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Investment strategies and risk management for participating life insurance contracts

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Abstract

This paper proposes an asset allocation strategy for the risk management of the broad category of participating life insurance policies. The nature of the liability implied by these contracts allows us to treat them as options written on the reference portfolio backing the policy; consequently, the valuation approach is based on classical contingent claim theory. This leads to the identification of additional safety loadings against the risk of default implied by these contracts, and the setting up of suitable investment strategies aimed at minimizing this risk. The impact on the solvency requirements for the capital of the insurer of the proposed asset allocation strategy is analyzed by means of Monte Carlo techniques. Stress testing is considered as well with respect to the key risk factors of the model, such as the equity volatility and the market interest rate. The numerical analysis shows that, for the specific policy design considered in this paper, a suitable choice of the participation rate combined with the proposed investment strategy minimizes the overall default risk of the insurance company, both in terms of probability of default and expected severity.

Keywords: asset allocation, fair valuation, Monte Carlo methods, participating contracts, solvency requirements, TVaR.

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1 Introduction

Participating contracts now comprise an important part of the life insurance markets of many countries, including the main developed economies in Europe, North America, Asia and Australasia. The history of these contracts can be traced back to the with profit policies offered by the Equitable Life in the UK in the early 1800s. The contracts now appear in many manifestations with a wide range of guarantees, options and ancillary benefit features included in the contract design. Correspondingly, they are subject to many risks (e.g. interest rates, financial markets, default and mortality) and so the modelling, valuation, pricing and risk management of participating life insurance contracts have become important subjects for investigation and analysis.

In recent years, there has been a growing literature which has addressed the application of fair valuation, using classical contingent claims theory, to participating life insurance contracts. This literature followed the pioneering work of Brennan and Schwartz (1976) on equity-linked life insurance and the extensions considered by the work of Briys and de Varenne (1994, 1997) to participating life insurance. One of the most recent contributions building on Briys and de Varenne (1994, 1997) is Ballotta et al. (2006.b) in which a simple Black-Scholes economy with constant interest rates is used to analyze the fair design of participating contracts in the case of a very general structure, which includes a minimum guarantee, a scheme for the distribution of the annual profit of the insurance company, a terminal bonus, the so-called default option and shareholders'contribution. Amongst some of the other most recent works on modelling and pricing for this class of contracts, we would also cite Bacinello (2001, 2003), who focuses in particular on the valuation of the surrender option in a discrete time framework by means of the CRR model (Cox et al., 1979); Grosen and Jørgensen (2000, 2002) and Bernard et al. (2005) who, instead, focus on the modelling of early default of the insurance company in an extended Black-Scholes economy. Further extensions aimed at including market shocks have been analyzed by Ballotta (2005) and Kassberger et al. (2008), whilst Ballotta (2009) focuses in particular on the impact of model misspecification on the capital requirements of the insurer.

This trend in the literature has been given added strength in the last few years by the development of new internationally accepted accounting standards, such as for example the International Financial Reporting Standard (IFRS) 4, which have placed an emphasis on "fair value" as a key concept. In this respect, Ballotta et al. (2006.a) offer a review of the fair value based accounting framework promoted by the International Accounting Standards Board (IASB) Insurance Project, with particular emphasis on the model and parameter errors; they also provide a comparison between the fair value principle and standard deterministic reserving methods. An additional aspect of this paper is the treatment which the authors propose for the so-called default option. Following the contributions of Briys and de Varenne (1994, 1997), Ballotta et al. (2006.a) identify the risk of default implied by the participating contract considered in their paper, quantify it via the premium of the default option, and consider it as the

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price of transferring the contract's default risk externally (if the insurance company had been given this possibility). Hence, Ballotta et al. (2006.a) use the premium of this embedded option to calculate a suitable solvency (or safety) loading to the initial premium, based on the principle that, by charging this amount to the policyholder, the insurer would ensure that the participating contract is issued at fair value, i.e. at a no-arbitrage price. This paper also addresses the issue of how this default option premium could be used by the insurance company for hedging and risk management purposes by considering the very simple case of reinvesting this premium in the portfolio backing the policy.

Hedging and reserving for participating contracts with minimum guarantee, though, have been explored with particular attention for example by Consiglio et al. (2006) and Bernard et al. (2006). In particular, Consiglio et al. (2006) consider the problem for an insurance company of structuring the portfolio underlying their with profit contracts, by means of non linear programming with stochastic variables generated by the Wilkie (1995) asset model. Bernard et al. (2006) instead adopt a model set á la Merton (1974) and develop three possible types of protection for the liability generated by this type of contract based on the premium for the default option. In particular, these protection types consist respectively of an increase in the assets backing the liabilities (as in Ballotta et al., 2006.a), Equity Default Swaps, and a synthetic put option representing the potential loss that the policyholders would suffer in the case of default of the insurance company.

Following this latter stream of contributions, the aim of this paper is to consider issues connected with the risk management of participating contracts by analysing the choice of a suitable allocation strategy for the insurance company's assets. In particular, our approach is to consider a common policy design used in the UK for accumulating (or unitized) with profits contracts; this particular policy design incorporates both reversionary and terminal type bonus elements, and hence it differs (in terms of both detail and complexity) from that considered by other authors. The contract design is also common in other European countries and Japan. We apply classical contingent claims theory for its valuation, which allows us to identify the appropriate safety loading; in this sense, the paper follows on from Ballotta et al (2006.b) in terms of choice of the contract design and identification of the corresponding embedded options. We then set up an asset allocation strategy which is aimed at minimizing the risk induced by the policy as measured by the volatility of the (guaranteed) reversionary component. Further, we analyze the impact of the choice of asset allocation on probabilities of default and capital requirements for solvency purposes, and test its robustness via stress testing techniques.

The analysis carried out in this paper shows that the proposed asset allocation, despite reducing the relevance of the default option premium for risk management purposes, generates a probability of default over a 1-year time horizon which lies within the boundaries proposed by the regulatory requirements of Solvency II, and, at the same time, minimizes the corresponding expected loss. Finally, the numerical evidence presented in this paper reveals the existence of an optimal policy design which minimizes the overall default risk of the company (with respect to the given asset allocation strategy) even under extreme market conditions.

The paper is organized as follows. Section 2 introduces the reader to the participating contract considered in this paper, the identification of the embedded options, the market model adopted for this study and the valuation procedure. Section 3 sets out the principles underpinning the asset allocation proposed in this paper. The results of the numerical experiments are presented in section 4, whereas section 5 is devoted to the analysis of the stress testing procedure. Section 6 provides some concluding comments.

2 Participating contracts with a minimum guarantee

Consider a participating contract initiated at time $t = 0$ with the payment of a single premium $\pi(0)$. Under the terms of the policy, the insurer invests the premium in a fund, F, in the financial market together with the capital provided by the shareholders, $E(0)$. In particular, we assume that $\pi(0) = \theta F(0)$ and $E(0) = (1-\theta)F(0)$, where $F(0)$ is the initial value of the reference fund and θ is the cost allocation parameter, in the sense that θ governs the distribution of the financial risk between the policyholders and the shareholders, and therefore determines the size of the debt in the company's balance sheet (in this respect, Ballotta et al. (2006.b) regard θ as a leverage coefficient). Finally, the insurer credits interest on the policy's account balance until the contract expires; the accumulated benefit is then payable if the policyholder dies during the policy term of T years, or (at maturity) after T years. In either event, the account is settled by a single payment from the insurer to the policyholder, unless the policyholder is entitled to terminate the contract at his/her discretion prior to expiration, through the so-called surrender option.

The single benefit payment is formed by a guaranteed component, π, which is represented by the terminal value of the policy's account balance, and a discretionary bonus, R. The design of the guaranteed component generally includes a minimum guaranteed annual rate, r_G, and a participation in the surplus earned by the insurance company. The exact specification varies from country to country. In this study, we focus on the case of a typical UK accumulating (unitized) with profit contract, so that π includes a regular periodic reversionary bonus reflecting the individual policyholder's shares of the company's profit. As noted by Booth et al. (2005), most participating life insurance contracts in the UK are now of the accumulating with profit type.

As the liability implied by these contracts is linked to the investment profile of the insurer, the sources of risk associated with this type of contract include the risk from the financial markets, from surrenders and from mortality. In this analysis, we focus only on the first type of risk and we ignore both the possibility that the policyholder sells back the contract to the insurer (the surrender option) and mortality, recognizing that these are both areas in which the analysis may be extended.

In the following sections, we introduce the features of the benefit paid by the participating contract considered in this paper, with particular focus on the identification of all the options embedded in the contract design, and their impact on the solvency profile of the insurance company. In particular, we adopt the same set up as in Ballotta et al. (2006.b).

2.1 Contract design

We assume that at the beginning of each period over the lifetime of the contract, the policy reserve, π, accumulates at rate r_π so that

$$\begin{cases} \pi(t) = \pi(t-1)(1 + r_\pi(t)) \quad t = 1, 2, ..., T \\ \pi(0) = \theta F(0). \end{cases} \tag{1}$$

The rate r_π incorporates the minimum annual guaranteed rate, r_G, and the reversionary bonus rate. Our detailed assumption is that the scheme for the distribution of the returns earned by the reference fund is based on the arithmetic average of the last τ period returns on the reference portfolio, so that

$$\begin{aligned} r_\pi(t) &= \max\left\{ r_G, \frac{\beta}{n}\left(\frac{F(t)}{F(t-1)} + ... + \frac{F(t-n+1)}{F(t-n)} - n \right) \right\} \\ &= r_G + \left(\frac{\beta}{n} \sum_{i=1}^{n} \frac{F(i)}{F(i-1)} - (\beta + r_G) \right)^+, \end{aligned} \tag{2}$$

where $\beta \in (0, 1)$ denotes the participating rate and n is the length of the smoothing period chosen as $n = \min(t, \tau)$.

Thus, the annual credited interest rate depends on a smoothed percentage of the annual returns generated by the reference fund. Insurance companies in the UK also use alternative smoothing schemes for the building up of the benefits, with the reversionary bonus based, for example, on the geometric average of the last τ period returns on the reference portfolio, or on the principle of the "smoothed asset share" (see, for example, Chadburn (1998) and Tillinghast-Towers Perrin (2001)).

It is clear from equation (2) that, on the one hand, the benefit π is guaranteed never to fall below the contractually specified rate r_G, offering in this way a downside protection to the policyholder. On the other hand, the benefit π also incorporates a sequence of Asian call options on the fund's returns, which affect the risk profile of the insurer's liabilities and, therefore, require particular attention in terms of pricing and reserving.

As mentioned above, at the maturity of the contract, T, the policyholder receives the terminal value of the policy reserve and any surplus generated by the reference portfolio in excess of the benefit in respect of his/her contribution to the assets of the insurance company. Hence, the full terminal payoff of the contract is

$$B(T) = \pi(T) + \gamma R(T),$$

where

$$R(T) = (\theta F(T) - \pi(T))^{+} \tag{3}$$

denotes the payoff of the terminal bonus, and $\gamma \in (0, 1)$ is a second participation parameter. We note the distinction between the two participation parameters, whereby β affects the annual credited rate $r_\pi(t)$, whereas γ only operates at maturity (or on earlier death). The terminal bonus is, therefore, a European call option on the reference portfolio, capturing the fact that the policyholders do not contribute to possible deficits of the insurance company. We also note that this call option has exotic features due to the floating nature of the strike price.

From equations (1) and (3), it follows that

$$\begin{aligned} \pi(T) &= \theta F(0) \prod_{t=1}^{T} (1 + r_\pi(t)) = \theta \pi^u(T), \\ R(T) &= (\theta F(T) - \pi(T))^{+} = \theta R^u(T), \end{aligned}$$

where $\pi^u(T)$ and $R^u(T)$ denote respectively the value at maturity of the policy reserve and the terminal bonus in the case in which the insurance company is "fully sponsored" by policyholders (i.e. $\theta = 1$). This implies that, in absence of default, the overall benefit function is

$$B(T) = \theta(\pi^u(T) + \gamma R^u(T)). \tag{4}$$

2.2 Default risk and solvency loading

Equation (4) describes the overall benefit which the policyholder is entitled to receive at maturity if the company is solvent, i.e. if the insurer's assets are enough to provide the payment of the guaranteed benefit π. If default occurs, instead, and the equityholders have limited liability, the policyholder takes those assets that are available. Under the assumption that default can only occur at maturity, the liability implied by the participating contract for the insurance company is

$$\begin{cases} F(T) & \text{if } F(T) < \pi(T) \\ \pi(T) & \text{if } \pi(T) \leq F(T) < \frac{\pi(T)}{\theta} \\ B(T) & \text{if } F(T) \geq \frac{\pi(T)}{\theta}, \end{cases} \tag{5}$$

or, in a more compact way,

$$B(T) - D(T), \tag{6}$$

where

$$D(T) = (\pi(T) - F(T))^{+}$$

represents the potential default of the insurance company. As already noted, in this framework, default can only occur at maturity; in practice, though, the regulators monitor the solvency of insurance companies on a regular basis over time (for example, through an annual analysis of statistical data and financial statements). Grosen and Jørgensen (2002) and Bernard et al. (2005) show how to extend the pricing framework in order to incorporate early default. In this paper, we do not explore this issue any further but we refer the interested reader to the above mentioned references.

The balance sheet equation implies that the overall equityholders' claim can be expressed as

$$F(T) - B(T) + D(T).$$

If we denote by $V_{\cdot}(0)$ the market consistent value of each contract component, then the market consistent value of the policyholder's and equityholders' payoff at maturity can be rewritten as

$$\begin{cases} \pi(0) = \theta\left[V_{\pi^u}(0) + \gamma V_{R^u}(0)\right] - V_D(0), \\ E(0) = F(0) - \theta\left[V_{\pi^u}(0) + \gamma V_{R^u}(0)\right] + V_D(0). \end{cases}$$

Further, we note that, since $E(0) = (1-\theta) F(0)$, the fair value equation for the equityholder is the same as the one for the policyholder, i.e. for no arbitrage opportunities to rise in the market, we require that

$$\pi(0) = \theta\left[V_{\pi^u}(0) + \gamma V_{R^u}(0)\right] - V_D(0). \tag{7}$$

Equation (7) provides the market consistent value of the contract considered in this analysis, in the sense that it returns the fair price at which the policy could be traded in an arbitrage-free market. We note, however, that since participating contracts are not fully tradeable in the market, as there is no secondary market for these policies, equation (7) can be used to determine a feasible set of contractual parameters which are consistent with the assumption of no-arbitrage opportunities in the market. It is clear from the previous discussion, in fact, that the value of the contract depends on the specification of the policy design parameters: the level of the guaranteed return, r_G, the participation coefficients, β and γ, the smoothing parameter, τ, and the term of the contract, T. The market parameters, like the reference portfolio volatility or the risk-free rate of interest, are also essential to complete the description of the contract; however, they are in general not under the control of the life insurance company. Ballotta et al. (2006.b) address in great detail the issue of a fair design of these contracts by analyzing the feasible set of the contract parameters leading to a no-arbitrage price, given prespecified market conditions, and the trade-offs between these parameters. For example, they observe that the two participation parameters, β and γ, are in general inversely proportional one to the other: a higher participation in the annual fund performance is, in fact, compensated by a lower terminal bonus rate in order to preserve a no-arbitrage premium (and viceversa).

Finally, we note that from equation (7), it also follows that

$$V_B(0) \doteq \theta\left[V_{\pi^u}(0) + \gamma V_{R^u}(0)\right] = \pi(0) + V_D(0). \tag{8}$$

Therefore, the no arbitrage price of the default option V_D can be considered as the extra "premium" that the insurer has to charge in order to offer the policyholder protection against default, and therefore avoid arbitrage opportunities (Ballotta et al., 2006.a and 2006.b). In this respect, in the sequel we follow Ballotta et al. (2006.a) and define a safety loading rate $\varphi \in \mathbb{R}^{++}$ such that $V_D(0) = \varphi\pi(0)$, so that

$$V_B(0) = (1+\varphi)\,\pi(0), \tag{9}$$

which is in the spirit of the "expected value" premium principle of actuarial theory (Daykin et al. (1994)). In particular, Ballotta et al. (2006.a) explore the consequences of charging this additional premium to the policyholder and investing it in the same reference fund underlying the life insurance policy. The results of their analysis show a significant reduction in the probability of default at maturity, even if the insurance company is, in a sense, "passive" in terms of implementing a carefully designed hedging strategy.

2.3 The reference portfolio

The final component of the participating policy is the reference fund, F, in which the policyholder's contribution is invested and used for the calculation of his/her benefit. In the subsequent sections of this paper, we assume that F is represented by a portfolio of equity shares, and zero coupon bonds with redemption date set equal to the maturity of the policy. Thus, setting $\alpha \in [0,1]$ as the percentage of equity held in the fund, and assuming that the asset allocation is kept constant over time and rebalancing is carried out once a year, it follows that

$$F(t) = \alpha F(t-1)\frac{S(t)}{S(t-1)} + (1-\alpha)F(t-1)\frac{P(t,T)}{P(t-1,T)},$$

where $S(t)$ and $P(t,T)$ are the prices at time t respectively of the equity share and the zero coupon bond expiring at time T.

In more detail, we consider a frictionless market, with continuous trading and perfectly divisible securities, in which the equity share, S, follows (under the real probability measure $\mathbb{P}$) a geometric Brownian motion

$$dS(t) = \mu S(t)\,dt + \sigma S(t)\,dW(t), \tag{10}$$

with $\mu \in \mathbb{R}$ and $\sigma \in \mathbb{R}^{++}$, and the term structure of interest rates is modeled within a Heath-Jarrow-Morton (HJM) framework (Heath et al., 1992) with exponentially decaying forward rate volatility. The corresponding dynamic of the short rate, under $\mathbb{P}$, is therefore (see for example Chiarella and Kwon, 2001) given by

$$dr(t) = \kappa\left(a(t) - r(t)\right)dt + v\,dZ(t), \tag{11}$$

where

$$a(t) = \frac{v^2}{2\kappa^2}\left(1 - e^{-2\kappa t}\right) + \frac{v}{\kappa}\lambda_t,$$

and λ is the market price of interest rate risk. The corresponding price of a T-zero coupon bond is

$$\begin{aligned} P(t,T) &= \frac{P(0,T)}{P(0,t)} e^{-C(t,T)-\delta(t,T)(r(t)-f(0,t))}, \qquad (12) \\ \delta(t,T) &= \frac{1-e^{-\kappa(T-t)}}{\kappa}, \\ C(t,T) &= \frac{v^2}{4\kappa}\delta(t,T)^2\left(1-e^{-2\kappa t}\right), \\ f(0,t) &= r_0 e^{-\kappa t}. \end{aligned}$$

Finally, we assume that the stock price and the short rate of interest are correlated, i.e. $dW(t)\,dZ(t) = \rho dt$, where $\rho \in (-1,1)$ is the instantaneous correlation coefficient between the driving Wiener processes.

3 Asset allocation strategy

In the light of the capital requirements imposed on insurance companies by regulatory authorities following the move towards the new Solvency II regime, the pricing and hedging of the options embedded in participating policies have become a crucial issue for the risk management of these contracts. However, the absence of a liquid secondary market for insurance contracts, the long maturities involved with these contracts and the nature of the corresponding reference portfolios make it challenging to find, in the financial market, suitable derivatives with which to match the risk profile of the liabilities implied by participating contracts like the one described in section 2. In this respect, however, the insurance company could implement an asset allocation strategy for the reference portfolio which is aimed at minimizing (in some sense) the financial risk induced by the insurance policy. Hence, in this section, we consider the problem of choosing a (static) asset allocation strategy which "stabilizes" the expected guaranteed benefit due at maturity, $\pi(T)$, with respect to some prespecified target and according to a prespecified optimality criterion. We then analyze the impact of this optimal allocation on the solvency requirements for the insurer and its behaviour when the design parameters are changed.

Thus, in our approach, we calculate the target as the value at maturity of the minimum guarantee (i.e. the fixed part of the benefit) increased by $h\%$ in order to take into account the participation of the policyholder in the returns generated by the reference fund, i.e.

$$\mathrm{t} \doteq \pi_0 \left(1 + r_G (1+h)\right)^T.$$

As far as the optimality criterion is concerned, our objective is to minimize the volatility, under the real probability measure, of the guaranteed part of the benefit with respect to the prespecified target, in order to reduce as much as possible the uncertainty of the insurer's fixed liability. In other words, we want to solve the following minimization problem

$$\min_{\alpha \in [0,1]} \mathbb{E}\left[\left(\pi\left(T\right) - \mathrm{t} \right)^2 \right]^{1/2}. \tag{13}$$

In particular, we note that

$$\mathbb{E}\left[\left(\pi\left(T\right) - \mathrm{t} \right)^2 \right] = \mathbb{V}ar\left[\pi\left(T\right) \right] + \left(\mathbb{E}\left[\pi\left(T\right) \right] - \mathrm{t} \right)^2.$$

Further, using the results obtained in section 2, the minimization problem (13) can be formulated as

$$\min_{\alpha \in [0,1]} \mathbb{E}\left[\left(\pi\left(T\right) - \mathrm{t} \right)^2 \right]^{1/2} = \theta \min_{\alpha \in [0,1]} \left[\mathbb{V}ar\left[\pi^u\left(T\right) \right] + \left(\mathbb{E}\left[\pi^u\left(T\right) \right] - \mathrm{t}^u \right)^2 \right]^{1/2}. \tag{14}$$

The previous equation shows, on the one hand, that the optimum value α^* does not depend on the financial leverage θ; on the other hand, the optimal allocation strategy attempts to reduce both the volatility of the guaranteed benefit, π, and the options embedded therein, and the distance of the expected guaranteed benefit from the chosen target.

Because of the absence of analytical formulae for both the price of the contract and the objective function, our analysis is based on Monte Carlo simulation and is organized as follows. Given an initial set of parameters, we firstly solve numerically the optimization problem (13) in order to obtain the asset allocation related to the prespecified contract. Secondly, we calculate the corresponding no-arbitrage values of the life insurance policy and the related embedded options using the results obtained in section 2. Finally, we analyze the impact of the optimal asset allocation on the default probability of the insurance company both at maturity and 1 year after the inception of the contract, i.e.

$$\mathbb{P}\left(\pi(T) > A(T) \right) \text{ and } \mathbb{P}\left(\pi(1) > A(1) \right),$$

where A represents the total assets available to the insurer and such that $A(0) = F(0) + V_D(0)$. For this part of the analysis, we need to specify a suitable investment strategy for the safety loading $V_D(0)$ as well. In the literature, this particular problem has been explored in some detail by Bernard et al. (2006) who consider three alternatives for the construction of protection against the risk of default: (a) investing the safety loading in the same fund backing the liability, (b) the purchase of an equity default swap, (c) the partial sale of the default put back to the policyholder. The results of their analysis show that the proposed investment strategies return similar level of protection in the case of low volatility, whilst the equity default swap proves to be the best option in case the of high volatility.

Market model			
Equity	$\mu = 10\%$ p.a.	$\sigma = 20\%$ p.a.	
Interest rate	$r_0 = 4.5\%$	$\upsilon = 0.2942\%$	$\kappa = 0.9866\%$
	$\lambda = -0.015$	$\rho = -0.2$	
Policy design			
$\beta = 70\%$	$\theta = 90\%$	$r_G = 4\%$	$h = 70\%$
$F(0) = 100$	$\tau = 3$ years	$T = 20$ years	

Table 1: Parameter set for the numerical example implemented in section 4.

In this paper, we adopt a different approach, in that we assume that the full safety loading is charged to the policyholder; further, we assume that the insurance company invests this premium either in the fixed income market to purchase an equivalent amount of zero coupon bonds with maturity at the expiration of the policy, or in the equity market. The third alternative that we consider is the same as strategy (a) proposed by Bernard et al. (2006), which also corresponds to the "passive" investment strategy discussed in Ballotta et al. (2006.a).

We further extend the analysis to the insurer's capital requirements as well, by calculating the solvency target capital. In particular, we follow Ballotta (2009) and calculate a suitable risk measure, such as Value at Risk (VaR) and Tail Value at Risk (TVaR), over a 1-year time horizon (CEIOPS, 2006) of the solvency index

$$s(t) = \frac{\widetilde{RBC}(t+1) - RBC(t)}{A(t)},$$

where $RBC(t)$ denotes the Risk Bearing Capital (RBC) at time $t \in [0, T]$ (FOPI, 2004), and $\widetilde{RBC}$ denotes its value discounted at the risk free rate. The RBC is defined as the difference between the total value of the company's assets, including the safety loading, and the market consistent price of the liabilities, i.e.

$$RBC(t) = A(t) - V_\pi(t) - \gamma V_R(t).$$

Discounting at the risk free rate is applied in order to reflect the implicit assumption that the target capital should represent the amount that, once invested in the money market account, guarantees an appropriate protection to the policyholders as well as the stability of the company with a certain confidence level.

4 Results

In this section, we present and discuss the results obtained from the numerical experiments described above. In particular, unless otherwise stated, we use the model parameters reported in Table 1. Hence, we analyze the case in which the policyholders

β	α^*	$\mathbf{V}_\pi(0)$	$\mathbf{V}_R(0)$	$\mathbf{V}_D(0)$	γ %	φ %
0.3	1	68.6183	35.8058	13.1643	96.48	14.63
0.4	0.8924	75.1433	29.0018	12.4418	94.13	13.82
0.5	0.6563	76.6864	22.1740	7.0943	92.04	7.88
0.6	0.4835	78.4642	16.9290	3.6285	89.57	4.03
0.7	0.3448	80.7334	12.3448	1.5056	87.26	1.67
0.8	0.2222	84.0793	7.7263	0.4614	82.60	0.51
0.9	0.1114	89.6243	2.7147	0.2836	24.28	0.32

Table 2: Market consistent price of the options embedded in the participating contract for the optimal asset allocation α^* corresponding to different levels of the participation rate β. The table also reports the "fair" value of the terminal bonus rate γ, which is obtained as the solution to Equation (7) and the safety loading rate as defined in Equation (9). Estimation based on 100,000 simulations with variance reduction techniques.

Investment strategy		$\mathbb{P}(\pi(T) > A(T))$ %						$\mathbb{P}(\pi(1) > A(1))$ %		
		a) $V_D(0)$ only			b) $A(0)$			$V_D(0)$ only		
β	α^*	$\alpha = 0$	α^*	$\alpha = 1$	$\alpha = 0$	α^*	$\alpha = 1$	$\alpha = 0$	α^*	$\alpha = 1$
0.3	1	12.49	15.15	15.15	0.00	15.15	15.15	0.02	0.15	0.17
0.4	0.8924	12.38	14.78	15.04	0.00	14.78	18.52	0.04	0.26	0.28
0.5	0.6563	8.69	10.13	10.80	0.00	10.13	21.11	0.02	0.04	0.07
0.6	0.4835	5.97	6.46	7.08	0.00	6.46	28.88	0.00	0.00	0.00
0.7	0.3448	3.58	3.78	4.00	0.00	3.78	30.45	0.00	0.00	0.00
0.8	0.2222	1.93	1.98	2.08	0.01	1.98	43.11	0.01	0.02	0.02
0.9	0.1114	2.98	3.10	3.10	3.35	3.10	36.81	0.53	0.54	0.49

Table 3: Probability of default at maturity and in one year from the contract's inception for different levels of the participation rate β. For the case of default at maturity, we consider both the cases in which a) the insurance company invests the reference portfolio according to the optimal strategy α^*, whilst the premium of the default option is invested either in bonds, in the optimal portfolio or in equity only; b) the overall company's assets are invested according to the $(\alpha = 0, \alpha^*, \alpha = 1)$ strategies. For the 1-year probability, we consider only case (a). Estimation based on 100,000 simulations.

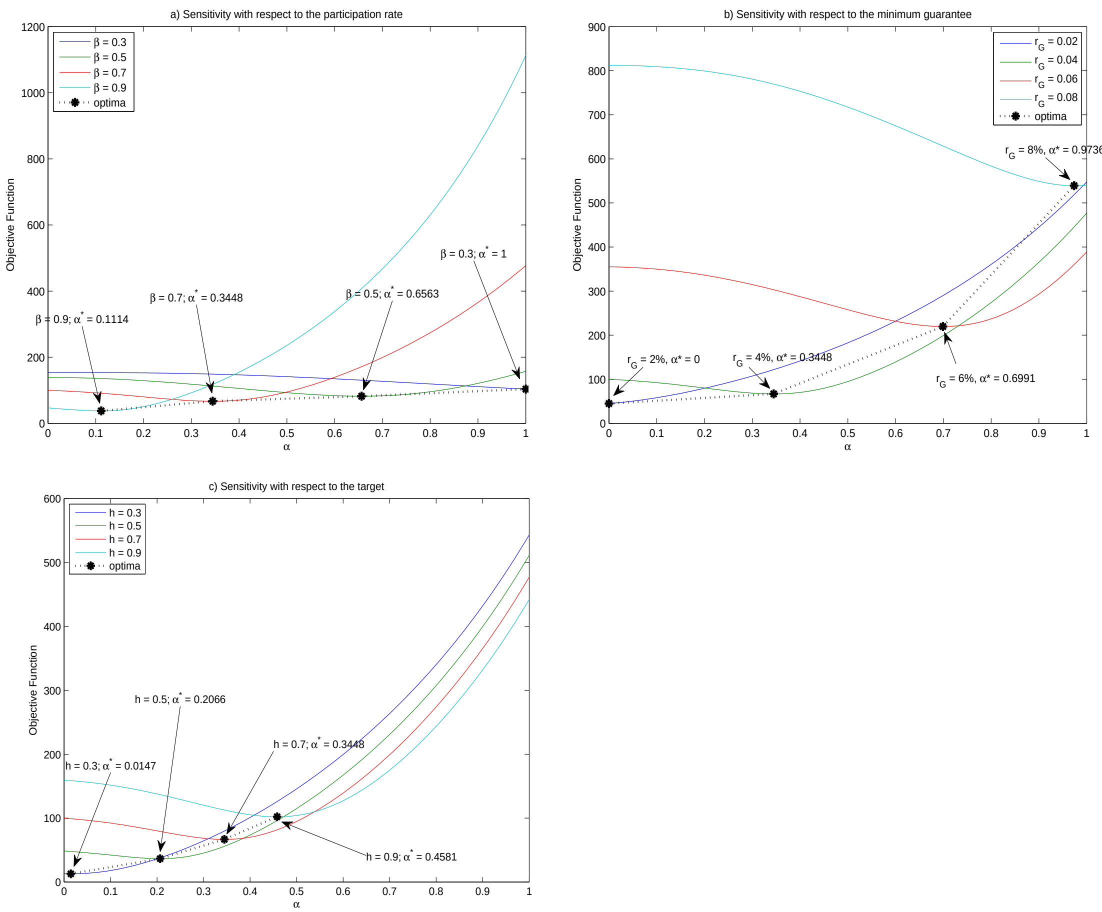

Figure 1: The optimal asset allocation for different values of the participation rate β (panel a), the minimum guarantee r_G (panel b) and the target parameter h (panel c). The Monte Carlo estimate of the objective function is based on 100,000 simulations. The optimization routine makes use of the Brent algorithm (see, for example, Press et al. (1997)).

contribute 90% of the initial value of the reference portfolio to enter a participating contract offering a 4% minimum guarantee and a 70% participation rate in the smoothed returns generated by the portfolio. Smoothing is operated on a 3-year time scale and the contract has a life span of 20 years. We also assume a 20% p.a. volatility for the equity investment with 10% expected return; further, the short interest rate is fixed at 4.5% whilst the parameters of the interest rate model are calibrated on the basis of a 6% p.a. 20 year yield to maturity and a 5.34% p.a. volatility on a 20-year zero coupon bond.

Figure 1 shows the optimal asset allocation for several combinations of the participation rate, β, the target, h, and the minimum guarantee, r_G. In Table 2, we report the corresponding market consistent values of the options embedded in the insurance policy, together with the value of the terminal bonus rate which guarantees the fairness of the contract according to the equilibrium condition given by Equation (7) and the size of the safety loading rate given by Equation (9).

The results from the numerical analysis show that the optimal asset allocation is a decreasing function of the participation rate, and an increasing function of the target parameter h and the minimum guarantee. This is consistent with intuition: ceteris paribus, a higher participation rate in the fund's returns makes the target easier to reach; therefore, the insurer can switch asset allocation by investing more in the less risky asset. On the other hand, the insurer will have to adopt a more aggressive investment strategy if the target becomes more demanding, due to either higher policyholder participation or a higher minimum guarantee.

Further, the fair values reported in Table 2 for the embedded options show that the value of the guaranteed benefit is increasing in β, irrespective of the composition of the reference fund. This is consistent with Equation (2): the level of the policyholder participation in the fund's returns is in fact the "spot price" of the underlying asset on which the embedded Asian option is written. The effect of the change in the portfolio mix is instead "averaged" by the smoothing mechanism in place. As far as the terminal bonus and the default option are concerned, we note that their price is a decreasing function of β. In the case of the terminal bonus, these findings are consistent with what has been observed previously by, for example, Ballotta et al. (2006.b); further, we note that for $\beta = 70\%$ the proposed asset allocation strategy leads to a terminal bonus rate $\gamma = 87.26\%$ which is lower than the $\gamma = 100\%$ which we would obtain from a bond-only reference portfolio, but higher than the $\gamma = 30.18\%$ originating from an investment in a equity-only reference fund. The case of the default option instead seems anomalous with respect to the existing literature. However, we note that the default option is essentially a put option on the company's assets with a path-dependent strike price. As the option price is an increasing function of the underlying's volatility, the observed pattern is justified by the fact that by reducing α, i.e. the equity component in the portfolio, the fund's overall volatility reduces as well. Hence, we note that the asset allocation operating on the reference portfolio reduces considerably the size of the safety loading compared to the initial company's fund: the safety loading corresponding to the ($\alpha = 100\%, \beta = 70\%$) combination is, in fact, 53.79%, whilst for the ($\alpha^* = 34.48\%, \beta = 70\%$) case, it is 1.67%.

The probability of default at maturity which is generated by the optimal asset allocation is shown in Table 3 and in Figure 2 for several values of the participation rate β. Specifically, in panel (a) of Figure 2 we show the default probabilities for the case in which the reference fund F is invested according to the optimal strategy α^*, while the safety loading is invested respectively in bonds ($\alpha = 0$), according to the strategy α^*, or in equities ($\alpha = 1$). In panel (b), instead, we consider an identical investment strategy for both the reference fund and the safety loading, and we compare the default probabilities generated by the optimal asset allocation with the bond-only and equities-only investment strategies. Table 3 and Figure 2 also report the probabilities of default 1 year after the inception of the contract corresponding to the investment strategies considered in panel (a).

For each row in Table 3 - panels (a) and (b), corresponding to a fixed value of β, we note that the probability of default increases as we move from a bond only investment strategy to an optimal investment strategy and then to an equities only investment strategy. This increase is relatively small in panel (a), i.e. when the reference fund is invested optimally, compared to the change observed in panel (b). Hence, this analysis of the default probability at maturity shows the importance of investing the reference fund optimally, especially because of the relatively small size of the safety loading (as observed in Table 2). This is indicated by the much higher default probabilities shown in the third column in Table 3 for each of panels (a) and (b). For instance, the default probability at maturity corresponding to a participation rate $\beta = 80\%$ is 2.08% for the case in which the fund is invested optimally and the safety loading is used to purchase additional equities (Figure 2.a), whilst it rises to above 43% in the case in which the total available assets are invested in the equity market (Figure 2.b).

This result contrasts, in a way, with the findings of Ballotta et al. (2006.a), in which the safety loading plays a significant role in reducing the probability of default. We note, however, that in their case the participating contract design is quite simple and does not incorporate smoothing, which means that the full annual return (and volatility) of the reference fund is transferred to the embedded option. Further, they do not apply any diversification and use an equity-only portfolio. Finally, we observe that the 1 year default probability for the case considered in Figure 2.a with $\beta = 80\%$ is 0.02% as reported in Table 3, which is within the limits suggested by the Solvency II review.

Figure 2 (and Tables 3) also shows a U-shaped pattern for these default probabilities, suggesting the existence of a value of the participation rate β which is optimal overall, in the sense that it minimizes both the 1 year probability of default and the corresponding probability at maturity. This optimal value lies between 60% and 80%.

Finally, we note that the bond-only strategy leads to the lowest default probabilities, independent of the chosen participation rate. However, as we could anticipate, Figure 3 shows that the bond-only strategy also offers the policyholder the smallest expected benefit at maturity; further, this expected benefit approaches the pre-specified target only for the case of $\beta = 100\%$. The optimal investment strategy, instead, provides on average a higher and more stable benefit at maturity.

Figure 4 reports the TVaR of the solvency index $s(t)$ defined in section 3 over the 1-year period after the start of the contract. Also in this analysis, we consider the case in which the reference fund is invested optimally, whilst the safety loading is used to purchase more bonds or equities, or an additional share of the same asset mix defining the underlying portfolio. The panels in Figure 4 show that for low values of β, the equities-only strategy generates the highest TVaR (for the case in which $\beta = 0.3$, the TVaR is actually the same for both the optimal and the equity-only strategy), whilst the bond-only strategy tends to produce the highest TVaR as β increases. This is due to the fact that the reference portfolio is invested optimally, and α^* decreases as the participation rate increases, i.e. bonds become the predominant component and therefore the main source of risk. The analysis of the reported target capital, however, confirms the results shown above for the default probability. Thus, for a participation rate of $\beta = 70\%$, the three investment strategies return a target capital which is very similar and in any case the lowest across the possible values of β that have been explored. The expected severity, in fact, lies between 8% and 9% (depending on how the safety loading is invested) at the AAA confidence level, and 7%-7.5% at the BBB confidence level. For $\beta = 50\%$, instead, the expected severity rises to 15%-18% at the AAA confidence level, and 12%-15% at the BBB confidence level. In the case in which $\beta = 90\%$, the expected severity varies between 20%-21% and 14%-15% for the AAA and BBB confidence levels respectively. Analogous conclusions can be reached when other risk measures are used, such as the Value at Risk or the Tail Conditional Median, as illustrated in Figure 5.

5 Stress testing

In order to assess the robustness of the asset allocation proposed in this paper, we test the impact of an adverse (and extreme) movement in some of the fundamentals of the financial market, such as the equity volatility or the risk free rate of interest, occurring one year after the sale of the participating contract.

In particular, we consider the two cases in which, at time $t = 1$, there is (a) a 10% increase in the volatility σ to 30% (with respect to the benchmark value given in Table 1), and (b) a 1% cut in the rate of interest prevailing in the market at time $t = 1$. The resulting default probabilities are presented in Table 4 and Figure 6, in which they are compared with the "base case scenario" that has been discussed in section 4; the corresponding $TVaR(s(1))$ is shown in Figure 7.

Having seen in section 4 that this is the most appropriate course of action, we consider only the case in which the reference fund is invested optimally, and hence we analyze the impact of the investment strategy for the safety loading. This justi-

Investment strategy		$\mathbb{P}(\pi(1) > A(1))$ %					
		a) $\Delta\sigma = +10\%$			b) $\Delta r(1) = -1\%$		
β	α^*	$\alpha = 0$	α^*	$\alpha = 1$	$\alpha = 0$	α^*	$\alpha = 1$
0.3	1	0.42	1.21	1.17	0.80	1.74	1.75
0.5	0.6563	1.44	1.90	2.44	1.69	2.22	2.76
0.7	0.3448	0.17	0.19	0.25	3.67	3.87	4.17
0.9	0.1114	1.25	1.20	1.19	99.92	99.92	99.90

Table 4: Stress testing: 1-year probability of default originated by a) a 10% increase in the equity volatility, and b) a 1% reduction in the interest rate. Both shocks are assumed to occur at the beginning of year 1. Estimation based on 100,000 simulations

fies the trend in the RBC shown in Figure 7. The RBC seems, in fact, quite stable across the three investment strategies. Thus, the same shock (i.e. either the change in the volatility or the interest rate cut) produces the same change in the expected severity irrespective of the investment strategy adopted for the safety loading, which is consistent with its relatively small size. Further, the volatility shock has a bigger impact on $TVaR(s(1))$ for low values of the participation rate β, but this reduces as β increases; the impact generated by the interest rate shock has an opposite effect. This is consistent with the change in the composition of the reference portfolio from equities to bonds which occurs for increasing values of β.

Similar considerations apply to the 1-year default probability reported in Table 4 and Figure 6. The probability of default increases as a result of the shocks; the impact of the interest rate cut, in particular, is predominant for higher values of β. For $\beta = 90\%$, in fact, the bond component in the reference portfolio is 88.86% whilst the safety loading is 0.32% of the initial capital. This leads to a 99.92% probability of experiencing a loss of 48% of the initial capital at the AAA confidence level, and a loss between 36%-40% at the BB/BBB confidence levels. The volatility shock, instead, only generates an expected loss of 23% of the initial capital at the AAA confidence level with probability 1.19% (the corresponding loss at the BB/BBB level is between 14% and 17%). These results are independent of the investment strategy applied to the safety loading.

Thus, this experiment shows that if diversification is applied, i.e. a suitable asset allocation is performed on the reference portfolio, the magnitude of the safety loading is contained and therefore how it is actually used for hedging purposes becomes of less relevance. The analysis carried out also confirms the result obtained for the base case scenario discussed in section 4, that a value of the participation rate β in the region of 70% is optimal overall. In this case, which corresponds to a portfolio mix 34.48% equity - 65.52% bonds and a 1.67% safety loading, a 10% increase in the equity volatility produces an expected loss of 19% at the AAA confidence level (or between 14% and 15% at the BB/BBB confidence level) with probability 0.19%. A 1% cut in the interest rate, instead, leads to a 29% expected loss at the AAA confidence level (or

20%-22% loss at the BB/BBB confidence level) with probability 3.87%.

6 Conclusions

The main contribution of this paper consists in the development of a suitable asset allocation strategy aimed at the risk management of the most common type of participating life insurance sold across Europe and the UK, and Japan. The choice of strategy is based on minimizing the risk induced by the policy, as measured by the mean squared departure of the guaranteed benefit at maturity relative to a pre-specified target. We also investigate the impact of this investment strategy on the capital requirements of the insurance company by means of indices such as the probability of default, and VaR and TVaR risk measures based on the Risk Bearing Capital. The analysis is based on a market consistent pricing methodology.

The principal numerical results show that the optimal asset allocation generates a probability of default over a 1 year time horizon that is consistent with the regulatory requirements imposed by the Solvency II regime. Further, because the default option premium is quite small, the asset allocation strategy applied to this premium plays a minor role as far as risk management is concerned. The results also indicate the existence of an optimal choice for the design parameter β, representing the participation rate, which minimizes the overall default risk of the insurance company.

The stress testing exercise investigates the impact of an increase in the equity market volatility or a decrease in the short rate of interest 1 year into the contract's lifetime, when the optimal asset allocation strategy is deployed. The effect of these shocks depends on the contract design and the optimal choice for the asset allocation - with a high equities content, the portfolio is sensitive to changes in the equity volatility. With a high bond content, the portfolio is extremely sensitive to reductions in the interest rate as measured by the 1 year probability of default and other risk measures. However, the optimal choice for the participation rate β detected in the base case scenario still proves optimal in terms of minimization of the overall default risk.

As noted in the earlier sections, the model does not take account of mortality risk. In normal circumstances, the life insurance benefit would be paid to the policyholders providing that they survive until maturity, T, and there might be a payment on earlier death. We, instead, assume that policyholders will survive to time T with probability 1; this choice is partly justified by the term of the contract considered in this paper, as mortality effects generally play a more important role over longer time horizons. Similarly, we also ignore the policyholder's option to surrender the policy. This would give the policyholder the possibility to cancel the policy before time T, if this seemed desirable. This additional "American-option" feature would make the contract more valuable to the policyholder. In order to compensate for this increase in value, it is common for insurance companies to impose a penalty charge if the policyholder opts to surrender the policy before time T. Both of these features represent possibilities for inclusion in the analysis and hence for future work.

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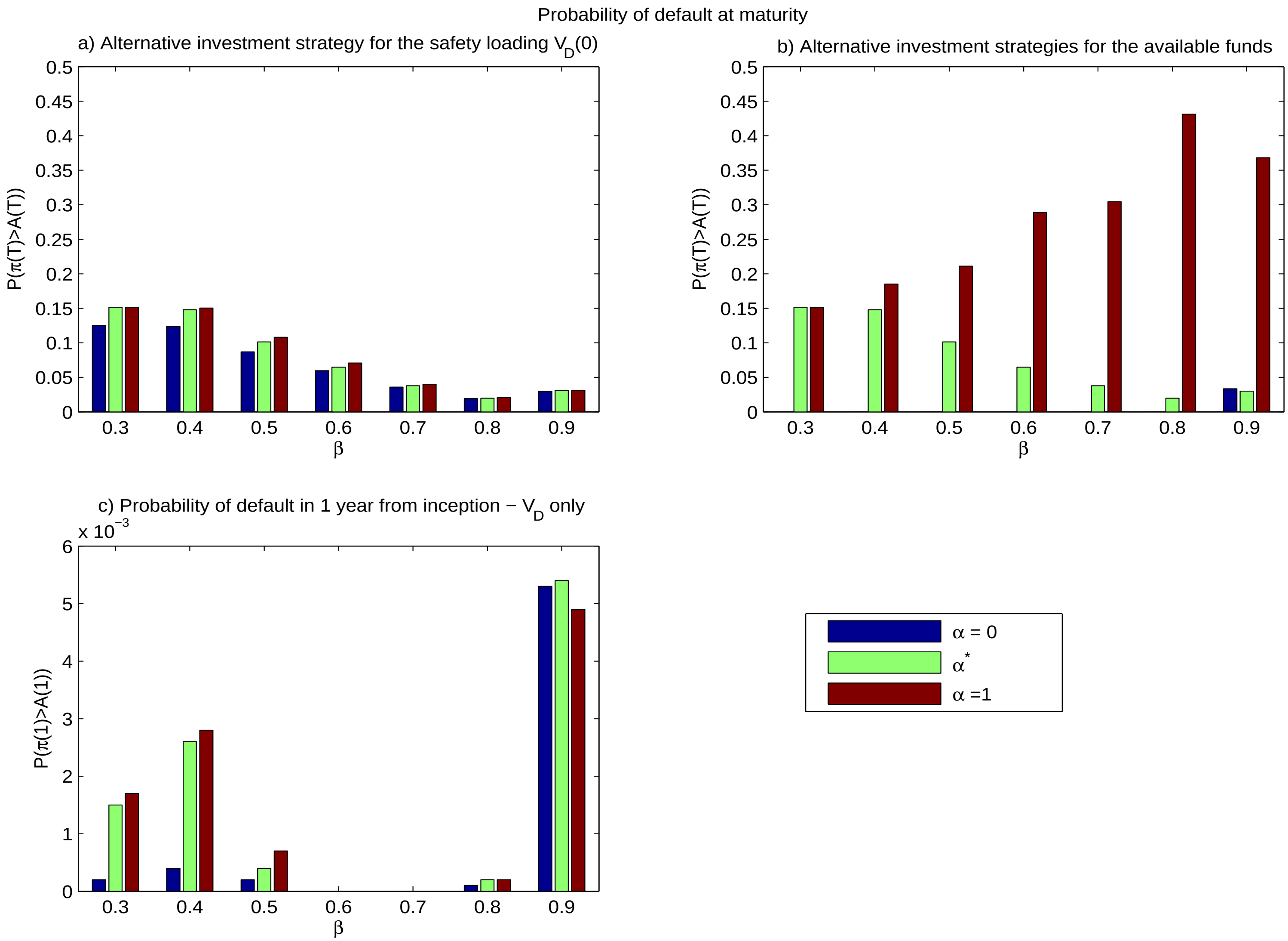

Figure 2: Probability of default for different combinations of the participation rate β and the asset allocation strategy α^*. The values correspond to the ones reported in Table 3.

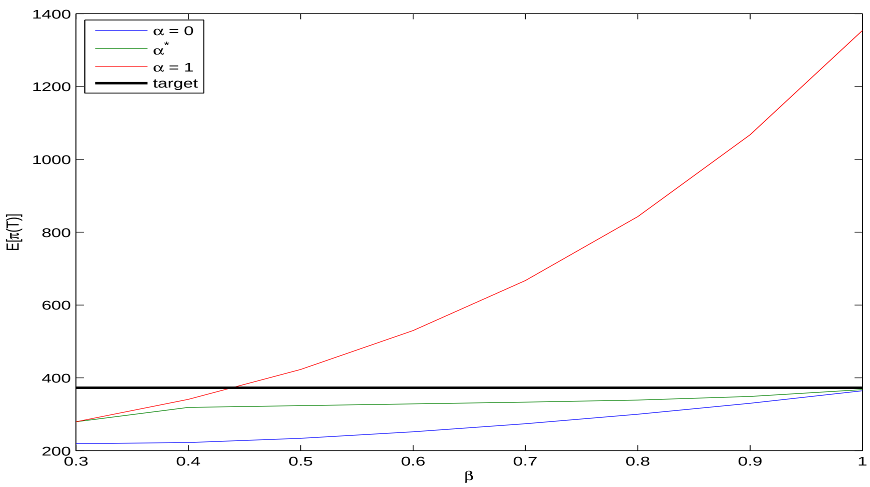

Figure 3: Expected value of the guaranteed benefit at maturity generated by the $(\alpha = 0, \alpha^*, \alpha = 1)$ possible investment strategies. In the plot these expected values are compared against the prespecified target $\mathrm{t} = \pi_0 \left(1 + r_G \left(1 + h\right)\right)^T$ calculated in accordance with the base parameter set reported in Table 1. Estimation based on 100,000 Monte Carlo simulation.

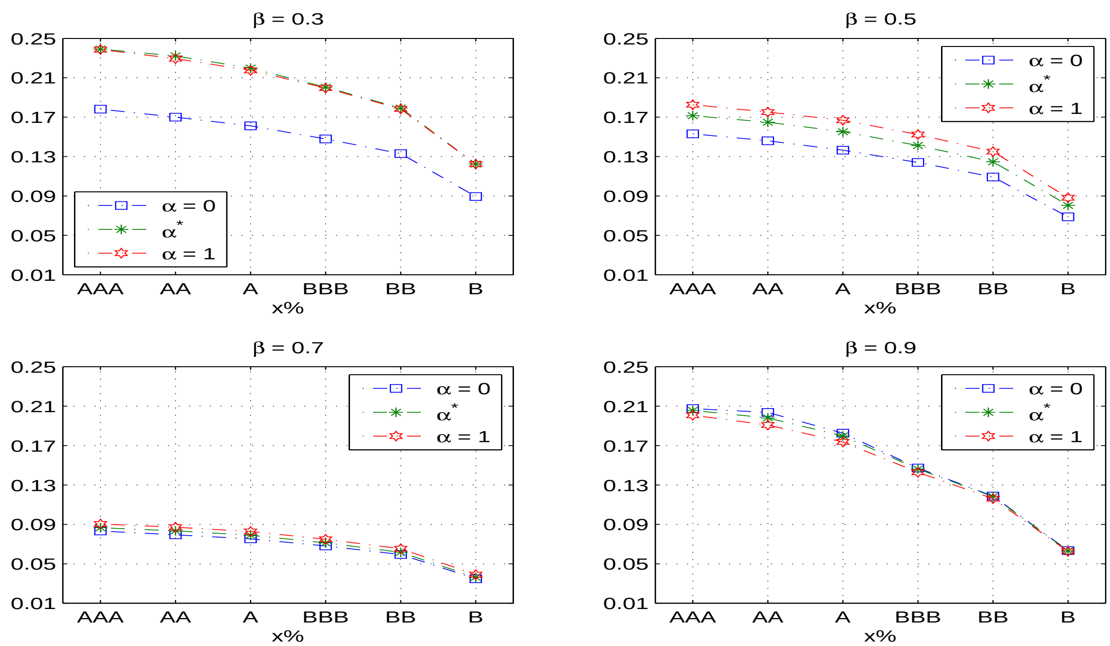

Figure 4: Risk Bearing Capital for the participating contract described in section 2 as measured by $TVaR(s(1))$. The reference fund is invested optimally, whilst the safety loading is invested according to the $(\alpha = 0, \alpha^*, \alpha = 1)$ investment strategies. The confidence levels are fixed on the basis of the Standard & Poor's classification (AAA = 99.99%; AA = 99.97%; A = 99.93%; BBB = 99.77%; BB = 99.31%; B = 93.46%). Estimation based on 100,000 simulations for the benchmark set of parameters given in Table 1.

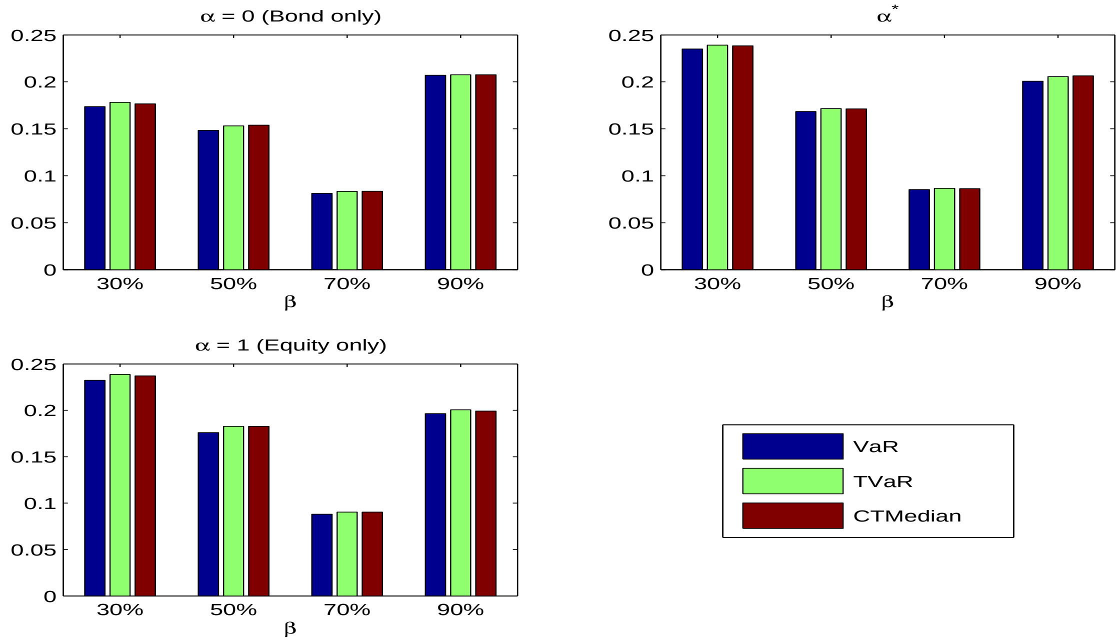

Figure 5: Target capital based on different risk measures in correspondence of the AAA confidence level. The reference fund is invested optimally, whilst the safety loading is invested according to the $(\alpha = 0, \alpha^*, \alpha = 1)$ investment strategies. Estimation based on 100,000 simulations for the benchmark set of parameters given in Table 1.

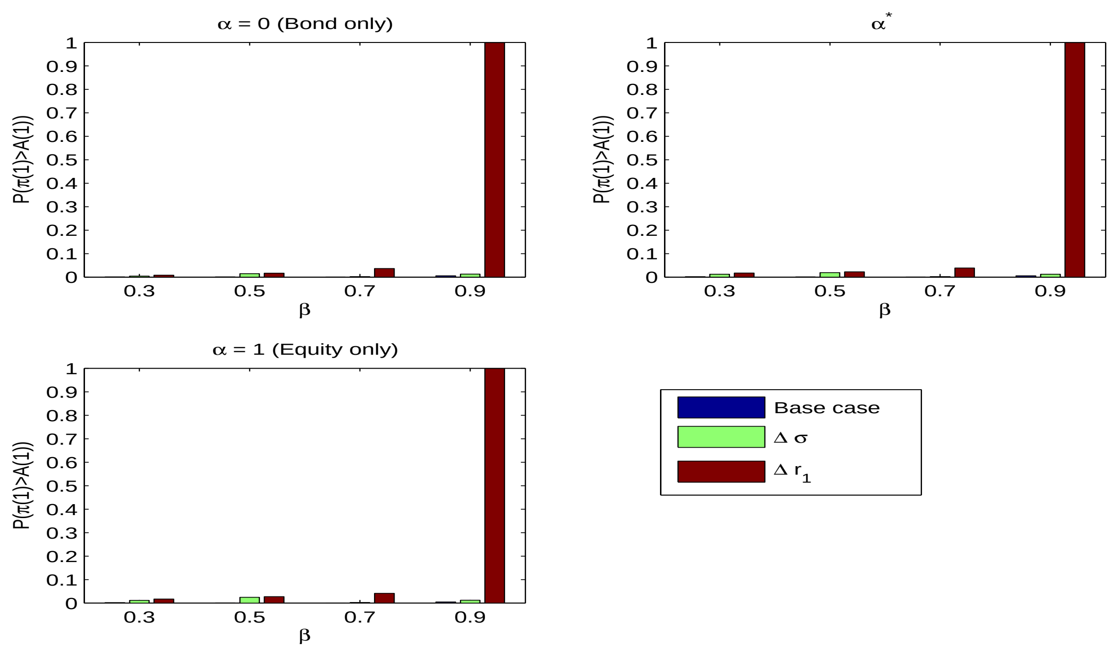

Figure 6: Stress testing: the 1-year default probabilities reported in Table 4 are compared against the corresponding 1-year probabilities for the base case scenario (Table 3).

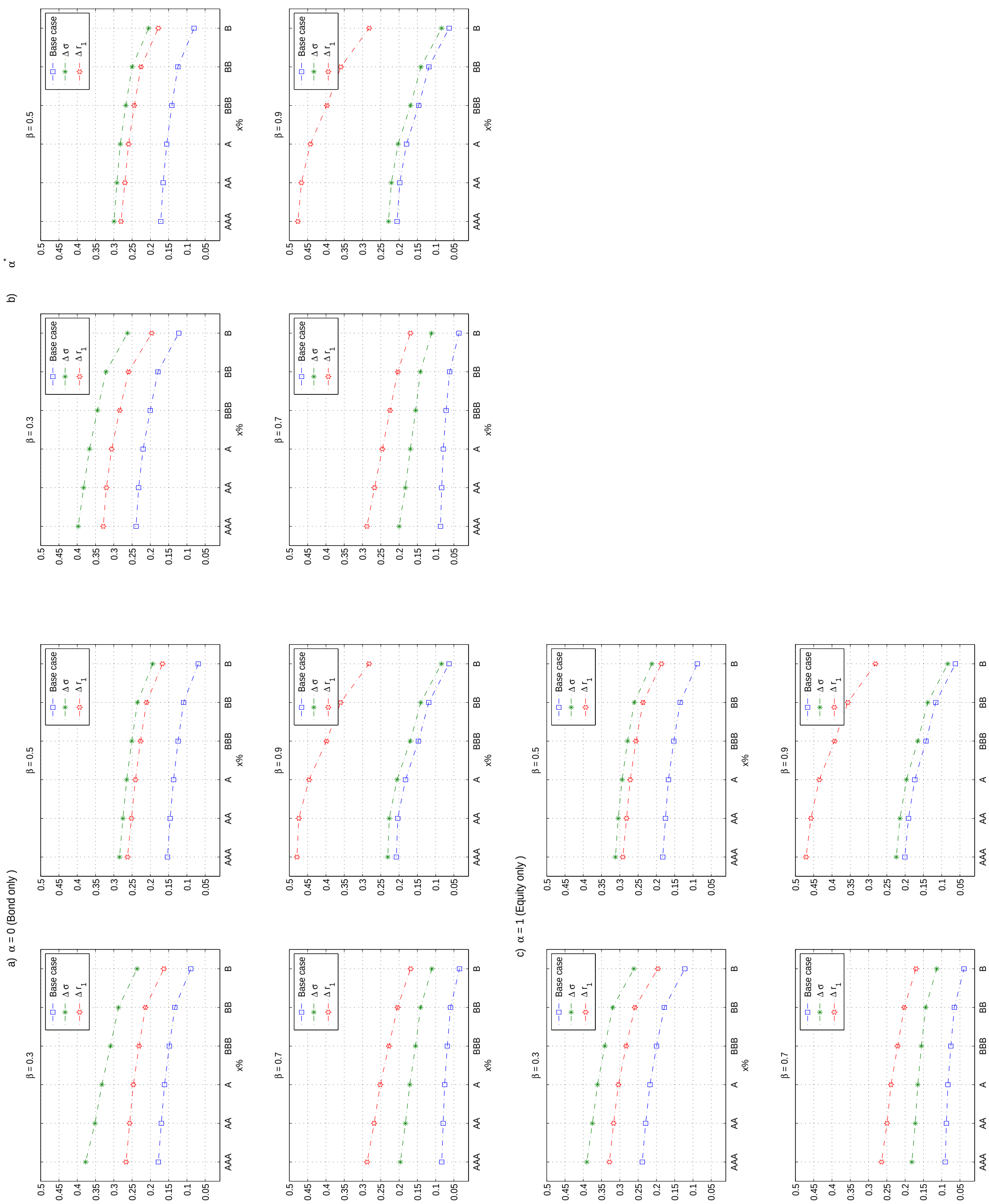

Figure 7: Risk Bearing Capital originated by the equity volatility shock and the interest rate shock, as measured by $TVaR(s(1))$. The capital requirements are compared against the corresponding values obtained for the base case scenario (Figure 4). Estimation based on 100,000 simulations.