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Computational Management Science
Computational framework for longevity risk management
--Manuscript Draft--

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	Steven Haberman, Full Professor
	Gabriella Piscopo, Researcher
	Maria Russolillo, Researcher
Order of Authors Secondary Information:	
Abstract:	Longevity risk threatens the financial stability of private and government sponsored defined benefit pension systems as well as social security schemes, in an environment already characterized by persistent low interest rates and heightened financial uncertainty. The mortality experience of countries in the industrialized world would suggest a substantial age-time interaction, with the two dominant trends affecting different age groups at different times. From a statistical point of view, this indicates a dependence structure. It is observed that mortality improvements are similar for individuals of contiguous ages (Wills and Sherris 2008). Moreover, considering the dataset by single ages, the correlations between the residuals for adjacent age groups tend to be high (as noted in Denton et al 2005). This suggests that there is value in exploring the dependence structure, also across time, in other words the inter-period correlation. In this research, we focus on the projections of mortality rates, contravening the most commonly encountered dependence property which is the "lack of dependence" (Denuit et al. 2005). By taking into account the presence of dependence across age and time which leads to systematic over-estimation or under-estimation of uncertainty in the estimates (Liu and Braun 2010), the paper analyzes a tailor-made bootstrap methodology for capturing the spatial dependence in deriving prediction intervals for mortality projection rates. We propose a method which leads to a prudent measure of longevity risk, avoiding the structural incompleteness of the ordinary simulation bootstrap methodology which involves the assumption of independence.
Response to Reviewers:	Dear Referees, we thank you for your reports, which were useful for us, in order to increase the value of our work. Please find below, the description of how we have changed the paper according to your suggestions. Reviewer #1: General comments

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Computational framework for longevity risk management

***Valeria D'Amato*[1]*, Steven Haberman*[2]*, Gabriella Piscopo*[3]*, Maria Russolillo*[1]**

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[2] *Faculty of Actuarial Science and Insurance, Cass Business School, City University, Bunhill Row London, UK, e-mail:* s.haberman@city.ac.uk

[3] *Department of Economics and Quantitative Methods, University of Genoa, Via Vivaldi, 2 (Darsena), 16126 Genoa, Italy, e-mail:* gabriella.piscopo@unige.it

Keywords: Longevity Risk Management, Bootstrap Techniques

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1. Introduction

The improvements of the longevity phenomenon over the time are noteworthy. According to Swiss Re (2011), "life expectancy at birth in the developed world has risen from around 65 years in 1950 to over 75 years now, or one extra year every six years, and is currently projected to rise to more than 88 years by the end of this century".
From the life insurance companies' point of view, the risk that people live longer than predicted, i.e. the so-called longevity risk, has to be carefully managed. Longevity projections are also a critical feature for sponsors of defined benefit pension plan and

government sponsored welfare systems and social security systems. On a global scale, the costs of ageing are a substantial threat to the financial stability of whole nations and make fiscal balance sheets more vulnerable, as pointed out by International Monetary Fund (2012).

Broadly speaking, our research is addressed to produce reliable mortality projections. In order to manage the mortality risk properly, we need to assess the uncertainty coming from the mortality dynamics carefully. In the literature, simulation techniques have been proposed to measure the mortality risk and confidence intervals are then calculated to obtain a measure of the risk arising from the uncertain mortality rates. With regard to the Lee Carter framework, which is a seminal work in terms of mortality projections, empirical studies reveal better performances under the bootstrap techniques rather than by implementing the Monte Carlo approach which is sensitive to the identifiability constraints (Renshaw and Haberman 2008).

Recently, various bootstrap methods have been proposed to measure mortality risk, as seen in Brouhns et al. (2005) for the parametric bootstrap, in Brouhns et al. (2005) for the semi-parametric bootstrap, and in Koissi et al. (2006) for the ordinary residual bootstrap. In these papers, the implicit assumption is that the residuals after fitting the model to the data are independent and identically distributed. However, as has been shown in the literature, correlations across age and year can be observed in the residuals. It should be highlighted that if a correlation structure between the residuals exists and it is not taken into account, then the resulting confidence intervals could be too narrow or too wide. In particular, when calculating confidence intervals by bootstrap methods, there may be an underestimation of the mortality risk if correlations in residuals are not properly handled. In the light of this consideration, in the context of mortality data, the re-sampling has to be carried out in such a way that the dependence structure is captured. One of the typical methods used for bootstrapping dependent data is the block bootstrap (Kunsch 1989). The basic idea of the block bootstrap is based on drawing observations with replacement. In the block bootstrap, however, instead of single observations, blocks of consecutive observations are drawn. This is done in order to capture the dependency structure of neighbouring observations (Liu and Braun 2010). In the literature, there is considerable evidence that the sieve bootstrap, initially proposed by Kreiss (1992) and Bulhmann (1997), usually outperforms the block bootstrap (Choi and Hall 2000). D'Amato et al. (2012) apply a sieve bootstrap on the residuals of the Lee Carter model; they take up the Lee Carter parametric model firstly and then re-sample a particular class of the residuals, the so-called centred residuals, according to the design of the typical autoregressive sieve bootstrap. According to this scheme, they are able to reproduce in the sampling the dependence structure that exists between the years of the dataset for each age.

In this work we try to capture a more complex structure, incorporating in the bootstrap procedure the whole error matrix. In the case of panel data with a complex dependence structure, there are two different way to implement a bootstrap scheme: the first one is to apply a vector autoregressive (VAR) bootstrap, which extends the autoregressive procedure to the multidimensional case (Trapani, 2011); the second one consists of a univariate AR sieve bootstrap, with the modification that the residuals are re-sampled jointly across units to preserve the cross-sectional dependence (Smeeks and Urbain, 2011). With regard to the former, the VAR bootstrap scheme becomes infeasible in panel data where the number of cross-sectional units is large and the dimension of the system is too high. With regard to the latter, Palm (1977) shows that any VAR model can be written as a system of ARMA equations for each unit; starting from this consideration and using the results of Kreiss et al. (2011), Smeeks and Urbain (2011) describe the AR sieve bootstrap algorithm for panel data. Chang (2004) has proven the validity of the AR sieve bootstrap in the context of panel data if there is only one contemporaneous source of dependence between the units; however, this condition is likely to be violated in many empirical applications. In this paper, we verify if the condition for the validity of the AR sieve bootstrap in panel data exists for mortality data in order to apply an opportune algorithm to the residuals of the Lee Carter model. The paper is structured as follows: in section 2, we provide a motivation for the paper; in section 3 dependency is discussed and in section 4 the panel sieve algorithm is described; section 5 provides an application to Italian male mortality data, articulated in two steps: first the condition of validity is verified and then the algorithm is applied; finally, some remarks and conclusions are presented in section 6.

2. Motivation

A key objective of for many of the aforementioned stakeholders is to ensure that longevity risk is well managed and is supported by adequate financial resources. An integrated method of risk assessment should help to protect policyholders' and pension plan members' interests more effectively, by making reliable evaluation of the uncertainty around longevity projections. This corresponds to having robust methods of calculation of the confidence intervals for the forecasted rates. The robustness has to be investigated with respect to both the statistical principles and the objective of consistent risk management. The increasing complexity of the real world imposes the necessity of modeling of dependent risks, so that, in the case of longevity data, the interactions between age and time cannot be neglected. Indeed, the presence of spatial dependence across age and time leads to systematic over-estimation or under-estimation of uncertainty in the estimates, caused by whether negative or positive dependence dominates (Liu et al. 2010). Thus, in order to produce accurate longevity projections, it is necessary to allow for the so-called dependency risk (D'Amato et al. 2012).

In light of these considerations, the aim is to develop an appropriate algorithm for deriving better forecasts of mortality rates, taking into account the dependency feature.

3. Dependence Framework

The leading statistical model for projecting mortality is represented by the Lee Carter model. Lee and Carter (1992) suggested a log-bilinear form for the force of mortality:

$$
\begin{aligned}
&m_{xt} = \exp(\alpha_x + \beta_x k_t + u_{xt}) \\
&\ln(m_{xt}) = \alpha_x + \beta_x k_t + u_{xt} \\
&\sum_t k_t = 0 \\
&\sum_x \beta_x = 1
\end{aligned}
\tag{1}
$$

where m_{xt} is the crude log-death rate at age x in calendar year t, which is the logarithm of the number of deaths occurred among individual aged x in calendar year t, divided by the corresponding exposure-to-risk and where the constraints ensure the model identification. The value of α_x corresponds to the average of $\ln(m_{xt})$ over time t. The actual forces of mortality change according to the overall mortality index k_t modulated by an age response β_x. The time factor k_t is intrinsically viewed as a stochastic process and Box-Jenkins techniques are then used to model and forecast k_t. Formally, the log mortality rate of the x-year-old at time t $\ln(m_{xt})$, based on the Lee Carter model , is represented by panel data, in other words multidimensional data. The panel under consideration has the form m_{xt},

$x=1,...,N$ and $t=1,...,T$, where the cross-sectional dimension is related to the ages and time series dimension to the observation periods.

Generally, panel data could reveal dynamics that are difficult to detect only with cross-sectional data. In the case of human population, each single unit is represented by a different age; the variable observed is the central mortality death rate and the observations are NT, consisting of time series of length T, on N parallel units-ages. Cross-sectional or "spatial" dependence is a problematic aspect of many panel data sets in which the cross-sectional units are not randomly sampled. The standard techniques can fail to account for the presence of spatial correlations, yielding inconsistent estimates of the standard errors of the model parameters.

In the mortality setting, consider a rectangular mortality data array (m_{xt}), with the log-bilinear structure, as composed by determinations from random vectors.

Let $\Omega, A, \wp$ the probability space where A the $\sigma-$algebra on Ω and $\wp$ a probability on A. Let us consider a random mortality vector M represented by a n-dimensional vector of $(M_1, M_2,..., M_n)$, where the random variables M_i are the components of the vector.

Note that in the case of a specific demographic population, for each n-dimensional vector of real numbers $m=(m_1, m_2,..., m_n)$, it is possible to write the following: $\{\omega \in \Omega : M_1(\omega) \le m_1, M_2(\omega) \le m_2,..., M_n(\omega) \le m_n\} = \cap_{i=1}^{n}\{\omega \in \Omega : M_i(\omega) \le m_i\}$ where this event is intersection of elements belonging to A.

For any random mortality vector M, let us define the joint probability function F_M from $R^n \to [0,1]$ by the following expression: $F_M(m) = P(M_1 \le m_1, M_2 \le m_2,..., M_n \le m_n)$ where $F_M(m)$ are marginal probability mass functions. In the rectangular mortality data array, it is essential to compare the random mortality vectors allowing for dependence. With this aim in mind, the standard tool is the correlation Pearson index, which we can arrange in the context under consideration as follows:

$$r_M = \frac{Cov(M_i, M_j)}{\sqrt{V[M_j]V[M_j]}} \qquad (2)$$

In this paper, we start to verify the validity of the assumption of lack of dependence or the presence of correlations across age and year in the residuals, because calculating confidence intervals by bootstrap methods may imply an underestimation of the mortality risk if correlations in residuals are not properly handled (D'Amato et al. 2012).

Hence, we investigate the autocorrelation structure in the matrix of residuals both through graphical analysis and statistical inference.

Let Σ(x,t) be the matrix of residuals obtained after fitting the Lee Carter model:

$$\varepsilon_{xt} = \ln(m_{xt}) - \alpha_x - \beta_x k_t \qquad (3)$$

Following Lee and Carter (1992), the parameters can be estimated according to the SVD of the matrix of the log age-specific observed death rates with suitable constraints (see eq. 1) to obtain a unique solution. The matrix can be viewed as being composed of some random vectors, where in the rows and columns the residuals are collected respectively by age and time and are realizations of different stochastic processes. In order to investigate the correlation in the residuals, we make use of the correlogram, a graphical tool to examine the strength of association between observations. In our mortality matrix, it is interesting to evaluate the correlation between both age and time, i.e. across rows and columns. In the former case, we are interested in the distance between neighboring observations, i.e. the residuals for consecutive ages. In the latter case, we look at each row as a time series and verify whether it is autocorrelated or not. The graphical results need to be supported by statistical inference. We have chosen to use the Ljung–Box test, a statistical test of whether any of a group of autocorrelations of a time series are different from zero, which tests the overall randomness based on a number of lags instead of testing randomness at each distinct lag.

4. The AR sieve algorithm for panel data

D'Amato et al. (2012) take up the older idea of first fitting Lee Carter parametric model, because of its well known properties (Deaton and Paxson, 2004) and then re-sampling a particular class of the residuals, the so-called centred residuals, according to the design of the typical sieve scheme: an autoregressive approximation for generating bootstrap replications of the data. As has been shown, the order of the autoregressive approximation increases at some appropriate rate with increases in the sample size (Kreiss 1992). In this paper, we explore the possibility of applying the AR sieve bootstrap algorithm adapted for panel data to the error matrix of the Lee Carter model. In order to describe this algorithm, we introduce below the adopted notation:

u_{xt}	error term
ε_{xt}	innovation term
r_{xt}	estimated innovation or residual
$\bar{r}_{xt}$	mean value of the residuals
$r_{xt} - \bar{r}_{xt}$	centred residuals
$\hat{F}_{xr}$	empirical cumulative distribution function of the residuals
u^*_{xt}	Bootstrap error
ε^*_{xt}	iid term from $\hat{F}_{xr}$

Let m_{xt} describe the matrix of central death rates; The LC model is fitted to the m_{xt} and the matrix of the residuals by age and time indicated by u_{xt} is computed, *x=1…N, t=1,…T.* The steps of the algorithm are the following:

1. <u>For each age *x=1,..,N*</u>, the error term is approximated by an $AR(q)$ representation:

$$u_{xt} = \sum_{j=1}^{q} \varphi_j u_{xt-j} + \varepsilon_{xt} \qquad (4)$$

We specify the value of the lag length $q(n)$ by Akaike's information criterion as suggested by Amemiya (1973) and calculate the autoregressive coefficients by using the Yule-Walker method:

$\hat{\phi}_j, \quad j = 1,\ldots,q(n)$

2. <u>For each age *x=1,..,N*</u>, we run an ADF (Augmented Dickey Fuller) regression with *q* lags to obtain residuals:

$$r_{xt} = \hat{u}_{xt} - \sum_{j=1}^{\hat{p}(n)} \hat{\phi}_j \hat{u}_{x\,t-j} \tag{5}$$

We highlight that the lag q needs to be selected for each equation individually by using information criteria.

3. <u>For each age *x=1,..,N*</u>, recentre the residuals to obtain $\tilde{r}_{xt}$
4. Resample with replacement from $\tilde{r}_{qt} = \left(\tilde{r}_{q1t}, \ldots, \tilde{r}_{qNT}\right)'$ to obtain bootstrap residuals $\varepsilon_t^{\,*} = \left(\varepsilon_{1t}^{\,*}, \ldots, \varepsilon_{NT}^{\,*}\right)'$
5. For <u>each age *x=1,..,N*</u>, construct $u_{xt}^{\,*}$ recursively as

$$u_{xt}^{\,*} = \sum_{j=1}^{p(n)} \hat{\varphi}_j \hat{u}_{xt-j} + \varepsilon_{xt}^{\,*} \tag{6}$$

On the basis of the values of ε_{xt}^{*} obtained by randomly sampling with replacement from $\tilde{r}_{q,t} = \left(\tilde{r}_{q,1,t}, \ldots, \tilde{r}_{q,N,T}\right)'$, the simulated u_{xt}^{*} are computed and consequently the m_{xt}^{*} are mapped. New matrices of central death rates are obtained as the difference between the observed death rates and the synthetic u_{xt}^{*}. Finally the estimates $\alpha_x^{*}, \beta_x^{*}, k_t^{*}$ are obtained by fitting the log-bilinear structure to the $m_{xt}^{\,*}$. In particular, for each of the B bootstrap samples, the ARIMA model is re-fitted to k_t^{*} and then re-projected. Bootstrap percentile intervals on the re-projected k_t^{*} are constructed. The validity of this AR sieve algorithm adapted to panel data is verified if the matrix r_{xt} is a white noise vector, which requires that there is only contemporaneous dependence between units.

To verify the forecast goodness of the bootstrap technique under consideration, some measures of forecast accuracy can be investigated. There are some commonly used accuracy measures whose scale depends on the scale of the data, like ME, RMSE and MAE, and others scale- independent, like MPE and MAPE. In the following numerical application we offer a comparative implementation of these measures in both the Lee Carter model (where is dependence not assumed) and in the panel sieve algorithm (which considers dependence) to show how the latter improves the accuracy in the mortality forecasts. Moreover, we set out a backtesting procedure for multi-ahead mortality projections (as in Dowd et al. 2010) to evaluate the forecast performance of the bootstrap algorithm.

5. Empirical evidence

Chang (2004) prove the validity of the AR sieve bootstrap in the context of panel data if there is only contemporaneous dependence between units; however, this condition is likely to be violated in many empirical applications. In this section, we verify if the condition of validity of the AR sieve bootstrap in panel data exists for mortality data, in order to apply a bootstrap algorithm to the residuals of the Lee Carter model. In other words, we have to verify if the residuals on which we will operate the bootstrap are distributed as a vector white noise.

We investigate the empirical evidence of the aforementioned condition by considering the Italian male mortality dataset, ranging from 1980 up to 2006, from ages 0 up to 100. The death rates, considered by single calendar year and by single year of age, are aggregated in an open age group 100+ for the class of age above 100 years. Before implementing the bootstrap algorithm, we proceed as follows:

1. we fit the Lee Carter model to the selected dataset;
2. we analyze the residuals: as has been well verified in the literature, the independence assumption is violated;
3. we operate an autoregressive approximation of the residuals for each age. We specify the value of the lag length $q(n)$ by Akaike's information criterion as suggested by Amemiya (1973) and calculate the autoregressive coefficients by using the Yule-Walker method.
4. we verify if the errors of the autoregressive approximation operated in the previous step are a vector white noise.

A *k* by 1 vector stochastic process $\{\varepsilon_t\}$ is said to be a vector white noise if

$$\begin{aligned} E[\varepsilon_t] &= 0_k \\ E[\varepsilon_t \varepsilon'_{t-s}] &= 0_{kxk} \\ E[\varepsilon_t \varepsilon'_t] &= \Sigma \end{aligned} \tag{7}$$

Figure 1 illustrates the evolution of the mortality dynamics over age, simultaneously highlighting the log death rate trends from 1980 to 2006.

Figure 1- log death rates - Italian male population, age: from 0 to 100 (the upper curve represents the year 1980 while the lowest curve shows the rates for 2006)

In order to produce mortality death rates projections, we implement the standard version of the Lee Carter model (1992). Figure 2 shows the estimates of the model parameters provided by the *demography* package developed in R software (Hyndman):

Figure 2- ax, bx, kt adjusted, basic Lee Carter model - Italian male population, age: from 0 to 100

As is shown in Figure 3, there are systematic patterns in the residual plots suggesting that the independence assumption is violated.

Figure 3- Residuals year vs age – basic Lee Carter model -Italian male population, age: from 0 to 100

Starting from the residuals represented, we subdivide the matrix of residuals into *n* vectors, where *n* corresponds to the number of ages being considered, and find an autoregressive approximation for the residuals for each age. For the sake of clarity, let us consider the first row vector of the residual matrix which corresponds to the age equal to 0. We represent it as an AR process and calculate the correspondent forecasted errors. We successively replicate the above operation for each row vector (each age) and construct a new matrix of errors, where, in each row, the errors of the AR processes derived exactly from the residuals of the Lee Carter model are allocated. Finally, by verifying if the composed matrix represents a vector white noise, we can check whether the conditions described in formula (7) are verified. On this basis, we can apply the AR sieve bootstrap for the panel data using this new error matrix.

Numerical results

In previous studies, the fitting of the Lee Carter model has been shown and the residuals have been represented, which reveal that there is dependence in the residuals. In the following, we fully investigate the particular dependence structures. Separately for each row and column, respectively representing age and calendar year, we have produced the correlograms shown, in order to highlight graphically the correlation between values of the process at different points in time and at different ages. The first group of correlograms, which is displayed in Figure 4, is constructed considering the correlation between years for each age. In this case, for each age, we are dealing with a time series generated from a stochastic process and verify the autocorrelations over time. In other words, we verify the existence of temporal dependence for each age during the years. The correlograms show the presence of temporal dependence for almost all ages and in particular for the younger ones.

Figure 4- Autocorrelation function of the residuals by age

The second group of correlograms, which is displayed in Figure 5, is constructed by considering the correlation between ages for each year of the dataset. Thus, time 1 corresponds to the year 1980, time t=2 to the year 1981 and so on. They show the persistence of spatial correlation in almost all cases between the years; in other words, in this case, spatial correlation means that there is a dependence structure between ages in the same year and this appears for each year that is separately considered: given t, we observe the correlation between the residuals of age x=0,1,2,…,p where p is the maximum lag considered.

Figure 5 - Autocorrelation function of the residuals by time

The previous graphical analysis is supported by the results of the Ljung-Box test, which have been implemented for each age separately. As shown in table 1, for almost all ages, the hypothesis of null correlation is rejected. In conclusion, we note that the presence of a dependence structure between residuals of the mortality model has been verified and so needs to be taken into account.

Table 1 - Ljung-Box test on the residuals

Furthermore, for formally testing the dependence structure into the residuals, we have considered also the standard measure of Pearson, since it is particularly suitable to the configuration model which assumes normality in the residuals. Table 2 and Figure 6 show a strong positive dependence.

Table 2 - Pearson's correlation coefficient test on the residuals

Figure 6 - Pearson's correlation coefficient on the residuals - contour map

Thus, the presence of a dependence structure in residuals of the mortality model has been verified and needs to be taken into account.

At this stage we compare two kinds of simulation scheme: a) the residual bootstrap on the Lee Carter residuals relying on the independence assumption; b) the Panel Sieve bootstrap algorithm that we have developed in the Lee Carter setting for capturing the dependence structures which we have assessed. Figures 7-10 display the simulated trajectories for the model parameters for α_x, β_x and k_t in the two different bootstrap schemes for different numbers of simulations $B = 100{,}500{,}1000$. The model is fitted to the Italian male mortality dataset, ranging from 1980 up to 2006, from ages 0 up to 100 and then the parameter kt is projected for h=1,...,15 years ahead.

We begin by examining the following Figures which illustrate the simulated patterns for the model parameters for α_x, β_x and k_t and for the projected k_t in the case of $B = 1000$

Figure 7 – Simulated paths for ax – Residual Bootstrap and Panel Sieve Bootstrap

Figure 8 - Simulated paths for bx – Residual Bootstrap and Panel Sieve Bootstrap

Figure 9 - Simulated paths for kt – Residual Bootstrap and Panel Sieve Bootstrap

Figure 10 - Forecasted kt – Residual Bootstrap and Panel Sieve Bootstrap

As is highlighted in the graphs, the Panel Sieve Bootstrap produces wider confidence intervals, since it allows for another source of risk: *the dependency risk* (D'Amato et al. 2012).

In our analysis, we find the following numerical results on the basis of the algorithm indicated in section 4. Table 3 illustrates different percentiles of the mean of projection of k_t obtained implementing different bootstrap algorithms for future times of valuation equal to h and for the number of simulations equal to $B = 100$. In particular, for $h = 1, \ldots, 15$ periods ahead, the performance of the residual bootstrap and panel sieve bootstrap is examined by calculating 5% and 95% confidence intervals, CI's.

Table 3 – Comparison among Residual Sieve Bootstrap (RSB) and Panel Sieve Bootstrap (PSB), B=100

As is clearly shown by Table 4, if we compare the different algorithms in terms of the distance between the 95% and 5% percentiles, we notice the wider CI's for the Panel Sieve Bootstrap. From this point of view, the residual bootstrap leads to less uncertain projections, with the dependency in the data being completely neglected. In the case of Panel Sieve bootstrap procedure we are able to capture the whole correlation structure and thereby obtain more reliable projections.

Table 4 – Comparison among Residual Bootstrap and Panel Sieve Bootstrap, in terms of the difference between 95% and 5%, B=100

The outcomes remain stable for the increasing the number of replications, as shown in tables 5-8, for the cases of $B = 500$ and $B = 1000$.

Table 5 – Comparison among Residual Bootstrap and Panel Sieve Bootstrap, B=500

Table 6 – Comparison among Residual Bootstrap and Panel Sieve Bootstrap, in terms of the difference between 95% and 5%, B=500

Table 7– Comparison among Residual Bootstrap and Panel Sieve Bootstrap, B=1000

Table 8 – Comparison among Residual Bootstrap and Panel Sieve Bootstrap, in terms of the difference between 95% and 5%, B=1000

To verify the forecast goodness of the panel sieve bootstrap technique, we investigate some measures of forecast accuracy in both Lee Carter model and in the panel sieve algorithm; table 9 shows how the panel sieve bootstrap improves the accuracy in the mortality forecasts with respect to widely used Lee Carter model.

Table 9– Comparison among Lee Carter and Panel Sieve bootstrap in terms of forecast accuracy

Finally, we set out a backtesting procedure for multi-ahead mortality projections to evaluate the forecast performance of the panel sieve bootstrap algorithm. Its implementation is based on the following steps:

- selection of the metric of interest: we have chosen to adopt the life expectancy at birth, which is a very useful metric in the actuarial practice;
- selection of the historical lookback window and the lookforward window to make the backtesting forecasts: we have considered the Italian male mortality dataset, ranging from 1980 up to 1996, from ages 0 up to 100; it is a reduced dataset with respect to that previously used, so that it is possible to produce projections from 1997 to 2006 and compare them with the realized values of the metric of interest.
- graphical results.

Figure 11 – Backtesting on life expectancy in LC and PSB

Figure 11 shows the results of the backtesting procedure; in it we compare the actual expectancy of life at birth, calculated on the realized mortality dataset for an Italian male ranging from 1980 to 2006, with those obtained with a backtesting on the Lee Carter model and the panel sieve boostrap. We observe that, even though both models produce a well-known underestimation of the life expectancy, the projections achieved with the panel sieve bootstrap are closer to the actual values, due to a wider projection interval.

6. Concluding Remarks

The complex structure of the longevity phenomenon means that, in order to produce reliable projections of mortality indices, the interactions between age and time cannot be neglected. Ignoring *dependency risk* (D'Amato et al. 2012) would lead to an inefficient risk management strategy for insurance companies.

In particular, the presence of spatial dependence across age and time leads to a systematic over-estimation or under-estimation of uncertainty in the estimates, caused by whether negative or positive dependence dominates (Liu et al. 2010).

As is well-known in the demographic literature, the Lee Carter model has become the seminal statistical model for projections of mortality. To obtain a measurement of the uncertainty in the forecasted mortality rates, reliable confidence intervals for the quantities of interest connected to the phenomenon under consideration can be calculated on the basis of simulation techniques.

Nevertheless, we propose a method which leads to a prudent measure of longevity risk, avoiding the structural incompleteness of the ordinary simulation bootstrap methodology which involves the assumption of independence. The algorithm that we have studied combines model-based predictions in Lee Carter framework (1992) with a bootstrap procedure for dependent data, and so both the historical parametric structure and the intra-group error correlation structure are preserved. D'Amato et al. (2012) apply a sieve bootstrap to the residuals of the Lee Carter model, according to the design of the typical autoregressive sieve bootstrap. According to this scheme, we develop a Panel Sieve Bootstrap in the Lee Carter setting, and are able to reproduce in the sampling the dependence structure that exists between the years of the dataset for each age. The methodology is sufficiently flexible to be extended to the whole family of the Lee Carter models in order to take into account additional issues. In particular one important question is represented by the cohort effect. In this paper, the benefit of introducing the cohort effect has not been studied, but certainly deserves a deeper investigation. Nevertheless, the method we have proposed, which utilises the basic version of the Lee Carter model, can incorporate a consideration of the cohort effect. In this context, the literature recognises the desirable properties and the good performance of the Renshaw and Haberman (2006) model which incorporates the cohort effect in the Lee Carter model. In this context, the literature recognises the desirable properties and the good performances of the Renshaw and Haberman Lee Carter version (2006) which allows for cohort effect.

Another additional issue to address is the higher variability in the older age groups due to small sample size, that influences the accuracy in the mortality. Hyndman and Ullah (2007) show a particular version of the LC methodology based on the combination of functional data analysis and nonparametric smoothing and D'Amato et al (2011c) offer a comparative analysis of LC and the Hyndman Ullah version. Further analysis could combine smoothing techniques and bootstrap procedure in the mortality setting to improve beyond the forecasts.

Future research will focus on detecting the dependence across different populations. The investigation about the factors driving changes in mortality, in particular across countries, requires us to handle the related high dimensional data question. The high dimensionality refers to the total number of data 'cells' that are modeled, and this is equal to the product of the numbers of categories for the factors classifying the data.

In order to address the dimensionality problem by extracting age patterns from the data we will take into account principal components approaches in future work.

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Figure

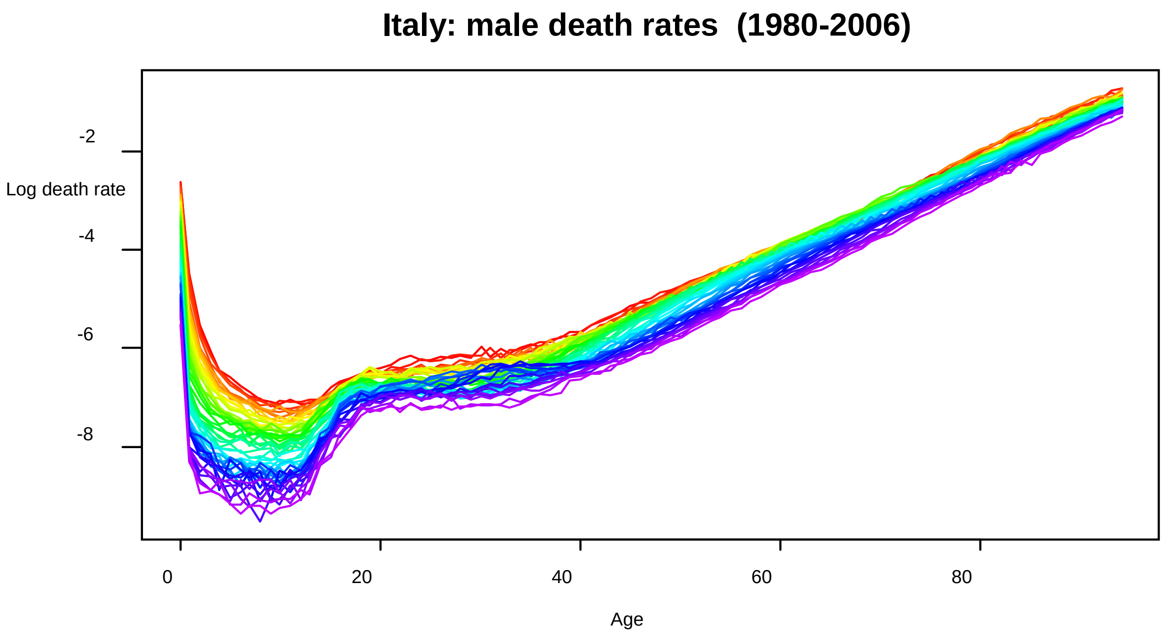


Figure 1- log death rates - Italian male population, age: from 0 to 100 (the upper curve represents the year 1980 while the lowest curve shows the rates for 2006)

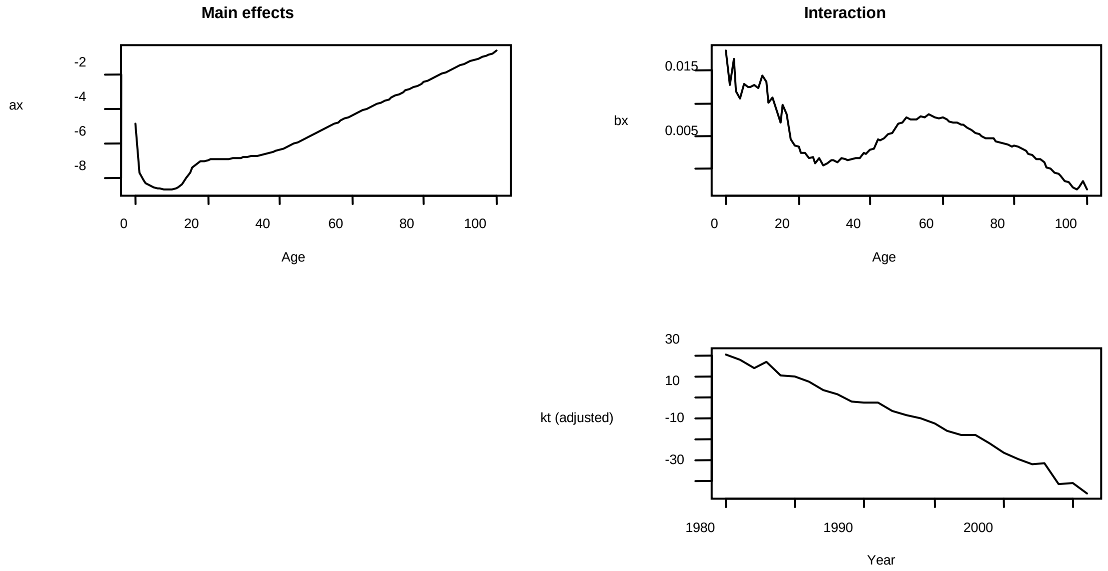


Figure 2- ax, bx, kt adjusted, basic Lee Carter model - Italian male population, age: from 0 to 100

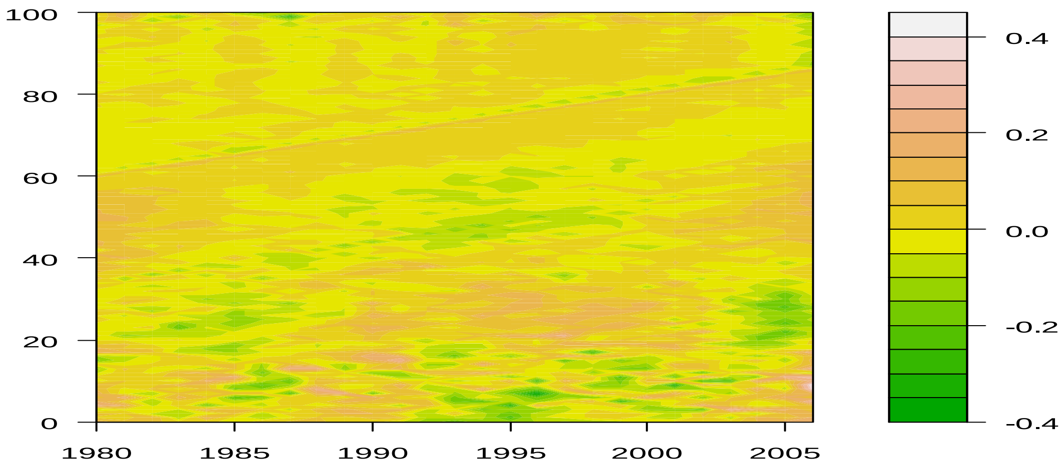
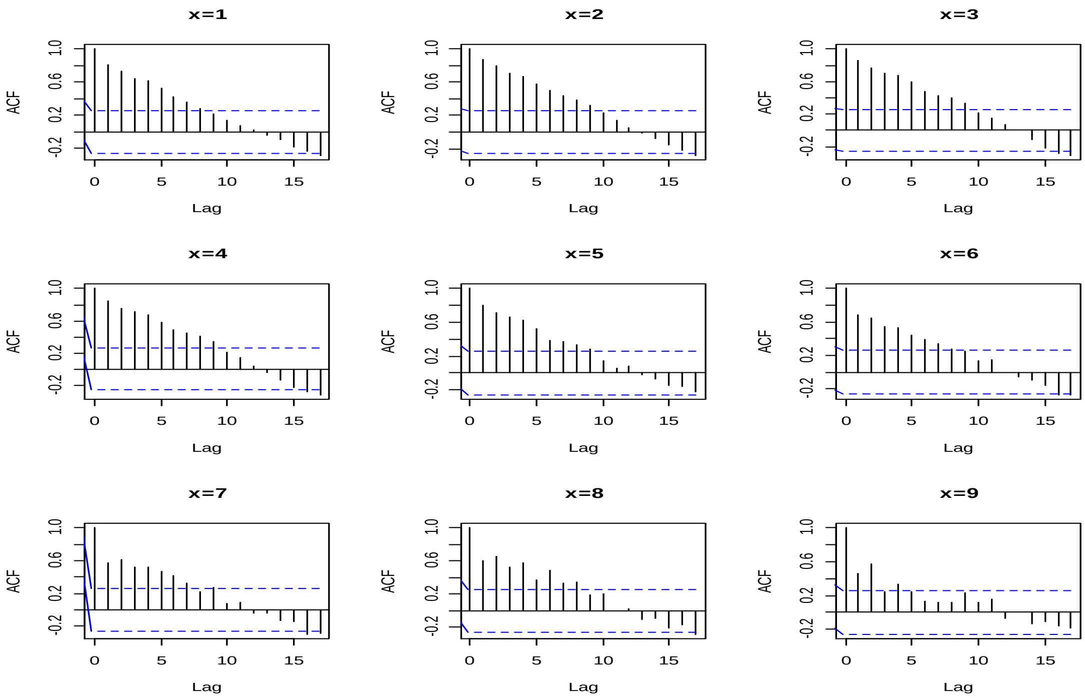


Figure 3- Residuals year vs age – basic Lee Carter model -Italian male population, age: from 0 to 100



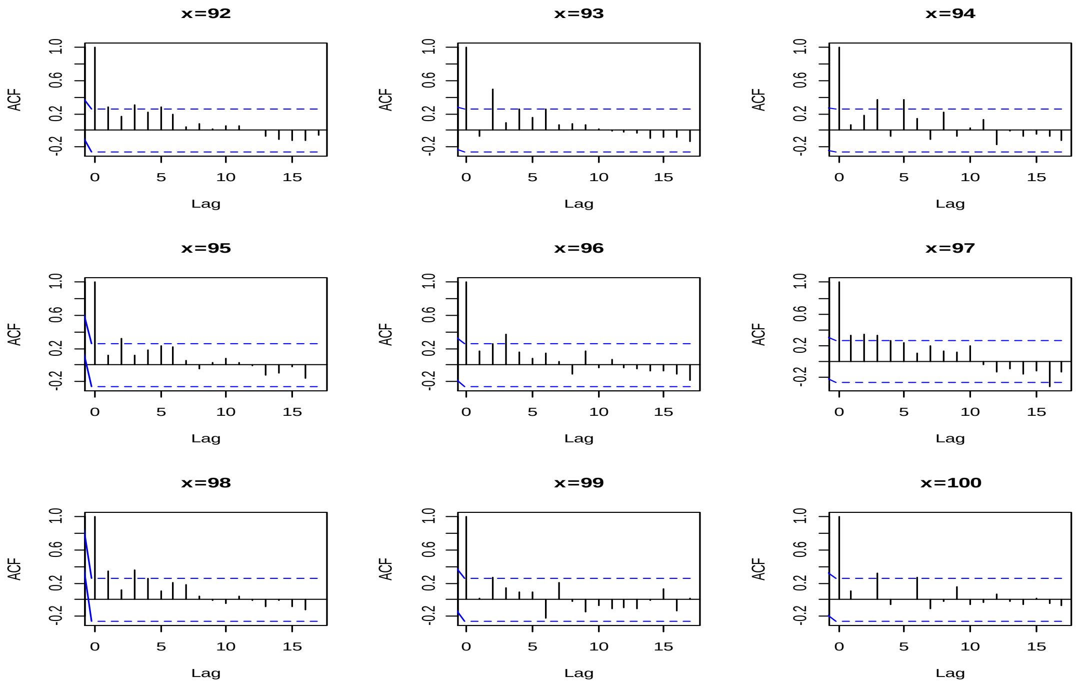
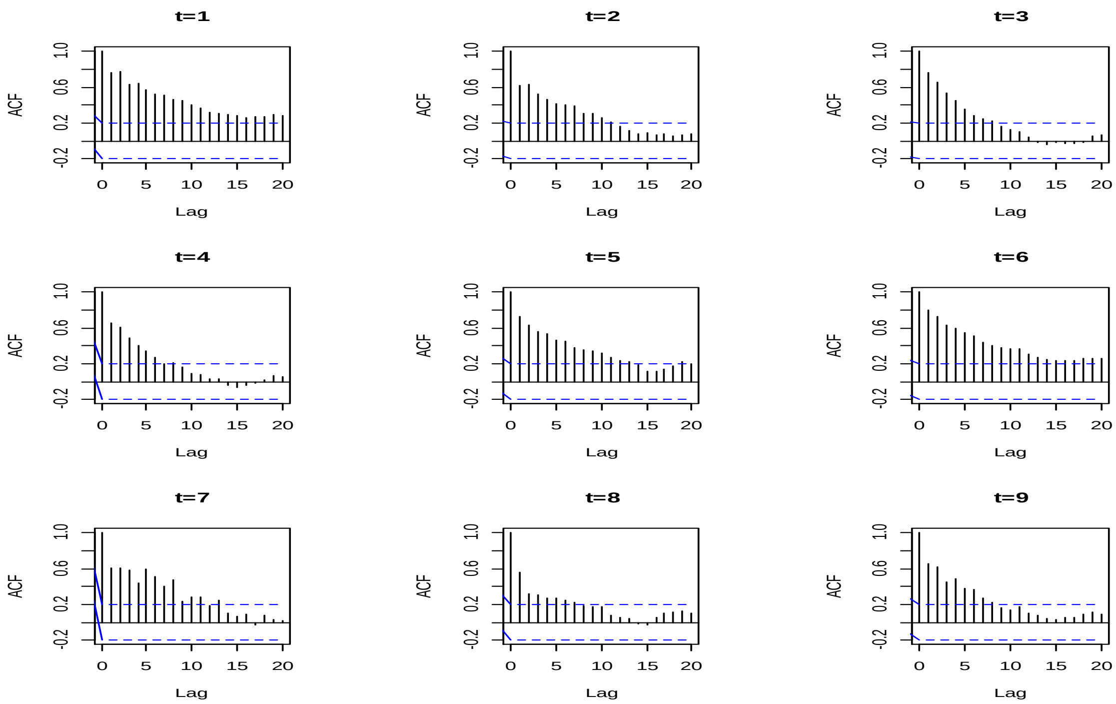


Figure 4- Autocorrelation function of the residuals by age



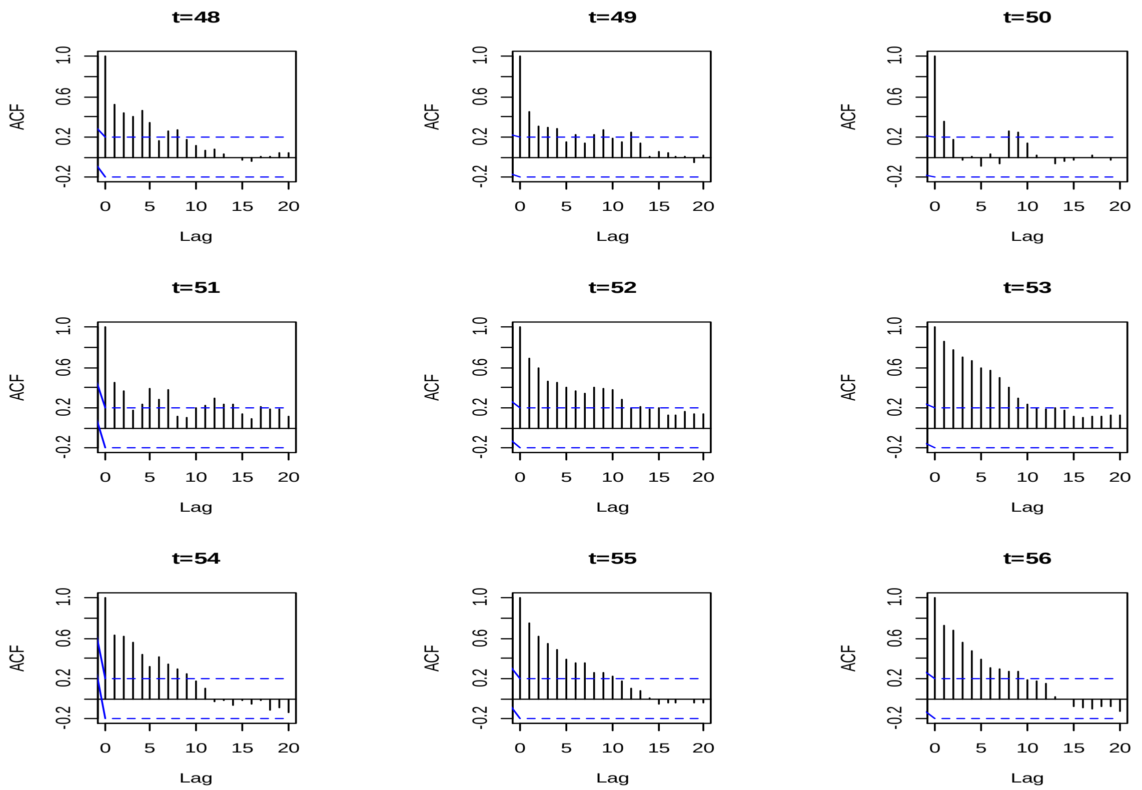


Figure 5 - Autocorrelation function of the residuals by time

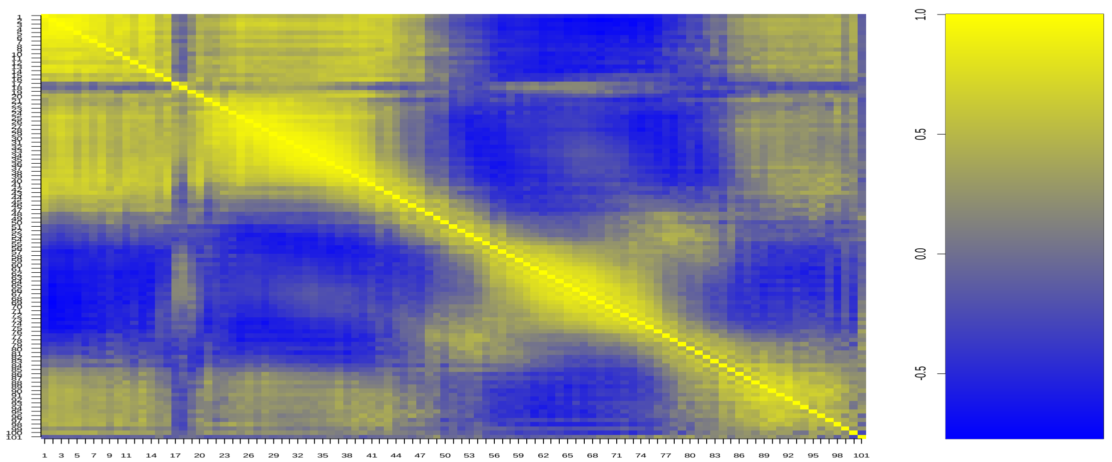


Figure 6 - Pearson's correlation coefficient on the residuals - contour map

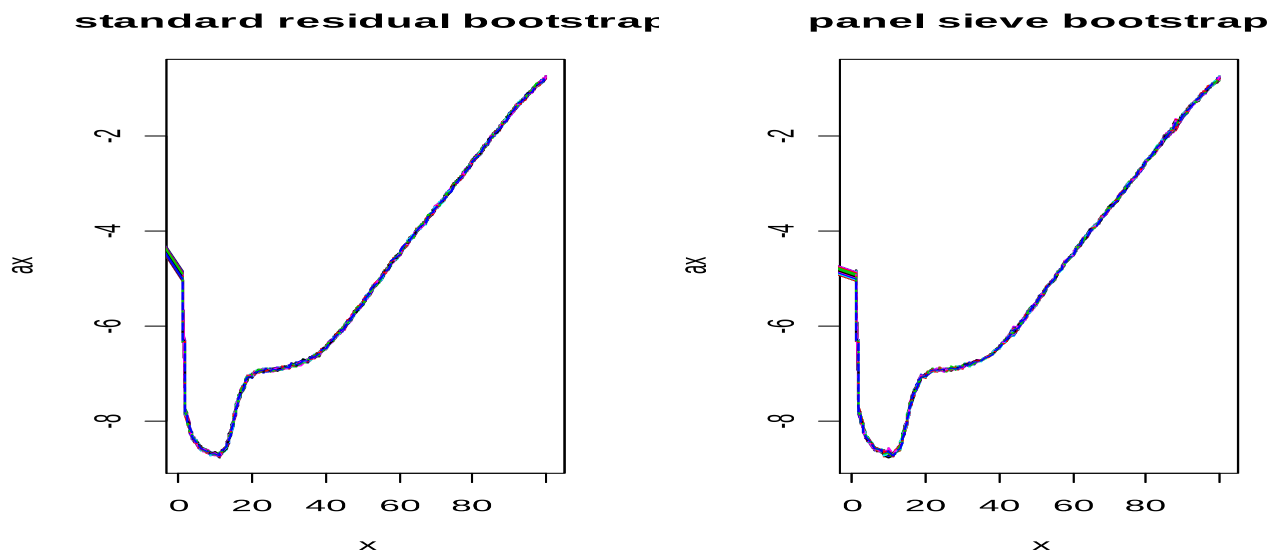


Figure 7 – Simulated paths for ax – Residual Bootstrap and Panel Sieve Bootstrap

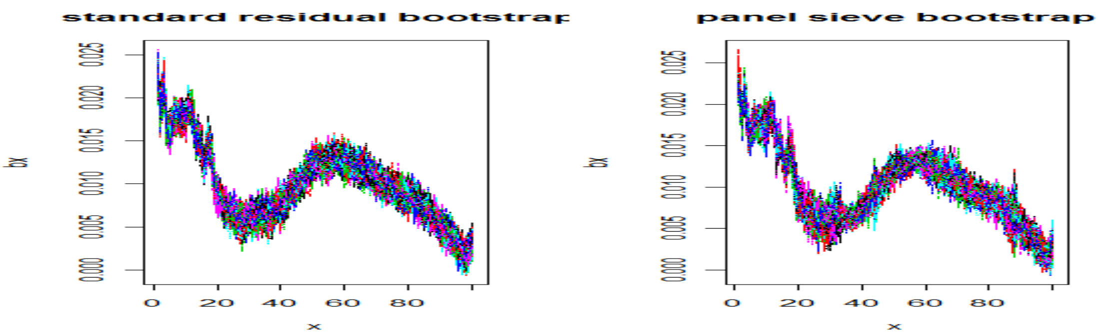


Figure 8 - Simulated paths for bx – Residual Bootstrap and Panel Sieve Bootstrap

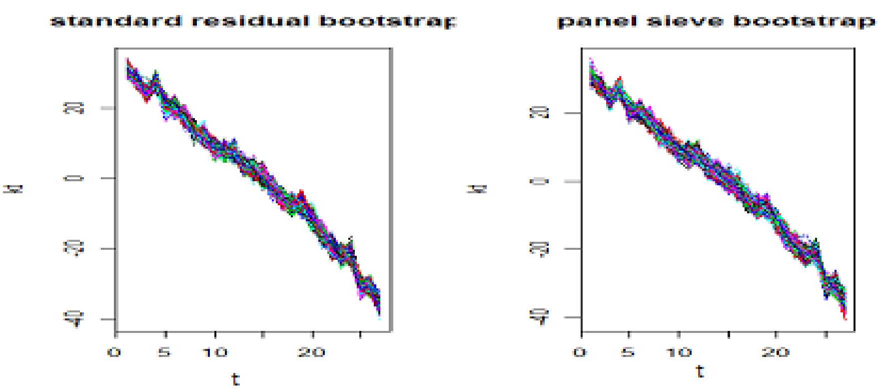


Figure 9 - Simulated paths for kt – Residual Bootstrap and Panel Sieve Bootstrap

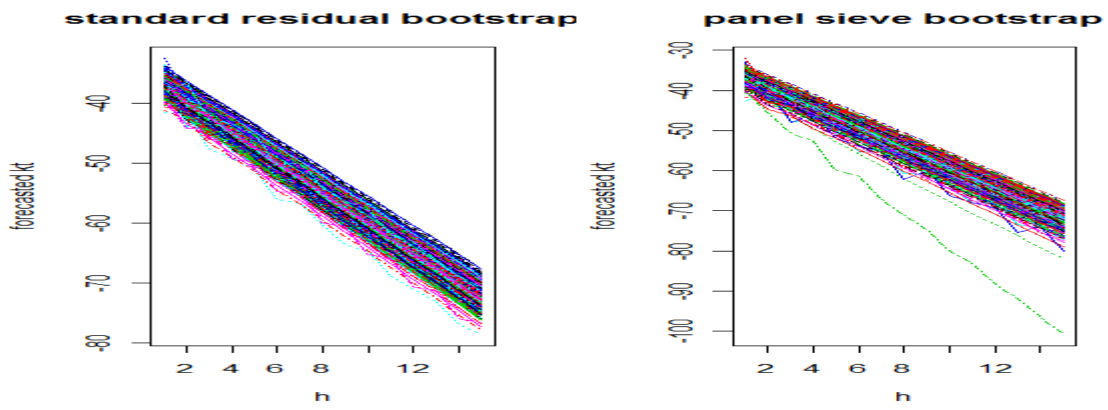


Figure 10 - Forecasted kt – Residual Bootstrap and Panel Sieve Bootstrap

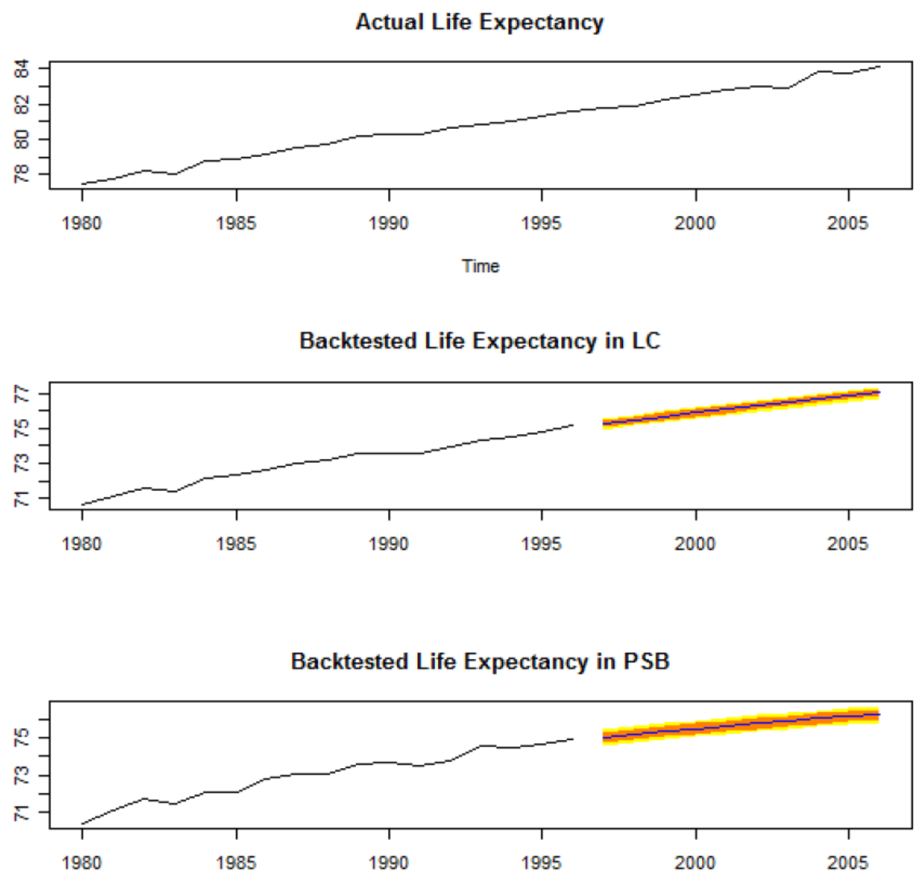


Figure 11 – Backtesting on life expectancy

Table

Ljung-Box test

x=age	X-squared, df=10	p-value	x=age	X-squared, df=10	p-value	x=age	X-squared, df=10	p-value	x=age	X-squared, df=10	p-value
0	171.4472	< 2.2e-16	**26**	143.561	< 2.2e-16	**52**	85.7832	3.653e-14	**78**	20.5523	0.02444
1	218.2905	< 2.2e-16	**27**	144.3126	< 2.2e-16	**53**	50.4564	2.2e-07	**79**	15.0216	0.1313
2	214.1386	< 2.2e-16	**28**	168.4717	< 2.2e-16	**54**	40.0539	1.658e-05	**80**	20.5557	0.02441
3	212.4354	< 2.2e-16	**29**	157.1803	< 2.2e-16	**55**	44.725	2.438e-06	**81**	21.3561	0.01874
4	174.1954	< 2.2e-16	**30**	154.4508	< 2.2e-16	**56**	70.2763	3.921e-11	**82**	20.9374	0.02153
5	133.0718	< 2.2e-16	**31**	162.6379	< 2.2e-16	**57**	77.6903	1.422e-12	**83**	6.409	0.7798
6	118.8049	< 2.2e-16	**32**	150.9264	< 2.2e-16	**58**	74.802	5.199e-12	**84**	16.3974	0.08881
7	133.5848	< 2.2e-16	**33**	163.961	< 2.2e-16	**59**	84.6718	6.062e-14	**85**	13.3461	0.205
8	55.0495	3.091e-08	**34**	152.1727	< 2.2e-16	**60**	87.3811	1.765e-14	**86**	14.0861	0.1691
9	90.1288	4.996e-15	**35**	156.5525	< 2.2e-16	**61**	95.115	5.551e-16	**87**	20.9962	0.02112
10	117.0806	2.2e-16	**36**	141.9757	< 2.2e-16	**62**	105.7261	< 2.2e-16	**88**	21.4929	0.01791
11	73.7842	8.2e-12	**37**	137.5273	< 2.2e-16	**63**	99.1519	< 2.2e-16	**89**	28.552	0.001472
12	126.8546	< 2.2e-16	**38**	133.7374	< 2.2e-16	**64**	107.2912	< 2.2e-16	**90**	21.2368	0.0195
13	94.2806	7.772e-16	**39**	115.5826	< 2.2e-16	**65**	108.3895	< 2.2e-16	**91**	24.1413	0.007234
14	47.1129	9.007e-07	**40**	135.2164	< 2.2e-16	**66**	100.9233	< 2.2e-16	**92**	27.0711	0.002538
15	56.4433	1.695e-08	**41**	80.119	4.757e-13	**67**	105.7413	< 2.2e-16	**93**	26.1229	0.003578
16	11.8187	0.2974	**42**	67.1395	1.576e-10	**68**	98.1969	< 2.2e-16	**94**	18.2301	0.0512
17	31.9824	0.0004031	**43**	49.8749	2.814e-07	**69**	89.2363	7.55e-15	**95**	20.8646	0.02206
18	18.6092	0.04552	**44**	55.8906	2.152e-08	**70**	82.5403	1.592e-13	**96**	37.5233	4.593e-05
19	38.2392	3.448e-05	**45**	35.8041	9.096e-05	**71**	67.5697	1.303e-10	**97**	26.8398	0.002761
20	79.3661	6.682e-13	**46**	48.9972	4.078e-07	**72**	100.8786	< 2.2e-16	**98**	14.9058	0.1355
21	53.1234	7.063e-08	**47**	56.3886	1.736e-08	**73**	74.967	4.829e-12	**99**	15.0975	0.1285
22	58.857	5.96e-09	**48**	38.7686	2.787e-05	**74**	91.887	2.22e-15	**100**	20.8864	0.0219
23	83.4226	1.068e-13	**49**	28.0167	0.001794	**75**	63.2996	8.561e-10			
24	146.1879	< 2.2e-16	**50**	48.7136	4.596e-07	**76**	49.4813	3.324e-07			
25	140.6023	< 2.2e-16	**51**	47.4033	7.974e-07	**77**	41.8919	7.838e-06			

Table 1 - Ljung-Box test on the residuals

Pearson's correlation coefficient

x=age	Value	x=age	Value	x=age	Value
0,1	0.9249975	25,26	0.8967373	51,52	0.5762156
1,2	0.9593225	26,27	0.9136936	52,53	0.608357
2,3	0.9616913	27,28	0.8928995	53,54	0.6347328
3,4	0.91959	28,29	0.8973545	54,55	0.5896141
4,5	0.8337363	29,3	0.9353529	55,56	0.4981563
5,6	0.8806236	30,31	0.9362392	56,57	0.5836698
6,7	0.7955352	31,32	0.9336044	57,58	0.6220767
7,8	0.7433184	32,33	0.940451	58,59	0.6545007
8,9	0.6569047	33,34	0.9311992	59,6	0.714403
9,10	0.6516526	34,35	0.9343669	60,61	0.7304692
11,12	0.6837091	35,36	0.9261032	61,62	0.7586733

12,13	0.6134128	36,37	0.8973486	62,63	0.8151193
13,14	0.7446103	37,38	0.880285	63,64	0.8005696
14,15	0.6364435	38,39	0.8352225	64,65	0.8044449
15,16	0.5834051	39,40	0.8086334	65,66	0.8336427
16,17	0.161985	40,41	0.853257	66,67	0.821401
17,18	0.6298621	41,42	0.6992298	67,68	0.8329737
18,19	0.3826222	42,43	0.6305726	68,69	0.7977524
19,20	0.6161803	43,44	0.6976953	69,7	0.7648516
20,21	0.537574	44,45	0.6622836	70,71	0.7532988
21,22	0.7511346	45,46	0.5847089	71,72	0.7505663
22,23	0.6995258	46,47	0.5756761	72,73	0.7210286
23,24	0.7612173	47,48	0.564031	73,74	0.7078396
24,25	0.82023	48,49	0.4790489	74,75	0.692018

Table 2 - Pearson's correlation coefficient test on the residuals

k	h	1	2	3	4	5	6	7	8	9	10
RSB	95%	-35,1029	-37,8412	-40,2909	-42,7406	-45,1903	-47,6399	-50,0896	-52,5393	-54,9890	-57,4386
	5%	-37,9485	-41,6024	-43,6126	-46,3549	-48,7447	-51,4305	-53,9951	-56,6297	-59,2171	-61,8310
PSB	95%	-34,4772	-37,3907	-39,9745	-42,2592	-44,7042	-47,1557	-49,5995	-52,0901	-54,4603	-56,9359
	5%	-38,3828	-41,2725	-43,8084	-46,3553	-48,8723	-51,5311	-54,0284	-56,7043	-59,2498	-61,8862

Table 3 – Comparison among Residual Sieve Bootstrap (RSB) and Panel Sieve Bootstrap (PSB), B=100

d=percentile difference	1	2	3	4	5	6	7	8	9	10
RSB	2,8457	3,7611	3,3217	3,6143	3,5544	3,7906	3,9055	4,0904	4,2281	4,3923
PSB	3,9057	3,8819	3,8339	4,0962	4,1680	4,3754	4,4289	4,6142	4,7895	4,9503

Table 4 – Comparison among Residual Bootstrap and Panel Sieve Bootstrap, in terms of the difference between 95% and 5%, B=100

k	h	1	2	3	4	5	6	7	8	9	10
RSB	95%	-34,8267	-37,4444	-39,8900	-42,3531	-44,8069	-47,2672	-49,7144	-52,1682	-54,6224	-57,0763
	5%	-38,1847	-41,4835	-43,7844	-46,3902	-48,8436	-51,4966	-53,9663	-56,6517	-59,1648	-61,7945
PSB	95%	-34,7849	-37,3747	-39,7985	-42,2644	-44,6976	-47,1526	-49,6018	-52,0752	-54,5231	-56,9483
	5%	-38,3525	-41,5316	-43,9232	-46,4729	-49,0212	-51,5929	-54,0779	-56,7117	-59,2265	-61,8516

Table 5 – Comparison among Residual Bootstrap and Panel Sieve Bootstrap, B=500

d=percentile difference	1	2	3	4	5	6	7	8	9	10
RSB	3,3580	4,0391	3,8944	4,0370	4,0367	4,2294	4,2518	4,4835	4,5424	3,3580
PSB	3,5676	4,1569	4,1246	4,2085	4,3236	4,4402	4,4761	4,6365	4,7034	3,5676

Table 6– Comparison among Residual Bootstrap and Panel Sieve Bootstrap, in terms of the difference between 95% and 5%, B=500

k	h	1	2	3	4	5	6	7	8	9	10
RSB	**95%**	-34,8927	-37,6057	-40,0222	-42,5391	-44,9649	-47,4626	-49,8737	-52,3705	-54,8035	-57,2775
	5%	-38,3747	-41,6093	-43,9257	-46,5947	-48,9216	-51,7172	-54,0910	-56,7676	-59,2577	-61,8552
PSB	**95%**	-34,7843	-37,3706	-39,7948	-42,2834	-44,7101	-47,1804	-49,6229	-52,0777	-54,5262	-56,9918
	5%	-38,5647	-41,7132	-43,9634	-46,6553	-49,0909	-51,8127	-54,2415	-56,9547	-59,3713	-61,9779

Table 7– Comparison among Residual Bootstrap and Panel Sieve Bootstrap, B=1000

d=percentile difference	1	2	3	4	5	6	7	8	9	10
RSB	3,4821	4,0036	3,9035	4,0555	3,9567	4,2546	4,2173	4,3972	4,4543	4,5777
PSB	3,7804	4,3426	4,1686	4,3719	4,3808	4,6324	4,6186	4,8770	4,8451	4,9861

Table 8– Comparison among Residual Bootstrap and Panel Sieve Bootstrap, in terms of the difference between 95% and 5%, B=1000

Accuracy Forecast Measures in the Lee Carter setting:

ME	RMSE	MAE	MPE	MAPE
-5.406879e-04	1.961126e+00	1.334911e+00	-1.825702e+02	1.939768e+02

Accuracy Forecast Meausres in the panel sieve setting:

ME	RMSE	MAE	MPE	MAPE
-0.0002705448	1.9668560658	1.3391544329	19.6005880675	27.6438442311

Table 9– Comparison among Lee Carter and Panel Sieve bootstrap in terms of forecast accuracy