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The Costs of Adjusting Labor: Evidence from Temporally Disaggregated Data

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The Costs of Adjusting Labor: Evidence from Temporally Disaggregated Data

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Abstract

I estimate the costs for establishments of hires and separations using a dynamic labor demand framework and matched employer-employee data from Germany, which records the exact dates of start and end of an employment spell. I estimate adjustment costs under different assumptions of adjustment frequencies. Under the assumption that establishments revise their labor demand every month, GMM estimates suggest hiring costs per employee of approximately 5,000 Euros, and costs of separations of 1,000 Euros. Hiring costs vary considerably between skilled (8,000 to 28,000 Euros per hire) and unskilled (4,000 to 8,000 Euros) labor. Spatial aggregation (large establishments) is associated with lower cost estimates, and only monthly adjustment frequencies yield estimates consistent with theoretical predictions.

1 Introduction

Estimating the costs associated with adjusting employment is a notoriously difficult endeavor. In particular, three difficulties in estimating adjustment costs have been ubiquitous. Can we infer anything about the structure of adjustment costs from data above the plant or firm unit, say from sectoral data? The answer, it turns out, is no, a point made most forcefully in Hamermesh (1989), where employment fluctuations aggregated over just 7 plants suggest frequent and incremental adjustment, while the individual plants exhibit intermittent adjustment. Only the former would be compatible with a purely convex cost structure of adjustment. The wide availability of micro-data now easily solves this problem. The second difficulty – temporal aggregation – is more difficult to tackle (see Hamermesh (1993)). We can reasonably assume that the appropriate unit to look at is the individual firm or plant, but at what frequency it revises its labor demand is unclear. One may look at the time period that elapses between two adjustments, but the employer or manager might have decided any number of times not to adjust within that period. Most data used in the literature dictate what time frame is used by the frequency of its waves, mostly quarterly or annual. This paper is distinct in the labor adjustment literature in using matched employer-employee data which identify the exact point of time of every employment flow into and out of an establishment, thus allowing the use of any adjustment frequency, from daily to annual. Out of four different time specifications (monthly, quarterly, half-annual, annual), I find that only the monthly results are compatible with a dynamic labor demand model allowing for asymmetric adjustment costs similar to the ones used in the literature. Finally, the third difficulty has been getting actual point estimates of the money-value of costs. Many studies have focused on inference of the structure of costs, but were not able to identify cost values due to identification issues (e.g. only parameters normalized by residual variances were identified), a recent example being Nilsen et al. (2006).

Despite these difficulties finding ways to estimate and infer adjustment costs is a worthwhile and important task and a starting point for answering a myriad of interesting

questions: Why did unemployment surge in the US but not in Germany during the 2009 recession, despite comparable contractions in GDP? Do we expect the labor market to adjust through wages or employment, and how is unemployment affected when adjusting employment is costly? Are technological parameters of production functions estimated consistently when adjustment costs are not accounted for? In any case, when modeling adjustment costs one needs to start with some idea of how they are incurred (are they fixed or variable, symmetric or asymmetric?), of their magnitude, and of the frequency of adjustment by employers. All of those issues have been reflected on earlier, and Hamermesh and Pfann (1996) provide a review of the earlier literature.

Typically the identification of fixed and/or non-convex adjustment costs has been achieved by exploiting the periods of inactivity and using some kind of latent variable model to maximize a likelihood function on the probabilities of expansion and contraction (Hamermesh (1989), Hamermesh (1992), Nilsen et al. (2006), and Varejão and Portugal (2007) using a hazard function estimation). The difficulty of this approach is how exactly the thresholds between the different regimes should be specified. Abowd and Kramarz (2003) provide the most direct method of estimating adjustment costs, using reported costs and employment changes using a cross-section of French establishments. Reported costs are regressed on a quadratic of hires and separations, where the constant is interpreted as fixed costs, and the coefficient on the quadratic as a measure of the convexity of costs. This has the advantage of circumventing temporal aggregation issues, but can capture only measured/recorded and reported costs. They find substantial costs of terminations, the fixed component often estimated at more than 150,000 Euros (1992 values). Hall (2004) uses annual sectoral data and identifies convex adjustment costs by the response of factor input ratios to factor price ratios. In the presence of adjustment costs, input ratios should be less responsive to changes in price ratios. Estimation results cast some doubt on this method, since the coefficients carry "wrong" signs for 10 out of 18 sectors (being estimated imprecisely, Hall interprets those as *no costs* for the said industries). Finally, Caballero et al. (1997) and Cooper et al. (2004) exploit information on the correlations of hours worked per employee and number of employees to estimate

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adjustment costs, the former relying on a theoretical and possibly wrongly measured construct (the "gap" between desired and actual employment), and the latter using iterative methods with the full dynamic problem of the establishments being solved numerically at every step of the iteration. While the latter approach maps closely from theory to estimation, it is computationally intensive and difficult to extend to more complicated cases, such as the inclusion of different types of labor.

This paper uses generalized method of moments estimation of the first-order-condition of the dynamic labor demand problem of an establishment. While GMM has become a standard application to dynamic panel models, to my knowledge it has not been applied in the adjustment cost framework. It has the advantage of being easily extended to heterogeneous labor applications by simply estimating as many first-order-conditions as there are factor inputs. All of the three difficulties mentioned above are – to some extent – dealt with. I obtain and compare results under different assumptions of adjustment frequencies, and coefficients for a model with constant marginal adjustment costs can be interpreted as Euro-costs per employee.

2   Framework

The establishment maximizes its expected present value of current and future profits by choosing how much labor to employ today, taking as given all prices. That decision will matter for its labor demand in the next period, because the cost of changing employment tomorrow will depend on employment today. This is intuitive and easy to see in a recursive formulation:

$$V(\mathbf{l_{t-1}}, \mathbf{I_t}) = \max_{\mathbf{l_t}} p_t q(\mathbf{l_t}) - \mathbf{w_t} \cdot \mathbf{l_t} - c(\mathbf{l_{t-1}}, \mathbf{l_t}) + \beta EV(\mathbf{l_t}, \mathbf{I_{t+1}}) \qquad (1)$$

Here c is the cost of adjusting labor, $\mathbf{w}$ is a wage vector for different kinds of labor. Labor types will be indexed by j, and I assume that adjustment costs are separable for types of labor: $c = \sum_j c_j(l_{t-1,j}, l_{t,j})$ The expectation here is formed over any kind of

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information (**I**) that will be revealed at the beginning of next period, including wages, prices, productivity, and demand. The establishment enters the period with its employment from the previous period, it observes this period's wages, prices, etc. and decides how many new employees to employ. Importantly, adjustments are NET adjustments. There are two reasons for this: The first is theoretical. In the framework above it makes no sense for the establishment to hire and separate from one labor type in the same period, since every worker of a certain type is assumed to be equal. Second, comparing small and big establishments will provide some information on how aggregation within an establishment affects estimates of adjustment costs. Big establishments can be thought of as consisting of subdivisions, some of which might expand, and others contract within the same time period. Costs will be incurred, but in the aggregate no adjustments will be recorded. For small establishments net adjustment will coincide with gross adjustment most of the time. To be sure, legal and technological differences between small and big establishments make it impossible to attribute differences in estimates solely to aggregation effects, and results should be interpreted as suggestive. A priori I would hypothesize that adjustment is more costly for larger establishments, because large establishments are subject to stricter legal rules concerning labor adjustment and relations. Furthermore, they probably command and employ more resources in the hiring process. However, if adjustment costs are linear (a discussion of German labor relations and different cost specifications follows below), and aggregation will suggest frequent adjustment, the marginal cost of adjustment is likely to be underestimated. Aggregation and structural/institutional factors would work in opposite directions, so that smaller estimates for larger establishments would be conservative evidence for aggregation bias.

To keep the exposition simple, I solve equation (1) with one type of labor in the main text. For heterogeneous labor I will make explicit any additional assumptions.

2.1 Convex costs

To start with the simpler model, I first derive labor demand for a convex adjustment cost function from Hamermesh and Pfann (1996) allowing for asymmetric costs for hiring and separations:

$$c(l_{t-1}, l_t) = \frac{a}{2}(l_t - l_{t-1})^2 - b(l_t - l_{t-1}) + \exp(b(l_t - l_{t-1})) - 1$$

If $b > 0$, the marginal cost of a positive adjustment exceeds that of a negative adjustment, and vice versa if $b < 0$. Solving (1) with this is simple, since the value function is differentiable. Using the envelope theorem I get

$$p_t q_t' - w_t - a(l_t - l_{t-1}) + b(1 - \beta) - be^{b(l_t - l_{t-1})} - \beta a l_t$$
$$+\beta E\left(al_{t+1} + be^{b(l_{t+1} - l_t)}\right) = 0$$

Assume that $E(l_{t+1}|l_t) = l_t$. Given all information today, let the establishment treat next period's optimal labor demand as a random variable distributed normally with mean l_t. Figure 1 plots a histogram of $l_{t+1} - l_t$. The distribution is symmetric around zero, lending some justification for this assumption. Then $e^{bl_{t+1}}$ is distributed log-normally with $E(e^{bl_{t+1}}) = e^{bl_t + (b^2\sigma^2)/2}$. We can thus rewrite the above equation as

$$p_t q_t' - w_t - a(l_t - l_{t-1}) + b(1 - \beta) - be^{b(l_t - l_{t-1})} + \beta be^{(b^2\sigma^2)/2} = 0 \tag{2}$$

2.2 Linear, asymmetric costs

This case is a little more complicated because of the non-differentiability of the value function at $l_t = l_{t-1}$. Write the problem now as:

$$V(l_{t-1}, \mathbf{I_t}) = \max_{l_t} p_t q(l_t) - w_t l_t - \tau^+ \mathbf{1}_{l_t > l_{t-1}}(l_t - l_{t-1}) - \tau^- \mathbf{1}_{l_{t-1} > l_t}(l_{t-1} - l_t) + \beta EV(l_t, \mathbf{I_{t+1}}) \tag{3}$$

The cost of hiring a worker is τ^+, the cost of a separation is τ^-. The indicator functions take on the value one, if the expression in the subscript becomes true. For example,

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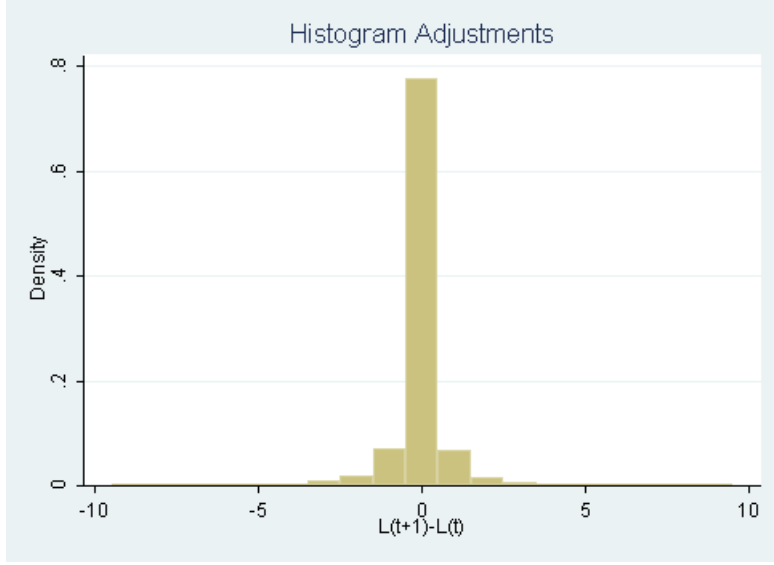
Figure 1: Distribution of $L_{t+1} - L_t$, cut at adjustments ≥ 10

the direct cost of hiring three more workers is $3\tau^+$. The way to solve this is by solving two constrained optimization problems. If we let the establishment solve its problem restricting it to hiring or not changing its employment, this is equivalent to solving

$$\max_{l_t} p_t q(l_t) - w_t l_t - \tau^+(l_t - l_{t-1}) + \beta EV(l_t, \mathbf{I_{t+1}}) \quad \text{s.t.}$$

$$l_t \geq l_{t-1}$$

The solution is

$$p_t q'(l_t) - w_t - \tau^+ + \beta E \frac{\partial V(l_t, \mathbf{I_{t+1}})}{\partial l_t} = 0 \quad \text{if} \quad l_t > l_{t-1} \tag{4}$$

$$p_t q'(l_t) - w_t - \tau^+ + \lambda_t^+ + \beta E \frac{\partial V(l_t, \mathbf{I_{t+1}})}{\partial l_t} = 0 \quad \text{if} \quad l_{t-1} \geq l_t \tag{5}$$

Here, λ^+ is the Lagrange multiplier for this problem. Similarly, the solution when constrained not to hire, is

$$p_t q'(l_t) - w_t + \tau^- + \beta E \frac{\partial V(l_t, \mathbf{I_{t+1}})}{\partial l_t} = 0 \quad \text{if} \quad l_{t-1} > l_t \tag{6}$$

$$p_t q'(l_t) - w_t + \tau^- - \lambda_t^- + \beta E \frac{\partial V(l_t, \mathbf{I_{t+1}})}{\partial l_t} = 0 \quad \text{if} \quad l_t \geq l_{t-1} \tag{7}$$

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The establishment must have satisfied equation (4) if it hired, equation (6) if it contracted, and equations (5) and (7) if it didn't change its employment. It remains to find an expression for $\frac{\partial V(l_t, \mathbf{I_{t+1}})}{\partial l_t}$. Assuming the value function is differentiable in its state, denote the solution to equation (4) by $l_{t,h}^*$ (for hiring) and its associated value by V_h^*, to equation (6) by $l_{t,s}^*$ (for separation) and V_s^*. If inactivity is optimal, denote this by l_{t-1}^* and V_{in}^*. Then

$$V(l_{t-1}, \mathbf{I_t}) = \max\{V_h^*, V_{in}^*, V_s^*\}$$

where

$$V_h^* = p_t q(l_{t,h}^*) - w_t l_{t,h}^* - \tau^+ \left(l_{t,h}^* - l_{t-1}\right) + \beta E V \left(l_{t,h}^*, \mathbf{I_{t+1}}\right)$$

$$V_{in}^* = p_t q(l_{t-1}^*) - w_t l_{t-1}^* + \beta E V \left(l_{t-1}^*, \mathbf{I_{t+1}}\right)$$

$$V_s^* = p_t q(l_{t,s}^*) - w_t l_{t,s}^* + \tau^- \left(l_{t,s}^* - l_{t-1}\right) + \beta E V \left(l_{t,s}^*, \mathbf{I_{t+1}}\right)$$

Equation (3) becomes the first line if hiring is optimal, the second line if inactivity is optimal, and finally the third line if separation is optimal. Taking derivatives for each line with respect to the state variable l_{t-1} we get:

$$\frac{\partial V_h^*}{\partial l_{t-1}} = \left(p_t q'(l_{t,h}^*) - w_t - \tau^+ + \beta E \frac{\partial V(l_{t,h}^*, \mathbf{I_{t+1}})}{\partial l_{t,h}^*}\right) \frac{\partial l_{t,h}^*}{\partial l_{t-1}} + \tau^+$$

$$\frac{\partial V_{in}^*}{\partial l_{t-1}} = p_t q'(l_{t-1}^*) - w_t + \beta E \frac{\partial V(l_{t-1}^*, \mathbf{I_{t+1}})}{\partial l_{t,h}^*}$$

$$\frac{\partial V_s^*}{\partial l_{t-1}} = \left(p_t q'(l_{t,s}^*) - w_t + \tau^- + \beta E \frac{\partial V(l_{t,s}^*, \mathbf{I_{t+1}})}{\partial l_{t,s}^*}\right) \frac{\partial l_{t,s}^*}{\partial l_{t-1}} - \tau^-$$

The terms in parentheses in the first and third lines will be zero - the familiar principle of optimality. Note that the second line equals $\tau^+ - \lambda_{t-1}^+$ and $\lambda_{t-1}^- - \tau^-$ from equations (5) and (7). Call this expression ζ_{t-1}. Denote the establishment's probability of hiring next period by π_t^+, and its probability of separation by π_t^-. We then have

$$E \frac{\partial V(l_t, \mathbf{I_{t+1}})}{\partial l_t} = \pi_t^+ \tau^+ - \pi_t^- \tau^- + (1 - \pi_t^+ - \pi_t^-) \zeta_t$$

Finally, writing the first order conditions from equations (4) to (7) in one equation and substituting the expression for the expected value, we can write:

$$p_t q'(l_t) - w_t + \tau^- (\mathbf{1}_{l_{t-1}>l_t} - \beta \pi_t^-) - \tau^+ (\mathbf{1}_{l_t>l_{t-1}} - \beta \pi_t^+) + \zeta_t (\mathbf{1}_{l_{t-1}=l_t} - \beta(1 - \pi_t^- - \pi_t^+)) = 0 \quad (8)$$

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Thus, if the cost parameters τ can be treated as coefficients, this specification predicts a positive coefficient on $\mathbf{1}_{l_{t-1}>l_t} - \beta\pi_t^-$ and a negative coefficient on $\mathbf{1}_{l_t>l_{t-1}} - \beta\pi_t^+$, while the coefficients have the nice interpretation of being marginal costs of separations and hires *relative to inactivity*, respectively. The intuition is straightforward: A hire costs τ^+, but reduces the chance of incurring hiring costs next period. Separations enter with a positive sign because the marginal effect on sales is negative. The term multiplying ζ_t is collinear with the terms multiplying the τ and is dropped in the subsequent analysis. Hires and separations are observed, but once again the establishment's expectations (the π) are not. I will have to infer these probabilities. Append an error term to (8) to account for any deviations from the establishment's "true" expectations from the author's best guess and for any other measurement and simplification errors.

2.3 Fixed costs

The model does not include fixed costs of labor adjustment. The reason for this omission is weak - if any - identification of fixed costs from linear costs as specified in the previous section. Convex and linear variable costs lead to distinct predictions. The first should lead to frequent and small adjustments, the second to long periods of inactivity and consequently to a large fraction of inactivity for any cross section. However, inactivity will also be a property of fixed costs. There remains some hope for identification from the size of adjustment conditional on any adjustment happening. However, this is complicated by two factors: First, one needs assumptions on what labor demand would be in the absence of any adjustment costs, since linear variable costs are likely to dampen both upward and downward adjustments, while the marginal cost of hiring or separating the *second* worker is zero in the case of purely fixed costs. Presumably, adjustments in this latter scenario would be bigger. Second, the data for this paper include many establishments with fewer than ten employees, and employment change is not continuous, not even approximately so for small establishments, amplifying the aforementioned problem. Thus, one needs to be cautious in interpreting results. If costs are a mix of fixed and linear variable costs,

and the specification attributes all to variable costs, then the cost coefficients will be approximate measures of average, but not of marginal adjustment costs.

2.4 Establishments' adjustment frequencies

A major advantage of the matched employer-employee data used in this paper is that every start and every termination of an employment spell can be exactly dated for all employees of the establishments in this panel. Most of the literature has been restricted to temporal aggregation at the frequency at which the panel was available. Employment adjustment has been identified off the changes in stocks of employment between two points of time given by the panel frequency - the net adjustment. While this seems reasonable for a sufficiently short time interval, it is a clear limitation for low-frequency, e.g. annual panels. An establishment hiring x employees in March and dismissing them in September will not have a recorded adjustment from January to December, yet both decisions must have been responses to something and presumably have been costly. The data at hand allow distinguishing between net and gross flows, and with a different framework an identification strategy using gross changes might be possible. Here, I chose to follow the literature in keeping the general dynamic programming framework, but to estimate the model for different time periods separating t from $t + 1$. This has three advantages in regard to checking robustness and sensitivity of results: First, for shorter time periods net adjustment will be closer to gross adjustment, and running estimations for different time periods will give us an idea of how sensitive results are to timing assumptions. Second, I can compare results from the full sample to a sample for which gross and net adjustments are the same, providing yet another test of aggregation bias. Third, the above formulation can in principle be extended to the case where an establishment incurs costs from current adjustment, but also still from adjustment in the previous period.

2.5 Production function

I use the translog production function for all estimates:

$$\ln q = \alpha_0 + \sum_j \left(B_j \ln l_j + (1/2) B_{jj} (\ln l_j)^2 + \sum_{k \neq j} B_{jk} \ln l_j \ln l_k \right)$$

Taking the derivative of q with respect to l_j and multiplying this by the price I arrive at

$$pq'_j = R \times \left(\frac{B_j}{l_j} + \frac{B_{jj} \ln l_j}{l_j} + \sum_{k \neq j} B_{jk} \frac{\ln l_k}{l_j} \right) \qquad (9)$$

where R denotes the sales of the establishment. This production function has the obvious and well-known property of being very flexible (see Berndt and Christensen (1973) for a discussion). On the other hand, the estimates will not be informative for any technological inferences. When more than one type of labor is considered ($j > 1$), equation (9) might not be defined for some establishments (due to logs of and divisions by zero) and for estimation purposes dummies for this case will be included in the regression equations, introducing 'kinks' into the production function without any concrete interpretation. Furthermore, the quadratic and interaction terms will very likely introduce ranges of production with some and other ranges with different properties, e.g. increasing returns to scale for some labor-type combinations, and decreasing returns for others. Yet, the use of a flexible production function is better suited for the problem at hand. The production technology is not interesting in itself for the purpose of this paper, and it is more important to avoid biasing adjustment costs results by imposing technological restrictions.

A more serious problem could be the omission of any other factor of production, notably capital. Suppose capital is costly to adjust and it is complementary in production to labor. If labor is adjusted together with capital, some or much of capital adjustment costs might be attributed to costs of adjusting labor (see for example Asphjell et al. (2010) for a study of interrelated factor demand). Table 1 reports statistics relating investment to net labor adjustment. Investments are reported for a year, so the statistics

Table 1: Investment and Labor Adjustment

		fraction hires	fraction separations
small	I>0	3.2	3.6
	I=0	3.3	4.3
large	I>0	20.7	22.7
	I=0	15.9	21.4

fraction: Fraction of establishments in %, correlation: Correlation between size of adjustment and size of investment

relate to annual adjustments. Large establishments are more likely to adjust employment when they invest. This is particularly true for hires. 20.7% of large establishments have hired in a year in which they have reported investment, compared to 15.9% of hiring by establishments which did not invest. However, investing firms have also been slightly more likely to contract than those firms which did not invest. For small establishments, hiring does not seem to be effected much by investment, while separations occur more frequently at establishments with zero investment. These statistics suggest that investment might have been labor-complementing. Furthermore, small establishments seem to have exploited complementarities by fewer separations, while large establishments have adjusted through hiring. While labor-capital complementarities are present, they seem to be more important for bigger establishments.

3 Institutional characteristics

A full characterization of the German labor market and labor relations are out of the scope of this paper, but a general knowledge of institutions governing or influencing adjustment costs is helpful. Apart from a relatively high coverage of employees by collective agreements (in 2009 56% of all employees in West and 38% in East Germany were covered by sectoral collective agreements, see IAB (2010)), three aspects about labor markets in

Germany are important for adjustment costs:

1. Establishments in Germany engage in training youth entering the labor market. This happens in the framework of the *dual system*: Besides working and learning at an establishment, apprentices attend school for one or two days of the week. Costs are covered by employers, but net costs of training are notoriously difficult to calculate. Harhoff and Kane (1997) list studies putting the annual costs of training one apprentice at a range of 5,000$ to 10,000$ in 1990 dollars (see the same study for a more detailed overview of this training system).

2. At establishments with at least five permanent employees, workers' councils can be set up at the initiative of the employees. These councils need to be informed and consulted for certain decisions, including restructuring of employment and recruitment, and enjoy certain co-determination rights for procedures related to dismissals. See Addison et al. (2001) for a lengthier discussion.

3. Employees at establishments with at least five (since 2004 at least ten) permanent employees are protected by certain provisions of the *Kundigungsschutzgesetz* - dismissal protection bill: Roughly, employees can be dismissed only as a consequence of misconduct or special business situations (drop in sales, organisational changes). See BMAS (2010).

In general, adjustment is more regulated – and appears to be more costly – for large establishments, tenured personnel, and high-wage personnel, since compensation/severance payments are based on these measures.

4 Data

The data are the matched employer-employee panel (LIAB), of the German IAB (Institute for employment research). Several versions of the LIAB exist, distinguished by

cross-sectional and longitudinal coverage. This paper uses longitudinal version 3, with 14 years of observations for some 4,200 establishments. The data are described at length in Jacobebbinghaus (2008). Here are the basics: The data consist of two sources, the establishment panel (short: panel), and the employment history (short: history). Once a year a survey for the establishment panel is conducted, and establishments are asked to report sales, employment, investments et cetera. This panel is then matched with administrative data of the German federal employment agency (history). Every new employee and separation has to be reported by the employer to this agency. Wages are also reported. The wage data here are known to be more reliable than survey data, since they are based on the actual payments reported to the agency. Misreporting is unlawful. Occupations not subject to social security contributions (certain state employees, doctors with private practices, firm-owners et cetera) are not covered by the data, but some 80% of the German employment is. Thus, I know for any given day how many people are employed in any establishment covered by the LIAB, how much they earned, their education, and age. The education variable is reported by employers, and reporting it is not mandatory. As a consequence, it has more missing values and for some observations exhibits inconsistencies over time. Throughout this paper, I am using the unbalanced panel.

4.1 Sales

Sales are from the panel and reported as the total amount of sales in the last year. Establishments which reported budgets instead of sales are excluded, because they are probably non-profits. Since I am mostly using a shorter time-frame than years (the reporting frequency of the panel), yearly sales volumes have to be divided to shorter time periods (months in this example). One way to do it would be to divide equally across all twelve months, but there would be unrealistic jumps from December to January. Another possibility would be to check quarterly aggregate sales for different industries and adjust the establishments' sales accordingly. I chose to do the following: I minimize the sum of
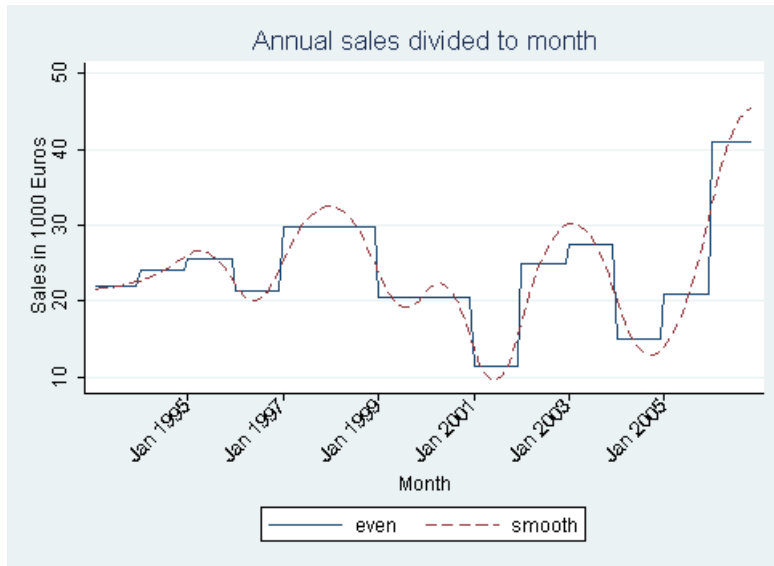
Figure 2: Annual sales divided to months, even and smooth

squared sales distances from month to month over the choice of monthly sales, restricting the sum of sales for a year to equal the sales reported by the establishment:

$$\min_{\{s_t\}_{t=1}^{T}} J = \sum_{t=2}^{T}(s_t - s_{t-1})^2 \quad \text{s.t.}$$

$$\sum_{t=1}^{12} = S_1$$

$$\sum_{t=13}^{24} = S_2$$

$$\vdots$$

with S_1 the reported sales in the first year, S_2 in the second year, and so forth. This is equivalent to smoothing the jumps between monthly sales. Figure 2 depicts an establishment with 10 years of data.

The blue, solid line shows the sales data divided evenly across the months of a year, the dashed red line presents the sales data after the described procedure. I estimated adjustment costs for both of the constructed monthly sales data series with quantitatively

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very similar results, lending some support to the robustness of the results to the choice of how to divide the reported annual sales to months. The results presented in this paper are for the smoothed sales series.

4.2 Employment

I estimate adjustment costs for different labor aggregates and for different time perspectives (month, quarter, semester, year). I have aggregate labor, and labor divided into four categories - old and young, skilled and unskilled. Old workers are workers over 40, skilled workers are workers with higher education or college-qualifying degrees (at least 12 years of primary and secondary education) together with professional training. Only full-time workers are considered, and workers with very low (less than 5 Euros daily wage in January 1993 and inflated by an annual 2.5% thereafter) or high wages (more than 400 Euros daily wage in January 1993 and inflated by an annual 2.5% thereafter) are excluded. A problem arises if some type of worker is not employed. First, I do not observe wages for that type (see next section). Also, the production function becomes undefined, because of logs of zero and division by zero. Thus, for non-employment, in the estimation I include dummies for non-employment, and the undefined parts of the first order conditions are set to zero. Obviously, this problem does not occur if I have only one type of labor, because the establishments always employ at least one worker.

A hire is recorded if an employee starts working at the establishment and was not working there the previous day. A separation is recorded if an employee works at an establishment one day, but not the next. Note that with this definition recalls and temporary quits will count as adjustment, but immediate contract renewals will not.

4.3 Wages

If an establishment employs at least one worker of a certain type j, I take the median wage paid for that type of labor in that establishment as $w_{j,t}$. I take the median rather than the

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mean, because of some censoring issues with the wage data, but for the whole sample the mean and median wages are very close for the different types of labor. More serious is the problem of establishments which have not employed a certain type of labor. Note that this is not just a non-response or random missing value problem. Since there are many small establishments in the data, and since net and gross adjustments for small establishments will overlap much more than for bigger establishments, dropping establishments with unobserved wages would be undesirable. Furthermore, those establishments have chosen not to employ, even though they could have done it for some (presumably) observed wage. It remains to "guess" what that wage was. I take that wage to be the median wage of that type of labor in the observed state, industry, year, and size category. Here is an example: A small establishment in the consumer goods manufacturing sector located in the state of Bavaria has not employed any unskilled, old worker in may 2000. What was the wage it observed for unskilled, old workers? The median wage of unskilled, old workers in Bavaria in the consumer goods industry in small establishments in 2000.

4.4 Expectations

Recall the final estimable optimality conditions in the linear cost case in equation (8), all variables are observed or imputed, except the establishments' expectations for hiring, separating, or inactivity for the next period (π^+, π^-), and the β. I set β to a number to make the yearly discounting factor equal to 0.95. For the expectations I pursue the following strategy: I predict the probabilities using an ordered probit model where I regress adjustment today (the categories being hiring, inactivity, separation) on adjustment last period, size of adjustment last period, wages last period, sales last period, and a full set of state and sectoral dummies. The predictions are made separately for small (ten or less employees) and large establishments. As a robustness check, I create cells of establishments of equal period, size, sector, state, and adjustment, with potentially T*2*14*16*3 cells, where T is the number of time periods. The fraction of establishments in a cell hiring the next period is π^+ for establishments in that cell (the fraction is calculated

excluding the establishment to which π^+ is being assigned). Put differently, given observable characteristics of an establishment and its adjustment today, I guess that next period it will behave like similar establishments which have made the same adjustment in this period. The latter approach has the advantage of avoiding any distributional assumptions, but creates a good number of empty or single-element cells. Moreover, the establishments in the single-element cells are likely to be a non-random sub-sample of all establishments. The results with this approach did not differ qualitatively, but estimates were comparable for hiring and distinctly higher for separation costs (by a magnitude of four).

5   Estimation

Estimation is by GMM and follows the procedure outlined in Arellano and Bond (1991) – a workhorse in estimating dynamic panel models – with little differences due to nonlinearities in this application. Denote establishments by i and rewrite the first-differences of equations (2) and (8)

$$\Delta(p_{i,t}q'_{i,t}) - \Delta w_{i,t} - a\Delta(l_{i,t} - l_{i,t-1}) - b\Delta(e^{b(l_{i,t}-l_{i,t-1})}) + \Delta(\varepsilon_{i,t}) = 0 \qquad (2.2')$$

$$\Delta(p_{i,t}q'_{i,t}) - \Delta w_{i,t} + \tau^-\Delta(\mathbf{1}_{l_{i,t-1}>l_{i,t}} - \beta\pi^-_{i,t}) - \tau^+\Delta(\mathbf{1}_{l_{i,t}>l_{i,t-1}} - \beta\pi^+_{i,t}) + \Delta(\varepsilon_{i,t}) = 0 \quad (2.8')$$

Let $\varepsilon_{i,t} = \mu_i + \delta_t + u_{i,t}$ The first difference takes out any establishment-fixed effects. With a vector of instruments Z such that $E(\Delta u|Z) = 0$ the equations above can be estimated by GMM. The typical assumption to qualify endogenous variables lagged two periods and more as instruments is no serial correlation in the $u_{i,t}$. Endogenous variables in $t-2$ will be uncorrelated with $u_{i,t}$ and $u_{i,t-1}$ and valid instruments for $\Delta(l_{i,t} - l_{i,t-1})$, $\Delta(\mathbf{1}_{l_{i,t}>l_{i,t-1}} - \beta\pi^+_{i,t})$ et cetera, and possibly weak instruments for $\Delta(p_{i,t}q'_{i,t})$, since the second lags do not enter into this last expression. Summarizing, since wages and period dummies are assumed exogenous, the instrument vector Z for equation (2.2') consists of: Differenced wages and period dummies, second and third lags of the components

of $p_{i,t}q'_{i,t}$ (levels), of l_t, and of e^{-l_t}. For (2.8') Z consists of: Differenced wages and period dummies, second and third lags of the components of $p_{i,t}q'_{i,t}$ (levels), second and third lags of $(\mathbf{1}_{l_{i,t-1}>l_{i,t}} - \beta\pi^-_{i,t})$ and $(\mathbf{1}_{l_{i,t}>l_{i,t-1}} - \beta\pi^+_{i,t})$. Where I used different lags for robustness checks, this is made explicit. Equations (2.2') and (2.8') are then estimated by 2-step GMM using Z as instruments.

6 Results

The benchmark results are for the asymmetric, linear cost model with monthly adjustments. I also discuss results for the convex model and for different frequencies of adjustment. The convex model implies sensible adjustment costs for only a small adjustment range, and longer than monthly frequencies contradict theoretical predictions.

Table 2 reports results for the model with one type of labor and monthly adjustments. The first panel lists results using all time periods, but dummies for quarters (due to computer memory constraints). The second uses quarter dummies for the middle third of the panel (from September 1997 to April 2002). The third panel covers the same period but uses monthly dummies. Finally, the fourth panel uses monthly dummies but only observations from the year 2000, corresponding to the middle of the time covered by the panel. The reason for comparing these four sets of data is the unusual long dimension of the panel (166 months) and the uncertainties associated with estimates and test procedures for long panels. Specification 1 is on the full sample, specification 2 excludes establishments for which gross adjustment is different from net adjustment (hiring and separations taking place within the month), 3 uses only small establishments (less than 10 employees), and 4 only big establishments.

All coefficients had the predicted signs (adjustment costs are actually positive). Hiring is costly, at around 4,000 Euros per hire. Note that neither prices nor wages have been adjusted at any point, so we might think of these as end of 1999 - beginning of 2000 Euros, as this is the middle of our sample. This holds for specifications 1 to 3, but the

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Table 2: Homogeneous Labor - Adjustment costs

	1 All		2 Gross=Net		3 Small		4 Big	
	τ^+	τ^-	τ^+	τ^-	τ^+	τ^-	τ^+	τ^-
quarter dummies	3,745***	877***	4,447***	181	5,164***	1,015***	1,715***	396***
	(224)	(202)	(274)	(215)	(410)	(387)	(108)	(110)
Sargan $\chi^2(df=5)$	53.2***		37.4***		0.83		67.5***	
n	468,628		441,738		295,837		172,791	
quarter dummies	3,754***	1,132***	4,511***	344	5,068***	1,030*	1,677***	533
(Sep 97-Apr 02)	(334)	(339)	(389)	(374)	(604)	(596)	(192)	(185)
Sargan $\chi^2(df=5)$	20.6***		11.2**		4.8		7.5	
n	184,118		173,208		116,925		67,193	
month dummies	3,926***	1,073***	4,537***	420	5,173***	987*	1,722***	429***
(Sep 97-Apr 02)	(331)	(325)	(376)	(350)	(594)	(572)	(182)	(163)
Sargan $\chi^2(df=5)$	18.8***		15.7***		5.2		12.0**	
n	184,118		173,208		116,925		67,193	
month dummies (2000)	3,374***	1,613**	4,704***	53	6,829***	151	959***	635***
	(659)	(662)	(789)	(686)	(1,552)	(1,173)	(229)	(233)
Sargan $\chi^2(df=5)$	5.0		3.0		0.7		9.6*	
n	44,482		41,876		29,119		15,363	

Note: Standard errors in parentheses. P-values:***<0.01, **<0.05, *<0.1

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estimate drops for big establishments. I suggested above that for legal and institutional reasons we should expect higher costs for big establishments, but that aggregation might bias results downward. Comparing this estimate to the second specification, which excludes establishments which hire and face separations within the same month, suggests that aggregation bias for hiring costs is substantial, at over 50%. Third, separations seem to be very inexpensive. Significant estimates of separation costs range around 1,000 Euros, and often enough the costs are insignificant.[1] This might be surprising at first glance, but mind that every separation is counted, including retirements[2] and voluntary quits, which are both presumably costless or associated with small costs for the employer. Survey results from the German Micro-census suggest that one fifth to one fourth of all separations are involuntary on the part of the employee (e.g. firings, own calculations), so that 4,000 to 5,000 Euros might be a more accurate estimate of firing costs. Fourth, the over-identification tests in the full samples with close to half a million observations almost always reject the exogeneity assumption for the instruments, except for small establishments, very likely due to more homogeneity within this sub-sample, whereas heteroscedasticity is likely to be an issue for the big establishments and for the full sample. Going to subsequently smaller samples (panels two to four), the Sargan test statistics become smaller without changing the cost estimates, and for the sample covering observations from the year 2000 only, exogeneity is not rejected in any of the specifications, except weakly for big establishments. What explains this sensitivity of overidentification tests to the sample size? I suspect that the high number of observations leads to more type I errors - or the reverse might be true: lower number of observations result in the test NOT rejecting exogeneity when it should. I circumvent this technical problem here by checking the robustness of the results to the use of different lags. Using higher order lags allows for some autocorrelation in the residuals $u_{i,t}$, but comes at the expense of becoming less "informative" for the endogenous variables. So far, second and

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[1] The non-parametric approach of predicting adjustment probabilities estimates separation costs of 2,400 Euros for the full sample.

[2] Pensions in Germany are for the most part through a mandatory governmental, pay-as-you-go system. Thus, most employers do not pay pensions for retired personnel.

Table 3: Homogeneous Labor - Different lags

	1		2		3	
	All		Small		Big	
	τ^+	τ^-	τ^+	τ^-	τ^+	τ^-
Lags 3,4	3,326***	1,551***	5,882***	405	1,731	413
	(429)	(471)	(491)	(391)	(287)	(369)
Lags 4,5,6	5,172***	-814*	6,122***	0	2,643***	-765*
	(500)	(492)	(489)	(405)	(368)	(421)

Note: Standard errors in parentheses. P-values:***<0.01, **<0.05, *<0.1

third period lags of endogenous variable levels have been used as instruments.

Table 3 reports estimates for two different sets of instruments. The upper panel uses the third and fourth lags, and the lower panel the fourth to sixth lags of the endogenous variables. The use of lags 3 and 4 gives results in the ballpark of the previous estimation. With lags 4, 5, and 6 estimated hiring costs increase somewhat, but the results for separation costs become nonsensical. The instruments will become weaker the further we go back in the lags for the instruments.

Table 4 reports results for different time periods – a comparison for which the data set is perfectly suited. Flows and stocks of employees are calculated over quarters, semesters, and years, possibly successively increasing the inaccuracy of measured adjustment.

We see that – except for semesters – in absolute values the coefficients increase for longer time periods, which is not surprising given that this effectively allows adjustment costs to be incurred over a longer time interval. Looking at small establishments, hiring costs seem to increase proportionally to the length of the time interval (by a factor of five for quarters and semesters, and by a factor of 10 for the year). However, the model in months is the only one which always yields coefficients in accordance with theoretical predictions. Note in particular the frequent negative sign for τ^-. This does not neces-

Table 4: Homogeneous Labor - Different time periods

	1		2		3	
	All		Small		Big	
	τ^+	τ^-	τ^+	τ^-	τ^+	τ^-
Monthly	3,745***	877***	5,164***	1,015***	1,715***	396***
	(224)	(202)	(491)	(391)	(287)	(110)
Quarterly	14,073***	-2,764***	25,085***	-2,801***	8,369***	-2,569***
	(1,328)	(607)	(2,253)	(852)	(749)	(432)
Half-annual	1,585**	-869	23,171***	-7,167***	3,296***	-1,904***
	(777)	(544)	(2,574)	(956)	(271)	(172)
Annual	52,464***	-22,252***	47,781***	-4,961**	-13,276*	6,879*
	(15,232)	(6,822)	(12,898)	(2,151)	(7,426)	(3,782)

Note: Standard errors in parentheses. P-values:***<0.01, **<0.05, *<0.1. Monthly specification includes quarter dummies. All other specifications include corresponding time dummies.

sarily reject the use of lower than monthly frequencies. One could as well argue that the model specification is rejected. While this might be true, it would amount to a conceptual rejection of the setup of many dynamic labor demand models. Yet, the results are at odds with papers which have used quarterly and annual data but have not found cost estimates inconsistent with theoretical predictions, such as Lapatinas (2009), Ejarque and Portugal (2007), and Cooper et al. (2004). However, these studies impose symmetry in the variable components and sometimes fixed component of hiring and firing costs. I have repeated the estimations with different time periods under the constraint $\tau^+ = \tau^-$. The results are reported in table 5. The results obtained under the constraint $\tau^+ = \tau^-$ are consistent with our expectation of positive adjustment costs, and the temporal aggregation issue is still apparent. Quarterly adjustment results are somewhat close to and higher than monthly adjustment results, but going to lower frequency adjustment results in a dramatic drop of the estimated costs. While this seems to lend more credibility to the use of quarterly data, one should keep in mind that allowing for asymmetry

Table 5: Homogeneous Labor - Different time periods, Symmetric costs

	1	2	3
	All	Small	Big
	τ	τ	τ
Monthly	2,345***	3,062***	1,178***
	(117)	(155)	(48)
Quarterly	2,833***	5,304***	1,465***
	(370)	(512)	(90)
Half-annual	100**	567	129***
	(45)	(384)	(11)
Annual	212***	492	348
	(59)	(329)	(360)

Note: Standard errors in parentheses. P-values:***<0.01, **<0.05, *<0.1. Monthly specification includes quarter dummies. All other specifications include corresponding time dummies.

in hiring and separation costs resulted in *negative* separation costs only when quarterly adjustment was assumed.

How do the estimates compare with a model of convex costs? Note first, that the actual behavior of inactivity of most establishments is more consistent with a linear cost model than with a convex cost specification. Since one model does not nest the other, formal tests of misspecification can not be performed. However, with comparable sample sizes, and the same number of parameters and moment conditions, first step minimized GMM objective function values were higher for the convex model for the full sample, for small establishments, and for establishments with gross adjustment equaling net adjustment. Not surprisingly, in this sense the convex model had a worse "fit" than the linear model. Table 6 reports estimates of the convex model. The first column of any of the four samples restricts the parameter b in equation (2) to be zero, imposing symmetry of hiring and separations. The second column adds the parameter b, allowing

Table 6: Homogeneous Labor - convex costs

| | (1) | | (2) | | (3) | | (4) | |
	All		Gross=Net		Small		Big	
a	241**	-0.007***	9***	-0.002***	382**	29	40**	-2.6
(quadratic)	(101)	(0.001)	(3)	(0.0004)	(165)	(38)	(19)	(4.6)
b		-0.002***		1,730***		1.3***		-20.9***
(exponential)		(0.0001)		(77)		(0.1)		(0)

Note: Standard errors in parentheses. P-values:***<0.01, **<0.05, *<0.1. Regressions include quarter dummies.

for asymmetry. In the symmetric case, the coefficient is always significantly positive, as it should be, and we see the familiar pattern that cost estimates are much lower for big than for small establishments. The coefficient for establishments with gross=net adjustment is lower in absolute value than the one for big establishments, which is rather puzzling. The estimates for small (big) establishments imply adjustment costs of 191 (20) Euros for an adjustment by one, 4775 (500) Euros for an adjustment by five, and 19,100 (2,000) Euros for an adjustment by ten employees. The asymmetric model fails to yield sensible estimates for big establishments, most likely due to the difficulty of fitting a cost curve which becomes very steep very fast to a sample with a great degree of size and adjustment heterogeneity. The coefficients imply *negative* – if tiny – costs for expanding the pool of employees. For small establishments the *qualitative* results from the linear model are replicated: Hiring is more costly than separations. The estimates imply that hiring one worker costs 16 Euros, hiring 5 workers costs 204 Euros per hire, and hiring 10 workers costs 44,000 Euros per hire. Going beyond that leads to a rapid explosion of costs. These numerical examples illustrate a major problem with convex cost models: The model fits reasonably well for a certain range, but becomes unrealistic for big (and small) adjustments. Thus, a comparison of results and implied adjustment costs seem to favor the linear adjustment cost model.
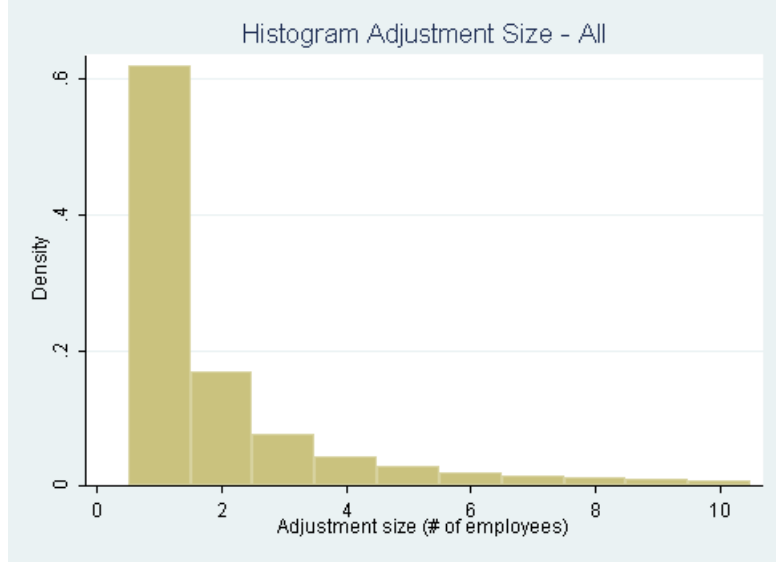
Figure 3: Size of adjustment if any - all establishments

I turn to the case of heterogeneous labor with linear costs. Unfortunately, the derivations from section 2 do not go through without further assumptions, due to possible complementarities in different types of labor. The first order condition for the employment of type j labor becomes:

$$p_t \frac{\partial q_t}{\partial l_{j,t}} - w_{j,t} - \tau_j^+ \left(\mathbf{1}_{l_{j,t}>l_{j,t-1}} - \beta \pi_{j,t}^+\right) + \tau_j^- \left(\mathbf{1}_{l_{j,t-1}>l_{j,t}} - \beta \pi_{j,t}^-\right) + \beta \sum_{k \neq j} \frac{\partial l_{k,t}}{\partial l_{j,t}} \left(\pi_{k,t}^+ \tau_k^+ - \pi_{k,t}^- \tau_k^-\right) = 0$$

This condition differs from the previous equation in the inclusion of the last summation (see appendix A for a full derivation). Note that the last term includes the change in the optimal choice of type k labor, when the optimal choice of type j labor changes. For the estimation I will assume the terms in the summation to be zero, while the cross-terms in the production function are preserved. To gauge how restrictive this assumption is I have plotted histograms for adjustment size conditional on adjustment taking place, cut – not truncated – at an adjustment size of 11. Figure 3 shows the histogram for all establishments. It is clear that most adjustment is of only one employee. Adjustments of two or more employees are not uncommon, but the right tail of the histogram becomes very thin. The pattern is more pronounced for small establishments, depicted in figure 4.
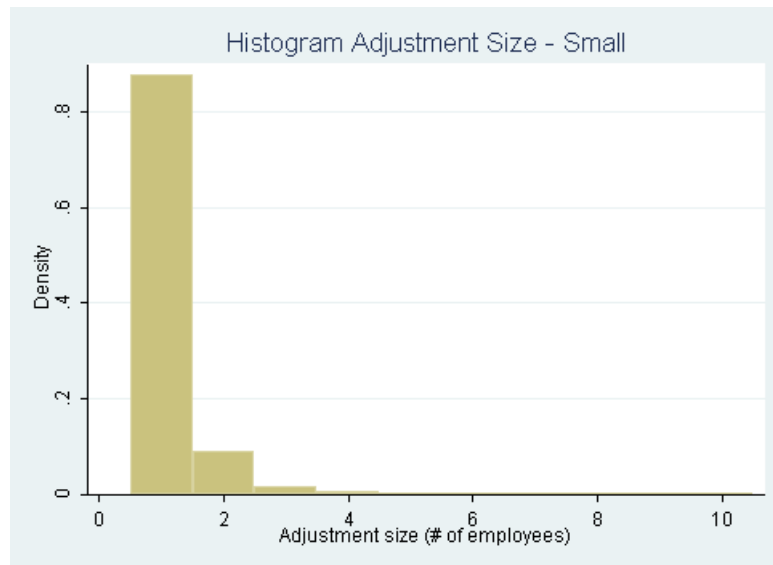
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Figure 4: Size of adjustment if any - small establishments

Building on this, table 7 reports results for heterogeneous labor. We would expect skilled labor to be more expensive to adjust, and the estimation results confirm this. Hiring skilled, young workers is twice as costly as hiring unskilled ones, and the difference between old skilled and unskilled workers is even stronger. Indeed, hiring skilled and old workers seems quite expensive, at more than 28,000 Euros. Separations, too, are more costly for skilled workers. The high test statistics for the over-identification tests are discomforting, but the possible roles of large sample sizes and establishment heterogeneity have been discussed. I have run the same estimation by sectors (not reported). In only 2 of 51 cases is the exogeneity of the instruments rejected, while the general pattern of hiring and separation costs is confirmed.

7 Conclusion

What has been learned from this exercise? The old news is: First, adjustment costs are present, and second, convex cost models provide sensible approximations only for small ranges of adjustment. The third result might not have been anticipated for Germany:

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Table 7: Heterogeneous labor - All establishments

	1		2		3		4	
	Young skilled		Old skilled		Young unskilled		Old unskilled	
	τ^+	τ^-	τ^+	τ^-	τ^+	τ^-	τ^+	τ^-
All	8,479***	2,276***	28,551***	3,773*	4,011***	1,972***	8,088***	306*
	(515)	(687)	(2,430)	(2,037)	(273)	(268)	(761)	(303)
Sargan $\chi^2(df=12)$	27.0***		20.8*		96.5***		117.9***	
n	423,103							

Note: Standard errors in parentheses. P-values:***<0.01, **<0.05, *<0.1 Regressions include a full set of quarter dummies.

Hiring is more costly than separations – but maybe not more than firings or layoffs. This last point can not be settled given the absence of the reason for separations in the data. Fourth, aggregation in time and "space" matter a great deal, and this finding is the major novelty of this paper. Under asymmetric costs, longer time intervals result in higher estimates of hiring costs, possibly due to capturing a longer actual incidence of costs, but reject the model predictions for separation costs frequently. If adjustment costs are assumed to be symmetric, monthly and quarterly adjustment frequencies yield comparable results, but in comparison annual adjustment cost estimates drop by 90% and are economically negligible. Costs are always estimated as lower for big establishments, for which gross changes in employment are more likely to differ from net changes. Finally, and reassuringly, in general the model yields higher cost estimates for skilled and for older labor.

The third result – hiring costs exceeding separation costs – may come as a surprise and contrasts with findings for Norway by Nilsen et al. (2006). I offer some tentative explanations: As mentioned earlier, voluntary separations on the side of the employee may be costless or cheap for the employer. Only a fraction of all separations are involuntary to the employee (20%-25% according to own estimates from Micro-census data). A more subtle explanation might be the treatment of costs as coefficients. Employers have some, maybe a lot of discretion on how much to invest into searching new employees, and a determinant of hiring costs might be how expensive it is to dismiss an employee

who turns out to be a bad match. Employers might then invest in search, to minimize involuntary separations. As a result, hiring is costly and separations are not, because they happen voluntarily most of the time (as matches should be good as a consequence of costly search).

The work here could be extended in several ways to deepen our understanding of the working and consequences of adjustment costs. Hiring costs could be endogenized in a search-framework and welfare analyzes based on the costs and benefits – in terms of unemployment and risk-reduction – are possible extensions.

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A   Demand for heterogeneous labor

Take equation (3) and extend it to K types of labor.

$$
\begin{aligned}
V(\mathbf{l_{t-1}}, \mathbf{I_t}) \quad = \quad & \max_{\mathbf{l_t}} p_t q(\mathbf{l_t}) - \mathbf{w_t} \cdot \mathbf{l_t} - \sum_k \tau_k^+ \mathbf{1}_{l_{k,t} > l_{k,t-1}}(l_{k,t} - l_{k,t-1}) \\
& - \sum_k \tau_k^- \mathbf{1}_{l_{k,t-1} > l_{k,t}}(l_{k,t-1} - l_{k,t}) + \beta E V(\mathbf{l_t}, \mathbf{I_{t+1}})
\end{aligned}
$$

Setting up a Lagrangian with constraints $l_{k,t} \geq l_{k,t-1}$ for all k, we get the following first order conditions:

$$
p_t \frac{\partial q}{\partial l_1} - w_{1,t} - \tau_1^+ + \lambda_{1,t}^+ + \beta E \frac{\partial V(\mathbf{l_t}, \mathbf{I_{t+1}})}{\partial l_{1,t}} \quad = \quad 0
$$

$$
\vdots
$$

$$
p_t \frac{\partial q}{\partial l_K} - w_{K,t} - \tau_K^+ + \lambda_{K,t}^+ + \beta E \left(\frac{\partial V(\mathbf{l_t}, \mathbf{I_{t+1}})}{\partial l_{K,t}} \right) \quad = \quad 0
$$

This needs to be done for all hiring and separation combinations. Similarly, the expression $E \left(\frac{\partial V}{\partial l_{k,t}} \right)$ can be split into the sum of all different hiring-separation combinations multiplied by the probabilities of that event happening. For example, for hiring every

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type of labor *next* period, the last term of the first order condition for labor type j in *this* period would be

$$
\begin{aligned}
\frac{\partial V(\mathbf{l_t}, \mathbf{I_{t+1}})}{\partial l_{j,t}} &= p_{t+1}\left(\sum_k \frac{\partial q}{\partial l_{k,t+1}}\frac{\partial l_{k,t+1}}{\partial l_{j,t}}\right) \\
&\quad - \sum_k w_{k,t+1}\frac{\partial l_{k,t+1}}{\partial l_{j,t}} - \sum_k \tau_k^+ \left(\frac{\partial l_{k,t+1}}{\partial l_{j,t}} - \frac{\partial l_{k,t}}{\partial l_{j,t}}\right)
\end{aligned}
$$

Collecting all terms $\frac{\partial l_{k,t+1}}{l_{j,t}}$ and using envelope theorems, we and up with

$$
\frac{\partial V(\mathbf{l_t}, \mathbf{I_{t+1}})}{\partial l_{j,t}} = \sum_k \tau_k^+\left(\frac{\partial l_{k,t}}{\partial l_{j,t}}\right)
$$

Do this for all possible hiring-separation combinations, multiply them by the corresponding probabilities to get $E\left(\frac{\partial V}{\partial l_{j,t}}\right)$, and plug this back into the first-order conditions, which then can be written in one equation as in the text.