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# **Department of Economics**

# Detecting the Presence of Informed Price Trading Via Structural Break Tests

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# Detecting the Presence of Informed Price Trading Via Structural Break Tests

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#### Abstract

The occurrence of abnormal returns before unscheduled announcements is usually identified with informed price movements. Therefore, the detection of these observations beyond the range of returns due to the normal day-to-day activity of financial markets is a concern for regulators monitoring the right functioning of financial markets and for investors concerned about their investment portfolios. In this article we introduce a novel method to detect informed price movements via structural break tests for the intercept of an extended CAPM model describing the risk premium of financial returns. These tests are based on the use of a U-statistic type process that is sensitive to detecting changes in the intercept that occur very early in the evaluation period and that can be used to construct a consistent estimator of the timing of the change. As a byproduct, we show that estimators of the timing of change constructed from standard CUSUM statistics are inconsistent and therefore fail to provide useful information about the presence of informed price movements.

**Keywords and Phrases**: CUSUM tests; ECAPM; Informed Price Movements; Insider Trading; Linear Regression Models; Structural Change; *U*-statistics.

**JEL classification**: C14, G11, G12 G14, G28, G38.

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#### 1 Introduction

Recent research of Grégoire and Huang (2009) has analyzed, from a theoretical perspective, some of the consequences that informed trading and insider information have on the cost of issuing new equity. In particular, their model indicates that such information can cause the market to demand a higher premium over the risk-free rate on newly issued equity. In this model, the informed trader, in some situations - absence of noise trading in stocks, has incentive to disclose some information to the public. Under the likely scenario that many stocks involve noise trading there is little incentive to disclose material information. When information of this nature is not disclosed to financial markets in a timely fashion, Vo (2008) has shown that the price of equity is positively correlated with this information. Because insider information can negatively impact the price of equity, financial regulators have incentive to detect such trading and when appropriate prosecute.

Detection of informed trading is important then for the well functioning of financial markets. One regulatory body, the British Financial Services Authority (hereafter FSA) has the statutory objective to maintain confidence in the British financial system. In particular, it is responsible for detecting market abuse and when detected to prosecute. Dubow and Monteiro (2006 hereafter DM) in an article published in the FSA Occasional Paper Series developed a measure of market cleanliness based on the extent to which share price move ahead of regulatory announcements which issuers to financial markets are required to make. Share price movements observed ahead of significant announcements made by issuers to financial markets may reflect insider trading. DM (2006) examine two kinds of announcements: trading statements made by FTSE 350 issuers and public takeover announcements by companies to which takeover code applies. Their measure of market cleanliness is based on the proportion of significant announcements where the announcement was preceded by an informed price movement - an informed price movement is an instance where there is an abnormal stock return before an announcement.

The authors implement a capital asset pricing model to model the dynamics of risky returns and use a definition of abnormal returns as the residuals of the corresponding time series regression. Their method of detecting an informed price movements is via bootstrap techniques to approximate the finite-sample distribution of the sequence of abnormal returns before an unscheduled announcement and compare this distribution against the magnitude of four-day and two-day cumulative returns taken four days before the announcement and on and one day after

the announcement to see if these observations are in the tails of the bootstrap distribution. This method is further refined in Monteiro, Zaman and Leitterstorf (2007 hereafter MZL) to allow for serial correlation and conditional heteroscedasticity in the data by modeling the risky returns using an extended capital asset pricing model.

This article takes a different view on the statistical detection of insider trading and informed price movements. Under informed price movements, we expect a positive shift in the mean of the abnormal return sequence. Using a capital asset pricing model to describe the dynamics of asset returns, we observe that this change is reflected in an increase in the value of the intercept of the model. Therefore, we argue that a natural methodology to detect informed price movements is the use of structural break tests for changes in the intercept of LRMs.

Chow (1960) was one of the first to develop tests for structural breaks in LRMs. In particular, he constructed two test statistics capable of detecting a one-time change in regression parameters at a known time. Work by Brown, Durbin and Evans (1975, hereafter BDE) and Dufour (1988) extended Chow's test to accommodate multiple changes in regression parameters that may occur at unknown times. Other tests, called fluctuation tests, of Ploberger, Kramer and Kontrus (1988, hereafter PKK) have also been developed. An interesting contribution to the literature is that of Altissimo and Corradi (2003) who develop a statistic that tests for any number of break-points. CUSUM tests of BDE (1975), however, have been shown to be biased to a one-time change in intercept of linear regression models. Even the fluctuation tests of PKK (1988) have not performed well in finite samples see for example Olmo and Pouliot (2008) because of excessive inflation of nominal coverage probabilities.

The aim of this article is twofold. First, we propose a novel method to detect informed price movements via structural break tests for the intercept of LRMs. These tests are based on the use of a U-statistic type process that is sensitive to detecting changes in the intercept that occur early and late on in the evaluation period and can be used to construct a consistent estimator of the timing of the change; and second, the paper shows via simulation experiments the outperformance of this new method for change-point detection compared to the CUSUM test of BDE (1975) and the fluctuation test of PKK (1988).

The article is structured as follows. Section 2 introduces the definition of abnormal returns and sets out the novel hypothesis tests based on a U-statistic type process for detecting informed price movements. Section 3 discusses alternatives for change point detection based on standard CUSUM tests and shows via different Monte Carlo simulation experiments the deficiencies in

terms of size and power of these tests when the structural break is in the intercept of LRM. Section 4 illustrates these methods with an application to detect informed price movements using real data on financial returns of an anonymous database of firms trading on the London Stock Exchange and provided by Financial Services Authority. Section 5 concludes.

## 2 Detecting Informed Price Movements

Standard methodologies for detecting informed price movements and insider trading define abnormal stock returns as

$$AR_{it} := R_{it} - \mathbb{E}[R_{it}] = \varepsilon_{it},$$

where  $R_{it}$  refers to returns on stock i at time t,  $AR_{it}$  refers to abnormal returns and  $\mathbb{E}$  is the expectation. The expected return can be modeled using time series or cross-section methods. We follow the literature on market cleanliness, see DM (2006) and MZL (2007), and describe the dynamics of the expected return via an extended CAPM (hereafter ECAPM) given as follows;

$$\mathbb{E}[R_{it}] = \alpha + \beta_1 R_t^M + \beta_2 R_{it-1} + \beta_3 R_{t-1}^M, \tag{1}$$

where  $R_t^M$  refers to the market return at time t,  $\alpha$  the intercept and  $\beta_1$ ,  $\beta_2$  and  $\beta_3$  the slope parameters. The authors substitute lagged variables into the model to filter the presence of serial dependence in the data. Given that in the empirical application we are using the same dataset as in MZL (2007) it makes sense to use the same econometric model as these authors.

The estimated abnormal returns are obtained from the following time series regression model;

$$R_{it} = \alpha + \beta_1 R_t^M + \beta_2 R_{it-1} + \beta_3 R_{t-1}^M + \varepsilon_{it}, \tag{2}$$

with  $\varepsilon$  such that  $\mathbb{E}[\varepsilon_{it}] = 0$  and  $\mathbb{E}[\varepsilon_{it}^2] = \sigma_{it}^2$  for  $t = 1, \dots, T$ . The model accommodates the presence of conditional heteroscedasticity, modeled for our purposes and following MZL (2007), as

$$\sigma_{it}^2 = \omega_0 + \omega_1 \varepsilon_{it-1}^2 + \omega_2 \sigma_{it-1}^2, \tag{3}$$

with  $\boldsymbol{\omega} = (\omega_0, \omega_1, \omega_2)$  the vector of parameters.

One can argue that a time varying volatility structure can be due to frequent breaks in the variance or in the intercept of the process. For the purposes of this work we rule out the presence of breaks in the parameters of the conditional volatility process and assume that the process is genuinely changing over time, depends on past information and can be modelled with the

GARCH structure introduced above. Note, however, that the occurrence of a structural break in the intercept of equation (1) can produce a change in the variance parameters. To see why consider the following nonlinear version of the ECAPM:

$$R_{it} = \begin{cases} \alpha + \beta_1 R_t^M + \beta_2 R_{it-1} + \beta_3 R_{t-1}^M + \varepsilon_{it} & 1 \le t \le t^* \\ \alpha + \Delta + \beta_1 R_t^M + \beta_2 R_{it-1} + \beta_3 R_{t-1}^M + \varepsilon_{it} & t^* + 1 \le t \le T \end{cases}, \tag{4}$$

and assume that model (1) is instead considered. Then

$$\mathbb{E}[\varepsilon_{it}^{2}] = \begin{cases} \mathbb{E}[(R_{it} - \alpha - \beta_{1}R_{t}^{M} - \beta_{2}R_{it-1} - \beta_{3}R_{t-1}^{M})^{2}] = \sigma^{2}, & 1 \leq t \leq t^{*} \\ \mathbb{E}[(R_{it} - \alpha - \beta_{1}R_{t}^{M} - \beta_{2}R_{it-1} - \beta_{3}R_{t-1}^{M})^{2}] = \sigma^{2} + \Delta^{2}, & t^{*} + 1 \leq t < T \end{cases}$$
(5)

indicating a structural break in the variance parameter.

In the framework set out above we will identify a structural break in the intercept of (1) that occurs before an unscheduled announcement with informed price movements. Under the presence of these events one would expect to observe an increase in the risk premium of the risky asset that is not explained by the systematic component and that is, therefore, reflected in the intercept of model (1). Further, if there were some forthcoming announcement of negative news that the market anticipates, the structural break should be in the slope  $\beta_1$  component, not in the intercept. Therefore, it is important to construct a test for structural breaks that is able to detect changes only affecting the intercept. Standard CUSUM tests applied to LRMs are able to detect changes in the generating process but without identifying whether the rejection is due to a one-time change in the intercept or slope. In order to correct for this oversight, in the forthcoming subsections we propose hypothesis tests based on U-statistic type processes that permit us to use statistics tailored for detecting breaks in the intercept. Furthermore, this novel hypothesis test is more sensitive than standard CUSUM tests when these breaks are early and late on in the evaluation period. This is because these U-statistics can accommodate the presence of weight functions that can be tuned to have more power against specific alternatives.

#### 2.1 A U-Statistic Type Process for Detecting a One-time Change in Intercept

We begin by illustrating the U-statistic type process considered by Gombay, Horváth and Hušková, (1996, hereafter GHH). These authors develop a statistic that can be used to test a sequence of independent and identically distributed (iid) random variables for constant variance. They consider the following setting; given a set of observations  $\{Y_1, \ldots, Y_T\}$  for  $T \geq 2, 3, \ldots$ , one

might be interested in testing for the presence of at most one change in variance at a distinct, yet unknown time. With positive constants  $\sigma$  and  $\sigma^*$ , let

$$Y_t = \begin{cases} \mu + \sigma \varepsilon_t, & 1 \le t \le t^*, \\ \mu + \sigma^* \varepsilon_t, & t^* < t \le T. \end{cases}$$
 (6)

where

 $\varepsilon_t$  are independent and identically distributed with  $\mathbb{E}[\varepsilon_1] = 0$ ,  $\mathbb{E}[\varepsilon_1^2] = 1$  and  $\mathbb{E}|\varepsilon_1|^4 < \infty$ ,  $t = 1, \dots, T$ .

(7)

The values of the parameters  $\mu$ ,  $\sigma$ ,  $\sigma^*$  and  $t^*$  are unknown. Assuming that  $\sigma \neq \sigma^*$ , the no change in variance null hypothesis can be formulated as

$$H_O: t^* \geq T$$

versus the at-most-one change (AMOC) in variance alternative

$$H_A: 1 \le t^* < T.$$

To test the null hypothesis GHH use the change in mean framework to develop a statistic suited to testing for AMOC in the variance. Their statistic is reproduced below;

$$M_T^{(1)}(\tau) := T^{1/2}\tau(1 - \tau) \left\{ \frac{1}{T\tau} \sum_{t=1}^{[(T+1)\tau]} (Y_t - \mu)^2 - \frac{1}{T - T\tau} \sum_{t=[T\tau]+1}^T (Y_t - \mu)^2 \right\}, \quad 0 \le \tau < 1, \quad (8)$$

which compares two estimators of the variance. One estimator is fashioned from the first  $[(T + 1)\tau]$  observations and then compared to the estimator constructed from the last  $T - [(T+1)\tau]$  observations. After some simple algebra, the above process can be re-expressed as,

$$M_T^{(1)}(\tau) := T^{-1/2} \left\{ \sum_{t=1}^{[(T+1)\tau]} (Y_t - \mu)^2 - \tau \sum_{t=1}^T (Y_t - \mu)^2 \right\}, \quad 0 \le \tau < 1.$$
 (9)

This representation of  $M_T^{(1)}(\tau)$  will be used in what follows as it is simpler to manipulate. GHH substitute  $\bar{Y}_T = \frac{\sum_{t=1}^T Y_t}{T}$  for  $\mu$  and arrive at,

$$\widetilde{M}_{T}^{(1)}(\tau) := T^{-1/2} \left\{ \sum_{t=1}^{[(T+1)\tau]} (Y_t - \bar{Y}_T)^2 - \tau \sum_{t=1}^T (Y_t - \bar{Y}_T)^2 \right\}, \quad 0 \le \tau < 1.$$
 (10)

Our setting here is to generalize the framework of GHH to the following: Let  $\{(Y_t, \mathbf{X}_t)'\}_{t=1}^T$  be a sequence of multivariate random variables (hereafter rvs) such that

$$\mathbb{E}\left[\begin{array}{c} Y_t \\ \mathbf{X}_t \end{array}\right] = \boldsymbol{\mu} = \left[\begin{array}{c} \boldsymbol{\mu} \\ \boldsymbol{\mu}_{\mathbf{X}} \end{array}\right],\tag{11}$$

and

$$\mathbb{E}\left[\begin{array}{c} Y_{t} - \mu \\ \mathbf{X}_{t} - \mu_{\mathbf{X}} \end{array}\right] \left[\begin{array}{c} Y_{t} - \mu, & \mathbf{X}_{t}' - \mu_{\mathbf{X}}' \end{array}\right] = \mathbf{\Sigma}, \tag{12}$$

where  $\Sigma$ , the variance covariance matrix, is nonsingular and all diagonal terms nonzero and less than infinity. Let the scalar random variable  $Y_t$  have the following representation in terms of the multivariate random variables:

$$Y_t = \begin{cases} \mu(\mathbf{X}_t) + \sigma \varepsilon_t, & 1 \le t \le t^*, \\ \mu(\mathbf{X}_t) + \sigma^* \varepsilon_t, & t^* < t \le T, \end{cases}$$
(13)

with  $\mu(\cdot)$  the conditional mean process, and where the sequence  $\{\varepsilon_t\}_{t=1}^T$  satisfy conditions in (7). Note that if  $\mu(\mathbf{X}_t) = \mu$  for t = 1, ..., T the setting of GHH is reproduced.

Furthermore, GHH explored the use of weight functions to improve the statistical power of related tests to detect changes in the parameters produced at specific subsamples of the evaluation period. They study, in particular, the following family of processes;

$$q(\tau;\nu) := \{ (\tau(1-\tau))^{\nu}; \ 0 \le \nu \le 1/2 \}. \tag{14}$$

For the sake of this paper,  $\nu$  is restricted to the interval [0, 1/2).

In a risk management setting Olmo and Pouliot (2008) show that this family of weight functions is sensitive to a change that occurs both early and later on in the sample. Some additional definitions and notation are required before any statement can be made regarding the sequence of partial sum processes obtained from standardizing the above processes by  $q(\tau; \nu)$ . First, we introduce a class of functions Q.

**Definition 2.1.** Let Q be the class of positive functions on (0,1) which are non-decreasing in a neighborhood of zero and non-increasing in a neighborhood of one, where a function  $q(\cdot)$  defined on (0,1) is called positive if

$$\inf_{\delta < \tau < 1 - \delta} q(\tau) > 0 \quad \text{for all } \delta \in (0, 1/2).$$

**Definition 2.2.** Let  $q(\cdot)$   $\epsilon$  Q. Then define  $I(\cdot,\cdot)$  as  $I(q,c) := \int_0^1 \frac{1}{\tau(1-\tau)} \exp^{-\frac{c}{(\tau(1-\tau))q^2(\tau)}} d\tau$  for some constant c > 0.

Now as a special case of Theorem 2.1 of Szyszkowicz (1991) the following statements can be made regarding the process defined in (10).

**Proposition 2.1.** Assume  $H_O$  holds; let the rvs  $Y_t$ , for t = 1, ..., T, follow the model specified in (6), assume (7) holds, let  $q \in Q$  and set  $\gamma^2 = V[(Y_1 - \mu(\mathbf{X}_1))^2]$ . Then we can define a sequence of Brownian bridges  $\{B_T(\tau); 0 \le \tau \le 1\}$  such that, as  $T \to \infty$ ,

$$(i) \sup_{0 < \tau < 1} \frac{\left| \frac{1}{\gamma} \widetilde{M}_{T}^{(1)}(\tau) - B_{T}(\tau) \right|}{q(\tau; \nu)} = \begin{cases} o_{P}(1), & \text{if and only if } I(q, c) < \infty & \text{for all } c > 0 \\ O_{P}(1), & \text{if and only if } I(q, c) < \infty & \text{for some } c > 0, \end{cases}$$

$$(ii) \sup_{0 < \tau < 1} \frac{\left| \frac{1}{\gamma} \widetilde{M}_{T}^{(1)}(\tau) \right|}{q(\tau; \nu)} \xrightarrow{\mathcal{D}} \sup_{0 < \tau < 1} \frac{|B(\tau)|}{q(\tau)},$$

only if  $I(q,c) < \infty$  for some c and where  $B(\tau)$  refers to standard Brownian bridge.

Weighted versions of the *U*-statistic type process  $\widetilde{M}_{T}^{(1)}(\tau)$  converge in distribution to the corresponding weighted versions of the Brownian bridge.

#### 2.2 Tests for Structural Change

The purpose of this section is to use the above results on U-statistic type processes to design a hypothesis test alternative to the CUSUM type tests able to detect a change in intercept. The process entertained now is the following piecewise linear regression model;

$$Y_{t} = \begin{cases} \beta_{0}^{(1)} + \boldsymbol{\beta}' \mathbf{X}_{t} + \sigma \varepsilon_{t}, & 1 \leq t \leq t^{*}, \\ \beta_{0}^{(2)} + \boldsymbol{\beta}' \mathbf{X}_{t} + \sigma \varepsilon_{t}, & t^{*} < t \leq T. \end{cases}$$

$$(15)$$

where the  $\varepsilon_t$ 's satisfy conditions detailed in (7), and  $\beta_0^{(1)} \neq \beta_0^{(2)}$ .

The null and alternative hypothesis are as follows;  $H_O: t^* \geq T$ , versus the alternative hypothesis of at-most-one change (AMOC) in intercept;  $H_A: 1 \leq t^* < T$ . As advertised, the task here is to construct a test to detect such deviations.

Replacing  $(Y_t - \mu)^2$  in equation (8) with  $(Y_t - \boldsymbol{\beta}' \mathbf{X}_t)$  yields the resulting *U*-statistic type process is

$$M_T^{(2)}(\tau) := T^{-1/2} \left\{ \sum_{t=1}^{[(T+1)\tau]} (Y_t - \boldsymbol{\beta}' \mathbf{X}_t) - \tau \sum_{t=1}^T (Y_t - \boldsymbol{\beta}' \mathbf{X}_t) \right\}$$
(16)

Corollary 2.1. Assume  $H_O$  holds; let the rvs  $Y_t$ , for t = 1, ..., T, follow the model specified in (13), assume (7) holds; let  $q \in Q$  and set  $\gamma^2 = V[(Y_1 - \beta' \mathbf{X_1})^2]$ . Then we can define a sequence of Brownian bridges  $\{B_T(\tau); 0 \le \tau \le 1\}$  such that, as  $T \to \infty$ ,

(i) 
$$\sup_{0<\tau<1} \frac{\left|\frac{1}{\gamma}M_T^{(2)}(\tau) - B_T(\tau)\right|}{q(\tau;\nu)} = \begin{cases} o_P(1), & \text{if and only if } I(q,c) < \infty & \text{for all } c > 0 \\ O_P(1), & \text{if and only if } I(q,c) < \infty & \text{for some } c > 0, \end{cases}$$
(ii) 
$$\sup_{0<\tau<1} \frac{\left|\frac{1}{\gamma}M_T^{(2)}(\tau)\right|}{q(\tau;\nu)} \xrightarrow{\mathcal{D}} \sup_{0<\tau<1} \frac{|B(\tau)|}{q(\tau;\nu)},$$

if and only if  $I(q,c) < \infty$  for some c and where  $B(\tau)$  refers to standard Brownian bridge.

Here, interest centers on how large this process can be for  $0 < \tau < 1$ . If there is in fact a change in intercept the value of the supremum of the above process should be large. This consideration leads to the following test statistic:

$$\sup_{0 < \tau < 1} \frac{|M_T^{(2)}(\tau)|}{q(\tau; \nu)} \tag{17}$$

to test for a one-time change in intercept. The asymptotic distribution of this test statistic depends on the weight function  $q(\cdot)$  and the unknown parameters  $\beta_0$  and  $\beta$ . This, however, poses no problem. Simply replace these parameters with any sequence of consistent estimators,  $\{\widehat{\beta}_{0,T}\}_{T\geq K+1}$  and  $\{\widehat{\beta}_T\}_{T\geq K+1}$ . This slightly altered version of process (17) will be denoted  $\widehat{M}^{(2)}(\tau)$ . Said substitution leads directly to the following corollary to Proposition 2.1, as the proof involves only simple algebra, none will be provided.

Corollary 2.2. Assume  $H_O$  holds; let the rvs  $Y_t$ , for t = 1, ..., T, follow the model specified in (13), assume (7) holds; let  $q \in Q$  and set  $\gamma^2 = V[(Y_1 - \beta' X_1)^2]$ . Then we can define a sequence of Brownian bridges  $\{B_T(\tau); 0 \le \tau \le 1\}$  such that, as  $T \to \infty$ ,

$$(i) \quad \sup_{0 < \tau < 1} \frac{\left| \frac{1}{\gamma} \widehat{M}_{T}^{(2)}(\tau) - B_{T}(\tau) \right|}{q(\tau; \nu)} = \begin{cases} o_{P}(1), & \text{if and only if } I(q, c) < \infty & \text{for all } c > 0 \\ O_{P}(1), & \text{if and only if } I(q, c) < \infty & \text{for some } c > 0, \end{cases}$$

$$(ii) \quad \sup_{0 < \tau < 1} \frac{\left| \frac{1}{\gamma} \widehat{M}_{T}^{(2)}(\tau) \right|}{q(\tau; \nu)} \xrightarrow{\mathcal{D}} \sup_{0 < \tau < 1} \frac{|B(\tau)|}{q(\tau; \nu)},$$

only if  $I(q,c) < \infty$  for some c and where  $B(\tau)$  refers to standard Brownian bridge.

Using process (16), a consistent estimator of the time of change can be fashioned. Following GHH (1996) and Anotch, Hušková and Veraverbeke (1995) let the estimator of  $t^*$  given by  $\hat{t}^*$ ,

then

$$\widehat{t^*} = \min \left\{ t : \frac{|\widehat{M}_T^{(2)}(\frac{t}{T})|}{q(\frac{t}{T};\nu)} = \max_{1 \le t < T} \frac{\widehat{M}_T^{(2)}(\frac{t}{T})|}{q(\frac{t}{T};\nu)} \right\}. \tag{18}$$

Theorems 1 and 2 of Antoch, Hušková and Veraverbeke (1995) detail consistency and the limiting distribution of the above estimator using weight function  $q(\tau; \nu)$ .

# 3 CUSUM type tests and Instability of Intercept Detection

Influential works in the change point detection literature claim that CUSUM methods are inconsistent to detect changes in the intercept of regression models. Thus, it has been documented by Maddala (1999, Chapter 13, page 393) and others (cf. McCabe and Harrison (1980)) that the CUSUM tests of BDE (1975) have asymptotically low power against instability in intercept but not against instability of the entire coefficient vector.

The CUSUM test of BDE is based on recursive residuals, standardized appropriately. In particular, the cumulative sum of recursive residuals is given by

$$W^{(r)} := \frac{1}{\hat{\sigma}} \sum_{t=K+2}^{r} w_t, \tag{19}$$

where  $w_t$  is the recursive residual. This leads to an equivalent test statistic detailed by the following formula

CUSUM Test := 
$$\max_{K+1 < r \le T} \frac{\frac{|W_t^{(r)}|}{\sqrt{T-K-1}}}{1 + 2\frac{r-K-1}{T-K-1}}.$$
 (20)

In this formula, T refers to the sample size and K to the number of slope parameters, one in our case. The null hypothesis of parameter constancy is rejected whenever BDE statistic exceeds some critical value.

Another interesting test statistic for change point detection is the fluctuation test of Kramer, Ploberger and Alt(1988, hereafter KPA). This statistic is based on estimates of the parameters from a linear regression model. Define  $\mathbf{X}^{(t)} = [\mathbf{x}_1, \dots, \mathbf{x}_t]', \mathbf{Y}^{(t)} = [Y_1, \dots, Y_t]', t = 1, \dots, T$  and  $\boldsymbol{\beta}^{(t)} = (\mathbf{X}^{(t)'}\mathbf{X}^{(t)})^{(-1)}\mathbf{X}^{(t)}\mathbf{y}^{(t)}$  for  $t = K, \dots, T$ . Their test statistic is defined as

$$S^{(T)} = \max_{t=K,\dots,T} \frac{t}{\widehat{\sigma}T} || (\mathbf{X}^{(T)}'\mathbf{X}^{(T)})^{1/2} (\widehat{\boldsymbol{\beta}}^{(t)} - \widehat{\boldsymbol{\beta}}^{(T)} ||_{\infty},$$
(21)

where  $||\widehat{\boldsymbol{\beta}}^{(t)} - \widehat{\boldsymbol{\beta}}^{(T)}||_{\infty} = \max_{k=1,\dots,K} |\widehat{\boldsymbol{\beta}}_k^{(t)} - \boldsymbol{\beta}_k^{(T)}|$ . The test statistic  $S^{(T)}$  rejects  $H_O$ , given below, of a one-time change in  $\boldsymbol{\beta}$  of the LRMs whenever it is too large, i.e. the parameter estimates fluctuate too much.

The following subsection explores the statistical properties, size and power, of these tests and aims to shed some light on the failure of these CUSUM type tests to detect changes in intercept and hence to provide support to the choice of structural break test methods based on *U*-statistics and described above. This is done via different Monte-Carlo simulation experiments.

#### 3.1 CUSUM tests for a One-time Change in Intercept

The proposed model is given by

$$Y_t = \begin{cases} \beta_0^{(1)} + \beta X_t + \sigma \varepsilon_t, & 1 \le t \le k^*, \\ \beta_0^{(2)} + \beta X_t + \sigma \varepsilon_t, & k^* < t \le T. \end{cases}$$
 (22)

where the  $\varepsilon_t$ 's satisfy the conditions given in (7).

The first Monte Carlo experiment concerns comparison of the statistical power for the fluctuation tests developed by KPA (1988) with the CUSUM test of BDE (1975) for a one-time change in intercept of the entertained linear regression model (22). Our aim is to study the sensitivity of the different tests to alternatives that involve a small change in intercept in small sample sizes when the one-time change in intercept occurs early on, middle of and later on in the sample. Within the LRMs specified in (22), the specific alternatives considered for the intercept are  $\beta_0^{(1)} = \beta = 1$ , while  $\beta_0^{(2)} = 1.25$ , 1.5, 1.75, 2. The change in intercept considered in this simulation increased from 25% - a small change, to 75% - a moderate change, to 100% - a large change. The sample size considered here ranged from T = 75 - a small size, to T = 100 - a moderate size and then T = 125 - a large size. We will assume  $\varepsilon_t$  follows a N(0,1) for  $t = 1, \ldots, T$ .

Table 1 records the nominal coverage of the entertained tests when the errors of LRMs are standard normal random variables. All tests, except the fluctuation test, have nominal coverage probabilities that are less than 6% - the fluctuation test's nominal coverage exceeds 20%.

Table 1: Nominal Coverage

	T = 75	T = 100	T=125
CUSUM Test	0.050	0.056	0.040
FLUCTUATION	0.224	0.224	0.208

Table 2: Empirical Power (EP)

				MIDDL	MIDDLE OF SAMPLE $(\tau^*$	IPLE $(\tau^{\star})$	= 0.5					
	7	$\beta_0^{(2)} = 1.25$			$\beta_0^{(2)} = 1.5$	).		$\beta_0^{(2)} = 1.75$	5		$\beta_0^{(2)} = 2$	
Statistic	T = 75	T = 100	T=125	T=75	T = 100	T=125	T=75	T = 100	T=125	T = 75	T = 100	T=125
CUSUM Test	90.0	0.07	80.0	0.10	0.13	0.16	0.21	0.26	0.34	0.4	0.45	0.58
$ \widehat{t^{\star}}_{CUSUM} $	0.53	99.0	0.79	0.88	0.55	0.74	0.84	0.92	0.57	0.75	0.88	0.93
FLUCTUATION	0.23	0.20	0.22	0.29	0.39	0.43	0.53	0.62	0.71	92.0	0822	06.0
	-			LATE .	LATE DETECTION $(\tau^*)$		= 0.85					
	T = 75	T = 100	T=125	T=75	T = 100	T=125	T=75	T = 100	T=125	T = 75	T = 100	T=125
CUSUM Test	0.05	0.04	0.04	0.05	0.04	0.05	0.04	0.04	0.04	0.04	90.0	90.0
$\widehat{t^{\star}}_{CUSUM}$	0.44	0.48	0.51	0.57	0.46	0.49	0.53	0.59	0.45	0.51	0.54	0.61
FLUCTUATION	0.25	0.17	0.16	0.23	0.23	0.19	0.28	0.33	0.27	0.31	0.36	0.5
				EARLY	EARLY DETECTION $(\tau^*$	10N (7*	= 0.15					
	T = 75	T = 100	T=125	T=75	$T = 100 \mid T = 125$	T=125	T=75	T = 100	T = 125	T = 75	T = 100	T=125
CUMSUM Test	90.0	90.0	0.05	0.12	0.16	0.20	0.25	0.31	0.41	0.47	0.56	0.61
$\widehat{t^{\star}}_{CUSUM}$	0.46	0.47	0.49	0.52	0.45	0.48	0.50	0.52	0.52	0.50	0.52	0.53
FLUCTUATION	0.22	0.21	0.17	0.23	0.22	0.20	0.22	0.26	0.23	0.35	0.36	0.37

It is clear from Table 2 that the CUSUM test of BDE (1975) can only detect a change in intercept when it occurs early on in the sample. It does so only because this test has sufficient time to detect the change. Of course, one could reverse the order of the observations, calculate the recursive residuals then compute the CUSUM test of BDE (1975). This procedure will allow the test to detect the one-time change in intercept that occurs later on in the sample but it cannot be used to estimate the timing of the rejection; the timing of the rejection will occur much earlier than that estimated via the CUSUM statistic of BDE (1975). This last point regarding inconsistent estimation of the change fraction can be seen from the second row in the Early Detection section of Table 2; the change fraction is 0.08 while the estimate ranges from a small of 0.45 to a large of 0.53. Clearly the estimate of the change fraction cannot be extracted via the CUSUM statistic of BDE (1975).

The second Monte Carlo experiment consists of repeating the above simulation but comparing now against the test statistic  $\sup_{0<\tau<1}\frac{|\widehat{M}_{T}^{(2)}(\tau)|}{q(\tau;\nu)}$  introduced in this paper. This comparison allows a more realistic assessment of the ability of the newly fashioned statistic to detect a one-change in intercept and follows closely the criteria used by KPA (1988, p. 1359). Even though power is an important criteria for comparison, the accuracy of the nominal size of the tests should also be considered. Both criteria, power and accuracy of nominal coverage, were adopted by KPA to evaluate performance of the BDE CUSUM with their fluctuation test within a dynamic linear regression model. As in their study, both criteria will be adopted here as well as an additional criteria: how accurate the estimate of the timing of rejection is. Hence there will be three measures upon which the tests will be judged on. Table 3 estimates the nominal coverage for no change in the intercept. From this table, we observe that the newly fashioned statistic performs well in terms of nominal size attaining the level of 0.067 when T = 125 for a nominal size of 0.05. By contrast, the nominal coverage of the CUSUM test is smaller than 0.05.

Table 3: Nominal Coverage

	T = 75	T = 100	T=125
$\sup_{0 < \underline{\tau} < 1} \frac{ \widehat{M}_{T}^{(2)}(\tau) }{q(\tau, \nu = \frac{50}{218})}$	0.087	0.079	0.067
$\widehat{T^{\star}}_{\frac{ \widehat{M}_{T}^{(2)}(\tau) }{q(\tau,\nu=\frac{50}{218})}}$	0.010	0.368	0.401
CUSUM	0.050	0.029	0.040
$\widehat{ au^{\star}}_{CUSUM}$	0.461	0.450	0.444
FLUCTUATION	0.242	0.209	0.184

Table 4 tabulates the empirical power for the model entertained in (22) using the same changes in intercept and sample sizes discussed above. Two interesting observations can be made from the simulation. In particular,  $\sup_{0<\tau<1}\frac{|\widehat{M}_T^{(2)}(\tau)|}{q(\tau,\nu=\frac{50}{128})} \text{ is much better than the CUSUM test} for detecting a one-time change in intercept that occurs in the middle of the sample. The empirical power (hereafter EP) of the statistic developed here ranged from a small of 0.1 (25% change in intercept and <math>T=75$ ) to a maximum EP of 1 (100% change in intercept and T=125), compared to the CUSUM which had a minimum EP of 0.05 (25% change in intercept and T=75) to a maximum EP of 0.89 (100% change in intercept and T=125). The second interesting observation concerns the estimator of  $\tau^*$ , the break fraction. For the statistic developed here,  $\widehat{\tau}^*$  ranged from a minimum of 0.5 (25% change in intercept and T=75) to a maximum of 0.92 (100% change in intercept and T=125). This emphasizes the inconsistency of the estimator for the change fraction which should be near 0.5, for a one-time change in intercept that occurs in the middle of the sample. We see that the estimator of  $\tau^*$  based on  $\frac{|\widehat{M}_T^{(2)}(\tau)|}{q(\tau,\nu=\frac{50}{128})}$  had smaller variability and was near 0.5, especially for moderate to larger sample sizes; the estimator was 0.49 when T=125 and there was a 100 % change in intercept.

For a change late in the sample, i.e  $\tau^*$  at 0.9,  $\frac{|\widehat{M}_T^{(2)}(\tau)|}{q(\tau,\nu=\frac{50}{128})}$  has higher EP than the CUSUM, and the estimator of  $\tau^*$  based on this process estimates the break fraction to be 0.77 which is much closer to the true value of  $\tau^*$  than the CUSUM that estimated the break fraction to be 0.62 which is well off the true break fraction of 0.90. For Early detection the CUSUM has higher EP, especially for small T and small changes in the intercept. The reason for this is explained in the previous paragraphs. The estimate of  $\tau^*$  based on the CUSUM, however, does not do very well. Indeed, ranged from a minimum of 0.45 to a maximum of 0.54 well off the change fraction of 0.05. This, however, does not hold for  $\frac{|\widehat{M}_T^{(2)}(\tau)|}{q(\tau,\nu=\frac{50}{128})}$  for T=125 and a 100% change in intercept, the estimate of  $\tau^*$  based on it is 0.23. While still above 0.05, it is much closer than the estimate produced via the CUSUM. Hence, even when the CUSUM proves useful for early detection, it cannot be used to manufacture an estimate of the break fraction  $\tau^*$ , whereas the process manufactured here provides a good estimate and certainly much better than one constructed via the CUSUM. An accurate estimate of the break fraction is important here as it will allow correct diagnosis of a structural break in the intercept.

Table 4: Empirical Power (EP)

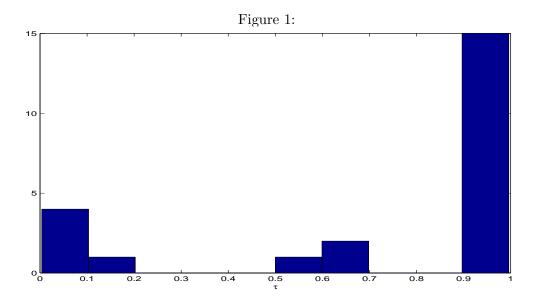
				MIDDL]	MIDDLE OF SAMPLE	$\overline{\text{PLE}}$ $(\tau^*)$	= 0.5					
		$\beta_0^{(2)} = 1.25$	ರ		$\beta_0^{(2)} = 1.5$			$\beta_0^{(2)} = 1.75$	2		$\beta_0^{(2)} = 2$	
Statistic	T = 75	N = 100	T = 125	T = 75	T = 100	T = 125	T = 75	T = 100	T = 125	T = 75	T = 100	T=125
$\sup_{0 < t < 1} \frac{ \widehat{M}_T^{(2)}(\tau) }{q(\tau, \nu = \frac{50}{218})}$	0.098	0.095	0.149	0.273	0.432	0.504	0.644	8.0	0.92	0.92	0.978	1
$\overbrace{T^*}_{q(\tau,\nu = \frac{50}{218})}$	0.4056	0.4054	0.4157	0.4226	0.4518	0.4612	0.4682	0.4786	0.4837	0.47	0.4941	0.4907
CUSUM	0.047	0.061	0.078	0.136	0.205	0.254	0.366	0.459	0.626	0.627	0.797	0.893
$\widehat{\tau^{\star}}_{CUSUM}$	0.5017	0.5232	0.5696	0.6732	0.7165	0.7612	0.8093	0.8469	0.8864	0.8702	0.9111	0.9229
FLUCTUATION	0.289	0.254	0.293	0.462	0.574	0.622	0.784	98.0	0.956	0.949	0.986	1
				LATE	DETECTION $(\tau^*)$	$ION (\tau^* =$	= 0.9)					
	T = 75	T = 100	T=125	T=75	T = 100	T=125	T=75	T = 100	T=125	T = 75	T = 100	T=125
$\sup_{0 < t < 1} \frac{ \widehat{M}_T^{(2)}(\tau) }{q(\tau, \nu = \frac{50}{218})}$	0.089	0.064	0.084	960.0	0.105	0.11	0.146	0.203	0.281	0.247	0.382	0.574
$\overbrace{T^*}_{q(\tau,\nu = \frac{50}{218})}$	0.3817	0.4026	0.4151	0.4268	0.4727	0.5297	0.5262	0.5936	0.6416	0.629	0.6938	0.7661
CUSUM	0.042	0.044	0.043	0.052	0.044	0.043	0.039	0.054	0.051	0.051	0.051	0.077
$\tau^*_{CUSUM}$	0.4727	0.4567	0.4519	0.4916	0.476	0.4909	0.5158	0.5177	0.5412	0.5502	0.5904	0.6156
FLUCTUATION	0.224	0.224	0.179	0.227	0.242	0.186	0.252	0.324	0.238	0.277	0.283	0.476
				EARLY	$^{\prime}$ DETECTION $( au^{\star}$	$_{ au^{\star}}$ .	= 0.05					
	T = 75	T = 100	T = 125	T=75	T = 100	T=125	T=75	T = 100	T=125	T = 75	T = 100	T=125
$\sup_{0 < t < 1} \frac{ \widehat{M}_T^{(2)}(\tau) }{q(\tau, \nu = \frac{50}{218})}$	0.091	0.081	0.059	60.0	0.086	0.105	0.093	0.123	0.175	0.171	0.253	0.359
$\overbrace{T^{\star}}_{q(\tau,\nu = \frac{50}{918})}^{(2)}$	0.4065	0.4074	0.4038	0.4098	0.3919	0.3765	0.384	0.3619	0.3135	0.3453	0.2873	0.2341
CUSUM	0.05	0.08	0.088	0.103	0.163	0.196	0.251	0.305	0.427	0.448	0.516	0.634
$\widehat{\tau^{\star}}_{CUSUM}$	0.4457	0.464	0.444	0.4858	0.4807	0.4798	0.4994	0.5127	0.5144	0.5155	0.5323	0.5417
FLUCTUATION	0.231	0.188	0.19	0.223	0.216	0.202	0.252	0.228	0.221	0.291	0.291	0.274

## 4 Application to Return Series

The insightful works of DM (2006) and MZL (2007) show clear evidence of informed price movements before unscheduled announcements. These announcements are trading statements made by FTSE350 issuers and public takeover announcements made by companies to which the takeover code applies. Given the nature of the techniques employed and statistical methods, there is no certainty about their results, however these authors assert that there is a high probability that most of the informed price movements detected in their study and covering the period 1999-2005 are due to insider trading. They also conclude that there seems to be a reduction in this practice after the introduction of the Financial Services and Market Act in 2001; that is more apparent in the years 2004 and 2005.

Their methodology was discussed in previous sections but is repeated here for sake of exposition. These authors obtain a sequence of announcements affecting the different stocks comprising the FTSE350. Their experiment consists on fitting the ECAPM featured in (2) to the latest 240 daily observations up to ten days before the announcement, in order to obtain the corresponding sequence of abnormal returns. They use these observations to construct the bootstrap approximation of the distribution of the cumulative returns; the corresponding right tail critical value is compared against the value of four-day and two-day cumulative returns constructed from the 10-day window just before the announcement in order to see if these abnormal returns can be statistically derived from the process generating the returns or are due to some external intervention.

FSA has granted us partial access to the data of the study by MZL (2007). In particular, we have 371 announcements on the 350 FTSE350 companies with 251 return observations per company per announcement and standardization of the announcement on the 250th day. We only have information concerning the timing of the announcement but not on the nature of this announcement or the name of the company under study. The number of announcements (371) implies that for some companies there is more than one announcement. After fitting the regression model (2) and computing the test statistic (17) developed in this paper with the parameter  $\nu = 50/128$ , we detect 23 series where there is at least one break in the intercept. We have chosen the parameter  $\nu$  that maximizes the number of break detections in the intercept and that is tailored to detect breaks that occur early, in the middle as well as late in the sample period.



The following histogram, Figure 1, shows the frequency of the timing of the rejections.

There are 13 series out of 23 with the break after  $\tau=0.985$  (observation 248), and 12 series with the break on the announcement day. In these series the break, given by a change in the intercept parameter, is interpreted as an increase in the idiosyncratic risk as a result of the announcement. There is a group of eight companies in the middle of the histogram with breaks between  $\tau=0.50$  and  $\tau=0.70$  (observations 125 and 175); this corresponds to between four and two months before the announcements. We claim that, due to the timing of the break, these are the firms that should be further investigated for insider trading practices. The remaining two break events between  $\tau=0.00$  and  $\tau=0.20$  are very far from the announcement day to be considered as indicators of some irregular activity.

For illustration purposes we report our study for five series of abnormal returns. These are chosen to lie in each of the three groups according to the above histogram: firms with changes due to announcements, firms suspect of informed price movements and finally, spurious detections. Figure 1 plots the sequence of abnormal returns for each of the five series studied in detail. The timing (fraction of the sample) of the detection of each break are [0.640 0.068 0.690 0.992 0.992] and are signaled with an arrow.

The parameters of the regression model (2);

$$R_{it} = \alpha + \beta_1 R_t^M + \beta_2 R_{it-1} + \beta_3 R_{t-1}^M + \varepsilon_{it},$$

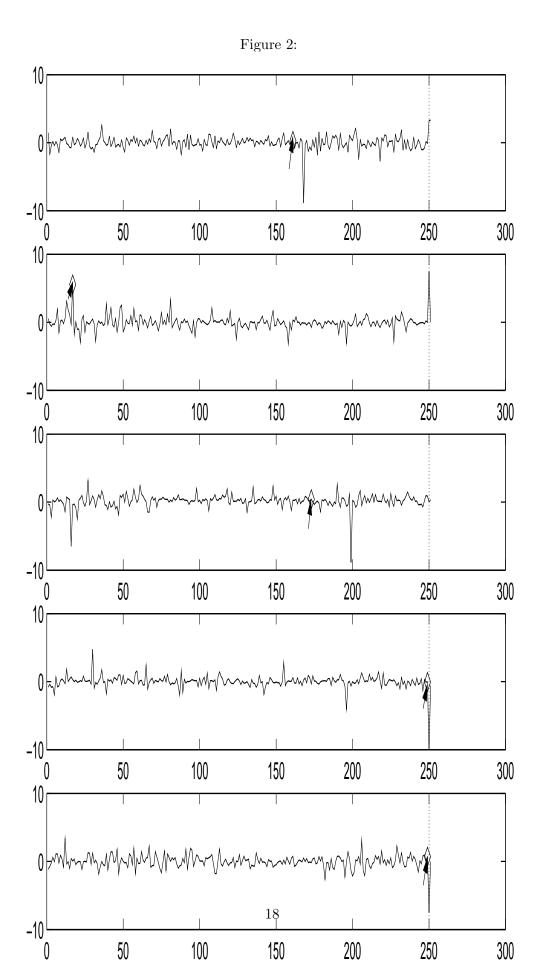


		Table 5:	Estimate	of Param	eters		
Series/coeff	$\alpha$	$\beta_1$	$\beta_2$	$\beta_3$	$\omega_0$	$\omega_1$	$\omega_2$
Series 9	-0.002	0.490	0.139	-0.290	0.000	0.101	0.000
	(0.001)	(0.129)	(0.066)	(0.131)	(0.001)	(0.037)	(0.290)
Series 30	0.006	0.258	0.205	-0.070	0.000	0.000	0.986
	(0.001)	(0.104)	(0.060)	(0.106)	(0.000)	(0.005)	(0.051)
Series 38	-0.002	0.399	0.018	0.013	0.000	1.000	0.000
	(0.001)	(0.177)	(0.063)	(0.178)	(0.000)	(0.112)	(0.080)
Series 65	-0.001	0.454	0.266	0.140	0.000	0.000	0.028
	(0.001)	(0.139)	(0.062)	(0.142)	(0.000)	(0.026)	(0.000)
Series 88	0.000	0.660	0.021	-0.032	0.000	0.169	0.000

Table F. Estimate of Danamatons

and

$$\sigma_{it}^2 = \omega_0 + \omega_1 \varepsilon_{it-1}^2 + \omega_2 \sigma_{it-1}^2,$$

(0.063)

(0.116)

(0.000)

(0.111)

(0.393)

accommodating for conditional heteroscedasticity are located in Table 5.

(0.108)

(0.001)

Table 5 estimates the parameters of model (2) with standard errors recorded in brackets. Series # determines its location in the FSA database provided. The results from this table show that the lagged market portfolio return is not significant for any of the series. The idiosyncratic lagged variable, however, is significant for all cases. Finally, we also observe that the presence of conditional heteroscedasticity seems not to be instrumental in this study.

## 5 Conclusion

The occurrence of abnormal returns before unscheduled announcements is usually identified with informed price movements and in particular with the presence of insider trading. This practice is banned in most of worldwide financial markets. For example, the British FSA as part of the statutory objective of maintaining confidence in the British financial system is responsible for detecting market abuse and when detected to prosecute.

It is not obvious that price movements before sensitive market announcements are due to the effect of insiders attempting to take profit of private information, therefore and following the work of MZM (2007) we concentrate on detecting the presence of informed price movements before unscheduled announcements and leave the detection of insider trading to more sophisticated techniques involving detailed investigation of the company under scrutiny.

We show that the presence of informed price movements can be detected by running structural break tests for the intercept of an extended capital asset pricing model. In particular, we propose a test statistic that is based on a *U*-statistic type process which can be tailored to have more power against detections that occur early or later on in the evaluation period and is robust to the estimation of model parameters. As a by-product, we show that standard CUSUM type tests have no statistical power and more importantly, are not well suited in general to handle changes in the intercept of LRMs.

The application of our method to data on returns on companies comprising the FTSE350 detects twenty three breaks in the intercept of the model during the evaluation period. From these breaks we find evidence of informed price movements for ten companies, the break in intercept being more significant for eight companies where the break occurs somewhere between four and two months before the announcement. Our recommendation from this study is to monitor more closely these eight companies to see if these statistical detections are due to fraudulent use of sensitive information or are simply the result of our statistical device. Further research includes the detection of more than one break in the evaluation period and of breaks in the volatility process.

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