Entry deterrence and Experimentation under Demand Uncertainty[1]

Neelam Jain[2]

September 7, 2010

[1] I thank the editor, the anonymous referees, participants at Midwest Economics Association, 2008, Society of Economic Design, 2008, and seminar participants at University of Montreal and University of Kansas for valuable comments. All remaining errors are mine.

[2] City University London, Department of Economics, Northampton Square, London EC1V 0HB, U.K.; Email: Neelam.Jain@city.ac.uk; Tel: 44 (0)20 7040 3271; Fax: 44 (0)20 7040 8580.

Abstract

We examine the effect of a threat of entry on experimentation about demand by an incumbent monopolist when there is a fixed cost of entry. We show that experimentation may itself be used as a tool for entry deterrence and derive conditions under which experimentation reduces the probability of entry. These conditions depend on the entry rule which in turn depends on entry costs. We show that if experimentation does not deter entry, the monopolist incumbent experiments less. We also characterize experimentation and entry in the linear-uniform example, and show that cost of entry and experimentation do not have a monotonic relationship.

1 Introduction

This paper studies experimentation in a market with potential entry that entails a fixed cost. Experimentation, or active learning, refers to a firm adjusting its choices away from the myopically optimal levels in order to learn about parameters of interest. In the absence of entry, it has been shown (see Mirman, Samuelson and Urbano, 1993 (MSU)), that a monopolist has an incentive to actively learn about demand so that it can make greater profits in the future. We examine the impact of potential, costly entry on this incentive. In addition, we identify and analyze a new rationale for experimentation, namely entry-deterrence, since due to fixed cost of entry, entry need not always occur.

The issues addressed in this paper are important because firms face constant pressures of existing or potential competition, while making decisions under uncertainty. In addition to the standard decisions such as setting prices or choosing output, a firm also aims to acquire information about various elements relevant to its profits, such as demand, cost of production and the nature of competition. However, just as with prices or quantities, the firm must consider strategic effects in determining the extent of active learning. That is, the firm must take into account the effect of its experimentation on decisions of other firms in the market or firms considering entry into the market. On one hand, experimentation may reveal good information to other firms and encourage further entry thereby reducing future benefit to the experimenting firm but on the other hand, it may reveal bad news discouraging entry. In order to understand pricing in such markets, it is important to analyze experimentation and entry in the same model. Indeed, since experimentation itself may lead to a lower price in the market, just as limit pricing, it is important to incorporate the effect of experimentation in a market with threat of entry. It is then an empirical issue whether the observed low prices are a result of limit pricing or experimentation, given other aspects of the market, such as the demand structure. Further, the learning behavior of the incumbent is also important for understanding overall welfare effects of entry. For example, if a threat of entry leads to more information generation, which in turn leads to better economic decisions by agents, welfare effects of entry are strengthened.

This work has empirical implications for emerging markets that are characterized by increasing deregulation and therefore, potential entry and at the same time, by uncertainty and asymmetric information. Similarly, there are empirical implications for markets where experimentation is common such as the health industry or the movie industry. Firms introduce new products, such as drugs, or movies, in limited markets to learn about demand or firms may adjust their advertising expenditure in order to learn about demand. This paper provides insights into the effect of potential entry on various experimentation variables. In particular, we show that in such markets, the level of fixed costs and therefore, the extent to which entry is a threat, determines the market outcomes including the extent to which information is generated and used in setting prices and quantities.

Traditionally, the literature on entry-deterrence has focused on limit pricing, financial structure, capacity expansion or advertising etc., as a tool for deterring entry. We show that an incumbent operating under uncertainty can use experimentation as a tool for entry-deterrence for some parameter values. While the literature on entry-deterrence (see the pioneering works of Milgrom and Roberts, 1981, and Matthews and Mirman, 1983) as well as experimentation is extensive, the role of experimentation as an entry-deterrence tool has not been analyzed. The existing literature has examined experimentation in different contexts, such as a duopoly market structure (see Mirman, Samuelson and Schlee, 1994, Alepuz and Urbano, 1999 and Belleflamme and Bloch, 2001) or learning by a principal in the context of entry-deterrence (Jain, Jeitschko and Mirman (2002, 2003 and 2005))[1] or information acquisition by an incumbent facing entry when there are no entry costs (Dimitrova and Schlee[2] (DS), 2003 and Patron, 2001)). This paper generalizes the work of DS by incorporating entry costs and thereby permitting the analysis of the effects of experimentation on entry-deterrence. With no entry costs, entry cannot be deterred and therefore, the new role of experiementation cannot be studied.

We also provide insights into how entry affects the incumbent's incentive to experiment when there are entry costs. In this respect, we generalize results of DS. They show that entry reduces information acquisition, under the conditions of linear demand, increasing demand dispersion and no entry costs. Since most models of entry assume fixed costs of entry, reflecting the conclusions of empirical research, it seems important to study experimentation and entry when the entrant faces fixed entry costs, as we do in this paper. DS provide an example with entry costs where entry increases information acquisition. Our contribution is to derive conditions under which the incumbent experiments less even when there are entry costs. Further, our results are robust to the direction of demand dispersion, and are consistent with the DS example of information increasing entry. We do not address the other useful examples in DS where entry increases information acquisition since our focus is on conditions under which entry reduces information acquisition.

The reason to expect that an incumbent threatened with entry may experiment to a different

[1] These authors do not emphasize the role of experimentation as an entry-deterrence tool. Their focus is on the role of debt in deterring entry.

[2] They provide an interesting example with entry costs.

extent is that entry reduces future profits of the incumbent and thus, reduces the marginal benefit from experimentation. If this were the only effect, the monopolist facing entry will reduce experimentation since the marginal benefit is smaller and the marginal cost of experimentation, incurred in the first period in the form of lower profits than in the static case, remains the same.[3] However, when entry does not occur surely, experimentation also influences the potential entrant's decision to enter the market, and may deter entry. The intuition behind this is that experimentation increases the probability of a high price being associated with high demand and a low price being associated with low demand. Since entry is only profitable when demand is high, experimentation can lower the probability of entry. This is not guaranteed and only occurs for some parameter values because better information can either reinforce beliefs that market is good or that the market is bad. For example, if entry only occurs when demand is good, then experimentation, by increasing this probability, increases the probability of entry. This scenario depends on the cost of entry and other parameters of the model.

We show that if experimentation does not deter entry, the threatened incumbent experiments less. The precise conditions that lead to a higher probability of entry and less experimentation depend on the entry rule, which in turn depends on the size of the fixed cost. We also show that these conditions become weaker as fixed cost increases. That is, as fixed cost increases, experimentation increases probability of entry and thus, the incumbent experiments less due to entry for a larger set of parameters. If entry is deterred, which occurs when entry cost is sufficiently low, results are

[3] We use the term 'myopic 'in the sense standard in the experimentation literature - that the incumbent ignores the effect of its current actions on the future outcomes altogether. The implication is that the myopic incumbent chooses the same quantity with or without the threat of entry. Then, greater experimentation by the incumbent can be simply measured by the difference between the output choice of the isolated incumbent and that of the threatened incumbent.

unclear in a general setting because there is a trade-off between the reduced marginal benefit from experimentation (given entry) and the increased benefit of experimentation as an entry-deterrence tool. We work out an example in which this trade-off becomes clear. In the example, the price shock is uniformly distributed and the demand function is linear, satisfying the assumption of increasing dispersion. Results cover the entire range of possibilities, depending on entry costs. That is, probability of entry may increase or decrease and experimentation by the threatened incumbent may or may not be less than that of the isolated incumbent. Further, we find that the effect of fixed cost of entry on experimentation is non-monotonic. As entry cost increases from low to moderate, the incumbent experiments more but as entry cost increases from moderate to high, the incumbent experiments less. The intuition behind this is that when entry cost is moderate, so that entry does not occur surely, entry-deterrence becomes possible. The implication for welfare effects of entry is that at moderate levels of entry cost, the competitive effects of entry are strengthened whereas for high levels, the informational effect offsets the competitive effects.

We assume the standard environment (as in MSU) to study this problem. Demand is uncertain. It can be 'high' or 'low', and in each case, there is a random shock, independent of the state of demand, that determines the price of the good. Firms only know the probabilities of demand being high or low and of the random shock. Price of the good at the end of the first period is assumed to be observable and used to update beliefs about the state of demand by all market participants. In this paper, it is assumed that output of the incumbent is also observable so that the posterior beliefs are identical across market participants.[4] The incumbent is the only firm in the market

[4]This assumption is standard in the experimentation literature. While this assumption does not apply to all economic situations, the resulting analysis is useful in understanding certain economic situations, for example, when

in the first period and therefore is the only firm that can experiment. We assume that all costs other than entry costs are zero, for convenience. We also assume a demand structure in which a higher output provides more information about true demand, that is, demand structure satisfies increasing demand dispersion, though results can be shown to hold even when the opposite holds. We then consider different entry rules, corresponding to different levels of fixed cost.

The paper is organized as follows: in section 2, the model is presented, with both the benchmark no-entry case and the entry case; in section 3, we present an example with linear demand and uniformly distributed demand shock; in section 4, the effect of potential entry is analyzed in general; finally, in section 5, we conclude. The Appendix contains some derivations and proofs.

2 Model

There are two time periods, $t = 1, 2$. In each period, output is chosen by firms in the market, given the inverse demand function, to maximize their expected profit over the two periods. We assume no discounting for simplicity. Let $p = g(q, \gamma) + \epsilon$ denote the inverse demand function, where p is the market price of the good, publicly observed at the end of the first period; q is the first period output; $g(., \gamma)$ is a twice-continuously differentiable function, decreasing in q, and γ is a time-invariant demand parameter in $\{\overline{\gamma}, \underline{\gamma}\}$. The upper bar indicates high demand and the lower bar indicates low demand. We shall use $\overline{g}$ and $\underline{g}$ to denote the expected price under high demand and low demand states respectively. The state of demand is unknown to the firms. Instead, firms

the choice variable is advertising expenditure, or research and development expenditure, rather than output. In some situations, even output may be observed simply because the firm may be required to report it. Further, the analysis and results of the observable case provide guidance for the analysis of the situation where the output is not observed and hence, signal jamming occurs.

have a prior belief that with probability ρ_0 demand is high and with probability $1-\rho_0$, demand is low. The random component of the market price ϵ is distributed according to the density function $f(\epsilon)$. We assume that the density function f satisfies the monotone likelihood ratio property strictly ($\frac{f'(\epsilon)}{f(\epsilon)}$ is a continuous and strictly decreasing function), is continuously differentiable on the entire real line and has zero mean. The random component is assumed to be uncorrelated over the two time periods.

At the end of the first period, the market price is realized and observed by all market participants. In this paper, it is also assumed that output q is observed by all participants. These two observations are used to update beliefs about the state of the demand, according to Bayes' rule. Denoting the posterior belief that demand is high by ρ, we obtain:

$$\begin{aligned} \rho(p,q) &= \frac{\rho_0 f(p-g(q,\overline{\gamma}))}{\rho_0 f(p-g(q,\overline{\gamma}))+(1-\rho_0)f(p-g(q,\underline{\gamma}))} \\ &\equiv \frac{\rho_0\overline{f}}{\rho_0\overline{f}+(1-\rho_0)\underline{f}}. \end{aligned} \tag{1}$$

It should be noted that MLRP is equivalent to ρ being non-decreasing in p. It is also equivalent to $\underline{f}/\overline{f}$ decreasing in p.

After updating beliefs, all firms in the market choose the second period output, given the expected inverse demand function, to maximize the second period expected profits. Expected profits of the incumbent in the first period are given by $\pi(q) = q(\rho_0 g(q,\overline{\gamma}) + ((1-\rho_0)g(q,\underline{\gamma}))$. For simplicity, we assume that costs are zero for the incumbent. The entrant incurs a fixed cost of entry, F.

2.1 The benchmark: No-Entry

This is the standard experimentation model (see MSU). The second period expected profits of the incumbent are the same as in the first period except for possibly different beliefs, denoted by ρ. Let $V_M(\rho(p,q))$ denote the interim value function of the incumbent under the no-entry scenario, that is, the maximized expected second period profits, given ρ. Let $h(p,q)$ denote the probability that p is realized at the end of the first period given the choice of q. That is, $h(p,q) = \rho_0 \overline{f} + (1-\rho_0)\underline{f}$. Finally, let $W_M(q)$ denote the value function, that is the interim value function integrated over the first period price, namely, $\int V_M(\rho(p,q))h(p,q)dp$. The first period maximization problem of the monopolist, then, is to choose the first period output q to maximize the sum of expected profits in each period:

$$\Pi(q) = \pi(q) + W_M(q).$$

A myopic incumbent ignores the future and thus, chooses the first period output to maximize $\pi(q)$. Experimentation is measured by the difference between the first period output chosen by a myopic incumbent and that of an experimenting incumbent who takes the effect of its first period choice on the future beliefs. We impose Assumption 1 of MSU to ensure that quantity-increasing experimentation occurs when the incumbent does not face the threat of entry. For convenience, this assumption is stated below and illustrated in Figure 1. Let $\widehat{q}$ be the highest quantity chosen by the incumbent who does not face the threat of entry, to be referred to as an isolated incumbent.

Assumption 1 (MSU): For all $q \in [0, \widehat{q}]$, $\underline{g} < \overline{g}$ and $\underline{g}' < \overline{g}' < 0$.

That is, the 'high demand' curve lies above and is flatter than the 'low demand' curve. In other words, mean demand curves become further apart as q increases so that a higher q leads to better information in the second period. Furthermore, information is valuable because different outputs are optimal corresponding to the states of high and low demand under Assumption 1. MSU show that under this assumption, the monopolist chooses a higher q than in the myopic case because the marginal benefit of doing so is positive at the myopic output level. This in turn is based on two conditions: one that the interim value function is convex in ρ, implying that information is valuable (see Blackwell, 1951) and second that the incumbent's actions can influence information because demand curves become further apart with a higher q. MSU provide examples of linear demand functions where either information is not valuable or the monopolist is unable to learn. We rule out those cases so that experimentation occurs in the absence of the threat of entry. Note that experimentation is costly in the current period. That is, due to the choice of an output that is different from the static optimal level, the monopolist sacrifices current profits in exchange for higher profits in the future on account of better information. It is this trade-off that drives the decision to experiment.

2.2 Potential Entry

Expected profit of the entrant upon entry is $\pi_e(q_e, q_i) = q_e(\rho g(q_e + q_i, \overline{\gamma}) + ((1-\rho)g(q_e + q_i, \underline{\gamma}))$, where q_e is the entrant's output and q_i is the incumbent's output in the second period. The entrant enters if and only if $\pi_e(q_e^*(\rho), q_i^*(\rho)) = \pi_e(\rho) \geq F$, where q_e^* and q_i^* are the Cournot-Nash equilibrium outputs under incomplete information. We assume that the demand function satisfies conditions to generate a unique equilibrium in which the entrant's profits are increasing in ρ[5] and therefore, the entry condition reduces to $\rho \geq \rho_e$, where ρ_e is a constant determined from the values of $F, \underline{\gamma}$

[5] The Appendix contains the precise conditions required. These conditions are satisfied when demand is linear.

and $\overline{\gamma}$.

Let $\pi(q_e, q_i) = q_i(\rho g(q_e + q_i, \overline{\gamma}) + ((1-\rho)g(q_e + q_i, \underline{\gamma}))$ denote the incumbent's second period profits if $\rho \geq \rho_e$. Let $V_D(\rho) = \pi(q_e^*(\rho), q_i^*(\rho))$ denote the incumbent's interim value function under Cournot duopoly. Specifically, the interim value function of the incumbent, denoted by $V_i(\rho)$, is given by:

$$
\begin{aligned}
V_i(\rho) &= V_M(\rho), \rho < \rho_e, \\
V_i(\rho) &= V_D(\rho), \rho \geq \rho_e.
\end{aligned}
$$

Obviously, $V_M(\rho) > V_D(\rho) \forall \rho \geq \rho_e$ and therefore, the interim value function is discontinuous at ρ_e.[6]

Substituting ρ_e in (1) for ρ yields a cut-off value for the observed market price p, as a function of q. Then, by MLRP, the entry rule 'enter if and only if $\rho \geq \rho_e$' is equivalent to 'enter if and only if $p \geq \widehat{p}(q)$' where $\widehat{p}(q)$ is the price that corresponds to the cut-off belief ρ_e, given q. That is, the entrant enters for all prices above $\widehat{p}(q)$ and stays out otherwise. This implies that the expected value function of the threatened incumbent takes the following form:

$$
W_i(q) = \int_{-\infty}^{\widehat{p}(q)} V_M(\rho(p,q))h(p,q)dp + \int_{\widehat{p}(q)}^{\infty} V_D(\rho(p,q))h(p,q)dp.
$$

The problem of the myopic incumbent remains the same and thus, the threatened incumbent is said to experiment more if and only if it chooses a higher output than the isolated incumbent. Since

[6]Note that this discontinuity implies that we can not use convexity of the value function to demonstrate that the threatened incumbent learns or the convexity of the difference $V_M - V_i$ to show that the isolated monopolist learns more, as DS do, since $V_M - V_i$ is discontinuous at the belief when entry occurs. However, we can and do use convexity of $V_M - V_D$ in our main result.

the two types of incumbent face the same cost in the first period, it suffices to consider the effect of the first period choice on the value function. In particular, the isolated incumbent experiments more if and only if the marginal future benefit from the first period output is larger without the threat of entry.

3 An Example

In this section, we examine the case where the inverse demand is linear and the random term in the inverse demand function is uniformly distributed. We derive closed-form solutions in this example and therefore, characterize the relationship between experimentation and entry-deterrence. We find that depending on the fixed cost of entry and therefore the entry rule, probability of entry may increase or decrease with experimentation. The uniform distribution leads to either complete learning or no learning. Due to this extreme learning outcome, the effects of experimentation on entry become transparent.

Let the inverse demand function be given by $p = a - bq + \epsilon$, where a and b are unknown, strictly positive, parameters and ϵ is a random, unobservable term that is known to be distributed uniformly on the interval $[-\eta, \eta]$, where $\eta > 0$. The intercept a and slope b take two different values: $(\overline{a}, \underline{a})$ and $(\overline{b}, \underline{b})$, where $\overline{a} \geq \underline{a}$ and $\overline{b} < \underline{b}$, so that demand displays increasing dispersion as q increases. Let $\overline{p}$ and $\underline{p}$ be the mean prices under high demand and low demand respectively. The prior belief that demand is high (the upper bars) is ρ_0. Let $\widehat{a}(\rho)$ and $\widehat{b}(\rho)$ denote the mean values of the intercept and slope respectively, at the posterior belief ρ.

Bayes' rule generates the following posterior distribution:

$$\rho = \left\{ \begin{array}{c} 1, prob = \rho_0 \frac{\overline{p}-\underline{p}}{2\eta}, \\ 0, prob = (1-\rho_0) \frac{\overline{p}-\underline{p}}{2\eta} \\ \rho_0, prob = \frac{2\eta - (\overline{p}-\underline{p})}{2\eta} \end{array} \right\}$$

The interim value function of the isolated monopolist is $V_M(\rho) = \left(\widehat{a}(\rho)\right)^2 / 4\widehat{b}(\rho)$ and that of a monopolist who faces sure entry is $V_D(\rho) = \left(\widehat{a}(\rho)\right)^2 / 9\widehat{b}(\rho)$. However, since ρ takes only three values, probabilities of which are easily calculated, we obtain the following value function of the isolated monopolist:

$$\begin{aligned} EV_M = & \frac{\underline{a}^2}{4\underline{b}}(1-\rho_0)\left(\frac{\overline{a}-\underline{a}-(\overline{b}-\underline{b})q}{2\eta}\right) + \frac{\overline{a}^2}{4\overline{b}}\rho_0\left(\frac{\overline{a}-\underline{a}-(\overline{b}-\underline{b})q}{2\eta}\right) + \\ & + \left(\frac{\widehat{a}(\rho_0)^2}{4\widehat{b}(\rho_0)}\right)\left(\frac{2\eta - \left(\overline{a}-\underline{a}-(\overline{b}-\underline{b})q\right)}{2\eta}\right). \end{aligned}$$

For the incumbent facing entry, the value function depends on the entry rule, which in turn depends on the size of the fixed cost of entry. Since expected profits of the entrant are increasing in ρ,[7] as F increases, entry occurs for a smaller set of beliefs. Setting $\pi_e(\rho) = F$ yields the level of posterior belief, in terms of F, such that the entrant breaks even. Precise conditions for the three entry rules can now be presented:

[7] To see this, note that expected profits of the entrant are $\pi_e(\rho) = \widehat{a}(\rho)^2/9\widehat{b}(\rho)$. Using $\triangle a$ for $\overline{a}-\underline{a}$ and $\triangle b$ for $\overline{b}-\underline{b}$, we obtain,

$$\frac{d\pi_e(\rho)}{d\rho} = \frac{\widehat{b}(2\widehat{a})(\triangle a) - \widehat{a}^2\triangle b}{9\widehat{b}^2}.$$

This expression is positive if and only if $\rho\triangle b\triangle a + 2\underline{b}\overline{a} + \underline{ab} \geq 0$. All terms in this expression are positive. Thus, $\pi_e(\rho)$ is increasing in ρ.

1. If $\pi_e(\rho_0) < F$ and $\pi_e(1) = \frac{\overline{a}^2}{9\overline{b}} \geq F$, entry occurs if and only if $\rho = 1$;

2. If $\frac{\underline{a}^2}{9\underline{b}} < F$ and $\pi_e(\rho_0) \geq F$, entry occurs if and only if $\rho \in \{\rho_0, 1\}$;

3. If $\pi_e(0) = \frac{\underline{a}^2}{9\underline{b}} \geq F$, entry occurs for all beliefs.

We consider these cases next.

3.1 Entry when $\rho = 1$: High Entry Cost

If entry occurs only when demand is revealed to be high, then, the value function of the threatened incumbent equals:

$$\begin{aligned} EV_i =& \frac{(\underline{a})^2}{4\underline{b}}(1-\rho_0)\frac{\overline{a}-\underline{a}-(\overline{b}-\underline{b})q}{2\eta} + \frac{(\overline{a})^2}{9\overline{b}}\rho_0\frac{\overline{a}-\underline{a}-(\overline{b}-\underline{b})q}{2\eta} + \\ &+ \left(\frac{\widehat{a}(\rho_0)^2}{4\widehat{b}(\rho_0)}\right)\frac{2\eta - \left(\overline{a}-\underline{a}-(\overline{b}-\underline{b})q\right)}{2\eta}. \end{aligned}$$

Thus, the only change is in the second term. Due to possible entry, the incumbent's profits are reduced in the high-demand state. Note that in this case, the probability of entry is $\rho_0\left(\frac{\overline{a}-\underline{a}-(\overline{b}-\underline{b})q}{2\eta}\right)$, which increases as q increases.

To see how entry affects experimentation, we calculate the impact of the first period output on value functions in the two cases (dropping the argument of the mean intercept and slope for convenience):

$$\frac{dEV_M}{dq} = \frac{-(\bar{b}-\underline{b})}{2\eta}\left(\frac{(\underline{a})^2}{4\underline{b}}(1-\rho_0)+\frac{(\bar{a})^2}{4\bar{b}}\rho_0-\frac{(\widehat{a})^2}{4\widehat{b}}\right),$$

$$\frac{dEV_i}{dq} = \frac{-(\bar{b}-\underline{b})}{2\eta}\left(\frac{(\underline{a})^2}{4\underline{b}}(1-\rho_0)+\frac{(\bar{a})^2}{9\bar{b}}\rho_0-\frac{(\widehat{a})^2}{4\widehat{b}}\right).$$

The isolated incumbent experiments more if and only if $\frac{dEV_M}{dq}-\frac{dEV_i}{dq}\geq 0$, that is,

$$\frac{-(\bar{b}-\underline{b})}{2\eta}\left(\frac{5(\bar{a})^2}{36\bar{b}}\rho_0\right)\geq 0.$$

This inequality holds because $-(\bar{b}-\underline{b})>0$.

Thus, the threatened incumbent experiments less than the isolated incumbent. The intuition is that experimentation increases the probability of entry and thus, the standard experimentation effect is reinforced by the entry-deterrence effect. Further, note that the threatened incumbent experiments if and only if $\frac{dEV_i}{dq}\geq 0$ which can be shown to hold only for some parameter values. For all other values, the incumbent reduces information and therefore, reduces the probability of entry compared to the myopic case.

Next, we consider the case where entry occurs when demand is revealed to be high or not revealed at all.

3.2 Entry when $\rho = 1$ or ρ_0 : Moderate Entry Cost

The value function of the incumbent changes to:

$$\begin{aligned} EV_i =& \frac{(\underline{a})^2}{4\underline{b}}(1-\rho_0)\left(\frac{\overline{a}-\underline{a}-(\overline{b}-\underline{b})q}{2\eta}\right)+\frac{(\overline{a})^2}{9\overline{b}}\rho_0\left(\frac{\overline{a}-\underline{a}-(\overline{b}-\underline{b})q}{2\eta}\right)+ \\ &+\left(\frac{(\widehat{a})^2}{9\widehat{b}}\right)\left(\frac{2\eta-\left(\overline{a}-\underline{a}-(\overline{b}-\underline{b})q\right)}{2\eta}\right), \end{aligned}$$

and,

$$\frac{dEV_i}{dq} = \frac{-(\overline{b}-\underline{b})}{2\eta}\left(\frac{(\underline{a})^2}{4\underline{b}}(1-\rho_0)+\frac{(\overline{a})^2}{9\overline{b}}\rho_0-\frac{(\widehat{a})^2}{9\widehat{b}}\right).$$

The isolated incumbent learns more if and only if $\frac{dEV_M}{dq}-\frac{dEV_i}{dq}\geq 0$. This can be simplified to obtain,

$$\rho_0 \geq \frac{\widehat{a}^2}{\widehat{b}}\frac{\overline{b}}{\overline{a}^2}.$$

Since $\widehat{a}$ and $\widehat{b}$ are functions of the prior, we substitute for $\widehat{a}$ and $\widehat{b}$ and obtain the following quadratic inequality in ρ_0 :

$$\rho_0^2(\overline{a}^2\left(\overline{b}-\underline{b}\right)-\overline{b}\left(\overline{a}-\underline{a}\right)^2)+\rho_0\left(\overline{a}^2\underline{b}-2\left(\overline{a}-\underline{a}\right)\underline{a}\overline{b}\right)-\overline{b}\underline{a}^2\geq 0.$$

It is straightforward to show that this inequality holds for ρ_0 sufficiently high provided $\underline{a}/\underline{b}<\overline{a}/2\overline{b}$, that is, if and only if, low demand is sufficiently lower than high demand. If this is the case, the isolated incumbent learns more for sufficiently high values of ρ_0. Otherwise, the inequality does not hold except weakly at $\rho_0 = 1$, implying that entry increases experimentation.

Note that probability of entry in this case is $1-(1-\rho_0)\left(\frac{\overline{a}-\underline{a}-(\overline{b}-\underline{b})q}{2\eta}\right)$, which is decreasing in q. Thus, the interpretation of the result above is that when high demand and low demand are not too different, the entry-deterrence role of experimentation dominates the reduced benefit from experimentation and hence, the threatened incumbent experiments more. Thus, when the noise is uniformly distributed and demand is linear, there are conditions under which a threatened incumbent experiments more.

Further, in this case, $\frac{dEV_i}{dq} \geq 0$ implying that the threatened incumbent experiments for all parameter values and therefore, the probability of entry is less with experimentation.

3.3 Sure Entry: Low Entry Cost

In this case, the profits of the incumbent are lower in all states and thus, the marginal future benefit from experimentation changes to:

$$\frac{dEV_i}{dq} = \frac{-(\bar{b} - \underline{b})}{2\eta} \left(\frac{(\underline{a})^2}{9\underline{b}}(1 - \rho_0) + \frac{(\bar{a})^2}{9\bar{b}}\rho_0 - \frac{(\widehat{a})^2}{9\widehat{b}} \right).$$

The isolated incumbent experiments more if and only if $\frac{dEV_M}{dq} - \frac{dEV_i}{dq} \geq 0$ or,

$$\frac{-(\bar{b} - \underline{b})}{2\eta} \frac{5}{36} \left(\frac{(\underline{a})^2}{\underline{b}}(1 - \rho_0) + \frac{(\bar{a})^2}{\bar{b}}\rho_0 - \frac{(\widehat{a})^2}{\widehat{b}} \right) \geq 0,$$

which holds by strict convexity of the value function. Thus, the isolated incumbent experiments more. This is intuitive since entry cannot be deterred so that entry reduces benefits from experimentation unambiguously. Further, in this case, $\frac{dEV_i}{dq} \geq 0$ implying that the threatened incumbent experiments for all parameter values

Let q_d^* and q_m^* denote the optimal first period output of the threatened incumbent and the isolated incumbent respectively. Let q_0^* be the optimal output chosen by a myopic incumbent. We can summarize the results for the Example in the following Proposition:

Proposition 1 *When demand is linear, satisfies increasing dispersion and the random component of price is uniformly distributed, then,*

1. *If entry occurs if and only if $\rho = 1$, (i) the probability of entry increases with experimentation; (ii) $q_m^* > q_d^*$ and (iii) $q_d^* > q_0^*$ for some parameter values.*

2. *If entry occurs for sure, $q_m^* > q_d^* > q_0^*$.*

3. *If entry occurs if and only if $\rho \in \{1, \rho_0\}$, (i) the probability of entry decreases in q. (ii) $q_m^* > q_d^*$ if and only if $\underline{a}/\underline{b} < \overline{a}/2\overline{b}$ and $\rho_0 \geq \left(\widehat{a}^2/\widehat{b}\right)\left(\overline{b}/\overline{a}^2\right)$ and (iii) $q_d^* > q_0^*$.*

Summarizing, if fixed cost is high so that profitable entry requires demand to be high for sure, probability of entry increases with experimentation and the threatened incumbent experiments less than the isolated incumbent. Indeed, for some parameter values, the threatened incumbent reduces information in order to deter entry. In contrast, when fixed cost is at a moderate level, probability of entry decreases with experimentation and the threatened incumbent may, for some parameter values, experiment more than the isolated incumbent. The incumbent never reduces information in this case. Finally, when fixed cost is low so that entry occurs for all possible beliefs, there is no entry-deterrence effect. Consequently, the threatened incumbent experiments but less than the isolated incumbent. Note that the relationship between fixed cost of entry and experimentation is non-monotonic. The precise relationship is presented in the following Corollary:

Corollary 1 *q_d^* increases as F increases from low to moderate and decreases as F increases from moderate to high.*

The proof is straightforward and therefore, omitted.

The intuition for this result is that as F increases from low to moderate, the entry-deterrence role comes into play to offset the standard experimentation effect. As a result, experimentation increases. On the other hand, as F increases further so that profitable entry requires high demand, the entry-deterrence role of experimentation reverses. As a result, experimentation decreases. Figure 2 illustrates Proposition 1 and Corollary 1.

The linear-uniform case provides insights into the factors that determine whether the threatened incumbent experiments more or less. In the next section, we exploit these insights in a model where the assumptions of linearity of demand and uniform distribution of the demand shock are relaxed.

4 General Analysis

In the general model, learning is never complete and given our assumptions on f and g, the posterior belief ρ is a differentiable function of p and q, unlike the linear-uniform case. Now, if F is higher than the expected profits under high demand, the threat of entry is vacuous. On the other hand, if F is less than expected profits under low demand then entry occurs for all beliefs. In this case, there is no entry-deterrence possible and the standard experimentation effect prevails, leading the incumbent to experiment less (as shown by DS, and as seen above in the linear-uniform case). So, it remains to consider values of F that generate entry for some beliefs but not all.

Let $\overline{\rho}_m$ denote the posterior resulting from the observation of price $\overline{p}_m$, the unique *modal* price under high demand *(it equals the mean price $\overline{p}$, if the distribution is symmetric)*. MLRP holding strictly implies that $\underline{f}/\overline{f}$ decreases as p increases, implying that there can be only one price at which $\underline{f}/\overline{f} = 1$. Existence of such a price is implied by the assumptions of infnite support and zero (finite) mean. This is because f cannot be monotonic and must have a global maximum. We define $\underline{\rho}_m$ and $\underline{p}_m$ similarly. By Assumption 1, the density function $\overline{f}$ lies to the right of the density function $\underline{f}$ since $\overline{g} - \underline{g} > 0 \Rightarrow \underline{p}_m < \overline{p}_m$. We start with moderate values of F, values that yield the cut-off belief for entry to be between $\underline{\rho}_m$ and $\overline{\rho}_m$, and then consider the extreme cases where $1 > \rho_e > \overline{\rho}_m$ or $0 < \rho_e < \underline{\rho}_m$.

4.1 Moderate Fixed Cost: $\rho_e \in [\underline{\rho}_m, \overline{\rho}_m]$

We first show that the cut-off observed price, above which the entrant enters, is a decreasing function of the first period output.

Lemma 1 *: For $\rho_e \in [\underline{\rho}_m, \overline{\rho}_m]$, $\frac{d\widehat{p}(q)}{dq} \leq 0$.*

Proof. Substituting ρ_e in (1) for ρ yields (arguments are suppressed for convenience),

$$\begin{aligned} \rho_e &= \frac{\rho_0 \overline{f}}{\rho_0 \overline{f} + (1-\rho_0)\underline{f}}, \\ \overline{f} &= \alpha \underline{f}, \alpha \equiv \frac{\rho_e(1-\rho_0)}{\rho_0(1-\rho_e)}. \end{aligned} \tag{2}$$

Differentiating implicitly with respect to q yields,

$$\frac{d\widehat{p}}{dq} = \frac{\overline{f}\underline{f}'\underline{g}' - \underline{f}\overline{f}'\overline{g}'}{\overline{f}\underline{f}' - \underline{f}\overline{f}'}. \tag{3}$$

By MLRP, the denominator is negative. To show that the numerator is positive, note that by Assumption 1, both $\underline{g}'$ and $\overline{g}'$ are negative. Further, $\underline{f}' < 0$ and $\overline{f}' > 0$, at $\widehat{p}(q)$, in the range of beliefs considered, since $\underline{p}_m$ and $\overline{p}_m$ are modal prices for the two densities. ■

Increase in q is equivalent to mean demand curves being further apart, by Assumption 1. Thus, the set of prices for which the potential entrant enters increases as expected prices are set further apart. However, this does not necessarily imply that the probability of entry increases as we show below in the next lemma.

Lemma 2 *For $\rho_e \in [\underline{\rho}_m, \overline{\rho}_m]$, experimentation reduces the probability of entry if and only if the following condition is satisfied:*

$$\rho_0 \leq \frac{\underline{f}^2 \overline{f}'}{\underline{f}^2 \overline{f}' - \overline{f}^2 \underline{f}'}. \tag{4}$$

Proof. Probability of entry $= \Pr\{p \geq \widehat{p}(q)\} = \int_{\widehat{p}(q)}^{\infty} h(p,q)dp$. Now,

$$\frac{d}{dq} \int_{\widehat{p}(q)}^{\infty} h(p,q)dp \leq 0 \Leftrightarrow$$

$$\rho_0 \overline{f}\overline{g}' + (1-\rho_0)\underline{f}\underline{g}' \leq \left(\rho_0 \overline{f} + (1-\rho_0)\underline{f}\right) \frac{d\widehat{p}}{dq}. \tag{5}$$

Substituting for $\frac{d\widehat{p}}{dq}$ from (3), we obtain,

$$\rho_0(\overline{f}\overline{g}' - \underline{f}\underline{g}') + \underline{f}\underline{g}' \leq (\rho_0(\overline{f} - \underline{f}) + \underline{f}) \frac{\overline{f}\underline{f}'\underline{g}' - \underline{f}\overline{f}'\overline{g}'}{\overline{f}\underline{f}' - \underline{f}\overline{f}'}.$$

Since $\overline{f}\underline{f}' - \underline{f}\overline{f}' < 0$ by MLRP, we obtain the following condition after some simplifying,

$$\rho_0(\overline{g}' - \underline{g}')\left(\overline{f}^2\underline{f}' - \underline{f}^2\overline{f}'\right) \geq -\underline{f}^2\overline{f}'\left(\overline{g}' - \underline{g}'\right).$$

Now, by assumption, $\overline{g}' - \underline{g}' > 0$ and we can show that $\overline{f}^2\underline{f}' - \underline{f}^2\overline{f}' < 0$ by MLRP[8]. The result follows. ■

There are two effects on the probability of entry of an increase in q. On one hand, holding the cut-off price constant, a higher q implies that lower prices are more likely, which in turn implies that demand is more likely to be low, and therefore, the probability of entry decreases. On the other hand, the potential entrant rationally revises the cut-off price downward, after observing a higher output, and this increases the probability of entry. Lemma 2 shows that if ρ_0 is not too high, the negative effect on entry dominates. The logic of the proof is that the increase in q shifts both distributions to the left but the low-demand distribution shifts more than the high-demand distribution. The result is that the two distributions intersect at a lower price (and by MLRP, a lower density) implying an increase in the probability of entry if demand turns out to be high and a decrease in the probability of entry if demand turns out to be low. The lower the prior that demand is high, the lower the weight on the increase. The intuition behind this is simple: entry occurs if and only if the entrant believes that demand is sufficiently likely to be high. Experimentation increases the probability that higher prices are a result of high demand and lower prices the result of low demand. The former increases the probability of entry and the latter decreases it. The greater the

[8] $\overline{f}^2\underline{f}' - \underline{f}^2\overline{f}' < 0 \Leftrightarrow \overline{f}^2\underline{f}' < \underline{f}^2\overline{f}' \Leftrightarrow \frac{\overline{f}^2}{\underline{f}^2}\underline{f}' < \overline{f}'$. This is true because MLRP implies that $\frac{\overline{f}}{\underline{f}}\underline{f}' < \overline{f}'$. Since the RHS is positive and the LHS is negative, multiplying the LHS by $\alpha > 0$ preserves the inequality.

prior belief that demand is high, the greater the weight on the former possibility and hence, the probability of entry increases for sufficiently high values of the prior belief.[9]

Next, we examine the marginal benefit from experimentation by comparing the effect of a change in q on the value function.

$$\begin{aligned}
\frac{dW_i(q)}{dq} &= \frac{d}{dq}\left[\int_{-\infty}^{\widehat{p}(q)} V_M(\rho(p,q))h(p,q)dp + \int_{\widehat{p}(q)}^{\infty} V_D(\rho(p,q))h(p,q)dp\right] \\
&= \int_{-\infty}^{\widehat{p}(q)} \frac{d}{dq}\left[V_M(\rho(p,q))h(p,q)\right]dp + \int_{\widehat{p}(q)}^{\infty} \frac{d}{dq}\left[V_D(\rho(p,q))h(p,q)\right]dp \\
&\quad + \left[V_M(\rho_0) - V_D(\rho_0)\right] h(\widehat{p}(q),q)\frac{d\widehat{p}}{dq}. \qquad (6)
\end{aligned}$$

Similarly,

$$\frac{dW_M(q)}{dq} = \int_{-\infty}^{\infty} \frac{d}{dq}\left[V_M(\rho(p,q))h(p,q)\right]dp. \qquad (7)$$

To compare the two derivatives, given by Equations (6) and (7), we fix q at the level chosen when the incumbent is myopic, say q_0, and let $\widehat{p}(q_0) = p_0$. Then, subtracting (6) from (7) yields,

$$\begin{aligned}
&\frac{d\left(W_M(\rho)\right)}{dq} - \frac{d\left(W_i(\rho)\right)}{dq}\rfloor_{q_0} \\
= &\int_{p_0}^{\infty} \frac{d}{dq}\left[V_M(\rho(p,q))h(p,q)\right]dp - \int_{p_0}^{\infty} \frac{d}{dq}\left[V_D(\rho(p,q))h(p,q)\right]dp \\
&- \left[V_M(\rho_0) - V_D(\rho_0)\right] h(p_0,q)\frac{d\widehat{p}}{dq}\rfloor_{q_0}. \qquad (\ 8)
\end{aligned}$$

[9] Note that this is an insight not obtained in the Example. There, learning is either complete or zero and consequently, probability of entry is either unaffected, decreases unambiguously or increases unambiguously, regardless of the value of the prior. Nevertheless, the Example does show that probability of entry may either increase or decrease with experimentation, depending on F.

Using the derivation in the Appendix, Equation (8) becomes,

$$\begin{aligned}
& \frac{d\left(W_M(\rho)\right)}{dq}-\frac{d\left(W_i(\rho)\right)}{dq}\rfloor_{q_0} \\
= & \left(\overline{g}'-\underline{g}'\right)(1-\rho_0)\left(V_M'-V_D'\right)\rho_0\underline{f}(p_0) \\
& +\left(\overline{g}'-\underline{g}'\right)(1-\rho_0)\int_{p_0}^{\infty}(V_M''-V_D'')\frac{d\rho}{dp}\rho\underline{f}dp \\
& +\left(V_M(\rho_0)-V_D(\rho_0)\right)\left(\rho_0\overline{f}\overline{g}'+(1-\rho_0)\underline{f}\underline{g}'\right) \\
& -\left[V_M(\rho_0)-V_D(\rho_0)\right]h(p_0,q)\frac{d\widehat{p}}{dq}\rfloor_{q_0}.
\end{aligned} \tag{9}$$

Now, we can prove the main result of this paper.

Proposition 2 *The isolated incumbent learns more, by choosing a higher q, than the threatened incumbent if $V_M - V_D$ is an increasing and convex function of ρ, and the prior satisfies:*

$$\rho_0 \geq \frac{\underline{f}^2\overline{f}'}{\underline{f}^2\overline{f}'-\overline{f}^2\underline{f}'}. \tag{10}$$

Proof. The isolated incumbent learns more if and only if $\frac{d(EV_M(\rho))}{dq}-\frac{d(EV_i(\rho))}{dq}\rfloor_{q_0} \geq 0 \Rightarrow R.H.S.$ of (9) ≥ 0. Now, by assumption of increasing demand dispersion, that is, $\overline{g}'-\underline{g}' > 0$ and by assumptions on $V_M - V_D$, the first and the second terms in the RHS of (9) are positive. The third term is negative because $\overline{g}'$ and $\underline{g}'$ are both negative. And the fourth term is positive by Lemma 1 and by the fact that $V_M - V_D \geq 0$. The Proposition follows if the sum of the last two terms is positive. That is,

$$(V_M(\rho_0) - V_D(\rho_0))\,(\rho_0\overline{f}\overline{g}' + (1-\rho_0)\underline{f}\underline{g}' - h(p_0, q)\frac{d\widehat{p}}{dq}\rfloor_{q_0}) \geq 0,$$

$$\rho_0\overline{f}\overline{g}' + (1-\rho_0)\underline{f}\underline{g}' - h(p_0, q)\frac{d\widehat{p}}{dq}\rfloor_{q_0} \geq 0.$$

This inequality is the opposite of (5) and therefore, the result follows. ■

Condition (4) is necessary and sufficient for a decrease in probability of entry. Thus, condition (10) in Proposition 1 implies that if the probability of entry is increasing in q, which occurs if the prior on high demand is high enough, and if conditions on value functions are satisfied, then the isolated incumbent will experiment more than the threatened incumbent.[10]

This proof provides insight into the trade-off that a threatened incumbent faces when choosing output in the first period. The first two terms in (9) represent the reduced expected marginal benefit from experimentation given a certain probability of entry and the last two terms represent the expected gain/loss due to a change in the probability of entry induced by experimentation. Since entry reduces the expected marginal benefit from experimentation, the first two terms are positive (meaning that the isolated incumbent enjoys a higher expected marginal benefit). On the other hand, the effect of experimentation on entry deterrence is ambiguous. If it increases the probability of entry, as is the case when (10) is met, the entry-deterrence effect and the standard experimentation effect reinforce each other, implying that the threat of entry reduces information acquisition. This corresponds to the scenario of high fixed costs in the Example. On the other

[10] It can be easily verified that the assumptions on $V_M - V_D$ are satisifed for linear demand, that is, $g(q, \gamma) = a - bq$, where $\gamma = (a, b)$. There, $V_M - V_D = 5\,(\widehat{a}(\rho))^2\,/36\widehat{b}(\rho)$.

hand, if (10) is not met, entry may or may not reduce information acquisition, depending on the demand function and the nature of uncertainty. This corresponds to the case of moderate fixed costs in the Example.

We next show that the sufficient condition for entry to reduce experimentation by the incumbent becomes weaker as entry becomes more costly. That is, the incumbent experiments less in the presence of a threat of entry for a larger set of prior beliefs. This result corresponds to the monotonic effect of F on experimentation in the Example, as F increases from moderate to high.

Let $\rho_0^* \equiv \frac{\underline{f}^2 \overline{f}'}{\underline{f}^2 \overline{f}' - \overline{f}^2 \underline{f}'}$, the bound given in Proposition 2, by Inequality (10).

Proposition 3 *The bound ρ_0^* decreases as F increases.*

The proof is in the Appendix.

We next discuss cases where cost of entry is either low or high so that the effect of experimentation on entry is unambiguous, that is regardless of the value of the prior. These cases correspond to the scenarios of moderate and high fixed cost in the Example.

4.2 Extreme Fixed Cost

In this subsection, we discuss the extreme cases. There are two possibilities to consider: one, when profitable entry requires that the cut-off belief for entry satisfy $1 > \rho_e > \overline{\rho}_m$, due to a high fixed cost and two, when profitable entry occurs even when $0 < \rho_e < \underline{\rho}_m$, due to a low fixed cost of entry. It turns out that experimentation has an unambiguous effect on probability of entry in these two cases. When fixed cost is high, experimentation increases probability of entry regardless of the prior. To understand the intuition, note that in this range of prices, both $\underline{f}$ and $\overline{f}$ slope downwards. As a result, as experimentation spreads the distributions apart and to the left, the cut-off price has a higher likelihood of occurring under high demand but then to maintain the cut-off likelihood ratio, the cut-off price must also entail a higher likelihood under low demand. Thus, the probability of entry increases regardless of the state of demand. This has a straightforward implication for experimentation: the threatened incumbent has less of an incentive to experiment because both the entry-deterrence effect and the marginal benefit effect reinforce each other.

In contrast, in the other extreme case, when $\rho_e < \underline{\rho}_m$, experimentation reduces probability of entry regardless of the value of prior and therefore, the effect on experimentation of the threat of entry is ambiguous since the entry-deterrence effect and the standard benefit from learning go in the opposite direction. The intuition behind entry deterrence is that now both $\underline{f}$ and $\overline{f}$ slope upwards, and thus as experimentation spreads the distributions apart and to the left, probability of entry falls regardless of the state of demand. As a result, we do not have an obvious sufficient condition to ensure less experimentation by an incumbent facing entry. Note that this is the case in which the trade-off between standard marginal benefit from experimentation and entry-deterrence benefit is most severe. This would seem to be the case in which entry may increase experimentation. On the other hand, this is also the case when entry occurs more often and therefore the expected marginal benefit from experimentation (given a probability of entry) is lower. Thus, overall, one cannot conclude that entry increases experimentation.

The linear-uniform example illustrates these results. When entry cost is high, the probability of entry is increasing in experimentation and experimentation is less under a threat of entry. When entry cost is moderate, probability of entry decreases in experimentation but the entry-deterrence role of experimentation may or may not be dominated by the reduced benefit from experimentation in the future due to entry.

Thus, the overall conclusion of this section, which assumes increasing demand dispersion, is that when demand displays increasing dispersion, as the cut-off belief for entry increases, the role of experimentation in entry-deterrence becomes weaker and therefore, the sufficient condition for entry to reduce experimentation becomes weaker. Since the level of fixed cost drives the cut-off belief in our model, the implication is that, the higher the fixed cost of entry, the more likely it is that the incumbent facing entry experiments less than the incumbent not facing entry.[11]

5 Conclusion

This paper provides another rationale for experimentation, namely, that it can be used as a tool for entry-deterrence, and thus, enriches the literature both on entry-deterrence and experimentation. We have analyzed the learning behavior of an incumbent who faces a threat of entry with entry costs. The main question addressed here is whether the threat of entry reduces or increases experimentation and if so, under what conditions. We have derived sufficient conditions under which the incumbent facing a threat of entry experiments less. In doing so, we have also shown that experimentation need not reduce the probability of entry and identified conditions under which entry becomes more likely or less likely due to experimentation. Further, the linear-uniform example provides a complete characterization of the effects of entry on experimentation and vice-versa.

[11] As mentioned in the Introduction, results continue to hold if demand satisfies decreasing dispersion.

References

[1] Alepuz, M.D. and A. Urbano (1999), "Duopoly Experimentation: Cournot Competition", Mathematical Social Sciences, v37, pp. 165-188.

[2] Belleflamme, P. and F. Bloch (2001), "Price and Quantity Experimentation: A Synthesis", International Journal of Industrial Organization, v19, pp 1475-1626.

[3] Blackwell, D., 1951. Comparison of experiments. In: Neyman, J. (Ed.), Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability. University of California Press.

[4] Dimitrova, M. and E. E. Schlee (2003), "Monopoly, Competition and Information Acquisition", International Journal of Industrial Organization, v21, pp 1623-42.

[5] Jain, N. T. D. Jeitschko, and L. J. Mirman, (2002), "Strategic Experimentation in Financial Intermediation with Threat of Entry," Annals of Operations Research, Vol. 114 (1-4), pp. 203-227.

[6] Jain, N., T. D. Jeitschko, and L. J. Mirman, (2003), "Financial Intermediation and Entry Deterrence," Economic Theory, 22(3), 793-815.

[7] Jain, N. T. D. Jeitschko, and L. J. Mirman, (2005), "Entry-Deterrence under Financial Intermediation with Private Information and Hidden Contracts", Review of Economic Design, vol 9, n3, pp 203-225.

[8] Matthews, S. and L. J. Mirman, (1983), "Equilibrium Limit Pricing: The Effects of Private Information and Stochastic Demand," Econometrica, Vol. 51, No. 4, pp. 981–96.

[9] Milgrom, R. and J. Roberts, (1981), "Limit Pricing under Incomplete Information," Econometrica, Vol. 50, pp. 443–60.

[10] Mirman, L., Samuelson, L., Schlee, E., 1994. Strategic information manipulation in duopolies. Journal of Economic Theory 62, 363–383.

[11] Mirman, L., Samuelson, L., Urbano, A., 1993. Monopoly experimentation. International Economic Review 34, 549–564.

[12] Patron, H., 2001. Monopoly experimentation and entry deterrence: limit pricing through learning, mimeo.

[13] Tirole, J., 1988. The Theory of Industrial Organization, the MIT Press, Cambridge, Massachussetts.

Appendix

Conditions underlying Second Period Equilibrium

The entrant maximizes the expected profits, $\pi_e(q_e, q_i, F)$, by choosing output q_e, given the incumbent's output in the second period, q_i, and the fixed cost of entry F :

$$\pi_e(q_e, q_i, F) = q_e(\rho g(q_e + q_i, \overline{\gamma}) + ((1-\rho)g(q_e + q_i, \underline{\gamma})) - F.$$

The entrant enters if and only if $\pi_e(q_e^*, q_i^*) = \pi_e(\rho, F) \geq 0$, where q_e^* and q_i^* are the Cournot-Nash equilibrium outputs under incomplete information. Since this is a standard symmetric Cournot duopoly problem, except that the demand function relates expected price to quantity and the entrant faces an entry cost, a sufficient condition for a unique, pure-strategy, equilibrium (see Tirole (1988)) is that $0 > \frac{\partial^2 \pi_i}{\partial q_i q_j} > \frac{\partial^2 \pi_i}{\partial q_i^2}$, that is the effect of the other firm's output on firm $i's$ expected marginal profit is negative and smaller in absolute value than the effect of its own output. In our model, this condition reduces to assuming that $\frac{\partial^2 \pi_i}{\partial q_i q_j} = q_i \widehat{g}_{11} + \widehat{g}_1 < 0$ where $\widehat{g}(q_e + q_i, \rho) = \rho g(q_e + q_i, \overline{\gamma}) + ((1-\rho)g(q_e + q_i, \underline{\gamma}), \widehat{g}_1$ denotes $\frac{\partial \widehat{g}}{\partial q_i}$ and $\widehat{g}_{11}$ denotes $\frac{\partial^2 \widehat{g}}{\partial q_i^2}$. Now, $\frac{\partial^2 \pi_i}{\partial q_i^2} = q_i \widehat{g}_{11} + 2\widehat{g}_1 < 0$ and smaller than $\frac{\partial^2 \pi_i}{\partial q_i q_j}$ because $\widehat{g}_1 < 0$ by assumption. If demand curve is concave or not 'too convex', so that $q_i \widehat{g}_{11} + \widehat{g}_1 < 0$, a unique pure-strategy Cournot equilibrium exists. To ensure that the equilibrium is interior, we further assume that the monopoly output is larger than the output that induces the other firm to produce zero. The equilibrium outputs q_e^* and q_i^* are functions of ρ and parameters $\underline{\gamma}$ and $\overline{\gamma}$. Note that F does not enter the solutions.

Suppressing the parameters, $\pi_e(q_e^*(\rho), q_i^*(\rho), F)$ is an increasing function of ρ if and only if $q_e^* \left(\rho \frac{\partial \overline{g}}{\partial q_i^*} q_i^{*\prime} + (1-\rho) \frac{\partial \underline{g}}{\partial q_i^*} q_i^{*\prime} + \overline{g}(\rho) - \underline{g}(\rho)\right) \geq 0$. This condition requires that $\widehat{g}_1 q_i^{*\prime} + \widehat{g}_\rho \geq 0$. That is, the direct and indirect effect of ρ on expected price is positive. Using the first order conditions and exploiting symmetry of the firms, one can show that $q_i^{*\prime} = \frac{q_i^{*\prime} \widehat{g}_{1\rho} + \widehat{g}_\rho}{2 q_i^{*\prime} \widehat{g}_{11} + 3\widehat{g}_1} \geq 0$ so that the sufficient condition for expected profits to be an increasing function of ρ is as follows:

$$
\begin{aligned}
\widehat{g}_\rho &\geq -\frac{\widehat{g}\widehat{g}_{1\rho}}{2(q_i \widehat{g}_{11} + \widehat{g}_1)} \\
\overline{g} - \underline{g} &\geq \frac{\widehat{g}\left(\overline{g}_1 - \underline{g}_1\right)}{2(q_i \widehat{g}_{11} + \widehat{g}_1)}.
\end{aligned}
$$

We impose these conditions on the demand function (linear demand satisfies these conditions, for example) and therefore, the entry condition reduces to $\rho \geq \rho_e$, where ρ_e is a constant determined from the values of $F, \underline{\gamma}$ and $\overline{\gamma}$. Further, ρ_e is an increasing function of F.

Derivation of Equation (9)

Suppressing arguments where there is no scope for confusion, we first derive the expression for the first term on the right hand side of (8).

$$
\begin{aligned}
& \int_{p_0}^{\infty} \frac{d}{dq} \left[V_M(\rho(p,q)) h(p,q)\right] dp \\
= & \int_{p_0}^{\infty} V_M' \frac{d\rho}{dq} h dp - \int_{p_0}^{\infty} V_M(\rho_0 \overline{f}' \overline{g}' + (1-\rho_0) \underline{f}' \underline{g}') dp
\end{aligned}
$$

Integrating the second term by parts yields,

$$\begin{aligned}
& \int_{p_0}^{\infty} \frac{d}{dq}\left[V_M(\rho(p,q))h(p,q)\right]dp \\
= \ & \int_{p_0}^{\infty} V_M'\left(\frac{d\rho}{dq}h + \frac{d\rho}{dp}(\rho_0\overline{f}\overline{g}' + (1-\rho_0)\underline{f}\underline{g}')\right)dp \\
& + V_M(\rho_0)(\rho_0\overline{f}\overline{g}' + (1-\rho_0)\underline{f}\underline{g}')
\end{aligned}$$

This expression can be reduced to (see Mirman, Samuelson and Schlee (1994) for the first term):

$$\begin{aligned}
& -\left(\overline{g}' - \underline{g}'\right)\left[\int_{p_0}^{\infty} V_M' \frac{d\rho}{dp}(1-\rho_0)\underline{f}dp + \int_{p_0}^{\infty} V_M'\underline{f}'\rho(1-\rho_0)dp\right] \\
& + V_M(\rho_0)(\rho_0\overline{f}\overline{g}' + (1-\rho_0)\underline{f}\underline{g}')
\end{aligned} \tag{A1}$$

Now,

$$\begin{aligned}
& \int_{p_0}^{\infty} V_M'\underline{f}'\rho(1-\rho_0)dp \\
= \ & (1-\rho_0)\left[-V_M'(\rho_0)\rho_0\underline{f}(p_0) - \int_{p_0}^{\infty} V_M'' \frac{d\rho}{dp}\rho\underline{f}dp - \int_{p_0}^{\infty} V_M' \frac{d\rho}{dp}\underline{f}dp\right]
\end{aligned}$$

Substituting this in Equation (A1), we obtain,

$$\begin{aligned}
& \int_{p_0}^{\infty} \frac{d}{dq}\left[V_M(\rho(p,q))h(p,q)\right]dp \\
= \ & \left(\overline{g}' - \underline{g}'\right)(1-\rho_0)V_M'(\rho_0)\rho_0\underline{f}(p_0) \\
& + \left(\overline{g}' - \underline{g}'\right)(1-\rho_0)\int_{p_0}^{\infty} V_M'' \frac{d\rho}{dp}\rho\underline{f}dp \\
& + V_M(\rho_0)(\rho_0\overline{f}\overline{g}' + (1-\rho_0)\underline{f}\underline{g}')
\end{aligned}$$

A similar expression can be obtained for $\int_{p_0}^{\infty} \frac{d}{dq}\left[V_D(\rho(p,q))h(p,q)\right]dp$.

Proof of Proposition 3

Proof. Let $\Delta = \underline{f}^2\overline{f}' - \overline{f}^2\underline{f}'$. Then $\rho_0^* = \frac{\underline{f}^2\overline{f}'}{\Delta}$, and,

$$
\begin{aligned}
\frac{d\rho_0^*}{dF} &= \frac{1}{\Delta^2}\left[\Delta\left(\underline{f}^2\overline{f}'' + 2\underline{f}\overline{f}'\underline{f}'\right) - \underline{f}^2\overline{f}'\left(\underline{f}^2\overline{f}'' + 2\underline{f}\overline{f}'\underline{f}' - \overline{f}^2\underline{f}'' - 2\overline{f}\overline{f}'\underline{f}'\right)\right] \\
&= \frac{\underline{f}\overline{f}}{\Delta^2}\left[\underline{f}\overline{f}\left(\overline{f}'\underline{f}'' - \underline{f}'\overline{f}''\right) - 2\overline{f}'\underline{f}'\left(\overline{f}\underline{f}' - \underline{f}\overline{f}'\right)\right]
\end{aligned}
$$

$$
\leq 0 \Leftrightarrow \underline{f}\overline{f}\underline{f}'\left(\frac{\overline{f}'}{\underline{f}'}\underline{f}'' - \overline{f}''\right) - 2\overline{f}'\underline{f}'\left(\overline{f}\underline{f}' - \underline{f}\overline{f}'\right) \leq 0. \tag{A2}
$$

Now, MLRP implies that $\frac{f'}{f}$ is decreasing in p. Thus, using the fact that in the assumed range, $\underline{f}' < 0$ and $\overline{f}' > 0$, we obtain,

$$
f'' \leq \frac{f'^2}{f} \tag{A3}
$$

$$
\left(\frac{\overline{f}'}{\underline{f}'}\underline{f}'' - \overline{f}''\right)\underline{f}' \leq \left(\frac{\overline{f}'}{\underline{f}'}\underline{f}'' - \frac{\overline{f}'^2}{\overline{f}}\right)\underline{f}'. \tag{A4}
$$

Inequality (A2) follows if,

$$
\underline{f}'\left(\frac{\overline{f}'}{\underline{f}'}\underline{f}'' - \overline{f}''\right) \leq \frac{2\overline{f}'\underline{f}'\left(\overline{f}\underline{f}' - \underline{f}\overline{f}'\right)}{\underline{f}\overline{f}}
$$

which holds because of (A4) if,

$$
\begin{aligned}
\left(\frac{\overline{f}'}{\underline{f}'}\underline{f}'' - \frac{\overline{f}'^{2}}{\overline{f}}\right)\underline{f}' &\leq \frac{2\overline{f}'\,\underline{f}'\left(\overline{f}\,\underline{f}' - \underline{f}\,\overline{f}'\right)}{\underline{f}\,\overline{f}} \\
\frac{\underline{f}''}{\underline{f}'} - \frac{\overline{f}'}{\overline{f}} &\geq \frac{2\left(\overline{f}\,\underline{f}' - \underline{f}\,\overline{f}'\right)}{\underline{f}\,\overline{f}} \\
\underline{f}'' &\leq \frac{\underline{f}'}{\underline{f}}\left[2\underline{f}' - \frac{\underline{f}}{\overline{f}}\overline{f}'\right] = 2\frac{\underline{f}'^2}{\underline{f}} - \underline{f}'\frac{\overline{f}'}{\overline{f}}.
\end{aligned}
$$

By Inequality (A3),

$$
\underline{f}'' \leq \frac{\underline{f}'^2}{\underline{f}} \leq 2\frac{\underline{f}'^2}{\underline{f}} \leq 2\frac{\underline{f}'^2}{\underline{f}} - \underline{f}'\frac{\overline{f}'}{\overline{f}}.
$$

Last inequality follows because in the assumed range, $\underline{f}' < 0$ and $\overline{f}' > 0$. Hence, the proof. ■

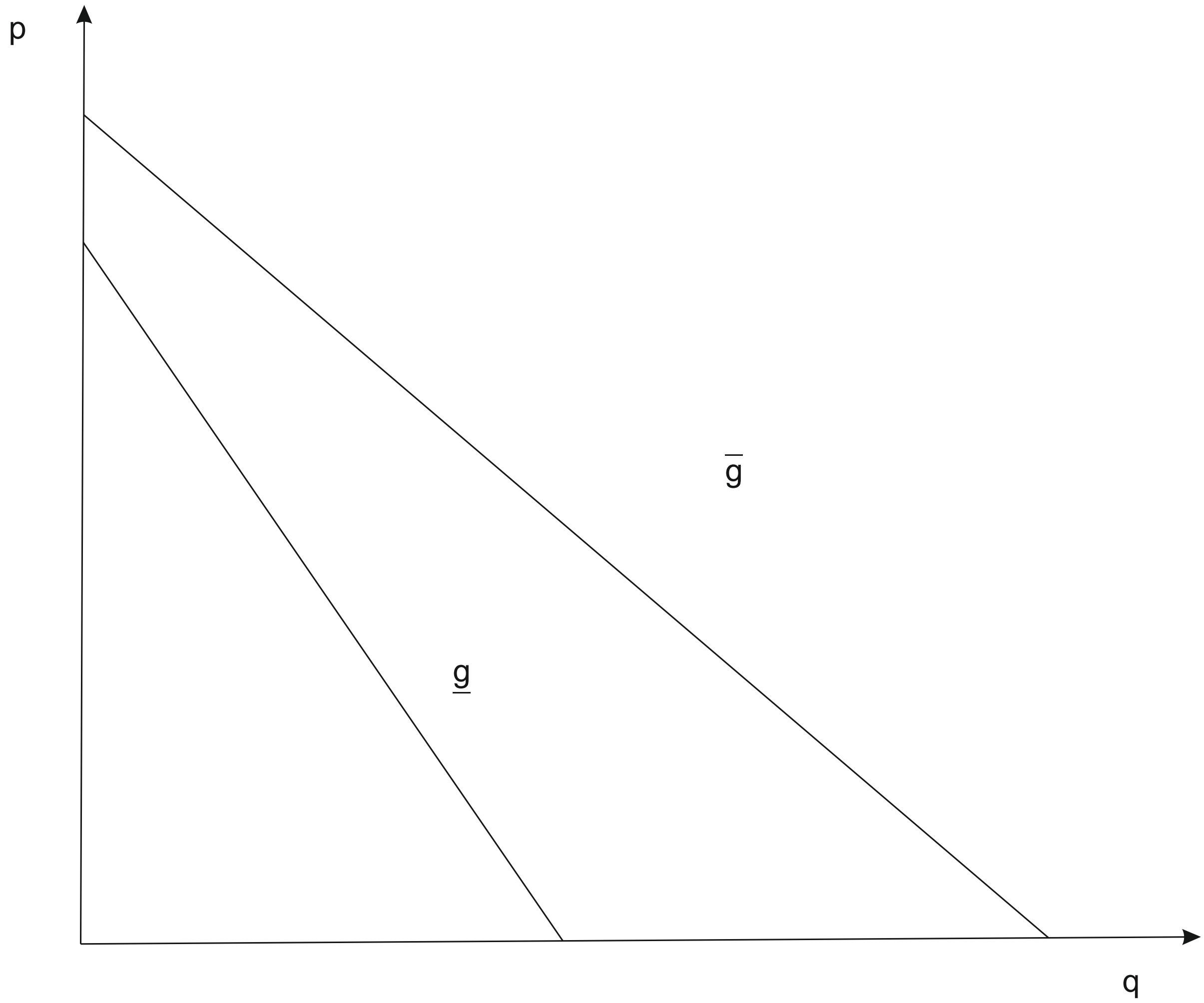

Figure 1: Two Types of Demand

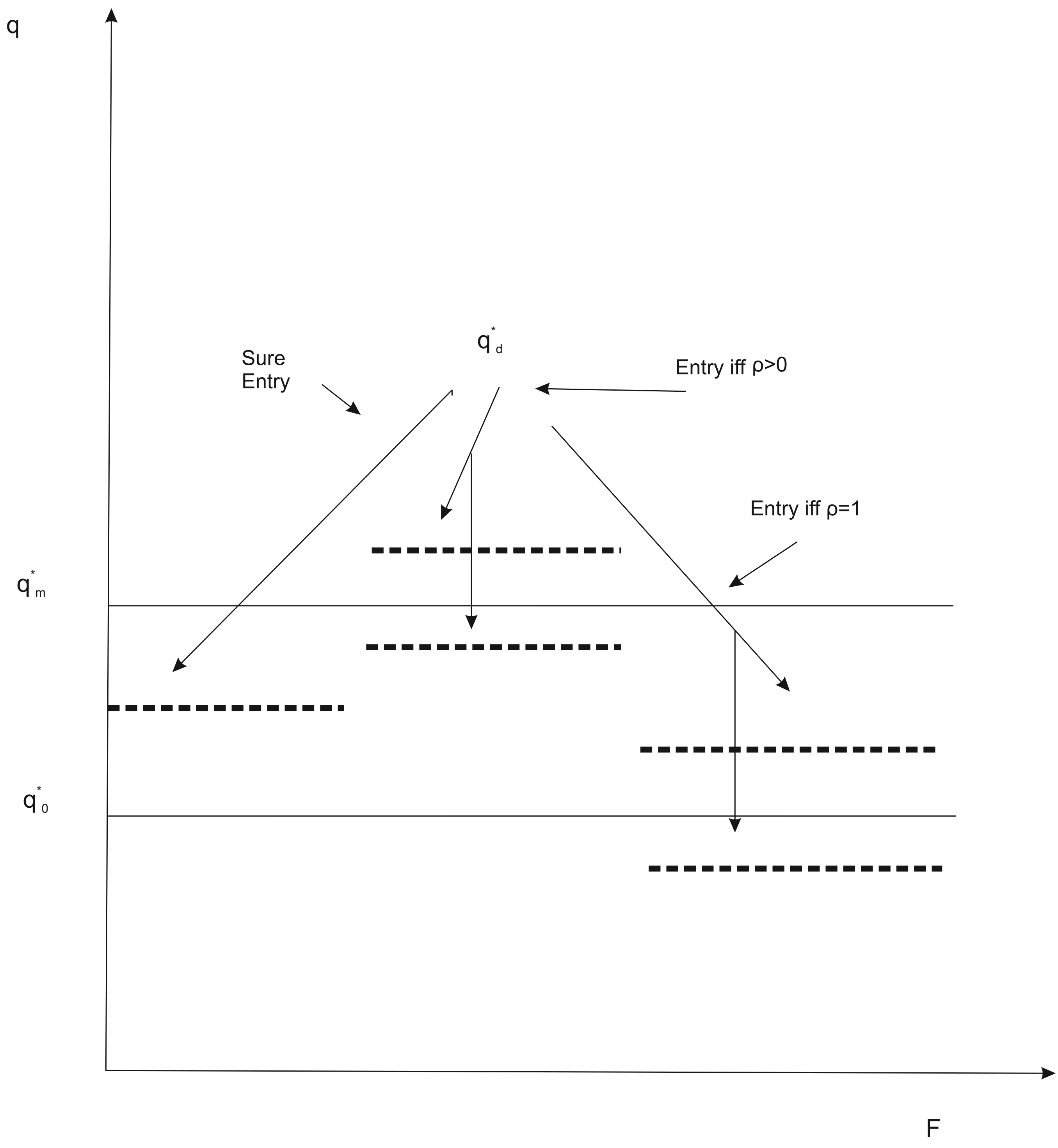

Figure 2: Relationship between F and Experimentation (Proposition 1 and Corollary 1)

.