



City Research Online

## City, University of London Institutional Repository

---

**Citation:** Asenov, M., Mojsilović, N. & Micic, T. (2016). Reliability of masonry walls subjected to in-plane loading: A slip failure along head joints / Zuverlässigkeit von Mauerwerkswänden bei Belastung in der Ebene: Versagen entlang der Stoßfugenflucht. *Mauerwerk*, 20(4), pp. 271-283. doi: 10.1002/dama.201600700

This is the accepted version of the paper.

This version of the publication may differ from the final published version.

---

**Permanent repository link:** <https://openaccess.city.ac.uk/id/eprint/15706/>

**Link to published version:** <https://doi.org/10.1002/dama.201600700>

**Copyright:** City Research Online aims to make research outputs of City, University of London available to a wider audience. Copyright and Moral Rights remain with the author(s) and/or copyright holders. URLs from City Research Online may be freely distributed and linked to.

**Reuse:** Copies of full items can be used for personal research or study, educational, or not-for-profit purposes without prior permission or charge. Provided that the authors, title and full bibliographic details are credited, a hyperlink and/or URL is given for the original metadata page and the content is not changed in any way.

---

City Research Online:

<http://openaccess.city.ac.uk/>

[publications@city.ac.uk](mailto:publications@city.ac.uk)

---

## **Reliability of masonry walls subjected to in-plane loading: A slip failure along head joints**

Masonry structures represent a sustainable, economical and traditionally widely used type of construction. Surprisingly, there is a limited consensus among experts and as a result current masonry design codes are rather conservative. Hence, there is a growing need for the revision i.e. calibration of safety factors to improve material resources allocation. For such calibration, it is of great importance to develop a reliable and comprehensive approach to assess masonry reliability and ensure rational masonry design. In this paper, we investigate the probability of occurrence of slip failure along head joints (predominantly overlooked in analysis) in masonry subjected to in-plane loading using failure criterion based on Mohr-Coulomb law. Thus, appropriate limit state function is established and masonry material properties and loads are defined as random variables in order to simulate likelihood of the occurrence of head joints slip failure regime as relevant to structural masonry design. To illustrate the approach used in this research, an example of masonry wall with probabilistic analysis outcomes using Monte Carlo simulation is presented and recommendations for further work are provided.

### **1 Introduction**

Masonry structures are widely used type of construction that are mostly subjected to in-plane loads. By definition, masonry consists of units and mortar joints that may cause highly anisotropic and nonlinear behaviours. Furthermore, these structures can experience different modes of failure i.e. flexural, shear failure etc. The failure modes mostly depend on masonry geometry, material properties, applied loads, as well as on workmanship and the way masonry walls are constructed on site. For instance, in some countries, e.g. Switzerland, Germany, structural masonry walls are built with unfilled head joints that may lead to an increasing probability of occurrence of slip failure along head joints. However, such failures were observed in masonry with filled head joints, too. In this regard, masonry modelling can be challenging and complex. According to author's knowledge there is little published research material on head mortar joints impact on masonry failure modes and strengths, [1-7]. Mortar joints are the most vulnerable parts of masonry walls and joint slip failures tend to occur along the interface between mortar joints and units rather than through mortar joints, [8]. Series of clay block masonry elements with unfilled head joints were tested in [1] and a considerable reduction of masonry compressive strength has been observed. Comparative experimental and numerical analyses of grouted and un-grouted concrete masonry walls have shown that filled head joints have significant contribution to masonry shear strength and deformation capacity, see [2]. In-plane strength of masonry can be investigated applying the theory of the single plane of weakness [3-5]. It was also shown, using diagonal tensile test [2] that un-grouted concrete masonry assemblages experience failure mode characterised by step-wise crack at the block mortar interfaces. On the other hand, filled grouts tend to enhance masonry wall and to force crack to the units. Clay brick masonry walls subjected to in-plane loading that have been constructed with different quality of filled and unfilled head mortar joints are analysed in [2], [6]. Depending on the quality of masonry constituents filled head joints can contribute to brick masonry shear strength by up to 50 %. It was also shown that the wall with unfilled vertical mortar joints subjected to in-plane loading developed clean step-wise bond failure along the compression diagonal, see [6]. An influence of unfilled head joints for autoclaved aerated concrete (AAC) masonry walls under axial compression was investigated in [7] and it was concluded that unfilled vertical joints do not influence significantly masonry compressive strength.

Structural masonry codes are often based on expert opinion rather than on consistent uncertainty quantification. Therefore, there is a disparity between safety factors used in design codes worldwide, i.e. safety factors on material properties are, respectively, in Germany 1.5, Switzerland 2.0, England 2.5-3.0 etc. Furthermore, Swiss and European masonry codes do not provide provisions for the slip failure along head joints that may lead to unsafe masonry design in some cases. Hence, it is of crucial importance to investigate the probability of occurrence of this type of masonry failure with the aim to provide recommendations for possible structural code revisions.

In this paper, an appropriate limit state function together with probabilistic model is established according to theory of plasticity in order to determine the probability of slip failure occurrence along head joints for masonry wall, Fig. 1 when uncertainties are present in masonry material properties and in-plane loads.

*Figure 1. Slip failure along head joints, Versagen entlang der Stossfugenflucht*

## 2 Current state of the research in the field

Most previous research work on masonry reliability was focused on the probabilistic modelling of masonry properties, its constituents and on defining different limit state functions corresponding to specific load situations, see [9-17]. Although many authors have carried out research on reliability of masonry structures, there is still significant lack of consensus in this in comparison to the reliability evaluation for steel and reinforced concrete structures. For example, the most recent and advanced study on assessment of the compressive strength of structural masonry has shown that current approaches for masonry compressive strength do not distinguish between epistemic and aleatory uncertainties and that they neglect material heterogeneity [17]. Furthermore, masonry walls subjected to in-plane loads in general can experience six different failure regimes depending on wall geometry and applied load. To the authors' knowledge, the most advanced probabilistic model up to date takes into account only four failure regimes, see [16]. In addition, one of the major limitations of current probabilistic models is that the in-plane strength of masonry is not determined according to the theory of plasticity. In order to develop both reliable and efficient probabilistic model for masonry walls, it is of great importance to consider consistently possible failure modes within the failure criterion as defined according to the theory of plasticity.

## 3 Failure criterion

Failure criterion based on Mohr-Coulomb theory for in-plane loaded masonry walls was formulated by [18]. However, an additional failure regime, namely sliding failure along head joints, within the failure criterion without tensile strength, is introduced by [19].

*Figure 2. Compression field in shear wall [19], Druckstrebe in einer Schubwand [19]*

Assuming that applied in-plane loads, which in general can act eccentrically, could be transferred through the masonry wall of length  $l$  by means of distinct inclined uniaxial compressed stress field (strut), of length  $l_s$ , see Fig. 2, the masonry strength is then determined by using discontinuous stress field according to the lower-bound theorem of theory of plasticity. The inclination and dimensions of the stress field are dependent on wall aspect ratio, applied loads and static boundary conditions. In addition, the resulting principal compressive stress,  $\sigma_2$ , in the strut depends on the angle of inclination of the strut,  $\alpha$ , and must not violate the failure criterion. Therefore, such criterion distinguishes six different failure regimes that are given by the following set of inequalities:

$$\tau_{xy}^2 - \sigma_x \sigma_y \leq 0 \quad (1)$$

$$\tau_{xy}^2 - (\sigma_x + f_x)(\sigma_y + f_y) \leq 0 \quad (2)$$

$$\tau_{xy}^2 + \sigma_y(\sigma_y + f_y) \leq 0 \quad (3)$$

$$\tau_{xy}^2 - (c - \sigma_x \tan \varphi)^2 \leq 0 \quad (4)$$

$$\tau_{xy}^2 + \sigma_x \left[ \sigma_x + 2c \tan \left( \frac{\pi}{4} + \frac{\varphi}{2} \right) \right] \leq 0 \quad (5)$$

$$\tau_{xy}^2 - \left( \frac{c_b}{2} - \sigma_y \tan \varphi_b \right)^2 \leq 0 \quad (6)$$

where the failure criterion depends on six material properties, masonry compressive strengths perpendicular and parallel to bed joints  $f_x$  and  $f_y$ , (mortar) bed joint and block/brick cohesion  $c$  and  $c_b$ , angle of friction for (mortar) bed joint  $\varphi$  and brick/block internal angle of friction  $\varphi_b$ . Structural design of masonry wall is rational if at least one of these material properties is fully utilized.

Failure criterion, as defined in [18] does not include slip failure regime along head joints. Thus, it is defined by the set of inequalities (1) to (5), see Fig. 3.

Figure 3. Failure criterion [18], Bruchbedingung [18]

Assuming a head joint line as a slip line, no dissipation of energy along head joints, and that the masonry units material satisfies Coulomb's failure criterion [20], equation (7) for a principal compressive stress,  $\sigma_2$  ( $\sigma_2 < 0 = \sigma_1$ ) is derived by equating the work  $W$ , equation (8) and dissipation  $D$ , equation (9), see [19].

$$\sigma_2 = \frac{c_b}{2 \sin^2 \alpha (\tan \varphi_b - \cot \alpha)} \quad (7)$$

$$W = -\sigma_2 \tan \alpha \cos(\alpha + \varphi_b) \quad (8)$$

$$D = \frac{c_b \cos \varphi_b}{2 \cos \alpha} \quad (9)$$

Where  $\sigma_2$  and  $\alpha$  denote resulting principal compressive stress in the strut and its angle of inclination to head joints respectively.  $c_b$  is block/brick cohesion whereas  $\varphi_b$  is block/brick internal angle of friction. For  $\alpha = \pi/4 - \varphi_b/2$ , derived equation (7) exhibits a minimum value, which equals to

$$\sigma_{2,min} = \frac{-c_b \cos \varphi_b}{1 - \sin \varphi_b} \quad (10)$$

The variation of uniaxial strength within the extended failure criterion in strut depending on its inclination to head joints is shown in Fig. 4, [19].

Figure 4. Uniaxial strength in strut [19], Einachsige Druckfestigkeit [19]

Function for  $\sigma_2$ , equation (11), describes the uniaxial strength in strut for  $\alpha \leq (\pi/4 + \varphi/2)$  and for  $\alpha$  larger or equal to the strut angle of inclination corresponding to the intersection of functions equation (7) and equation (11), see Fig. 4.

$$\sigma_2 = \frac{c}{\cos^2 \alpha (\tan \varphi - \tan \alpha)} \quad (11)$$

Here  $c$  is the mortar (bed joint) cohesion whereas  $\varphi$  is its angle of internal friction. Slip failure regime along head joints i.e. inequality (6) is obtained combining equations (7), (12) and (13).

$$\sigma_y = \sigma_2 \sin^2 \alpha \quad (12)$$

$$\tau_{xy} = \sigma_2 \sin \alpha \cos \alpha \quad (13)$$

Enhanced failure surface representation [19] is shown in Fig. 5, where the plane (6) represents slip failure along head joints.

*Figure 5. Extended failure criterion [19], Erweiterte Bruchbedingung [19]*

Masonry wall will not fail, if all inequalities (1) to (6) are fulfilled. On the contrary, the failure of the wall will occur as soon as the resultant stress vector is out of the failure surface.

## 4 Probabilistic modelling

### 4.1 Limit state function

Failure along head joints in masonry walls subjected to in-plane loading is defined through inequality (6) i.e. plane, see (Fig. 5). This failure regime is relevant to structural design when plane (6) intersects the cylindrical surface defined by (3). In this case safe domain is moved downwards compared to the safe domain when the plane (6) cannot cut off the cylinder (3), see (Fig. 5). This means that the limit state function, equation (14) reflects difference between the cylinder, (3) and the plane, (6).

$$g = \sigma_y (\sigma_y + f_y) + \left( \frac{c_b}{2} - \sigma_y \tan \varphi_b \right)^2 \quad (14)$$

Using the following equations (15), (16), (17) and (18) the limit state function, equation (14) can be transformed into more appropriate function, equation (19) for probabilistic analysis.

$$\tan \alpha = \frac{V}{N} \quad (15)$$

$$l_s = \left[ \frac{l}{2} - e - (h + d_c) \cdot \tan \alpha \right] \cdot 2 \quad (16)$$

$$\sigma_2 = \frac{N}{l_s \cdot d \cdot \cos^2 \alpha} \quad (17)$$

$$\sigma_y = \sigma_2 \sin^2 \alpha \quad (18)$$

$$g = \left( \frac{f_y}{8} - \frac{V^2 \cdot \tan \varphi_b}{(Nl - 2Ne - 2hV - 2d_c V)d} \right)^2 + \left( \frac{V^2}{(Nl - 2Ne - 2hV - 2d_c V)d} \right)^2 + \frac{V^2 \cdot f_y}{(Nl - 2Ne - 2hV - 2d_c V)d} \quad (19)$$

$\alpha$ ,  $l_s$  and  $\sigma_2$  denote the angle of inclination, length and principal stress of the compressive stress field, respectively.  $\sigma_y$  is a component stress. Where  $V$  and  $N$  are horizontal and vertical in-plane force and  $e$  is the eccentricity respectively, whereas  $l$ ,  $h$ ,  $d$ , are wall length, height and thickness, respectively. The slab thickness is denoted by  $d_c$ .

Thus, the reliability of the wall against the slip failure along head joints is given as

$$P_f = P(g(X) \leq 0) = \int_{\text{failure domain}} f_g dX \quad (20)$$

Considering the complexity of the limit state and that many variables within this formulation are uncertain it is almost impossible to obtain in explicit format the joint probability density function  $f_g$  we need to consider approximate methods to evaluate this probability. In the following the uncertainty associated with relevant physical and loading parameters is represented by using random variable models.

## 4.2 Probabilistic modelling for load variables

Load action can be categorised based on time, origin and spatial variation, as well as on its intrinsic nature. Therefore, taking into account time variation criteria we can distinguish between permanent, variable and accidental load actions. Permanent actions are characterised by small and gradual variations around their mean values (e.g. self-weight), while variable actions by frequent and large time variations (e.g. regular occupancy, wind, snow). The magnitude of accidental load actions is considerable with a small probability of occurrence for the anticipated life time of structural use, (e.g. explosions), see [21]. Furthermore, load actions can be divided into direct (e.g. forces) and indirect (e.g. temperature) depending on their origin. In respect of spatial variation load actions can be fixed (e.g. self-weight) or free (e.g. regular occupancy). Concerning the nature of the structural response, static and dynamic load actions are normally identified.

To create the probabilistic models of load actions, it is necessary to identify their physical and statistical properties. Physical description refers to physical data of the action model, e.g. vertical forces distributed over a given area, while statistical description with the statistical properties of the variables, i.e. a probability distribution function for the intensity, etc. [21]. In general, a load action can be represented as a random variable, a random process or a random field [16]. However, probabilistic modelling of load actions using random processes and fields is quite complex and not practical for general design case.

In this paper, it is assumed that masonry building is located in region with low seismic activity and an average snowfall. Therefore, for considered case relevant in-plane design load is permanent, vertical load is variable whilst wind represents horizontal load.

Permanent load includes mostly the self-weight of masonry wall and the slab. Self-weight of members is derived from the material density and member volume. Density of masonry units mainly contributes to self-weight of masonry walls. According to [16] variability of masonry unit density is small, e.g. 3 %. Self-weight of reinforced concrete slabs mostly depends on concrete density and slab thickness whereas density of reinforcement can be neglected. Hence, the variability of self-weight of masonry walls and reinforced concrete slabs can be considered equal and represented by the variation of permanent load coefficient, [16]. Such load can be considered as a deterministic force on masonry wall which is mostly favourable to the wall shear carrying capacity. In the case of common residential buildings, this load

represents approximately 70% of the total load. Normal distribution function may be assumed for the modelling of permanent loads, see [12], [16], [21].

Variable loads in buildings are defined by the weight of furniture, equipment, people, and stored materials. This type of load varies randomly in time and space. Consequently, modelling of variable loads becomes challenging. Combined long-term and short-term variable loads yield total variable load which represents the basis of design, [16]. Statistical parameters of variable loads depend on building usage, for instance office, residential, library etc. [21]. According to [21] variation of such load over time can be modelled using a Poisson-process. In general, it is common approach in literature that variable load is statistically modelled using random variable with Gumbel distribution function.

Wind load mainly represents horizontal load that acts on masonry walls. Furthermore, wind load is mostly defined by wind velocity, air density, building location, building exposure, shape and dimensions of structures, and building dynamic properties. The wind velocity and the gust intensity have predominantly impact on wind load. The reference wind velocity is obtained as an average wind velocity for 10 min time interval at an elevation of 10 m above ground, [21]. As stated in [21] Weibull probabilistic distribution is appropriate for representation of wind velocity. According to [16], Weibull probabilistic distribution best fits set of wind load data for an observation period of 50 years.

### 4.3 Probabilistic modelling for masonry material properties

In order to describe masonry parameters, e.g. compressive strength parallel to bed joints, cohesion, module of elasticity etc., masonry compressive strength perpendicular to bed joints is used as a reference characteristic [13]. Hence, masonry compressive strength perpendicular to bed joints can be regarded as a key material characteristic. Few alternative procedures can be applied to determine characteristic masonry compressive strength perpendicular to bed joints. The European standard [22] provides a detailed explanation on how to define masonry specimen, how to load it up to failure and how to determine characteristic compressive strength from the test results is given. On the contrary, according to [23] the masonry compressive strength can be represented as a function of the unit,  $f_b$ , and mortar compressive strength,  $f_m$  correlated with three coefficients  $\alpha$ ,  $\beta$ ,  $K$ , equation (21).

$$f_x = K \cdot f_b^\alpha \cdot f_m^\alpha \quad (21)$$

Constituent's strength can be obtained from (compulsory) tests that are carried out during their production as specified by relevant standard. Values of above-mentioned coefficients are determined based on the statistical analysis of numerous test results, [14]. Thus, approach for modelling of masonry compressive strength perpendicular to bed joints that is based on tests of masonry specimens is more accurate, but more expensive as well. On the other hand, the procedure for determination of masonry compressive strength using test results of masonry components is convenient and inexpensive. Considering advantages and disadvantages of both approaches for obtaining masonry compressive strength third procedure is proposed at ETH Zurich, see [14]. This procedure enables the assessment of the masonry compressive strength based on components test results and its update on the basis of few large scale tests using Bayesian updating. Since other masonry properties are correlated with masonry compressive strength, they can be straightforwardly obtained by applying regression analysis.

For instance, the explicit relations between reference masonry property and other material parameters for Swiss masonry are proposed, equations (22) to (26), see [14].

$$f_y = \frac{1}{\eta} \cdot f_x \quad (22)$$

$$E_x = 1000 \cdot f_x \quad (23)$$

$$E_y = 1000 \cdot f_y \quad (24)$$

$$c = \frac{f_y}{16} \quad (25)$$

$$c_b = \frac{f_y}{4} \quad (26)$$

Where  $\eta$  is the coefficient determined from the masonry produced in Switzerland and depends on type of masonry units. Thus,  $\eta$  is 3, 1.6 or 1.4 for clay-block, calcium-silicate, or concrete block masonry, respectively.  $f_x, f_y$  are masonry compressive strengths perpendicular and parallel to bed joints,  $c$  and  $c_b$  denote (mortar) bed joint and block/brick cohesion, respectively. Module of elasticity parallel and perpendicular to bed joints are denoted by  $E_x$  and  $E_y$ . Internal angles of friction for (mortar) bed joint and brick/block cannot be correlated with masonry compressive strength and they are usually assumed to be deterministic.

Masonry compressive strength perpendicular to bed joints might be represented by lognormal distribution function. The same distribution function for modelling masonry compressive strength is used by other authors, [9-12], [16]. Thus, lognormal distribution may be appropriate for probabilistic modelling of masonry compressive strength parallel to bed joints and both for (mortar) bed joint and block/brick cohesion.

It can be concluded that simplified models for masonry material probabilistic modelling and masonry workmanship can cause main sources of uncertainties associated with masonry material properties. According to [24] masonry workmanship can have a significant impact on masonry material properties. This sort of impact can be alleviated by establishing better site supervision. In addition, better calibration with numerous experimental results and using more accurate approach for the assessment of masonry material parameters can decrease uncertainties. However, most of masonry material properties are defined in correlation with the reference material parameters that unavoidably introduce new uncertainties. Such uncertainties should be taken into account within the analysis, see [14].

## 5 Case of masonry reliability analysis: A slip failure along head joints

To illustrate the probabilistic modelling explained above, the following example of in-plane loaded masonry wall is considered. It is assumed that the wall has the same geometric and material properties, Table 2, as the walls used in the full-scale experiment carried out at the ETH Zurich, see [25]. In that experiment seven typical Swiss clay hollow block masonry walls with different boundary conditions and pre-compression levels were tested up to failure. For these tests masonry compressive strength parallel to bed joints,  $f_y$ , was determined according to [26] whereas masonry shear bond strength according to [27].

In this example, the masonry wall subjected to three different vertical and five lateral in-plane forces is analysed. In all cases, it is assumed that the vertical force value includes 70% of dead load and 30% of variable load. Values of lateral forces are adopted so that the length of compressive stress field (strut)  $l_s$  is less than a wall length,  $l$ . Therefore, fifteen analyses with different load scenarios for the sample wall are carried out, see Table 1.

Table 1. Random variables for wall loading parameters in limit state function

Case 1 (Total vertical force 150 kN)	Variable	Distribution	Mean value	Cov
	$N_{d.load}$ [kN]	Normal	105	6%
	$N_{l.load}$ [kN]	Gumbel	45	20%
	$V_{wind.load}^a$ [kN]	Weibull	30,45,50,55,60	7%
Case 2 (Total vertical force 160 kN)	$N_{d.load}$ [kN]	Normal	112	6%
	$N_{l.load}$ [kN]	Gumbel	48	20%
	$V_{wind.load}^a$ [kN]	Weibull	30,45,50,55,60	7%
Case 3 (Total vertical force 170 kN)	$N_{d.load}$ [kN]	Normal	119	6%
	$N_{l.load}$ [kN]	Gumbel	51	20%
	$V_{wind.load}^a$ [kN]	Weibull	30,45,50,55,60	7%

<sup>a</sup>observation period of 50 years,  $\tau = 0.073$

Table 2. Random variables for wall material and geometry parameters in limit state function

Variable	Distribution	Mean value	Cov
$f_y$ [MPa]	Lognormal	1.6	11%
$\tan\phi$ [-]	Constant	0.48	-
$c$ [MPa]	Lognormal	0.26	11%
$\tan\phi_b$ [-]	Constant	0.78	-
$c_b$ [MPa]	Lognormal	0.4	11%
$l$ [mm]	Constant	2700	-
$h$ [mm]	Constant	2600	-
$d$ [mm]	Constant	150	-
$d_c$ [mm]	Constant	200	-
$e$ [mm]	Constant	30	-

Probability of occurrence of slip failure along head joints for considered wall and load scenarios, probabilistic model (Table 1 and Table 2), is determined by applying limit state function (19) and Monte-Carlo method. Limit state function (19) is defined as a function of geometric, material and load random variables, see paragraph 4.1 above. Traditional alternatives for evaluation of the probability in equation (19) are first order second moment methods and simulation methods. Due to the nature of the governing limit state function the simulation approach is followed here. Therefore, for the derived limit state function (19) and defined probabilistic model, (Table 1 and Table 2) probability of failure event is calculated in line with the equation (27)

$$p_f = \frac{n(g(x_i) \leq 0)}{N} \quad (27)$$

where  $n$  is the number of simulations for which  $g(x_i) \leq 0$  is achieved and  $N$  is the number of all simulations that depends on desired accuracy. All simulations, approx. one million are carried out using Comrel 9 software [28] and the obtained results are shown in Table 3.

*Table 3. Probability of occurrence of slip failure along head joints*

Vertical force $N$ [kN]	Lateral force $V$ [kN]	Monte Carlo pr [%]	Reliability index $\beta$
Case 1 (Total vertical force 150 kN)	30	$3 \cdot 10^{-6}$	4.527
	45	$1.92 \cdot 10^{-4}$	3.551
	50	$7.45 \cdot 10^{-4}$	3.177
	55	$2.78 \cdot 10^{-3}$	2.773
	60	$9.79 \cdot 10^{-3}$	2.334
Case 2 (Total vertical force 160 kN)	30	$2 \cdot 10^{-6}$	4.612
	45	$7.8 \cdot 10^{-5}$	3.781
	50	$2.96 \cdot 10^{-4}$	3.435
	55	$1.05 \cdot 10^{-3}$	3.074
	60	$3.43 \cdot 10^{-3}$	2.703
Case 3 (Total vertical force 170 kN)	30	$1 \cdot 10^{-6}$	4.754
	45	$3.5 \cdot 10^{-5}$	3.976
	50	$1.32 \cdot 10^{-4}$	3.648
	55	$4.35 \cdot 10^{-4}$	3.33
	60	$1.35 \cdot 10^{-3}$	2.999

*Figur 6. Minimum reliability index against lateral forces for different level of vertical forces, Verlauf des minimalen Zuverlässigkeitsindexes in Abhängigkeit vom Niveau der vertikalen Last*

It can be noted in Fig. 6, that vertical force acts favourably on reliability of masonry walls for considered failure regime. On the other hand, lateral forces decrease reliability index, e.g. increase likelihood that the slip failure along head joints is relevant to masonry structural design. Reliability index decrease is nonlinear and indicates significant likelihood of slip failure occurring for high lateral load. In other words, there is high probability that target reliability index for residential buildings,  $\beta_{min} = 3.2$ , see JCCS [21] can be violated in case of acting of unfavourable loads combination.

It is important to note that the above outcome is related to only one failure regime (albeit predominantly overlooked) as defined earlier in the text. It should be investigated what the likelihood of other failure regimes occurring is and if there is any correlation between defined failure regimes.

## **6 Conclusion and outlook**

Reliability of masonry structures is a challenging research field that is not sufficiently investigated in comparison to reliability of steel or concrete structures. Unexpectedly so considering that masonry structures are very widespread. Thus, it is of great importance to further advance the research in this area in order to improve masonry design. This paper is focused on investigating the likelihood of the slip failure along head joints as a relevant failure mechanism to structural design when uncertainties are present in material properties and in-plane loads. Hence, the physical model is established using the theory of plasticity to define slip failure occurrence along head joints for masonry wall. Slip failure along head joints (based on Mohr-Coulomb theory) is one of the six failure regimes within failure criterion for in-plane loaded masonry walls. It is evident from the simulation results that this type of failure could be associated with high probability of occurrence. Therefore, for masonry walls with high uncertainties subjected to large lateral in-plane forces under low pre-compression this type of masonry failure should be considered.

Further work will include a detailed parametric probabilistic analysis of various masonry wall configurations in order to identify parameters and load scenarios which mostly contribute to the occurrence of slip failure along head joints. Following that any system behaviour could be investigated as well. The findings might have impact on current design codes provisions and as a consequence code recommendations could be revised.

## **7 References**

- [1] *Barth, M., Marti P.*: Tests on clay brick masonry with dry head joints. Report no. 230., Institute of Structural Engineering, ETH Zurich, 1997. (In German)
- [2] *Bolhassani, M., Hamid, A., Lau, A., Moon F.* : Simplified micro modelling of partially grouted masonry assemblages, *Construction and Building Materials* 83 (2015), p. 159-173.
- [3] *Hamid, A., Drysdale R.* : Proposed failure criteria for concrete block masonry under biaxial stresses. *Journal of Structural Division* 107 (1981), Nr. 8. p. 1675-1687.
- [4] *Hamid, A., Drysdale, R.*: Proposed failure criteria for brick masonry under combined stresses. In: 2nd North American Masonry Conference. College Park, USA, 1982.

- [5] *Hamid, A., Bolhassani, M., Turner, A., Minaie, E, Moon, F.L.*: Mechanical properties of ungrouted and grouted concrete masonry assemblages. In: 12th Canadian Masonry Symposium. Vancouver, Canada, 2013.
- [6] *Maheri, M., Najafgholipour, M., Rajabi, A. R.* :The influence of mortar head joints on the in plane and out-of-plane seismic strength of brick masonry walls. In: 14th World Conference on Earthquake Engineering. Beijing, China, 2008.
- [7] *Kubica, J.*: Unreinforced AAC Block's masonry under axially compression -an influence of not filled head joints. In: 8th International Masonry Conference. Dresden, Germany, 2008.
- [8] *El-Sakhawy Raof, H. A., Gouhor, A.*: Shearing Behavior of Joints in Load Bearing Masonry Walls. *Journal of Materials in Civil Engineering* 14 (2002) , Nr. 2, p. 145-150.
- [9] *Elingwood, B.* :Analysis of Reliability for Masonry Structures. *Journal of Structural Division* 107 (1981), Nr. 5, p. 757-773.
- [10] *Elingwood, B., Tallin, A.*: Limit States Criteria for Masonry Construction. *Journal of Structural Engineering* 111 (1985). Nr. 1, p. 108-122.
- [11] *Stewart, M. G., Lawrence, S.* : Model Error, Structural Reliability and Partial Safety Factors for Structural Masonry in Compression. *Masonry International* 20 (2007), Nr. 3, p. 107-116.
- [12] *Glowienka, S., Brehm, E.*: Probabilistic Analysis of Unreinforced Masonry Consisting of Large Units. In: 6th International Probabilistic Workshop. Dramstadt, Germany, 2008.
- [13] *Mojsilović, N., Faber, M. H.*: Probabilistic Model Framework for the Design of Structural Masonry. In: Inaugural International Conference of the Engineering Mechanics Institute (EM08). Minneapolis, USA, 2008
- [14] *Mojsilović, N., Faber, M. H.*: Probabilistic Assessment of Masonry Compressive Strength. In: 10th International Conference on Structural Safety and Reliability. Osaka, Japan, 2009.
- [15] *Mojsilović, N.* :Tensile strength of clay blocks: An experimental Study. *Construction and Building Materials* 25 (2011), Nr. 11, p. 4156-4164.
- [16] *Brehm E.*: Reliability of Unreinforced Bracing Masonry Walls. PhD Thesis, Technical University Darmstadt, 2011.
- [17] *Nagel, B. J., Mojsilović, N., Sudret, B.*: Bayesian Assessment of the Compressive Strength of Structural Masonry. In: 12th International Conference on Applications of Statistics and Probability in Civil Engineering. Vancouver, Canada, 2015.
- [18] *Ganz, H. R.*: Masonry Walls Subjected to Normal and Shear Forces. PhD Thesis, Institute of Structural Engineering, ETH Zurich, 1985, (In German).
- [19] *Mojsilović, N.* : Strength of masonry subjected to in-plane loading: A contribution. *International Journal of Solids and Structures* 48 (2011), Nr. 6, p. 865-873.
- [20] *Mojsilović N.*: On the Response of Masonry Subjected to Combined Actions. PhD. Thesis, Institute of Structural Engineering, ETH Zurich, 1995, (In German).
- [21] JCSS, Probabilistic Model Code. Joint Committee on Structural Safety, Zurich, 2006

- [22] EN 1052-1. Methods of test for masonry, Part 1: Determination of compressive strength. European Committee for Standardisation (CEN), Brussels; 2002.
- [23] Eurocode 6: Design of masonry structures. Part 1-1: General rules for reinforced and unreinforced masonry structures (EN 1996-1-1), European Committee for Standardization (CEN), , Brussels; 2005.
- [24] *Fyfe, A. G., Middleton, J., Pande, G. N.*: Numerical Evaluation of the Influence of Some Workmanship Defects on Partial Factor of Safety for Masonry. *Masonry International* 13 (2000), Nr. 2, p. 48-53.
- [25] *Salmanpour, H. A., Mojsilović, N., Schwartz, J.*: Displacement capacity of contemporary unreinforced masonry walls: An experimental study. *Engineering Structures* 89 (2015), p. 1-16
- [26] SIA 266/1. Mauerwerk-Ergänzende Festlegungen. Society of Engineers and Architects, Zurich, 2015. (In German).
- [27] EN 1052-3. Methods of test for masonry, Part 3: Determination of initial shear strength. European Committee for Standardisation (CEN), Brussels; 2007.
- [28] Comrel 9, User's manual, RCP Consult GmbH, Munich, 2015.

**Author(s) of this article:**

MEng Miloš Asenov, University of Niš, Faculty of Civil Engineering and Architecture, Aleksandra Medvedeva 14, 18000 Niš, Serbia, [asenov.milos@gmail.com](mailto:asenov.milos@gmail.com)

Corresponding author: Dr. sc. techn. Nebojša Mojsilović, ETH Zurich, Institute of Structural Engineering, 8093 Zurich, Stefano-Frascini-Platz 5, Switzerland, +41446333763, [mojsilovic@ibk.baug.ethz.ch](mailto:mojsilovic@ibk.baug.ethz.ch)

Dr. Tatjana Mičić, City University London, Department of Civil Engineering, Northampton Square, London EC1V 0HB, United Kingdom, [t.micic@city.ac.uk](mailto:t.micic@city.ac.uk)

**Keywords:** *in-plane resistance, head joints slip failure, masonry walls, probabilistic model, probability of failure, structural reliability, unreinforced masonry (URM)*