INELASTIC DISPLACEMENT RATIOS OF DEGRADING SYSTEMS

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Abstract

Seismic code provisions in several countries have recently adopted the new concept of performance-based design. New analysis procedures have been developed to estimate seismic demands for performance evaluation. Most of these procedures are based on simple material models though, and do not take into account degradation effects, a major factor influencing structural behavior under earthquake excitations. More importantly, most of these models can not predict collapse of structures under seismic loads. This study presents a newly developed model that incorporates degradation effects into seismic analysis of structures. A new energy-based approach is used to define several types of degradation effects. The model also permits collapse prediction of structures under seismic excitations. The model was used to conduct extensive statistical dynamic analysis of different structural systems subjected to a large ensemble of recent earthquake records. The results were used to propose approximate methods for estimating maximum inelastic displacements of degrading systems for use in performance-based seismic code provisions. The findings provide necessary information for the design evaluation phase of a performance-based earthquake design process, and could be used for evaluation and modification of existing seismic codes of practice.

CE Database subject headings: Displacement; Seismic analysis; Degradation; Hysteresis; Nonlinear response; Inelastic actions.

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Introduction

The seismic design provisions of building codes in several countries have recently adopted the concept of performance based design. A Performance Based Earthquake Engineering design process is a demand/capacity procedure that incorporates multiple performance objectives. The procedure consists of four main steps. In the first step, performance objectives of a structural system at different hazard levels are defined (e.g. immediate occupancy, life safety, and collapse prevention). In the second step, a conceptual design of the structure is performed in order to meet the objectives defined in step 1. The third step is a design evaluation phase needed in order to evaluate the conceptual design previously developed in step 2. Finally, in the fourth step, the socio-economic consequences of the earthquake excitations are evaluated in the form of cost/benefit analysis. In the design evaluation phase, seismic demands of the structure need to be evaluated as accurately as possible at different hazard levels for demand/capacity comparison. Most codes rely on approximate methods that predict the desired seismic demand parameters. Two methods were established in that sense, the capacity spectrum method developed originally by Freeman (1978) and adopted by ATC-40 (1996), and the method of coefficients developed by Seneviratna and Krawinkler (1997) and used by FEMA-356 (2000). Both methods are similar in the sense that they are based on a nonlinear static push-over of the structure. They are different, however, in the way they estimate the maximum "target" inelastic displacement. The first method is based primarily on superimposing capacity diagram plots on demand diagram plots, and estimating the target displacement with an iterative procedure using elastic dynamic analyses. Several modified versions were introduced to improve the originally developed method (e.g. Paret et al. 1996, WJE 1996, Bracci et al. 1997, Fajfar and Fischinger 1999, and Chopra and Goel 1999). In the second method used by FEMA-356, the target roof displacement δ_t of a building is obtained from the elastic spectral displacement S_d using several modification factors derived from SDOF analysis as follow:

$$\delta_t = C_0 C_1 C_2 C_3 S_d \tag{1}$$

where C_0 is a modification factor that relates spectral displacements of SDOF systems to roof displacements of MDOF systems, and is computed using any of the following three procedures: a) the first mode participation factor at the roof, (b) the modal participation factor at the roof using a shape vector corresponding to the deflected shape of the building at the target displacement, (c) values given in table 3-2 of the FEMA 356 document, which are based on the type of load pattern used. C_1 is a factor that accounts for the ratio of maximum inelastic to maximum elastic displacements, C_2 is a factor that accounts for degradation effects, and C_3 is a factor that accounts for dynamic second-order effects. These coefficients were based on extensive statistical analysis of SDOF systems. The factor C_2 was derived by considering models that degrade only in strength, and does not account for strength softening behavior. An improved procedure for nonlinear seismic analysis of buildings with new expressions for these modification factors was proposed in FEMA-440 (2005).

Several researchers attempted to develop procedures for estimating maximum inelastic displacements to be used within a performance-based design process. In most of these studies though, the material models used followed simple hysteretic non-degrading rules. Only few of these studies considered degradation effects. Even in these studies, degradation was still not based on clear physical reasoning. Furthermore, none of these studies considered collapse prediction of the structures. A brief summary of earlier studies in this field is given below.

The first research work in this field is the one by Veletsos and Newmark (1960) who analyzed SDOF systems using 3 earthquake records. The models were assumed elasto-plastic. They concluded that in the regions of low frequency, the maximum inelastic deformation is equal to the maximum elastic deformation, which is known as the equal displacement rule. They also concluded that this rule doesn't hold true for regions of high frequency, where the inelastic displacement considerably exceeds the elastic one.

Shimazaki and Sozen (1984) conducted a similar numerical study on a SDOF system using five different hysteretic models. The models used were either bilinear or of Clough type (1966), and only El Centro earthquake record was used for the analysis. No degradation was considered in their study. In their work, they developed a relation between maximum inelastic displacements and corresponding maximum elastic displacements for different values of strength and period ratios. The conclusion of their work is that for periods higher than the characteristic period, defined as the transition period between the constant acceleration and constant velocity regions of the response spectra, the maximum inelastic displacement equals approximately the maximum elastic displacement regardless of the hysteresis type used, confirming the equal displacement rule. For periods less than the characteristic period, the maximum inelastic displacement exceeds that of the elastic displacement and the amount vary depending on the type of hysteretic model and on the lateral strength of the structure relative to the elastic strength. Their conclusion was confirmed later by Qi and Moehle (1991).

Miranda (1991, 1993a and 1993b) analyzed over 30,000 SDOF systems using a large ensemble of 124 earthquake ground motions recorded on different soil types. He developed ratios of maximum inelastic to elastic displacements for 3 types of soil conditions. He also studied the limiting period value where the equal displacement rule applies. The material model used in his study is also elasto-plastic. Miranda (2000) extended his earlier work, and developed displacement ratio plots for different earthquake magnitudes, epicenter distance, and soil conditions. Later, Miranda (2001) showed that maximum inelastic displacements could be related to maximum elastic displacements either through inelastic displacement ratios, the so-called direct method, or through strength reduction factors, the so-called indirect method. He also showed that the second method is a first order approximation of the first, and that both methods yield similar results in the absence of variability. In addition, he proved that the indirect method typically produces un-conservative results compared to the direct method of analysis. A comparison between the displacement ratios for peak-oriented and bilinear models was presented by Miranda and Ruiz-Garcia (2002a). In addition, Miranda and Ruiz-Garcia (2002b) evaluated six different methods for predicting maximum inelastic displacements. Four methods are based on equivalent linearization techniques, while two are based on multiplying maximum elastic displacements by modification factors. Another evaluation of existing approximate methods was discussed by Akkar and Miranda (2005). The effect of strength softening was investigated by Miranda and Akkar (2003). Finally, Ruiz-Garcia and Miranda (2004, 2006) developed inelastic displacement ratio plots for structures on soft soils. It is worth mentioning that in all the research work conducted by

Miranda and his co-workers, cyclic degradation effect was not accounted for, and collapse potential was not considered.

Krawinkler and his co-workers (1991, 1993 and 1997) conducted similar studies to the ones by Miranda. The material models used were either bilinear, Clough or of pinching type. Degradation effects were included, but in the form of strength degradation only, or stiffness degradation only. Gupta and Kunnath (1998) conducted a similar study on SDOF systems subjected to 15 ground motions. They included degradation effects using a 3 parameters model. Whittaker et al. (1998) conducted a numerical study on SDOF systems using 20 earthquake records. They used the Bouc-Wen model (1976) in their analysis and neglected degradation effects. They developed mean and mean+1sigma ratio plots of maximum inelastic to elastic displacements for different strength values. Song and Pincheira (2000) developed inelastic displacement ratios for strength and stiffness degrading systems using a set of 12 earthquake records. Their degrading model however was explicitly based on the number of cycles rather than the hysteretic dissipated energy. Furthermore, it did not account for collapse potential.

The purpose of this study is to conduct a thorough investigation of the effect of degradation on the behavior of SDOF systems, and to develop new inelastic displacement ratios of SDOF and first mode-dominant degrading building structures. The findings of the study will provide necessary background for the design evaluation phase of a performance-based earthquake design process. The newly-developed degrading material models are presented first.

Material Models

Two material models were used in this research. The models considered were bilinear model to represent steel structures, and modified Clough model as per Clough and Johnston (1966) to represent concrete structures.

The main skeleton for the bilinear, and modified Clough models is shown in Figures (1) and (2) respectively along with a numbering that shows the progress of the hysteresis path. Both models consist of an elastic branch, a strain hardening branch, and a softening branch referred to as a cap. A residual strength is assumed in all models. However, the loading-reloading rules under cyclic loading differ from a model to another. For the bilinear model, the initial unloading is parallel to the initial slope. The reloading curve is then bounded by the positive and negative strain hardening branches. As shown in Figure (1), these branches form two main asymptotes for the model. For the modified Clough model, the initial unloading is parallel as well to the initial slope. As shown in Figure (2), the behavior under cyclic loading is characterized by targeting the maximum previous displacement point.

Degradation

It is well known from experimental verification that all materials deteriorate as a function of the loading history. Rahnama and Krawinkler (1993) discuss with details the different types of degradation observed during experimental tests. Each inelastic excursion causes damage and the damage accumulates as the number of excursions increases. Therefore, it is essential to include degradation effects in modeling hysteretic behavior.

There are three common methods to consider degradation. In the first method, degradation is related to the element ductility. This method does not always produce accurate results. In particular, the method fails to simulate the degrading behavior of specimens subjected to loading cycles producing constant ductility. In the second method, degradation is a function of both the element ductility and the dissipated hysteretic energy. The main disadvantage of this method lies in its complexity, since too many factors are required for calibration of the degradation parameters. The third method uses only the hysteretic energy dissipation to account for degradation. This method has proven to provide results that match well with experimental evidence, while requiring in general simple procedures for calibration of the degradation parameters. The method represents a good compromise between accuracy and simplicity and hence was selected in the current study.

An 8 parameters energy-based criterion is adopted in the current study to account for degradation effects. The model is based on the work by Rahnama and Krawinkler (1993) and was used in several earlier studies (Ayoub et al. 2004a, 2004b; Ibarra et al. 2005). In this model, four types of cyclic degradation are considered: (1) Yield (Strength) degradation, (2) Unloading stiffness degradation, (3) Accelerated stiffness degradation, and (4) Cap degradation. The four types of degradation are simultaneously implemented for both bilinear and modified Clough models.

Yield (Strength) Degradation

Yield degradation refers to the decrease of the yield strength value as a function of the loading history. The yield degradation is derived through the following equation:

$$F_y^i = F_y^{i-1}(1-\beta_{str}^i) \qquad (2)$$

Where

$F_y^i =$ Yield strength at the current excursion i,

$F_y^{i-1} =$ Yield strength at the previous excursion $i-1$, and

$\beta_{str}^i =$ Scalar parameter, ranging from 0 to 1, that accounts for degradation effects at the current excursion i.

The parameter β_{str}^i is defined through the following equation:

$$\beta_{str}^i = \left(\frac{E_i}{E_{capacity} - \sum_{j=1}^{i} E_j} \right)^{C_{str}} \qquad (3)$$

Where

$E_i =$ Hysteretic energy dissipated in the current excursion i;

$\sum_{j=1}^{i} E_j =$ Total hysteretic energy dissipated in all excursions up to the current one; and

$E_{capacity} =$ Energy dissipation capacity of the element under consideration;

$C_{str} =$ Exponent defining the rate of deterioration.

The term $E_{capacity}$ represents the resistance of the material to cyclic degradation. The structure can be considered totally degraded once the total dissipated hysteretic energy due to cyclic loading, attains a value equals to the energy dissipation capacity. The term $E_{capacity}$ is calculated as a function of the strain energy up to yield through the following equation:

$$E_{capacity} = \gamma_{str} F_y \delta_y \qquad (4)$$

where F_y and $\delta_y =$ Initial yield strength and deformation respectively and

$\gamma_{str} =$ Constant.

The values of γ_{str} and C_{str} are calibrated for each material by means of experimental data. The degradation defined this way follows simple physical reasoning.

Figure (3) represents the degraded envelope and corresponding decrease in yield force due to strength degradation.

Unloading Stiffness Degradation

Unloading stiffness degradation refers to the decrease in unloading stiffness as a function of the loading history. The parameter β_{unl}^{i} used for unloading stiffness degradation is also energy dependent but differs from the one of the strength degradation in the values of C and γ. These are referred to as C_{unl} and γ_{unl}. The modified unloading stiffness can be calculated through the following equation:

$$k_{unl}^{i} = k_{unl}^{i-1}(1-\beta_{unl}^{i}) \qquad (5)$$

where $k_{unl}^{i} =$ Unloading stiffness at current excursion i.

Figure (4) represents the effect of unloading stiffness degradation on the hysteretic response.

Accelerated Stiffness Degradation

It was observed from experimental results that the reloading stiffness degrades as a function of cumulative loading in peak-oriented models. This effect can be taken into consideration in the analytical hysteretic model by modifying the target point to which the loading is directed, which is referred to as accelerated stiffness degradation. The accelerated stiffness degradation parameter β_{acc}^{i} is similar to the one used for strength and unloading stiffness degradation except that different values for C and γ are used, and are referred to as C_{acc} and γ_{acc}. The displacement value of the target point can be calculated through the following equation:

$$\delta_{tar}^{i} = \delta_{tar}^{i-1}(1+\beta_{acc}^{i}) \qquad (6)$$

where $\delta_{tar} =$ Displacement of the target point.

The effect of the accelerated stiffness degradation on the hysteretic behavior is represented in Figure (5).

Cap Degradation

From experimental results, it was also observed that the point of onset of softening moves inwards as a result of cumulative damage. This is referred to as cap degradation. The cap degradation parameter β_{cap}^{i} is similar to the one used for strength and stiffness degradation except that C_{cap} and γ_{cap} values are used. The point of onset of softening can be modified through the following equation:

$$\delta_{cap}^{i} = \delta_{cap}^{i-1}(1-\beta_{cap}^{i}) \qquad (7)$$

where $\delta_{cap} =$ Displacement of the point of onset of softening.

The modified envelope due to cap degradation is represented in Figure (6).

Collapse of Structural Elements

A structural element is assumed to have experienced complete collapse if any of the following two criteria is established:

(a) The displacement has exceeded the value of that of the intersection point of the softening (cap) slope with the residual strength line, which is referred to as cap failure, or

(b) The scalar parameter β has exceeded a value of 1, which is referred to as cyclic degradation failure.

Experimental Verification of Material Models

Several studies were performed in order to calibrate the degrading material models' parameters versus data obtained from experimental specimens. As explained earlier, each material model represents the characteristics of a specific material, steel or concrete. The goal of the calibration procedure is to define γ and C values that represent the behavior under cyclic loading. The coefficient γ consists of four sub-coefficients each describing a type of degradation. For simplicity, γ will be assumed to be equal for all four types of degradation (i.e. $\gamma_{str} = \gamma_{unl} = \gamma_{acc} = \gamma_{cap} = \gamma$). The same assumption was used for the parameter C. As an example, the modified Clough model was used to simulate the cyclic behavior of the reinforced concrete column tested by Lynn et al. (1996). The experimental and analytical cyclic load-deformation plots for the test specimen are shown in Figures (7a) and (7b) respectively. The degradation parameters γ, and C for all four types of degradation were selected to be equal to 50, and 1 respectively. These values were found to provide the better match with the experimental results. From the figures, it is rather obvious that the eight-parameter degrading material model successfully described the global behavior, and the decay in strength under large load reversals. A similar numerical study was performed on the steel specimen tested by Krawinkler and Zohrei (1983). The study showed that degradation parameters of γ = 100 and C = 1 proved to provide the best fit with the experimental results. Since the value of the parameter C equals to 1 for both materials, the rate of degradation is typically defined as a function of the parameter γ only.

Degradation Effect on SDOF Systems under Seismic Excitations

Figure (8) investigates the effect of degradation on SDOF systems. A bilinear system with a period $T = 0.294$ sec and a damping ratio $\zeta = 5\%$. The strain hardening ratio α equals 3%, and the strength reduction factor R of the system equals 4. The cap displacement is assumed to equal 4 times the yield displacement, and its slope is negative and equals 6% of the initial slope. The degradation parameters C and γ were assumed to equal 1 and 50 respectively for all degradation types, which corresponds to a severe degradation case. The Imperial Valley earthquake record recorded at station El Centro 1 was used in the analysis. Figure (8) shows the behavior of both a degraded and an equivalent non-degraded system. From the figure, it is observed that the non-degraded system doesn't experience collapse, while the degraded system experienced collapse after 8.6 sec, which is denoted by a '*' symbol in the plot. The force-displacement diagrams for both non-degraded and degraded cases are shown in Figures (9) and (10) respectively. The maximum displacement for the non-degraded system was 1.71 in., while the degraded system experienced collapse at 2.03 in. In this case, the behavior reached the cap in the first few cycles, and was eventually driven to collapse.

Earthquake Records

A large database set of earthquake records is used to derive the inelastic displacement ratios. The records were used in several earlier studies (e.g. Krawinkler et. al. 2000), and are documented in the report by Medina and Krawinkler (2003). The database consists of four bins representing different M (Moment Magnitude), and R (Shortest Distance from Fault) pairs as follows:

- Bin-I: small M-small R: $5.8 < M \leq 6.5$ and 13 km<R<30 km

- Bin-II: small M-large R: $5.8 < M \le 6.5$ and $30\ \text{km} \le R \le 60\ \text{km}$
- Bin-III: large M-small R: 6.5<M<7.0 and 13 km<R<30 km
- Bin-IV: large M-large R: $6.5 < M < 7.0$ and $30\ \text{km} \le R \le 60\ \text{km}$

Each bin constitutes of 20 earthquake records. The records were all recorded in California, and correspond to NEHRP soil type D (soft rock and stiff soil).

An earlier study by Shome et al. (1999) showed that scaling of earthquake records to a common spectral acceleration value does not introduce any bias to the response, and therefore reduces the necessity of the number of analysis needed for statistical evaluation. Furthermore, proper scaling ensures that all records used fall within the same hazard level defined by codes of practice. A new study by Ayoub and Chenouda (2006) investigated this approach for different degrading material models, and for different degrees of degradation. The conclusion was that the approach holds true for degrading systems in terms of both response measures and failure estimation. The prior scaling approach was therefore used in this study for all records in order to reduce the total number of analysis required for statistical evaluations, and to ensure that all records fall within the same hazard level.

Inelastic Displacement Ratios of Degrading Structures

The purpose of this study is to develop inelastic displacement ratios for degrading systems. A large set of structures is selected for the study. The periods of these structures range from 0.1 to 2.0 sec. Three values for the strength reduction factor (*R*) were also used in this study: 4, 6, and 8. This wide range of periods and strength reduction factors allows a thorough evaluation of the behavior of SDOF systems. The 4 bins of earthquake records recorded in California and described earlier, are used to conduct the numerical study. The material models used are the bilinear, and modified Clough models described earlier. The damping ratio ζ for all systems is assumed to equal 5% and the strain hardening ratio α to equal 3%. The cap displacement is assumed to equal 4 times the yield displacement, and its slope equals 6% of the initial slope. The residual strength is assumed to equal zero. Three different degradation cases are considered and compared to a corresponding non-degrading system. These cases represent low ($\gamma = 150$, C=1), moderate ($\gamma = 100$, C=1), and severe degradation ($\gamma = 50$, C=1) respectively for all degradation types. Plots of ratio of maximum inelastic displacements to maximum elastic displacements for different period values and for the different strength reduction factors R are generated for all degradation cases. The results for the case of Bins I-IV scaled to a common spectral acceleration according to USGS values LA 10/50 are shown in Figures (11) to (16). In these plots, the set of curves with low inelastic ratios represent median values, and the set of curves with high inelastic ratios represent 84th percentile values. The last point before collapse of the system is identified with a '*' in the plots, and no corresponding point for non-degraded systems exist. Median collapse is defined when more than 50% of the records failed.

The ratios of maximum inelastic to maximum elastic displacements for a strength reduction factor $R = 4$ are shown in Figures (11) and (12) for bilinear and modified Clough models respectively. The same set of plots is repeated for a strength reduction factor value of $R = 6$ in Figures (13) and (14), and for $R = 8$ in Figures (15) and (16). Several conclusions can be extracted from those graphs to better understand the effect of the different variables on the ratio of maximum inelastic to maximum elastic displacements.

From all figures, it is clear that degradation did not affect the behavior of long period structures. Furthermore, it was observed that, in this period range, the equal displacement rule still applies even for degraded systems. The effect of degradation becomes apparent for short period structures ($T < 0.5$ sec). In this range, degradation increases the maximum inelastic displacements for both material models. This conclusion applies as well for the different strength reduction factors. For very short periods ($T < 0.2$ sec), degraded system typically collapse at any level of degradation. The difference between median and 84th percentile values on the behavior and collapse potential is also evident. For example, when examining Figure (11) for a period $T = 0.4$ sec, it is observed that severely degraded bilinear systems with R=4 collapse only when considering 84th percentile values, but not when considering median values. This finding is justified by the fact that the 84th percentile values are more stringent than the median values. Higher values of strength reduction factors also influence the collapse criteria. For $R = 4$, collapse for a moderate degradation case for a modified Clough model occurs at $T = 0.2$ sec while it occurs at T = 0.3 sec for $R = 6$ and $T = 0.4$ sec for $R = 8$. This is due to the fact that increasing the R value results in a weaker system which consequently escalates the collapse probability.

From Figures (11) and (12), it was observed that the median ratio of maximum inelastic to elastic displacement for severely degraded systems for a case with strength reduction factor $R = 4$ and period $T = 0.3$ sec equals to 1.48 and 2.04 for bilinear, and modified Clough models respectively. For $R = 6$ and $T = 0.5$ sec in Figures (13) and (14), this ratio equals 1.21 and 1.50 for bilinear, and modified Clough models. Similarly, at $R = 8$ and $T = 0.8$ sec, the ratio in Figures (15) and (16) equals to 0.97 and 1.01. From this discussion, it is observed that the ratio for bilinear models tends to be lower than its corresponding value for modified Clough models. This observation is mainly due to the fact that the behavior of peak-oriented models is dominated by accelerated degradation which increases the inelastic displacements.

The difference in material models characteristics is also noticed when examining collapse of severely degraded systems for the different cases of strength reduction factor. For bilinear models in Figure (11), collapse occurs at $T = 0.3$ sec for $R = 4$. For the same conditions but for $R = 6$, collapse takes place at $T = 0.5$ sec as shown in Figure (13) with a 66% increase in the period value. This value equals to 0.8 sec in Figure (15) when R reaches a value of 8 denoting a 60% increase from the previous value. For modified Clough models in Figures (12), (14) and (16) collapse occurs at $T = 0.2$, 0.3 and 0.4 sec for $R = 4$, 6 and 8 respectively with 50% and 33% increase. These results imply that bilinear models are more susceptible to collapse than peak-oriented models. This observation is justified by the fact that the hysteretic energy dissipation of bilinear models is typically higher than that of modified Clough models.

The preceding discussions confirm the fact that degradation has a major effect on the inelastic behavior of structures, particularly those in the short period range, and on their potential for collapse.

Proposed Equations for Evaluation of Inelastic Ratios of Degrading Systems

The preceding results were used to develop approximate equations for the evaluation of median inelastic displacement ratios of degrading structural systems. Three equations are proposed for both bilinear and modified Clough models. The first equation is a modification to the expression originally proposed by Krawinkler and Nassar (1992) as follow:

$$c = \frac{T^a}{1+T^a} + \frac{b}{T} \qquad (8)$$

$$\frac{\delta_{inelastic}}{\delta_{elastic}} = \frac{1}{R}\left[1 + \frac{R^c - 1}{c}\right] \qquad (9)$$

Where the constant values of the coefficients *a* and *b* depend on the strain hardening ratio α, *T* is the fundamental period of the structure, and *R* is the strength reduction factor. In this work, the values of the coefficients *a* and *b* were recalibrated using a least square fit procedure for the degrading systems considered for a value of α=3%. The proposed new values are as follow:

a= 0.6, and $b = 0.32 - \frac{R}{100} + \frac{0.026R^2}{\sqrt{\gamma}}$ for bilinear systems (10)

and a= 0.7, and $b = 0.39 - \frac{R}{50} + \frac{0.033R^2}{\sqrt{\gamma}}$ for modified Clough systems (11)

where γ is the degradation parameter defined in (4).

The proposed new expressions of the coefficient *b* recognize the fact that degradation, represented by the parameter γ, has a greater effect on the displacements of systems with higher values of *R*. The preceding proposed equations are only valid for systems with period values higher than the collapse period, defined as the period less than which structures are expected to collapse. The values of the collapse periods for the different systems considered are shown in Table (1a) and (1b) for bilinear and modified Clough systems respectively.

The second equation used to estimate the inelastic ratios of degrading systems is based on the expression proposed by Chopra and Chintanapakdee (2004) as follow:

$$L_R = \frac{1}{R}\left(\frac{R-1}{\alpha}+1\right) \tag{12}$$

$$\frac{\delta_{inelastic}}{\delta_{elastic}} = 1+\left[\left(L_R-1\right)^{-1}+\left(\frac{a}{R^b}+c\right)\left(\frac{T}{T_c}\right)^d\right]^{-1} \tag{13}$$

Where T_c is the period at the start of the acceleration sensitive region of the response spectrum, and is assumed to equal 0.41s for NEHRP soil type D. Using nonlinear regression analysis of response data, but ignoring data with inelastic ratios smaller than 1, the following coefficients were proposed by the authors: a=61, b=2.4, c=1.5, and d=2.4. Since the previous equation ignores data with inelastic ratios smaller than one, it typically provides values larger than the exact earthquake response data, and is therefore considered a conservative approach for estimation of maximum inelastic displacements that could be used for design purposes. Surprisingly, the equation also provided conservative values for degrading bilinear systems with fundamental periods larger than the collapse period. For modified Clough systems though, the parameter c needed to be recalibrated, and a value of c=0.5 was found to provide conservative estimates for inelastic displacements.

The third equation used to estimate the inelastic ratios of degrading systems is based on the expression proposed by Ruiz-Garcia and Miranda (2003) as follow:

$$\frac{\delta_{inelastic}}{\delta_{elastic}} = 1+\left[\frac{1}{a(T/T_s)^b}+\frac{1}{c}\right]\left(R-1\right) \tag{14}$$

Where T_s is assumed to equal 1.05 for NEHRP site class D, a=50, b=1.8, and c=55.

The preceding equation was derived for elastic-perfectly plastic bilinear systems only. Since the displacements of elastic-hardening systems are typically smaller by only a small amount than those of elastic-perfectly plastic systems, the equation can be used to provide conservative estimates for the formers. It was also proved that the equation provides conservative estimates for degrading bilinear systems as well. For peak-oriented modified Clough systems though, the coefficient b had to be recalibrated, and a value of b=2.2 was found to provide conservative estimates for systems with periods larger than the collapse period.

The proposed three equations were used in a comparative study for the following systems: a system with R=4 and γ=150, a system with R=6 and γ=50, and a system with R=8 and γ=100. Figure (17) shows the results for a bilinear system with R=4 and γ=150. The equation of Krawinkler-Nassar with the newly proposed b expression seems to provide accurate estimates for the inelastic displacement ratios. The expressions by both Chopra-Chintanapakdee and Ruiz Garcia-Miranda both provided conservative estimates for the inelastic ratios with the former providing smaller values for long period systems, while the latter providing smaller values for short period systems. Figure (18) shows the same results for a bilinear system with R=6 and γ=50. The same conclusion held true except that the Ruiz Garcia-Miranda expression provided much more conservative values than the others. Figure (19) shows the results for a bilinear system with R=8 and γ=100. The same conclusion was also observed except that the Chopra-Chintanapakdee expression provided slightly un-conservative values for periods less than 0.5 sec. Figures (20-22) show the same results but for a modified Clough system. The equation based on the modified expression by Krawinkler-Nassar in general provided reasonably accurate results, although the error was slightly higher than for bilinear systems, but did not exceed 15% in most cases. Since this expression is based on regression analysis conducted on the ratio of inelastic to elastic displacements, it is considered a direct method following the description of Miranda (2001). The expressions for Chopra-Chintanapakdee and Ruiz Garcia-Miranda with adjusted coefficients were able to provided conservative estimates.

Summary and Conclusions

The study presents a new model that incorporates degradation effects into seismic analysis of structures. An energy-based approach is adopted to define several types of degradation effects, and to predict collapse under seismic excitations. The model was calibrated versus experimental results, and was used to conduct extensive statistical analysis of different structural systems under earthquake excitations. The results were used to propose approximate methods for estimating maximum inelastic displacements of degrading systems for use in performance-based seismic code provisions. Three different methods were proposed and were evaluated for a series of degrading systems. The study resulted in the following conclusions:

- For SDOF systems, degradation had a great effect on the inelastic displacement ratios, especially for short period structures where the inelastic displacements were quite larger than the corresponding displacements of non-degraded systems. For very short period structures, collapse is typically observed even for systems with low strength reduction factors. For long period structures, the well-known equal displacement rule is preserved even for degrading systems. In this case, collapse is not expected even for systems with large strength reduction factors.

- The effect of degradation on the maximum inelastic displacements is lower for bilinear models than for modified Clough models. This is due to the fact that the behavior of peak-oriented models is dominated by accelerated degradation which strongly increases the inelastic displacements.
- For short period structures, bilinear models have a faster collapse rate than peak-oriented models. This is due to the fact that bilinear models dissipate the largest hysteretic energy and hence reach their capacity earlier. The strength reduction factor *R* also has a great influence on the collapse potential of these structures.
- Three methods were proposed to estimate median inelastic displacement ratios of degrading systems. The expression by Krawinkler and Nassar (1992) was modified to account for degradation. The expression results in general in accurate estimates, although an error of up to 15% was observed for a few modified Clough degrading systems. The expressions by Chopra and Chintanapakdee (2004) and Ruiz-Garcia and Miranda (2003) in general provide conservative estimates for inelastic ratios, and can be therefore used for design purposes. New coefficients for both expressions were developed for degrading modified Clough models.

Acknowledgement

The second author would like to express his deepest gratitude to Prof. Helmut Krawinkler, his post-doctoral advisor at Stanford University, for several fruitful discussions regarding the seismic behavior and analytical implementation of degrading structural systems, which constituted the basis of this work.

This material is based upon work supported by the National Science Foundation under Grant No. 0448590. The authors greatly acknowledge the support of NSF.

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Notation

δ_t : Target roof displacement

S_d : Elastic spectral displacement

C_0 : Modification factor that equals the first mode participation factor at the roof

C_1 : Factor that equals the ratio of maximum inelastic to maximum elastic displacements

C_2 : Factor that accounts for strength degradation

C_3 : Factor that accounts for dynamic second-order effects

F_y : Yield strength

β_{str} : Scalar that accounts for strength degradation

E : Hysteretic dissipated energy

$E_{capacity}$: Energy dissipation capacity

C_{str} : Factor that defines rate of strength degradation

k_{unl} : Unloading stiffness

C_{unl} : Factor that defines rate of unloading stiffness degradation

δ_{tar} : Displacement of target point for peak-oriented models

C_{acc} : Factor that defines rate of accelerated stiffness degradation

δ_{cap} : Displacement of the onset point of softening

C_{cap} : Factor that defines rate of cap degradation

$\gamma_{str}, \gamma_{unl}, \gamma_{acc}, \gamma_{cap}$: Constants to calibrate strength, unloading stiffness, accelerated stiffness, and cap degradation effects respectively.

T_c is the period at the start of the acceleration sensitive region of the response spectrum.

Figure Captions

Fig. 1 Bilinear Model

Fig. 2 Modified-Clough Model

Fig. 3 Strength Degradation for Modified Clough Model

Fig. 4 Unloading Stiffness Degradation for Modified Clough Model

Fig. 5 Accelerated Stiffness Degradation for Modified Clough Model

Fig. 6 Cap Degradation for Modified Clough Model

Fig. 7a Experimental Behavior of Lynn Reinforced Concrete Specimen

Fig. 7b Analytical Behavior of Lynn Reinforced Concrete Specimen

Fig. 8 Time History Response for Roof Displacement; T=0.294sec, ζ=5%, α=3%, Cap-Slope=-6%

Fig. 9 Force-Displacement Behavior, No Degradation; T=0.294sec, ζ=5%, α=3%, Cap-Slope=-6%

Fig. 10 Force-Displacement Behavior, Severe Degradation; T=0.294sec, ζ=5%, α=3%, Cap-Slope=-6%

Fig. 11 Inelastic Displacement Ratio, Bilinear Model-R=4

Fig. 12 Inelastic Displacement Ratio, Modified Clough Model-R=4

Fig. 13 Inelastic Displacement Ratio, Bilinear Model-R=6

Fig. 14 Inelastic Displacement Ratio, Modified Clough Model-R=6

Fig. 15 Inelastic Displacement Ratio, Bilinear Model-R=8

Fig. 16 Inelastic Displacement Ratio, Modified Clough Model-R=8

Fig. 17 Inelastic Displacement Ratio, Bilinear Model-R=4, Low Degradation

Fig. 18 Inelastic Displacement Ratio, Bilinear Model-R=6, Severe Degradation

Table 1a. Median Collapse Period for Bilinear Systems

	Low Degradation	Moderate Degradation	Severe Degradation
R=4	0.2	0.2	0.3
R=6	0.3	0.3	0.5
R=8	0.4	0.4	0.8

Table 1b. Median Collapse Period for Modified Clough Systems

	Low Degradation	Moderate Degradation	Severe Degradation
R=4	0.2	0.2	0.2
R=6	0.2	0.3	0.3
R=8	0.3	0.4	0.4

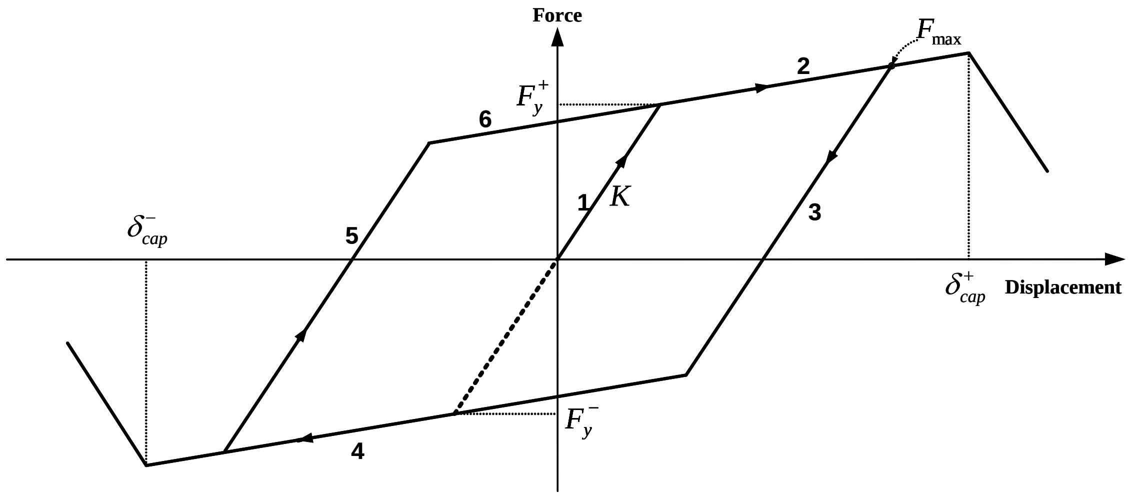

Fig. 1 Bilinear Model

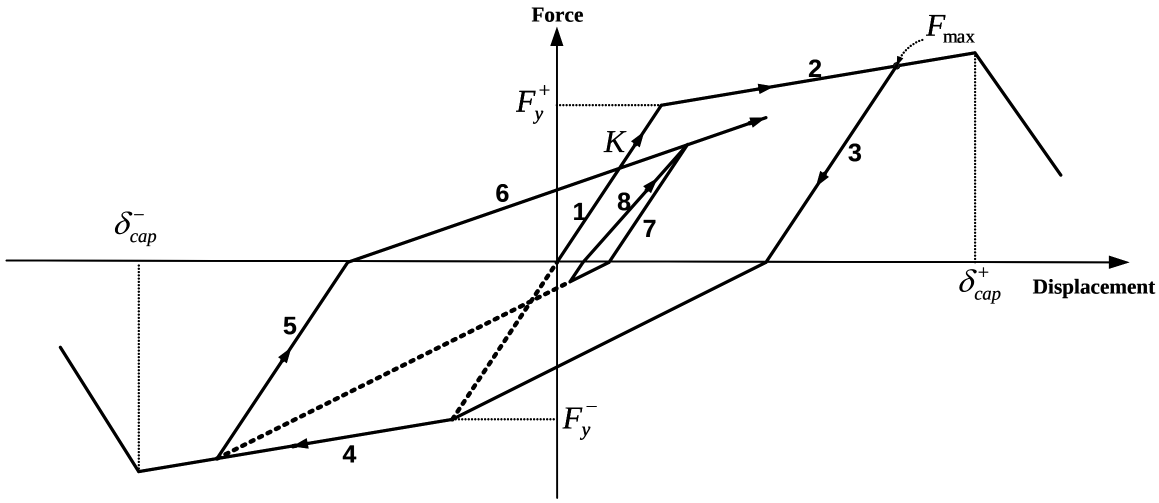

Fig. 2 Modified-Clough Model

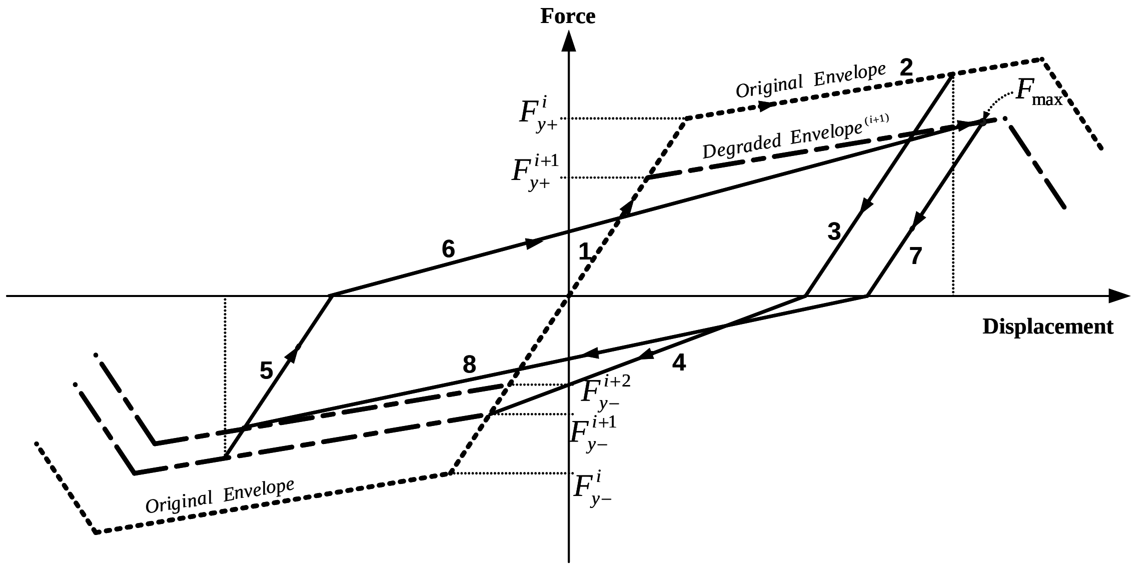

Fig. 3 Strength Degradation for Modified Clough Model

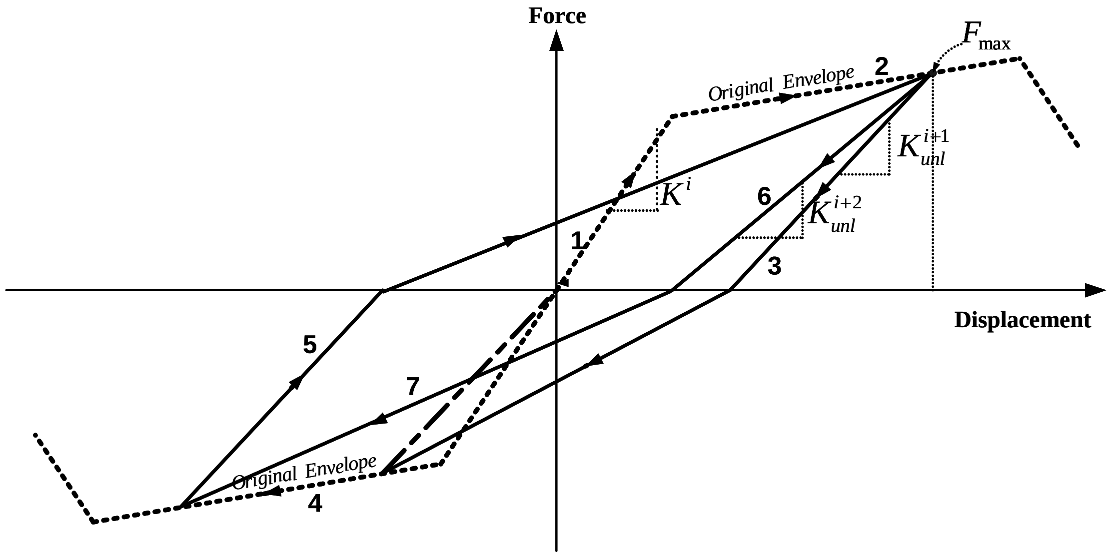

Fig. 4 Unloading Stiffness Degradation for Modified Clough Model

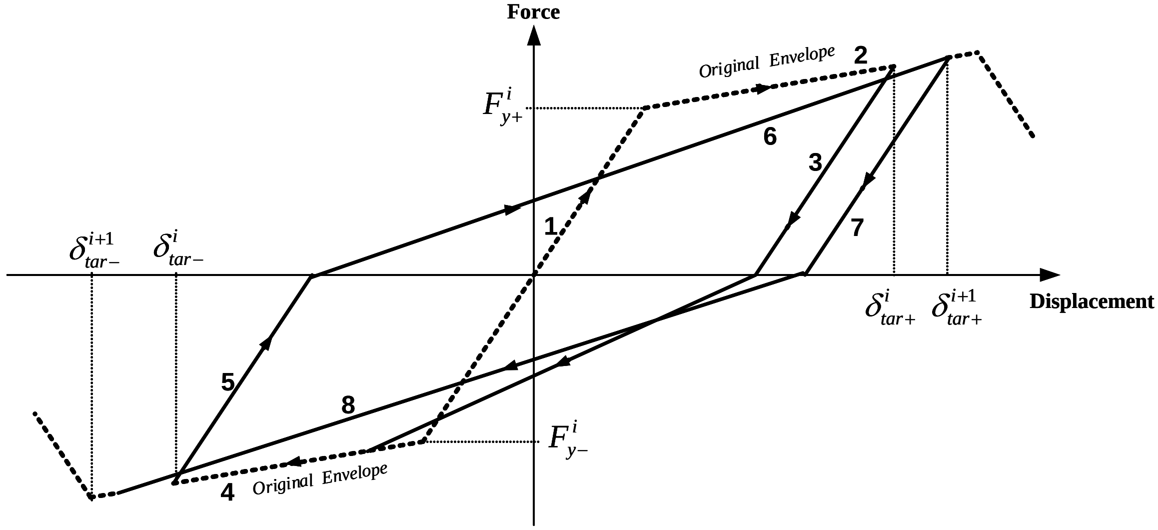

Fig. 5 Accelerated Stiffness Degradation for Modified Clough Model

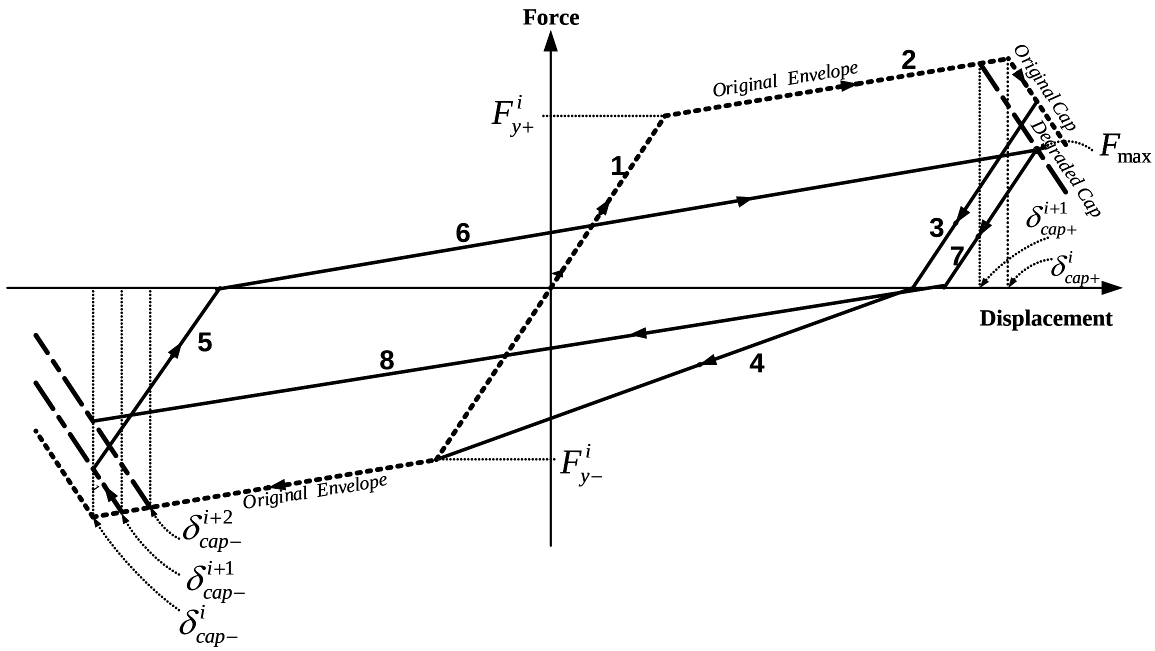

Fig. 6 Cap Degradation for Modified Clough Model

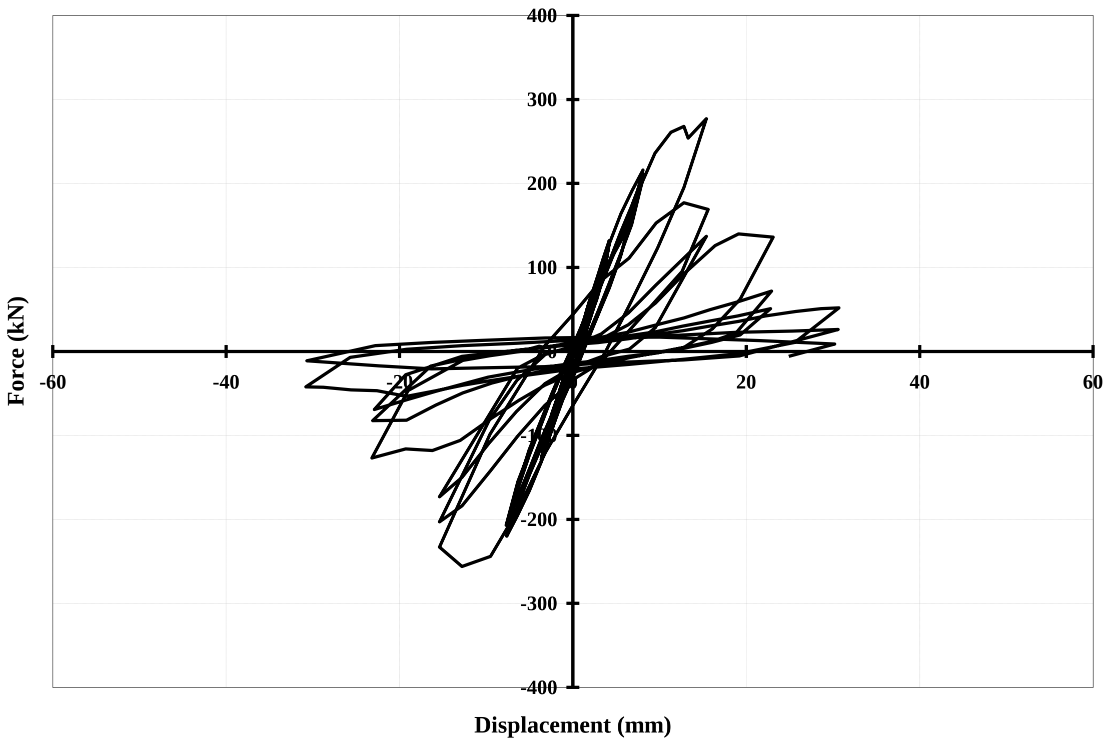

Fig. 7-a Experimental Behavior of Lynn Reinforced Concrete Specimen

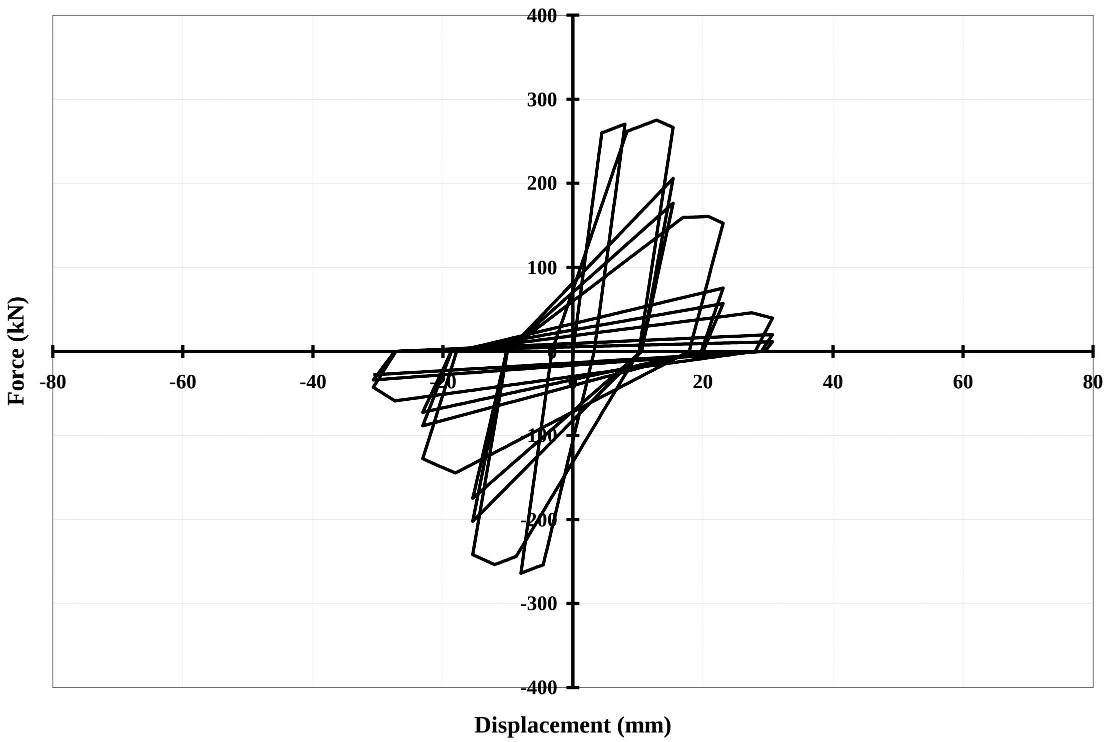

Fig. 7-b Analytical Behavior of Lynn Reinforced Concrete Specimen

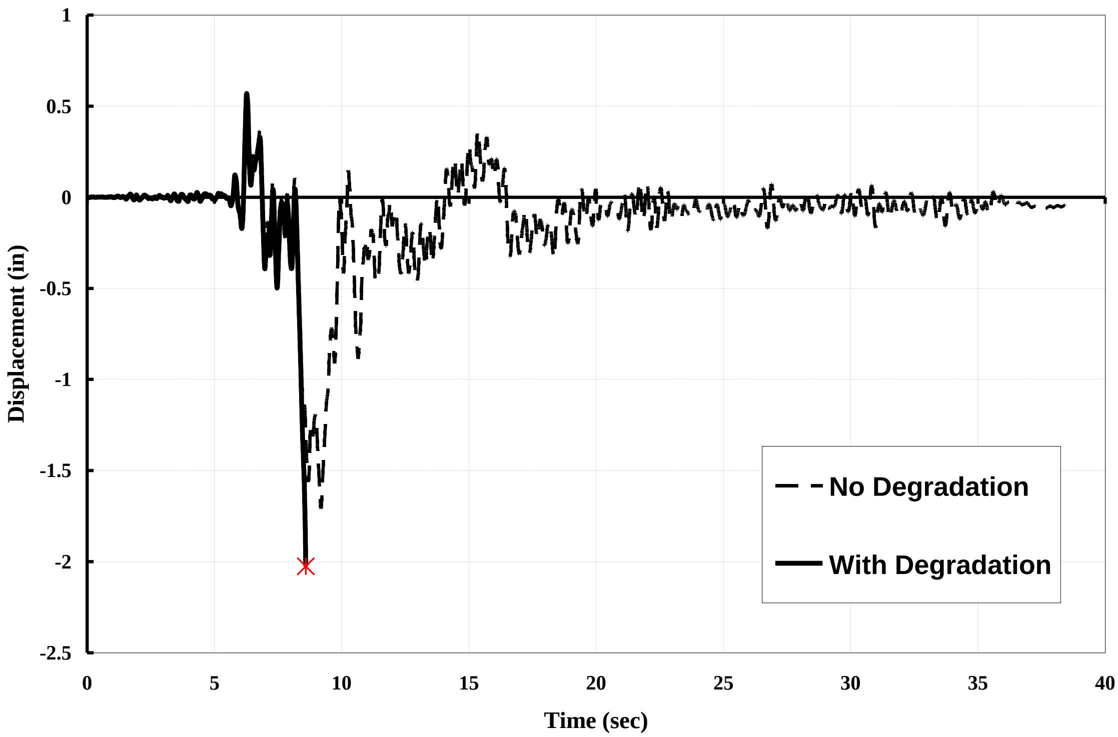

Fig. 8 Time History Response for Roof Displacement
T=0.294sec, ζ=5%, α=3%, Cap-Slope=-6%

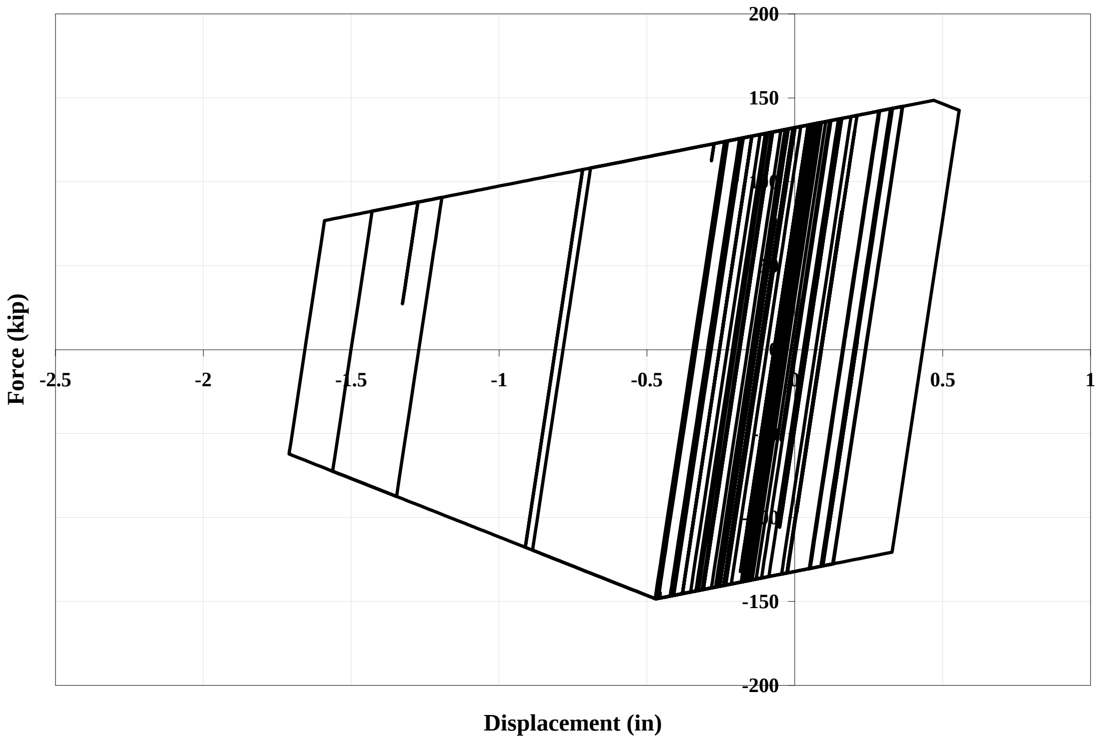

Fig. 9 Force-Displacement Behavior, No Degradation
T=0.294sec, ζ=5%, α=3%, Cap-Slope=-6%

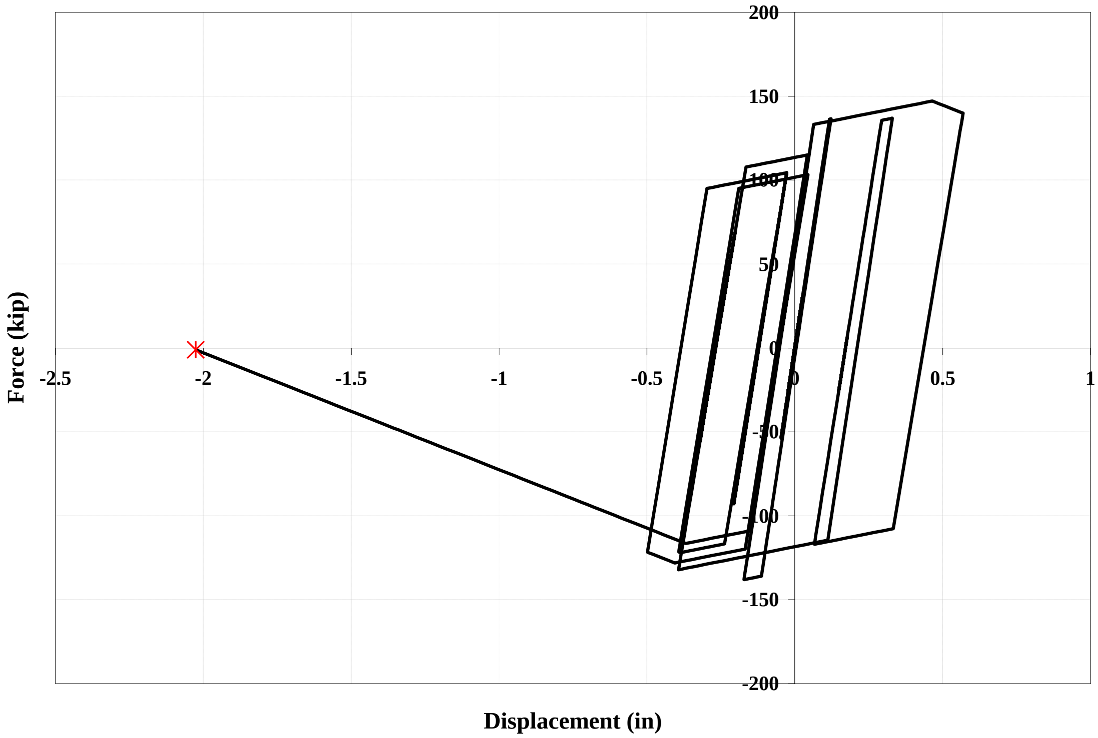

Fig. 10 Force-Displacement Behavior, Severe Degradation
T=0.294sec, ζ=5%, α=3%, Cap-Slope=-6%

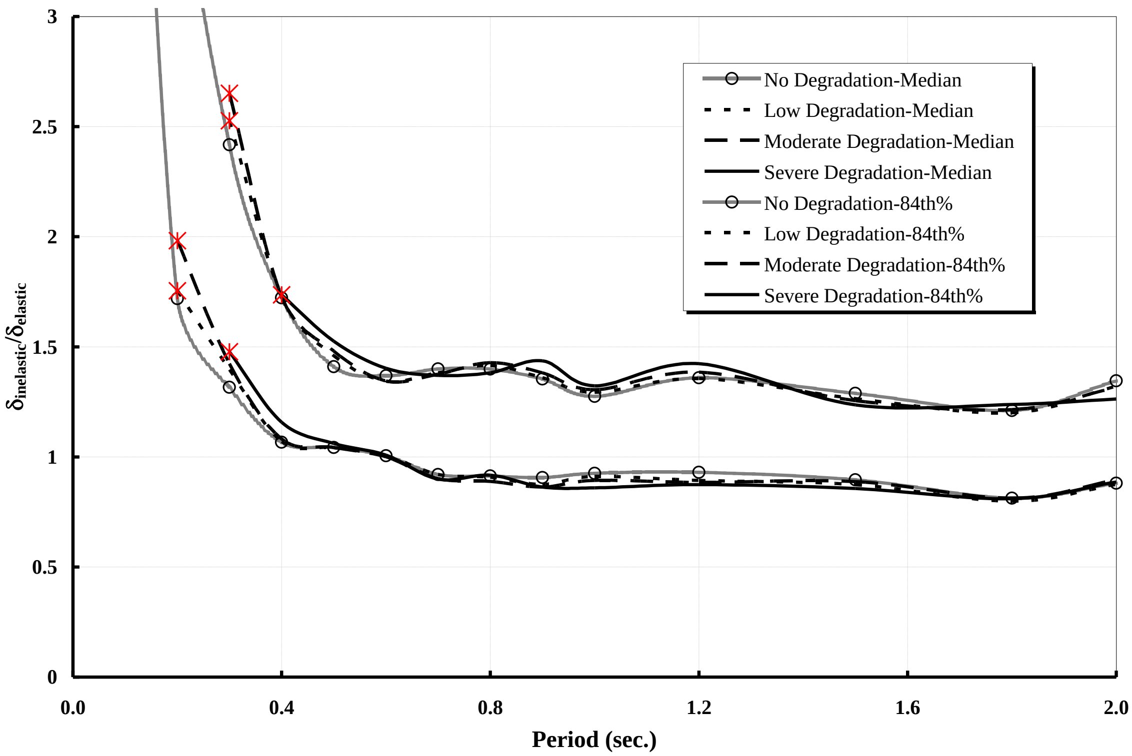

Fig. 11 Inelastic Displacement Ratio, Bilinear Model-R=4

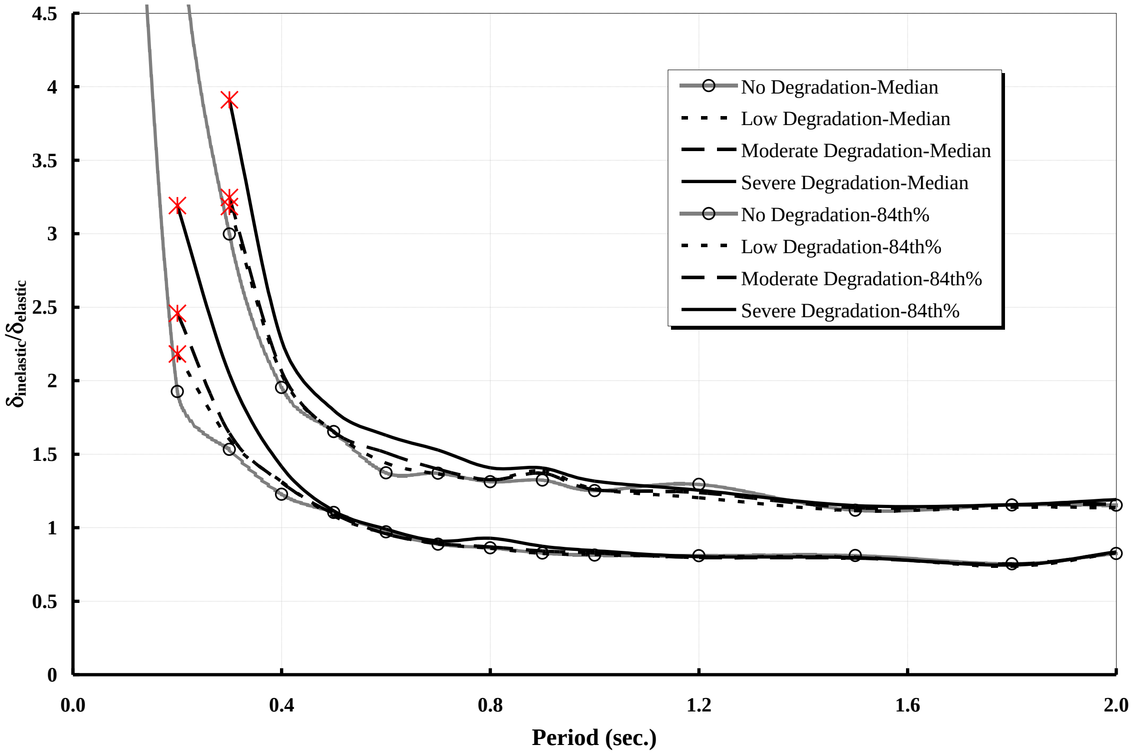

Fig. 12 Inelastic Displacement Ratio, Modified Clough Model-R=4

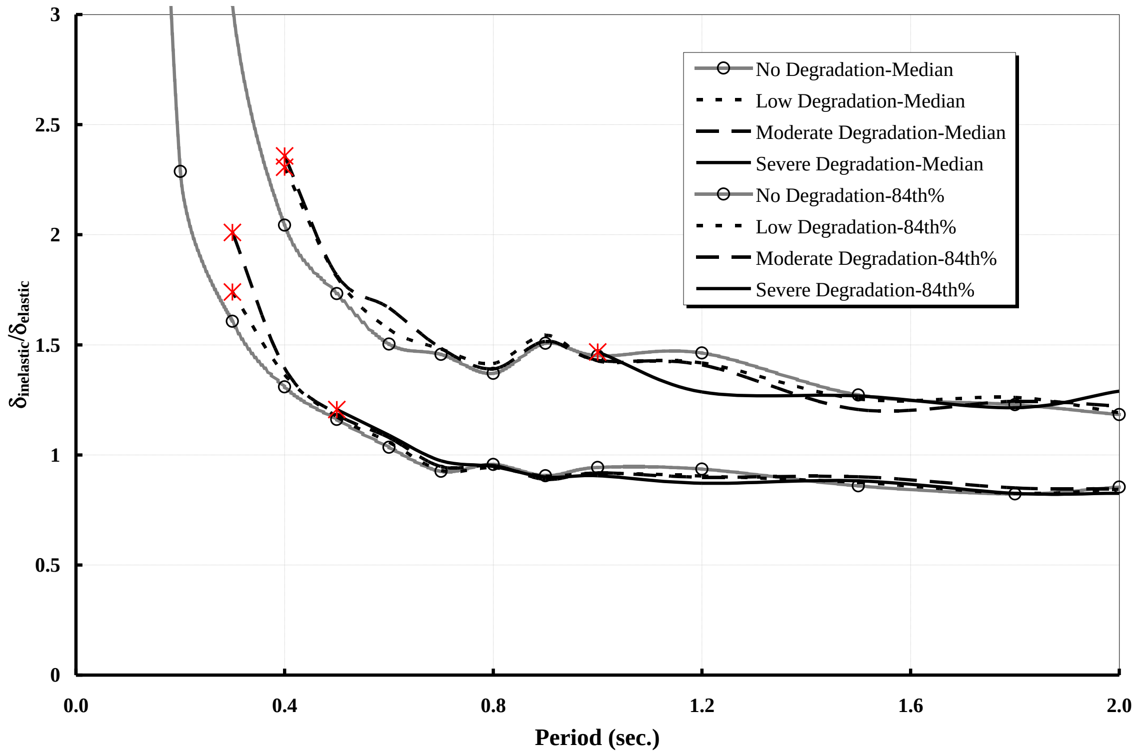

Fig. 13 Inelastic Displacement Ratio, Bilinear Model-R=6

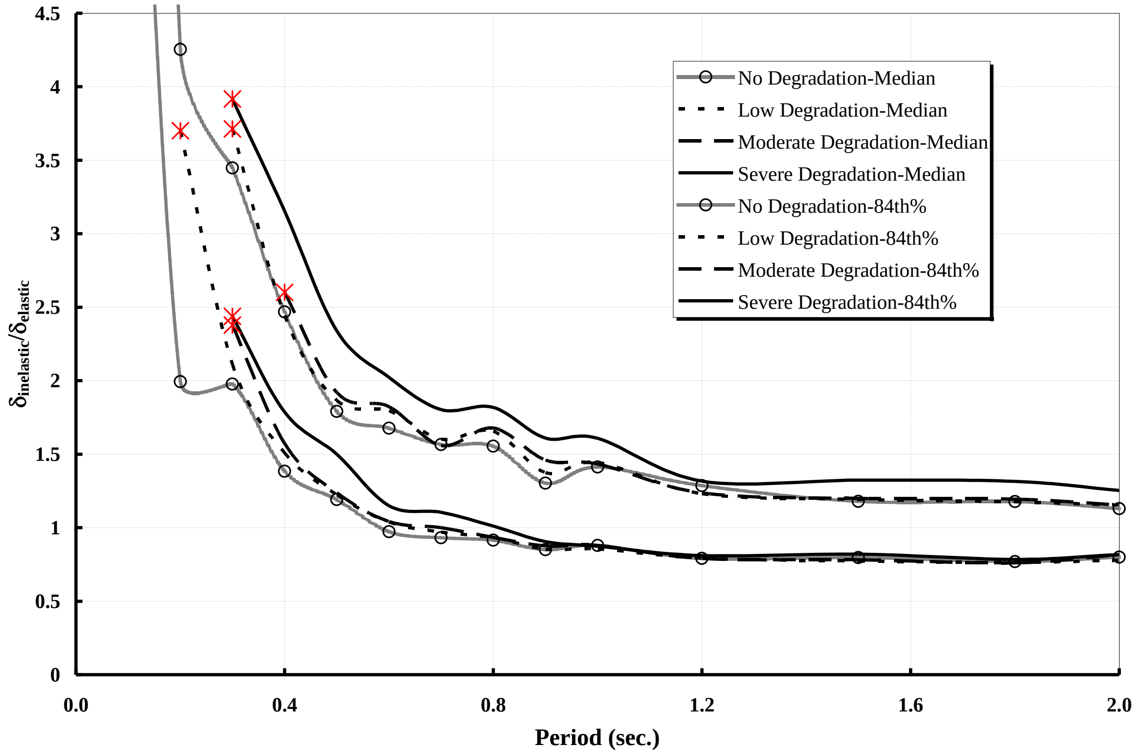

Fig. 14 Inelastic Displacement Ratio, Modified Clough Model-R=6

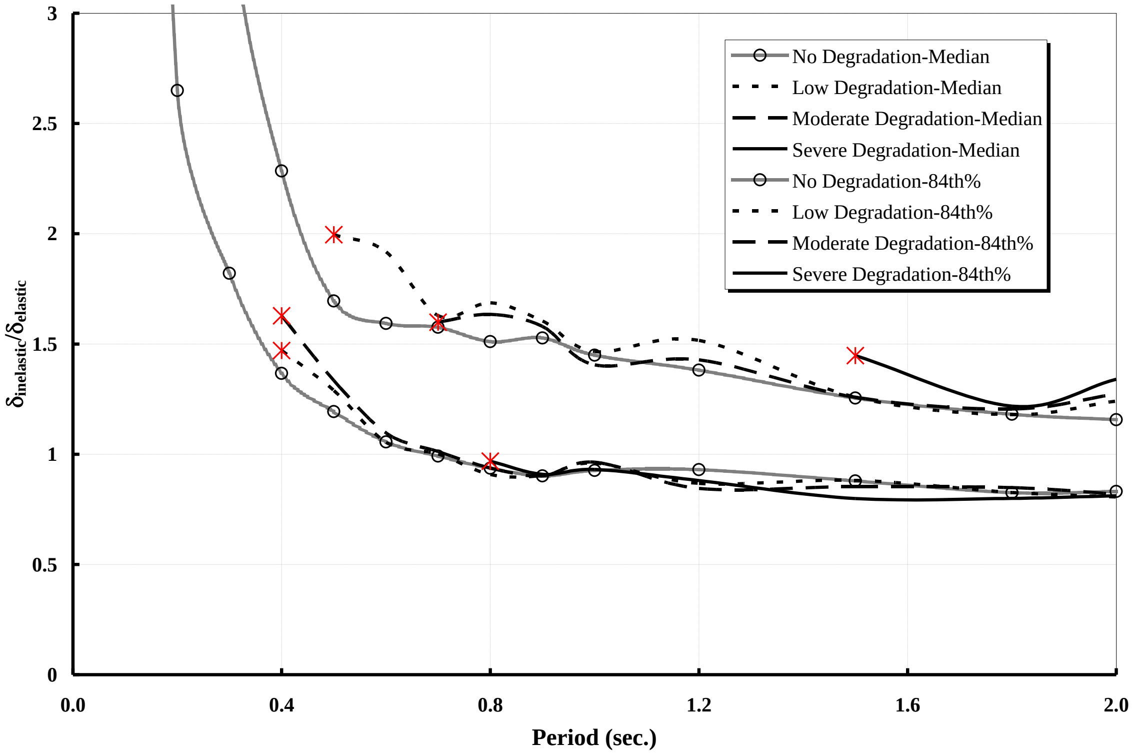

Fig. 15 Inelastic Displacement Ratio, Bilinear Model-R=8

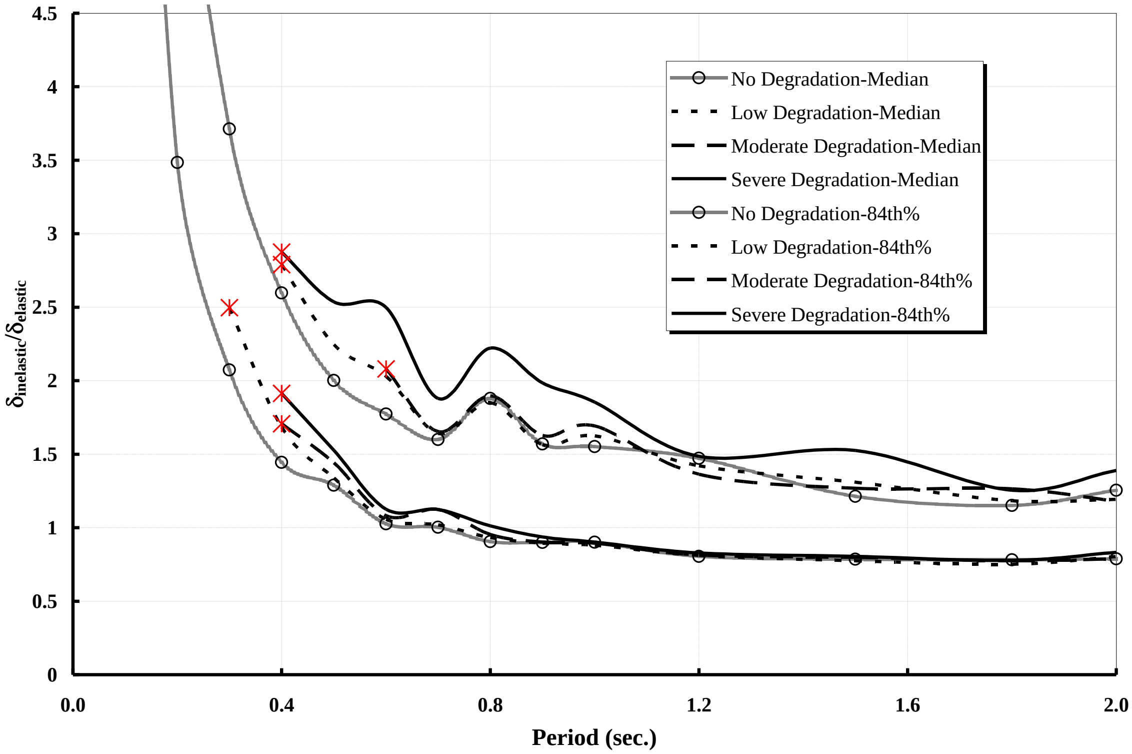

Fig. 16 Inelastic Displacement Ratio, Modified Clough Model-R=8

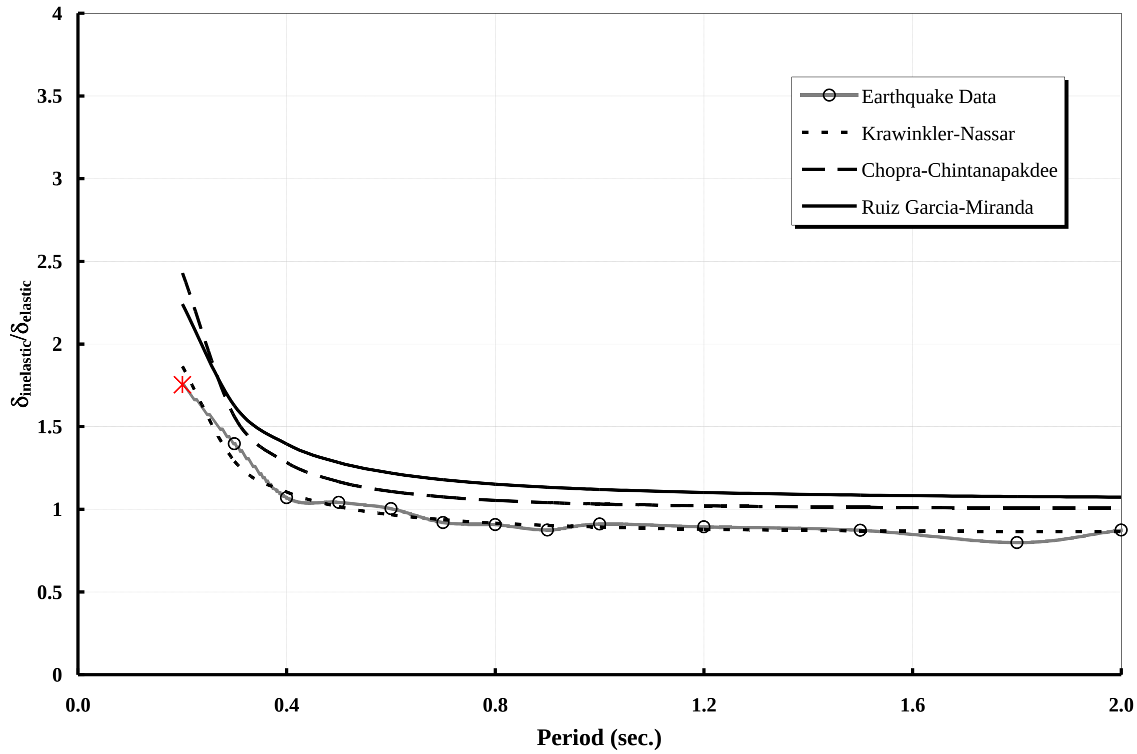

Fig. 17 Inelastic Displacement Ratio, Bilinear Model-R=4, Low Degradation

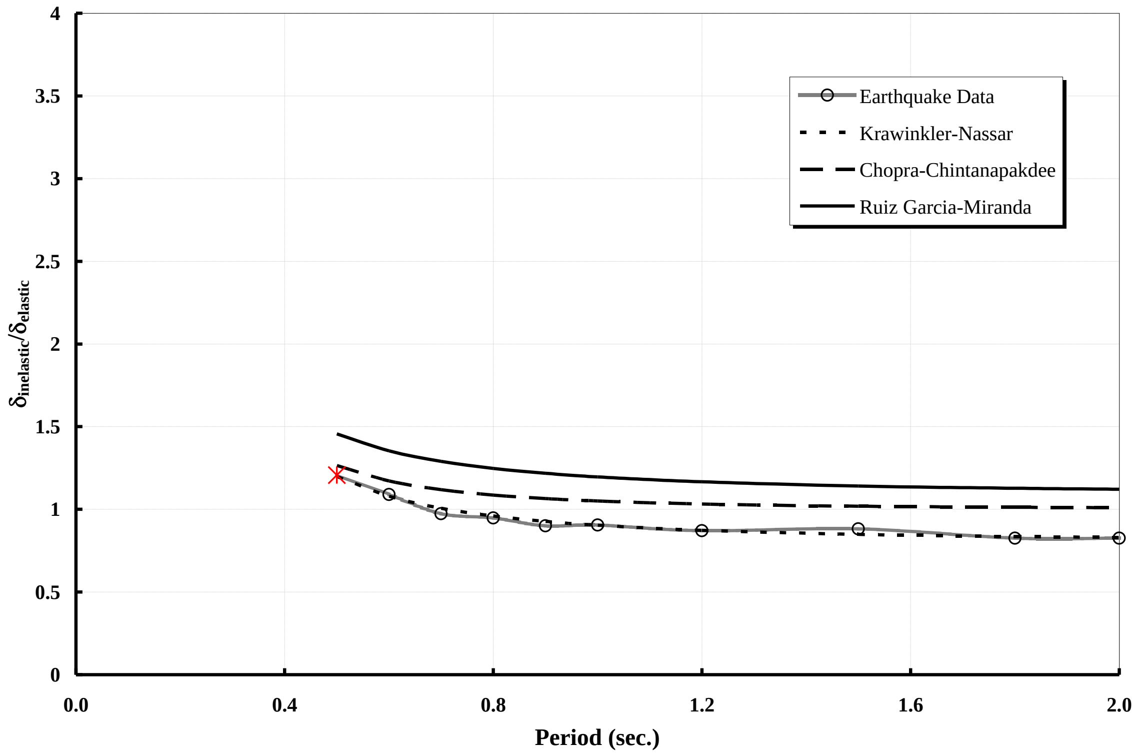

Fig. 18 Inelastic Displacement Ratio, Bilinear Model-R=6, Severe Degradation

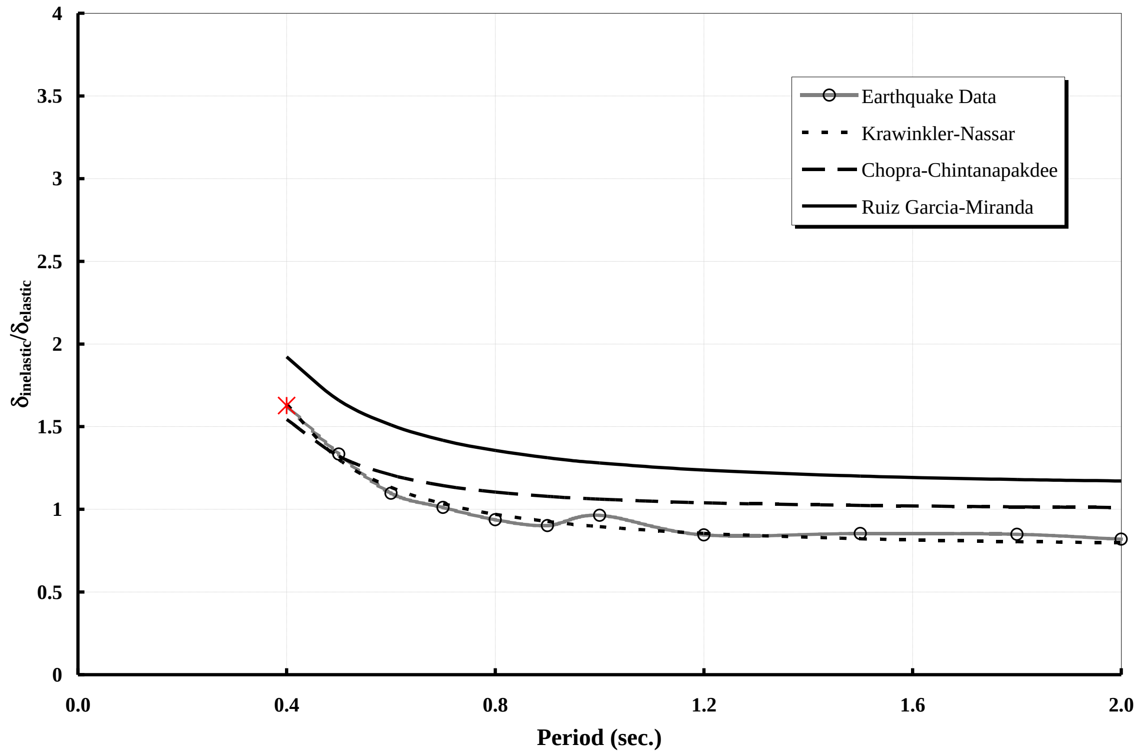

Fig. 19 Inelastic Displacement Ratio, Bilinear Model-R=8, Moderate Degradation

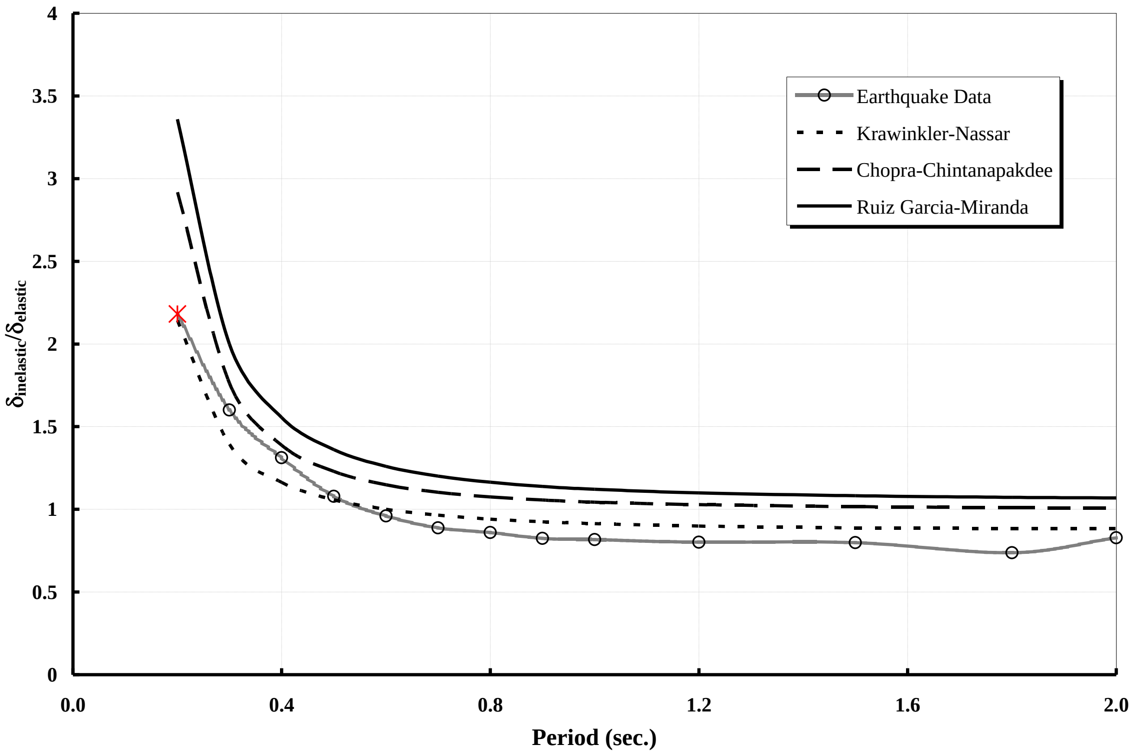

Fig. 20 Inelastic Displacement Ratio, Modified Clough Model-R=4,Low Degradation

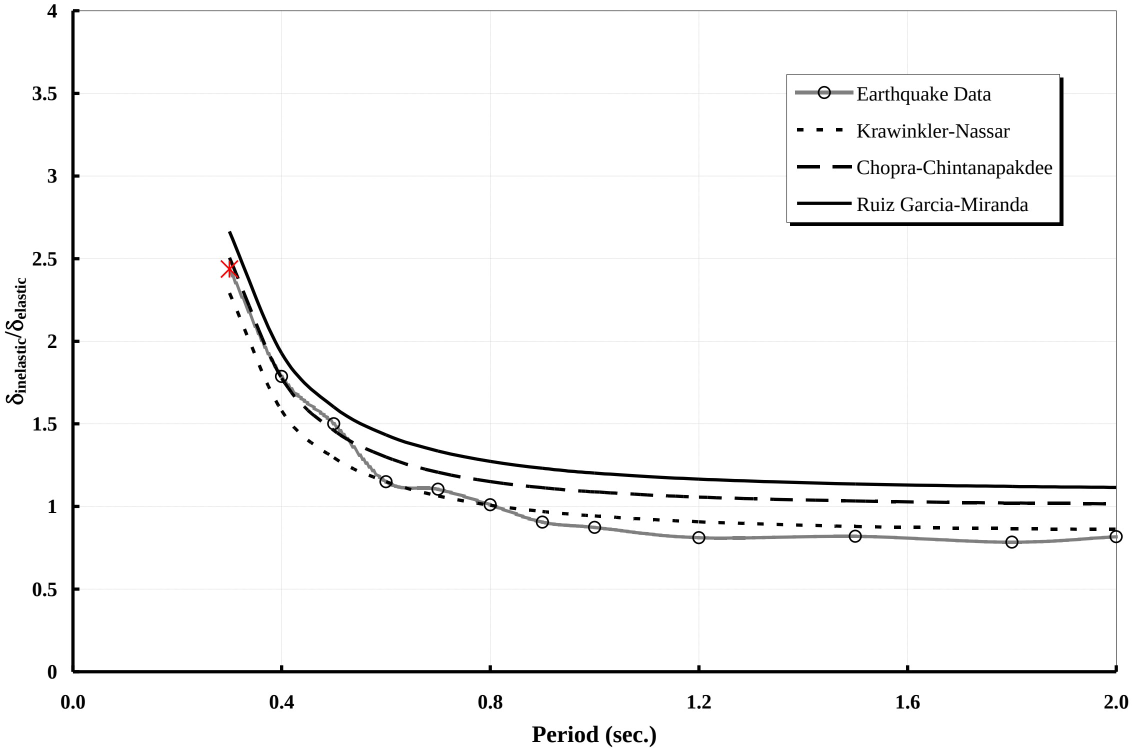

Fig. 21 Inelastic Displacement Ratio, Modified Clough Model-R=6, Severe Degradation

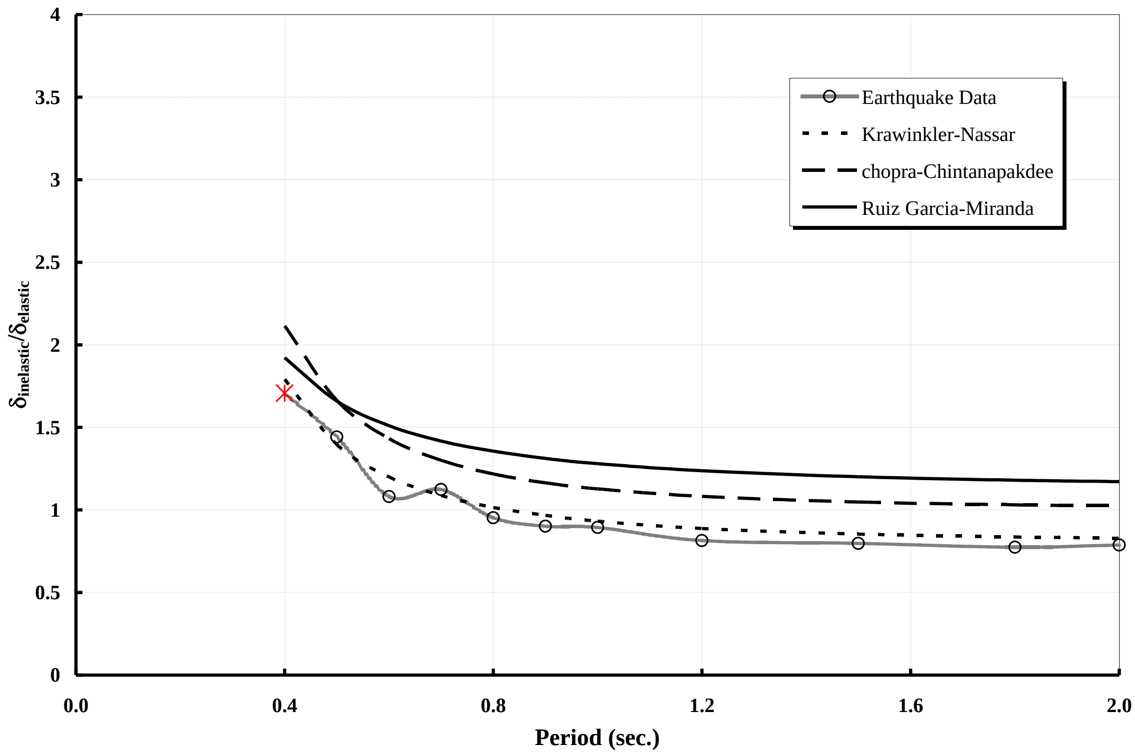

Fig. 22 Inelastic Displacement Ratio, Modified Clough Model-R=8, Moderate Degradation