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# Modal analysis and experimental research into improved centering–leveling devices

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## A B S T R A C T

Centering–leveling devices are often used together with rotary tables to improve measuring process in various fields of metrology. The most important factor of measurement quality – accuracy – is affected by numerous external and internal factors. To ensure the optimum quality of measurements, several factors have to be well known and thus taken into account in the final measurement to minimize their influence. Analysis of structural dynamics provides data on sensitivity as well as an appropriate method to verify the analytical model. The paper deals with an analysis of structural dynamics of a plain structure centering–leveling device by performing appropriate modal analysis. The experimental setup for vibration monitoring and measurement principle underpinning the work is described in the paper. Measurement results of table dynamics as well as a comparison of theoretical and experimental modal shapes are discussed.

## 1. Introduction

The monitoring of the position of an object is very important while performing a wide range of other measurements on the object. Coordinate tables, which come in a wide range of sizes, have been developed to adjust the position of a measured object. Some are produced with small dimensions and are designed to have high sensitivity to the adjustment of the object displacement. However, coordinate tables are designed to adjust the position of measured object only in one plane and this causes some problems in practical measurements if there is a need to set up the devices coaxially or to perform a displacement of the object in a vertical plane.

In order to address these issues, the centering–leveling device developed here and investigated in this work can be implemented in association with other precision measurement systems, i.e., it can be attached to a rotary or indexing table with a circular scale on the top of it. One of the advantages of the device is its simplified structure and therefore it simplifies the operational alignment procedure. Moreover, the measured object can be adjusted in both horizontal and vertical planes [9,19] which is an important advantage.

Analysis of structural dynamics using modal response to vibration modes is a critical component of analytical model verification as well as sensitivity analysis [1,2]. Detailed modal analysis allows the determination of the fundamental vibration mode shapes and the corresponding frequencies [20]. Such information is crucial for the further measurements that can be performed by using a centering–leveling device, along with other components.

## 2. Means and methods for modal analysis

### 2.1. Overview of the state-of-the-art modal analysis methods

Although operational modal analysis is often used in the fields of earthquake engineering and structural engineering including buildings and other great structures to determine modal parameters under real operational conditions [8,12,13], it can also be applied in laboratories for testing the individual devices or various sets of laboratory equipment.

Modal analysis is a process in which the structure is characterized by dynamic features, such as an eigenvalue (resonance) frequency, damping and modal shapes. Most of widely used constructions and machinery are affected by some specific form of fluctuation. To understand the problems and the nature of such fluctuations in different types of construction, it is necessary to evaluate the system resonance frequencies. It is common practice to acquire these data by defining the key modal parameters which leads to a mathematical model of the dynamics in the system. Such a model consists of the appropriate mode shapes, where each represents a frequency and the modal damping. It is these parameters which provide the dynamic description of the construction.

Modal analysis is categorized into two main sections – classical modal analysis (experimental modal analysis) and operational modal analysis. Of these, classical modal analysis allows the calculation of the frequency response function, based on the measured forces of excitation and the response of the construction. Operational modal analysis is based on the measurement of the output signals, including environmental noise and forces as a non-measured input signal [5]. The latter is used when there is a physical construction and controlling of artificial construction excitation is complicated or impossible. There are many methods for the application of modal analysis described in literature. One of these – fully automated (operational) modal analysis – was developed to avoid parameters that need to be specified or tuned by the user. This modal analysis produced similar results as in a manual analysis. Also, an improved modal analysis for three-dimensional problems using face-based smoothed finite element method is discussed in [10], where the main focus is on the Finite Element Method which is used together with meshfree methods and the generalized gradient smoothing techniques [3,13,14,17].

In every research there are certain circumstances which have to be dealt with while performing an experiment. Therefore, the method and means for modal analysis are chosen depending on measurement object shape, size, material, etc. The nature of the proposed experimental approach is based on the principles of the experimental modal analysis. Observing the performance of the proposed experimental approach provides sufficient data to conclude the experimental approach results and therefore, to compare theoretical and experimental mode shapes.

Also, a brief review and analysis of the current state-of-the-art of circular scale measurement research has revealed the key problematic areas to be scale precision,

as well as taking steps to increase measurement effectiveness [9,19].

### 2.2. The centering–leveling table

To determine the angular position of raster scale grating, the centering–leveling device is very useful when scanning the surface of the scale. Raster scales can be made of glass, plastic, etc. The research undertaken here reports a novel technique, using a standardized Optical Disk [7] as a measuring scale as well as several other issues originating from this idea. Also, the experimental modal analysis performed on this novel setup provides essential data for measurement accuracy improvement. Most significantly, the positioning of an object (its centering and leveling) which has the highest impact on the accuracy of the measurement results is addressed. The research carried out reports on a new centering–leveling precision positioning table for raster scale measurement, which has the following key features:

- A main frame (outer ring) on which the rest of the components are mounted;
- A centering–leveling disk (inner ring);
- A lower ring allowing mounting of the system on other measuring systems;
- Two micro-traverses for centering;
- Additional supporting components (springs, spheres and cone-shaped heads) for motion transfer.

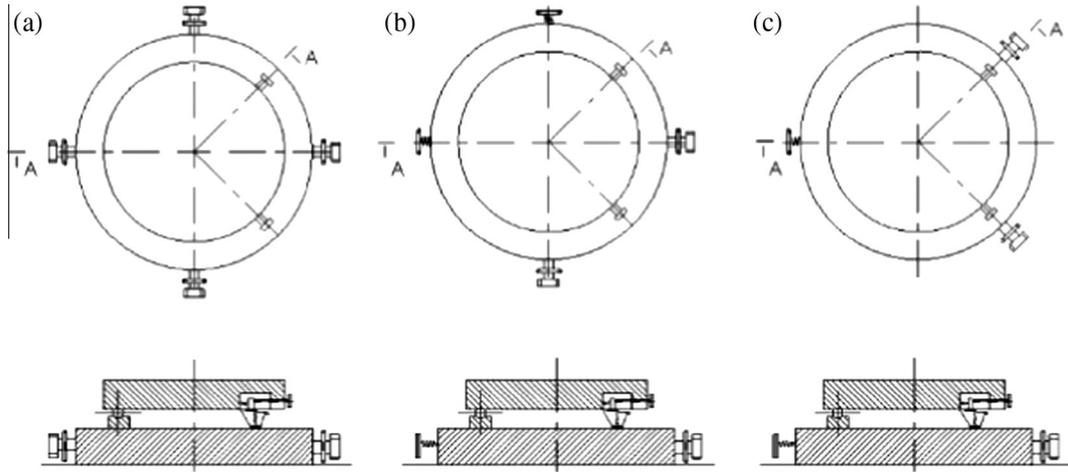
The initial layout for the centering–leveling system, designed to increase the accuracy of the measurements that can be made, is presented in Fig. 1.

The position of centering and leveling can be adjusted on the same surface, along two perpendicular axis. The mechanisms for this process are positioned concentrically and the motion is transferred to two supports of the disk. The two mechanisms are specifically positioned to improve the accuracy of the motion and consequently the centering and leveling pitches, using mechanisms that are independent from each other. The centering position is adjusted using two circular elements with a 0.2 mm slope, and to do so this operation includes moving the main circular element. Further, the leveling position is adjusted using two cams with cone-shaped endings mounted on their end. Thus rotating the cams to one side or the other provides the motion required for leveling. The device developed can be easily utilized for other measuring systems to eliminate the type of errors caused by the position of the measured object.

## 3. Experimental approach

### 3.1. Modeling and system design

The experimental identification of modal parameters was carried out and is described further in the paper. This experiment is based on an empirical approach, where the data is gathered from monitoring the experimental performance.



**Fig. 1.** Centering-leveling layouts: (a) centering using four bolts; (b) centering using a pair of bolts and a pair of springs; and (c) centering using a pair of bolts and a single spring.

The dynamic parameters of the table can be described by the following formula given by Eq. (1):

$$[A]\{\ddot{q}(t)\} + [B]\{\dot{q}(t)\} + [C]\{q(t)\} = 0 \quad (1)$$

where (A), (B), (C) are matrices of inertia, damping and stiffness;  $\{q(t)\}$ ,  $\{\dot{q}(t)\}$ ,  $\{\ddot{q}(t)\}$  are vectors of velocity and acceleration.

The solution for the above of this phenomenon can be expressed as is shown in Eq. (2):

$$\mathbf{q}(t) = \sum_{k=1}^{2n} \mathbf{z}_{(k)} e^{\lambda_k t} \quad (2)$$

Therefore, the reaction of the system is modal with the base  $[\mathbf{z}_{(1)} \dots \mathbf{z}_{(2n)}]$ . Considering the time momentum,  $2n$ , is being evaluated, the data are collected in matrix the  $Q$  given by:

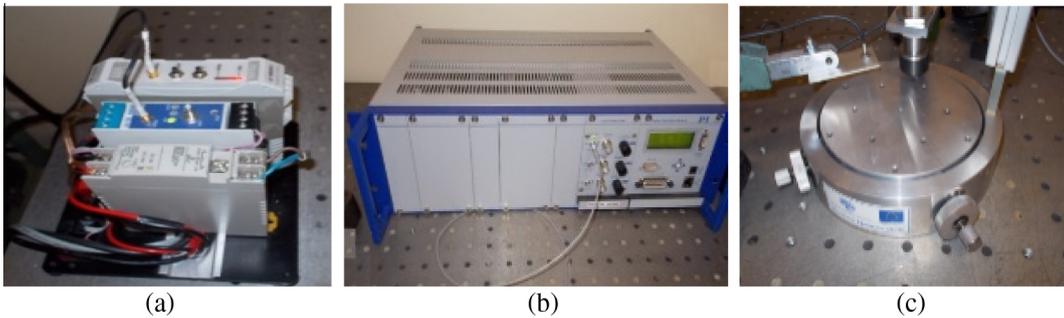
$$\underbrace{[\mathbf{q}(t_1) \dots \mathbf{q}(t_{2n})]}_Q = \underbrace{[\mathbf{z}_{(1)} \dots \mathbf{z}_{(2n)}]}_Z \underbrace{\begin{bmatrix} e^{\lambda_1 t_1} & \dots & e^{\lambda_1 t_{2n}} \\ \vdots & \ddots & \vdots \\ e^{\lambda_{2n} t_1} & \dots & e^{\lambda_{2n} t_{2n}} \end{bmatrix}}_A \quad (3)$$

Considering the same expression but with a delay of interval  $t$ , it may be expressed by the following:

$$\begin{aligned} \underbrace{[\mathbf{q}(t_1 + \Delta t) \dots \mathbf{q}(t_{2n} + \Delta t)]}_{Q_{\Delta t}} &= [\mathbf{z}_{(1)} \dots \mathbf{z}_{(2n)}] \cdot \begin{bmatrix} e^{\lambda_1(t_1 + \Delta t)} & \dots & e^{\lambda_1(t_{2n} + \Delta t)} \\ \vdots & \ddots & \vdots \\ e^{\lambda_{2n}(t_1 + \Delta t)} & \dots & e^{\lambda_{2n}(t_{2n} + \Delta t)} \end{bmatrix} \\ &= [\mathbf{z}_{(1)} e^{\lambda_1 \Delta t} \dots \mathbf{z}_{(2n)} e^{\lambda_{2n} \Delta t}] \cdot \begin{bmatrix} e^{\lambda_1 t_1} & \dots & e^{\lambda_1 t_{2n}} \\ \vdots & \ddots & \vdots \\ e^{\lambda_{2n} t_1} & \dots & e^{\lambda_{2n} t_{2n}} \end{bmatrix} \\ &= \underbrace{[\hat{\mathbf{z}}_{(1)} \dots \hat{\mathbf{z}}_{(2n)}]}_{\hat{Z}} \cdot \underbrace{\begin{bmatrix} e^{\lambda_1 t_1} & \dots & e^{\lambda_1 t_{2n}} \\ \vdots & \ddots & \vdots \\ e^{\lambda_{2n} t_1} & \dots & e^{\lambda_{2n} t_{2n}} \end{bmatrix}}_A \end{aligned} \quad (4)$$

The third reaction of the system, with a delay of interval,  $2t$ , is expressed as follows:

$$\underbrace{[\mathbf{q}(t_1 + 2\Delta t) \dots \mathbf{q}(t_{2n} + 2\Delta t)]}_{Q_{2\Delta t}} = \underbrace{[\hat{\hat{\mathbf{z}}}_{(1)} \dots \hat{\hat{\mathbf{z}}}_{(2n)}]}_{\hat{\hat{Z}}} S \underbrace{\begin{bmatrix} e^{\lambda_1 t_1} & \dots & e^{\lambda_1 t_{2n}} \\ \vdots & \ddots & \vdots \\ e^{\lambda_{2n} t_1} & \dots & e^{\lambda_{2n} t_{2n}} \end{bmatrix}}_A \quad (5)$$



**Fig. 2.** The means for vibration analysis: amplifier and power source (a), control unit (b), "Lion precision" shift sensors U20B and U3B (c); Piezo-ceramic excitation source (c).

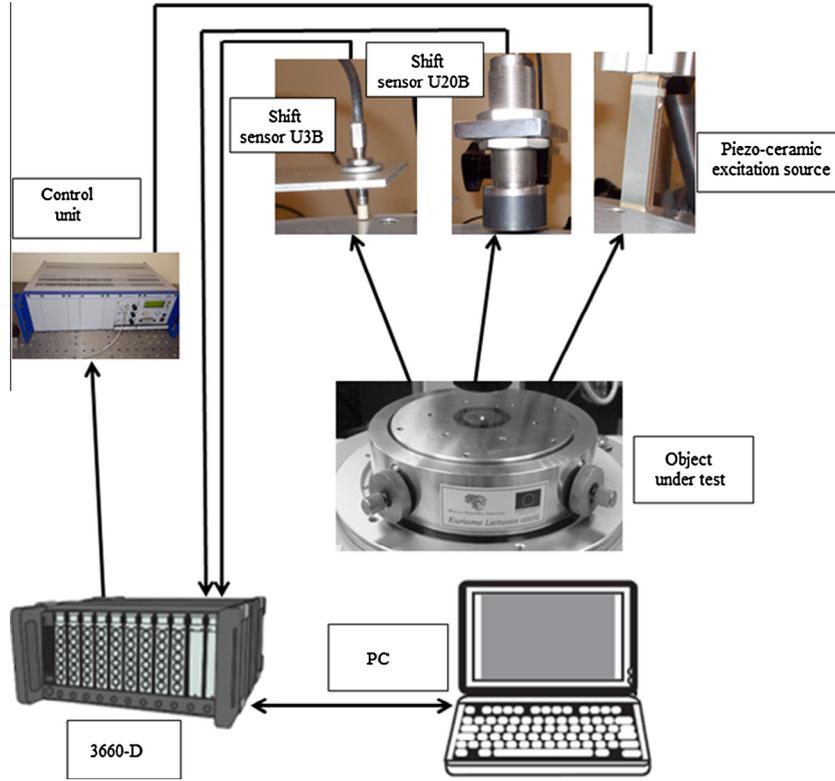


Fig. 3. General overview of the instrumental setup used in this work.

where  $\hat{z}_{(i)} = z_{(i)} e^{2\lambda_i \Delta t}$ .

Merging Eqs. (3) and (4) results in:

$$\begin{bmatrix} Q \\ Q_{\Delta t} \end{bmatrix} = \begin{bmatrix} Z \\ \hat{Z} \end{bmatrix} A \quad \text{ir} \quad \Phi = AA \quad (6)$$

In an analogous way, merging Eqs. (4) and (5), results in the following, Eq. (7):

$$\begin{bmatrix} Q_{\Delta t} \\ Q_{2\Delta t} \end{bmatrix} = \begin{bmatrix} \hat{Z} \\ \hat{Z} \end{bmatrix} A \quad \text{ir} \quad \hat{\Phi} = \hat{A}A \quad (7)$$

Eliminating the matrix between Eqs. (6) and (7) results in the following:

$$\hat{\Phi} \Phi^{-1} A = \hat{A} \quad (8)$$

If  $A = [a_1 \dots a_n]$  and  $\hat{A} = [\hat{a}_1 \dots \hat{a}_n]$ , the expression given in Eq. (9) provides association between elements  $a_i$  from  $A$  to  $\hat{a}_i$  from  $\hat{A}$

$$\hat{\Phi} \Phi^{-1} a_i = \hat{a}_i \quad (9)$$

Additionally, due to  $\hat{a}_i = e^{2\lambda_i \Delta t} a_i$ ,

$$\hat{\Phi} \Phi^{-1} a_i = e^{2\lambda_i \Delta t} a_i \quad (10)$$

This relationship given in Eq. (10) is expressed as an eigenvalue equation, where  $n$ , the first coordinate of vector  $a_i$  is similar to the shape of the construction of Eq. (6). The eigenvalue allows for equality to  $e^{2\lambda_i \Delta t}$ , where  $\lambda_i$  is the value of the complex structure. Eq. (8) reveals that the order of matrices  $\hat{\Phi} \Phi^{-1}$  should be equal to double the mode, this acquired by measuring after excitation. To simplify this procedure that is used, the method of least squares for error minimization can be used. It enables more data to

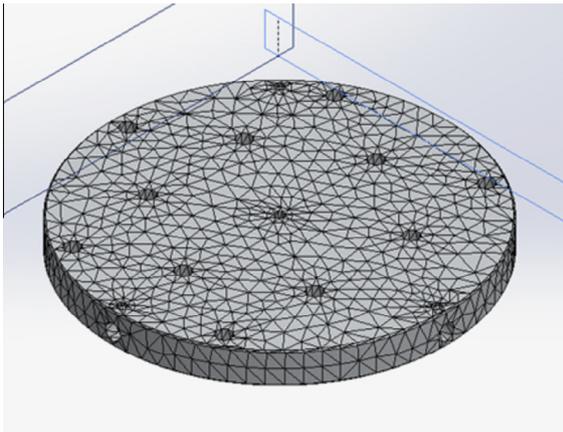


Fig. 4. Schematic view of the upper plate of the centering-leveling device divided into finite elements.

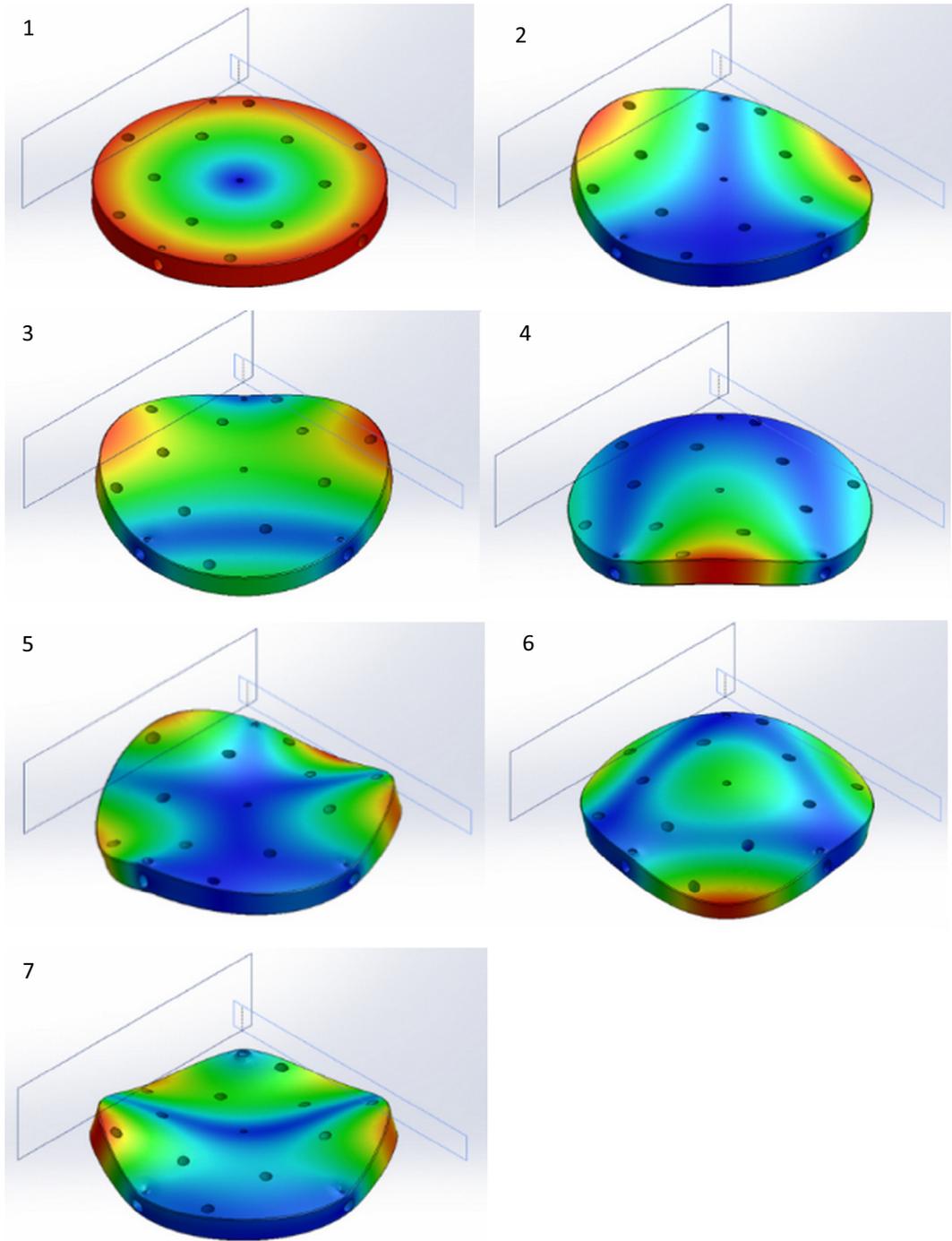


Fig. 5. Resultant modal shapes of upper plate of the centering-leveling device.

be taken into account by using a greater number of modes.  $\Phi$  and  $\hat{\Phi}$  are square matrices and therefore the Eq. (10) can be transformed into Eq. (11) as follows:

$$\left[ \hat{\Phi} \Phi^T \right] \left[ \Phi \Phi^T \right]^{-1} a_i = e^{\lambda_i \Delta t} a_i \quad (11)$$

### 3.2. The dynamics of the table

The mechanical instability, influencing the qualitative parameters of the system is next discussed in this research based on modal analysis.

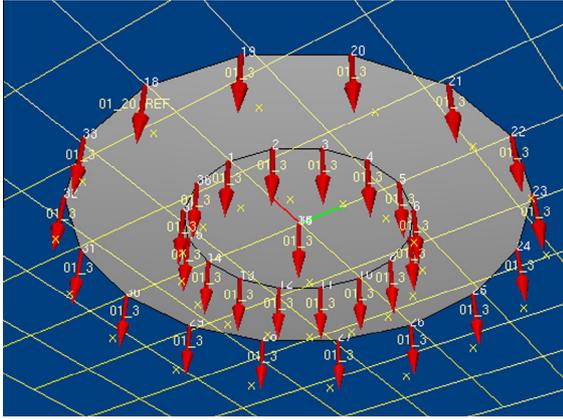


Fig. 6. Measurement points of the centering-leveling device.

The function of the response to a certain frequency is one of the most effective means of acquiring a frequency-related model of the system. The frequency response function is the spectrum of response, expressed as a spectrum of excitation with a system specific ratio given in Eq. (12):

$$X(\omega) = H(\omega) \cdot F(\omega) \quad (12)$$

The system specific ratio  $H(\omega)$  may be expressed as:

$$H(\omega) \equiv H(\omega)/F(\omega) \quad (13)$$

This describes a complex relation between the excitation and response as a function of frequency  $\omega$ . The complex ratio leads to introduction of multiplier  $|H(\omega)|$  and phase  $\times H(\omega) = \varphi(\omega)$ .

From the physical point of view, the response function reveals that the sinusoidal force of excitation with a frequency,  $f$ , would generate a sinusoidal response force of the same frequency. The amplitude of the response force will be magnified by  $|H(\omega)|$ , and the difference between the excitation and response force phases will be equal to  $\times H(\omega)$ . The result response function provides the modal

data, the eigenvalue frequencies and the errors and the damping ratios of the system [4].

### 3.3. Experimental arrangement and measurements

The key parameters of the system vibration were measured using instrumentation produced by Brüel & Kjær and this has included:

- A Mobile measurement result storing and processing module 3 (TYPE 660-D) as shown in Fig. 2,
- A PC (DELL), as shown in Fig. 3,
- Shift sensors U20B and U3B (Lion Precision), as shown in Fig. 2c with amplifiers and adapters (Fig. 2a), a piezo-electric excitation source (Fig. 2c) and the inclusion of a control unit (Fig. 2b).

A general overview of the instrumental setup used for the determination of the dynamic features is presented in Fig. 3.

For convenience, portable equipment for the oscillation measuring system control, data processing and measurement result storage was used for the experiment as this could be readily used in other applications.

## 4. Results of the investigation

The results of research into the dynamics of the centering-leveling device are presented later in this section. However, in order to simulate the most accurate response of the upper plate of centering-leveling device to excitation and thus to determine the possible resonant frequencies, a finite element method (FEM) analysis was performed [11,16]. In this, the upper plate of centering-leveling device is simulated being divided into finite elements is shown in Fig. 4. Fig. 5 represents first seven mode shapes of the upper plate of the centering-leveling device.

After performing the modal analysis by using the finite element method, seven different modal shapes as well as the correct frequencies and their shifts were determined.

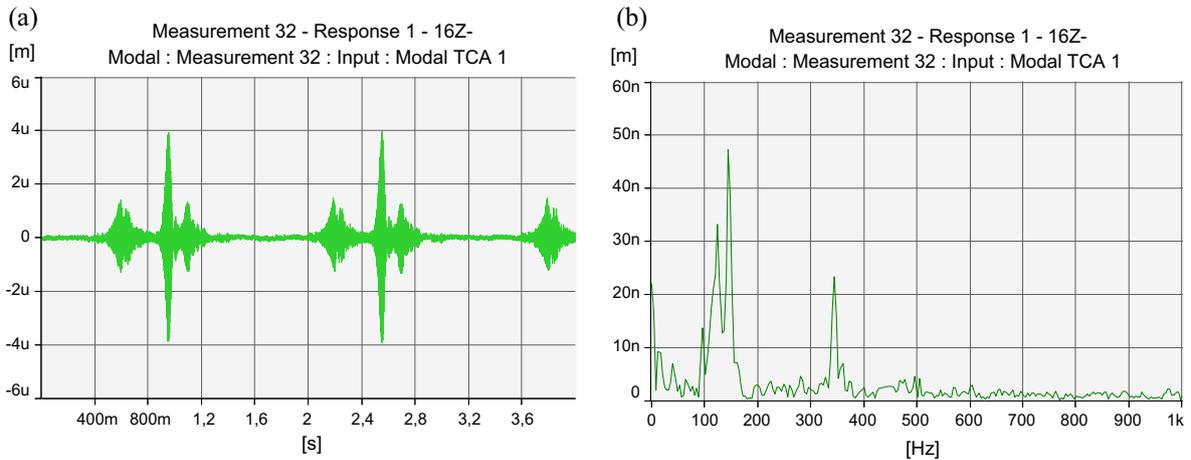
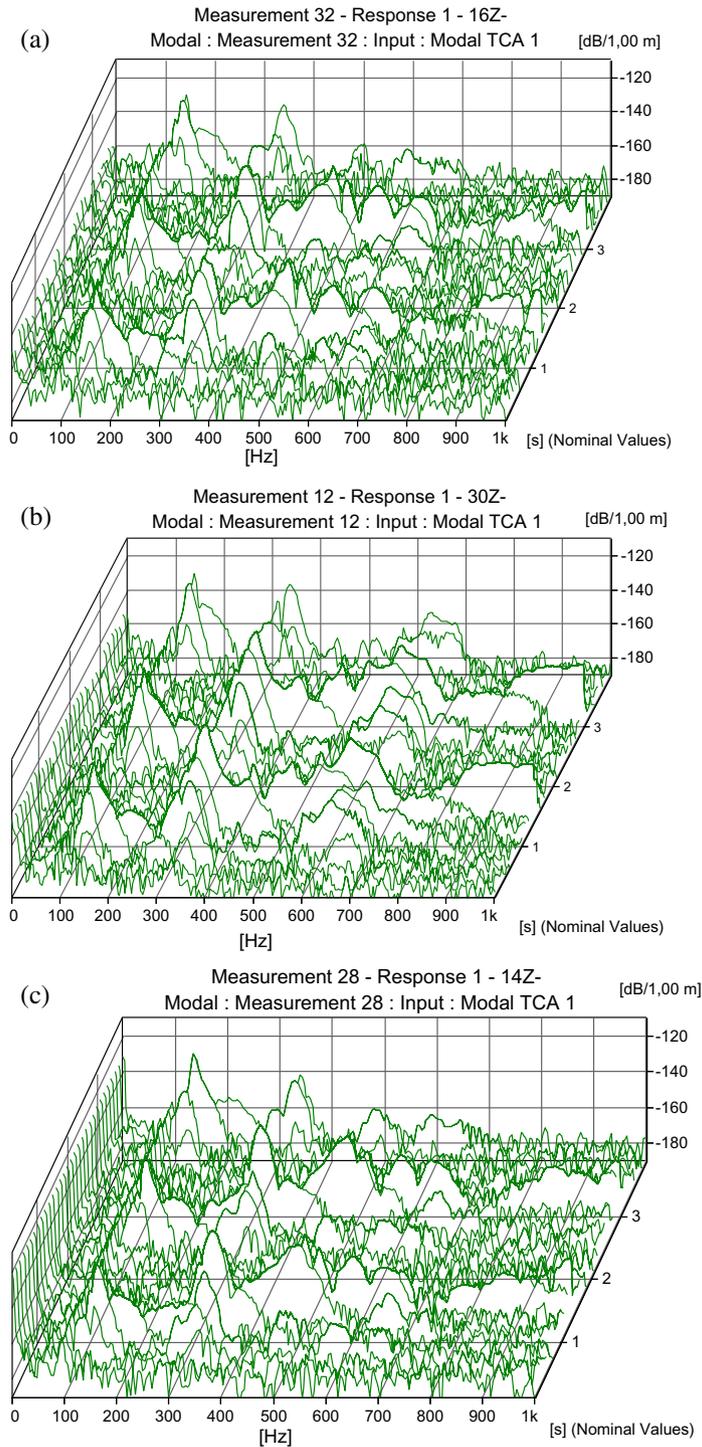


Fig. 7. Illustration of the movement of the center point of centering-leveling table (a) and power spectral density of the noise (b).



**Fig. 8.** Frequency fluctuations of the center point (a), inner circle (b) and outer circle (c) of the centering-leveling device.

The frequencies determined from the analysis were 298 Hz, 431 Hz, 593 Hz, 1121 Hz, 2312 Hz, 2510 Hz and 2623 Hz for 1, 2, 3, 4, 5, 6 and 7 modal shapes respectively. The measured points for the modal analysis of the centering-leveling device are shown in Fig. 6.

The structure of the centering-leveling device as well as the improvement of measurement process, thus avoiding harmful working modes which excite oscillations of the device, were determined, based on the research data developed in the model and illustrated in Fig. 5.

**Table 1**  
Comparison of theoretical and experimental modal analysis.

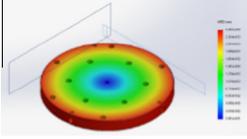
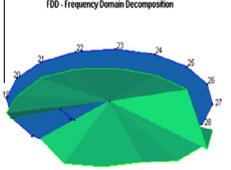
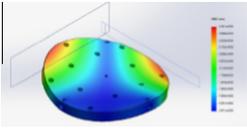
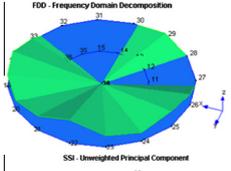
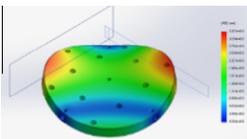
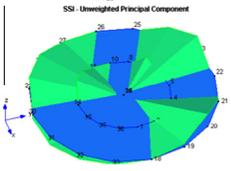
Mode	Theoretical		Experimental	
	Frequency Hz	Shape	Frequency Hz	Shape
1	298		312	
2	431		423	
3	593		570	

Fig. 7 shows the motion amplitude of the center point of the device under testing and power spectral density of the noise which both show the response to the external excitation. The shape of the excitation signal generated by piezo ceramic excitation source by using the swept-sine method is shown in Fig. 7 where  $x$  axis represents the time during which the signal was measured and  $y$  axis shows the range of motion of the center point.

Fig. 8 shows a series of 3-D plot of noise spectrum. Here frequency fluctuations that relate both to time and shift amplitudes are displayed. Fig. 8a–c shows greater levels of detail and represents the parameters of center point, inner circle and outer circle of the centering-leveling device respectively.

It is inevitable in simulations of this type that minor differences will appear between the results of the theoretical model and the outputs that represent the experimental data. The comparison between the experimental and the theoretical results is affected by any differences between the actual and the simulated material properties and any imperfections in geometry that may result from the fabrication of the device [19,18]. However, it is important to compare both the theoretical and the experimental mode shapes obtained from these two analyses of the centering-leveling device.

As shown in Figs. 5 and 8, the dominant frequencies are 312 Hz, 423 Hz and 570 Hz for the first, second and third modes respectively. The biggest portions of the surface of the device are affected by these frequencies and therefore it could have a direct impact on the comparison between the results of actual measurement and of simulation. This comparison is displayed in Table 1. For the figures shown for the experimental work, the green color in the diagrams in the Table 1 represents the experimental modal analysis

and shows the deformed surface of the instrument when under test. By contrast, the blue color represents the non-deformed parts of the system geometry. The results of the experimental modal analysis for mode 1 and mode 2 were obtained by using the frequency domain decomposition technique and, for mode 3, the stochastic subspace identification technique with unweighted principal component [15,16].

During the measurement process random errors were unavoidable and an electrical signal was obscured by various sources of noise – shot noise, thermal noise, partition noise, generation-recombination noise, flicker noise, etc. [6].

## 5. Conclusion

The research undertaken has enabled data to be obtained from both experimental and theoretical studies of a centering-leveling system and this has showed that shapes, higher than the first mode can be excited in vertical plane. Moreover, the comparison of experimental data to that from the theoretical approach yields a very acceptable deviation in the range of 1.9–4.5%. Although the experiment deals with the three first shapes of the table, bounded by 1000 Hz, further research of these frequencies during the operational period should be considered and this is the subject of on-going work.

## Acknowledgement

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