A variational study of the random-field XY model

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A disorder-dependent Gaussian variational approach is applied to the d-dimensional ferromagnetic XY model in a random field. The randomness yields a non extensive contribution to the variational free energy, implying a random mass term in correlation functions. The Imry-Ma low temperature result, concerning the existence ($d > 4$) or absence ($d < 4$) of long-range order is obtained in a transparent way. The physical picture which emerges below $d = 4$ is that of a marginally stable mixture of domains. We also calculate within this variational scheme, disorder dependent correlation functions, as well as the probability distribution of the Imry-Ma domain size.

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I. INTRODUCTION

The effect of quenched disorder on systems with continuous symmetry has recently attracted a lot of theoretical as well as experimental interest. Experimental realizations include, among others, arrays of flux-lines in type II disordered superconductors, crystalline surfaces with a disordered substrate, spin or charge-density waves subject to random pinning, etc... Most theoretical studies of this model have focused on the random-field vortex-free XY case, which is equivalent to the random phase sine-Gordon model. [1–17]

Recently, a disorder dependent variational approach has been proposed for this problem [18]. In this approach, the disorder enters only through a unique variable $u = \int d\vec{x}\cos(2\pi d(\vec{x}))$, where $d(\vec{x})$ is a random phase. This random variable has a Gaussian distribution, and for $u < 0$, one recovers the results of the replica Gaussian variational principle, with Parisi symmetry breaking scheme [7].

One major advantage of this approach is that it is genuinely variational, thus providing a true upper bound to the free energy of the system, unlike replica based methods, which are plagued by the $n = 0$ limit.

In this paper, we shall use this variational approach in the framework of the (full) XY model in a random magnetic field, for which few results are available [19,20]. Its Hamiltonian reads:

$$\mathcal{H} = -J \sum_{<\vec{r},\vec{r}'>} \cos(\theta_{\vec{r}} - \theta_{\vec{r}'}) - \sum_{\vec{r}} h_{\vec{r}} \cos(\theta_{\vec{r}} - \phi_{\vec{r}}) \tag{1}$$

where $\theta_{\vec{r}}$ denotes the phase angle of the XY spin $\vec{S}_{\vec{r}}$, J is the ferromagnetic nearest-neighbor coupling constant, and $(h_{\vec{r}}, \phi_{\vec{r}})$ are the polar coordinates of the random field $\vec{h}_{\vec{r}}$. The probability distribution of the (site uncorrelated) random field is Gaussian, given by:

$$P(\vec{h}_{\vec{r}}) = \left(\frac{1}{2\pi h^2}\right)^N \exp\left(-\sum_{\vec{r}} \frac{\vec{h}_r^2}{2h^2}\right) \tag{2}$$

where $N = L^d$ is the total number of sites on a d-dimensional hypercubic lattice of linear size L and lattice spacing a; h is the variance of the random field.

The physics of the Hamiltonian (1) is believed to be well captured by the Imry-Ma argument, which we briefly summarize [19]. Consider, at low temperature, a magnetized domain of size L, with magnetization in the θ_0 direction. To study the stability of such a region, we imagine that we construct inside a subdomain of size ξ, in which the magnetization is aligned with the average local magnetic field. The energetic balance reads:

$$\Delta E \sim J\xi^{d-2} - h\xi^{d/2} \tag{3}$$

where the first term represents the spin-wave distortion energy, whereas the second represents the magnetic energy gain due to the random field. Following [19], we conclude that the lower critical dimension for the system is $d_c = 4$. Above d_c, the energetic cost is prohibitively high, so that such domains of size ξ cannot exist: ferromagnetic long range order is stable. On the contrary, for $d < d_c$, the whole system will break into subdomains of smaller size, so as to maximize its magnetic energy gain. Consequently, no long range ferromagnetic order may exist below d_c.

Usually, in quenched disordered systems, extensive thermodynamical quantities, such as the free energy, are identified to their average over the disorder [21]. This can be understood in the following way: one divides the macroscopic system into mesoscopic subsystems , each subsystem corresponding to a particular disorder configuration. For short-range forces, the free energy is additive, and thus the total free energy is the sum of the free energies of the subsystems. This procedure clearly neglects all correlations or domain-wall energies between neighboring subsystems. In the present case, these non-extensive contributions are precisely of the same order of magnitude than the terms of equation (3). Therefore, in the following, we will not perform quenched averages, but rather keep the disorder variables throughout the calculations.

The layout of this paper is the following. In section II, we define the variational Hamiltonian, and calculate the corresponding variational free energy. The variational method yields two solutions. In this approximation, we find that the transition temperature is the same for both solutions, equal to that of the pure system. In section III, we discuss the issue of long-range order as a function of space dimension. We find long-range ferromagnetic order at low temperature in dimensions $d > 4$. For lower dimensions, the discussion is postponed to the next section. In section IV, we discuss the non-extensive corrections to the free energy, and study the stability of the variational solutions. For dimensions $2 < d < 4$, we argue that the physical solution is the marginally stable one, shedding light on the Imry-Ma domain picture. In the conclusion, we calculate the probability distribution for the Imry-Ma length, and discuss the issue of correlation functions.

II. THE VARIATIONAL FREE ENERGY

We consider Hamiltonian (1) and its associated variational Gaussian companion:

$$\beta\mathcal{H}_0 = \frac{1}{2}\sum_{\vec{r},\vec{r}'}(\theta_{\vec{r}} - \theta_0)\ g(\vec{r} - \vec{r}')^{-1}(\theta_{\vec{r}'} - \theta_0) \tag{4}$$

where we have restricted the variational kernel g to be translationally invariant, the direction of magnetization θ_0 is space independent and $\beta = 1/T$ is the inverse temperature.

The true free energy F satisfies the bound:

$$F \leq \Phi(\{\vec{h}_{\vec{r}}\}) = F_0 + < \mathcal{H} - \mathcal{H}_0 >_0 \tag{5}$$

Using equations (1,4), we obtain the disorder dependent variational free energy:

$$\Phi(\{\vec{h}_{\vec{r}}\}) = -\frac{1}{2\beta}\sum_{\vec{k}}\ \ln\tilde{g}(\vec{k}) - \frac{J}{2}L^d e^{-g(0)}\sum_{\alpha=1}^{2d} e^{g(e_\alpha)} - e^{-g(0)/2}\sum_{\vec{r}} h_{\vec{r}}\cos(\phi_{\vec{r}} - \theta_0) \tag{6}$$

where $\tilde{g}(\vec{k})$ is the Fourier transform of $g(\vec{r})$, $\{e_\alpha\}, \alpha = 1, ..., 2d$ denote the lattice unit vectors and $\{h_{\vec{r}}, \phi_{\vec{r}}\}$ are the polar coordinates of the random field $\vec{h}_{\vec{r}}$ at site $\vec{r}$.

The variational equation with respect to θ_0 reads:

$$e^{-g(0)/2}\sum_{\vec{r}} h_{\vec{r}}\sin(\phi_{\vec{r}} - \theta_0) = 0 \tag{7}$$

If we define ϕ as the polar angle of the total magnetic field $\vec{H} = \sum_{\vec{r}}\vec{h}_{\vec{r}}$, we see that the solutions to (7) are $\theta_0 = \phi$, corresponding to a magnetization aligned with the total magnetic field $\vec{H}$, and $\theta_0 = \phi + \pi$ corresponding to a magnetization opposite to $\vec{H}$.

The minimization of $\Phi(\{\vec{h}_{\vec{r}}\})$ with respect to $\tilde{g}(\vec{k})$ then yields, for the two solutions:

$$\frac{1}{\beta\tilde{g}(\vec{k})} = J\sum_{\alpha=1}^{2d}(1-e^{ik_\alpha})e^{-(g(0)-g(e_\alpha))} \pm e^{-\frac{1}{2}g(0)}L^{-d}\sum_{\vec{r}} h_{\vec{r}}\cos(\phi_{\vec{r}}-\phi) \quad (8)$$

the $+$ (resp. $-$) sign corresponding to $\theta_0 = \phi$ (resp. $\theta_0 = \phi+\pi$). For the sake of simplicity, we shall stick below to a unique notation $\tilde{g}(\vec{k})$ for the two solutions. In the following, we shall refer to these solutions as the $(+)$ solution (for $\theta_0 = \phi$) and the $(-)$ solution (for $\theta_0 = \phi+\pi$).

Defining the components $h_x = \sum_{\vec{r}} h_{\vec{r}}\cos(\phi_{\vec{r}})$, $h_y = \sum_{\vec{r}} h_{\vec{r}}\sin(\phi_{\vec{r}})$, we see that:

$$\sum_{\vec{r}} h_{\vec{r}}\cos(\phi_{\vec{r}}-\phi) = H \quad (9)$$

The total magnetic field $\vec{H}$ is a random variable, and the distribution of its modulus is given by the central limit theorem. Defining the positive random variable u by:

$$H = L^{d/2}\ hu \quad (10)$$

we have:

$$P(u) = u\exp(-u^2/2) \quad (11)$$

Each disorder configuration is thus specified by a single positive random variable u. The u-dependent free energy per site (the upper sign corresponding to the $(+)$ solution, and the lower sign to the $(-)$ solution).

$$\phi(u) = \frac{\Phi(u)}{L^d} = -\frac{1}{2\beta}\int\frac{d^dk}{(2\pi)^d}\ \ln\tilde{g}(\vec{k}) - \frac{J}{2}e^{-g(0)}\sum_{\alpha=1}^{2d}e^{g(e_\alpha)} \mp e^{-g(0)/2}L^{-d/2}hu \quad (12)$$

where we have replaced $\sum_{\vec{k}}$ by $L^d\int\frac{d^dk}{(2\pi)^d}$ (see below).

Using the symmetry of the problem, we set $\gamma_1 = g(0)-g(e_\alpha)$ independent of α. Summing over α, we get:

$$d\gamma_1 = \sum_{\alpha=1}^{2d}(g(0)-g(e_\alpha)) = \sum_{\alpha=1}^{2d}\int\frac{d^dk}{(2\pi)^d}\tilde{g}(\vec{k})(1-e^{ik_\alpha}) \quad (13)$$

Inserting equation (8) in equation (13), we obtain the (disorder dependent) critical temperature through:

$$2dJe^{-\gamma_1}\gamma_1 = T \mp g(0)e^{-\frac{1}{2}g(0)}L^{-d/2}hu \tag{14}$$

For large system size ($L \to \infty$), we see that the second term of the r.h.s is always negligeable, and is therefore identical for the two solutions. The critical temperature is the same as in the pure system $T_c^{\rm pure} = \frac{2dJ}{e}$, where $e = 2.718...$ The quantity γ_1 varies from 0 (at $T = 0$) to 1 (at $T_c^{\rm pure}$).

As is clear from equation (8), the form of $\tilde{g}(\vec{k})$ and therefore of $\phi(u)$ is very different for the two solutions. For the (+) case, the replacement of $\sum_{\vec{k}}$ by $L^d \int \frac{d^dk}{(2\pi)^d}$ is straightforward, whereas the (−) case requires a more careful treatment, due to the existence of poles in $\tilde{g}(\vec{k})$.

III. LONG RANGE ORDER AND THE IMRY-MA ARGUMENT

A. The (+) solution

In this case, equation (8) shows that:

$$\beta g(\vec{r}) = \int \frac{d^dk}{(2\pi)^d} \frac{e^{i\vec{k}\vec{r}}}{\frac{J}{2}\sum_{\alpha=1}^{2d}(1-e^{ik_\alpha})e^{-(g(0)-g(e_\alpha))} + e^{-\frac{1}{2}g(0)}L^{-d/2}hu} \tag{15}$$

implying (for $d > 2$) the existence of long range order both in the angular and spin variables. Indeed, we have:

$$< (\theta_{\vec{r}} - \theta_0)^2 >= g(0) \tag{16}$$

$$< S^x_{\vec{r}} >= \cos\phi \ \exp(-\frac{g(0)}{2}) \tag{17}$$

and $g(0)$ is finite (infrared convergent) for $d > 2$.

For $d = 2$, one finds, as in the pure case, algebraic order, but with different exponents. For example, the total magnetization reads:

$$M^x = \frac{1}{N}\sum_{\vec{r}} < S^x_{\vec{r}} >= \cos\phi \ e^{-\frac{g(0)}{2}} \sim \left(\frac{a^2}{L\xi}\right)^{\frac{\gamma_1/\pi}{2-\gamma_1/\pi}} \cos\phi \tag{18}$$

with $\xi = \frac{J}{hu}e^{-\gamma_1}$, the pure system behaving as

$$M^x_{pure} \sim \left(\frac{a}{L}\right)^{\gamma_1/\pi} \tag{19}$$

Note the appearance of the (u dependent) Imry Ma length scale ξ in equation 18. We do not investigate further the case (+), since it will be shown, in section IV, that the physically relevant case ($2 < d < 4$) is the ($-$) one.

B. The ($-$) solution

In this case, one has to be more careful in taking the continuous limit in (12), since the poles may yield a finite contribution to thermodynamic quantities (in close analogy to the Bose condensation). For instance, we may write (for small wave vectors):

$$\beta J L^d g(0) = \sum_{\vec{k}} \frac{e^{\gamma_1}}{\vec{k}^2 - \mu^2} \tag{20}$$

where

$$\mu^2 = u\frac{he^{\gamma_1 - g(0)/2}}{JL^{d/2}} \tag{21}$$

In equation (20), there may arise a singular contribution from the $2d$ smallest wavevectors $\vec{k} = \frac{2\pi}{L}(1,0,0,...)$ (plus permutations). One therefore has

$$\beta J L^d g(0) = \frac{2de^{\gamma_1}}{\frac{4\pi^2}{L^2} - \mu^2} + L^d \int_{k > \frac{2\pi}{L}} \frac{d^d k}{(2\pi)^d} \frac{e^{\gamma_1}}{\vec{k}^2 - \mu^2} \tag{22}$$

A detailed discussion of equation (22) requires separate treatments for $d > 4$, for $2 < d < 4$ and for $d = 2,\ 4$.

1. Dimension $d > 4$

According to (21), we have $\mu^2 << L^{-2}$. We may thus neglect the μ^2 contribution in the denominators of (22), yielding a finite magnetization:

$$M^x = \frac{1}{N}\sum_{\vec{r}} < S^x_{\vec{r}} >= -e^{-\frac{AT}{J(d-2)}} \cos\phi \tag{23}$$

where $A = \frac{1}{2^d \pi^{d/2}} \frac{e^{\gamma_1}}{\Gamma(d/2) a^{d-2}}$

2. Dimension $2 < d < 4$

Since the integral in (22) is infrared (IR) convergent, it may be neglected compared to the pole contribution. In this case, one finds:

$$\beta J L^d g(0) \simeq \frac{2de^{\gamma_1}}{\frac{4\pi^2}{L^2} - \mu^2} \tag{24}$$

This last equation shows that (when $L \to \infty$), μ is very close to $2\pi/L$. Using equation (21), we get:

$$g(0) = (4-d) \ln \frac{L}{\xi_d(u)} \tag{25}$$

where

$$\xi_d(u) = \left(\frac{4\pi^2 J e^{-\gamma_1}}{hu} \right)^{2/(4-d)} \tag{26}$$

Since $g(0)$ diverges for large L, we conclude that there is no long range order in this case.

3. Dimensions $d = 2$ *and* $d = 4$

For $d = 2$, one may easily see that both terms in the r.h.s. of equation (22) are of the same order of magnitude. Indeed, the integral has a logarithmic divergence, and performing the calculation, we obtain the same result as above, namely:

$$g(0) = 2 \ln \left(\frac{L}{\xi_2(u)} \right) \tag{27}$$

with

$$\xi_2(u) = \left(\frac{4\pi^2 J e^{-\gamma_1}}{hu} \right) \tag{28}$$

For $d = 4$, one gets:

$$g(0) = 2 \ln \left(\frac{hu}{4\pi^2 J e^{-\gamma_1}} \right) \tag{29}$$

implying disorder induced fluctuations in the magnetization.

IV. MARGINAL STABILITY AND FINITE SIZE CORRECTIONS

Whenever there are several solutions to the variational equations, one ought to pick the one with the lowest free energy. In our case, it is easily seen that the extensive part is the same for both solutions, and that the difference shows up only in non-extensive corrections.

A. Dimension $2 < d < 4$

To leading order, the variational free energy reads,

$$\Phi_+(u) = F_0 L^d - A_+ \; hu \; L^{d/2} \tag{30}$$

$$\Phi_-(u) = F_0 L^d + (4-d) \; A_- \; J e^{-\gamma_1} \; L^{d-2} \; \ln\left(\frac{L}{\xi_d(u)}\right) \tag{31}$$

where

$$F_0 = -\frac{1}{2\beta} \ln\left(\frac{e^{\gamma_1}}{\beta J}\right) - dJe^{-\gamma_1} + \frac{1}{2\beta} \int \frac{d^d k}{(2\pi)^d} \ln(\vec{k}^2) \tag{32}$$

and A_+ and A_+ are positive constants, independent of L, depending on geometrical factors as well as on the temperature of the system.

The above selection criterion holds only if the two minima are not degenerate. In the opposite case, the selection rule is provided for by the fluctuations since they may contribute to the extensive part of the free energy. In other words, one has to check for the stability of the solutions, with respect to variations of $\tilde{g}(\vec{k})$ and local variations $\delta_{\vec{r}}$ of θ_0.

We consider first the $\delta_{\vec{r}}$ fluctuation contribution to the free energy and get:

$$\Delta F = \sum_{\vec{r}} \left(\frac{J}{4} e^{-\gamma_1} \sum_{\alpha=1}^{2d} (\delta_{\vec{r}} - \delta_{\vec{r}+\vec{e_\alpha}})^2 + \frac{1}{2} \; e^{-g(0)/2} \; h_{\vec{r}} \; \cos(\phi_{\vec{r}} - \theta_0) \; \delta_{\vec{r}}^2 \right) \tag{33}$$

The positivity of ΔF is determined by the spectrum of the kernel in (33). This kernel is analogous to that of a tight-binding model in a random potential:

$$\left(-\frac{J}{2} \Delta_{\vec{r}\vec{r}'} + V_{\vec{r}}\right) \Psi = \lambda \Psi \tag{34}$$

where

$$V_{\vec{r}} = e^{-g(0)/2} e^{\gamma_1} h_{\vec{r}} \cos(\phi_{\vec{r}} - \theta_0) \tag{35}$$

It is easy to see that $V_{\vec{r}}$ is a random Gaussian distributed potential with:

$$\overline{V_{\vec{r}}} = e^{-g(0)/2} \sqrt{\frac{\pi}{2}} h L^{-d/2} \rightarrow 0 \tag{36}$$

$$\overline{V_{\vec{r}} V_{\vec{r}'}} = e^{-g(0)} h^2 \delta(\vec{r} - \vec{r}') \tag{37}$$

where the bar stands for an average over the random field.

Using standard perturbation theory to second order, the ground state energy of (34) can be expanded around the zero energy mode $\vec{k} = 0$ as:

$$E_0 = <\vec{0}|V|\vec{0}> - \sum_{\vec{k} \neq \vec{0}} \frac{|<\vec{k}|V|\vec{0}>|^2}{J\vec{k}^2/2} \tag{38}$$

where $|\vec{k}>$ denote the normalized plane waves.

Since V is a random potential of mean given by (36), the first term of the r.h.s. of (38) is of order $\overline{V} \sim L^{-d/2}$. The second order term is negative and finite, and thus the energy E_0 is negative, leading to a fluctuation induced instability of the finite magnetization (+) solution.

This instability is in agreement with the Imry-Ma picture, which shows that for $d < 4$, a uniformly magnetized system is unstable with respect to domain formation.

On the contrary, for the zero-magnetization (−) solution, the disordered potential vanishes. The spectrum is that of a free particle, implying the existence of zero energy modes.

Furthermore, one may check that the (−) solution is stable with respect to variations of $\tilde{g}(\vec{k})$, since its stability properties with respect to these variations are the same as the pure system.

Putting all these results together, we may conclude that the (−) solution is marginally stable, and thus appears as the physical solution.

B. Dimension $d > 4$

It is easily seen that both solutions having finite magnetization, (aligned or opposite to the field), they are unstable. This is again in accord with the Imry-Ma argument, which shows that for $d > 4$, the magnetization (i.e. θ_0) is not determined by the random field, but rather by a standard infinitesimal uniform field. This in turn ensures the stability of the uniform magnetisation with respect to small fluctuations.

C. Dimensions $d = 2$ and $d = 4$

According to section 3, for $d = 2$, both solutions are marginally stable. This is a borderline case, due to the existence of a Kosterlitz-Thouless transition in the pure system.

For $d = 4$, it can be seen that the random potential of equation (34) is not Gaussian. This dimension is likely to be a special dimension for the localization problem. This fact is also present in the framework of the $d \to d-2$ dimensional reduction theory [22,23].

V. CONCLUSION

We have presented a disorder dependent variational method for the full XY model in a random field. We recover the Imry-Ma results concerning the existence ($d > 4$) or absence ($2 < d < 4$) of long range order. In particular, we find in the latter case, a variational solution which is marginally stable with respect to local magnetization rearrangements. Using eqs.(11) and (26), this solution can be further characterized by the probability distribution of the Imry-Ma domain length ξ_d, which reads

$$P(\xi_d) = \frac{(4-d)}{2} \frac{B}{\xi_d^{5-d}} \exp\left(-\frac{B}{2\xi_d^{4-d}}\right) \tag{39}$$

with $B = (\frac{4\pi^2 J e^{-\gamma_1}}{h})^2$. Eq.(39) shows in particular, that the Imry-Ma domains are characterized by multiple length scales; for d=3, the average domain size is found to be divergent, whereas the most probable domain size is of order B.

In a similar way, one may deduce the spin-spin correlation function. We have

$$K(\vec{r}) =< \vec{S}_0 \vec{S}_{\vec{r}} >= e^{-(g(0)-g(\vec{r}))} \tag{40}$$

Considering only the case $2 < d < 4$, we have

$$K(\vec{r}) = e^{-\left((\frac{4-d}{d}) \ w(\frac{x_\alpha}{L}) \ln \frac{L}{\xi_d(u)} \right)} \tag{41}$$

where

$$w(\frac{x_\alpha}{L}) = \sum_{\alpha=1}^{d} (1 - \cos \frac{2\pi x_\alpha}{L}) \tag{42}$$

and x_α denote the coordinates of $\vec{r}$. The disorder dependence of the correlation function is contained in the Imry-Ma length scale $\xi_d(u)$: one may therefore get its probability distribution. Here we just point out two limiting cases for the disorder dependent spin-spin correlation function. At small distances ($x_\alpha << L$), $K(\vec{r})$ is Gaussian. At large distances ($x_\alpha \sim L$), $K(\vec{r})$ decreases like a power law

$$K(\vec{r}) \sim (\frac{\xi_d(u)}{r})^{\frac{w(4-d)}{d}} \tag{43}$$

where w is the value of $w(\frac{x_\alpha}{L})$ for $x_\alpha \sim L$. This behaviour is in broad agreement with the results of (i) a real space renormalization group [3] (ii) a replica variational calculation [7], both calculations pertaining to a vortex free model. The full XY model in a random field has been recently studied [20] by Monte Carlo calculations for $d = 2, 3$, indicating (for $d = 3$) the possibility of a phase transition to a pinned vortex free phase. Our variational approach emphasizes the existence of a probability distribution for the Imry-Ma domain size, which renders the comparison with this work rather delicate. Finally, our results should be of interest in other Imry-Ma like situations [24,25].

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