W_i = \left(\tfrac{4}{3}, \tfrac{7}{3}, -4\right)_i + d_{ijk}\ell^j\ell^k \geq 0, \quad i = 1, 2, 3 \tag{56a}$$

$$d_{ijk}\ell^i a^j a^k - \frac{g_s^2 l_s^4}{v^{2/3}}\left(d_{ijk}\ell^i\ell^j\ell^k + \ell^i(2,2,0)_i\right.$$
$$\left. -\left(\tfrac{1}{2}+\lambda\right)^2\ell^i W_i\right) = 0, \tag{56b}$$

$$d_{ijk}a^i a^j a^k - 3\frac{g_s^2 l_s^4}{v^{2/3}}\left(-\left(\tfrac{8}{3}a^1 + \tfrac{5}{3}a^2 + 4a^3\right)+\right.$$
$$\left. +2(a^1+a^2) - \left(\tfrac{1}{2}-\lambda\right)^2 a^i W_i\right) > 0, \tag{56c}$$

$$d_{ijk}a^i a^j a^k - 3\frac{g_s^2 l_s^4}{v^{2/3}}\left(d_{ijk}a^i\ell^j\ell^k+\right.$$
$$\left. +2(a^1+a^2) - \left(\tfrac{1}{2}+\lambda\right)^2 a^i W_i\right) > 0. \tag{56d}$$

These constraints have to be solved simultaneously with the condition (35) for the slope-stability of the observable $E_8$ non-Abelian vector bundle. That is,

$$\begin{pmatrix} a^1 < a^2 \leq \sqrt{\tfrac{5}{2}}a^1 & \text{and} & a^3 < \dfrac{-(a^1)^2 - 3a^1 a^2 + (a^2)^2}{6a^1 - 6a^2} \end{pmatrix} \quad \text{or}$$
$$\begin{pmatrix} \sqrt{\tfrac{5}{2}}a^1 < a^2 < 2a^1 & \text{and} & \dfrac{2(a^2)^2 - 5(a^1)^2}{30a^1 - 12a^2} < a^3 < \dfrac{-(a^1)^2 - 3a^1 a^2 + (a^2)^2}{6a^1 - 6a^2} \end{pmatrix} \tag{56e}$$

We now seek simultaneous solutions to eqns. (56a) to (56e).

## 4.2 An $E_7 \times U(1)$ Hidden Sector

The first observation is that the system of equations (56b) to (56e) is homogeneous with respect to the rescaling[1]

$$\left(a^1, \ a^2, \ a^3, \ \frac{g_s^2 l_s^4}{v^{2/3}}\right) \mapsto \left(\mu a^1, \ \mu a^2, \ \mu a^3, \ \mu^2 \frac{g_s^2 l_s^4}{v^{2/3}}\right), \quad \mu > 0. \tag{57}$$

Therefore, one can absorb the coupling constants into the Kähler moduli $a^i$. In other words, for a single $U(1)$ the $FI = 0$ equation (56b) fixes one Kähler modulus to a certain numerical value measured in multiples of $\left(\frac{g_s^2 l_s^4}{v^{2/3}}\right)^{1/2}$. This is tantamount to setting $\frac{g_s^2 l_s^4}{v^{2/3}}$ to unity, which we will do henceforth for simplicity.

Next, let us concentrate on the particular hidden line bundle

$$V^{(2)} = L = \mathcal{O}_X(1,2,3), \tag{58}$$

---

[1]Note that the coupling constants appear only in the combination $\frac{g_s^2 l_s^4}{v^{2/3}}$, which is a positive number.

that is,
$$(\ell^1, \ell^2, \ell^3) = (1, 2, 3). \tag{59}$$

This choice cancels the anomaly with the effective five-brane curve class

$$W = (16, 10, 0) \geq 0. \tag{60}$$

Having fixed the line bundle, we proceed to solve the Fayet-Iliopoulos equation (56b) for $a^3$, giving us

$$a^3 = \frac{-2(a^1)^2 - (a^2)^2 - 24a^1 a^2 - 108\lambda^2 - 108\lambda + 117}{6\left(2a^1 + a^2\right)}. \tag{61}$$

This can then be substituted into equations (56c), (56d) and (56e) to obtain a system of polynomial inequalities in $a^1, a^2$ and $\lambda$.

Finally, one can scan through the range $-\frac{1}{2} < \lambda < \frac{1}{2}$ and plot the region of validity in the $a^1$-$a^2$ plane to find solutions. For example, with the numerical value of $\lambda = 0.496$, which is close to the hidden wall, we do indeed find solutions which satisfy all of the constraints. The region where all physical constraints equations (56c) to (56e) are satisfied simultaneously is shown in yellow in Figure 2. For clarity, and to make contact with the visible sector stability region plotted in Figure 1, we present the same data in Figure 3 as a subset of the Kähler cone—that is, the positive octant $a^1, a^2, a^3 > 0$. This gives a multi-dimensional visualization of the facts that:

- The visible sector stability region (red) is a three-dimensional sub-cone of the Kähler cone.

- The FI-term stabilizes one particular combination of angular and radial Kähler moduli, which is why the hidden sector $U(1)$ constrains us to a two-dimensional surface (blue) in the Kähler cone.

## 4.3 Hidden Sector Matter Spectrum

We can now compute the low energy $U(1) \times E_7$ particle spectrum from the cohomology of the line bundle $L$. From the breaking pattern in (52) and (53), one can read off the representation content (54). The spectrum is then determined by the cohomology groups of the corresponding tensor power of the line bundle. These are tabulated in the middle column of Table 1. They are valid for any line bundle $L$ appropriately embedded in the $SU(2) \subset E_8$. In determining the bundles associated with a given representation, we used the fact that the line bundle $L$ induces a rank two bundle $L \oplus L^*$ with structure group $S(U(1) \times U(1)) \subset SU(2)$. Recalling that $L \otimes L^* = \mathcal{O}_X$, it follows that

$$\mathrm{End}\left(L \oplus L^*\right) = (L \oplus L^*) \otimes (L \oplus L^*) = (L^2 \oplus L^{*2} \oplus \mathcal{O}_X) \oplus \mathcal{O}_X. \tag{62}$$
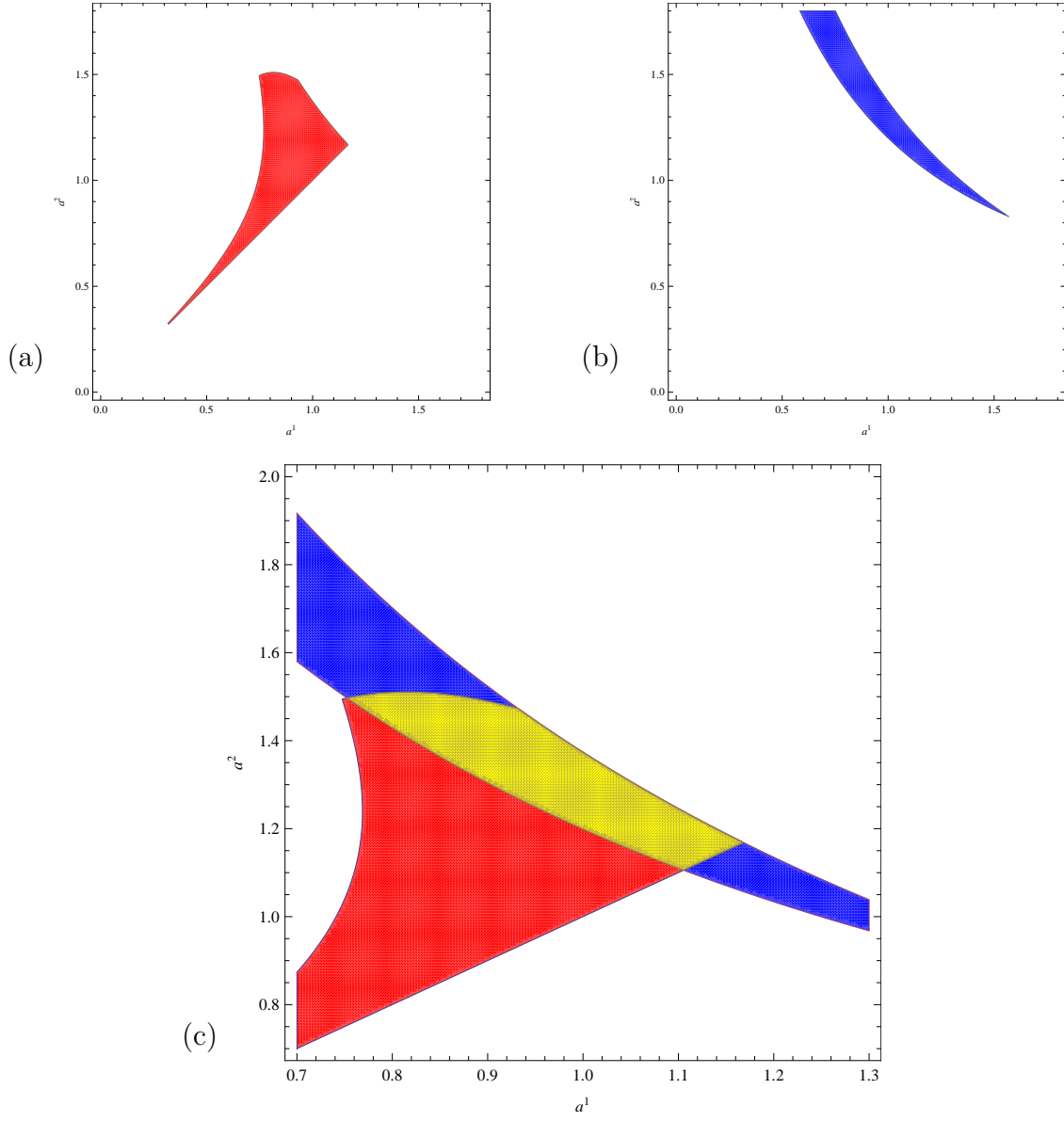
**Figure 2:** *The two-dimensional slice through the Kähler cone where the FI-term of the hidden line bundle $L = \mathcal{O}_X(1,2,3)$ with five-brane position $\lambda = 0.496$ vanishes. The slice is parametrized by $(a^1, a^2)$ with $a^3$ given by (61). In red, the visible sector stability condition, see sub-figures a) and c). In blue, the region where the both the visible and hidden sector gauge couplings are positive, see sub-figures b) and c). Their intersection is drawn in yellow, see sub-figure c).*
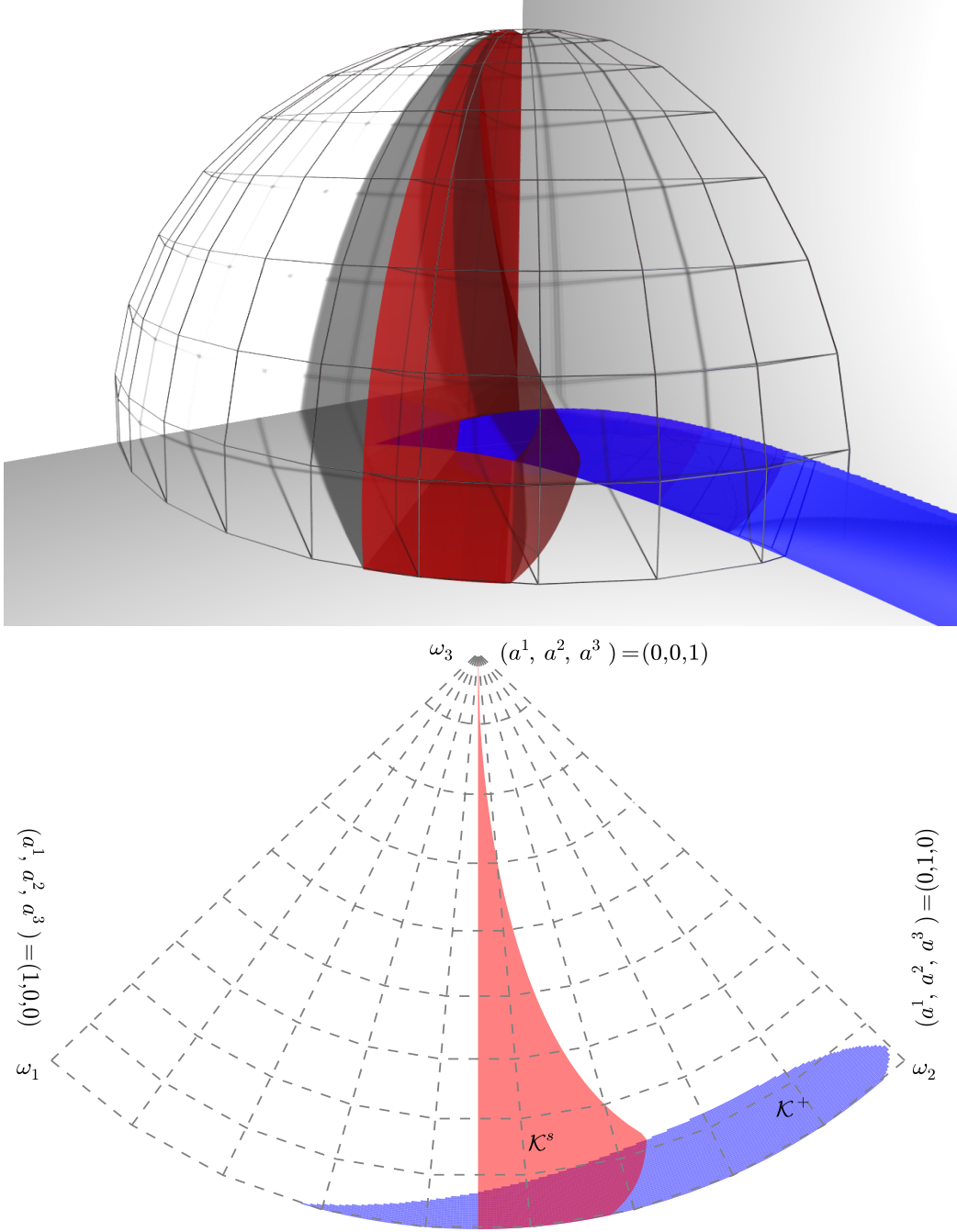
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**Figure 3:** *The Kähler cone, in 3 dimensions (top) and the projection in radial directions (bottom). The blue region $\mathcal{K}^+$ is our hidden sector solution for $L = \mathcal{O}_X(1,2,3)$ at $\lambda = 0.496$. It shows the Kähler moduli $\omega = a^1\omega_1 + a^2\omega_2 + a^3\omega_3$ simultaneously satisfying the $FI = 0$ condition and the positivity of the visible and hidden sector gauge couplings. The red region $\mathcal{K}^s$ is the stability region of the visible sector bundle from Figure 1. The intersection $\mathcal{K}^s \cap \mathcal{K}^+$ is where all physical constraints are satisfied.*

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| $U(1) \times E_7$ | Cohomology | Index $\chi$ |
|:---:|:---:|:---:|
| $(0, \mathbf{\underline{133}})$ | $H^*(X, \mathcal{O}_X)$ | $0$ |
| $(1, \mathbf{\underline{56}})$ | $H^*(X, L)$ | $8$ |
| $(-1, \mathbf{\underline{56}})$ | $H^*(X, L^*)$ | $-8$ |
| $(2, \mathbf{\underline{1}})$ | $H^*(X, L^2)$ | $58$ |
| $(-2, \mathbf{\underline{1}})$ | $H^*(X, L^{*2})$ | $-58$ |
| $(0, \mathbf{\underline{1}})$ | $H^*(X, \mathcal{O}_X)$ | $0$ |

**Table 1:** *The chiral spectrum for the $E_8 \to U(1) \times E_7$ breaking pattern with a line bundle L. The index $\chi$ counts the number of left-chiral minus the number of right-chiral fermionic zero-modes with the given gauge charge.*

Therefore

$$H^*(\mathrm{End}(L \oplus L^*)) = \left(H^*(L^2) \oplus H^*(L^{*2}) \oplus H^*(\mathcal{O}_X)\right) \oplus H^*(\mathcal{O}_X), \qquad (63)$$

corresponding to the $(2, \mathbf{\underline{1}}), (-2, \mathbf{\underline{1}}), (0, \mathbf{\underline{1}})$ and $(0, \mathbf{\underline{133}})$ representations respectively.

For each representation, the number of chiral fermionic zero-modes of the Dirac operator is determined from the corresponding index $\chi$. For an arbitrary vector bundle $V$ on a Calabi-Yau threefold, the Atiyah-Singer index theorem tells us that

$$\chi(V) = \sum_{i=0}^{3} (-1)^i h^i(X, V) = \int_X \mathrm{ch}(V) \wedge \mathrm{td}(TX)$$

$$= \int_X \left( \frac{1}{12} c_1(V) \wedge c_2(TX) + \mathrm{ch}_3(V) \right). \qquad (64)$$

In the case of *any* single line bundle of the form $V = \mathcal{O}_X(\ell^1, \ell^2, \ell^3)$, this simplifies to

$$\chi(\mathcal{O}_X(\ell^1, \ell^2, \ell^3)) = \frac{1}{3} \ell^1 + \frac{1}{3} \ell^2 + \frac{1}{6} \sum_{i,j,k=1}^{3} d_{ijk} \ell^i \ell^j \ell^k. \qquad (65)$$

Using this, the index of any of the tensor powers of $L$ appearing in the middle column of Table 1 is easily computed.

Before restricting to the specific example specified in (58), let us present some important generic results. To begin with, for any bundle $V$

$$\chi(V) = -\chi(V^*) . \qquad (66)$$

Therefore, for $V$ carrying a real representation of its structure group, that is, $V = V^*$, it follows that $\chi(V) = 0$. For example

$$\chi(\mathcal{O}_X) = 0 , \qquad (67)$$

as indicated in the first and last entries of the third column of Table 1. Hence, any possible fermionic zero-modes associated with the representations $(0, \mathbf{133})$ and $(0, \mathbf{1})$ in the decomposition (54) must be non-chiral. In fact, one can determine the exact number of conjugate pairs of fermions there are in any such representation. Recall that the trivial line bundle $\mathcal{O}_X$ on a Calabi-Yau threefold $X$ has cohomology groups of dimension

$$h^0(X, \mathcal{O}_X) = h^3(X, \mathcal{O}_X) = 1 \tag{68}$$

and 0 otherwise. Note that this is consistent with a vanishing Atiyah-Singer index since $\chi = h^0 - h^1 + h^2 - h^3$. It follows that for the trivial bundle $\mathcal{O}_X$, there is exactly one left-chiral fermion zero-mode specificed by $h^0 = 1$ and one conjugate right-chiral fermion zero-mode specified by $h^3 = 1$. These are the conjugate fermion gauginos in a vector supermultiplet. Specifically, our effective theory has one vector multiplet in the $\mathbf{133}$ adjoint representation of $E_7$ and one vector multiplet in the adjoint $\mathbf{1}$ representation of $U(1)$.

A second generic result is that because of relation (66), one needs to compute the index of only one bundle in each conjugate pair appearing in Table 1. We will choose $L$ and $L^2$, corresponding to the representations $(1, \mathbf{56})$ and $(2, \mathbf{1})$ respectively. Consider, for example, the bundle $L$. Then $\chi(L) = n_L - n_R$ gives the number of chiral families of fermionic zero-modes transforming in the $(1, \mathbf{56})$ representation. In principle, there can also be some number of vector-like pairs of such zero-modes. However, these pairs are naturally massive and, hence, we will ignore them. In fact, within the context of an *ample* line bundle, such as (58), one can go further and actually compute the number of positive and negative chirality modes. To see this, note that by definition an ample line bundle has only positive entries $\ell^1, \ell^2, \ell^3$. An immediate consequence is that

$$h^i(X, L) = h^i(X, L^2) = 0, \quad i \neq 0 \tag{69}$$

by the Kodaira vanishing theorem. Therefore, only $h^0(X, L)$ and $h^0(X, L^2)$ can be non-zero. Recalling that $\chi = h^0 - h^1 + h^2 - h^3$, it follows from (69) that for each of $L$ and $L^2$ there is exactly $\chi = h^0$ left-chiral fermion zero-modes and no right-chiral zero-modes. That is, there are no vector-like pairs. Note that this phenomenon of a non-vanishing index arising from $h^0$ (and, generically, $h^3$)–as opposed to $h^1, h^2$–is due to the fact that $c_1(L) \neq 0$. That is, the hidden sector bundle is chosen not to be supersymmetric classically. $N = 1$ supersymmetry is only restored by the non-vanishing one-loop corrections to the Fayet-Iliopoulos term rendering the D-term zero. Phrased another way, for stable $SU(n)$ bundles $V$ one has $c_1(V) = 0$ and $h^0 = h^3 = 0$. Hence, the chiral spectrum is coming from $h^1$ and $h^2$ only. However, our bundles do not have vanishing first Chern class. It follows that $h^0$ and, in general, $h^3$ need not vanish. In principle, therefore, all 4 cohomology groups can contribute to the index.

To proceed, one must specify the line bundle $L$. Choose this to be (58). Using expression (65) for the Atiyah-Singer index, we find

$$\chi(X, L) = 8 \quad \text{for} \quad L = \mathcal{O}_X(1, 2, 3) \tag{70}$$

and, similarly, that

$$\chi(X, L^2) = 58 \ \text{ for } \ L^2 = \mathcal{O}_X(2,4,6), \tag{71}$$

as indicated in the third column of Table 1. That is, the effective $U(1) \times E_7$ theory has 8 chiral supermultiplets transforming as $(1, \underline{\textbf{56}})$ and 58 chiral supermultiplets transforming as $(2, \underline{\textbf{1}})$. In summary, the complete $U(1) \times E_7$ hidden sector spectrum of our model is

$$1 \times (0, \underline{\textbf{133}}) + 8 \times (1, \underline{\textbf{56}}) + 58 \times (2, \underline{\textbf{1}}) + 1 \times (0, \underline{\textbf{1}}). \tag{72}$$

## 4.4   Constraints for Two Line Bundles

In the previous section, we discussed the case of the hidden sector being a single line bundle and presented a detailed solution. One can easily move on to bundles of higher rank and show that indeed there is a plenitude of solutions. This is clearly desirable for model building since each independent $U(1)$ imposes one Fayet-Iliopoulos vanishing equation and, therefore, stabilizes one Kähler modulus. In this section, we will consider the next simplest hidden sector consisting of the direct sum of two line bundles

$$V^{(2)} = L_1 \oplus L_2, \qquad L_1 = \mathcal{O}_X(\ell_1^1, \ell_1^2, \ell_1^3), \ L_2 = \mathcal{O}_X(\ell_2^1, \ell_2^2, \ell_2^3) \tag{73}$$

where

$$\ell_r^1 + \ell_r^2 = 0 \bmod 3, \quad r = 1, 2. \tag{74}$$

Furthermore, $L_1 \oplus L_2$ will be given the simplest simplest embedding into $E_8$, namely via

$$U(1) \times U(1) \times SO(12) \ \subset \ SU(2) \times SU(2) \times SO(12) \ \subset \ E_8. \tag{75}$$

The branching rules easily follow from those of the $SU(2) \times SO(12)$ maximal subgroup of $E_7$ in conjunction with (52). In particular, the adjoint representation of $E_8$ decomposes under $SU(2) \times SU(2) \times SO(12)$ as

$$\begin{aligned}
\underline{\textbf{248}} \longrightarrow \ & (\underline{\textbf{1}}, \underline{\textbf{3}}, \underline{\textbf{1}}) \oplus (\underline{\textbf{3}}, \underline{\textbf{1}}, \underline{\textbf{1}}) \oplus (\underline{\textbf{2}}, \underline{\textbf{1}}, \underline{\textbf{32}}) \oplus (\underline{\textbf{1}}, \underline{\textbf{2}}, \overline{\underline{\textbf{32}}}) \\
& \oplus (\underline{\textbf{2}}, \underline{\textbf{2}}, \underline{\textbf{12}}) \oplus (\underline{\textbf{2}}, \underline{\textbf{2}}, \underline{\textbf{12}}) \oplus (\underline{\textbf{1}}, \underline{\textbf{1}}, \underline{\textbf{66}}).
\end{aligned} \tag{76}$$

One now has to choose the embedding of the generator $Q_r$ of each of the two $U(1)$ structure groups into the corresponding $SU(2)$. We again pick the simplest embedding, identifying the structure group of $L_1$ with the center of the first $SU(2)$, as in (53), and similarly for $L_2$. Consequently, under $U(1) \times U(1) \times SO(12)$ we have the branching rule

$$\begin{aligned}
\underline{\textbf{248}} \longrightarrow \ & (0, 2, \underline{\textbf{1}}) \oplus (2, 0, \underline{\textbf{1}}) \oplus (0, -2, \underline{\textbf{1}}) \oplus (-2, 0, \underline{\textbf{1}}) \oplus 2 \times (0, 0, \underline{\textbf{1}}) \\
& \oplus (1, 0, \underline{\textbf{32}}) \oplus (-1, 0, \underline{\textbf{32}}) \oplus (0, 1, \overline{\underline{\textbf{32}}}) \oplus (0, -1, \overline{\underline{\textbf{32}}}) \oplus (1, 1, \underline{\textbf{12}}) \\
& \oplus (1, -1, \underline{\textbf{12}}) \oplus (-1, 1, \underline{\textbf{12}}) \oplus (-1, -1, \underline{\textbf{12}}) \oplus (0, 0, \underline{\textbf{66}}).
\end{aligned} \tag{77}$$

As before, one can read off from (25) the group-theoretic embedding coefficients $a_1$ and $a_2$. They are given by

$$a_1 = a_2 = 1. \tag{78}$$

Again assuming that we have only a single fivebrane, the constraints for the case of the direct sum of two line bundles—analogous to equations (56a) to (56e)—are

$$W_i = \left(\tfrac{4}{3}, \tfrac{7}{3}, -4\right)_i + \sum_{r=1}^{2} d_{ijk}\ell_r^j \ell_r^k \geq 0, \quad i = 1,2,3 \tag{79a}$$

$$d_{ijk}\ell_r^i a^j a^k - \frac{g_s^2 l_s^4}{v^{2/3}}\left(d_{ijk}\ell_r^i \ell_r^j \ell_r^k + \ell_r^i (2,2,0)_i \right.$$
$$\left. -\left(\tfrac{1}{2}+\lambda\right)^2 \ell_r^i W_i\right) = 0, \quad r = 1,2 \tag{79b}$$

$$d_{ijk}a^i a^j a^k - 3\frac{g_s^2 l_s^4}{v^{2/3}}\left(-\left(\tfrac{8}{3}a^1 + \tfrac{5}{3}a^2 + 4a^3\right)\right.$$
$$\left. +2(a^1 + a^2) - \left(\tfrac{1}{2}-\lambda\right)^2 a^i W_i\right) > 0, \tag{79c}$$

$$d_{ijk}a^i a^j a^k - 3\frac{g_s^2 l_s^4}{v^{2/3}}\left(d_{ijk}a^i \ell^j \ell^k + 2(a^1 + a^2)\right.$$
$$\left. -\left(\tfrac{1}{2}+\lambda\right)^2 a^i W_i\right) > 0, \tag{79d}$$

$$\begin{pmatrix} \left(a^1 < a^2 \leq \sqrt{\tfrac{5}{2}}a^1 \quad \text{and} \quad a^3 < \dfrac{-(a^1)^2 - 3a^1 a^2 + (a^2)^2}{6a^1 - 6a^2}\right) \quad \text{or} \\ \left(\sqrt{\tfrac{5}{2}}a^1 < a^2 < 2a^1 \quad \text{and} \quad \dfrac{2(a^2)^2 - 5(a^1)^2}{30a^1 - 12a^2} < a^3 < \dfrac{-(a^1)^2 - 3a^1 a^2 + (a^2)^2}{6a^1 - 6a^2}\right) \end{pmatrix} \tag{79e}$$

We point out that only the first two sets of equations differ from the single line bundle case in the previous section. We must now find simultaneous solutions to equations (79a) to (79e).

## 4.5 An $SO(12) \times U(1) \times U(1)$ Hidden Sector

We will now focus on the specific direct sum of two line bundles given by

$$V^{(2)} = L_1 \oplus L_2 = \mathcal{O}_X(2,1,1) \oplus \mathcal{O}_X(0,3,2). \tag{80}$$

In terms of our basis choice for the first Chern class, see (20), this is

$$(\ell_1^1, \ell_1^2, \ell_1^3) = (2,1,1), \quad (\ell_2^1, \ell_2^2, \ell_2^3) = (0,3,2). \tag{81}$$

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For simplicity, we assume that there is only a single five-brane. In order to cancel the heterotic anomaly for this choice of bundle and embedding, it must then wrap the effective curve class

$$W = (20, 9, 0) \geq 0. \tag{82}$$

As in the previous section, we use the homogeneous rescaling (57) to set $\frac{g_s^2 l_s^4}{v^{2/3}} = 1$. We will also use the same five-brane position for convenience; namely $\lambda = 0.496$. The only remaining parameters are now the Kähler moduli $a^1$, $a^2$ and $a^3$, subject to two FI-term constraints and a number of inequalities. We first consider the two FI-term constraints (79b), which are two quadratic equations that stabilize two of the three Kähler moduli. The standard strategy to parametrize the solution set is to pick one variable and then compute a lexicographic Gröbner basis with the chosen variable last. If we pick $a^3$ as the parameter, then the result is that $a^2$ is a real solution of the quartic equation

$$(a^2)^4 + 6.7058 a^3 (a^2)^3 + 4.2352 (a^3)^2 (a^2)^2 - 0.3955 (a^2)^2 - 1.5808 a^3 a^2 - 1.1801 = 0 \tag{83a}$$

and, finally,

$$a^1 = 1.2651 (a^2)^3 + 8.4839 a^3 (a^2)^2 + 5.3582 (a^3)^2 a^2 - 0.83377 a^2 - 4 a^3. \tag{83b}$$

It remains to impose all inequalities; namely, the positivity of all Kähler moduli, the gauge couplings equations (79c) and (79d), as well as the visible sector stability conditions (79e). The numerical result is that the free Kähler modulus has to lie in the interval

$$0 < a^3 < 0.0701743. \tag{84}$$

It then follows that the unique positive root $a^2$ of (83a) and $a^1$ determined by (83b) satisfy all physical constraints. For example, if we pick $a^3 = 0.06$ then the remaining Kähler moduli and gauge couplings are

$$
\begin{aligned}
(a^1, a^2, a^3) &= (0.95, 1.06, 0.06), \\
\left( \frac{4\pi}{g^{(1)2}}, \frac{4\pi}{g^{(2)2}} \right) &= (0.65, 0.02).
\end{aligned} \tag{85}
$$

The entire one-dimensional solution set is plotted in Figure 4.

Finally, as in the one line bundle case, the particle spectrum of this low energy $SO(12) \times U(1) \times U(1)$ hidden sector can be computed from the cohomology of the various tensor products of $L_1$ and $L_2$. Since this is similar to the discussion in Subsection 4.3, but rendered more complicated by the presence of two line bundles, we will not discuss the results here.
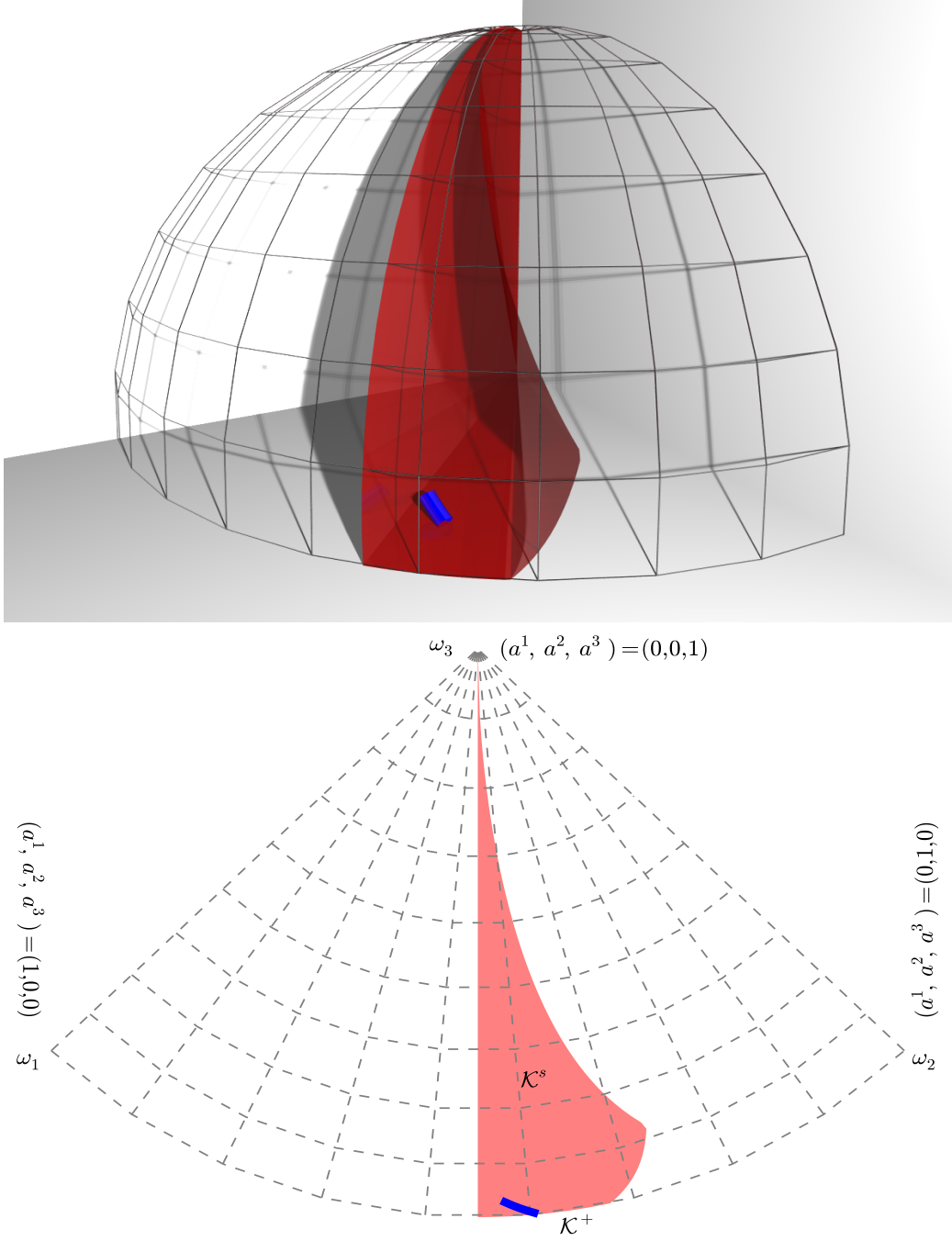
**Figure 4:** *The Kähler cone, in 3 dimensions (top) and the projection in radial directions (bottom). The 1-dimensional blue region $\mathcal{K}^+$ is our hidden sector solution for $L_1 \oplus L_2 = \mathcal{O}_X(2,1,1) \oplus \mathcal{O}_X(0,3,2)$ at $\lambda = 0.496$. It shows the Kähler moduli $\omega = a^1\omega_1 + a^2\omega_2 + a^3\omega_3$ simultaneously satisfying the two independent $FI = 0$ conditions for the two $U(1)$ factors, as well as the positivity of the isible and hidden sector gauge couplings. The red region $\mathcal{K}^s$ is the stability region of the visible sector bundle from Figure 1. The intersection $\mathcal{K}^s \cap \mathcal{K}^+$ is where all physical constraints are satisfied.*

# Acknowledgments

# Appendix A   Line Bundles and $E_8$

## A.1   Induced Bundles

The usual approach of constructing $E_8$ bundles for heterotic strings is to first construct a $G$-bundle for a smaller group $G$ and then use a map (group homomorphism) $\psi : G \to E_8$ to build an induced $E_8$ bundle. Really, this is just the composition of the $G$-bundle presentation $[X, BG]$ with $B\psi : BG \to BE_8$. Explicitly, we can think of the $G$-bundle as a collection of transition functions $\varphi_{\alpha\beta} : U_{\alpha\beta} \to G$ on overlaps $U_{\alpha\beta} = U_\alpha \cap U_\beta$. The transition functions of the induced $E_8$ bundle are then simply given by the composition $\tilde{\varphi}_{\alpha\beta} = \psi \circ \varphi_{\alpha\beta}$. A popular choice for $G$ is $SU(n)$, equivalent to a rank-$n$ vector bundle of vanishing first Chern class. This is usually combined with a group homomorphism $SU(n) \subset SU(9) \to SU(9)/\mathbb{Z}_3 \subset E_8$ that factors through the $SU(9)/\mathbb{Z}_3$ subgroup,[2] corresponding to removal of a node in $\tilde{E}_8$ affine Dynkin diagram.

However, there is no need to pick a special unitary group and, for the purposes of this section, we are considering $G = U(1)$. As a first example, the easiest group homomorphism to $E_8$ can be obtained from embedding $U(1)$ as a maximal torus in $SU(2)$ and then embedding it further in $SU(9) \to E_8$. Up to a choice of coordinates, this is the homomorphism

$$U(1) \to SU(2), \quad e^{i\phi} \mapsto \begin{pmatrix} \exp(i\phi) & 0 \\ 0 & \exp(-i\phi) \end{pmatrix} \tag{86}$$

or $1 \mapsto \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$ in the Lie algebra. This construction cannot yield the most general $E_8$ bundle, but rather only one whose structure group can be reduced to $SU(9)/\mathbb{Z}_3 \subset E_8$. This is of course desirable for phenomenological applications, so that not all of the $E_8$ gauge group is broken. In this example, the commutant of $SU(2)$ in $E(8)$ is $E_7/\mathbb{Z}_2$ and

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[2] Because it will be important in this appendix, we will break with the physics tradition and be careful about discrete quotients of groups. A good reference for the relevant group theory is [54].

the embedded $U(1) \subset SU(2)$ commutes with it self. Hence, the overall commutant and unbroken gauge group of $U(1) \subset E_8$ is $(E_7 \times U(1))/\mathbb{Z}_2$.

## A.2  Consistency Conditions

It is occasionally claimed that a $U(n)$ bundle, for example, a line bundle, has to satisfy extra divisibility conditions in order to define an induced $E_8$ bundle. However, this is not true and any bundle together with a $U(n) \to E_8$ homomorphism is admissible. In particular, the first Chern class of the line bundle need *not* be even.[3] An $E_8$ bundle is automatically spin since $E_8$ is simply connected. While it is true that a line bundle with an odd first Chern class is not spin, any induced $E_8$ bundle is well-defined and admits adjoint spinors.[4] Since the $U(n)$ bundle is purely auxiliary in this construction, there is no need for it to be spin. This is related to the fact that the contribution to the heterotic anomaly from a line bundle is always an even multiple of its second Chern character, as we will discuss in Subsection A.3.

There is a related, but different, context where an even first Chern class does play a role [55, 56]. In an effort to clear up any confusion, let us review it in the remainder of this subsection. First, recall the usual conformal field theory construction of the $E_8 \times E_8$ heterotic string. There are 36 real left-moving fermions, 16 for each $E_8$. The GSO projection acts as a minus sign, so the 16 fermions transform under $Spin(16)/\mathbb{Z}_2$. Only this group, and not the whole $Spin(16)$, is a subgroup of $E_8$. Since it is still difficult to construct the most general $Spin(16)/\mathbb{Z}_2$ bundle, it is tempting to combine the 16 real spinors into 8 complex ones and construct a $U(8)$ gauge bundle for them. The trouble is that this constructs a $U(8) \subset SO(16)$ gauge bundle which need not be a $Spin(16)/\mathbb{Z}_2$ bundle. A sufficient condition to avoid this problem is if the $U(8)$ bundle is spin, that is, has even first Chern class. In that case it lifts to a $Spin(16)$ bundle, which we can divide to obtain a $Spin(16)/\mathbb{Z}_2$ bundle. The key difference is that this does not use a $U(8) \to Spin(16)/\mathbb{Z}_2 \subset E_8$ homomorphism to construct an induced $E_8$ bundle, but rather relies on a lifting that might not exist.

## A.3  Chern Classes and the Anomaly

The heterotic anomaly cancellation condition for two $E_8$-bundles $V^{(1)}$, $V^{(2)}$ is

$$- c_2(V^{(1)}) - c_2(V^{(2)}) + c_2(TX) = W \quad \in H^4(X, \mathbb{Z}), \tag{87}$$

where $W$ is the the class of the five-brane(s). Here, $c_2$ of a $E_8$ bundle means its degree-4 characteristic class. Completely analogous to the usual Chern classes, it

---

[3]That is, divisible by 2.

[4]We remark that this is different in the $Spin(32)/\mathbb{Z}_2$ heterotic string if one uses a line bundle and $U(1) \to SO(32)$ homomorphism. In this case, the line bundle does have to satisfy additional constraints for the induced bundle to lift to a $Spin(32)/\mathbb{Z}_2$ bundle.

is unambiguously defined in integral cohomology as the pull-back of the generator $c_2 \in H^4(BE_8, \mathbb{Z}) \simeq \mathbb{Z}$ by the map $[X, BE_8]$ defining the bundle. Fortunately, we never have to actually evaluate the homotopy-theoretic definition. For an $E_8$ bundle whose structure group reduces to the $SU(9)/\mathbb{Z}_3$ subgroup, which is the case we are interested in for phenomenolgical reasons, the degree-4 characteristic class of the $E_8$ bundle coincides with the usual second Chern class of the $SU(9)$ bundle. Hence, the contribution to the anomaly of such an induced bundle $V_\rho$ defined by a $U(n)$ bundle $V$ and homeomorphism $\rho : U(n) \to SU(9)$ can be computed in terms of the Chern classes of $V$ and the group theory of $\rho$.

Let us consider the case where $V^{(1)} = L_\rho$ is induced from a line bundle $L$ and $\rho : U(1) \to SU(9)$. On general grounds, the anomaly cancellation condition then must be of the form

$$a_\rho \, c_1(L)^2 - c_2(V^{(2)}) + c_2(TX) = W \quad \in H^4(X, \mathbb{Z}) \tag{88}$$

since the only available characteristic class is $c_1(L)$ with some group-theoretic numerical coefficient $a_\rho$. In de Rham cohomology the visible sector contribution is represented by

$$a_\rho \, c_1(L)^2 = 2a_\rho \, \mathrm{ch}_2(L) = \frac{1}{16\pi^2} \, \mathrm{tr}_\rho \, F \wedge F \tag{89}$$

It is suggestive, but wrong, that the coefficient $a_\rho$ should be half-integral such that the contribution of the line bundle to the anomaly is always an integer multiple of its second Chern character. In fact, the coefficient $a_\rho$ is always integer which is twice what one would naively expect.[5] This is also required for the anomaly contribution $a_\rho c_1(L)^2$ to define an integral cohomology class. As the simplest example, let us return to the $\rho : U(1) \to SU(2) \subset SU(9)$ embedding from (86). The induced $SU(9)$ bundle is

$$L_\rho = L \oplus L^{-1} \oplus \mathbf{1}_7, \tag{90}$$

so its second Chern class is $-c_2(L_\rho) = c_1^2(L)$, that is, $a_\rho = 1$.

## A.4 Example

As a more complicated example, we now consider a combination of line bundle and non-Abelian bundle. Let us start with a $SU(3)$ bundle $V$ and a line bundle $L$. We can use this data with different group homomorphisms to construct the same $SU(9)$ (and, therefore, $E_8$) bundle in two different ways:

(A) Use the group homomorphism

$$\rho_A : SU(3) \times U(1) \to SU(4) \subset SU(9),$$

$$\left(g_{3\times3}, e^{i\phi}\right) \mapsto \begin{pmatrix} e^{i\phi}g_{3\times3} & 0 & 0 \\ 0 & e^{-3i\phi} & 0 \\ 0 & 0 & \mathbf{1}_{5\times5} \end{pmatrix} \tag{91}$$

---

[5]For discrete Wilson lines, this observation was already made in the footnote on Page 88 of [57].

(B) The direct sum $(V \otimes L) \oplus L^{-3}$ is a rank-4 bundle with vanishing first Chern class. We combine it with the trivial embedding $\rho_B : SU(4) \subset SU(9)$.

Both constructions yield the same induced $SU(9)$-bundle, namely

$$E = (V \otimes L) \oplus L^{-3} \oplus \mathbf{1}_5 = (V \oplus L)_{\rho_A} = \left((V \otimes L) \oplus L^{-3}\right)_{\rho_B}. \qquad (92)$$

Its contribution to the heterotic anomaly is $-c_2(E) = -c_2(V) + 3c_1(L)^2$. In other words, the group-theoretic coefficient $a_\rho = 3$. It is again an integral class, as it must be.

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