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ADVANCES IN STRUCTURAL DYNAMICS, AEROELASTICITY AND MATERIAL SCIENCE

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An overview of research and a list of publications submitted to
City University London
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Doctor of Science Submission

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Summary

This submission for the degree of Doctor of Science includes all the publications by the author and a description of his research, covering the period 1969-2015. The main contributions to knowledge made by the author concern his new approaches to structural dynamics, aeroelasticity, material science and related problems. In particular, the major activities of his research relate to the (i) free vibration and buckling analysis of structures, (ii) dynamic stiffness formulation, (iii) response of metallic and composite structures to deterministic and random loads, (iv) aeroelasticity of metallic and composite aircraft, (v) a unified approach to flutter, dynamic stability and response of aircraft, (vi) aeroelastic optimisation and active control, (vii) application of symbolic computation in structural engineering research, (viii) development of software packages for computer aided structural analysis and design and (ix) thermal properties of polymer nanocomposites and hot ductility of steel.

The free vibration analysis of structures is a research topic which has been an age old companion of the author ever since he was working for his Master's degree in Mechanical Engineering in the early 1970s, when he chose a crankshaft vibration problem of the Indian Railways as the research topic for his Master's thesis. With increasing maturity and experience, he provided solutions to vibration and buckling problems ranging from a simple single structural element to a high capacity transport airliner capable of carrying more than 500 passengers and a large space platform with a plan dimension of more than 30 metres. To provide these solutions, he resorted to an elegant, accurate, but efficient method, called the dynamic stiffness method, which uses the so-called dynamic stiffness matrix of a structural element as the basic building block in the analysis. The author has developed dynamic stiffness matrices of a large number of structural elements including beams, plates and shells with varying degrees of complexity, particularly including those made of composite materials. Recently he published the dynamic stiffness matrices of isotropic and anisotropic rectangular plates for the most general case when the plate boundaries are free at all edges. Computation of natural frequencies of isotropic and anisotropic plates and their assemblies for any boundary conditions in an exact sense has now become possible for the first time as a result of this development. This ground-breaking research

has opened up the possibility of developing general purpose computer programs using the dynamic stiffness method for computer-aided structural analysis and design. Such computer programs will be vastly superior to existing computer programs based on the finite element method, both in terms to accuracy and computational efficiency. This is in line with the author's earlier research on free vibration and buckling analysis of skeletal structures which led to the development of the computer program BUNVIS (Buckling or Natural Vibration of Space Frames) and BUNVIS-RG (Buckling or Natural Vibration of Space Frames with Repetitive Geometry) which received widespread attention.

Numerous research papers emerged using BUNVIS and BUNVIS-RG as research tools. The author's main contributions in the Aeronautical Engineering field are, however, related to the solutions of problems in aeroelasticity, initially for metallic aircraft and in later years for composite aircraft. He investigated the aeroelastic problems of tailless aircraft for the first time in his doctoral studies about 40 years ago. In this research, a unified method combining two major disciplines of aircraft design, namely that of stability and control, and that of flutter and response, was developed to study the interaction between the rigid body motions of an aircraft and its elastic modes of distortion. The computer program CALFUN (CALculation of Flutter speed Using Normal modes) was developed by the author for metallic aircraft and later extended to cover composite aircraft. The associated theories for composite aircraft were developed and the allied problems of dynamic response to both deterministic and random loads were solved.

With the advent of advanced composite materials, the author's research turned to aeroelasticity of composite aircraft and then to optimization studies. New, novel and accurate methods were developed and significant inroads were made. The author broke new ground by applying symbolic computation as an aid to the solution of his research problems. The computational efficiency of this new approach became evident as a by-product of his research. The development of software based on his theories has paved the way for industrial applications. His research works on dynamic stiffness modelling of composite structures using layer-wise and higher order shear deformation theory are significant developments in composites engineering. Such pioneering developments were necessitated by the fact that existing methodologies using classical lamination theory are not sufficiently accurate, particularly when the structural components made from composite materials are thick, e.g. the fuselage of a transport airliner. Given the close relationship between structural engineering and material science, the author's research has broadened into polymers and nano-composites, functionally graded materials and hot ductility of steel. His research activities are continuing and expanding with further diversification of his interests.

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Finally, the author wishes to dedicate this thesis to the memory of his parents.

CONTENTS

1. INTRODUCTION

The author's research career began when he joined the Master of Technology Course in Mechanical Engineering at the Indian Institute of Technology, Kharagpur in 1969 after receiving his Bachelor of Engineering degree (First Class) in Mechanical Engineering from the Bengal Engineering College of Calcutta University (now Indian Institute of Science and Technology, Shibpur). In chronological order, the break-down of his research experience consists of 2 years (1969-71) at the Indian Institute of Technology, Kharagpur as a Master of Technology student in Mechanical Engineering, 5 years (1971-75) at the Indian Space Research Organisation, Trivandrum, first as a Structural Engineer (1971) and then as a Senior Structural Engineer (1975), 3 years (1975-78) in the then College of Aeronautics, Cranfield University as a Commonwealth Research Scholar, 6 years (1979-85) in Cardiff University, first as a Research Associate (1979) and then as a Senior Research Associate (1982), and 30 years (1985-present) in the Department of Mechanical Engineering and Aeronautics at City University London, first as a Lecturer (1985), then as a Senior Lecturer (1994) and Reader (1998) and finally as a Professor (2003), the position he currently holds. The research carried out by the author has culminated in 199 publications to date (104 Journal papers, 90 Conference papers and 5 Departmental reports, Google Scholar Citations: 3033, h-index: 28) which form the basis of this thesis. An overview of his work history and research experience is given in Appendix A and a list of his publications is arranged in Appendix B (Refs. 1-199). Contributed papers in published books are mentioned in Appendix C. The publication record with the number of journal papers published and citations in each year is given in Appendix D (Source: Web of Science).

During the author's research and academic career spanning more than four decades, Aeronautical Engineering has undergone great changes which the author has witnessed as a student, researcher and a teacher. Today the emphasis is on safety, the efficient use of fuel and structure, reduction of noise and environmental pollution and importantly, achieving minimum life cycle or direct operating cost. In particular, the use of fibre-reinforced composite materials has stimulated the research interest of aeroelasticians, giving birth to the subject of aeroelastic tailoring. There are two main reasons for undertaking this research. First composite structures have high specific strength and secondly, they have directional properties that can be aligned in a beneficial way to achieve desirable aeroelastic effects. The author anticipated this change and adopted advanced composite materials in his research although the root of his aeroelastic experience at Cranfield lies in metallic aircraft. The experience at Cranfield was gained at a time when the aerobatic aircraft Cranfield-A1 was designed, built and successfully flown during his doctoral studies. In the 1980s, most of the aeroelastic tailoring work was conducted in the USA, and the UK activities in this area were relatively inconsequential. Against this background the author began to initiate a programme of research by combining the power of the dynamic stiffness method (which he developed over many years as one of the most important and extensive focus areas of his research) with that of symbolic computing for aeroelastic tailoring of composite wings.

In today's competitive world of research in science and technology, cost effectiveness criteria play a relatively greater role than hitherto, particularly in the planning, prioritization and execution of research. With the demise of the National Physical Laboratory, the Aeronautical Research Council and the Royal Aircraft Establishment (later Royal Aerospace Establishment), the UK focus on conceptual research has moved away from the research establishments to the universities. The author responded to this change by initiating a series of innovative research programmes concentrating on practical applications in aircraft industry. Foremost amongst these are in the areas of dynamic stiffness formulation and free vibration analysis of structures, response of metallic and composite structures to deterministic and random loads, aeroelasticity of metallic and composite aircraft, a unified approach to dynamic stability and response of aircraft, aeroelastic optimisation and active control, application of symbolic computation in structural engineering research, development of software packages for computer aided structural analysis and design and some aspects of research in material science. A substantial part of these activities has been supported by the Engineering and Physical Research Council (EPSRC) of UK, Airbus UK, Royal Academy of Engineering, the American Air Force Base, Embraer Aircraft Company, amongst others. The author's research, particularly on advanced theories for elastodynamic, aeroelasticity and other problems has had an impact well beyond the UK and his research findings have continued to arouse considerable international interest, paving the way for further research.

2. CONTRIBUTIONS TO STRUCTURAL DYNAMICS AND AEROELASTICITY

The main contributions of the author to the advancement of knowledge concern his new approaches to (i) free vibration and buckling analysis of structures, (ii) dynamic stiffness formulation, (iii) response of metallic and composite structures to deterministic and random loads, (iv) aeroelasticity of metallic and composite aircraft, (v) a unified approach to flutter, dynamic stability and response of aircraft, (vi) aeroelastic optimisation and active control, (vii) application of symbolic computation in structural engineering research, (viii) development of software packages for computer aided structural analysis and design and (ix) research in polymer nanocomposites and hot ductility of steel. In all the joint works reported in the list of publications in Appendix B, the author made the primary contribution wherever his name appears first. In many of the papers published by the author with his research students, research fellows and other colleagues, when the names of coauthor(s) appear before him, it is merely a matter of courtesy and encouragement shown by the author to his coauthor(s).

The complete research work has been divided into the foregoing eight sections (2.1–2.8) for structural dynamics and aeroelasticity and two sections (3.1–3.2) for material science. However it should be recognised that a certain amount of overlap is inevitable between two or more sections because of the interdisciplinary nature of the work. For example, the subjects of free vibration, response, aeroelasticity and the dynamic stiffness formulation are all related and therefore, reference to some published work may appear in more than one section. In all his published work the author has been guided by two important principles. First, he has always endeavoured to give sufficient details (data) in each of his publications to ensure that interested researchers are in a position to reconstruct his results if necessary. Secondly, he has made every attempt to validate results from his theory wherever possible, by comparing his own results with those already published in the literature. However, in the absence of published results, he resorted to various novel techniques by investigating degenerate cases of his advanced theories and making sure that they converge back to simpler theories under certain restricted conditions. Alternatively in order to ensure the efficacy of his advanced theories, he often relied on numerical simulation by pushing simpler theories to the limit (for example, when developing exact dynamic theories for a tapered beam, the author obtained comparative results using a stepped approximation to the beam using 500 uniform elements).

2.1 Free Vibration and Buckling Analysis of Structures

The interest of the author in this area has grown continuously ever since he was an undergraduate and this interest has remained with him to this day. His investigation of structural vibration problems ranges from a single structural element to a high-capacity transport airliner and a large space platform.

Papers [1], [2] and [3] report on the free vibration and force analysis of the crankshaft of a diesel-electric locomotive engine following the work described in the author’s master’s thesis. The finite element method was then in its infancy and Myklestad’s and Holzer’s methods were used for flexural and torsional vibration analyses respectively. Alongside the theoretical analysis, the author conducted a series of experiments to verify the flexural and torsional natural frequencies and mode shapes obtained from the theoretical analysis.

Paper [4] focuses on the stress wave patterns in a solid propellant rocket subjected to an axial thrust. Essentially the paper deals with the free vibration characteristics of the elastic rocket structure and then determines the dynamic deflections and stresses due to the pulse loading. The usefulness and originality of the work stemmed from the idealization of the rocket structure to represent its mass and stiffness properties and yet obtain sufficiently accurate results. This enabled some important engineering judgments to be made concerning the structural safety of the launch vehicle. Paper [5] extends this work, giving the response of a rocket structure to transverse random loads using the finite element method. The author computed the power spectral density (PSD) of the responses of the rocket structure when subjected to an ideal white noise and to a peaked type PSD input. The response of randomly excited orthotropic sandwich plates is described in [6]. The mean square response of the displacement was computed for variation of various non-dimensional parameters, including the damping ratio, core shear stiffness, core orthotropy, face orthotropy and aspect ratio of the plate.

Paper [7] highlights the danger of using approximate and semi-analytical methods when predicting the natural frequencies of a structure, noting some errors in a published paper. Papers [8] and [9] give details of the computer program BUNVIS developed using exact member theory. BUNVIS was subsequently used to predict the torsional rigidity of a racing car frame [23]. Paper [10] describes a novel approach to the design of a self-expanding stayed column for use in space. One of the main objectives of this research was to achieve a lighter structure without compromising its buckling strength. An important feature of BUNVIS [8, 9] is that it can handle stayed columns as substructures in a computationally efficient manner while retaining the exactness of the computed results. Its application provides an insight into the free vibration characteristics of self-expanding stayed columns for use in space [11].

Paper [12] gives analytical expressions for stiffness coefficients of axially loaded Timoshenko beams and presents FORTRAN codes to perform exact free vibration and buckling analysis of plane frames. Paper [15] reports accurately computed modal densities for plates and cylinders including corrugations and stiffeners. The object of the work was to co-relate results with experimental observations and also to provide data for analysis using the statistical energy method. The author initiated an exhaustive and detailed free vibration analysis of large space structures. Paper [16] illustrates the exact free vibration characteristics of a space platform with plan dimension of more than 30 metres and assembled from stayed columns. The investigation showed that for a given computational effort, the dynamic stiffness method gives the most accurate (exact) results.

Papers [17] and [18] give approximate (but sufficiently accurate) results for natural frequencies of axially loaded tapered beams with linear and parabolic distributions of taper ratios. The investigations focused on a wide range of cross-sections covering the majority of engineering applications. Subsequently, the author developed the exact dynamic stiffness matrix for tapered beams [20] using Bessel functions. Paper [21] describes the BUNVIS-RG software (an extended version of BUNVIS [8, 9]), which was widely acknowledged as a sophisticated research tool for free vibration and buckling analysis of space structures. The development of BUNVIS-RG was a collaborative project with NASA Langley Research Center, and a substantial amount of the original research and writing and debugging of the code were carried out by the author.

In Paper [25], the author published a computer program for studying the buckling behaviour of plane frames consisting of tapered members. Later work included the effect of shear deformation on the critical buckling of columns [41] and provided an insight into the buckling behaviour of shear deformable columns, particularly in the context of future applications using fibre-reinforced anisotropic composites which have low shear moduli.

The investigation of the free vibration characteristics of bending-torsion coupled beams using the dynamic stiffness method and the development of associated computer programs were reported in [29] and [33] respectively. Based on this work, free vibration characteristics of an axially loaded bending-torsion coupled beam [34] and a bending-torsion coupled Timoshenko beam [35] were investigated using the dynamic stiffness method. Later, the effects of shear deformation, rotatory inertia and axial load were combined in a unified manner when investigating the free vibration characteristics of bending-torsion coupled beams [44].

The author’s work on the free vibration of bending-torsion coupled beams is of particular significance in relation to the dynamic characteristics of aircraft wings. Here the coupling between the axial and torsional motion is relatively unimportant, but for certain structures such as helicopter blades, it has considerable importance. Paper [42] develops the dynamic stiffness matrix of extension-torsion coupled structural members using a general theory whose applications include free vibration analysis of helical springs, composite beams and helicopter blades. A physical approach to this analysis is reported in [37]. The effect of warping stiffness on bending-torsion coupled free vibration using the dynamic stiffness method was reported in [39] and [55]. Paper [69] describes, within the context of flutter and sensitivity analyses, the modal behaviour of a large transport airliner capable of carrying more than 500 passengers.

Next the author turned his attention to free vibration analysis of composite structures which allow coupling between various modes of deformations to be manipulated to produce desirable dynamic consequences. The research has significant applications in aeronautical design. Paper [38] reports studies on the free vibration characteristics of composite beams that exhibit coupling between extensional and torsional motion as applicable to helicopter blades. Paper [50] is the first report on exact free vibration analysis of composite beams using the dynamic stiffness method and symbolic computation. In particular the paper deals with bending-torsion coupling that generally occurs in composite wings having symmetric anisotropic lay-ups. From an aeroelastic tailoring point of view, this work is significant because the free vibration analysis of a composite wing is a preliminary step before carrying out the aeroelastic analysis. Paper [56] extends this work to form the basis of free vibration analysis of composite Timoshenko beams. The work is particularly important for composite beams (wings) because fibrous composites have generally low shear moduli. Later an integrated approach was used in [73] to investigate the free vibration characteristics of an axially loaded composite Timoshenko beam.

The author's intention has always been to provide, wherever possible, exact solutions to the free vibration problems of structural elements in explicit analytical form. Paper [78] gives explicit analytical expressions for the frequency equation and mode shapes of bending-torsion coupled beams of which an aircraft wing is a classic example. Derivation of such expressions is a formidable task and has become possible because of the advances in symbolic computation in recent years. This first analytical approach to the problem opens up the prospects for enhanced developments in aeroelastic optimisation because of its computational efficiency. A further significant advance is reported in [80], which used symbolic computation to derive the frequency equation and mode shapes of an axially loaded bending-torsion coupled Timoshenko beam in explicit analytical form. This research was extended to more complex composite beam problems [86, 87].

Next the author focused his attention to rotating structural elements. The free vibration analysis of centrifugally stiffened uniform and tapered beams using the dynamic stiffness method was reported in [79]. This work was later extended to rotating Timoshenko beams in [89] which is significant because a serious error found in the existing literature was rectified in its formulation. The research paved the way for further research on rotating tapered beams using Bernoulli-Euler beam theory [120] and Rayleigh beam theory [161], respectively. At high rotational speeds, additional effects such as the effects of Coriolis forces which couples the in-plane flexural motion with axial motion of a rotating beam can become significant. To address the problem, the author investigated the effects of Coriolis forces on the free vibration of a rotating beam in a recent publication [177].

The author made extensive investigations on the free vibration of beams. He studied the free vibration problems of twisted beams [90, 108], sandwich beams [100, 115, 126], triply coupled beams [118], moving beams [125, 131], cracked beams [134], spinning beams made of both isotropic [106] and anisotropic [119] materials and functionally graded beams [158, 159, 162, 186]. An important aspect of author's research on beam dynamics concerns an integrated approach wherein he studied coupled systems combining continuous beam elements with discrete single and multi-degree of freedom systems [98, 99, 113, 152]. All these investigations on the free vibration behaviour of beams and coupled beam structures are believed to be the first of their kind. Alongside these activities, the author made an effort to transfer his knowledge to other disciplines. He thus applied his aeronautical engineering knowledge to civil engineering structures by investigating the free vibration of bridge decks in an aeroelastic context [101].

Having studied the free vibration characteristics of a range of isotropic and anisotropic beam structures including tapered, twisted, rotating, coupled, spinning, moving, cracked and functionally graded beams, the author turned his attention to free vibration problems of isotropic and anisotropic plates. In Papers [136, 139], first order shear deformation theory (FSDT) has been used for the first time in explicit analytical form to investigate the free vibration behaviour of plates and plate assemblies using the dynamic stiffness method. The theory presented in these papers also includes the effects of rotatory inertia. Extensive results for the free vibration characteristics of uniform and non-uniform plates are illustrated with significant conclusions drawn when important parameters are varied. Paper [140] represents a breakthrough in the investigation of in-plane free vibration of plates using the dynamic stiffness method. A distinctive and useful set of solutions which went unnoticed by eminent researchers for well over four decades was discovered for the first time and the inadequacies and limitations of earlier works were redressed. New theories and results were presented in this paper and all available published results were used for comparison. Paper [146] reports a detailed parametric study on the free vibration of plates and stringer panels with mass and spring attachments. The paper is particularly helpful in solving frequency attenuation problems and the results form useful checks on approximate methods. The methodologies described in these papers are further developed to cover the free vibration of anisotropic composite plates. Paper [149] makes a novel contribution by providing a comprehensive theory for the free vibration analysis of composite Mindlin plates (accounting for shear deformation and rotatory inertia). Applications of this theory are covered in paper [150] which gives extensive results for composite plates including stringer panels. The paper also highlights the huge advantages of the proposed theory, both in terms of numerical accuracy and computational efficiency, when compared with the traditional finite element method. The Mindlin plate theory, accurate though it is when compared with classical plate theory, has limitations in that it presupposes a fictitious (and often controversial) shear correction factor to satisfy the zero stress condition at the free boundary. Paper [160] is a significant refinement, which overcomes the shear correction limitation by incorporating higher order shear deformation theory in the free vibration analysis of composite plates using the dynamic stiffness method. The paper thus provides free vibration results for a range of uniform and non-uniform composite plates. This novel theory is especially important for thick composite plates because fibre-reinforced composites have generally low shear moduli. When presenting results a wide range of parameters such as thickness ratio, orthotropy ratio, step ratio for the stepped plate, number of layers, lay-ups, stacking sequence, and boundary conditions of the plate were used. A companion paper [163] deals with the buckling behaviour of composite plates using higher order shear deformation theory. The paper provides a novel theory using the dynamic stiffness method and symbolic computation, covering a wide range of problems on the buckling strength of composite plates with varying degrees of geometric complexity. Results were obtained by changing significant parameters (thickness ratio, orthotropy ratio, step ratio, number of layers, lay-ups and stacking sequence and boundary conditions). In the literature, the free vibration analysis of rectangular plates using different methods has been generally carried out in an approximate manner except for the two obvious cases when either two opposite sides of the plate or all of its four sides are simply supported (the former is described as Levy type plate whereas the latter is known as Navier plate and exact results are available only for these two cases). It was thus not possible to obtain exact natural frequencies of plates for any arbitrary boundary conditions, for example completely free or fully clamped plates. The difficulty to seek exact results for arbitrary boundary conditions arises from the fact that the bi-harmonic equation which governs the plate vibration problem is not easily amenable to an exact general solution. The equation has been around for more than two centuries, but it was not until very recently when its general solution in the context of solving the free vibration problem of rectangular plates became possible. The author developed the dynamic stiffness matrix of completely free isotropic [188] and anisotropic [187, 189, 190] rectangular plates for the first time by solving the bi-harmonic equation in an exact manner. Computed exact natural frequencies and mode shapes of isotropic and anisotropic plates for any arbitrary boundary conditions including completely free and fully clamped plates and their assemblies are now available as a consequence of the author’s research. Papers [187-190] give extensive free vibration results for a wide range of isotropic and anisotropic plates and plate assemblies and compare and contrast them with results obtained from various other methods. Having extensively researched the free vibration and buckling behaviour of isotropic and anisotropic beams and plates, the author turned his attention to shell structures [164, 167, 171, 183, 185]. The results given in these papers can be used as bench-mark to validate finite element and other approximate methods.

In recent years, the author established collaboration with Professor Erasmo Carrera of Polytechnic of Turin, Italy in order to integrate the dynamic stiffness method (DSM) with that of the Carrera Unified Formulation (known as CUF in the literature). The collaboration was extremely promising and led to the publications of new theories and many interesting results. This was possible as a consequence of combining DSM and CUF for free vibration analysis of isotropic [166] and anisotropic [173, 176] beams. These investigations are significant because they capture all important three dimensional effects inherent in CUF providing cross-sectional deformations of the beam in its free vibratory motion. The natural frequencies and mode shapes were checked using the results obtained from detailed finite element analysis carried out with the help of commercially available codes, utilizing a large number of three dimensional solid elements [166, 173]. The enormous advantages of using the integrated theory based on both DSM and CUF have been demonstrated in these papers. The associated problem of buckling was solved in [181] by using this novel approach.

2.2 Dynamic Stiffness Formulation for Structural Elements

For the past 35 years the author has been developing the dynamic stiffness properties for a new generation of structural elements made of both isotropic and anisotropic materials. He has developed dynamic stiffness matrices of commonly used structural elements such as beams, plates and shells. His work on beams includes tapered beams [20, 22, 25], twisted beams [90, 108], bending-torsion [29, 33, 34, 35, 44, 50, 55, 56, 73, 118] and extension-torsion coupled beams [38, 42], sandwich beams [100, 115, 126], rotating beams [79, 89, 120, 161, 177], moving beams [125, 131, 196], spinning beams [106, 119], cracked beams [134], functionally graded beams [158, 159, 162, 186, 195] and beams carrying single and multi-degree of freedom systems [99, 112, 113, 152]. Alongside these investigations, the dynamic stiffness developments for plates have been explored by the author in great depth [139, 140, 149, 150, 160, 163, 187, 188, 189, 190]. Foremost amongst these developments is the formulation of the dynamic stiffness matrix of a completely free plate in an exact sense which hitherto had not been possible. Due to the difficulty in obtaining the general solution of the bi-harmonic equation, the dynamic stiffness development of plates using exact theory was earlier restricted to the case when the plate was simply supported at its two opposite edges. This restriction continued for well over four decades. Recently, the author overcame this restriction by formulating the dynamic stiffness matrices of isotropic [188] and anisotropic [187, 189, 190] plates for the most general case through the exact solution of the bi-harmonic equation. Furthermore, he made significant efforts to extend his dynamic stiffness research to shells [164, 167, 183]. A brief description of the salient features of the dynamic stiffness research of the author is given below.

Paper [20] gives explicit expressions for dynamic stiffness elements of tapered beams for which the governing differential equations in free vibration were solved using Bessel functions. The corresponding (static) stiffness coefficients for buckling analysis were reported in [22]. One of the next significant developments was that of a bending-torsion coupled beam [29]. This has an important application in flutter analysis of aircraft wings and the theory constitutes an important module for the modal analysis of aircraft wings coded in CALFUN [13, 30]. This code [33] is a fundamental prerequisite for the computation of flutter speed, flutter frequency and flutter mode of an aircraft wing. Further work on the dynamic stiffness development of bending-torsion coupled beams included the important effects of an axial load [34] and of shear deformation and rotatory inertia [35]. These efforts culminated in the development of a unified dynamic stiffness theory [44] for a coupled beam, which combined the effects of shear deformation, rotatory inertia and an axial load. Years of accumulated experience enabled the author to give a general theory [48, 60] which illustrated the dynamic stiffness development procedure in its entirety. This comprehensive approach was later used for the first time to develop the dynamic stiffness matrix of twisted beams using Bernoulli-Euler [90] and Timoshenko [108] theories. The applications of this research include compressor and helicopter blades, amongst others. In order to improve the efficacy of structural analysis of sandwich beams, the author developed the dynamic stiffness properties of a range of sandwich beams using different theories [100, 115] and importantly verified his results by experimental means [126]. These investigations were significant and some of the most interesting modal deformations were captured by using a combination of light and heavy materials such as rubber, aluminium, steel and lead in the core and face materials respectively. Papers [99], [113] and [152] which report the dynamic stiffness developments of beams coupled with spring-mass systems are important pieces of research with several practical applications. For instance the research is directly relevant for the prediction of human-structure interactions and also for the solution of frequency attenuation problems.

The author initiated a programme of research on the dynamic stiffness formulation of rotating structural elements. Papers [79] and [89] give the dynamic stiffness matrices of centrifugally stiffened beams using Beronoulli-Euler and Timoshenko beam theories, respectively, and accounting for an outboard force at the free end, making the applications sufficiently general. The comprehensive set of results reported in these papers demonstrated the effects of rotational speed, hub radius and other beam parameters including slenderness ratios on the dynamic behaviour of rotating beams. The research led to further developments of the dynamic stiffness method for the corresponding rotating tapered beams [120, 161]. The types of taper considered included a majority of practical cross-sections. The variations of natural frequencies and mode shapes in tapered beams reported in these papers would enable designers to use engineering judgment as to their suitability in distributing strength and stiffness, for saving mass and accommodating aesthetic considerations. At exceptionally high rotating speeds, the effects of Coriolis forces which couples the in-plane flexural and axial motions of a rotating beam can be significant. This was investigated in detail in paper [177]. In a rotating beam, the centrifugal force induces tension which increases the stiffness and hence has a stabilizing effect, whereas for a spinning beam the effect can be counter-productive as the advancing and retreating modes can be very different. The latter causes instability and there is a critical spinning speed at which the natural frequency tends to zero and the beam becomes unstable. The author recognized the need to develop the dynamic stiffness matrices for spinning beams and carried out an in-depth investigation on the subject [106]. Instability of beams in dynamic motion was again captured by the author when he developed the dynamic stiffness matrices of moving beams [125, 128]. Engineering applications of the theories he developed include chain drives, belt drives and robotics amongst others. These investigations are significant because there is a critical moving speed at which the moving beam becomes unstable. The author captured and quantified this instability and showed how the dynamic stiffness method can be used to solve such problems.

Next the author began fundamental research on the dynamic stiffness formulations for plate elements. Paper [139] presents the dynamic stiffness elements for plate elements with the inclusion of the effects of shear deformation and rotatory inertia. One of the significant features of this research was that explicit expressions for the dynamic stiffness elements were generated for the first time. A parallel research for dynamic stiffness properties of plates for in-plane free vibration was published [140] which was indeed, a breakthrough because the paper identified a new set of solution that was missed earlier for well over four decades by eminent dynamic stiffness researchers. Despite all these advancements, the dynamic stiffness research for plates lacked generality in that there was always an inherent assumption in all previous investigations that two opposite sides of the plate must be simply supported when deriving the dynamic stiffness matrix in an exact sense. Recently this restriction has been removed by the author when he developed the dynamic stiffness matrix of a rectangular plate for the general case which allows any arbitrary boundary conditions. He achieved this by solving the bi-harmonic equation and taking advantage of the symmetry of the rectangular plate [188]. Generalised Fourier series and limitant theory were employed in this research [187].

While establishing a general procedure for dynamic stiffness development of metallic structural elements, the author also focused his attention on composite structural elements. This was against the background that advanced composite materials were making their headway through their high specific strengths and directional properties. Particularly from an aeroelastic standpoint, the author took full advantage of such materials and used dynamic stiffness development as the foundation to investigate the dynamic characteristics of composite structures. Papers [38] and [42] report for the first time the extensional-torsional vibration behaviour of a composite beam using the dynamic stiffness method. As a result of ply orientation composite structures exhibit directional properties and the paper demonstrates these effects on the natural frequencies of extension-torsion coupled composite beams. Following this work, a significant development in the context of the aeroelastic tailoring of composite wings took place [50]. This paper presents in explicit analytical form the dynamic stiffness matrix of a composite beam (wing) which exhibits coupling between bending and torsional modes of deformation. One of the important aspects of this work is its model accuracy. As in all cases with dynamic stiffness formulation, higher natural frequencies and mode shapes can be computed from the theory without the need for further discretisation of the structure and, importantly, without any loss of accuracy in the analysis. This, in sharp contrast to finite element and other approximate methods, is significant, particularly from a computational standpoint because research in areas such as aeroelastic tailoring is generally computer intensive.

It has long been recognised by the author and others that the effects of shear deformation and rotatory inertia, which are relatively less important for metallic structures, may have significant effects on the free vibration characteristics of composite structures which generally have very low shear moduli. The author therefore, carried out a number of investigations on composite beams using the dynamic stiffness method to examine the effects of shear deformation, rotatory inertia and axial load [56, 73]. Further research was instigated on the dynamic stiffness development of composite beams including spinning beams [119] and aircraft wings [130]. Although the application of advanced composite materials is overwhelmingly promising, there are however some potential problems, particularly associated with the delamination of composite laminates. Functionally graded materials for which the properties vary continuously using a predetermined formula have no such delamination problems and the author seized this opportunity and formulated the dynamic stiffness matrices of functionally graded beams using Bernoulli-Euler [158, 162] and Timoshenko [159, 186] theories.

Next the author turned his attention to dynamic stiffness formulations of composite plate elements. Paper [149] gives the dynamic stiffness theory of composite Mindlin plates for the first time, and a comprehensive set of results is given in [150]. As is well known, the Mindlin plate theory accounts for the effects of shear deformation and rotatory inertia, but the theory relies on the use of a fictitious (and often controversial) shear correction factor to compensate for the free stress boundary condition on the surface of the plate on a rather ad-hoc basis. To overcome this shortcoming, the author extended his research to investigate the problem by using higher order shear deformation theory through the application of the dynamic stiffness method [160, 163] which dispenses with the previously unavoidable shear correction factor used in the Mindlin theory. The author's research also led to the development of the dynamic stiffness theories for sandwich and/or composite plates using layer-wise theory [157, 172]. This development is ground breaking because it eliminates the shortcomings and inadequacies of the classical lamination theory in an innovative way. Nevertheless, all these dynamic stiffness developments for composite plates were restricted to simply supported boundary conditions of opposite edges. Recently a break-through came when the author developed the dynamic stiffness matrices of composite plates for the general case with no restriction at all on boundary conditions [189, 190]. Alongside this research, the author turned his attention to the dynamic stiffness development of composite shells [164, 167, 183, 185] to provide accurate results for their free vibration characteristics, but this research at present is not applicable for the most general case with any arbitrary boundary conditions, and hence requires further developments.

The author's research on the dynamic stiffness formulation for beam elements was somehow restricted for some applications because the elements were basically considered to be one-dimensional. Thus all allowable displacements were confined as variables of only one direction, i.e. along the length of the beam and no cross-sectional deformations of the beam were taken into account in the formulation. The author became aware of the Carrera Unified Formulation (CUF) in the literature which was earlier applied in a finite element context. In a collaborative endeavor he seized the opportunity to utilize CUF when developing the dynamic stiffness matrices of isotropic [166] and anisotropic [173, 176] beam elements which provided cross-sectional deformations of the beam in its free vibratory motion in a three-dimensional manner. These contributions are enormously important and have far reaching consequences, particularly in aeroelastic studies. Earlier research on aeroelasticity of aircraft wings using stick model based on conventional beam element idealisation was unable to capture the chord-wise deformations of the wing, but now the incorporation of CUF in the DSM has paved the way for a much more comprehensive aeroelastic analysis using stick model.

2.3 Response of Metallic and Composite Structures to Deterministic and Random Loads

The author's interest in the solution of response problems dates back to the early 1970s when his first paper on randomly excited orthotropic sandwich plates [6] was published. In his doctoral studies he investigated the response of aircraft structures to deterministic and random loads. In particular he studied aircraft response to discrete gusts following the time domain analysis, whereas his investigation of aircraft response to continuous atmospheric turbulence was focused on the frequency domain analysis using the power spectral density (PSD) method. The same procedure has been used in [54, 57] in the context of metallic aircraft wings and later in the context of composite wings [52, 65]. The author showed for the first time that the usually adopted (displacement based) criteria of choosing normal modes in the response analysis of bending-torsion coupled beams such as an aircraft wing could lead to erroneous conclusions. In characterising normal modes, Paper [81] reports a novel approach based on the concept of generalised mass which has been proved to be the most effective way of choosing the appropriate subset of normal modes for a response analysis. The response analysis of structures to various types of loading is a natural extension of the author's experience and expertise on dynamic stiffness formulation and free vibration analysis.

2.4 Aeroelasticity of Metallic and Composite Aircraft

Significant time was devoted by the author in developing theories and software which can be used as research tools when dealing with the aeroelastic problems of metallic or composite aircraft. CALFUN [13] is such a research tool that has received considerable attention and has undergone major improvements and modifications.

Paper [13] in essence illustrates the original capability of CALFUN for metallic aircraft with a companion user guide [30]. Following the author's doctoral studies in the late 1970s, Paper [14] reports the flutter characteristics of a tailless aircraft, comparing results with those of a tailed aircraft of similar class. This investigation is believed to be the first of its kind and is directly relevant to the current interest in blended wing-body aircraft. The complexity of the investigation demanded a novel, but unified, approach combining two important disciplines of aircraft design, namely that of stability and control and that of flutter and response. In particular the emphasis was placed on the interaction between the rigid-body motion of the aircraft and its elastic modes of distortion. For a tailless aircraft, the interaction between the short period oscillation and the first wing bending mode can be serious, giving rise to an overall instability at a low speed. Paper [26] describes a unified method for solving the aeroelastic problems of a tailless aircraft, whereas [27] illustrates the flutter modes and contrasts them with those of a tailed aircraft.

The effect of axial force on the flutter characteristics of high aspect ratio aerofoil blades is examined in [24]. By contrast [28] reports the effects of tail plane aerodynamics and fuselage flexibility on the flutter characteristics of high aspect ratio aircraft. Paper [31] explores the validity of strip theory in flutter analysis.

The important paper [32] examines the aeroelastic behaviour and response to turbulence of the Cranfield aerobatic aircraft (Cranfield A1). After establishing the flutter speed, the paper gives the response of the aircraft to discrete gusts and continuous atmospheric turbulence. The former is carried out in the time domain whereas the latter is carried out in the frequency domain using the PSD method. Response studies showed an overlap between the two methods: when interpreted properly, the two methods give comparable results. Paper [69] reports predictions of free vibration and flutter behaviour of a high capacity transport airliner and extends the work further for sensitivity analysis as required for optimisation studies.

The author's attention was focused next on the flutter investigation of aircraft wings made of composite materials. Papers [46] and [58] examine the flutter speed of composite wings using an exact dynamic stiffness matrix method. Following these investigations, a parametric study established that the flutter speed of a composite wing is significantly influenced by both ply orientation and sweep angle [51, 59]. Paper [62] provides a critical account of the flutter modes of composite wings exhibiting wash-in and wash-out. These papers are internationally significant as they provide the first aeroelastic application of the dynamic stiffness method to composite wings.

Paper [63] focuses on a wide range of composite wings and identifies the circumstances when either the torsional or the bending-torsion (material) coupling rigidity, or both, can have pronounced effects on the flutter and divergence speeds of composite wings. A representative metallic wing was included in the analysis to compare, contrast and assess the results.

The author initiated and maintained a number of research programmes on the aeroelasticity of composite wings. Paper [66] demonstrates the importance of including the higher order modes and is the first reporting of the role of modal interchange on the flutter of composite wings. The author's integrated approach of using the dynamic stiffness method in flutter analysis was particularly valuable in the investigation. For example, a complete misunderstanding and an erroneous concept continued to exist amongst an elite research community in aeroelasticity, mostly in the USA. A number of research workers there concluded that from an aeroelastic point of view, the effect of wash-in enhances the flutter characteristics of a composite wing, whereas the opposite conclusions are applied to the effect of wash-out. Paper [72] is the first of its kind to refute completely this commonly held view. The author showed conclusively that under certain circumstances wash-out can be beneficial for a composite wing from the flutter standpoint.

Although a wing structure is very different from a bridge structure, the author recognized that some of the fundamental aspects of aeronautical engineering research, particularly in aeroelasticity, can be applied to civil engineering structures and thus a knowledge transfer is possible. This encouraged him to investigate the aeroelastic behaviour of bridge decks [101], arriving at many useful conclusions.

The author's earlier research in aeroelasticity was predominantly confined to low (subsonic) speeds because the unsteady aerodynamic model used was linearized small deflection theory or potential flow based on Theodorsen type unsteady aerodynamics or Multhopp type lift surface theory [14, 24, 26, 27, 28, 31, 32, 47, 49, 64, 69] which do not account for non-linear effects at transonic speed arising from shock waves and compressibility. Thus the investigations carried out by the author were accurate in the low to medium speed range, but the results (flutter speed, flutter frequency, divergence speed etc) cannot be sufficiently relied upon around the transonic speed region where the unsteady aerodynamics is highly non-linear. The task of carrying out an in-depth investigation in the transonic speed region has always been a major challenge to the aeroelasticians. The author responded to this challenge and substantially extended his research to transonic speed aeroelasticity by using the computational fluid dynamics (CFD) code OpenFOAM [184, 197]. Active research is currently going on in this area in full vigour.

2.5 A Unified Approach to Flutter, Dynamic Stability and Response of Aircraft

While investigating the aeroelastic characteristics of a tailless aircraft during his doctoral studies, the author felt the need to develop a unified method that combines the stability and control of an aircraft with that of its flutter and response characteristics. In Papers [14, 27] he included the rigid-body freedoms in his investigation of the flutter and stability problems of a tailless aircraft and that of a tailed aircraft of similar category. The investigation revealed that for the tailed aircraft the rigid-body modes were not of much significance whereas they were the main contributors (along with elastic modes) to the flutter mode of the tailless aircraft. In essence the instability of the tailless aircraft was primarily due to the coupling between the short-period oscillations of the aircraft with that of its first wing bending mode. The interaction was serious and gave rise to body-freedom flutter causing an unacceptably low flutter speed of the aircraft. Based on this earlier work, the unsteady aerodynamic theory of Theodorsen was extended for non-harmonic (arbitrary) aerofoil motion in order to investigate the short period oscillation characteristics of an aircraft including flexible modes as well as unsteady aerodynamics. Paper [26] gives details of the theory and numerical results in which extensive computation of Bessel and Hankel functions using complex arguments became necessary. This integrated approach is believed to be entirely novel.

2.6 Aeroelastic Optimisation and Active Control

The author carried out the necessary ground work for optimisation studies by studying the flutter sensitivity of high aspect ratio aircraft wings [36] using the dynamic stiffness method and Theodorsen type unsteady aerodynamics. As a result of two collaborative EPSRC research grants during 1993-1998 between the author's institution and the University of Bath the computer program CALFUN [13, 30] was further extended under the author's leadership to solve aeroelastic optimisation problems of both metallic and composite aircraft structures. All publications arising from these EPSRC projects were the outcome of intensive collaborative exercises. Development of new theories and associated computer programs were carried out jointly although the main strategy and decisions were those of the author, particularly in respect of the analytical aspects of the work. Paper [40] describes the initial version of a new program CALFUNOPT (CALFUN with OPTimisation) which determines minimum mass designs of aircraft wings subject to a minimum flutter speed constraint, using the dynamic stiffness method. The paper presents typical optimisation results for metallic and composite wings, illustrating various design trends together with the accuracy of results and the computational efficiency. Paper [47] demonstrates new features added to CALFUNOPT which are particularly suitable for conceptual design purposes. The paper emphasises that, with both divergence and flutter speeds as constraints, the solution time for optimum designs is shorter in the author's method than when using the conventional finite element method.

Paper [53] presents results for optimum design of non-uniform wings constrained by either a minimum value of the fundamental natural frequency or a minimum separation between the first bending and first torsional natural frequencies. The investigation showed, that in terms of computational effort, the frequency separation approach is much more efficient and, in conjunction with the flutter analysis of the final design, may be used to establish an upper bound to the acceptable values of design mass for a given design flutter speed.

Paper [64] reports a new and rapid sensitivity analysis method and its use in aeroelastic optimisation studies. Paper [69] examines the free vibration and flutter characteristics of a large transport aircraft (capable of carrying more than 500 passengers) using the dynamic stiffness method and compares results obtained using the traditional finite element method. The paper also reports the computed flutter sensitivities of the aircraft. An extremely important contribution [76] focuses on ply angle optimisation of non-uniform composite beams (wings) subject to aeroelastic constraints. A novel further optimisation study [77] utilizes two independent codes, one based on the dynamic stiffness matrix of aircraft wing elements (CALFUNOPT) and the other on the dynamic stiffness matrix of plate elements. Both metallic and composite wings were optimised for three combinations of constraints, namely static strength, aeroelastic roll efficiency (represented by limiting the twist of the wing for an aileron loading) and aeroelastic divergence, respectively. As expected, the investigation showed that composite wing designs are significantly lighter than metallic ones, due to the well-known tailoring capability of composite materials. The upper-skin compression panels produced by

CALFUNOPT were further optimised using the more detailed and accurate panel sizing code VICONOPT [77] which takes (elastic) buckling of structures into account. Paper [83] describes the optimisation of non-uniform composite beams subject to aeroelastic constraints and shows that the optimum design may be sensitive to small changes in ply angles.

In an extension of the author's research interests, paper [49] reports an in-depth investigation on gust alleviation and flutter suppression of an optimised composite wing using active control. Although to date the number of papers published by the author in this area is small, the work is of considerable importance.

2.7 Application of Symbolic Computation in Structural Engineering Research

The author has made rigorous use of symbolic computation as an aid to solve many structural engineering problems. Papers [44], [50], [56], [73] and [149, 150] demonstrate that there is substantial computational advantage in using explicit expressions for the dynamic stiffness elements obtained from symbolic computation as opposed to the standard numerical approach of matrix manipulation. The use of symbolic computation has also enabled him to derive the frequency equations and mode shapes of complex vibrating structures including that of an idealised whole aircraft [82]. Papers [78], [80], [86], [87], [88], [139], [140], [149], [160] and [164] are some other examples of related, but different investigations which couple structural engineering research with symbolic computation. One of the most significant contributions of the author in this area is his analytical, but explicit, solution to the flutter problem for metallic [75] and composite [85] wings using symbolic computation. In view of the enormous analytical complexity, flutter problems are generally solved numerically, but the author has shown that symbolic computation offers a novel approach.

2.8 Development of Software for Computer Aided Structural Analysis and Design

The author has been developing software ever since he was a research student. A majority of his research papers are associated with a substantial amount of software development, particularly using FORTRAN. Details of these computer programs are far too extensive to report. However, a brief description of the capability of two of his major computer programs called **CALFUN** and **BUNVIS,** and their derivatives **CALFUNOPT** and **BUNVIS-RG** are given below.

CALFUN (**CAL**culation of **F**lutter speed **U**sing **N**ormal modes) is a FORTRAN program with about 9000 lines of code. It gives flutter speeds of metallic or composite aircraft from its stick model using dynamic stiffness or finite element methods in the structural idealisation and strip theory or lifting surface theory in the unsteady aerodynamic idealisation. The program has the option to compute the divergence speed and carry out a wing only analysis for both metallic and composite aircraft. The effects of offset mass and inertia (for example aircraft engines mounted on the wing) as well as tail-plane and fin aerodynamics can optionally be accounted for.

CALFUNOPT (**CALFUN** with **OPT**imisation) is an extended version of **CALFUN** with about 30000 lines of code and has the important added feature of optimisation. The program permits an efficient optimisation of metallic or composite aircraft structures at the conceptual design stage and allows the designer to consider the effects of constraints on flutter speed, divergence speed, material strength and wing twist displacements. The variables that are modified are front and rear spar thicknesses and upper and lower skin thicknesses. Engine position and spar position can also be selected as design variables. Optimisation is carried out using one of the following gradient based methods: (i) modified method of feasible directions, (ii) sequential linear programming and (iii) sequential quadratic programming. To allow for the rapid calculation of the sensitivity of flutter speed to a change in a design variable, a determinant interpolation method, which reuses data obtained during analysis, has been incorporated in the program.

BUNVIS (**BU**ckling or **N**atural **VI**bration of **S**pace frames) is a FORTRAN program with about 850 lines of code and was developed with the aim of using as few instructions as possible to achieve low memory requirement and fast execution. The intention at the time of its original development during the late 1970s was to use small computers to solve large structural problems. The program evaluates natural frequencies in vibration problems (or critical load factors in buckling problems) using the dynamic stiffness method and the Wittrick-Williams algorithm with the option of finding the corresponding mode shapes. It essentially deals with space structures idealised as a collection of beam-column and string members. A very important feature of the program is its ability to include stayed columns as substructures with very little calculation and data. Other features include allowance for independent mass, inertia or spring stiffness at any particular node.

BUNVIS-RG (**BUNVIS** with **R**epetitive **G**eometry) has been developed from the earlier program **BUNVIS.** It has about 11000 lines of code with many added features such as handling repetitive geometry, use of tapered structural members, preliminary static calculations, automatic node numbering to reduce bandwidth, a user friendly data preparation scheme, a plotting facility and provisions for flexible joints and offset connections at nodes. All such features are treated exactly for eigenvalue analysis, whereas conventional finite element programs usually make some approximations even when additional nodes are added. The program is especially suited for investigating the free vibration and buckling characteristics of pre-stressed structures used for space applications. Structures having linearly or rotationally repetitive geometry are treated in a very efficient manner.

3. CONTRIBUTIONS TO MATERIAL SCIENCE

The author's interests in material science originated from his work on steel structures and composites made of fibre-reinforced plastics. This was stimulated by collaboration with his colleague Emeritus Professor Barrie Mintz at City University London and through a project with Cracow University of Technology, Poland when one of author's PhD students working on composite materials was awarded a Marie Curie Fellowship during 2003-2004 to carry out further research. The author was also the Co-Investigator of a material science project funded by EPSRC during 2005-2008.

3.1 Polymer Nanocomposites

Recent research in polymers indicates that the introduction of layered silicate –montmorillonite-into the matrix increases the thermal stability of a number of polymer nanocomposites. Paper [123] presents a detailed examination of many factors influencing the thermal stability, including the role of chemical constitution of organic modifier, composition and structure of nanocomposites, and mechanisms of improvement of thermal stability in polymer/montmorillonite nanocomposites. The paper draws many useful conclusions for the changes in thermal properties and has continued to arouse considerable interests amongst the material science community. Paper [124] complements [123] by describing the basic changes in thermal behaviour of different polymeric matrices (including polyolefins, polyamides, poly (vinyl chloride) and styrene –containing polymers) when montmorillonite layers are added. The kinetics of the decomposition process in inert and oxidative environments and the analysis of volatile and condensed products of degradation are also discussed in this paper.

3.2 Hot Ductility of Steel and Related Research

The author was fortunate to be able to exploit the research base and experimental facilities in material science at City University London. He continued to work for a considerable period of time with Professor Barrie Mintz, Emeritus Professor of Material Science, who is internationally known for his research in micro and macro mechanical behaviour of steel and is an acknowledged authority in the subject. The author's interest in steel structures can be dated back to his presence in the Civil Engineering Department at Cardiff University for six years (1979-85). The main emphasis of research on material science at City University London is however, on the hot ductility of steel and associated areas of` fracture mechanics.

Paper [135] discusses the use of hot ductility curves in predicting the likelihood of transverse cracking that occurs in commercial straightening operation. The investigation was carried out against the background that the influence of C and Mn on the hot ductility curve is not always clear and the paper clarifies their influence by examining past work. It is concluded that the strain during straightening is usually so low that the curves have to be used with great caution and with an understanding of their limitations. It was also ascertained that increasing the start temperature of the austenite transformation (Ar_3) helps to avoid transverse cracking and thus decreasing C and/or Mn levels in steel should be generally beneficial.

Paper [141] is a significant contribution in that it develops a regression equation for the Ar_3 temperature and describes the major influence of C, Mn and Nb in decreasing the Ar_3 temperature. The formula given in this paper can be used by practising metallurgists and steel engineers.

Paper [142] was inspired by the fact that Twin Induced Plasticity (TWIP) steels are increasingly being used and further developed by the automotive industry because of their high tensile strength and large elongation rate. A thorough investigation of the hot ductility behavior was carried out with varying composition and many useful conclusions were drawn. The complementary paper [143] examines the hot ductility behaviour of high Al content TWIP steels comprising Nb/V. Both optical microscopy and transmission electron microscopy were used when measuring the ductility, and microstructures showing dynamic recrystallisation and austenite grain size resulting from the investigation were illustrated.

Previous research on the hot ductility of steel focusing on the influence of Al on both NB containing and Nb free Transformation Induced Plasticity (TRIP) steel showed that worse ductility occurred for an Al level of around 1% weight, resulting in a wide and deep trough. In order to underpin the reason for this finding, an in-depth scanning electron microscope and transmission electron microscope study was undertaken. Paper [144] highlights the results in a detailed and comprehensive manner.

It is well recognized that from an engineering material point of view, the influence of directionality on strength and impact behavior of steels is important. Paper [148] provides a detailed insight by providing experimental results examining the effect of directionality on the strength and impact behavior of control rolled high strength steels. Tensile specimens tested along the rolling direction showed considerably lower strength than the ones tested transversely of the rolling direction. This is in sharp contrast to the impact test results obtained from the charpy V notch samples which showed lower impact transition temperatures in the rolling direction than in the transverse direction. Paper [151] examines the hot ductility of a high Al, twin induced plasticity austenitic steel with varying S levels after heating the sample to 1250^0C and cooling it at 60^0 C per minute to test temperature in the range of 700-1100^0C. It was observed that the ductility was poor for the high S content steels, as a consequence of the change from coarse hexagonal plate AlN to very long dendritic rod precipitates, which are situated at the dendritic and austenite grain boundaries. The investigation on the influence of B and Ti on hot ductility of high Al and Nb containing TWIP steels is reported in paper [165]. For tensile specimens reheated to 1250^0C, the presence of B with sufficient Ti to combine with the entire N improved ductility over the temperature range of 700–950^0C, the reduction in area (RA) values was more than 40%. For the higher strength more complex high Al, TWIP steels having Nb present, there was no improvement in ductility with a similar B and Ti addition, when the average cooling rate after melting to the test temperature was 60 K per minute. Reducing the cooling rate to 12 K per minute resulted in the RA values being close to the minimum required to avoid transverse cracking throughout the temperature range 800–1000^0C. Transverse cracking was found not to be a problem when continuously casting these high Al containing TWIP steels with the addition of B and Ti. It was further established [174] that provided B was fully protected by adding sufficient Ti to combine with all the N, then B could segregate to the austenite grain boundaries to improve ductility and the improvement is pronounced within the temperature range 700–900^0C. It was noted that Ti additions in themselves were beneficial to the hot ductility of the steel specimens as precipitations of AlN at austenite boundaries were avoided, but only if the cooling rate was sufficiently slow to allow the TiN particles to coarsen. An addition of B was also required to ensure freedom from cracking. Paper [175] examined the influence of P and N on hot ductility of high Al, boron containing TWIP steels and concluded that even at 0.02% of P, the detrimental influence on hot ductility was quite small, but the ductility decreased as the P content was increased. The high N level resulted in good ductility because although there was a large volume fraction of TiN in the specimen, the high Ti/N ratio encouraged the growth of the TiN precipitates, causing precipitation to take place at high temperatures which was too coarse to influence the hot ductility.

4. CONCLUSIONS

The entire series of publications relates to explicit or implicit knowledge (both in a qualitative and a quantitative sense) in the interdisciplinary fields of structural dynamics, aeroelasticity and material science. Throughout his demanding research career, the author has managed to combine the disciplines of structural dynamics, aeroelasticity, composite materials, symbolic computation, structural optimization and material science. In the early stages of his career, he studied the free and forced vibration characteristics of aerospace structures using the finite element method and subsequently replaced it, wherever possible, by a superior method, called the dynamic stiffness method. One of the novelties of his work lies in the fact that he has successfully completed dynamic stiffness formulations for a wide range of new and non-standard metallic and composite structural elements, including beams, plates and shells. One of the most significant contributions that he made in this area is the development of the dynamic stiffness matrix of a completely free plate with no restriction on the boundary conditions. He achieved this by solving the bi-harmonic equation for a rectangular plate in an exact sense. He made use of symbolic computation wherever possible to generate explicit stiffness expressions. The author has always given preference to analytical, but elegant, solutions to engineering problems as opposed to repetitive and routine numerical studies. His derivation of the frequency equation and mode shapes of complex structural elements is, indeed, the outcome of his continuing interest and active participation in symbolic computation applications. The author's research in aeroelasticity has allowed the solution of the flutter and response problems of tailless aircraft. He combined two major disciplines of aircraft design, namely stability and control with flutter and response, in order to study the interaction between rigid body motion and the elastic modes of distortion of the aircraft. He extended his aeroelastic research to the demanding case of transonic speed aeroelasticity by taking advantage of computational fluid dynamics (CFD) codes which incorporates Navier-Stokes equations of unsteady aerodynamics. With the advent of advanced composite materials, his continued research on the aeroelasticity of composite aircraft has aroused international interest. On the question of flutter characteristics of laminated composite wings, exhibiting wash-in and wash-out, the author's findings have differed from those of eminent researchers in the field. In this respect he has uncovered some of the commonly held misconceptions on the aeroelasticity of composite wings and provided conclusive evidence to this effect. The development of the dynamic stiffness theory for composite wings, coupled with symbolic computation, resulted in its application to aeroelastic optimisation studies with commendable efficiency and speed. His collaborative efforts have resulted in combining the dynamic stiffness method with that of Carrera Unified Formulation to capture all essential three dimensional effects showing cross-sectional deformations when solving the free vibration problems of isotropic and anisotropic beams. This will enable researchers for the first time to account for the chord-wise deformation of wings in aeroelastic studies using stick models. During his research career the author has developed a suite of computer programs capable of dynamic and aeroelastic analysis of metallic and composite structures which are of practical help to the industry. His collaborative research in material science established the thermal properties of polymer nano-composites and provided an enhanced understanding of the hot ductility behaviour of steels with wide ranging constituents.

The author has found the work (described in his research papers) to be intellectually challenging and always very satisfying. It has been his privilege to have initiated and developed fundamental research in the field of structural dynamics, aeroelasticity and material science. The development of computer programs such as **CALFUN** and **BUNVIS** by the author bears the hall mark and represents the flagship of his research. It is gratifying that the work has never been far from potential engineering applications.

APPENDIX A

AN OVERVIEW OF WORK HISTORY AND RESEARCH EXPERIENCE

A research career spanning 46 years has been possible for the author because he has always worked in a research environment, either in a research establishment or in a University. He has, of course, spent almost all his career in the latter. Over these four decades, he has visited a large number of research establishments and universities, and their facilities, worldwide. This was boosted by the fact that he was privileged to have been awarded a Commonwealth Scholarship in 1975 to study for a PhD degree in the UK. He has visited almost all the leading institutions in the UK dealing with Aeronautical, Mechanical and Civil Engineering research. He was a British Council visitor to India in 1989 (visiting the Indian Institutes of Technology at Madras, Delhi, Kanpur and Kharagpur), to Canada in 1995 (visiting Universities in Victoria, British Columbia, Carleton and Toronto) and to Italy in 1993 (visiting Universities in Rome and Pisa). He has visited a number of leading institutions and their facilities all over the world. These include MIT, Harvard University, Stanford University, Virginia Polytechnic Institute and State University, Georgia Institute of Technology, University of Maryland at College Park, Purdue University, Wright State University and the University of Alabama at Huntsville in the USA, University of Sydney and the University of New South Wales in Australia, University of Kassel, Chalmers Technical University, Technical University of Delft, Technical University in Athens, Aristotle University, Technical University of Prague in Europe, Indian Institutes of Technology in Asia and the University of Cape Town and the British University of Egypt (Africa), amongst others. A substantial number of these visits were supported by the EPSRC and the Royal Academy of Engineering. The author was a distinguished visitor to the United States Air Force Base, Dayton, Ohio under the auspices of the US Air Force funded “Window on Science” scheme in 1992. He has attended and presented papers in over 50 national and international conferences to date and acted as a session chairman and/or a keynote speaker on more than 15 occasions. He has given over 20 invited seminars worldwide. He has been the Principal Investigator of 10 EPSRC projects over the years and has been a member of its Peer Review College ever since its inception in 1992. He is on the editorial board of Journal of Sound and Vibration, International Journals of Aeronautical and Space Sciences, Advances in Aircraft and Spacecraft Science, Shock and Vibration and Curved and Layered Structures. He has continued to act as a reviewer for around 20 engineering and applied science journals. He is also a reviewer for proposals from EPSRC, British Council and other national and international organisations. He has acted as a member of the Engineering Programme Grants Interview

Panel for individual grants in excess of £12m. He was external examiner for the undergraduate and postgraduate programmes for Cranfield University, University of Manchester and University of Hertfordshire and currently for Queen Mary University of London and the University of Bath. To date he has supervised 9 successful PhDs at City University London, examined 15 PhD theses in UK and abroad as external examiner and on numerous occasions, he was internal examiner for PhD candidates at City University London. He acted as a panel member for appointment and promotion at senior level (Reader and Professor) and also as an assessor for promotion from Senior Lecturer/Reader to Professor in British universities on countless occasions. The support extended by City University, London to promote and foster his research activities played a key role in the development of this thesis. The breakdown of the author's 46 years research experience is given below in chronological order.

A 1.1 Master of Technology Student, Indian Institute of Technology, Kharagpur, 1969-71

While a Master of Technology (MTech) student at the Indian Institute of Technology, Kharagpur the author developed a strong interest in free vibration and dynamic analysis of structures, which has remained to this day. The topic of his MTech thesis was both practical and academical, being concerned with the investigation of crankshaft failures of diesel-electric loco-engines of the Indian Railways. The work was carried out during the two-year period of the MTech study. The Nagpur and Kalka Divisions of the South-Eastern Railways in India which operated the loco-engines, suffered crankshaft failures on a massive scale. The exact reason for these failures was not known and so the need for the investigation was established. The probable cause of the crankshaft failures was suspected to be excessive vibration arising from the engine and its transmission system. The manufacturer of the engines, Mercedes-Maybach Benz of Germany was made aware of the problem, but insisted that the crankshafts were supplied in perfect quality in respect of both material and its heat treatment. The problem was compounded by the fact that the company would not supply any design details of the engines because of their commercial policy.

The research team consisted of one professor (the project supervisor), two MTech students including the author, and two supporting staff from the Indian Railways. In the preliminary stage, the pressure-volume diagram of the engine was obtained by drilling a hole in one of the cylinders of the engine block and recording the pressure distributions from outside using a vibrograph. From these data a detailed force analysis was then carried out on the crankshaft, connecting rod and crankpins. The natural frequencies and mode shapes of the crankshaft in flexural and torsional vibration were established both theoretically and experimentally. The position of the vibration damper used in the transmission system was examined critically and one of the conclusions drawn was that the damper was incorrectly placed because it was not reducing sufficiently the vibration generated by the engine. Ideally its location should have been nearer to an anti-node of the crankshaft. However, there were physical constraints which prevented the damper being moved.

The author played a full part in the whole investigation. Three research papers [1-3] were published from this work. These were all presented at the Sixteenth Congress of the Indian Society of Theoretical and Applied Mechanics in 1972.

A 1.2 Structural Engineer and Senior Structural Engineer, Indian Space Research Organisation, Trivandrum, 1971-75

After completing the MTech course, the author joined the Structural Engineering Division of the Indian Space Research Organisation (ISRO), Trivandrum in 1971 where he studied the free and forced vibration characteristics of rocket structures. Particular emphasis was placed on the development of theoretical techniques and associated computer programs when predicting the dynamic behaviour of solid propellant rocket structures. Despite commercial restrictions the author and his research colleagues managed to publish part of the theoretical developments on certain unclassified aspects of the investigations, for example, stress waves in rockets [4], response of rocket [5] and sandwich [6] structures to random loads. The author was also involved in resonance testing work on structural systems, but this was mainly used for validation of the finite element method. In recognition of his contributions to the work on various dynamical aspects of rocket structures, he was promoted to the position of Senior Structural Engineer in 1975. Later that year he won a Commonwealth Scholarship to study for a PhD degree at Cranfield, UK. This was, and still is, a highly competitive award and only a very few candidates are chosen (on an all India basis) by the Association of the Commonwealth Universities. The author believes that his academic background coupled with his success at ISRO contributed to this award. Some of his major unpublished works during this period with ISRO include design, development and fabrication of an acoustical horn (which was used for experimental determination of response and acoustic fatigue characteristics of rocket structures) and the finite element analysis of a four stage solid propellant rocket launcher. The author left ISRO in late 1975 and the four stage solid propellant rocket called Satellite Launch Vehicle-3 (SLV-3) was successfully launched in 1980.

A 1.3 Commonwealth Research Scholar, Cranfield University, 1975-78

At the end of 1975, the author joined the then College of Aeronautics, Cranfield Institute of Technology (now Cranfield University) as a Commonwealth Scholar to work for his doctoral thesis in Aeroelasticity and Structural Dynamics. The research was focused on investigating the aeroelastic behaviour and response to turbulence of a class of tailless aircraft, now relevant to current interest in the design of blended wing-body aircraft. It was clear that (classical) conventional methods were not adequate to deal with the problem because the interaction between the rigid-body motion and the elastic modes of distortion is a much more serious consideration for a tailless aircraft than for its tailed counterpart.

The author developed a unified method by combining two important disciplines of aircraft design, namely, the stability and control on the one hand and the flutter and response on the other, as necessitated by the investigation. The method involved extending the Theodorsen function used for computation of the unsteady aerodynamics of aircraft wings, to the case of non-harmonic arbitrary aerofoil motion. While working on this problem, he provided an accurate and reliable theory and associated computer programs to deal with the static and dynamic stability (divergence and flutter) of deformable aircraft. The investigation covered response analysis of aircraft to discrete gusts (in the time domain) as well as to continuous atmospheric turbulence (in the frequency domain). It was shown that the two methods give similar results when interpreted properly, and thus it was possible to predict an overlap between the two methods. The software developed by the author during his doctoral research later took the name CALFUN (CALculation of Flutter sped Using Normal modes) in the mid 1980s. The particular development of this software using accurate analytical theories is by far the most significant practical application of his doctoral studies.

During the period of his doctoral studies, the Cranfield Aerobatic Aircraft, called Cranfield A1, was designed, built and successfully flown. The author was responsible for its aeroelastic calculations. He was indeed very fortunate to take an active part in the complete design process. The main investigation of the author was, however, focused on the aeroelastic behaviour of a tailless sailplane, called the Ricochet. Incidentally, a tailed aircraft, the Kestrel-22m sailplane (Kestrel with 22 metre span) had a flutter problem at about the same time, and this aircraft was made available to the author by the manufacturer (Slingsby) for a detailed analysis and flight tests. The author took full part in this project and his investigation contributed towards the successful solution of the flutter problem of Kestrel-22m. He published three papers [13, 14, 27] from the work carried out in his doctoral studies which was completed at the end of 1978.

A 1.4 Research Associate and Senior Research Associate, Cardiff University, 1979-85

The author joined the Department of Civil Engineering and Building Technology, University of Wales Institute of Science and Technology (now Cardiff University), in February 1979 as a Research Associate to work in the field of vibration and buckling analysis of space structures under a Science and Engineering Research Council (SERC, now EPSRC) project. An important aspect of the work was to investigate the effects of using stayed columns on the free vibration and (elastic) buckling characteristics of space structures. Use of stayed columns can increase the load carrying capacity of a structure significantly, but it was very important to ensure that the free vibration and buckling characteristics were not affected adversely. The research was promising and its application was particularly relevant where space was not a limitation (for example, offshore structures, certain structures in deserts, and importantly, structures in space).

An important focus area of the research was to use accurate (exact) structural member theory so that model accuracy was not unduly compromised particularly when studying the high frequency vibration characteristics. It was an essential requirement of the project to develop a computer program based on exact theory for an accurate prediction of the free vibration and buckling behaviour of large space structures. The development of the FORTRAN computer program BUNVIS (BUckling or Natural VIbration of Space) by the author began in earnest at this stage. While writing, developing, debugging and successfully running various parts of the program, he was indeed, conscious of the need for the development of more refined theories [12] which would enhance the capability of the program. Preliminary results from BUNVIS showed the inadequacy of using simple methods which can miss some genuine natural frequencies of a structure [7]. The capability of BUNVIS and its user's guide were published [8, 9].

The originality and usefulness of BUNVIS stem from the fact that in contrast to the conventional finite element theory, it uses exact member theory for both the vibration and buckling analysis of structures. During the time when BUNVIS was developed, NASA researchers were considering the application of self-expanding stayed columns in space. The type of stayed column considered was initially developed by the ASTRO Research Corporation for NASA to evaluate, and was designed to be deployed in space after forming a light, compact package in the Space Shuttle. The buckling behaviour of such columns was of great significance and constituted a major part of both theoretical and experimental investigations by NASA. The author made noteworthy contributions towards refining and extending the original design of ASTRO by providing a significantly lighter and more compact package for the Space Shuttle and a better understanding of how such columns would behave [10]. NASA carried out a detailed free vibration analysis and performed tests on such stayed columns for use in space. The author's analytical work on this problem was more advanced than that of NASA because recent developments were implemented in the analysis technique [11]. Using the computer program BUNVIS, he investigated the free vibration characteristics of a space platform assembled from such stayed columns [16]. This particular structure (space platform) had plan dimensions of more than 30 metres and yet its first 4978 exact natural frequencies were located using only 2 hours of VAX-11/780 CPU time [16] which was remarkably impressive at the time. During the period of this research the author's investigation on modal densities for panels and cylinders, including corrugations and stiffeners [15], led to many useful conclusions.

As a result of supplying a substantial set of vibration results using BUNVIS (on spacecraft structures which NASA researchers were testing in their laboratory at the time) a relationship of mutual cooperation with their scientists was established. Later, NASA acquired BUNVIS under mutually beneficial terms. This fruitful collaboration resulted in a new research project in 1982 on the efficient design of large structures for use in space, jointly funded by SERC (now EPSRC) and NASA. The author was promoted to the position of Senior Research Associate in recognition of his contributions to the research he carried out in the earlier project.

During the period 1982-85 as a Senior Research Associate, the author was engaged in the analysis, design and development of large structures in space. In order to accomplish this task effectively, further developments of BUNVIS became necessary. The work was thus carried out in close collaboration with the Structural Concepts Branch of NASA at Langley Research Center, Virginia. The author was concerned with the dynamical behaviour of large space structures to be deployed, erected, assembled or fabricated in space. The original program BUNVIS was further developed into BUNVIS-RG (BUNVIS with Repetitive Geometry) which could deal with lattice structures with repetitive geometry and included a substructuring option [21]. Several additional features, including the provision for tapered members [17, 18, 20, 22] were added to the program, which was later used to analyse the robot arm of the Space Shuttle.

The author's role in the development of BUNVIS-RG [21] was central. He was always conscious of the need for developing basic and fundamental theories on free vibration and response analysis of structures [19, 20, 22]. The BUNVIS project ended in August 1985 and was judged hugely successful. The computer program BUNVIS-RG is used by NASA and other organisations. It has been released by the agency COSMIC in USA at a nominal cost in order to promote and stimulate research in dynamic analysis of aerospace structures.

A 1.5 Lecturer, Senior Lecturer, Reader and Professor, City University London, 1985 to date

The author joined City University, London as a Lecturer in Aircraft Structures in 1985 where he started developing his teaching and research interests in aircraft structures, fibre-reinforced composites, structural dynamics, aeroelasticity and material science. Virtually the entire programme of work in almost all of these areas was initiated, prepared and conducted under his leadership. He introduced finite element method and composite structures in the undergraduate curriculum for the first time in 1985 at City University London by substantially revising the Aircraft Structures syllabus. Soon after his arrival, he prepared new teaching material and introduced computer aided structural analysis and design in the curriculum and modernised the course.

The most significant change in the author's research activities that took place after leaving Cardiff University (where the author was in a Civil Engineering environment) was the revival and invigoration of his research interest in aeroelasticity. This was exploited to advantage by carrying out interrelated work on the subjects of aircraft structures and structural dynamics. Within two years of joining City University London, he published four research papers on four different topics. These were on an exact stiffness matrix for tapered beam columns [22], the torsional rigidity of racing car frames [23], the effects of axial force on the flutter of high aspect ratio aerofoil blades [24] and the development of a computer program for buckling analysis of frames consisting of tapered members [25]. Subsequently the author extended his earlier work on dynamic stability of tailless aircraft [26]. He established an aeroelastic capability in the Department completely on his own initiative and further enhanced his aeroelastic program CALFUN [30]. This led to the study of the effects of tail-plane aerodynamics and fuselage flexibility on the flutter of high aspect ratio aircraft structures [28]. In order to predict the free vibration characteristics of high aspect ratio aircraft wings such as that of a transport airliner, he developed the dynamic stiffness matrix for a beam element [29] coupled in bending and torsion which was later integrated in aeroelastic studies as an essential part of CALFUN [30]. When using simple theories, the author was always aware of the severity of all assumptions made and thus ensured that the computed results were sufficiently accurate and reliable. For instance, strip theory has often been used when modelling unsteady aerodynamics in aeroelastic studies, particularly for high aspect ratio aircraft, but importantly, its suitability for application in flutter analysis was further examined by the author [31], by demonstrating the circumstances when it can (or cannot) be used safely.

As mentioned earlier the author was involved in the design and development of the Cranfield Aerobatic Aircraft (Cranfield A1) during his doctoral studies which enabled him to acquire the structural and other data of the aircraft in great detail. Thus it was possible for him to study further the aeroelastic behaviour of Cranfield A1 [32] using refined theories [29, 33]. While studying the coupled bending-torsional free vibration characteristics of aircraft wings, he developed new theories based on the earlier ones by accounting for the effects of axial load [34] and shear deformation and rotatory inertia [35]. The author's later work combined all these effects into a unified theory [44]. Much computational efficiency was achieved as a result of using symbolic computation [34, 35, 44] in this research.

In the early 1990s the author began to take an interest in structural optimisation studies, particularly in the area of aeroelastic optimisation. The importance of sensitivity analysis [36] in these studies was the first investigation undertaken by him.

The author has always been keen to study coupled structural systems because of their importance in aeroelastic investigations. Although bending-torsion coupled systems as applicable to aircraft wings were his main concern, he was aware of the importance of other related problems and studied the free vibration characteristics of coupled extension-torsional structural systems [37, 38, 42] that are particularly relevant to helicopter, compressor and wind turbine blades as opposed to fixed wing aircraft.

The author has continued to use an established algorithm (known as the Wittrick-Williams algorithm in the literature) on numerous occasions [20, 22, 29, 33, 34, 35], particularly in connection with the applications of his dynamic stiffness work. Identification or isolation of clamped-clamped natural frequencies of a structural member is an essential requirement for an appropriate and proper application of this algorithm. For a bending-torsion coupled beam (such as an aircraft wing) this problem was extremely difficult to overcome. The author developed a theory for this problem in an elegant and computationally efficient manner [43]. Later he extended his modal analysis work to response problems by using normal mode methods employing generalised coordinates. In this way, the dynamic response of bending-torsion coupled beams (as applicable to aircraft wings) to deterministic and random loads was thoroughly investigated [45, 54, 57]. Alongside these investigations, he continued his research on buckling problems, providing insights into the effects of shear deformation on the critical buckling behaviour of columns [41].

The author's research interest in the dynamics of composite structures started in the early 1990s and was first concentrated on free vibration [38, 50, 56, 73] and then on aeroelasticity [46, 51, 58, 59, 62, 63, 66, 72, 84, 85] and subsequently on response analysis [52, 61, 65, 68, 70]. In the meantime he continued his work on the dynamic stiffness development of structural elements [48, 55, 60, 89, 90] by making use of symbolic computation [50, 56, 73]. Aeroelastic optimisation was studied through the use of an integrated approach applying the dynamic stiffness methods and symbolic computation. A number of publications [40, 49, 53, 64, 76, 83] resulted from this activity. His recent research in aeroelasticity is focused in the transonic region using computational fluid dynamics [184, 197].

The dynamic stiffness development of structural elements has always been one of the most intensive focus areas of the author's research. In different stages of this research, he focused on the dynamic stiffness developments of beam elements made of both isotropic and anisotropic materials which included tapered beams [20, 22, 25], bending-torsion coupled beams [29, 34, 35, 44, 55, 118], composite beams [50, 56, 73], rotating beams [79, 89, 120, 161, 177], twisted beams [90, 108], sandwich beams [100, 115, 126], spinning beams [106, 119], moving beams [125, 131], cracked beams [134] and functionally graded beams [158, 159, 162, 186], amongst others. In all these developments, the complexity of the problems demanded that the author formulated simpler theories first and then replaced them by more and more refined and advanced theories. Alongside these investigations, he gave a unified formulation combining the dynamic stiffness properties of continuous beam elements with those of discrete elements/systems [98, 99, 113, 152]. He established collaboration and made it possible to apply Carrera Unified Formulation in dynamic stiffness developments of isotropic [166] and anisotropic [173, 176] structural elements.

The author's work on beam dynamics is widely acknowledged as innovative and broad ranging and has fuelled many other areas of research. The author has always been mindful that complex structures such as aircraft wings cannot be always modelled accurately by beam elements. Accordingly he diversified his research interests and turned his attention to dynamic stiffness formulations for isotropic [139,140] and anisotropic [149, 150] plate elements. Later work on the dynamics of plates was advanced by including layer-wise [157, 172] and higher order shear deformation [160, 163] theories. Subsequent research on plate theories led to pioneering developments when the dynamic stiffness elements for isotropic [188] and anisotropic plates [187, 189, 190] were formulated for the most general case with no restriction on boundary conditions. The author also carried out preliminary research on the dynamic stiffness formulation and free vibration analysis of composite shells [164, 167, 183, 185].

Because of the close relationship and/or dependency between structural engineering and material science, it is essential for structural engineers to acquire adequate knowledge of material science, particularly when dealing with anisotropic composites and/or functionally graded materials. Understandably, the knowledge required of a structural engineer in material science is generally basic and rudimentary, and not necessarily of research standard. It is difficult and time consuming for a structural engineer to grasp the detailed and often intricate concepts of material science and then to carry out in-depth research. This is almost an impossible task because the subject area is so vast. The author took exception to this usually adopted notion in that he attempted to break the barrier between material science and structural engineering. Thus he began a programme of collaborative research in material science. As a result of this initiative, a number of original papers in polymer science [123, 124] and hot ductility of steel [135, 137, 141, 142, 143, 144, 148, 151, 165, 174, 175] were published. From a historical perspective his research in material science stemmed from his interests in composites and steel structures.

It has been a huge privilege and indeed, tremendously gratifying to the author that the majority of his research has been supported by the EPSRC, Airbus UK, Royal Academy of Engineering, American Air Force Base and Embraer Aircraft Company, amongst others. Understandably the overwhelming bulk of the research reported in this thesis originated at City University London where the author devoted almost all his entire academic career, spanning more than 30 years.

APPENDIX B

LIST OF PUBLICATIONS

1. Rao J.S., Bhattacharya B. and Banerjee J.R. Natural frequencies and mode shapes in flexural vibration of the ZDM2 diesel loco engine crank-shaft. Paper presented at the *Sixteenth Congress of the Indian Society of Theoretical and Applied Mechanics*, Allahabad, India, 29th March-1st April, 1972.
2. Rao J.S., Bhattacharya B. and Banerjee J.R. Natural frequencies and mode shapes in torsional vibration of the ZDM2 diesel loco engine drive. Paper presented at the *Sixteenth Congress of the Indian Society of Theoretical and Applied Mechanics,* Allahabad, India, 29th March-1st April, 1972.
3. Rao J.S., Bhattacharya B. and Banerjee J.R. Determination of crank-pin forces and torques in the ZDM2 diesel loco engine. Paper presented at the *Sixteenth Congress of the Indian Society of Theoretical and Applied Mechanics*, Allahabad, India, 29th March-1st April, 1972.
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15. Williams, F.W. and Banerjee, J.R. Accurately computed modal densities for plates and cylinders including corrugations and stiffeners. ***Journal of Sound and Vibration***, Vol. 93, No. 4, pp. 481-488, 1984.
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APPENDIX C

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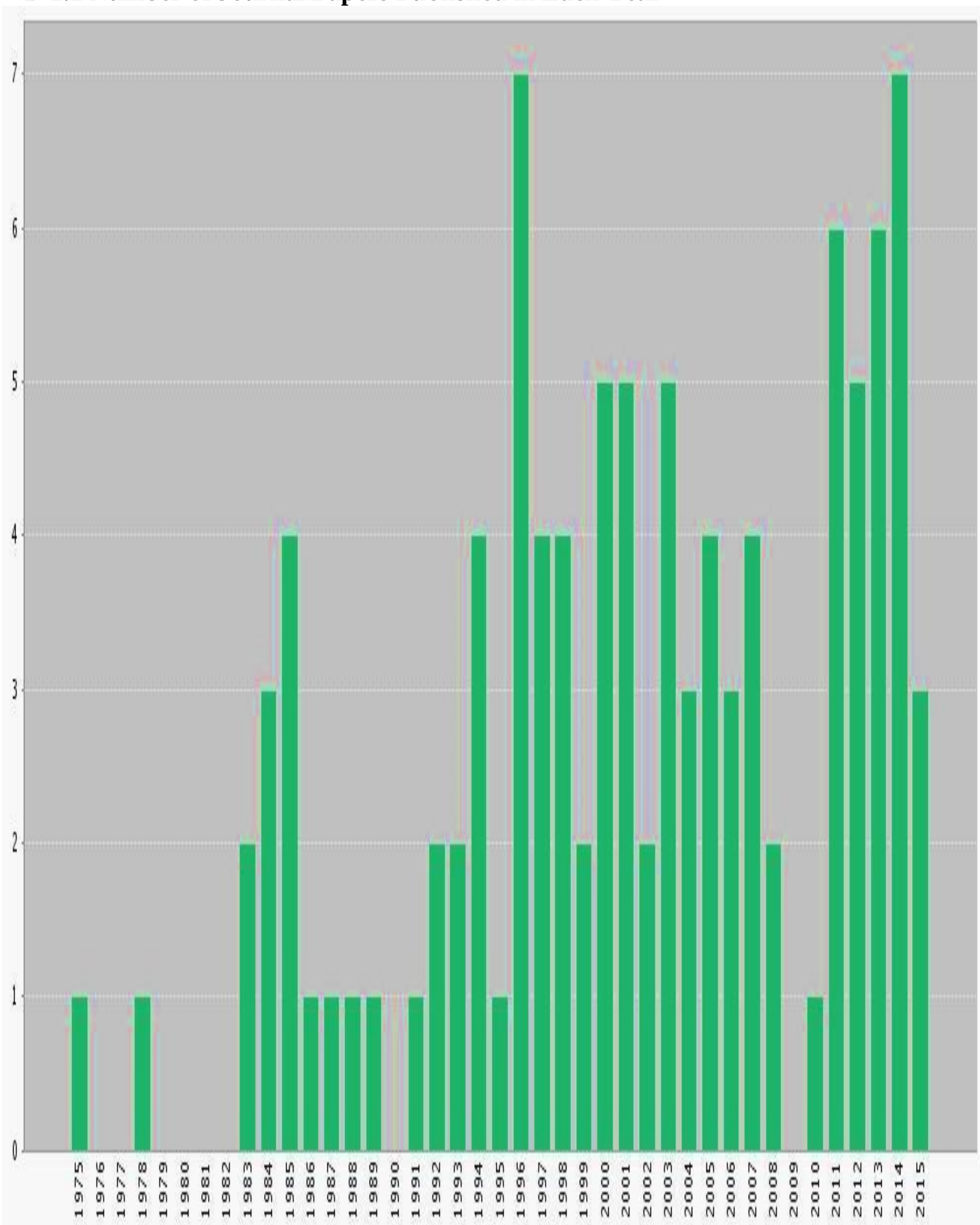
APPENDIX D

PUBLICATION RECORD

D 1.1 Number of Journal Papers Published in Each Year



D 1.2 Citations of Papers in Each year