Pricing of Reinsurance Contracts in the Presence of Catastrophe Bonds

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Abstract

A methodology for pricing of reinsurance contracts in the presence of a catastrophe bond is developed. An important advantage of this approach is that it allows for the pricing of reinsurance contracts consistent with the observed market prices catastrophe bonds on the same underlying risk process.

Within the proposed methodology, an appropriate financial pricing formula is derived under a market implied risk neutral probability measure for both a catastrophe bond and an aggregate excess of loss reinsurance contract using a generalised Fourier transform. Efficient numerical methods for the evaluation of this formula such as the Fast Fourier transform and Fractional Fast Fourier transform are considered.

The methodology is illustrated on several examples including Pareto and Gamma claim severities.

Key words: reinsurance, catastrophe bonds, risk neutral valuation, securitisation, Fast Fourier Transform, Fractional Fast Fourier Transform

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1 Introduction

The nature of the reinsurance industry is rapidly changing. Over the last decade large institutions and the financial markets have developed a range of new financial products that provide direct exposure to the risks that previously had been the sole interest of the insurance industry. These include catastrophe bonds, industry loss warranties, industry loss futures and a range of other insurance linked securities derived from special purpose vehicles such as sidecars.

The convergence of the insurance and capital markets appears to be accelerating. Considering the market for catastrophe (cat) bonds alone, the total outstanding issuance at end of 2007 was \$13.8 billion up 63 percent over the end of 2006. In fact \$7 billion of publicly disclosed cat bonds were issued in 2007, compared to \$4.7 billion in 2006 and \$2 billion in 2005 (see McGhee et al (2007)).

There are several reasons for the increasing popularity of securitised insurance instruments. Recently, the accelerated issuance of cat bonds was in response to limited catastrophe capacity in the reinsurance industry, following Hurricane Katrina, Rita and Wilma. During the early issues of cat bonds, prices were regarded as being high, compared to traditional reinsurance and the bonds acted to fill gaps in the market, where capacity was limited.

Over time, as the financial markets became comfortable with the concept of insurance linked securities, demand for them has increased. Many large investment entities such as hedge funds are constantly looking for investment vehicles that provide diversification from the rest of their portfolio. As most early cat bonds were written to cover high levels of losses, the majority did not trigger and provided rates of return well above LIBOR. This increased demand further and the insurance industry reacted by increasing issuance of cat bonds and other related securities.

More recently, as described by the Insurance Journal (2002), investment banks have moved into the reinsurance market and are now both capitalising and setting up reinsurance companies. The aim of this initiative is two-fold. Firstly it provides them with a direct exposure to the historically high levels of profitability in the reinsurance sector. Secondly, it allows them to exploit price differentials between the direct reinsurance market and insurance linked securities in the capital markets. This can be achieved by issuing insurance linked securities on the insured risks of the reinsurer. Typically, the design of the securitisation follows that of asset backed securities by forming a multi-tranche format of varying risk. This approach helps to make the securitised insurance instrument attractive to the widest range of investors and hence achieve greater prices for a given level of issuance.

As the trend of moving towards the capital markets for protection against catastrophe risk increases, an important question arises. Does the current market price of insurance linked securities imply something about how much reinsurers should be charging to insure similar risks? If investment banks are able to make profits by exploiting price differentials between reinsurance premiums and the price investors are prepared to pay for insurance linked securities, then the answer is a definite yes.

From the perspective of a direct insurer, it is important to be able to assess whether purchasing insurance linked securities or reinsurance cover provides better value for money. Similarly, a reinsurer should be able to judge whether the prices it is offering are higher or lower than that implied by the capital markets.

The aim of this paper will be to set up a framework in which the observed market prices of insurance linked securities can be used to assess a market consistent price for traditional reinsurance.

Rather than looking at the entire universe of insurance linked securities and reinsurance contracts, we will consider only the cat bond and the aggregate excess of loss contract. However, the approach taken could be extended to consider a wider range of securitisation vehicles and reinsurance policies.

The paper is organised as follows:

In Section 2 we describe a general approach for pricing reinsurance in the presence of a market for cat bonds. This provides the foundations upon which we build a consistent pricing methodology that is applied to both cat bonds and reinsurance contracts.

Sections 3 and 4 implement this pricing framework building upon the risk-neutral pricing approach originally developed for insurance by Delbaen & Haezendonck (1989). We apply the financial pricing approach first described for reinsurance by Sondermann (1991) and later for cat bonds by Baryshnikov et al (2001).

Following this approach leads to a pricing formula based around calculating the expected discounted value of the contract payoff under a risk-neutral probability measure.

Most theory relating to evaluating the risk-neutral pricing formula is derived from incomplete market derivative pricing theory in finance. The financial world has developed many techniques for dealing with pricing derivative contracts where the underlying stock price follows a Levy process. For example of such research we refer the interested reader to Cont & Tankov (2004). Since the compound Poisson model fits into this characterisation we can directly apply these methods.

The idea of applying risk-neutral valuation is not new to the actuarial profession. For example, Holtan (2004) describes applying option pricing techniques to insurance contracts. Further Muermann (2002) applies a Fourier transform based approach for pricing catastrophe derivatives and reinsurance based on a classic option pricing technique described by Carr & Madan (1999). We will follow the approach of risk-neutral valuation but will utilise a more recent pricing technique described by Lewis (2001): the generalised Fourier transformation method. This will provide us with an elegant mechanism for evaluating the theoretical prices of cat bonds and reinsurance contracts. It will lead us to an integral expression for the contract price in terms of the generalised Fourier transform of the payoff function and the aggregate loss characteristic function.

General inversion formulae which can be used for insurance and option pricing based on the Parseval's theorem have been recently obtained by Dufresne et al (2006). Additionally, a Fourier Space Time-stepping methodology is applied to the problem of pricing catastrophe equity put options is presented by Jaimungal (2007).

In Sections 5-7 we consider different numerical methods of evaluating the general pricing formulae for cat bonds and reinsurance contracts obtained in Sections 3 and 4. We begin by implementing the Fast Fourier Transform to provide an efficient numerical computation of contract prices, as suggested by Muermann (2003) as an extension of his work.

We then follow the approach of Chourdakis (2005) from derivative pricing theory to demonstrate how the Fractional Fast Fourier Transform can be used as an efficient numerical algorithm to evaluate cat bonds and reinsurance prices.

Section 8 considers two practical examples of applying the theoretical and numerical pricing techniques. The first example demonstrates that the pricing method works successfully in the case of a Gamma distribution for claim severity.

For the second example, we consider pricing contracts under the Pareto type II severity distribution. It is demonstrated that the pricing method is successful for cat bonds but fails for an aggregate excess of loss reinsurance contract due to the behaviour of the Pareto's characteristic function.

In order to resolve this issue we derive a put-call parity relationship for aggregate reinsurance contracts. This leads us to recover the price of the reinsurance contract in terms of the price of an put option on the aggregate claims process. This approach is a successful demonstration of the practicality of the generalised Fourier transform pricing method.

In Section 9 we compare the accuracy of the pricing method to that of Monte-Carlo simulation and conclude that for calibration purposes, the analytical formulae are preferable, since at least 2 million simulations are required to achieve accuracy within 0.1% of the theoretical prices.

Finally in Section 10 we consider a simple extension of the pricing formulae to allow for stochastic interest rates.

2 General approach

In this paper we utilise risk neutral valuation for pricing both cat bonds and reinsurance contracts. This differs to the standard actuarial approach of applying real-world premium principles to contracts involving insurance risk. We refer the read to Holtan (2004) for a comparison of the two methodologies and to Baxter & Rennie (1996) for a detailed introduction to risk neutral financial pricing.

Following Embrechts (1996), we describe the insurance market as a filtered probability space $(\Omega, F, (F_t)_{t\geq 0}, \mathbf{P})$, where (F_t) is an increasing family of σ-algebras, that represent all the information present in the history of the insurance risk process. We denote by $(S_t)_{0\leq t\leq T}$ the accumulated losses at time

t from an underlying insurance risk process. Throughout this paper we will assume that S_t can be adequately modelled as a compound Poisson process.

Using a (risk-neutral) equivalent martingale probability measure $\mathbf{Q}$ to the real-world probability measure $\mathbf{P}$, the arbitrage free price of a contingent claim with payoff $\phi(S_T)$ at time T is given by the fundamental theorem of asset pricing as

$$V_t = E_{\mathbf{Q}} \left\{ e^{-r(T-t)} \phi(S_T) | F_t \right\}, \tag{1}$$

where r is the continuously compounded risk-free interest rate. For more information about the fundamental theorem of asset pricing, we refer the reader to Delbaen & Schachermayer (1994).

Insurance risk modelling is usually carried out using incomplete market models, which means there is no unique equivalent martingale measure. Instead, the insurance market uses a wide range of risk-neutral measures that correspond to the many different actuarial premium principles. Delbaen & Haezendonck (1989) characterise the set of equivalent martingale measures $\mathbf{Q}$, under which the structure of the insurance risk process remains a compound Poisson process under the real-world measure $\mathbf{P}$. They also identify different risk-neutral probability measures that correspond to some of the actuarial premium principles.

Recently, Muermann (2002) characterises the market price of risk implied by different premium principles and describes a pricing technique for contracts with a European payoff (that is not path dependent) under the associated risk-neutral probability measure. In Muermann (2003) the author identifies the implied risk-neutral measure associated with different investor preferences when pricing catastrophe derivatives. In his most recent paper Muermann (2006) investigates whether the market price of catastrophe risk can be calculated by comparing the market price of reinsurance and catastrophe derivatives. He provides a method of calculating market price of catastrophe risk under the restriction that a single catastrophe event will be sufficient to bring an out-of-the-money catastrophe derivative into the money.

In this paper, rather than choosing an equivalent martingale measure that corresponds to a particular premium principle, we will follow Muermann (2006) and adopt a 'market implied' approach. That is, we will price reinsurance contracts using the (risk-neutral) equivalent martingale probability measure, implied by the observed market prices of cat bonds. This is analogous to how exotic stock options are usually priced using the risk-neutral measure implied by the observed market prices of vanilla European stock options, as described by Schoutens (2003).

We note that in the current secondary market for cat bonds, there is not sufficient liquidity to readily obtain market prices for variety of cat bonds on a specific region and peril. However, we believe that given the continuing growth in the insurance linked security market, it is only a matter of time before such a market develops. We will therefore proceed on the basis that the liquidity of the securitisation market will improve with time and we assume that prices are readily available for cat bonds at a range of expiration dates and trigger levels on the insured risk.

We will assume that under the market implied risk-neutral probability measure, the underlying loss process follows a compound Poisson distribution. This assumption is supported by Delbaen & Haezendonck (1989), who show that if the loss process is compound Poisson under the real-world probability measure $\mathbf{P}$, then it will remain compound Poisson under any equivalent risk-neutral measure $\mathbf{Q}$, provided that insurance premiums on the underlying risk are linear with respect to time. That is, the insurance premium for the outstanding period of cover, is a multiple of a premium density and the outstanding time period. Delbaen & Haezendonck (1989) conclude that provided sufficiently many reinsurance markets exist, premium linearity is satisisfied. We note that premium linearity will not necessarily hold when the underlying insurance risk process exhibits seasonality effects.

We will not tackle any of the issues surrounding calibration techniques in this paper. Instead we will focus on the actual practicalities of applying risk-neutral valuation to insurance processes. This is a necessary prerequisite to the calibration process. As such we return to the calibration problem in a future paper.

3 Pricing cat bonds

We will begin by providing some background information on how cat bonds operate. Cat bonds are a relatively new type of bond that provides a series of coupon payments and return of capital to an investor, contingent on a trigger event not occurring. The trigger event is defined to be where a measurable quantity related to an underlying insured risk exceeds a predetermined level. There are many different varieties of cat bond, where the trigger event could be based on modelled losses, industry losses or the severity of a natural disaster exceeding a specified limit.

In this paper we will only be considering cat bonds with an indemnity based trigger. The indemnity event is triggered when the actual losses of the bond issuer exceed a threshold trigger level (so the issuer is effectively fully indemnified against losses in a layer starting at the trigger level and extending to the trigger level plus the net present value of outstanding coupons and the redemption payment).

It will be assumed that under a risk-neutral probability measure $\mathbf{Q}$, the aggregate loss process S_t follows a compound Poisson process and

$$S_t = \sum_{j=1}^{N_t} X_j \text{ , with the convention that } S_t = 0 \text{ if } N_t = 0, \tag{2}$$

where N_t is the number of claims that have occurred by time t and X_j is a random variable representing the severity of the j-th claim.

We assume that N_t is a Poisson process with arrival rate λ and the X_j ($j = 1, ..., N_t$) are independent identically distributed absolutely continuous random variables with probability density function $f(x)$.

It is assumed also that the cat bond is of the indemnity type with trigger level D, $D > 0$. The bond is assumed to mature at time T and coupons are paid at rate C_{t_j} at times $t < t_1 < t_2 < ... < t_n = T$ ($n \geq 1$). We apply the financial pricing formula (1) to assert that the price at time t per \$1 nominal can be written as

$$\begin{aligned} V_t^{cat} &= V_t^{cat\_coup} + V_t^{cat\_cap} \\ &= \sum_{j=1}^{n} C_{t_j} E_{\mathbf{Q}} \left\{ e^{-r(t_j - t)} 1_{\{S_{t_j} < D\}} | F_t \right\} + E_{\mathbf{Q}} \left\{ e^{-r(T-t)} 1_{\{S_T < D\}} | F_t \right\}, \end{aligned} \tag{3}$$

where $1_{\{S_\tau < D\}}$ is the indicator function and $V_t^{cat\_coup}$, $V_t^{cat\_cap}$ denote the coupon and the capital parts of the bond price respectively.

For further details related to the derivation of this approach, we refer to Baryshnikov et al (2001).

In order to calculate the expectation in (3), we will use the generalised Fourier transform method. This was introduced to financial mathematics by Lewis (2001) who demonstrated its use for option pricing under a Levy process.

We define the generalised Fourier transform $\mathcal{F}$ of a function $w : \mathbf{R} \rightarrow \mathbf{R}$ as

$$\mathcal{F}\{w(x)\} = \hat{w}(z) = \int_{-\infty}^{\infty} e^{izx} w(x) dx, \text{ where } z = u + iv \text{ and } u, v \in \mathbf{R}$$

The inverse generalised Fourier transform is defined as

$$\mathcal{F}^{-1}\{\hat{w}(z)\} = w(x) = \frac{1}{2\pi} \int_{iv-\infty}^{iv+\infty} e^{-izx} \hat{w}(z) dz,$$

where integration is performed along a straight line parallel to the real axis, along which z stays within a strip of regularity.

The idea behind the generalised Fourier transform pricing method is to utilise the ability to switch the order of expectation and integration, when applying a consecutive Fourier and Fourier inverse transformation to the indicator process $1_{\{S_\tau < D\}}$. Switching the order of expectation and integration is equivalent to switching the order of integration under a double integral. This is permissible under Fubini's theorem provided that the real and imaginary parts of the integrand are both $L^1(\mathbf{R}^2)$ functions (see e.g. Weir (1973)). We will proceed under the assumption that this is satisfied. In practice, this will normally be the case, since the usual choices of severity distributions such as the Gamma, Pareto and Log-Normal are well behaved.

It is not difficult to see that the Fourier transform of the indicator process is given by

$$\mathcal{F}\left\{1_{\{S_T < D\}}\right\} = \int_{-\infty}^{\infty} 1_{(-\infty, D)}(S_T) e^{izS_T} dS_T = -\frac{ie^{izD}}{z} \text{ for } Im(z) < 0.$$

Let us now return to the pricing formula (3) and express the indicator process in terms of its Fourier transform. We begin by simplifying the capital part $V_t^{cat\_cap}$ of the bond price as follows

$$
\begin{aligned}
V_t^{cat\_cap} &= E_{\mathbf{Q}}\left\{e^{-r(T-t)}\mathcal{F}^{-1}\left(-\frac{ie^{izD}}{z}\right)|F_t\right\} \\
&= e^{-r(T-t)}E_{\mathbf{Q}}\left\{\frac{1}{2\pi}\int\limits_{iv-\infty}^{iv+\infty} e^{-izS_T}\frac{-ie^{izD}}{z}dz|F_t\right\} \\
&= \frac{-ie^{-r(T-t)}}{2\pi}\int\limits_{iv-\infty}^{iv+\infty}\frac{e^{izD}}{z}E_{\mathbf{Q}}\left(e^{-izS_T}|F_t\right)dz. \qquad (4)
\end{aligned}
$$

Now, in order to evaluate the integral given in (4) we will first need to simplify the expression $E_{\mathbf{Q}}\left(e^{-izS_T}|F_t\right)$.

Following definition (2) of the compound Poisson model the accumulated losses at the contract expiry time T can be expressed in terms of the accumulated losses at time t (i.e. using the information given by the filtration F_t) as

$$
S_T = S_t + \sum_{j=N_t+1}^{N_T} X_j = S_t + D_{t,T},
$$

where $D_{t,T} = \sum_{j=N_t+1}^{N_T} X_j$. Hence we have

$$
E_{\mathbf{Q}}\left(e^{-izS_T}|F_t\right) = E_{\mathbf{Q}}\left(e^{-izS_t}e^{-izD_{t,T}}|F_t\right) = e^{-izS_t}E_{\mathbf{Q}}\left(e^{-izD_{t,T}}|F_t\right).
$$

Now, $E_{\mathbf{Q}}\left(e^{-izD_{t,T}}|F_t\right)$ is actually the moment generating function of $D_{t,T}$ evaluated at $\tau = -iz$. A standard approach to evaluate the MGF yields

$$
E_{\mathbf{Q}}\left(e^{-izD_{t,T}|F_t}\right) = M_{N_T-N_t}\left\{\ln M_X(\tau)\right\},
$$

where we assume that X is such that $M_X(\tau)$ exists for imaginary τ.

This shows that

$$
\begin{aligned}
E_{\mathbf{Q}}\left(e^{-izS_T}|F_t\right) &= e^{-izS_t}M_{N_T-N_t}\left\{\ln M_X(-iz)\right\} \\
&= e^{-izS_t}M_{N_T-N_t}\left\{\ln \phi_X(-z)\right\} \\
&= e^{-izS_t}\exp\left(\lambda(T-t)\left(\phi_X(-z)-1\right)\right), \qquad (5)
\end{aligned}
$$

where $\phi_X(z)$ is the characteristic function of the claim severity distribution.

Applying the same approach to simplify the coupon payment part $V_t^{cat\_coup}$ of the pricing formula we arrive at the following general pricing formula for cat bonds

$$V_t^{cat} = -\sum_{j=1}^{n} C_{t_j} \frac{ie^{-r(t_j-t)}}{2\pi} \int_{iv-\infty}^{iv+\infty} \frac{e^{iz(D-S_t)+\lambda(t_j-t)[\phi_X(-z)-1]}}{z} dz$$
$$- \frac{ie^{-r(T-t)}}{2\pi} \int_{iv-\infty}^{iv+\infty} \frac{e^{iz(D-S_t)+\lambda(T-t)[\phi_X(-z)-1]}}{z} dz, \qquad (6)$$

where $Im(z) < 0$.

The integral in (6) will generally be computed numerically along a strip below and parallel to the real axis in the complex plane. The choice of severity distribution will impose restriction on the exact strip chosen, the criteria being to avoid passing through any points of singularity. It is unfortunate that realistic choices of the severity distribution will prevent the use of residue calculus to evaluate the integral explicitly due to the integrand not decaying fast enough as $|z| \to \infty$. However, most mathematical packages provide facilities to evaluate this type of integral and we will look at efficient algorithms for its computation later in the paper.

In order to use this pricing formula, we can estimate parameters of the Poisson rate λ and the distribution $f(x)$ of the individual claim sizes, X_i, that provide cat bond prices consistent with observed market prices and historical data for the loss distribution. Assuming that we have calibrated the compound Poisson model successfully (and therefore have described the loss process under the risk neutral probability measure), we will now proceed to look at market consistent reinsurance pricing. The calibration process for cat bonds under this type of model is described by Burnecki (2005) and we are not going to consider it here.

4 Cat bond consistent reinsurance pricing

There are many varieties of reinsurance contracts. For simplicity, we will only consider an aggregate excess of loss ('aggregate XL'). This is sometimes referred to as a stop-loss contract in the actuarial literature.

It is assumed that the aggregate XL contract will expire at time T and all claims are settled at the end of the contract. Under this assumption, the payoff from the contract at time T will be

$$X = \max(S_T - K, 0) = (S_T - K)^+ ,$$

where $K > 0$, is the priority of the contract. Other reinsurance contracts that depend only on the total loss at time T can be expressed in a similar way.

The analogies between the payoff function of this reinsurance contract and a European option are obvious and are a clear motivation for applying financial pricing methods. We refer the reader to Embrechts (1996) and Holtan (2004) for a stimulating discussion of the connections between the actuarial and financial fields in this context. Further insight into this duality can be gained from Dufresne et al (2006). he previous section.

As described in detail earlier in Section 2, we are adopting a market implied approach to determining the risk-neutral probability measure from the observed prices of cat bonds. Under assumptions of the framework of Delbaen & Haezendonck (1989), we are assured that that the insurance loss process follows a compound Poisson process under the implied risk-neutral probability measure.

We apply the fundamental theorem of asset pricing, to assert that the value of the reinsurance contract (on the same underlying insurance risk) at time t

is

$$V_t^{XL} = E_{\mathbf{Q}} \left\{ \exp\left(-\int_t^T r_s ds \right) (S_T - K)^+ |F_t \right\}, \tag{7}$$

or in the case of deterministic interest rates

$$V_t^{XL} = E_{\mathbf{Q}} \left\{ e^{-r(T-t)} (S_T - K)^+ |F_t \right\}. \tag{8}$$

It is worth noting that we can build more features into the aggregate XL contract by combining several simple contracts, in an analogous way to creating spreads using a portfolio of options. For instance, if we wish the XL contract to attach at level K_1 and have an upper limit of K_2, then we create the insurance equivalent of a Bull Spread. That is, we effectively buy an aggregate XL contract with priority K_1 and sell an aggregate XL contract with priority K_2. Thus the price at time t of the aggregate XL contract that attaches at K_1 with limit K_2 is

$$E_{\mathbf{Q}} \left\{ e^{-r(T-t)} (S_T - K_1)^+ |F_t \right\} - E_{\mathbf{Q}} \left\{ e^{-r(T-t)} (S_T - K_2)^+ |F_t \right\}.$$

In order to evaluate these expressions, we will again use the Fourier transform method that was introduced in Section 3. We begin by noting that the Fourier transform of the payoff function $(S_T - K)^+$ is easily seen to be

$$\mathcal{F}\left\{ (S_T - K)^+ \right\} = -\frac{e^{izK}}{z^2}, \text{ where } Im(z) > 0.$$

We therefore have that

$$\begin{aligned} V_t^{XL} &= e^{-r(T-t)} E_{\mathbf{Q}} \left\{ \mathcal{F}^{-1} \left(-\frac{e^{izk}}{z^2} \right) |F_t \right\} \\ &= e^{-r(T-t)} E_{\mathbf{Q}} \left\{ \frac{1}{2\pi} \int_{iv-\infty}^{iv+\infty} e^{-izS_T} \frac{-e^{izK}}{z^2} dz |F_t \right\} \end{aligned}$$

$$= -\frac{e^{-r(T-t)}}{2\pi} \int\limits_{iv-\infty}^{iv+\infty} E_{\mathbf{Q}} \left(e^{-izS_T} | F_t \right) \frac{e^{izK}}{z^2} dz$$

$$= -\frac{e^{-r(T-t)}}{2\pi} \int\limits_{iv-\infty}^{iv+\infty} \frac{e^{iz(K-S_t)+\lambda(T-t)[\phi_X(-z)-1]}}{z^2} dz, \tag{9}$$

where the integration must be computed along a strip above and parallel to the real axis. Again, this is performed numerically and will need to be within a strip of regularity (determined by the choice of severity function).

5 How to evaluate the integrals

While the financial pricing approach is attractive from a theoretical standpoint, it is of little value unless the complex integrals can actually be evaluated in a practical manner.

In this section we will consider different ways of computing the integrals in (6) for the price of a cat bond. Without loss of generality, we will only consider a zero coupon cat bond. This simplifies the problem to calculating a single integral, rather than one per coupon payment. The price of the coupon paying cat bond can then be calculated as a linear combination of zero coupon bonds of varying duration. The numerical techniques developed are also applied in computing the integral in (7) and pricing aggregate XL contracts (See Section 6 (14)).

The problem we are aiming to solve is to find a method of computing the following integral

$$V_t^{cat} = \frac{-ie^{-r(T-t)}}{2\pi} \int\limits_{iv-\infty}^{iv+\infty} \frac{e^{iz(D-S_t)+\lambda(T-t)[\phi_X(-z)-1]}}{z} dz, \tag{10}$$

where V_t^{cat} denotes the price of a zero coupon cat bond.

The simplest way to evaluate this type of integral is to represent it as a Riemann summation and then either compute it directly or apply an efficient numerical integration algorithm. For instance, adaptive Gauss-Kronrod integration described by Calvetti et al (2000) can be applied by making a substitution to remove the complex limits.

Alternatively, techniques such as the Fast Fourier Transform can be applied if efficient evaluation of the integral is required to be evaluated at a range of trigger levels. For instance, during the calibration process one will attempt to replicate observed cat bond price for a variety of different trigger levels by repeated re-evaluation at different parameter values. It is important that at each iteration of the calibration process the cat bond prices can be evaluated quickly.

The first step in developing the numerical computation of cat bond prices is to simplify the integral expression. It currently has complex limits that prevent it being represented as a summation, we therefore make the substitution $\hat{z} = z - iv$, which yields

$$V_t^{cat} = \frac{-ie^{-r(T-t)}}{2\pi} \int_{-\infty}^{\infty} \frac{e^{i(\hat{z}+iv)(D-S_t)+\lambda(T-t)[\phi_X(-\hat{z}-iv)-1]}}{\hat{z}+iv} d\hat{z}, \tag{11}$$

where the real and imaginary parts of the integrand function are assumed to be in $L^1(\mathbf{R})$. In (11) we truncate the limits of integration at $-A/2$, $A/2$, we have

$$\begin{aligned} V_t^{cat} &\approx \frac{-ie^{-r(T-t)}}{2\pi} \int_{-\frac{A}{2}}^{\frac{A}{2}} \frac{e^{i(\hat{z}+iv)(D-S_t)+\lambda(T-t)[\phi_X(-\hat{z}-iv)-1]}}{\hat{z}+iv} d\hat{z} \\ &= \frac{-ie^{-r(T-t)}}{2\pi} \int_{-\frac{A}{2}}^{\frac{A}{2}} e^{i\hat{z}(D-S_t)} \frac{e^{-v(D-S_t)+\lambda(T-t)[\phi_X(-\hat{z}-iv)-1]}}{\hat{z}+iv} d\hat{z}. \end{aligned}$$

Convergence of this integral is guaranteed following the property that integrals are continuous functions of their limits.

Finally, we define

$$
\begin{aligned}
\Delta &= \frac{A}{N-1}, \\
\hat{z}_m &= -\frac{A}{2} + m\Delta,
\end{aligned}
$$

where N is the number of steps in the numerical approximation, $\hat{z}_0 < \hat{z}_1 < \ldots < \hat{z}_N$ define a uniform partition of the interval $[-A/2, A/2]$ and Δ is the width of the partition. This is a classical Riemann approximation of the integral, which yields

$$
\begin{aligned}
V_t^{cat} &= -\frac{ie^{-r(T-t)}}{2\pi} \sum_{m=0}^{N-1} e^{i\left(-\frac{A}{2}+m\Delta\right)(D-S_t)} \frac{e^{-v(D-S_t)+\lambda(T-t)[\phi_X(-\hat{z}_m-iv)-1]}}{\hat{z}_m+iv}\Delta \\
&= -\Delta \frac{ie^{-r(T-t)-(\frac{iA}{2}+v)(D-S_t)}}{2\pi} \sum_{m=0}^{N-1} \frac{e^{im\Delta(D-S_t)+\lambda(T-t)[\phi_X(-\hat{z}_m-iv)-1]}}{\hat{z}_m+iv}. \quad (12)
\end{aligned}
$$

Clearly, as A and $N \to \infty$ this will converge to the required integral (10). The convergence of this approximation could be very easily improved by using a better numerical integration method such as a standard quadrature rule or a more advanced method such as the aforementioned adaptive Gauss-Kronrod approach.

The pricing formula can therefore be evaluated numerically using a simple computer program. However, as mentioned above, there is a more efficient way of calculating this type of integral, when the price is required for a range of trigger levels D: computation using the Fast Fourier Transform (FFT) or the Fractional Fast Fourier Transform (FFFT).

6 The Fast Fourier Transform

The Fast Fourier Transform (FFT) is an efficient algorithm that can be used to numerically evaluate the Discrete Fourier Transform (DFT). We refer the reader to Carr & Madan (1999) for a detailed discussion of the FFT for the application of option pricing.

Returning to equation (12), we can re-express it as a DFT by making the following substitution

$$D_n = S_t + \frac{2\pi n}{N\Delta},$$

as an approximation to D for a suitable choice of n. We then approximate the price of the cat bond as

$$V^{cat}_{t,D_n} = -\Delta \frac{ie^{-r(T-t)-\left(\frac{iA}{2}+v\right)(D_n-S_t)}}{2\pi} \sum_{m=0}^{N-1} f_m e^{\frac{2\pi inm}{N}}, \tag{13}$$

where $f_m = \frac{e^{\lambda(T-t)[\phi_X(-\hat{z}_m-iv)-1]}}{\hat{z}_m+iv}$.

This is in precisely the form of the DFT which means that we can use the Fast Fourier Transform to perform the required numerical computation. The FFT will return us with an array, $\{V^{cat}_{t,D_n}\}_{n=0}^{N-1}$, each element of which $V^{cat}_{t,n}$ represents the price of a cat bond with trigger $D_n = S_t + \frac{2\pi n}{N\Delta}$, where losses to date are S_t.

Thus, we are quickly able to price an entire range of cat bonds with a range of trigger levels $D_n = S_t$ to $S_t + \frac{2\pi N}{A}$, in a single computation. Note that any cat bonds for which D is below S_t will have already been triggered.

Similar computations yield the following approximation formula for the price of the aggregate XL contract in the earlier example

$$V^{XL}_{t,K_n} = -\Delta \frac{e^{-r(T-t)-\left(\frac{iA}{2}+v\right)(K_n-S_t)}}{2\pi} \sum_{m=0}^{N-1} f_m e^{\frac{2\pi inm}{N}}, \tag{14}$$

where $f_m = \frac{e^{\lambda(T-t)[\phi_X(-\hat{z}_m-iv)-1]}}{(\hat{z}_m+iv)^2}$ and $K_n = S_t + \frac{2\pi n}{N\Delta}$.

The FFT applied to (14) will yield aggregate XL reinsurance prices for a range of priority levels $K_n = S_t$ to $S_t + \frac{2\pi N}{A}$.

7 The Fractional Fast Fourier Transform

The Fast Fourier Transformation was shown in the previous section to provide a good method for evaluating the contract price for a range of triggers / priority levels. However, its main disadvantage is that the trigger / priority levels all lie on the mesh defined by $\left\{\frac{2\pi n}{N\Delta}\right\}_{n=0}^{N-1}$. This means that to price at particular points, interpolation must be used, which introduces additional error. In order to overcome this source of error the Fractional Fast Fourier Transform (FFFT) method suggested by Bailey (1990) and implemented for option pricing by Chourdakis (2005) can be useful.

As an example of applying the FFFT to pricing contracts, we consider the cat bond example from the previous section on the FFT. Suppose we wish to price cat bonds using (12) for a range of trigger levels $D = D_L$ to D_U. We define

$$D_n = D_L + \frac{D_U - D_L}{N-1} n, \text{ for } n = 0, 1, ..., N-1$$

and substitute into (12) yielding the following expression for cat bond prices,

$$V^{cat}_{t,D_n} \approx -\Delta \frac{ie^{-r(T-t)-\left(\frac{iA}{2}+v\right)(D_n-S_t)}}{2\pi} \sum_{m=0}^{N-1} e^{im\Delta\left(D_L+\frac{D_U-D_L}{N-1}n-S_t\right)} \frac{e^{\lambda(T-t)[\phi_X(-\hat{z}_m-iv)-1]}}{\hat{z}_m+iv}$$

$$= -\Delta \frac{ie^{-r(T-t)-\left(\frac{iA}{2}+v\right)(D_n-S_t)}}{2\pi} \sum_{m=0}^{N-1} e^{2\pi imn\gamma} f_m, \tag{15}$$

where $\gamma = \frac{\Delta}{2\pi}\left(\frac{D_U-D_L}{N-1}\right)$ and $f_m = e^{im\Delta(D_L-S_t)} \frac{e^{\lambda T[\phi_X(-\hat{z}_m-iv)-1]}}{\hat{z}_m+iv}$. Expression (15) can be computed for the exact range of required trigger levels in single application of the FFFT.

8 Example

We will now look at an example of pricing the cat bond and the aggregate excess of loss contract under a particular choices of severity function to demonstrate the method is feasible from a practical perspective.

8.1 Gamma Severity

It is assumed that S_t follows a compound Poisson distribution with frequency λ and loss size follows a Gamma distribution with parameters α and β. For simplicity, we assume that the cat bond pays no coupons.

The characteristic function of the Gamma distribution is

$$\phi_X(z) = \left(1 - \frac{iz}{\beta}\right)^{-\alpha} = \left(\frac{\beta}{\beta - iz}\right)^{\alpha}.$$

Applying formula (6) we have the value of a zero coupon cat bond at time t

$$V_t^{cat} = \frac{-ie^{-r(T-t)}}{2\pi} \int_{iv-\infty}^{iv+\infty} \frac{e^{iz(D-S_t)+\lambda(T-t)\left[\left(\frac{\beta}{\beta+iz}\right)^{\alpha}-1\right]}}{z} dz. \tag{16}$$

To evaluate this numerically, we require $Im(z) < 0$ and we must avoid the irregularity at $z = \beta i$ (which will make $\beta + iz = 0$). We therefore should integrate along a straight line that lies beneath the real axis. Note that since the point $z = \beta i$ lies above the real axis, it does not concern us.

Next, we consider the pricing formula (9) for the aggregate XL contract. Substituting in the formula for the characteristic function, we immediately see

$$V_t^{XL} = -\frac{e^{-r(T-t)}}{2\pi} \int\limits_{iv-\infty}^{iv+\infty} \frac{e^{iz(K-S_t)+\lambda(T-t)\left[\left(\frac{\beta}{\beta+iz}\right)^{\alpha}-1\right]}}{z^2} dz, \tag{17}$$

where integration is carried out on a straight line that lies above the real axis and below the point $z = \beta i$.

8.2 *Pareto Severity*

One of the more popular severity distributions in practical applications is the Pareto distribution. We will consider the two parameter version of the Pareto distribution and derive the pricing formulae for the two contracts.

The two parameter Pareto distribution with parameters k and α does not have a moment generating function, we only require the existence of its characteristic function, which can be shown to be

$$\phi_X(t) = k\,(-i\alpha t)^k\, \Gamma\left(-k, -i\alpha t\right), \tag{18}$$

where $\Gamma\left(a, z\right)$ is the incomplete upper Gamma function defined as

$$\Gamma\left(a, z\right) = \int\limits_{z}^{\infty} y^{a-1} e^{-y} dy.$$

The derivation of the characteristic function is standard but is rather lengthy and hence is omitted. For an example of similar calculations we refer to Bottazzi (2007) (see also Dufresne et al (2006)).

This representation of the characteristic function is only defined for z in the upper half of the complex plane, that is, where $Im(z) \geq 0$. This restriction will have a serious effect on our ability to price certain contracts where the pricing formula involves calculation of the characteristic function in the negative half of the complex plane.

Note that there is a numerically efficient method of evaluating the incomplete gamma function described in Numerical Receipes (1988), which is easily adapted to the complex parameter case.

Again, applying formula (6) we have the value of a zero coupon cat bond at time t to be

$$V_t^{cat} = \frac{-ie^{-r(T-t)}}{2\pi} \int_{iv-\infty}^{iv+\infty} \frac{e^{iz(D-S_t)+\lambda(T-t)\left[k(i\alpha z)^k \Gamma(-k, i\alpha z)-1\right]}}{z} dz. \tag{19}$$

To evaluate this numerically, we simply require $Im(z) < 0$. Since the Pareto characteristic function is being evaluated at $-z$, this will ensure that we are only evaluating the characteristic function in the upper half of the complex plane.

We now continue to look at the case of the aggregate XL contract. Using the same approach we have that the value of aggregate XL contract is

$$V_t^{XL} = -\frac{e^{-r(T-t)}}{2\pi} \int_{iv-\infty}^{iv+\infty} \frac{e^{iz(K-S_t)+\lambda(T-t)\left[k(i\alpha z)^k \Gamma(-k, i\alpha z)-1\right]}}{z^2} dz, \tag{20}$$

where integration is carried out on a straight line that lies above the real axis. This means that we need to evaluate the characteristic function in the lower half of the complex plane. However, as previously noted, the numerical form of the characteristic function is divergent in this region, which means that this pricing method fails.

We need to explore an alternative approach of evaluating the aggregate XL contract price in the case of Pareto severity. Fortunately, we can draw inspiration from the finance world once more. The key connection between the financial and actuarial fields exploited in this paper is the form of the aggregate XL payoff function, $\max(S_T - K, 0)$, which is identical to the payoff of a European option contract. An important formula in option pricing theory is the put-call parity relationship

$$c_t + Ke^{-r(T-t)} = p_t + s_t, \tag{21}$$

where c_t, p_t are the prices at time t of a call and put option expiring at time T on stock s, with strike price K.

We can derive a similar relationship in insurance. We begin by defining the insurance equivalent of a put option as described by Wacek (1997). This will be a contract that provides payoff

$$\max(K - S_T, 0) \tag{22}$$

at time T. We can construct a put-call parity relationship for insurance by considering no-arbitrage arguments under the assumption that the notional put contract exists. Consider the following portfolios.

Portfolio 1: At current time t, purchase a put contract at cost P_t and additionally purchase an insurance policy on the aggregate claims process for price ξ_t. By time T, if the aggregate claims process S_T has exceed priority K, then the put contract will provide zero payoff. If the aggregate claims do not exceed K then the put contract will provide payoff $K - S_T$. In both cases the insurance policy will provide payoff S_T. Thus the overall payoff from this portfolio at time T is

$$\begin{cases} S_T, \text{ if } S_T \geq K \\ K, \text{ if } S_T < K. \end{cases}$$

In a similar fashion, we set up a second portfolio below.

Portfolio 2: At current time t, purchase an aggregate XL contract at cost V_t^{XL} and additionally invest amount $Ke^{-r(T-t)}$ in risk-free cash. By time T, if the aggregate claims process S_T has exceeded priority K, then the aggregate XL contract will provide payoff $S_T - K$. If the aggregate claims do not exceed K then the contract will provide zero payoff. The cash will mature to amount K. Thus the overall payoff from this portfolio at time T is

$$\begin{cases} S_T, \text{ if } S_T \geq K \\ K, \text{ if } S_T < K, \end{cases}$$

which is precisely the same as Portfolio 1. Applying the principle of no-arbitrage, we assert that both portfolios have equal value at time t as well. Otherwise it would be possible to make risk-free profit by selling one portfolio short and taking a long position in the other.

We therefore have the following put-call relationship for insurance policies

$$V_t^{XL} + Ke^{-r(T-t)} = \xi_t + P_t.$$

The premium paid for the insurance policy on the aggregate claims process can be priced under a risk-neutral probability measure as follows

$$\xi_t = E_{\mathbf{Q}}\left(e^{-r(T-t)} S_T | F_t\right) = e^{-r(T-t)} E_{\mathbf{Q}}\left(S_T | F_t\right). \tag{23}$$

We can calculate this expectation using a similar approach and notation to that used in (5) for calculating the characteristic function of the aggregate claims process with respect to the filtration at time t. We have

$$\begin{aligned} E\left(S_T|F_t\right) &= E\left(S_t + D_{t,T}|F_t\right) \\ &= S_t + E\left(\sum_{j=N_t+1}^{N_T} X_j\right) \\ &= S_t + E(X)E(N_T - N_t|F_t) \\ &= S_t + \lambda E(X)\left(T-t\right), \end{aligned}$$

where in the last equality we have used the fact that the centred Poisson process $(N_T - \lambda T)$ is a martingale with respect to the filtration F_t.

This provides us with put-call parity relationship for insurance processes under the aggregate claims compound Poisson model

$$V_t^{XL} + Ke^{-r(T-t)} = e^{-r(T-t)}\lambda E(X)\left(T-t\right) + P_t. \tag{24}$$

We now return to the problem of evaluating the price of the aggregate XL contract in (20). Using the put-call parity relationship we can re-express V_t^{XL} as

$$V_t^{XL} = P_t - Ke^{-r(T-t)} + e^{-r(T-t)}\lambda E(X)\left(T-t\right).$$

So, provided we can successfully calculate the value of the put contract under the Pareto claims severity distribution, we will be able to work the price of the aggregate XL contract.

The generalised Fourier transform of the put payoff (22) is easily calculated to be

$$\mathcal{F}\left\{(K - S_T)^+\right\} = -\frac{e^{izK}}{z^2}, \tag{25}$$

where $Im(z) < 0$. This is virtually the same as the Fourier transform of the aggregate XL payoff function, except z is now restricted to the lower half of the complex plane. The put contract can therefore be valued as

$$P_t = -\frac{e^{-r(T-t)}}{2\pi} \int_{iv-\infty}^{iv+\infty} \frac{e^{iz(K-S_t)+\lambda(T-t)\left[k(i\alpha z)^k \Gamma(-k,i\alpha z)-1\right]}}{z^2} dz, \tag{26}$$

where integration is carried out along a straight line that lies beneath the real axis. This means that we will need to evaluate the characteristic function in the upper half of the complex plane, which is within the region of regularity. We therefore have successfully found a means of pricing aggregate XL contracts in the case of a Pareto severity distribution.

Let us note that an alternative expression for $E\left((S_T-K)^+ |F_t\right)$ has been obtained by Dufresne et al (2006) using Parseval's theorem.

8.3 Numerical Computation

As an illustration of the practicality of the FFFT approach, the price of cat bonds and aggregate XL contracts were computed using this algorithm and the parameterisation from example in Section 8.1, for a range of triggers / priority levels and durations between 0 and 1. A 3D plot of the aggregate XL prices is shown in Figure 1 and the corresponding plot for the cat bond is shown in Figure 2.

It is noticeable that at the boundary points where the trigger / priority level is close to zero, the integral approximation does not converge well. This is a well known problem in the application of the DFT to option pricing problems in finance. However, it is not a significant issue, since no cat bonds or reinsurance contracts are issued with a trigger / priority level close to zero. This would become a more serious problem for pricing direct insurance. For normal contracts arising in the reinsurance market the integral approximation

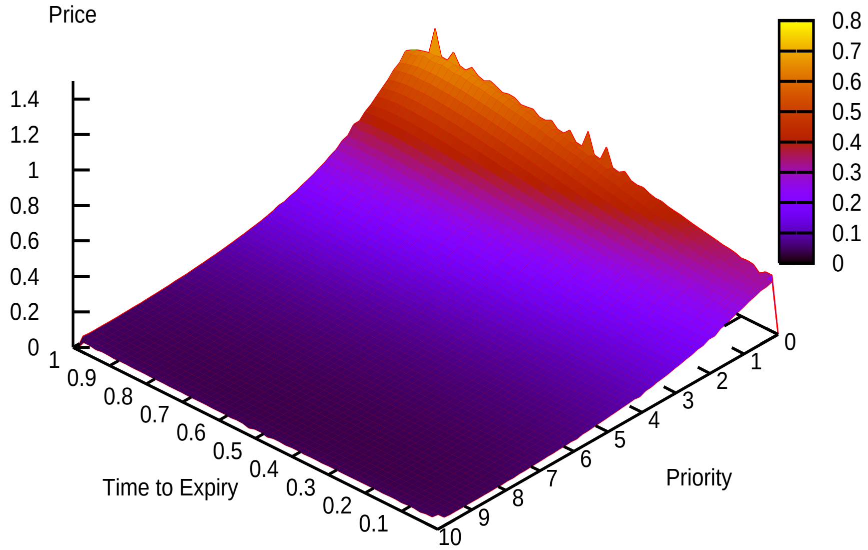

Fig. 1. Aggregate XL prices under Gamma(2,2) severity and $\lambda = 2$ claims frequency.

converges quickly and accurately.

9 Comparison with Monte Carlo simulation

To verify that the analytical formulae derived using the generalised Fourier transform method provide the anticipated results, they will be evaluated using the Riemann summation approximation and compared to that achieved using Monte Carlo simulation for a varying number of simulations.

For this purpose, we will set up a simple model in which losses are generated for a 1 year period according to a compound Poisson distribution with rate $\lambda = 2$ and severity distribution Gamma with parameters $\alpha = \beta = 1$. The generated losses are aggregated and then the recoveries are evaluated for both an aggregate XL contract and a cat bond. The aggregate XL contract has attachment point at 4.75 and has no limit of reinstatements or upper limit

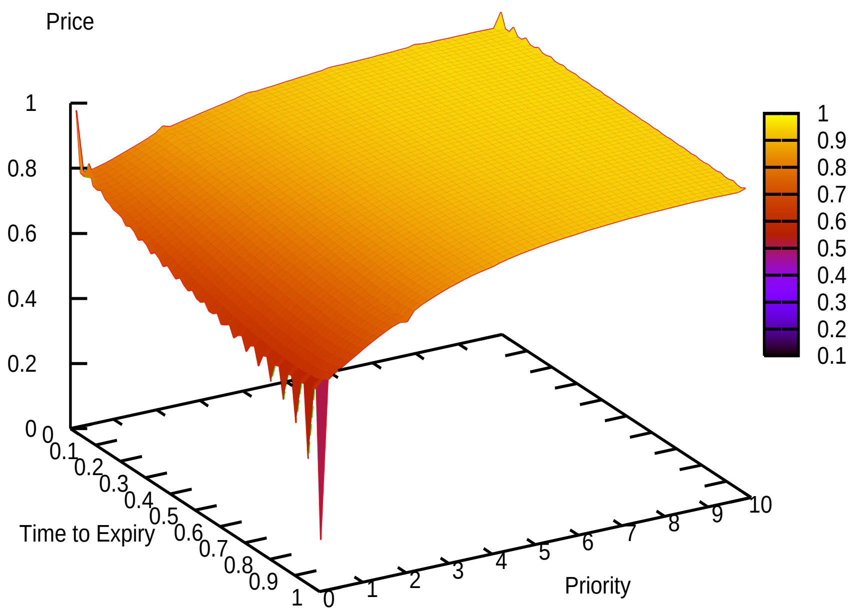

Fig. 2. Catastrophe bond prices under gamma(2,2) severity and $\lambda = 2$ claims frequency.

on recoveries. The cat bond is a simple zero coupon trigger based bond with priority level 4.75.

We will compare the price of these contract using the analytical formulae (16) and (17) to that achieved through simulation.

Calculating the analytical formulae numerically under the Riemann sum approximation to 7 decimal places of precision, we find the prices for the aggregate XL and cat bond contracts are 0.1625310 and 0.8658063 respectively assuming the risk free rate of interest is 0.04. Under Monte-Carlo simulation, the pricing results for different simulation sizes are shown in the table below:

Trials	XL Mean (MC)	Error %	Cat Bond Mean (MC)	Error %
10000	0.1519416	6.515%	0.8721086	0.728%
100000	0.1620345	0.306%	0.8665744	0.089%
500000	0.1612862	0.766%	0.8663496	0.063%
700000	0.1617444	0.484%	0.8660117	0.024%
1000000	0.1619830	0.337%	0.8658980	0.011%
2000000	0.1626854	0.095%	0.8657770	0.003%

It is interesting to observe that convergence under Monte-Carlo is quite slow and requires around two million trials to achieve an aggregate XL price within 0.1% of the analytical price. This suggests that for pricing applications the analytical methods of computing prices are more efficient than Monte Carlo simulation techniques. In particular, this makes the analytical method suitable for calibrating the model to observed market prices. This would not be possible to achieve in a reasonable time period using Monte-Carlo, since the calibration process usually involves an optimising routine recalculating modelled prices repeatedly using different parameter values.

10 Comments and conclusions

In this paper we have provided a framework for pricing reinsurance contracts in a way that is consistent with the prices of cat bonds on the same underlying loss process.

We have utilised existing work in this area by applying an option pricing technique developed by Lewis (2001) that applies the generalised Fourier transform to price derivative contracts on an underlying Levy process. We then demonstrated pricing an aggregate excess of loss contract and cat bond under this framework, for both the Gamma and Pareto Type II severity distribution.

While the mathematics involved under this approach is more complicated than traditional actuarial pricing methods, we have shown that it is relatively easy to compute the pricing formulae derived using efficient numerical methods.

In particular we have demonstrated that the Fractional Fast Fourier Transform (FFFT) can provide a useful role in actuarial science. Using the FFFT we have shown how insurance contracts can be priced in a single calculation for a range of priority / trigger levels. This provides a clear advantage over Monte-Carlo based methods, as it means that the modelled cat bond price can be computed at all required trigger levels in around one second (on a 3 Ghz Intel CPU). The source code implementing the methodology proposed in the paper is available upon request to the authors.

Undoubtedly, the most difficult part of applying this approach will be calibrating the compound Poisson distribution to the market prices of cat bonds and historical loss frequency / severity data. However, this is certainly achievable and is an extension of existing research that has focused on calibrating against historical data. Some work has already been carried out in this area by Burnecki (2005), who describes the calibration process for pricing cat bonds on the Property Claims Services (PCS) index in the United States.

We believe that the FFFT could provide an efficient method of calibrating the model by means of an optimisation algorithm. In particular we suggest to follow the calibration methods normally used in option pricing models. For example, an exhaustive algorithm such as adapted simulated annealing or a genetic algorithm could be applied to find the model parameters that minimise the total squared error between observed cat bond prices and modelled prices.

Finally, we note that the methodology presented in the paper can be easily generalised to the case of stochastic interest rates under the assumption of independence between the insurance and interest rate processes.

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