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Glass slippers and glass ceilings: A positive analysis of gender inequality and marriage

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ABSTRACT

This paper studies the combined effect of marriage and gender wage discrimination on female education and labour market participation. Given wage discrimination, marriage increases the proportion of time women spend in housework, biasing their education downwards. The bias is not just relative to men, but also relative to single women and is discontinuous in the gender wage gap. Furthermore, consensual marriages might restrict female labour force participation and education more than non-consensual ones and a proportionate increase in male and female wages could further restrict them. The latter prediction is consistent with the U-shaped relationship found in the empirical literature between female labour force participation and economic growth.

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1 Introduction.

It is generally true that in developing countries female children receive less education than male ones. This phenomenon is robust to differences in culture and levels of per-capita national income.[1] Indeed when tertiary education is factored in, it could be argued that such a bias exists even in the developed countries, remaining latent at lower levels of education.[2]

There is also evidence of anti-female bias in child nutrition and healthcare, important determinants of child survival probability (see, *e.g.* Khanna *et. al.* [2003]). Apart from many cultural explanations for such biases (such as son-preference) a common economic explanation is that they represent optimal responses of households to gender inequalities in returns to labour and human capital (see Rosenzweig and Schultz [1984] for a seminal investigation of this explanation). Faced with lower returns to females, parents shift resources towards males. At the same time, while it has been noted that gender biases in survival are mainly a matter of poverty and economic insecurity (see Deaton [1989] and Rose [1999]), those associated with education persist even when incomes rise above the poverty level. This could be because ensuring children's survival is cheaper and involves fewer tradeoffs than providing them with education, so that while escaping extreme poverty might be sufficient to erase gender inequalities in child survival, it might not be enough to erase the anti-female bias in education.

One factor likely to influence educational choice is the expected use of time each child will make when he or she grows up. In particular the prospect children will marry on reaching maturity can be an important influence on their (*i.e.* their parents') educational choices when they are young. This paper analyses this relationship, arguing that marriage can create a distinct anti-female bias in education which exacerbates that arising from labour market inequalities alone.

The argument is made in three steps. In the first step, we show that the presence of gender wage discrimination induces a marital division of labour which encourages specialisation: a male is more likely to work in the market, a female more likely to work at home and even when one partner does not specialise, the other does.

In the second step, we show that the anticipated marital division of labour exerts a multiplicative and discontinuous influence on children's education. The influence is multiplicative in that a female who is expected to get married might be subject to an even stronger anti-education bias than if she were expected to stay single, while a male who is expected to get married experiences an even stronger pro-education bias than if he were expected to stay single. The intuition is that a married female is expected, under the marital division of labour, to spend more time in housework than would her 'single self' and thus, receives less formal education and more training in homemaking skills as a child. The influence is discontinuous in that even a miniscule degree of gender wage inequality can create a discrete difference in male and female educational levels under the anticipation of marriage while this difference would remain miniscule if children were expected to grow into single adults.

[1] See Dreze and Gandhi-Kingdon [2000], Grootaert [1998], Ilahi [1999], Ilahi and Sedlacek [2000], Ray [2000]. As an exception, Munshi and Rosenzweig [2004] find that among lower-caste Marathas, girls are more likely than boys to receive a modern English education, as opposed to a traditional Marathi one. Both types of findings are broadly consistent with the arguments of this paper.

[2] In developed countries the bias tends to be reflected in fields of study at university rather than in enrollments, a dimension in which females can actually outnumber males. There is considerable evidence that women tend to pursue subjects that translate into lower paid careers than do men (see Folbre and Badgett [2003]).

In the final step, we analyse the marriage decision itself, under the assumption that each young adult can choose between getting married or staying single.[3] We show that under gender wage inequality a member of the privileged gender can demand a minimum level of homemaking skills from a prospective partner. The intuition here is that under the marital division of labour, the privileged spouse will provide an income subsidy to his partner so he expects something in return which, in the context of this paper, boils down to a minimum level of household skills. This 'marital desirability' constraint can limit female education even more than the downward bias that is found in step two. Indeed females with a high subjective desire to attain education might foresake marriage altogether rather than subject themselves to this constraint.

The argument that the prospect of marriage can create a downward pressure on female education and labour market participation is anecdotally familiar, especially in developing countries. But to our knowledge it has rarely been subject to economic analysis. In one exception, Lahiri and Self [2007] analyse a model of educational choice taking into account the prospect of marriage. While boys are expected to remain part of the parental household upon growing up, bringing brides into the fold, girls are expected to leave and join their husbands' parental household. Parents discount the future earnings of their daughters as they are expected to accrue to their in-laws and this leads to an under-investment in daughters' education.[4]

Lahiri and Self's explanation is complementary to but separate from the one made in this paper. Here, all children become financially independent of the parental household upon growing up so that the attribution of future income is not a concern. The source of anti-female bias is due to the household division of labour for married couples. Moreover, the anti-female educational bias in Lahiri and Self is a result of an inter-household externality and therefore inefficient, while in our model there is no inter-household externality and therefore the anti-female bias is not necessarily a matter of inefficiency. Indeed, if we assume that each individual receives a non-pecuniary, private benefit from education, it is possible for both male and female education to become inefficiently high for married couples, even while the anti-female bias exists in the positive sense.

Another paper to study the effect of marriage prospects on educational and occupational choice is Folbre and Badgett [2003]. Their argument is that the marriage market reinforces gender stereotypes with respect to occupational choice by penalising women for pursuing non-traditional occupations. Testing this hypothesis by measuring student responses to personal ads, they found that both men and women who reported holding non-traditional occupations were rated as less attractive, holding other attributes constant. Our own analysis both reinforces this insight and suggests an underlying reason why a female who holds a non-traditional occupation might lose desirability in the marriage market – her choice of occupation could reflect an educational background (coupled with unobservable personal traits) which detract from 'desirable' homemaking skills.[5]

[3]The matching framework in this paper is very simple and involves a yes/no decision by a single prospective bride and groom. A relatively simple extension to a multi-period, multi-agent model can conceivably be made without sacrificing most of the major insights, so long as it is assumed that all males and all females were identical. A more meaningful extension, with heterogeneity of potential partners along the lines of search theoretic models, is outside the scope of the present paper.

[4]More recently, Lahiri and Self [2008] extend this line of argument to explain the anti-female bias in survival ratios. They argue that in the presence of costly health care, both labour market discrimination and an inter-household externality can lead to such a bias.

[5]Our papers are different in several aspects but most importantly Folbre and Badgett take preferences over marital partners' characteristics as given while in our model these are derived

Apart from the insight that marital cohabitation induces a discontinuity in the educational and labour market gender gap, our model produces some other unexpected results. One is that labour market discrimination alone can create a basis for consensual marriage through inducing a pattern of 'comparative advantage' between the genders which is then reinforced by their respective educational choices when young. In other words, under circumstances in which a potential couple might reject marriage under conditions of perfect labour market symmetry, even a small amount of gender bias in wages, through its impact on educational choice, can create a non-trivial space for the same couple to agree to marriage.

Another result is that when female wages are low to begin with, economic growth which leads to a proportionate (discrimination-preserving) increase in all wages can lead to a tightening of the marital desirability constraint and a decline in labour force participation and education for married females. This is because, when female wages are low to begin with, an increase in their wages has little (or indeed no) effect on their contribution to the married household's income. Therefore, the implicit intra-household income subsidy from male to female can increase with an equi-proportionate increase in wages, making men more demanding of their prospective partner's housework skills. This suggests a decline in female education and labour force participation at the early stages of economic growth.

This prediction is consistent with findings that female labour force participation first falls and then rises as a result of economic growth (see, *e.g.* Psacharopoulos and Tzannatos [1987]). This non-monotonicity has usually been attributed to changes in sectoral composition during the early stages of growth. Agriculture, which is more conducive to female employment, declines while industry, which is less conducive at least initially, grows (Psacharopoulos and Tzannatos). The mechanism of our model is not inconsistent with this explanation and could well shed light on why industry, in which employment is more formal than in agriculture and the required skills more dependent on formal education, is more likely to be strengthened as the preserve of men during the early period of expansion.

The rest of the paper is organised as follows. Section 2 describes the model. Section 3 analyses the labour market and educational decisions of single individuals. Section 4 analyses the analogous decisions for married individuals. Section 5 analyses the marital decision and section 6 offers concluding remarks.

2 The model.

There are two households, labelled X and Y respectively. The households have one offspring each, intrinsically identical except for gender. There are two periods: in the first period, each offspring is a child and in the second, each is an adult. In each period, they each have a time endowment equal to unity.

In the childhood stage, each parent decides on how to allocate the child's time between formal schooling and domestic chores. This allocation determines the level of market versus household skills that the child grows up with.

endogenously, albeit with a slightly different characteristic in mind. While Folbre and Badgett abstract from labour market discrimination and assume that a partner's occupation is in itself a source of attraction, in our model marital desirability is shaped by labour market discrimination so what makes a male attractive is income-earning capacity while for females homemaking skills, either alone or in combination with income-earning capacity, play a role.

Labeling the time spent in schooling as s, and the level of human capital as e, we assume that the latter is a linear function of the former: $e = \sigma s$. σ represents the maximum level of education that can be attained. For expositional purposes, however, we shall assume from hereon that $\sigma = 1$ and therefore, $e = s$.[6]

2.1 Home skills and market skills.

Time *not* spent in education can now be measured as $1-e$. This time is spent on household chores which lead to the formation of adult domestic skills, labeled as α. We assume that the relationship between α and $1-e$ is represented by a function $\alpha = \alpha(1-e)$ such that:

(A-1) $\alpha(0) \equiv \alpha_0 \geq 0$, $\alpha'(0) > 0$; $\alpha(1) \equiv \alpha_1 > 0, \alpha'(1) < 0$.

(A-2) $\alpha''(1-e) < 0$; $\forall e \in [0,1]$.

(A-3) $\exists\ \hat{e} \in (0,1]$ such that $\alpha(1-\hat{e}) \equiv \bar{\alpha} = \max\{\alpha(1-e)\}\ \forall\ e \in [0,1]$.

Assumption (A-1) implies that even without any household training when young, each adult will possess some household skills, but that starting from this corner, time spent in household training will increase these skills.

Assumption (A-2) states that this relationship is strictly concave; thus, assumption (A-3) guarantees that a child who combines domestic training with some formal schooling can grow up having more household skills than one with either no schooling or too much schooling. This is because formal schooling can instill basic skills, such as literacy and numeracy, which are useful for housework. The degree of complementarity between formal education and household skills, and thus the turning point in the formation of household skills can vary according to the social stratum to which the children belong. At higher social levels, not only literacy and numeracy but also exposure to the arts, history and literature might be important for the development of domestic skills. What is important is that there is some point at which further development of market skills comes into conflict with the formation of household skills.[7]

At any value of s, then, α and e are uniquely related. This relationship is characterised by the function:

$$\alpha(e) = \alpha\,(1-e)$$

where $\alpha = \alpha_0$ when $e = 1$, $\alpha = \alpha_1$ when $e = 0$ and $\alpha = \bar{\alpha}$ when $e = \hat{e}$.

Figure 1 below shows the combinations of household skills and formal education that go together given the choice of s.

[**Figure 1** in here]

[6]In principle, allowing σ to remain a free parameter would be useful for carrying out comparative static experiments regarding the impact of changes in school quality on educational choice and subsequent time use as adults. However, none of the questions asked in this paper use such an experiment so we normalise σ to be unity.

[7]A-2 and A-3 and the assumption that $\alpha'(0) > 0$ are not necessary for the results of this paper which require only that there is some interval of values of s over which homemaking skills decline as s goes up.

The horizontal axis measures education, e, which lies between zero and unity. The vertical axis measures α. Time spent in housework as a child decreases as s rises. Accordingly, at $s = 1$, $1 - s = 0$ and $\alpha = \alpha_0$. At $s = 0$, $\alpha = \alpha_1$. For intermediate values of s, α first increases and then decreases after peaking at $\bar{\alpha}$.

In the second period, the adult decides whether to marry or not, moves out accordingly and makes a time-allocation decision between market work (ℓ) and housework ($1-\ell$). This decision is made conditional on the individual's marital status and hourly market wage. The latter in turn depends on an underlying wage ω_i and the individual's human capital e_i through the linear function:

$$w_i = \omega_i e_i \tag{1}$$

where w_i represents the hourly wage, and ω_i is an underlying wage per unit of human capital.

Wage inequality is said to exist if ω is not the same for both genders. In this paper we take this to be an exogenous possibility.

2.2 Preferences and constraints.

We assume that childhood utility is separable from adult utility and does not depend on the choice between schooling and household chores. Each adult has a utility function:

$$U_i = u(c_i) + h_i + \delta z + be_i \tag{2}$$

$i = X, Y$; $u(\cdot)$ is a concave function of the adult's own consumption of a market good, satisfying $u'(0) = \infty$; h_i is the utility from consumption of a household public good; z is an index variable which takes on the value 0 if the agent remains single, and the value 1 if the agent gets married; $\delta \geq 0$ is the utility of companionship in marriage. [8]

The last term in the utility function represents a non-marketable subjective benefit that the adult derives from education. Such a subjective benefit does not exclude the marketable pecuniary benefit; the latter is already incorporated in equation (1). The point is that education confers subjective benefits on top of marketable ones.[9] These benefits could be direct, *i.e.* education might confer pride and satisfaction for its own sake, or indirect, *i.e.* education might enable individuals to seek out information which leads to better consumer choices.

One example of the latter effect is the link between education and health, itself a source of subjective utility.[10]

[8] A positive value of companionship is not essential for this paper, indeed negative companionship in marriage would reinforce some of the comparisons between consensual and non-consensual marriage.

[9] see for example, Lazear [1977] and Schaafsma [1976] for early attempts to disentangle the two effects of education.

[10] As summarised in Cutler and Lleras-Muney [2006] a large body of research shows a robust positive relationship between health and education, even after the market returns of education have been controlled for. Their argument is that education leads to better decision-making and information-seeking and thus helps individuals maintain good health.

The subjective benefit from education could be partially shared with other members of the household, in which case it might enhance the companionship associated with marriage. The important thing is that it is not fully shared. For simplicity we assume that this benefit is linear and represented by the coefficient b.

Each adult possesses one unit of time which can be split between market work ℓ and housework $1-\ell$. The level of market consumption depends on the individual's market income. For a single individual, this is:

$$c_i = w_i \ell_i \tag{3}$$

where ℓ denotes the fraction of adult time in market work.

If married, the market good is assumed to be split equally between the two partners:[11]

$$c_X = c_Y = \frac{w_X \ell_X + w_Y \ell_Y}{2} \tag{4}$$

The utility from consumption of a household good depends non-linearly on the amount of effort put into household production according to a concave function. For a single individual, the utility is given by

$$h_i = h\left(\alpha_i(1-\ell_i)\right) \tag{5}$$

where α_i is the individual's level of household skills and $(1-\ell_i)$ is adult time spent in household production. We assume that $h' > 0$ $h'' < 0$ and that $h'(0) = \infty$.[12]

For married adults, we assume that there is (i) a single household production function per couple; (ii) their efficiency-adjusted effort levels are perfect substitutes for each other and (iii) the utility from the household good is equally shared, even when the good itself is rivalrous.[13]

$$h_X = h_Y = \gamma h\left(\alpha_X(1-\ell_X) + \alpha_Y(1-\ell_Y)\right) \tag{6}$$

where h_X, h_Y are the consumption shares of the household good by each partner, and the function $h\left(\alpha_X(1-\ell_X) + \alpha_Y(1-\ell_Y)\right)$ represents the output of the good. γ measures the degree of rivalrousness in the consumption of the household good. $\gamma = 1$ implies that the household good is completely non-rivalrous, so that each partner can consume the full output without crowing each other out, while $\gamma = 0.5$ implies it is completely rivalrous. Although much of the literature on household economics assumes that household goods and services are non-rivalrous, any value of γ strictly greater than 0.5 will meet this assumption, so we treat γ as an open parameter.[14]

2.3 Decision making.

We first impose the state of being married or remaining single on the adults and within each state we solve backwards for their time allocation decisions. In solving for adult time

[11]This would result anyway from unitary decision-making and homogeneous preferences, as is the case here.

[12]Note that function $h(\cdot)$ can be interpreted two ways: either that the household production technology is itself concave or that it is linear but the utility of household goods is concave.

[13]The last assumption is again the result of unitary decision making under symmetric preferences.

[14]Intuitively, some aspects of household production seem more rivalrous than others: for example, ironing or making sandwiches should be more rivalrous than gardening or house-cleaning.

allocation we take as given the level of childhood schooling; then in solving for the latter we take into account its effect on adult time allocation.

Once the time allocation decisions have been solved for each state of marital status we consider the marital decision itself. We consider a non-consensual situation in which marriage is imposed exogenously on the two individuals, with a mechanism in which either party can veto the match.

Under the latter mechanism, we solve the second-stage problem of the marital decision by evaluating the utility from getting married or remaining single for each adult, given optimal time allocation decisions in each marital state. For marriage to take place, neither party should be worse off getting married than remaining single. Otherwise, one or both of them will veto the match.

3 Time allocation when single.

On reaching adulthood, the agents human capital e is fixed; hence so are terms such as $\alpha(e)$, be and $w = \omega e$ (agent subscripts are suppressed since the problem is qualitatively identical for both). The adult maximises utility with respect to labour market participation, ℓ.

Plugging the adult budget constraints into the utility function, the maximisation problem is expressed as :

$$\max_{\ell} \; u(\omega e \ell) + h(\alpha(e)(1-\ell)) + be$$

which has first-order condition:

$$u'(\cdot)\omega e - \alpha(e)h'(\cdot) = 0 \tag{7}$$

For the single agent, specialisation between housework and market work is not an option because of the Inada conditions assumed above. Hence, the first-order condition holds as an equality.

The interpretation is analogous to the one in the standard case of endogenous labour supply when leisure counts for its own sake. Here, we ignore that aspect of leisure and the trade-off becomes one between market and home labour. A small increase in market labour increases utility from the market good by $u'(\cdot)\omega e$ but reduces that from the home good by $h'(\cdot)\alpha$. At the optimum, the two effects cancel out.

The resulting solution can be expressed as $\ell(\omega, e)$. As is well known, a non-monotonic relationship between labour supply and the market wage is possible. A necessary and sufficient condition to rule this out is:

(A-4) $u'(c) + u''(c) \cdot c > 0$

Under (A-4) it can be established that $\ell_\omega > 0$ (see Lemma 3, Appendix). Since changes in e can also affect equation (7) through the home production function, (A-4) is by itself not sufficient to rule out $\ell_e \leq 0$ but imposing a further sufficient condition ensures that $\ell_e > 0$:

(A-5) $h'(\alpha(1-\ell)) + h''(\alpha(1-\ell))\alpha(1-\ell) > 0$

(A-4) and (A-5) are in line with conventional restrictions imposed to prevent 'backward bending' labour supply and allow us to set benchmarks for comparing time allocation across genders and marital states.

The decision on education is taken by parents on behalf of the agent, taking into account the implications of childhood education on adult time use. The problem can be expressed as:

$$\max_{e} \quad u(\omega e \ell(e)) + h(\alpha(e)(1-\ell(e))) + be$$

The first-order condition is:

$$u'(\cdot)\omega\ell + b + h'(\cdot)(1-\ell)\alpha'(e) \geq 0 \tag{8}$$

The first-order condition can only be satisfied at $e \geq \hat{e}$. But since there is no incentive to choose $e < \hat{e}$, where $\hat{e}$ is the amount of education where household skill α is maximised, this is not a binding restriction. If the first-order condition is satisfied with equality, $e \leq 1$. If, as a strict inequality, $e = 1$. The intuition is that a small increase in education will increase the utility from consumption, at given wages and market labour supply by an amount $u'(\cdot)\omega\ell$ and the subjective utility from education by b, while the utility from home production will fall, at given levels of home work and production, due to a fall in home skills α by an amount $\alpha'(e)$. Again these marginal effects cancel out at an interior optimum.

Once again, (A-4) is necessary and sufficient for $\partial e/\partial\omega > 0$ (see Lemma 3, Appendix). It is also easy to show that $\partial e/\partial b > 0$.

It is established in the Appendix, Lemmas 1-3, that (i) the relationship between ℓ and e is upward sloping along both first-order conditions; (ii) under concavity of the maximisation problem, the $\ell - e$ locus is steeper along the first-order condition for education than under that for labour and (iii) an increase in the underlying wage causes the $\ell - e$ locus for optimal labour to shift upwards and that for optimal education to shift rightwards.

These results are depicted in Figure 2 below:

[**Figure 2** in here]

The solid lines represent the first-order conditions, $\text{FOC}_e(w)$ and $\text{FOC}_\ell(w)$ respectively, at the lower wage, w, and the broken lines represent the first-order conditions at a higher wage, w'. Since the shifts in the two curves reinforce each other it is unambiguous that an increase in the underlying wage will cause both labour supply and education to increase.

We can now state the benchmark result:

Proposition 1: Under (A-4) and (A-5), if $\omega_Y \leq \omega_X$, and agents remain single, then Y will have less education and do less market work than X, *i.e.* $e_Y \leq e_X$ and $\ell_Y \leq \ell_X$.

In other words, wage discrimination leads to asymmetries in education and time allocation across single people. At the same time, the degree of asymmetry is continuous in the degree of wage discrimination. As wage discrimination gets smaller so does the asymmetry in outcomes.

We shall refer to the "single self" of a married agent as the same agent had he or she not got married and likewise, the "married self" of a single agent as that agent had he or she got married. In referring to decisions made during childhood in anticipation of the future marital state, we shall refer to them as the "prospectively single" and "prospectively married" selves.

Optimal choices of a single self are denoted by e_i^s and ℓ_i^s, $i = (X, Y)$. Optimal choices of a married agent will be denoted as e_i^m and ℓ_i^m, $i = (X, Y)$.

4 Time allocation when married.

If married, the adults decide on their respective levels of labour market participation. The household decision-making follows the *unitary* household model which assumes that decision-making is based on joint utility maximisation.[15]

The household utility function is expressed as:

$$V = u(c_X) + u(c_Y) + h_X + h_Y + b \cdot e_X + b \cdot e_Y + 2\delta$$

Since education levels are given at this stage, the adult couple maximise with respect to the first four terms. With unitary decision-making both partners will consume equal amounts of both the market and the household good so the problem reduces to:

$$\max_{\{\ell_X, \ell_Y\}} V = 2u\left(\frac{\omega_X e_X \ell_X + \omega_Y e_Y \ell_Y}{2}\right) + 2\gamma h\left(\alpha(e_X)(1-\ell_X) + \alpha(e_Y)(1-\ell_Y)\right)$$

which has first-order conditions (expressed generically due to the qualitative symmetry of the problem):

$$u'(\cdot)\omega_i e_i - 2\gamma\alpha(e_i)h'(\cdot) \geq 0 \quad i = (X, Y) \tag{9}$$

In choosing education, we assume that the parents care only for the welfare of their own offspring. We also assume that parents take into account the effect of their own child's education on own adult labour supply but not that on the spouse's. The latter is not essential since the envelope condition would apply even to the cross effects from education to adult time-use. Each parent then maximises the following analogous function:

$$\max_{\{e_i\}} u\left(\frac{\omega_i e_i \ell_i(e_i) + \omega_j e_j \ell_j}{2}\right) + \gamma h\left(\alpha(e_i)(1-\ell_i(e_i)) + \alpha(e_j)(1-\ell_j)\right) + be_i; \quad i, j = (X, Y)$$

The first-order condition is (terms involving the effect of e on adult labour supply drop out when evaluated at the optimum):

$$u'(\cdot)\omega_i \ell_i + 2\gamma h'(\cdot)(1-\ell_i)\alpha'(e_i) + 2b \geq 0 \quad i = (X, Y) \tag{10}$$

In order to analyse the first-order conditions we study the case of wage equality and that of wage discrimination separately.

[15] We use the unitary approach mainly for analytical convenience as it allows a separation of the allocation of household resources from the contribution of individual household member to joint income. The non-unitary approach assumes that each member of the household makes strategic choices in their own self-interest. Basu [2005] has shown that with non-unitary decision-making, the allocation of household resources depends crucially on each members contribution to household income. Rainer [2006] has shown that a non-unitary mechanism is if anything even more likely to induce asymmetries within the household when combined with gender wage discrimination.

4.1 Equal wages:

Suppose that $\omega_X = \omega_Y$ and assume for now that this leads to an identical level of education for both children. Then the first-order conditions with respect to adult time can be combined for the two adults.

$$\frac{\omega_X e_X}{\alpha_X} = \frac{\omega_Y e_Y}{\alpha_Y} \geq \frac{2\gamma h'(\cdot)}{u'(\cdot)}$$

The weak inequality on the extreme right must always hold as an equality for at least one of the two adults, since both cannot specialise in the same kind of work due to the Inada conditions; but if it does hold as an equality, it must do so for both spouses implying that they divide their time identically. Hence, $\ell_X = \ell_Y$.

Given that $\ell_X = \ell_Y \equiv \ell$, the first-order conditions for education will also be identical and $e_X = e_Y \equiv e$. Hence the married couple will be identical in both respects.

An interesting question is how time allocation differs between married agents and their single selves. It is useful to distinguish two cases: $b = 0$ and $b > 0$, *i.e.* the case of no private subjective benefit of education and the case when such a benefit exists.

When $b = 0$, by comparing the first-order conditions for single agents, equations (7) and (8), with those for married agents, equations (9) and (10) suppose there exist solutions for these conditions such that $e^m = e^s$ and $\ell^m = \ell^s$. Label the common values as $\tilde{\ell}$ and $\hat{e}$.

By construction, $\tilde{\ell}$ and $\hat{e}$ satisfy the first-order conditions for each activity, labour participation and education, in each marital state. This means that $\tilde{\ell}$ and $\hat{e}$ jointly satisfy:

$$u'(\omega\tilde{e}\tilde{\ell})\omega\tilde{e} - 2\gamma\alpha(\tilde{e})h'(2\alpha(\tilde{e})(1-\tilde{\ell})) = u'(\omega\tilde{e}\tilde{\ell})\omega\tilde{e} - \alpha(\tilde{e})h'(\alpha(\tilde{e})(1-\tilde{\ell}))$$

$$u'(\omega\tilde{e}\tilde{\ell})\omega\tilde{\ell} + 2\gamma h'(2\alpha(\tilde{e})(1-\tilde{\ell}))(1-\tilde{\ell})\alpha'(\tilde{e}) = u'(\omega\tilde{e}\tilde{\ell})\omega\tilde{\ell} + h'(\alpha(\tilde{e})(1-\tilde{\ell}))(1-\tilde{\ell})\alpha'(\tilde{e})$$

Since the first terms on both sides of each equation are equal by construction, whether or not the two equations are valid depends on the second term on both sides of each equation. This is also equal in each equation if and only if a single condition holds:

$$\gamma = \frac{h'(\alpha(\tilde{e})(1-\tilde{\ell}))}{2h'(2\alpha(\tilde{e})(1-\tilde{\ell}))}$$

Suppose that the above is satisfied by a value of $\gamma \in [0.5, 1]$¿ Label this critical value as $\tilde{\gamma}$.[16]

For $\gamma < \tilde{\gamma}$, when evaluated at common levels of labour market participation, $\tilde{\ell}$, and education, $\tilde{e}$, the first-order condition for a married agent's labour supply will be strictly greater than that for their single self, implying that $\ell^m > \ell^s$. Similarly the first-order condition for a prospectively married agent's education will be strictly greater than that for their prospectively single self, implying that $e^m > e^s$. For $\gamma > \tilde{\gamma}$, analogous arguments show that the inequalities are reversed.

The intuition is that when the household good is sufficiently rivalrous ($\gamma < \tilde{\gamma}$), married couples face relatively lower marginal returns to housework than to market work and thus

[16] For functions of the form $h(x) = x^\beta$ where $0 < \beta < 1$, $2h'(2x) > h'(x) > h'(2x)$, thus the above condition holds for some value of $\tilde{\gamma}$ lying strictly between 0.5 and 1.

spend more time in market work.[17] This in turn raises the marginal incentive for parents to invest in their education. The same intuition applies in reverse for the case when the household good is sufficiently non-rivalrous ($\gamma > \tilde{\gamma}$).

To consider the effect of $b > 0$, suppose again that there exist values $\tilde{\ell}$ and $\tilde{e}$ such that each first-order condition in each marital state is satisfied at these common values. The first-order conditions for market work remains the same when $b > 0$ but for education it now is

$$u'(\omega\tilde{e}\tilde{\ell})\omega\tilde{\ell} + 2\gamma h'(2\alpha(\tilde{e})(1-\tilde{\ell}))(1-\tilde{\ell})\alpha'(\tilde{e}) + 2b = u'(\omega\tilde{e}\tilde{\ell})\omega\tilde{\ell} + h'(\alpha(\tilde{e})(1-\tilde{\ell}))(1-\tilde{\ell})\alpha'(\tilde{e}) + b.$$

Since $2b \geq b$, the first-order condition for a prospective married agent's education will be strictly greater than its counterpart for their prospectively single self, given that both are evaluated at $\tilde{\gamma}$. Thus, at the given value, $\tilde{\ell}$, $e^m > e^s$ purely as a result of $b > 0$. Given the positive dependence of labour supply on education, ensured by (A-5), then $\ell^m > \ell^s$. Stated differently, for the two first-order conditions to be satisfied at common values of labour supply and education, the critical value of γ will have to be higher than before. A subjective preference for education therefore increases the space of outcomes where married agents receive more education and work more than their single selves.

This pro-education effect of marriage appears to arise because of an externality involving positive subjective benefits from education. Such benefits will induce parents to raise the educational levels of both the prospectively married and the prospectively single child but more so for the former than the latter.

To see this, first note that parents are altruistic only towards their own child, not towards the prospective spouse. Concentrating on the case where γ is at the critical value, $\tilde{\gamma}$, suppose the optimal choices for the prospectively single child have been made as functions of b and evaluate the first-order condition for a prospectively married child's education at these values of $e^s(b)$ and $\ell^s(b)$. If there is an infinitesimal increase in education from this benchmark for the prospectively married child, such an increase will bring (i) a market benefit to the household, (ii) a cost in terms of reduced household productivity and (iii) a subjective benefit that is specific to the child. The subjective benefit will therefore be fully internalised by the parent but the market benefit and the reduced household productivity will only be partly internalised because they are to be shared with the prospective spouse. Even if the balance between effects (i) and (ii) is marginally negative, this balance will be somewhat discounted in favour of effect (iii) and this will leave some room to expand $\hat{e}^m(b)$ above $\hat{e}^s(b)$.

Of course, this is a local effect that operates close to the critical value of γ. For larger values of γ, married agents tend to have less education and labour supply than their single selves so it will take proportionately larger values of b to reverse this bias.

4.2 Unequal wages:

Suppose $\omega_X > \omega_Y$. As explained in the description of the model, this represents labour market discrimination according to gender. Let us therefore call agent Y the underprivileged

[17] This is based on the assumption that utility is separable between consumption of the market good and that of the household good. Complementarity between the two might reverse this argument but it would nonetheless be of interest to note that marriage affects incentives between housework and market work even for perfectly symmetric couples.

spouse (or prospective spouse, when describing childhood decisions) and X as the privileged spouse.

Let us for now assume that when $\omega_X > \omega_Y$, it follows that $\omega_X e_X/\alpha(e_X) \geq \omega_Y e_Y/\alpha(e_Y)$. This will be established later, for now accept it to be the case.

Rearranging the first-order condition for X and Y,

$$\begin{aligned} \frac{\omega_X e_X}{\alpha(e_X)} &\geq \frac{2\gamma h'(\cdot)}{u'(\cdot)} \\ \frac{\omega_Y e_Y}{\alpha(e_Y)} &\geq \frac{2\gamma h'(\cdot)}{u'(\cdot)}. \end{aligned}$$

The left-hand side will be identical for both partners since it includes identical consumption levels. Inada conditions ensure that at least one agent will do market work and one will do housework. The following possibilities exist:

$$\text{(i)} \quad \frac{\omega_X e_X}{\alpha_X} > \frac{\omega_Y e_Y}{\alpha_Y} = \frac{2\gamma h'(\cdot)}{u'(\cdot)};$$

$$\text{(ii)} \quad \frac{\omega_X e_X}{\alpha_X} > \frac{2\gamma h'(\cdot)}{u'(\cdot)} > \frac{\omega_Y e_Y}{\alpha_Y};$$

$$\text{(iii)} \quad \frac{\omega_X e_X}{\alpha_X} = \frac{2\gamma h'(\cdot)}{u'(\cdot)} > \frac{\omega_Y e_Y}{\alpha_Y}.$$

In case (i) $\ell_X = 1$ while $\ell_Y \geq 0$; in case (ii) $\ell_X = 1$ while $\ell_Y = 0$; in case (iii) $\ell_X \leq 1$ while $\ell_Y = 0$.

It is immediately obvious that the underprivileged spouse will do less market work than the privileged spouse. The latter either specialises in market work or when he does not, the former specialises in house work.

To establish that $\omega_X e_X/\alpha(e_X) \geq \omega_Y e_Y/\alpha(e_Y)$, it is sufficient to show that $e_X \geq e_Y$. In the first two cases, with $\ell_X = 1$, $e_X = 1$, so the inequality follows trivially. In the third case, with $\ell_X < 1$, note that $\ell_Y = 0$ so the optimal value of education for agent Y satisfies

$$2\gamma h'(\cdot)(1 - \ell_Y)\alpha'(e_Y) + 2b = 0$$

Plugging in the value of e_Y that satisfies the above into the first-order condition for agent X's education

$$u'(\cdot)\omega_X \ell_X + 2\gamma h'(\cdot)(1 - \ell_X)\alpha'(e_Y) + 2b > 0$$

Since $\omega_X > \omega_Y$, $\ell_X > \ell_Y = 0$, $(1 - ell_X) < (1 - \ell_Y) = 1$ and the other terms are identical for both spouses, it is clear the above equation is strictly positive when evaluated at e_Y. Thus $e_X > e_Y$.

How do the labour market effort and educational levels of the married couple compare with that of their single selves? We shall start with the under-privileged spouse and show that the underprivileged spouse always does less market work and, at least when the subjective preference for education is not too high, gets less education than her single self.

In the two cases where the underprivileged spouse does no market work, it is is obvious that her labour supply will be less than that of her single self since the latter self undertakes positive levels of market work no matter how low the wage. In the case where both the married and single selves do market work, it can first be shown that, controlling for education, the married self of agent Y will do less market work than her single self. This is derived in the Appendix (Lemma 4).

The comparison is less clear-cut for education because of the pro-education effect of marriage when $b > 0$. As discussed in the section on equal wages, a subjective benefit from education gives rise to a pro-education externality.

To isolate the effect of the marital division of labour on the education of the under-privileged spouse, however, let $b = 0$. In this case, her education will be at its minimum value, $\hat{e}$ in the cases where she does only housework; hence it will be trivially less than that of her single self.

When she does some market work as an adult, however, it can at least be shown (Lemma 5 in the Appendix) that, for given levels of market work, her education will be (i) less than that of her single self and (ii) this gap will be greater as the public benefit from home production gets greater.

Putting the results of Lemma 4 and 5, with those of Lemmas 1,2 and 3, it can then be shown diagramatically that the married self of the the under-privileged spouse does less market work and receives less education than her single self, when both are chosen optimally. This is shown in Figure 3.

[**Figure 3** in here]

According to Lemma 4, the locus depicting the married self's optimal labour ($\mathrm{FOC}^m_{\ell_Y}$) lies always below that for her single self ($\mathrm{FOC}^s_{\ell_Y}$) while according to Lemma 5, the analogous locus for the married self's education ($\mathrm{FOC}^m_{e_Y}$) lies always to the left of that for her single self ($\mathrm{FOC}^s_{e_Y}$). Thus the optimal values, $\ell^m_Y < \ell^s_Y$ and $e^m_Y < e^s_Y$.

Turning to the privileged spouse, he unambiguously supplies more labour and receives at least as much education than his single self when he specialises in market work. Indeed because the marital division of labour drives his labour supply to unity in these cases, his education will be at its maximum level which might not be the case for his single self.

The comparison becomes less straightforward, however, when both the married and the single self undertake some housework and some market work. This possibility tends to arise when returns to labour supply are relatively low and the public benefit of marital household production is relatively high. In this case, the comparison between the labour supply effort of a single and a married version of agent X is not clear-cut, even controlling for the level of childhood education.

If, however, we restrict attention to cases where the public good nature of marital household production is not too high, *i.e.* we impose the following additional assumption:

(A-6): Let ℓ^s_X denote the optimum labour supply of agent X's single self at given values of e_X and α_X. Then[18]

$$\gamma(\ell^s_X) \leq \frac{h'(\alpha_X(1-\ell^s_X))}{2h'(2\alpha_X(1-\ell^s_X))}$$

[18]The critical value of γ that satisfies (A-6) is expressed as a function of ℓ^s_X although for functions of the form $h(x) = x^\beta$, it will be independent of ℓ^s_X and depend only on β.

(A-6) is analogous to the defininition of $\tilde{\gamma}$ in the previous sub-section. Given (A-6), it can be shown that married agent X's labour supply does indeed exceed that of his single self. This is done in the Appendix (Lemma 6). As for education, (A-6) is again sufficient to ensure that, controlling for labour supply, he receives more education than his single self. The proof of this is analogous to that of Lemma 6 and is omitted.[19]

Combining these results, it can de shown using an analogous diagram to Figure 3, and with an analogous explanation that the married self of agent X does more market work and receives more education than his single self.

Finally note that the labour supply of married adults and the education of prospectively married children reacts discontinuously to gender wage discrimination. In case (i) above, both spouses work in the labour market, but X specialises in this activity so that $\ell_X = 1$, $e_X = 1$ while Y does not so that $\ell_Y < 1$ and $e_Y < 1$. Suppose now that wage discrimination becomes progressively less, so that $\omega_Y \longrightarrow \omega_X$ from below. While ℓ_Y and e_Y will rise, ℓ_X and e_X will remain as corner solutions. At $\omega_Y = \omega_X$, however, we have already seen that $1 > \ell_X = \ell_Y = 0$ and $1 > e_X = e_Y > \hat{e}$. This shows that due to the possibility of specialisation, even a small amount of wage discrimination can induce corner solutions on the part of married agents.

For the single selves of these agents, this is not true, since their labour market and education levels are identical functions, given the assumptions of our model, of their own wages and vary continuously with wages . Thus as $\omega_Y \longrightarrow \omega_X$, the time allocation and education of single agents converge to each other.

The above results are summarised in the following.

Proposition 2: When $\omega_X > \omega_Y$,
(i) a married female works less than her single self and, if the subjective benefit of education is not too large, a prospectively married female receives less education than her prospectively single self;
(ii) if the public good aspect of household production is not too large, *i.e.* (A-6) holds, a married male will work more than his single self and prospectively married male will receive more education than his prospectively single self;
(iii) the time allocation and education of married agents are discontinuous in the degree of wage inequality while those of single agents are not.

The last result implies that policy-makers should not be surprised if gender imbalances in time allocation appear to be resilient to policies that gradually reduce workplace discrimination.

5 The marital decision.

In this section we consider the marital decision. For marriage to take place, the following must be true:

$$U_i^m \geq U_i^s \quad i = X, Y;$$

[19]Note that, strictly speaking, a less stringent restriction on the magnitude of γ than (A-6) would suffice to ensure that $e_X^m > e_X^s$ at given levels of labour supply. This is because of the pro-education effect of $b > 0$ on married couples which strengthens the effect arising from the household division of labour.

i.e. neither agent should be made worse off by marrying than by remaining single.

Rather than provide an exhaustive account of all the possible cases that might arise, we shall make some general observations about the factors that influence this decision and then discuss some interesting possibilities in detail.

5.1 Gender wage equality:

In the case of gender wage equality, the two features which make marriage mutually attractive are (i) the magnitude of δ, the companionship bonus and (ii) that of γ, the degree of non-rivalrousness of household production. A high value of either increases the benefits of marriage.

Indeed, if $\delta = 0$ and $\gamma = 0.5$, both agents will (at least weakly) prefer to remain single. This is proven in the Appendix, Lemma 7. The explanation is that when γ is low, the single self of each agent could, by mimicking the optimal time allocation of his or her married self, achieve exactly the same utility from consumption of the market good, and more from consumption of the household good. This is because the household production function is concave in effort so that when married agents double up on housework they achieve no more than twice their individual efforts. Thus revealed preference suggests that the utility levels associated with remaining single weakly exceed those achievable from getting married.

5.2 Gender wage inequality:

In the case of gender wage inequality, the first point to note is that, all else equal, gender wage inequality makes it more likely that the agents will consent to marry. This is because marriage between asymmetric adults allows the exploitation of comparative advantage in a way that is not possible with symmetric adults. Recall that in our model, males and females were assumed to be symmetric in terms of inherent attributes. This was not to claim reality but in order to show how asymmetry can creep in through gender wage discrimination and then get reinforced by inequalities in education and time allocation. As a result of both the wage and the educational gap, the male partner has a comparative advantage in market work and the female in housework. This can make sharing a household mutually beneficial even absent companionship or any public good benefits from home production.

Of course, although induced comparative advantage can tilt the balance towards marriage it does not guarantee it. We have already seen that lack of companionship or public good benefits from cohabitation work against marriage so strictly speaking, for $\delta = 0$ and $\gamma = 0.5$, the gains from comparative advantage would need to be strong enough to overcome this. In order to isolate this effect, we shall henceforth assume in this part of the analysis that δ and γ are sufficiently high in order that a symmetric couple would be barely willing to get married. We shall also focus our attention on those cases where the male spouse specialises in market work and receives the maximum level of education.

Starting from this benchmark, as we shall show, the two factors that contribute to a rejection of marriage are (i) male wages are such that the option of remaining single is sufficiently attractive for males *and* (ii) there is a high subjective desire for education by females.[20]

[20] By assumption, the subjective desire for education is the same for both agents but the female's desire can affect her willingness to marry in a way that the male's desire does not.

This is established by way of diagrams.

[**Figure 4** in here]

In the upper panel of Figure 4, we have plotted agent X's utility, U_X, against Y's education, e_Y. If X remains single, his utility is depicted by the horizontal line, U_X^s, which is independent of e_Y. If he marries, his utility is depicted by the curve U_X^m which rises and then declines in e_Y, peaking at an optimal value labeled $\hat{e}_Y^m$. Note that $\hat{e}_Y^m$ is optimal from the male's point of view and does not coincide with the married female's own optimal level of education, which is denoted by e_Y^{m*}, unless $b = 0$.

Four cases are possible: (i) that U_X^s lies above U_X^m for all feasible values of e_Y; (ii) that U_X^s lies below U_X^m for all feasible values of e_Y; (iii) that U_X^s lies above U_X^m at $e_Y = 0$ but below it at $e_Y = 1$, (iv) that U_X^s lies below U_X^m at $e_Y = 0$ but above it at $e_Y = 1$ and finally, (v) that U_X^s lies above U_X^m at both extreme values of e_Y but below it at intermediate values. The diagram depicts the last case but note that this is qualitatively similar to case (iv).

In this case, agent X will reject marriage with agent Y if her education level is either below a minimum value, labeled $\underline{e}_Y^m$ or above a maximum, labeled $\bar{e}_Y^m$. We shall refer to these thresholds as marital desirability constraints, as outside them prospective spouse Y is considered as lacking either sufficient market or sufficient homemaking skill, or in the case of the lower threshold, possibly both.[21] As we shall see below the minimum level of education that agent Y will undertake is $\hat{e}_Y^m$, so the lower marital desirability constraint never binds.

In the lower panel of Figure 4, we depict agent Y's utility, U_Y against her own education, e_Y. The broken lines labeled U_Y^s depict her utility from remaining single while the solid lines labeled U_Y^m depict it from getting married. Two cases are depicted: $b = 0$ and $b = b' > 0$. (0) represent the first case and the parentheses (b') represent the second. When $b = 0$, in the case of marriage, Y^s optimal education level, $e_Y^{m*}(0)$ will be equal to $\hat{e}_Y^m$, the level that X would ideally want Y to have. This is because when $b = 0$, both spouses fully internalise the benefits of each other's education. Second, in the case depicted, Y's optimal level of education when single, $e_Y^{s*}(0)$, exceeds that when she marries $e_Y^{m*}(0)$ but the optimum utility obtained from getting married $U_Y^{m*}(0)$ exceeds that from staying single $U_Y^{s*}(0)$. This sets up a benchmark in which marriage is preferable to staying single for at least a very low value of b. Third, note that since, $e_Y^{m*}(0) < \bar{e}_Y^m$, agent Y's optimal educational choice is not subject to a marital desirability constraint.

The second case depicted in the lower panel of Figure 4 is for b' which is large enough that the upper marital desirability constraint (barely) binds. In this case, agent Y's (or her parents acting in her interest) her unconstrained optimum, $e_Y^{m*}(b')$ coincides with the upper marital desirability constraint, $\bar{e}_Y^m$. However, since $U_Y^{m*}(b') > U_Y^{s*}(b')$ as shown, agent Y prefers to get married to remaining single. We next show that as b rises further, $U_Y^s(b)$ rises

[21] For cases (iv) and (v) to arise, agent X should do relatively well by remaining single but not too well. Thus agent X's own wage should be high enough but not so high relative to the benefits of getting married that he prefers to stay single regardless of the household services he can enjoy from a relatively skilled spouse. This combination is more likely to arise when there are large absolute differences in male and female wages.

faster than $U_Y^m(b)$ so that for a high enough value of b, agent Y will prefer to remain single rather than submit to a marital desirability constraint. This is shown in Figure 5 below.

[**Figure 5** in here]

In Figure 5, the solid lines continue to show agent Y's utility while married and the broken lines her utility when single. When $b = b''$, agent Y's married self's utility increases only to the extent that at the constrained level of education, $\bar{e}_Y^m$, her subjective utility from education goes up due to the increase in b. This is indicated by the double-headed arrow labelled ΔU_Y^m. However, her single self's utility increases both because of the increase in subjective utility at the initial level of education and because she will be able to increase her educational level in response. This increase is depicted by the arrow ΔU_Y^s. As depicted, she is now indifferent between getting married or staying single. For a larger value of b, she would strictly prefer to remain single.

An observable implication of a subjective desire for education is that if it is strong enough, agent Y will choose to remain single but by doing so, she will command a higher income and work more than if she had a weaker desire – what appears to be a pecuniary ambition would be driven by a non-pecuniary motive.[22]

On the basis of the foregoing analysis note that for any given configuration of other exogenous parameters, there are two different thresholds b' and b'' which characterise how a female's attitude to marriage depends on her underlying taste for education. For values of b lying in the interval $[0, b']$ marriage does not impose any constraints on a female's level of education; for b in the interval (b', b'') a female prefers to get married even though it means choosing a constrained level of education and for b above b'' a female prefers to remain single in order to be free of constraints on her education.

The intermediate case is of interest because it suggests that when marriage is mutually consensual it can create even stronger barrier to female education than would arise under exogenously imposed marriages where the male cannot demand an "ideal" level of home-making skills from a prospective spouse. A female whose subjective preference for education lies within this interval can be said to priortise marriage over education; a female with a subjective preference above the interval analogously prioritises education over marriage and a female with a subjective preference below the interval faces no conflict between these choices. This result is captured in the following proposition.

Proposition 3: When $\omega_X > \omega_Y$, a female can face a binding marital desirability on her education arising from her prospective spouse's consent to marriage. Three possible situations can arise, depending on the female's subjective desire for education: for given value of other parameters there are two critical thresholds $b' > 0$ and $b'\prime > b'$ such that (1) a female with $b \in (b', b'')$ submits to this binding constraint; (2) a female with $b \leq b'$ is not bound by the constraint; (3) a female with $b \geq b''$ rejects the constraint and stays single.

How do the marital desirability constraints and the critical thresholds for b change with the underlying wages and the degree of wage discrimination? In practice, a reduction in wage

[22] As b crosses the critical value, b'' there will be a jump in educational level. Thus, there can be sharp differences in the education, marital status and work hours of a woman, depending on the underlying preference for education, even without any change in the underlying wage.

discrimination is usually implemented through affirmative action which raises the wages of under-privileged workers rather than reduce those of privileged ones.

5.3 Increase in female wages:

The analysis of this case proceeds by appeal to the diagrams drawn in Figures 4. If agent Y was doing some market work initially, an increase in female wages would shift U_X^m outwards in Figure 4 while leaving U_X^s unchanged (these shifts are not shown). This will lead to an increase in the upper marital desirability bound on female education. Thus marital desirability will be consistent with a higher level of education.

What this does to the likelihood that, given a value of b, a female is likely to be in case 1, 2 or 3 of Proposition 3 will depend on how the two thresholds b' and b'' change as both a direct result of the increase in female wage and the indirect effect of an increase in the marital desirability constraint $\bar{e}_Y^m$. Several factors are at play in these shifts and in general both thresholds could either increase or decrease, so not much can be said about whether a female's marital and education choices will become more likely to be in harmony with each other (case 3), or in conflict (cases 1 and 2) and if in conflict whether marriage will win (case 1) or the desire for education will prevail (case 2).

If the married self of agent Y was not doing market work initially, then a rise in female wages will have no effect on U_X^m or U_Y^m. Hence neither $\bar{e}_Y^m$ nor b' change but since the single self would always work, U_X^s rises for every value of b and this suggests that b'' falls. Thus the only effect is a greater likelihood that given b, a female will be in case 2 rather then case 1.

5.4 Proportionate increase in wages:

We now consider a proportionate increase in wages at a constant degree of discrimination. Figure 6 below depicts an interesting possibility that might arise.

[**Figure 9** in here]

The top half of Figure 9 depicts agent X's utility as a function of agent Y's education; the second half depicts agent Y's own utility as a function of her education. In the top half, an increase in both wages is shown as shifting both U_X^s and U_X^m upwards but the shift in the former is somewhat greater than that in the latter. As a result, $\bar{e}_Y^m$ falls and the marital desirability constraint becomes even more binding.

Why is such an effect on $\bar{e}_Y^m$ plausible? Because agent X is always the principal bread-winner in the marital household, any increase in his wages is going to have be to shared at the margin with his spouse. Even though her own basic wage has gone up proportionately, her overall market income will increase less than proportionately so that the marginal income earned by agent X will not accrue entirely to him. When single, both the absolute and the marginal subsidy are zero. Moreover, when married, agent X will already be up against feasibility constraints in both education and market work, limiting the extra market income that he can earn in response to his higher wage. His single self, on the other hand, can always choose a higher level of education and more work hours. This is the context in which the above occurs.

In the bottom half of Figure 6, we show how the decrease in $\bar{e}_Y^m$ affects agent Y's choices at different levels of her subjective preference for education. At the threshold value of b', agent Y was previously borderline-constrained by the upper bound on education but is now strictly constrained by the new one. This is because of two factors: (i) the fall in $\bar{e}_Y^m$ would induce this even if her married self does not work in the labour market; (ii) if, in addition, her married self works, then the increase in ω_Y will, increase $e_Y^{m*}(b')$. This is the case shown in Figure 6. The first factor is indicated by the leftward arrow from the line indicating that initially, $e_Y^{m*}(b') = \bar{e}_Y^m$ and the second by the rightward arrow from the same line.

Thus, given the tightening of the marital desirability constraint on female education, agent Y becomes more likely to face a tradeoff between education and marriage, i.e. to be in case 1 or 2 of Proposition 3, rather than in case 1. Which way this tradeoff is resolved will depend on how the upper threshold b'' changes and this is ambiguous.[23]

6 Some notes on robustness:

Three assumptions that might merit further discussion are (i) the absence of a preference for leisure; (ii) the manner in which the household production function is specified and (iii) the intrinsic symmetry between the agents.

Including a preference for leisure in utility functions would not affect the results concerning specialisation in the married household so long as the disutility of work was independent of the type of work undertaken, i.e. market work versus household chores. In that case, the leisure-work tradeoff would depend only on the total time in work, not how it was divided between market work and housework. Within the married household, maintaining the other assumptions such as ex-ante symmetry and unitary decision-making, each partner would spend the same amount of overall time in work but would continue specialising, either in a one-sided or mutual fashion, as in the benchmark model.

If the disutility of work depended on the type of work this could affect the types of specialisation but not radically eliminate the phenomenon. For example, if housework was more onerous than market work, it might be less likely for a female to specialise in housework, all else equal, but this might not change the result that a male specialises in market work, nor would it alter the biases induced by the marital division of labour between married females and their single selves.

The specification of household production in this paper is certainly unusual in that it combines concavity of the production function itself with the possibility of non-rivalrousness in output. Our justification is that, in the absence of compelling evidence on the nature of economies of scale in household production, casual empiricism suggests that duplication in the number of workers should not be the driving force for economies of scale in housework

[23]With a tighter marital desirability constraint on female education, the benefits from remaining single will be comparatively larger but this is balanced by the fact that as a married female, even if her own market participation is not that high, Y can share in the higher income her spouse brings in as a result of the increase in his wages. The first effect suggests that b'' will decrease while the second suggests that it will increase.

but rather that many outputs of home production are potentially non-rivalrous. In any case, assuming an increasing returns technology at the level of time input would, if anything, strengthen the tendency for the lower-earning spouse to specialise in housework.

Another aspect of household production worthy of comment is that male and female household labour are (efficiency-adjusted) perfect substitutes for each other. For wage-taking households, male and female market labour would already be efficiency-adjusted perfect substitutes for each other. It is this dual substitutability which drives the specialisation results and induces the discontinuity in the effects of wage discrimination on marital outcomes. If male and female labour were not perfect substitutes in housework then married men might not specialise in market work but married women might nonetheless specialise in housework. Hence some manner of discontinuity could still exist. In any case, even if the discontinuity did not exist, the multiplicative effect of marriage on gender wage inequality would still apply as would the possibility of females facing a marital desirability constraint.

The assumption of ex-ante symmetry is possibly the most unrealistic one of those made in the paper. In particular, it is often argued that the main reason why married women tend to specialise in housework is that they have an intrinsic advantage in it; *e.g.* in childbirth and post-natal child care.

While the above is valid, this paper shows that labour market discrimination combined with the constraints imposed by marriage, is sufficient to generate a pattern of comparative advantages on its own. Biological differences reinforce this pattern. Of course, if gender wage inequality were itself endogenous to women's biological advantage in childbearing the claim that the two are independent factors would not be valid.

Let us examine the above possibility more closely. Suppose in our model that the underlying wage, ω is determined competitively. If agent Y has a higher intrinsic productivity in housework while both agents have the same absolute productivity in market work, the underlying market wage would be the same, even though the resulting patterns of time allocation and education might be different, with women tending towards housework and men towards market work. While absolute advantage in childbearing might explain why married women work and study less than either men or single women, it does not explain why they earn less after controlling for their education and work hours. It also does not explain why single women face labour market and educational disadvantages relative to single men. To the extent that such disadvantages exist, it seems legitimate to isolate their impact on the variables of concern.

7 Conclusions:

This paper has analysed a model of educational choice, time-use and marital choice in which labour market discrimination creates a wedge between male and female education and labour market outcomes and marriage exacerbates it. Our results suggests that reducing labour market discrimination would not necessarily lead to a continuous decline in the gender education gap. This of course does not mean that reducing labour market discrimination is futile, which even by our model it is not, but only that it might have a smaller impact than expected.

Another interesting and cautionary result is that economic growth might not initially lead to an increase in female education or labour force participation. This result is consistent with empirical findings regarding the U-shaped relationship between economic growth and female labour force participation. The mechanism through which it works is different from traditional explanations in which the change in sectoral composition from agriculture to industry plays a role. Our explanation is that with economic growth, females might have to invest more in learning household skills to keep up with the growth in male demand for spouses with these skills. To the extent that industry is likely to reward education more than does agriculture; the male advantage in income-earning is strengthened by industrialisation and thus males are likely to demand more housekeeping from their prospective partners in exchange for playing breadwinners.

Our analysis has been positive in nature and we have refrained from using social welfare criteria as a basis for making policy recommendations. However, this does not mean that there are no welfare implications of the analysis. For example, it has been shown that labour market discrimination can lead to a second-best outcome from a female' point of view where she has to decide between marriage and her own education. Even if she is better off getting married, this comes at the subjective cost of receiving more household training and less education than she would have chosen.

Our paper has also not questioned the source of labour market discrimination. However, theories of endogenous discrimination often point to a link between low levels of market participation by members of a particular group and lack of information by employers of this group's abilities, leading to a vicious circle of discrimination. Our model suggests that marriage could play a role in strengthening this relationship. This is not to advocate an anti-marriage policy but to lend additional support to the case for pro-female education policies such as the female stipend programme in Bangladesh, female fellowships for primary education in Pakistan and voucher programmes for girls' education in Colombia. The combination of such programmes with policies aimed at eliminating labour market discrimination against women could be far more effective than attempting to end discrimination against adult women alone.

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APPENDIX

The following results pertain to the case of gender wage discrimination, $\omega_X > \omega_Y$.

Lemma 1: Under (A-4) and (A-5), the first-order conditions for both labour and education, and for both single and married couples imply that, *given interior solutions*

$$\left.\frac{\partial \ell_i^k}{\partial e_i^k}\right|_{\ell^*} > 0$$
$$\left.\frac{\partial \ell_i^k}{\partial e_i^k}\right|_{e^*} > 0$$

for $i = X, Y$ and $k = s, m$; $|_{\ell^*}$ indicates a derivative along the first-order condition for labour while $|_{e^*}$ indicates a derivative along the first-order condition for education.

Proof: Differentiating the relevant first-order conditions for ℓ_i and e_i, keeping in mind that the exercise applies only for interior solutions in the case of married agents, we get the following

$$\left.\frac{\partial \ell_i^s}{\partial e_i^s}\right|_{\ell^*} = \frac{(u''\cdot c_i^s + u')\omega_i - \alpha'\cdot(h''\alpha\cdot(1-\ell_i^s)+h')}{u''\cdot(\omega_i)^2(e_i^s)^2 + \alpha^2 h''}$$
$$\left.\frac{\partial \ell_i^s}{\partial e_i^s}\right|_{e^*} = \frac{u''\cdot(\omega_i)^2 + (\alpha')^2(1-\ell_i^s)^2 h'' + \alpha'' h'\cdot(1-\ell_i^s)}{(u''\cdot c_i^s + u')\omega_i - \alpha'\,(h''\cdot\alpha\cdot(1-\ell_i^s)+h')}$$
$$\left.\frac{\partial \ell_i^m}{\partial e_i^m}\right|_{\ell^*} = \frac{(u''\cdot(c_i^m - 0.5\omega_j e_j^m \ell_j^m) + u')\omega_i - 2\gamma\alpha'\cdot(h''\alpha\cdot(1-\ell_i^m)+h')}{u''(\omega_i)^2(e_i^m)^2 + \alpha^2 h''}$$
$$\left.\frac{\partial \ell_i^m}{\partial e_i^m}\right|_{e^*} = \frac{u''\cdot(\omega_i)^2 + (\alpha')^2(1-\ell_i^m)^2 h'' + \alpha'' h'\cdot(1-\ell_i^m)}{(u''\cdot(c_i^s - 0.5\omega_j e_j^m \ell_j^m) + u')\omega_i - \alpha'\cdot(h''\alpha(1-\ell_i^m)+h')}$$

where $_j$ indicates a variable related to the spouse in the married agent's case. There are only two cases where this is relevant: (i)$i = Y$ and $j = X$ when $\omega_X e_X \ell_X = \omega_X$; (ii) $i = X$ and $j = Y$ when $\omega_Y e_Y \ell_Y = 0$.

In all cases, (A-4) and (A-5) are sufficient for the above to all be positive.

Lemma 2: Concavity of the maximisation problem requires that

$$\left.\frac{\partial \ell_i^k}{\partial e_i^k}\right|_{e^*} > \left.\frac{\partial \ell_i^k}{\partial e_i^k}\right|_{\ell^*}$$

for $i = X, Y$ and $k = s, m$.

Proof: For single agents, the Hessian matrix formed by the own- and the cross-partial derivatives of each first-order condition with respect to each choice variable is two-dimensional. For married agents, there are four endogenous variables in principle. However, because *at least* one spouse is always in a corner with respect to labour supply, the Hessian is also two-dimensional, if it is defined at all.

The determinant of the Hessian is equal to:

$$\left.\frac{\partial \ell_i^k}{\partial e_i^k}\right|_{e^*} - \left.\frac{\partial \ell_i^k}{\partial e_i^k}\right|_{\ell^*}$$

where, in the married agent's case, i is the spouse, if any, whose labour supply is interior.

It is easily verified that the diagonal elements of the Hessian are negative so the principal minors alternate in sign as (required for concavity) if and only if the above expression is positive.

Lemma 3: Under (A-4),

$$\left.\frac{\partial \ell_i^k}{\partial \omega_i}\right|_{\ell} > 0$$
$$\left.\frac{\partial e_i^k}{\partial \omega_i}\right|_{e} > 0$$

$k = s, m$ and $i = X, Y$.

Proof: We show this for a single agent. The case of a married agent (assuming an interior solution) follows analogously.

Taking the derivative of the first-order condition for labour supply of a single agent with respect to ℓ_i^s and ω_i, solve for:

$$\left.\frac{\partial \ell_i^s}{\partial \omega_i}\right|_{\ell} = \frac{-[u''c_i^s + u']}{u''(\omega_i)^2(e_i^s)^2 + \alpha^2 h''} > 0 \quad \text{by (A-4)}$$

Taking the derivative of the first-order condition for education of a single agent with respect to e_i^s and ω_i, solve for

$$\left.\frac{\partial e_i}{\partial \omega_i}\right|_{e} = \frac{-[u''c_i^s + u']}{u''(\omega_i)^2 + (\alpha')^2(1-\ell_i^s)^2 h'' + \alpha'' h'(1-\ell_i^s)} > 0 \quad \text{by (A-4)}$$

Lemma 4: For a given value of e_Y, $\ell_Y^m < \ell_Y^s$.

Proof: This is obvious when $\ell_Y^m = 0$ so the only case in which it needs to be proven is when $\ell_Y^m > 0$. Note that if this is the case then $e_X = 1$ and $1 - \ell_X = 0$.

Now suppose that in fact $\ell_Y^m \geq \ell_Y^s$. Then $1 - \ell_Y^s \geq 1 - \ell_Y^m$. By the concavity of $h(\cdot)$ and the fact that $2\gamma \geq 1$, it must then be the case that $2\gamma\alpha_Y h'(\alpha_Y(1-\ell_Y^m)) \geq \alpha_Y h'(\alpha_Y(1-\ell_Y^s))$. From the respective first-order conditions for the labour supply of agent Y's married and single self (both of which hold as equalities in this case), it follows from comparing terms that

$$u'\left(\frac{w_X + w_Y e_Y \ell_Y^m}{2}\right) \geq u'\left(w_Y e_Y \ell_Y^s\right)$$

By the concavity of $u(\cdot)$, this implies that

$$\frac{w_X + w_Y e_Y \ell_Y^m}{2} \leq w_Y e_Y \ell_Y^s$$

But since our working hypothesis is that $w_Y e_Y \ell_Y^m \geq w_Y e_Y \ell_Y^s$, the above can only be true if $w_X \leq w_Y e_Y \ell_Y^s$ which is not possible since $w_X > w_Y$ and $1 \geq e_Y^s$. Thus $\ell_Y^m < \ell_Y^s$.

Lemma 5: Suppose $b = 0$. At given ℓ_Y, for any value of $\gamma \in [0.5, 1]$, (i)$e_Y^m < e_Y^s$.

Proof:

Recall equation (8). Taking $b = 0$, this can be written for agent $Y^{`prime}$s single self as

$$u'\left(\omega_Y e_Y^s \ell_Y^s\right)\omega_Y \ell_Y^s = -h'(\alpha(e_Y^s)(1-\ell_Y^s))(1-\ell_Y^s)\alpha'(e_Y^s)$$

Similarly recall equation (10). Taking $b = 0$and noting that in the relevant case, $e_X^m = 1$ and $\ell_X^m = 1$ it can be rewritten for the married self of agent Y as

$$u'\left(\frac{\omega_X + \omega_Y \ell_Y^m e_Y^m}{2}\right)\omega_Y \ell_Y^m = -2\gamma h'(\alpha(e_Y^m)(1-\ell_Y^m))(1-\ell_Y^m)\alpha'(e_Y^m)$$

Now let $\Lambda = -2\gamma h'(\alpha(e)(1-\ell))(1-\ell)\alpha'(e)$ for a generic agent. Differentiating Λ with respect to e:

$$\frac{\partial \Lambda}{\partial e} = -2\gamma h''(\cdot)(1-\ell)^2(\alpha(e))^2 - 2\gamma h'(\cdot)(1-\ell)\alpha''(e) > 0$$

since both h'' and $\alpha\prime\prime$ are negative. Also,

$$\frac{\partial \Lambda}{\partial \gamma} = -2\gamma h'(\cdot)(1-\ell)\alpha'(e) > 0$$

since $\alpha' < 0$ at the optimum.

Now let e_Y^s solve equation (8) and evaluate equation (10) at this value of e_Y. Then the LHS of equation (8) must be strictly greater than that of equation (10) since, at e_Y^s and for any given value of ℓ_Y, agent Y's claim on market income will be higher when married to the higher paid agent X than when single.

Furthermore, the RHS of equation (8) must be less than or equal to that of equation (10) since the two will be equal when $\gamma = 0.5$ and the RHS of equation (10) will be strictly greater than that of equation (8) when $\gamma > 0.5$.

Hence, evaluated at e_Y^s, the marginal cost of education for prospectively married agent Y will be strictly greater than the marginal benefit; implying that $e_Y^m < e_Y^s$ for given values of ℓ^Y.

Lemma 6: Let (A-6) hold. For given values of e_X and α_X, $\ell_X^m > \ell_X^s$.

Proof: The proof follows a similar strategy to that for Lemma A1. Suppose that $\ell_X^m \leq \ell_X^s$. Then clearly $(w_X e_X \ell_X^m)/2 < w_X e_x \ell_X^s$. By the concavity of $u(\cdot)$, it follows that

$$u'\left(\frac{w_X e_X \ell_X^m)}{2}\right) \geq u'\left(w_X e_X \ell_X^s\right)$$

Since the first-order conditions for both married and single labour supply hold as equalities, it follows that $2\gamma\alpha_X h'(\alpha_X(1-\ell_X^m) + \alpha_Y^m) \geq \alpha_X h'(\alpha_X(1-\ell_X^s))$, where α_Y^m is the level of household skill of the underprivileged spouse (her household effort is full-time in this case. Rearranging, this implies that

$$\gamma \geq \frac{h'(\alpha_X(1-\ell_X^s))}{2h'(\alpha_X(1-\ell_X^m) + \alpha_Y^m)}$$

We now show this is not possible given the restriction on γ.

Because the two agents are ex-ante identical, and agent Y specialises in housework, her educational level will be lower and household skill higher than that of either version of agent X. Thus $\alpha_Y^m > \alpha_X(1-\ell_X^s)$. Also by maintained hyposthesis, $\alpha_X(1-\ell_X^m) > \alpha_X(1-\ell_X^s)$. Thus $\alpha_X(1-\ell_X^m)+\alpha_Y^m > 2\alpha_X(1-\ell_X^s)$. By the concavity of $h(\cdot)$, this implies that $2h'(\alpha_X(1-\ell_X^m)+\alpha_Y^m) < 2h'(2\alpha_X(1-\ell_X^s))$. Thus,

$$\frac{h'(\alpha_X(1-\ell_X^s))}{2h'(\alpha_X(1-\ell_X^m)+\alpha_Y^m)} > \frac{h'(\alpha_X(1-\ell_X^s))}{2h'(2\alpha_X(1-\ell_X^s))}$$

Combining the above inequality with that following from the first-order conditions, both can be true if and only if

$$\gamma > \frac{h'(\alpha_X(1-\ell_X^s))}{2h'(2\alpha_X(1-\ell_X^s))}$$

which is ruled out by assumption (A-6).

Lemma 7: When $\omega_X = \omega_Y = \omega$ and $\gamma = \delta = 0$, $U_i^s \geq U_i^m$, $i = X, Y$.

Proof: Note that in the symmetric case, a married couple remains symmetric, while each agent's single self makes identical choices. Thus $\ell_X^s = \ell_Y^s = \ell^s$, $e_X^s = e_Y^s = e^s$ and using similar notation for ℓ_X^m, ℓ_Y^m:

$$\begin{aligned}
U^s(\ell^s, e^s) &= u\left(\omega e^s \ell^s\right) + h\left(\alpha(e^s)(1-\ell^s)\right) \\
&\geq u\left(\omega e^m \ell^m\right) + h\left(\alpha(e^m)(1-\ell^m)\right) = U^s(\ell^m, e^m) \\
&\geq u\left(\omega e^m \ell^m\right) + 0.5h\left(2\alpha(e^m)(1-\ell^m)\right) = U^m(\ell^m, e^m)
\end{aligned}$$

where the first inequality is due to revealed preference and the second due to the concavity of h.

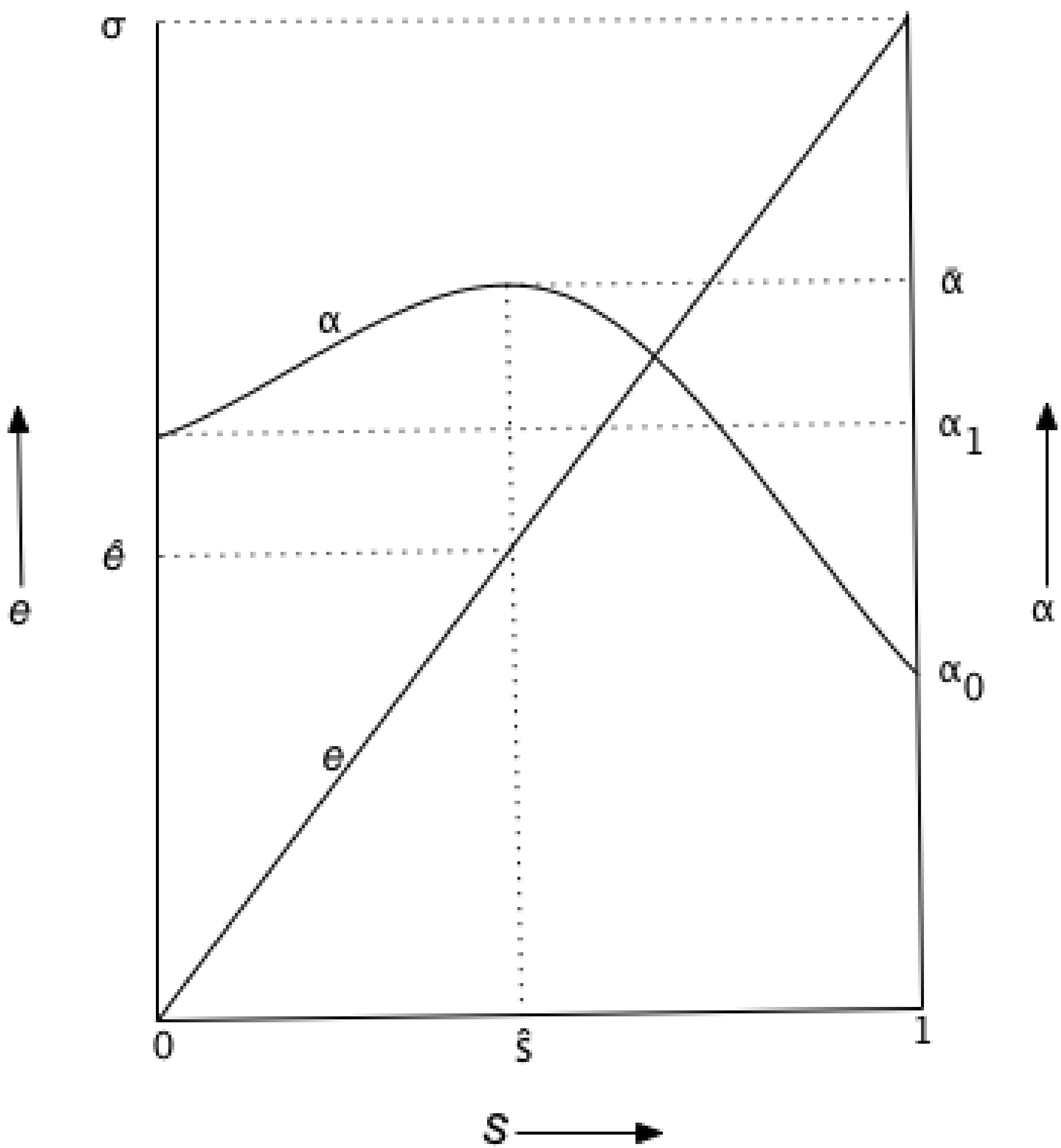


Figure 1

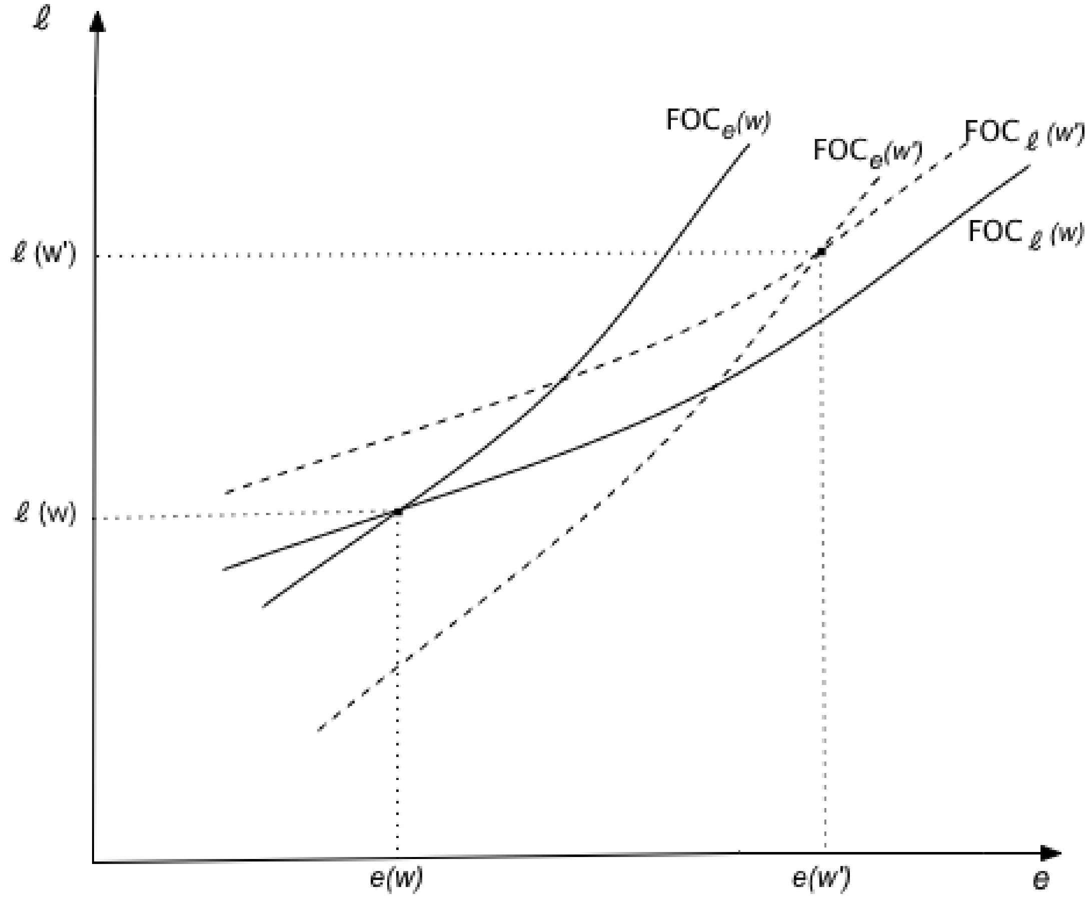
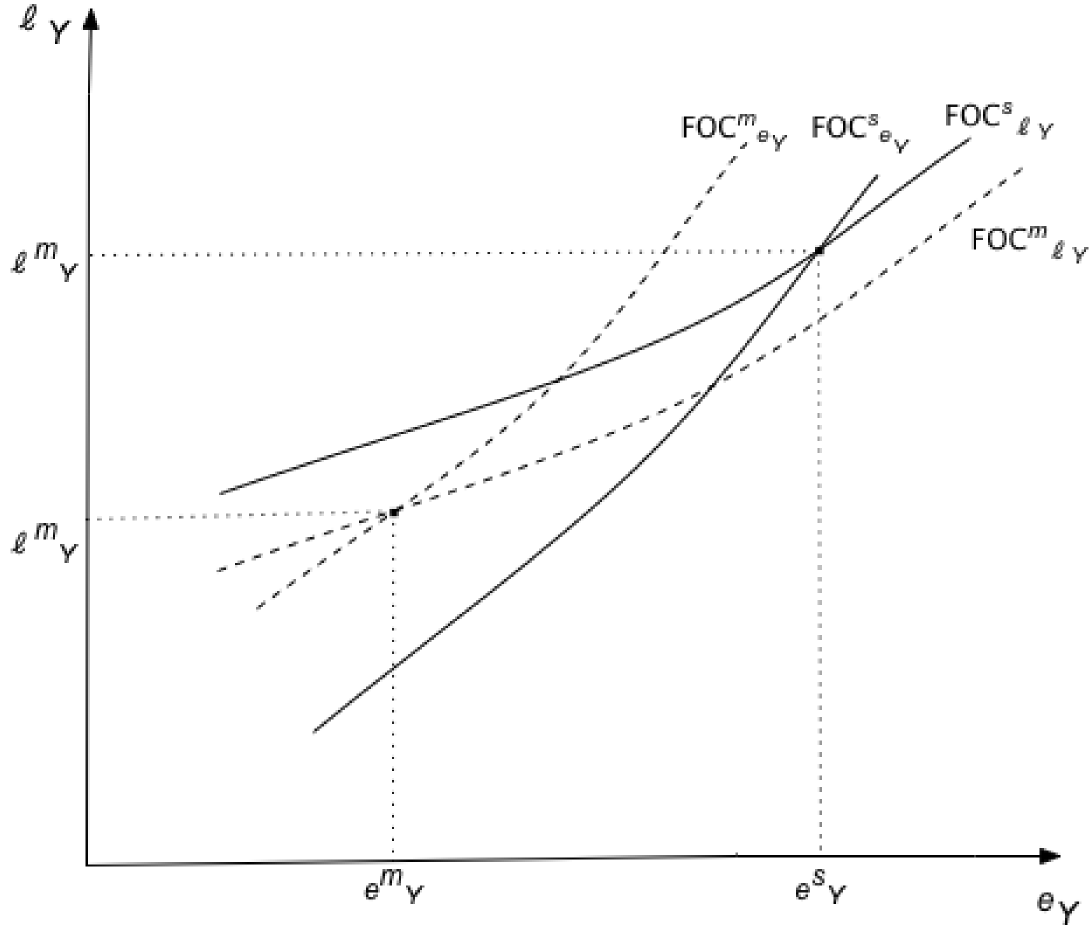
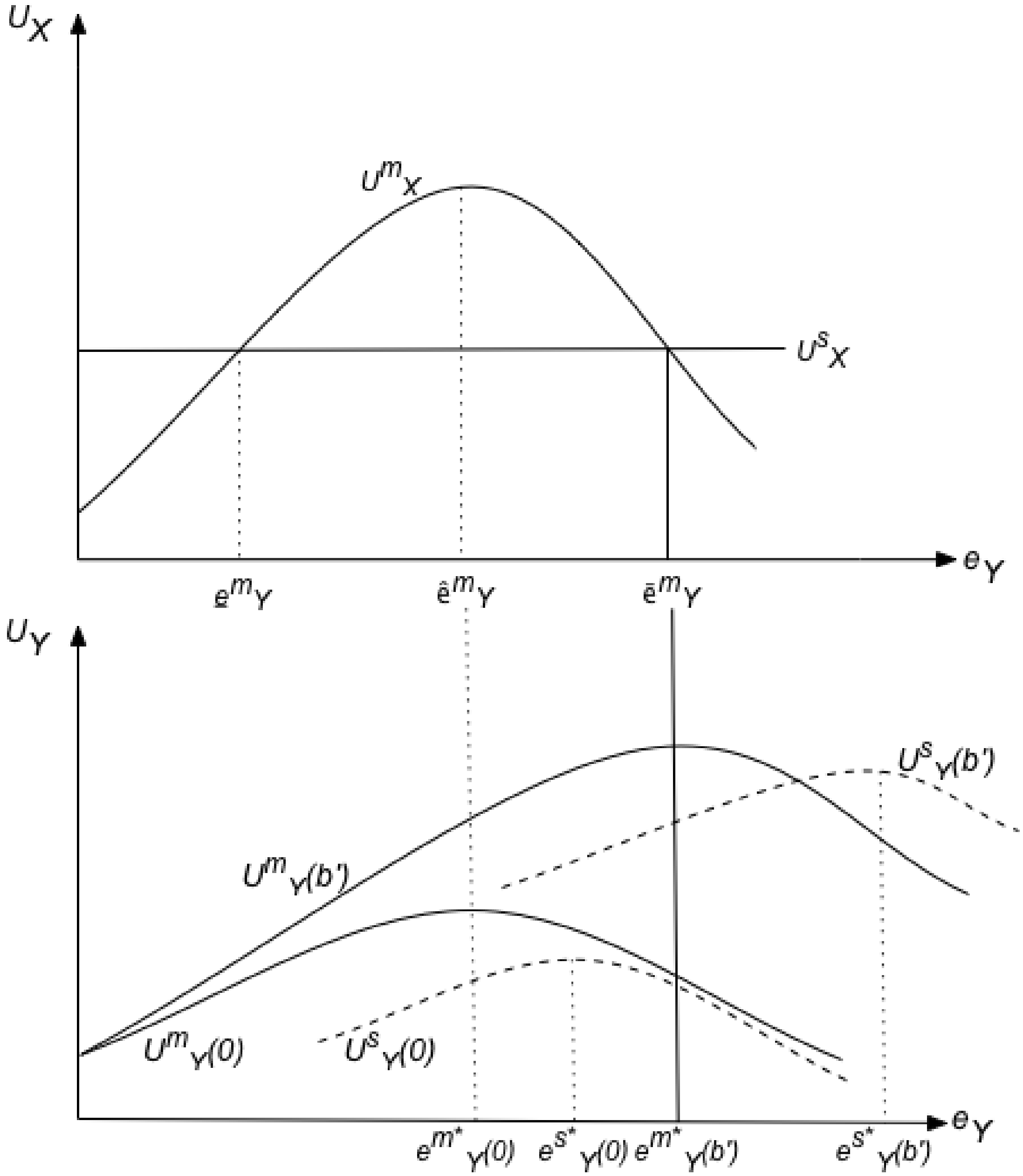


Figure 2



ℓ_Y
$FOC^m{}_{e_Y}$
$FOC^s{}_{e_Y}$
$FOC^s{}_{\ell_Y}$
$FOC^m{}_{\ell_Y}$
$\ell^m{}_Y$
$\ell^m{}_Y$
$e^m{}_Y$
$e^s{}_Y$
e_Y

Figure 3



U_X
$U^m{}_X$
$U^s{}_X$
e_Y
$\underline{e}^m{}_Y$
$\hat{e}^m{}_Y$
$\bar{e}^m{}_Y$
U_Y
$U^s{}_Y(b')$
$U^m{}_Y(b')$
$U^m{}_Y(0)$
$U^s{}_Y(0)$
e_Y
$e^{m^*}{}_Y(0)$
$e^{s^*}{}_Y(0)$
$e^{m^*}{}_Y(b')$
$e^{s^*}{}_Y(b')$

Figure 4

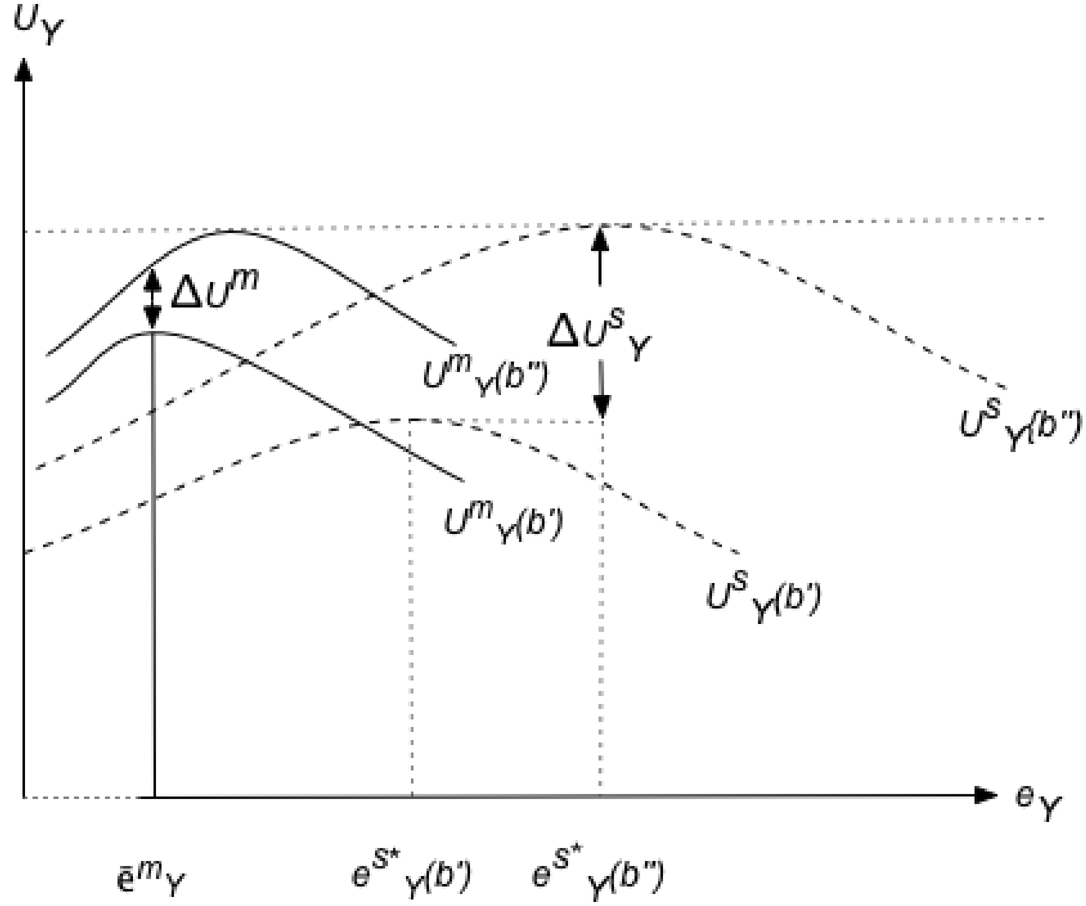
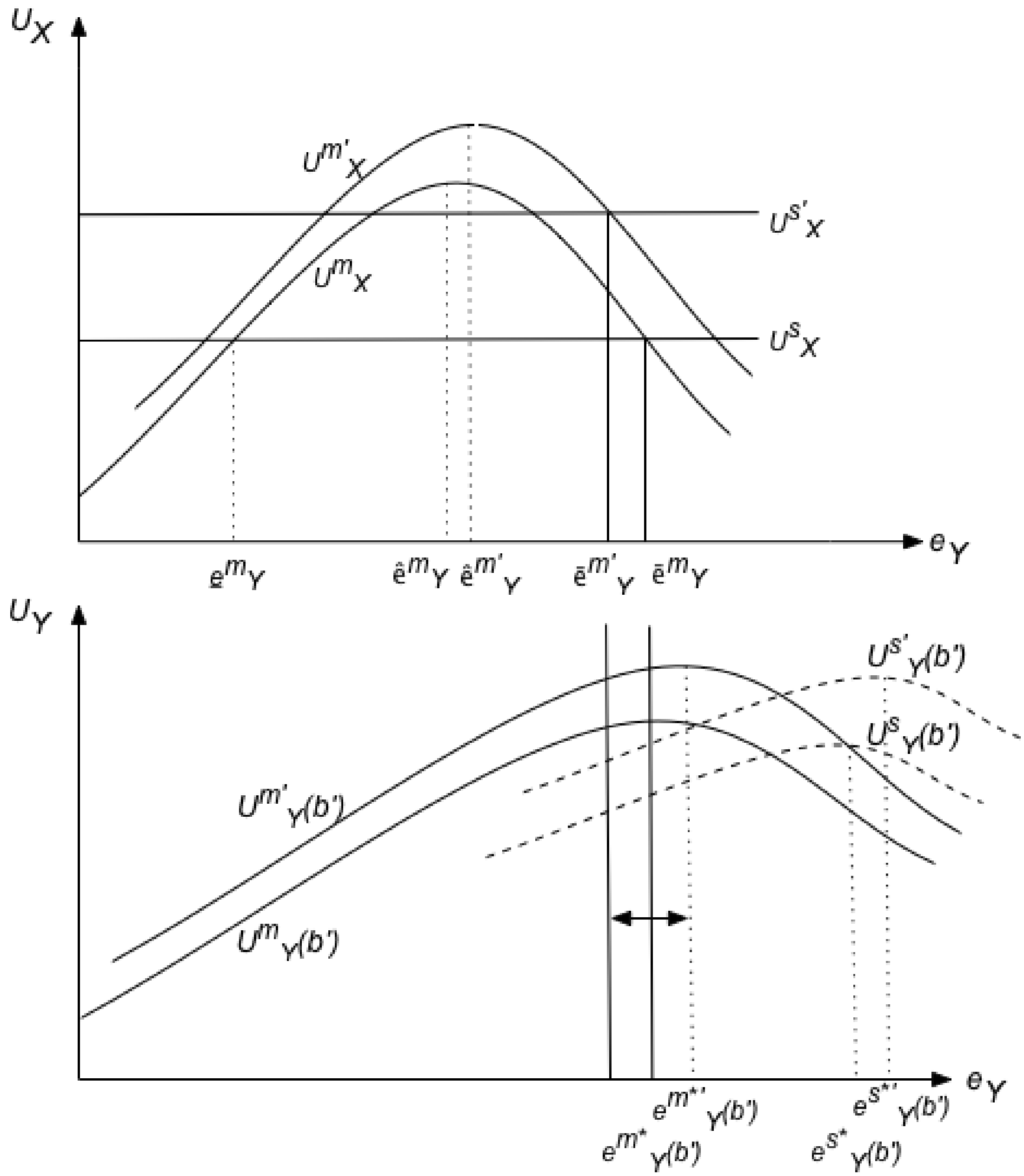


Figure 5



U_X
$U^{m'}_X$
$U^{s'}_X$
U^m_X
U^s_X
e_Y
$\underline{e}^m_Y$
$\hat{e}^m_Y$
$\hat{e}^{m'}_Y$
$\bar{e}^{m'}_Y$
$\bar{e}^m_Y$
U_Y
$U^{s'}_Y(b')$
$U^s_Y(b')$
$U^{m'}_Y(b')$
$U^m_Y(b')$
e_Y
$e^{m*'}_Y(b')$
$e^{m*}_Y(b')$
$e^{s*'}_Y(b')$
$e^{s*}_Y(b')$

Figure 6