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RISK, RETURN AND PORTFOLIO ALLOCATION UNDER ALTERNATIVE PENSION ARRANGEMENTS WITH IMPERFECT FINANCIAL MARKETS

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Abstract

This paper uses stochastic simulations on calibrated models to assess the steady state impact of different pension arrangements in an environment where financial markets are less than perfect. Surprisingly little is known about the optimal split between funded and unfunded systems when there are sources of uninsurable risk that are allocated in different ways by different types of pension system and where there are imperfections in financial markets (eg transactions costs or adverse selection) . This paper calculates the expected welfare of agents in different economies where in the steady state the importance of unfunded, state pensions differs. We estimate how the optimal level of unfunded, state pensions depends on rate of return and income risks and also upon the actuarial fairness of annuity contracts. We focus on the case of Japan where aging is rapid and unfunded pensions are currently generous.

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Risk, Return and Portfolio Allocation under Alternative Pension Systems with Imperfect Financial Markets

Abstract:

This paper uses stochastic simulations on calibrated models to assess the steady state impact of different pension arrangements in an environment where financial markets are less than perfect. Surprisingly little is known about the optimal split between funded and unfunded systems when there are sources of uninsurable risk that are allocated in different ways by different types of pension system and where there are imperfections in financial markets (eg transactions costs or adverse selection) . This paper calculates the expected welfare of agents in different economies where in the steady state the importance of unfunded, state pensions differs. We estimate how the optimal level of unfunded, state pensions depends on rate of return and income risks and also upon the actuarial fairness of annuity contracts. We focus on the case of Japan where aging is rapid and unfunded pensions are currently generous.

I. Introduction:

The old age dependency ratio in nearly all developed economies (the ratio of those of pensionable age to those of working age) will be substantially higher in the future. If unfunded (pay-as-you-go), state pensions are to continue to provide the larger part of retirement incomes, then contribution rates in most countries will have to be substantially higher to balance the system. The desirability of providing a significant proportion of retirement income from unfunded pensions is therefore a key policy issue. It has generated a large literature on the reform of pension systems (see, for example, Feldstein (1996); Feldstein and Samwick (1998); OECD (1996); Mitchell and Zeldes (1996); Disney (1996); Kotlikoff (1996); Huang, Imrohoroglu, and Sargent (1997); Miles and Timmerman (1999); Sinn (1999) and Campbell and Feldstein (2001)). If unfunded pensions have substantial advantages then it might be worth paying the costs of higher contribution rates to preserve them. But if greater reliance on other sources of pension income, most obviously income from funded pensions (or more generally from private saving), can replace unfunded pensions without adverse affects (for example on risk), then there would be associated long-run benefits of a higher capital stock and lower, potentially less distortionary, labor taxes. But funded and unfunded pension systems allocate risk in different ways, so any analysis of the long-run implications of different degrees of reliance on funded and unfunded pensions has to consider the welfare implications of different risk allocation mechanisms.

Key to this issue is how effectively financial markets allow individuals to insure against important risks. With pensions systems one of the most important risks reflects uncertainty over individual longevity. A crucial determinant of the optimal split between private, funded pensions (which we think of here as individual retirement accounts) and state, unfunded pensions is therefore the effectiveness of the annuities market. This paper models the performance of different pension arrangements allowing for imperfections in financial markets. We investigate the steady state properties of different pension arrangements allowing for agents to choose portfolios for funded pensions in an environment of less than perfect annuities. We illustrate the issues in the context of pensions policy in Japan. For Japan the pensions issues are particularly difficult: unfunded pensions are currently very generous; aging is projected to be unusually rapid and the government is already heavily in debt and running large deficits. We look ahead to a Japan where the demographic changes currently under-way have worked themselves through so that the population structure has stabilized – but to a position with very many more elderly people than exist today. We ask what pension arrangements would be desirable in an environment where life-expectancy is greater, the dependency ratio higher and where risk averse agents cannot insure fully against important risks. We consider what portfolios of assets agents hold and in doing so add to the literature on asset allocation over the life-cycle. Although we calibrate the model by using projected future Japanese mortality rates our results have general relevance since what is likely to happen to life-expectancy in Japan is common to developed countries.

Any model that wants to say something useful about risk and uncertainty must take account of several factors:

1. that individuals face substantial, largely idiosyncratic, uncertainty about their labor income.
2. that returns on most financial assets are volatile and uncertain.
3. that to the extent that individuals depend upon their own accumulated funds for retirement resources the way in which annuities markets work is important (state run, unfunded systems will be unaffected by the efficiency of annuities markets because the government is effectively providing insurance itself).

The central policy issue we address is one which is relevant in all economies: what is the desirable split between funded and unfunded systems when there are sources of uninsurable risk that affect risk averse agents and where those risks are allocated in different ways by different types of pension system? This issue has been analyzed in a rapidly growing literature. Merton (1983) addressed the issue and showed that in general a mixed system had benefits on standard portfolio allocation grounds. Feldstein and Ranguelova (1998) and Feldstein, Ranguelova and Samwick (1999) consider uncertainty about rates of return and how it affects the transition to funded systems. But they do not consider idiosyncratic risks that are important in practice and which individuals find it hard to insure against. Bohn (1999) analysed the impact of uncertainty about future demographic structure, but did not consider either rate of return or labor income uncertainty.

Imrohoroglu, Imrohoroglu and Joines (1995 and 1999a) investigate the role of social security in a general equilibrium setting with labor income uncertainty but non-stochastic rates of return and no annuities. Huang, Imrohoroglu and Sargent (1997) focus on the intergenerational impact of various social security systems on transition paths. They allow for stochastic labor income but there is no uncertainty on rates of return. (See Imrohoroglu, Imrohoroglu and Joines (1999b) for an excellent survey on computational models of social security)).

The nearest paper to our own is Campbell, Cocco, Gomes and Maenhout (2001). They consider the long-run pattern of lifetime savings and portfolio allocation in the presence of income and rate of return uncertainty and with various pension arrangements. They do not consider the impact of varying degrees of imperfection in annuity markets but do consider fixed costs of entering the equity market. Since the efficiency of annuity contracts is central to the desirability of private, funded pension systems - particularly of individual retirement accounts - we consider modeling the impact of different degrees of imperfection in this market to be important.

Personal funded pensions may allow people to insure perfectly against some risks – if annuities are available at actuarially fair rates then length of life risk can be avoided. But personal pensions mean that labor income risk from working years, which will have an impact on the contributions to a personal pension fund, have lasting effects upon pension income; such pensions obviously also generate rate of return risk. Given this we consider what role might be played by unfunded, state pensions that give varying degrees of insurance against labor income risk and are not dependent on rate of return risk. We look at two types of unfunded pension - flat rate pensions and unfunded pensions that depend on the income earned at the end of the individual's working life.

What is new about this paper is that it allows one to assess the impact of alternative pension systems in a world where there are multiple of sources of uncertainty none of which can be perfectly insured against in financial markets. We allow for deviations from efficiency in the annuities market and assess the impact of this both upon portfolio allocation and upon the welfare characteristics of different pension systems. There is now a large literature on portfolio allocation over the life cycle (see Campbell et al (2001) for extensive references). We add to that by showing the impact of imperfections in financial markets and the effects of social security in the form of state, unfunded pensions with varying degrees of redistribution.

The comparison between funded and unfunded pension systems depends not just on different risk sharing characteristics; it also depends upon demographic structure. Any unfunded system that is in balance will need to levy a higher contribution rate upon workers to give a certain level of benefits the higher is the old age dependency ratio. We look forward several decades and using information on the likely pattern of life expectancy in the middle of this century we first construct an estimate of the future steady-state population structure. We use this (steady state) population structure to work out what the required contribution rate would need to be to generate unfunded pensions of various levels of generosity. Given the level of unfunded pensions, we then aim to calculate what the optimal savings and investment behavior of cohorts of individuals would be. This allows us to calculate macroeconomic variables – the overall savings rate, the capital stock, levels of national income and so on – which are generated by different pension arrangements.

In calculating the optimal behavior of individuals we take account of all the sources of risk we noted above were essential. In particular, we assume that individuals face random, but persistent, shocks to labor income throughout their working life. We also assume that they face uncertainty about the returns they will earn on at least some sorts of financial assets. We assume that there is a safe asset but there are also risky assets which, on average, earn higher returns. We also assume individuals are risk averse and that they understand the risks of investing in different sorts of assets and are also aware of the uncertainty over how long they will live. We then use numerical techniques to calculate optimal profiles of consumption, saving and portfolio allocation for individuals over their lives. We aggregate these decisions to construct the macroeconomic aggregates and also construct measures of welfare. Solving this sort of model is difficult and we use numerical techniques to work out optimal paths. We set the critical parameters in the model (parameters of the utility functions such as degrees of risk aversion and rates of time preference, and characteristics of the labor income profile over life) by reference to recent data from the Japanese economy.

We then simulate the model using different settings for the key policy variables. We also consider how differences in the investment environment, particularly the efficiency of annuities contracts and the risk return trade-off on risky assets, affect the economy. We are able to show how different degrees of generosity of unfunded pensions affects the overall saving rate, the level of steady-state national income, and the allocation of savings across different assets. We are also able to make welfare comparisons. We construct a measure of welfare by estimating the expected utility of individuals born into a Japanese society whose population has settled down. We focus on the case where population is constant, which seems a relevant case given trends in fertility and life expectancy (arguably it is the only reasonable steady state to consider for a mature, developed country).

Our results indicate several things.

I. In steady state the capital stock, the level of national income and portfolio allocation are extremely sensitive to differences in the generosity of unfunded pensions. The steady-state aggregate stock of wealth might be twice as high if unfunded pensions were, on average, worth only 25 percent of average earnings, as opposed to 50 percent.

II. How much of financial wealth is invested in risky assets is very sensitive to both the level of state pensions and the efficiency of financial markets. Even with quite low risk aversion (a coefficient of relative aversion of 3) we can explain substantial holdings of safe assets (between 40% and 60% of portfolios) if state pensions are low. We do not need to assume extreme risk aversion or fixed costs of investing in risky assets to generate substantial investment in safe assets. This is so even though we use common assumptions about risk premia and the volatility of risky assets.

III. The effects of reducing the generosity of unfunded pensions upon welfare, savings, portfolio allocation and national income depend very much on the efficiency of annuities contracts. They are also sensitive to the size of the equity risk premium.

IV. Credit restrictions affect the answers substantially. Individuals find it difficult to borrow against future labor income (that is their human capital) and therefore any model with uncertainty over income and over length of life is one in which individuals naturally face borrowing constraints. We find that these constraints are likely to matter significantly. We also find that how serious borrowing constraints are, particularly amongst the elderly, depends very much on the pension environment.

II. The model:

We assume an economy where a given (large) number of agents are born each period and where mortality rates (probabilities of surviving to given ages) are unchanging. Such an economy will ultimately generate an unchanging demographic structure. We focus on steady state population structures.

Given the stochastic processes for labor income and for rates of return (and conditional on pensions arrangements and mortality rates) agents choose consumption (and therefore saving) and portfolio allocation in each period to maximise expected lifetime utility. We assume an additively separable form of the agent's lifetime utility function. We also assume a constant coefficient of risk aversion, the inverse of the intertemporal substitution elasticity. Agents are assumed to know the probabilities of surviving to given ages. Agent k who is aged j at time t maximises:

$$U_k = E_t \left[\sum_{i=0}^{i=T-j} s_{ij} \{ [c_{kt+i}]^{1-\zeta} / (1-\zeta) \} / (1+\rho)^i \right] \quad (1)$$

where T is the maximum length of life possible (which we take as 120 years of age) and the probability of surviving i more periods conditional on reaching age j is $s_{ij.}$ (s_{0j} =1). ρ is the rate of pure time preference; c_{kt+i} is consumption of the agent in period t+i.

ζ is the coefficient of relative risk aversion.

Agents face two constraints:
First there is a budget constraint governing the evolution of financial assets taken from one period to the next.

$$W_{k\,t+1} = [W_{k\,t} + \exp(y_{kt}).(1-\tau) - c_{kt} + b_{kt}].(\lambda \exp(r_{st}) + (1-\lambda)\exp(r_{ft})) \quad (2)$$

$W_{k\,t}$ is the stock of wealth of agent k in period t
y_{kt} is the log of gross labor income
τ is the tax rate on labor income. Tax paid is simply a proportion of gross income
b_{kt} is the level of the unfunded, state pension received by an agent, this pension is zero until age 65.
λ is the proportion of financial assets invested in risky assets.
r_{st} is the one period (log) rate of return on risky financial assets between period t and period t+1.
r_{ft} is the one period (log) rate of return on safe financial assets between period t and period t+1

For ease of notation we have not given agent-specific subscripts to asset returns but we will allow for returns to depend on characteristics of the investor; because probabilities of death are specific to agents of a given age rates of return on assets that might have annuities features will be agent specific. We will describe how rates of return on financial investments are determined shortly.

Agents also face a borrowing constraint; wealth cannot be negative:
$W_{kt} \geq 0$ for all k and t.

This constraint may bind in various periods. Whether it does so depends in a complex way upon the profile of the deterministic component of labor income, the realisations of income and rate of return shocks, portfolio choices, the degree of risk aversion and the volatility of shocks. It also depends on the tax rate and the generosity of state pensions.

We also assume agents cannot take short positions in either safe or risky assets:

$0 \leq \lambda \leq 1$

Agents work from age 20 to the end of their 64th year (if they survive that long) and are retired thereafter. We assume that the profile of gross of tax labor income reflects three factors. First, there is a time-related rise in general labor productivity. Second, there is an age-related element to the growth of labor income over an agent's life. This is modelled as a quadratic in age.

The age-specific part of the log of labor income is:

$$\alpha + \gamma age - \theta age^2 \qquad (3)$$

We set γ and θ so that the age-income profile matches patterns that have been typical (we discuss calibration issues in detail in the next section).

There is also an idiosyncratic (agent specific) stochastic element of labor income. The log of labor income for an agent is the sum of the age-related element, the time related element and the additive income shock. Denoting the log of gross labor income of agent k who is aged j in period t as y_{kt} we have:

$$y_{kt} = \alpha + gt + \gamma.j - \theta.j^2 + u_{kt} \qquad (4)$$

$$u_{kt} = \phi\, u_{kt-1} + e_{kt}$$

where $e \sim N(0, \sigma_e)$

α is a constant.

g is the rate of growth of labor productivity over time.

ϕ reflects the degree of persistence in idiosyncratic shocks to labor income; empirical evidence from a range of countries suggests ϕ is high and that idiosyncratic shocks to income typically have a high degree of persistence.

We assume that rates of return on risky financial wealth vary across periods due to random shocks that hit stock and bond markets. We assume that rates of return on savings at a particular time - both on safe and risky assets - differ between individuals because financial institutions take into account the probabilities of death of agents and offer age-related investment products. More specifically, financial institutions offer the following contracts. For every $ invested in period t, with a given risky/safe split, the investor receives the market return adjusted for a probability of survival to the next period. If markets are perfect the probability used in making this adjustment is the true survival probability. But we allow for imperfections (stemming from adverse selection or some other types of cost) which mean that the two are not equal. If the agent dies the institution keeps the funds. With no bequest motives agents will always chose these contracts over ones which just pay the market rate of return. *If* the insurance element of this contract is offered on actuarially fair terms the ex-post rate of return on a $ invested in the risky asset during period t by an agent k who is aged j and who survives to the next period is given by:

$$\exp(r_{st}) / s_{1j} \qquad (5)$$

s_{1j} is the probability of surviving one more year conditional on reaching age j

We assume r_s is the sum of the mean log return and an unpredictable shock.

$r_{st} = r + v_t$

r is the mean rate of return on risky assets

v_t is the random element of the rate of return on assets in period t.

We assume v is iid and normal: $v \sim N(0, \sigma_r)$

For a $ invested in the safe asset the return to an agent aged j is:

$$\exp(r_f) / s_{1j} \qquad (6)$$

We assume that the distribution of returns on assets is itself unaffected by pension arrangements; the most obvious justification for this is that there is a global capital market where claims on safe and risky assets can be traded.

We can write the log returns on actuarially fair investments in risky and safe assets respectively to an agent aged j at time t as:

$$r + v_t - \ln(s_{1j}) \tag{7}$$

$$r_f - \ln(s_{1j}) \tag{8}$$

If markets are perfect this financial contract can be offered at no risk by financial institutions because they pass on all the rate of return risk to investors and are assumed to be able to take advantage of the law of large numbers and face no uncertainty about the proportion of agents who will survive. It seems natural to assume that financial firms will offer insurance against risks that are idiosyncratic (individual length of life risk) but not offer insurance against systematic risk (rate of return risk). The contracts offered by financial institutions can be thought of as highly flexible individual retirement accounts (or personal pension schemes). Effectively agents have their own pot of assets into which they pay contributions and make deductions. Contribution rates and drawdowns from the fund are subject only to the constraint that the pot of assets can never fall below zero. The average rates of return on the fund increase with age since survival probabilities decline with age. Just as standard flat annuities available for a given sum rise with age, so the average rate of return offered by financial institutions increases with age.

In effect we are assuming that financial institutions offer one period annuities. These are the vehicles through which agents save for retirement. Agents are able to draw down such accounts in a flexible way in retirement. Individuals may decide to mimic the payments from standard flat annuities by having the "pot" size (ie. W) decline with age at a rate that is offset by rising average rates of return[1].

[1] A simple example shows how such assets can be used to mimic annuities. Suppose the probability of death is invariant with respect to age and is at a constant rate p. Assume a non-stochastic, constant rate of return rf. With an initial stock of wealth at retirement of W an agent could buy a standard annuity

To show how the one period contracts we assume are available allow agents to create standard annuities – should they so wish – consider someone who just invests in the safe asset (assumed to generate a constant return at rate r_f). As before we focus on an agent aged j at time t who has wealth W.

A standard, actuarially-fair annuity contract would promise to pay to a j year old an annual amount of A (until death) in exchange for a lump sum of W, where A satisfies:

$$W = A\ [\ 1 + s_{1j.}\ e^{-rf} + s_{2j.}\ e^{-2rf} + \ldots\ldots\ldots\ldots. + s_{nj.}\ e^{-nrf}\ ] \qquad n \rightarrow \infty \tag{9}$$

Here we are assuming payments are made at the start of each period and the first payment is made immediately. Let:

$$\Delta \equiv [\ 1 + s_{1j.}\ e^{-rf} + s_{2j.}\ e^{-2rf} + \ldots\ldots\ldots\ldots. + s_{nj.}\ e^{-nrf}\ ] \qquad n \rightarrow \infty \tag{10}$$

So $\quad A = W / \Delta$

Note the link between one period survival probabilities at different ages:

$$s_{2j} = s_{1j}\ s_{1j+1}$$

and more generally:

$$s_{nj} = s_{1j}\ s_{1j+1}\ s_{1j+2} \ldots\ldots\ldots.\ s_{1j+n-1} \tag{11}$$

If a j year old agent has wealth W in a fund and withdraws an amount $A= W/\phi$ and reinvests the remainder with our one-period contracts, their wealth at the start of the next peirod is:

$$W[\ 1 – 1/\Delta\ ]\ e^{rf} / s_{1j}$$

from a firm offering an actuarially fair deal which pays W(rf+p)/(1+rf) each period until death. We assume here that the first of these level payments is made immediately and then come at the start of each subsequent period so long as the agent is alive. One period savings contracts of the sort we envisage pay a return per dollar invested of (1+rf)/(1-p) if the agent survives and nothing otherwise. An agent starting with wealth of W could immediately take W(rf+p)/(1+rf) out and reinvest the rest for one period. If they survive their wealth at the start of the next period is : W [1 – (rf+p)/(1+rf)].{(1+rf)/(1-p)} = W. Obviously this policy can be sustained indefinitely. Thus the standard annuity contract can be replicated exactly by rolling forward one period contracts.

Withdrawing the same amount in the second period, when the agent is aged j+1, generates a fund at the start of the third period of:

$$W[\{(1 - 1/\Delta).\ e^{rf} / s_{1j} - 1/\Delta\}.\ e^{rf} / s_{1j+1}] = W[(1 - 1/\Delta)\ e^{2rf} / s_{2j} - (1/\Delta)\ e^{rf} / s_{1j+1}]$$

Assuming a constant per period withdrawal rate of W/ϕ the level of funds at the start of period n+1 is:

$$W[\ (1 - 1/\Delta)\ e^{nrf} / s_{nj}\ -\ (1/\Delta)\{\ e^{(n-1)rf} / s_{n-1\ j+1}\ +\ e^{(n-2)rf} / s_{n-2\ j+2}\ +\ e^{(n-3)rf} / s_{n-3\ j+3}\ + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ +\ e^{rf} / s_{1\ j+n-1}\ \}\ ] \qquad (12)$$

Which we can write:

$$W(e^{nrf} / s_{nj})\ [1 - (1/\Delta).\{1 + s_{1j.}\ e^{-rf}\ +\ s_{2j.}\ e^{-2rf}\ + \ldots\ldots\ldots\ +\ s_{n-1\ j.}\ e^{-(n-1)rf}\ \}] \qquad (13)$$

From (10) we have that for finite n

$$0\ <\ (1/\Delta).\{1 + s_{1j.}\ e^{-rf}\ +\ s_{2j.}\ e^{-2rf}\ + \ldots\ldots\ldots + s_{n-1\ j.}\ e^{-(n-1)rf}\ \}\ <\ 1$$

So (13) is always positive. This proves that the one period annuity contracts allow agents to mimic the returns from standard (open-ended) annuity contracts and satisfy budget constraints.

By assuming the availability of one period annuity contracts we are giving agents more options on lifetime accumulation and decumulation of assets than with standard annuities or with standard retirement accounts. Agents will value flexibility in annuitising and are unlikely to want a flat drawdown of their accumulated fund. The optimal rate of accumulation and decumulation of funds over time is a complicated function of all the parameters in the model and depends on the realisation of shocks; it can only be ascertained by simulations.

But in assuming that agents are offered these savings vehicles on actuarially fair terms we would be making a strong assumption that factors that seem to be important in the real world, and that make rates of return implicit in annuities contracts tend to be less than actuarially fair, are absent. (See, for example, Friedman and Warshawsky (1988); Mitchell, Poterba and Warshawsky (1997) and Brown, Mitchell and Poterba (1999)).

It is important to allow for problems that make annuities less than fair. We introduce a measure of the efficiency of annuities markets. When this measure, β , is 1 the annuities market work perfectly. When $\beta = 0$ annuities are, effectively, not offered. The survival probability implicit in the contract offered by a financial institution is a weighted average of the true survival probability, s_{1j} , and the rate when no annuity is offered, an effective survival probability of unity. β is the weight placed on the actuarially fair survival probability

The rate of return paid on one period savings invested with λ in the risky asset and (1-λ) in the safe asset for an agent aged j at time t becomes:

$$\{\lambda \exp[r + v_t] + (1-\lambda)\exp[r_f] \} / [\beta s_{1j} + (1-\beta)] \quad (14)$$

This way of modeling the efficiency of annuity contracts allows the departure from actuarially fair contracts to vary with age. The greater is age, the lower the probability of surviving and for all $\beta < 1$ the greater is the departure from actuarially fair contracts. Recent empirical evidence from the US suggests that annuity rates do become increasingly less favorable with age. Mitchell, Poterba and Warshawsky (1997) estimate that the average US annuity in 1995 delivered payouts with expected present value of between 80% and 85% of each $ annuity premium for 65 year olds; but the payout ratio was less for older people. A payout ratio of 80% of the actuarially fair value for a 65 year old corresponds in our simulations (based on projected future Japanese mortality rates[2]) to a value of β of about 0.3 if the rate of return on assets is a flat 6%. Friedman and Warshawsky (1988) report US payout ratios from the 1970's and 1980's of around 75% which corresponds to a β for a Japanese economy with 2050 life expectancies of about 0.2. Brown, Mitchell and Poterba (1999) provide some evidence that in the UK annuities average about 90% of the actuarially fair rates. This corresponds to a β of around 0.55. In both the US and the UK there is strong evidence of substantial variability in annuity rates across companies. In our simulations we consider 3 values for β: 1 (perfect annuities); 0.5 ("semi-perfect" annuities which represent a payout of about 89% of "fair" value); and $\beta = 0$ (no annuities).

State pensions

In the case we consider first the state pays a flat rate pension to all retired people. The system is financed by the proportional tax on labor income levied on all those working. In steady state the demographic structure is stable so the ratio of workers to pensioners is constant and is determined by the predicted Japanese survival survival probabilities for 2050. Since we assume all shocks to labor income are idiosyncratic, and are independent of the rate of return shocks, there is no uncertainty about the aggregate wage bill or about aggregate tax revenue. The tax rate is set to balance the unfunded state pension system in every period. We define the replacement rate of the state pension as the ratio between the pension paid in period t and the average gross income of those in the last year of their working life at period t-1. For a given replacement rate there is a constant tax rate which balances the system and allows the state pension to rise at the rate of aggregate labor productivity growth (g). Thus pensioners continue to benefit from aggregate labor productivity growth after leaving the work force. In this simple, constant population model the average (across all agents) implicit rate of return on contributions made to the unfunded (PAYGO) system is g.

The tax rate to finance state pensions of a given generosity is proportional to the replacement rate of the unfunded system. The factor of proportionality reflects the support ratio which in turn reflects mortality rates and life expectancy. We use data on anticipated Japanese mortality rates in 2050. For males these imply a life expectancy at birth of around 80; for women the figure is around 84. We take the average mortality rates across males and females to construct our survival probabilities. Using the figures the conditional life expectancy at different ages is shown in figure 1 and the attrition rate of pensioners from age 65 is shown in figure 2. In each figure we compare predicted (2050) mortality profiles with current (1999) Japanese life expectancies.

The steady state population structure of adults is easily calculated from the one period survival probabilities given in the predicted 2050 life tables. Given the life table data

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[2] using projected Japanese mortality rates for 2050.

we use, the steady state Japanese support ratio (the ratio between those aged 20 to 64 to those aged 65 and more) is around 2.8. This is substantially lower than the current Japanese support ratio which is almost 3.7. But the predicted steady state support ratio is much higher than that predicted for 2050 if very low fertility rates persist in Japan; that ratio could be under 2. Figure 3 illustrates the difference between steady state demographics and predictions for Japanese demographic structure for 2050 based on low fertility and a rapidly declining population.

We have defined the replacement rate as the ratio between the pension of a just retired person and the average gross income of those in their last year of work. We assume pension income is untaxed. The equilibrium tax rates for different gross (and net) replacement rates are shown in table 1.

III. Solving the Model:

The set of first order conditions from individual k's optimisation problem are:

if $c_{kt} < [W_{kt} + \exp(y_{kt}).(1-\tau) + b_{kt}]$

then (15)

$$U`(c_{kt}) = E_t\ [s_{1j}\ \{\ U`[c_{kt+i}].\{\ \lambda \exp(r+v_t) + (1-\lambda)\exp(r_{ft})\ \}/\ [\beta s_{1j} + (1-\beta)]\ \}\ /\ (1+\rho)]$$

else:

$c_{kt} = [W_{kt} + \exp(y_{kt}).(1-\tau) + b_{kt}]$

and (16)

$$U`(c_{kt}) \geq E_t\ [s_{1j}\ \{\ U`[c_{kt+i}].\ \{\ \lambda \exp(r+v_t) + (1-\lambda)\exp(r_{ft})\ \}\ /\ [\beta s_{1j} + (1-\beta)]\ \}\ /\ (1+\rho)]$$

where $U`(c_{kt})$ is $\partial U_k / \partial c_{kt}$

We also require a condition for optimal portfolio allocation:

Either :

$0 = E_t\ [\ U`[c_{kt+i}].\{\ \exp(r+v_t) - \exp(r_{ft})\ \}\ ]$

and $0 \leq \lambda \leq 1$

or

$0 < E_t$ [$U`[c_{kt+i}].\{ \exp(r + v_t) - \exp(r_{ft}) \}$]

and $\lambda = 1$ (17)

else

$0 > E_t$ [$U`[c_{kt+i}].\{ \exp(r + v_t) - \exp(r_{ft}) \}$]

and $\lambda = 0$.

(15) holds when the borrowing constraint is not binding. When the constraint binds complementary slackness implies that (16) holds. (17) is a standard condition for optimal portfolio allocation. Corner solutions may arise where agents wish to only invest in the safe asset or in the risky asset; for an internal solution the first equality at (17) must hold.

Although characterising optimal plans is easy enough solving explicitly for optimal consumption and for the optimal accumulation path for funds is not possible. Instead we have to turn to numerical methods. We solve the problem backwards in a now standard way (see Deaton (1990), Zeldes (1989) and Skinner, Hubbard and Zeldes(1995)). The resources available for consumption in any period are what Deaton calls cash in hand (given the borrowing constraint this is an upper limit on consumption). Cash in hand at time t for agent k is given by

$[W_{kt} + \exp(y_{kt}).(1-\tau) + b_{kt}]$ (18)

In the final possible period that an agent could survive to (age 120) consumption of all resources is optimal since we assume no bequest motive. We take a set of possible values for cash in hand available in the penultimate period to when the agent might be alive. For each of those values we solve a constrained optimisation problem. We seek a value of consumption and a portfolio choice value (λ) to satisfy the first order conditions (15) or (16) and (17) given the current state variable (cash in hand). We first solve (17) for given levels of saving to find λ, relying on the fact that consumption in the final period equals whatever cash in hand then is. Final period consumption will depend on the realization of v_T if any of the risky asset is held. Numerical integration can be used to estimate the RHS of (17) for any value of λ and a root finding algorithm then solves for the λ that satisfies the first order condition. With λ determined for various levels of consumption (or equivalently saving) we proceed to calculate the right hand side of (15) for a given level of consumption today at today's cash in hand. The right hand side of (15) is the expected product of marginal utility in the next period and the rate of return (adjusted for probabilities of death and for the rate of pure time preference). To do this calculation we need to consider the probability distribution of different values for cash in hand tomorrow *and* estimate what optimal consumption (and so marginal utility) will be at each realisation of cash in hand tomorrow. In general this involves integrating in two dimensions because there are two random components – a shock to rates of return and to labor income. (In fact after age 65 labor income is zero so then there is only one source of uncertainty). In the final period all cash in hand is consumed so finding the optimal level of consumption in the penultimate period – the value that solves the first order conditions – is relatively easy. We use the constrained optimisation routine in GAUSS to solve this problem.

Now we have a set of estimated values for optimal consumption and portfolio allocation at a finite set of cash in hand values in the penultimate period. We proceed to the second from last possible period and consider another set of possible cash in hand values. At each cash in hand value we again seek a level of consumption and portfolio allocation which satisfies the first order condition. To estimate what optimal consumption in the next period would be for a given a realisation for cash in hand we use the solved values at the finite set of cash in hand points for the penultimate period and interpolate between them.

We proceed backwards in this way storing solved values for optimal consumption and portfolio allocations for a given set of cash in hand values at each age. During the period of work there is another state variable - besides age and cash in hand - which is current income. Because income shocks are persistent we need to keep track of current income; this makes the problem much more computationally burdensome for the work years. We expand the dimension of the grid and solve the problem at combinations of age, cash in hand and income levels.

We continue back to the first period. At that stage we have a complete grid of optimal consumption and portfolio shares at various cash in hand values and income levels for every age an agent might reach.

What makes the process intensive of computer time is the need to perform many evaluations of the expected value of the product of marginal utility and the random rate of return. At each cash in hand value and at each income level we need to work out this expectation at each age and for many possible values of current consumption and portfolio choice until we find one that satisfies the first order conditions.

We use 10 point Gauss Hermite numerical integration which is the natural choice with normally distributed errors. With two sources of uncertainty this requires 100 function evaluations for the calculation of each numerical integral (at least until we reach retirement when integration in a single variable is sufficient). Each of these 100 evaluations requires 2-dimensional interpolation using the cash in hand / consumption pairs and portfolio shares/cash in hand pairs from the estimated solution grid in the next period.

A good deal of trial and error is needed to find the size of the grid that is required to get accurate solutions. We found that a grid that was fine at relatively low levels of cash in hand (regions where agents spend a good deal of time fairly early in life and when constraints are often binding) and becomes much coarser at high cash in hand values worked well. We found that 80 values for cash in hand were required to get accurate solutions. (Solutions were checked using simulations with large numbers of agents to ensure that first order conditions held).

Once we have a grid of vales for a given set of parameters running simulations is relatively easy. We draw 7000 paths of normally distributed random shocks to incomes and to rates of return over a life that could last until age 120. For each of the 7000 individuals (or sample lives) we use the first element of their vector of shocks to log income to calculate initial period income. For each agent we then use the calculated grids of optimal consumption and portfolio share values for various initial values of cash in hand and the income shock to estimate optimal first period consumption and optimal portfolio allocation. This involves 2 dimensional interpolating between the cash in hand and income shock points on the grid. Since the relations between cash in hand and both optimal consumption and portfolio allocation, and between the income shock and those choice variables, is smooth the interpolation does not pose any problems.

We now have a set of initial values for consumption from which we calculate saving and apply the portfolio shares to generate holdings of safe and risky assets. Second period wealth for each agent is initial saving multiplied by the portfolio rate of return which depends upon the random rate of return and the safe rate (both adjusted for the common survival probability). We then take another draw for the random income shock using a random number generator and, given the persistence parameter in the income process, generate a new realisation for income. After deducting tax we have second period net labor income which is added to the realised value for financial wealth (which depends on the shock to rates of return) to give second period cash in hand. We then use the grid solutions to calculate consumption and portfolio allocation at these new cash in hand values. The simulation proceeds in this manner.

What we end up with is 7000 profiles of consumption. Each profile shows what consumption and wealth holdings would have been for each agent facing a particular set of shocks and *if* they survived until the longest possible age. From this set of profiles we calculate expected utility of someone about to start their life. This calculation is straightforward and proceeds in stages. First at each age we calculate utility for the 7000 agents given their consumption. Second we average these utilities across the 7000 agents at each age. Third we take a weighted sum of these period averages using the overall discount factors as perceived at age 20 as weights. Thus the weight on the average utility for age 90 is the probability of surviving to age 90, conditional on a current age of 20, multiplied by a factor of $1/(1+\rho)^{90}$, which is a fairly small number. The weights attached to future average utilities (conditional on living) fall away very fast after age 90

IV. Calibration

We set the discount rate equal to -1.5% and the coefficient of relative risk aversion (ζ) equal to 3, which implies an intertemporal elasticity of substitution of 0.33. In the absence of bequests, in our model a negative discount rate is needed to generate the level of savings observed in Japan. A negative discount rate is not inconsistent with positive equilibrium real rates of return, see Benninga (1990), and Kocherlakota (1990). Other researchers have used a negative rate of pure time preference to model Japanese household decisions. Kato (1998) calibrates an OLG model to the Japanese economy with a -7.5% discount rate and Kato (2000) uses the rate of -3.5%. Imrohoroglu, Imrohoroglu and Joines (1999a) use a negative discount rate of just over 1% in their numerical simulations based on US data. The empirical work of Hurd (1989) is consistent with small negative rates of pure time preference (for the US).

The intertemporal elasticity of substitution is controversial; Cooley and Prescott (1995) use unity for their simulations whereas Auerbach and Kotlikoff (1987) use a coefficient of relative risk aversion of 4, implying the elasticity is only 0.25. Empirical work by Hansen and Singleton (1983) and Mankiw, Rotemberg and Summers (1985) suggest values a little over unity for intertemporal substitutability suggesting, in our framework, a coefficient of relative risk aversion a little under unity. Grossman and Shiller (1981), Mankiw (1985) and Hall (1980) found values between 0 and 0.4. for the intertemporal elasticity suggesting coefficients of risk aversion well in excess of 2. Hubbard, Skinner and Zeldes (1995) use a relative risk aversion of 3 in their simulations. Zeldes (1989) estimated the risk aversion coefficient as 2.3. Kato (1998) and Kato (2000) report relative risk aversion of 5 and 2.22 . We consider a value of 3 for the risk aversion coefficient is a central estimate but clearly the evidence makes it hard to be confident about what a plausible figure is.

Income profile

Cross section profiles of Japanese incomes suggest that it is typical for earnings to peak at around age 50 when average earnings are around double the earnings of new workers (20 year olds)[3]. We set the parameters of the earnings process so that on average the income of Japanese workers peaks at the age of 50 when it is double

[3] Japanese Ministry of Health, Labour and Welfare, "Basic Survey on Wage Structure" (2000).

earnings at age 20. With productivity growth at 2% this leads to the following average income profile

y_t = ln2 - (ln2)/900*(age-50)^2 + 0.02t.

Figure 6 shows the resulting pattern of labor income over the life cycle where there is 2% general labor productivity growth and for an agent who experiences no income shocks (e_{kt} = 0 for all t); here we set the state pension at 50% of gross pre-retirement average earnings (or at about 65% of net earnings). Pensions grow in line with aggregate labor productivity.

Income volatility

Setting the volatility of the shock to labor income is particularly important for the simulations. As noted above, a significant part of the shocks to individual incomes appear to be persistent. Our strategy is to set the variance of the iid shocks so as to generate an overall degree of income uncertainty which is typical, while keeping the persistence of shocks comparable to that of other studies. Hubbard, Skinner and Zeldes (1995) use a model of income dynamics to simulate the impact of social security which is based on characteristics of US household income data. Their model is similar to that used here except that they allow for both transitory and persistent shocks, while we only allow for persistent shocks. Their model for the log income of household k at time t is:

$$y_{kt} = f(age_{kt}) + u_{kt} + w_{kt}$$

$$u_{kt} = \phi\, u_{kt-1} + e_{kt}$$

where e and w are iid shocks that are not correlated and f(age_{kt}) is a deterministic function.

A measure of the unconditional volatility of log income is:

$$\sigma^2_w + \sigma^2_e / (1 - \phi^2)$$

Typical values for ϕ, σ_w and σ_v used by Hubbard et al are 0.955, 0.158 and 0.158. These imply that some income shocks are highly persistent. With these values their measure of the unconditional standard deviation of the shock to log income is 0.56[4].

[4] In fact Hubbard et al set different values of ρ, σ_w and σ_v for those with no high school, high school and College education. The implied unconditional standard deviation of the shocks to log income for these three groups are 0.64, 0.51 and 0.44 respectively.

The dispersion of Japanese wages is lower than in the US. We find that setting the unconditional standard deviation of shocks to log income between 0.4 and 0.5 for the Japanese workforce matches the empirical distribution quite well. Figure 4, for example, illustrates that with an unconditional standard deviation of 0.45 the implied distribution of the levels of income is very close to the actual 1996 distribution of Japanese household income. (For this comparison we adjust the intercept in our process for log income to match the mean of Japanese household income in 1996). Part of the inequality in the empirical distribution reflects the life cycle pattern of earning rather than the impact of persistent shocks to earnings or persistent differences in earnings power. To allow for this we assume an unconditional standard deviation of log income a little lower at 0.40. We set the persistence parameter at 0.95 implying a standard deviation of the random (iid) income shocks of around 0.125. (In terms of parameters in Hubbard et al we set $\phi = 0.95$, $\sigma_w = 0$ and $\sigma_e = 0.125$.)

Returns

The historical real returns on the NIKKEI 225 index over the past few decades have a mean of 7-8% with standard deviation of 22-25% depending on what period one considers (1960-2000, 1970-2000 or 1980-2000). These figures are for gross returns; net of charges annual returns to individuals are likely to be lower by at least 50 basis points, and perhaps by more.

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Bond portfolios are less volatile. Miles and Timermann (1999) suggest that a mixed bond and stock portfolio in developed countries would have generated a lower average real return than an equity portfolio and with a significantly lower annual volatility. In Japan such a mixed portfolio might generate an average real return of about 6% a year with annual standard deviations of around 17.5%. Stock returns with these characteristics have been typical in many developed countries in the past. The above figures are before any deductions for charges, for this reason we think of net returns on risky assets with means of 6% and volatility of 17.5% as relatively optimistic. We also consider a case with mean return 4% and volatility 17.5%. Feldstein and Ranguelova (2000) use a return for US risky assets with a mean annual return of 5.5% and a standard deviation of 17.5%

We set the risk-free rate at 2% making it equal to the productivity growth in our model.

Simulation with current survival rates:

Before considering in detail the results using projected steady state population structure based on 2050 life expectancies we first describe results of a simulation assuming that the Japanese population has stabilised at a level consistent with current survival rates (see Figure 5). This generates a support ratio 3.09 as opposed to the current support ratio of about 3.65. Guided by the historical distribution of NIKKEI 225 returns we set the mean return on risky assets at 8% and standard deviation at 25%. (We consider this too high a figure for future returns; for future steady state simulations we set the mean return on risky assets as either 6% or 4% and the standard deviation at 17.5%) The pension generosity is set at the current level (1999) in Japan which is about 50% (65% on a net of tax basis). Annuities are taken to be semi-perfect (β=0.5). This implies a money's worth ratio of slightly under 90%. All other parameters are set at levels described in previous section. This first simulation is designed to see if broad features of recent Japanese performance can be reproduced.

The implied average income profile is depicted in Figure 6.

We simulate random income shocks for 7000 individuals and calculate the corresponding optimal consumption-investment paths. In this illustrative simulation the number of credit constrained individuals is about 38% of 20 year olds, decreasing to virtually zero amongst those aged 50. The proportion credit constrained starts picking up again at retirement; half of those who survive to age 94, and nearly 80% of the small number who live beyond 100, are constrained. (See Figure 7).

Household assets start at zero. On average assets increase steadily until retirement when they typically represent 7.5 years of annual income of the age group 60-64 (see Figure 8). Savings rate mirror the accumulation and decumulation of assets. Young people under 30 on average save less than 10% of income whereas people aged 44 to

64 save (on average) more than 20%. With declining life expectancy, pensioners dissave most around age 85 (-25%), see also Figure 9.

Based on current mortality rates and with state pensions worth 50% of average (gross) final salary (65% of net) the aggregate wealth to national income ratio in steady state is 3.49, the labor share of national income is 74.7% and the aggregate saving rate is 10%. All these figures are net of depreciation. In aggregate 87% of wealth is invested in the risky asset. The life cycle evolution of risky investment is shown in Figure 10.

Note that we set the distribution of rates of return independently of saving and wealth accumulation in Japan; we consistently assume Japan is integrated with the world capital market and although a large economy it is assumed not to influence global capital market returns.

V. Results for future steady states with different pension arrangements:

We show results for various combinations of assumptions about the distribution of rates of return, annuity market efficiency and the structure of state pension arrangements. We begin with the case where the state pays flat rate pensions. Tables 2 to 7 summarise the results. One thing stands out immediately from the tables: the equilibrium stock of wealth, and the wealth to national income ratio, is highly sensitive to the generosity of unfunded, state pensions. When state pensions are worth 20% of (end -of -working-life) average gross incomes the aggregate stock of wealth is around twice the level when pensions are set to generate an average replacement rate of 50%. The wealth to national income ratios are somewhat less variable since with wealth so much higher when pensions are lower national income is also greater and the wealth to income ratio rises slightly less than the aggregate stock of wealth. A second general feature of the results is that with less generous state pensions a significantly higher proportion of wealth is invested in safe, rather than risky, assets. The scale of the difference depends upon how effective annuities contracts are and on the risk premium on risky assets, but the effect is never small. With a 4% risk premium (an average rate of return on risky assets of 6% and a 2% safe rate) and perfect annuities markets the share of wealth in risky assets falls from

94% with a replacement rate of 50% on flat-rate, state pensions to 60% if there are no unfunded pensions. The scale of the switch is even larger with less efficient annuities markets and is also greater with a lower risk premium. When the average return on risky assets is only 4%, and with no state pensions, only just under 40% of aggregate wealth is held in risky assets; that proportion rises to 85% or more if state pensions generate an average replacement rate of 50%. (See tables 5-7). Flat-rate state pensions are risk free so the more generous they are the more willing people are to take risks with their savings - but with no state pensions the retired are much less willing to invest a very high proportion of retirement savings in risky assets. The results suggest that there is a powerful link between the generosity of state, unfunded pensions that are independent of asset market risk and portfolios of wealth held by households.

Tables 2 to 4 summarise the results for a 6% average annual rate of return on risky assets with a standard deviation of 17.5%. If financial markets offer actuarially fair contracts (perfect annuity markets) steady state expected utility is maximised when there are no unfunded state pensions. The final row in the table ("equivalent prod. gain") shows by how much average labor income in a world with no state pensions would have to be different so as to generate the same expected utility as in a world with unfunded pensions of a given generosity. If state pensions were set at a low rate, such that someone with average income at the end of their working life only got a state pension at age 65 of 20% of their last labor income, expected lifetime utility would be significantly lower than with no pensions. Table 2 shows that this would reduce utility by the equivalent of a 2.4% cut in labor incomes. Higher state pensions further reduce expected utility: state pensions with an average replacement rate of 50% of gross income (around 65% on a net income basis) reduce welfare by the equivalent of a 10.4% permanent reduction in productivity. With perfectly efficient annuities contracts and a substantially higher return on risky assets than the growth in productivity (which equals the effective return on contributions to the state pension system) individuals would prefer to accumulate savings in their own retirement fund rather than rely significantly on state, unfunded schemes. They respond to lower contributions to the state pension and lower state pensions by saving substantially more. With no state pensions the stock of aggregate wealth is 738 and the wealth to national income ratio is about 7.5. The aggregate saving rate is about 16%. With 40%

safety net pensions the stock of wealth is 350 , the wealth to income ratio is 4.12 and the national saving rate is only just over 10%.

But if annuities are not available (β=0) then not only are agents much less well off but there is a welfare enhancing role for state (flat rate) pensions, even if the rate of return on risky assets (with a mean of 6%) is, on average, three times the implicit rate of return on contributions to unfunded pensions (2%). Table 4 shows that a flat rate, unfunded pension which generates a 20% replacement rate at retirement increases expected utility, relative to a world with no state pensions, by the equivalent of a 4.4% increase in labor productivity at all ages. Once again it also generates a very much smaller stock of aggregate assets than if no state pensions were paid: aggregate wealth is 915 with no pension, 566 with a 20% gross average replacement rate and 250 with a 50% replacement rate.

But even with completely inefficient annuities markets when the risk premium is 4% steady state expected utility is maximised with fairly small state pensions. If the replacement rate on the state unfunded pensions rises above 20% expected steady state welfare starts to fall. Table 3 shows that if annuities markets are semi-perfect (β=0.5) even state pensions generating a replacement rate of only 20% reduce welfare slightly relative to a situation with no state pensions.

But with a lower risk premium of only 2% there is a greater role for state pensions. Even if annuities markets are semi-perfect welfare is significantly higher with state pensions worth 20% of average gross final salaries than when there are no state pensions; welfare is higher by the equivalent of a 2% permanent gain in productivity. (See table 6).

With no state pensions agents would never chose to run down assets in retirement to zero, regardless of the efficiency of one period annuity contracts. But with 40% replacement rate state pensions a significant proportion of those who live longer than average would optimally run out of private savings. Table 8 shows the proportion of those of various ages who would chose to consume all their cash in hand (and are therefore credit constrained) at various ages and assuming various degrees of efficiency in annuities markets. Whatever the efficiency of annuity markets a substantial proportion of agents are credit constrained very early in life. The less perfect are annuity markets, the more people are credit constrained as they get older.

With no state pensions people never run out of savings as they age (though there remain substantial numbers of credit constrained among the young). With a 6% average rate of return and pensions worth 40% of average income at retirement about 15% of those who survived to age 90 would become credit constrained if $\beta = 0.5$. If pensions are worth 20% this proportion falls to almost zero. With a lower rate of return the proportion credit constrained is somewhat higher.

The pattern of average savings rates over the life cycle with different generosities of flat rate state pensions, and various degrees of efficiency of annuity contracts, is interesting. The profile of saving at the start of working life displays a pattern typical when precautionary motives are important and where income uncertainty is significant. In the first few periods in work most agents save a substantial fraction of wealth to establish a small stock of assets as a buffer to protect consumption against future labor income shocks. On average agents then dissave mildly for several years before starting to build up a stock of assets for retirement. When savings starts to become significant, and how great the stock of wealth at retirement is, are sensitive to both the generosity of state pensions and to the efficiency of annuity markets. With no state pensions and perfect annuity markets saving typically becomes significant from the mid 20's. With a state pension worth 40% of average labor income at retirement and completely imperfect annuity markets savings is typically not significant (after the initial establishment of a precautionary buffer at the start of work) until the 30's.

An alternative state pension system:

So far we have considered flat rate state pensions. These are highly redistributive and provide the greatest insurance against idiosyncratic labor income risk, but they come at a possible cost of labor supply distortions. How are the results different with state pensions that are related to income during the working life? We consider a state pension system which generates a given replacement rate *for each individual*, rather than generates an average replacement rate of a given level. This system makes pensions proportional to final salary and is therefore not redistributive. It also provides no insurance against shocks to labor income which are still having an effect on final period salaries. Given the highly persistent nature of income shocks, if state pensions are proportional to final salary they pass on much of the impact of income uncertainty from the working years into retirement income.

Tables 9 and 10 summarises the results of simulations where we have unfunded pensions linked to final salaries. We focus on a 6% expected return on risky assets and analyze "semi-perfect" annuities ($\beta = 0.5$) and no annuities (($\beta = 0.5$). It is natural to compare the salary-related pensions outcomes to those where pensions are paid at a flat rate (but with the same average generosity). Comparing the results in table 9 with those in table 3, and he results in table 10 with those in table 4, reveals four things:

1. With pensions linked to final salary agents face more idiosyncratic risk and they respond by investing slightly less of their wealth in risky assets. This reflects the well-known finding that more background risk reduces holding of risky assets, even if the risk is uncorrelated with returns on risky assets.
2. Agents also typically save rather more when pensions are less certain. With pensions worth 40% of final salary the aggregate stock of assets is about 16% higher than with flat rate pensions of the same average value. When the replacement rate is 50% the stock of wealth is about 19% higher with income-related pensions.
3. When viewed from the beginning of life - and from behind a veil of ignorance since first period income is not known - agents would strictly prefer flat rate pensions at the end of their lives. This is obvious because risk averse individuals will prefer pensions that offer insurance against bad draws of income while giving the same average value of benefits. But we do not attach much significance to the scale of the gap between expected lifetime utility in Tables 3 and 9. This is because we assume fixed labor supply so that any benefits of linking unfunded pensions to contributions (an implication of having pensions proportional to final salary) are assumed away. In a richer model with endogenous labor market effort the gains from smaller distortions to labor supply would need to be counted.

4. When annuities markets are semi-perfect (money's worth rations about 89%) there is no welfare enhancing roles for state, unfunded pensions. Only when annuities markets work poorly do unfunded pensions enhance welfare and even then only small pensions are warranted; when the replacement rate on state pensions moves above 20% welfare starts to fall sharply.

VI. Conclusions:

Two factors emerge as important to the design of pension systems with an optimal degree of reliance on personal retirement accounts relative to unfunded pensions. First, the nature of the distribution of rates of return. Second, how efficient are annuities contracts. The optimal size of unfunded pensions is sensitive to both factors. If average rates of return on risky assets are as high as 6% then even if volatility in returns is high (17.5% annual standard deviation) and annuity markets less than perfect ($\beta = 0.5$) there is no useful role for unfunded pensions. Only if annuities are absent altogether does the existence of any unfunded pensions generate higher steady state welfare. If rates of return on risky assets have a lower mean the situation is different. With 4% mean returns on risky assets then with semi-perfect annuity markets unfunded safety net pensions worth about 20% of average gross labor income at retirement are optimal. But with perfect annuity markets (β=1) there is still no welfare-enhancing role for unfunded state pensions.

One significant feature of the results is the sensitivity of portfolio allocation to the existence of state, unfunded pensions and, to a lesser extent, to the efficiency of annuity markets. With relatively generous unfunded pensions linked either to average wages or to individual earnings histories agents invest the great majority of their savings in risky assets. But when resources for retirement come largely from accumulated savings a substantial part (as much as 60% in some cases) is invested in safe assets. This dependence of investment patterns on the generosity of state pensions has been paid much less attention than the link between the *amount* of private saving and state pensions. Our results suggest this is very important.

We have assumed that rates of return are independently (and identically) distributed over time. This means that investment risk can, to a significant extent, be smoothed over time. There is evidence that rates of return are not independent over time; mean reversion at long horizons may exist (Poterba and Summers (1988)). This can have an important impact on the risk of funded pensions where investment horizons of thirty or more years is relevant. It can also mean that the iid assumption can underestimate the risk of funded pensions (see Miles and Timermann (1999) for illustrative calculations). An important area for future work is the modeling of optimal pension arrangements when asset markets are subject to sustained periods of above or below average returns; if there are bull and bear markets funded pensions are likely to be riskier than if returns come from the same distribution each period.

A key policy implication in this paper is that governments need to carefully consider the efficiency of annuity contracts before embarking on pension reform. The results reported here suggest that how efficient annuity contracts are, why deviations from actuarially fair contracts might exist, and how reform might change this are crucial. However we do not model the reasons why annuity contracts might be less than fair and to understand the policy implications of imperfections knowing why they exist is obviously important. But we can measure the welfare gains from making annuity contracts more efficient, and they are substantial. Table 11 shows the equivalent permanent increase in labor incomes which generates the same welfare gain as a given increase in the efficiency, or degree of actuarial fairness, of financial contracts. In each case we consider the gain which arises when β increases from 0.5 (semi-perfect annuity contracts) to1 (perfect contracts). The gain from this change is the equivalent of a permanent rise in productivity of between 3.5% and 4.5% of productivity when there are no state pensions. Those gains are smaller the greater is the generosity of unfunded pensions, though they are not trivial.
The results suggest that understanding why annuity contracts may not be actuarially fair is a major policy question.

By focusing on long run implications of demographic changes, and on the implications of different pension arrangements, we are, of course, abstracting from transitional issues. While we are able to say something about how different steady states look, that does not in itself give an unambiguous guide to policy. For example, by simply looking at steady states one might conclude that unfunded pensions should be substantially reduced and that a situation where individuals rely almost completely on funded pensions would be desirable. But given the existing generosity of unfunded pensions in most developed countries one faces a formidable problem in moving from the current situation to a largely funded system, and one would have to take account of existing obligations to pay pensions to those in or near retirement. The value of the kind of simulations we report in our research is that it nonetheless can tell us something about what the implications of making a particular transition are. For example, if it were to be the case that the long run benefits of reducing unfunded pensions and building up funded pension assets were to be very substantial, then that benefit would be weighed against the transitional costs. On the other hand, if it would be desirable even in a steady state to preserve very substantial levels of unfunded pensions (for example because of their desirable risk sharing characteristics) then it might not be worth undertaking any sort of transition to a funded system given that there are costs in moving there anyway. For these reasons we believe the sort of steady state analysis we undertake are a critical element of any assessment of optimal responses to demographic change.

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Table 1

Steady State Replacement Rates and Contribution Rates to State, Unfunded Pension System (%)

Gross replacement rate♣	5	10	20	30	40	50
Net replacement rate♦	5.1	10.5	21.9	34.5	48.4	63.9
Balanced contribution rate♠	2.18	4.36	8.72	13.1	17.44	21.8

Note: Figures are based on a system in balance with a population structure implied by anticipated Japanese 2050 mortality rates and assuming a constant population.

♣ ratio of age 65 pension to average gross earnings of age 64 agents one period earlier

♦ ratio of age 65 pension to average net earnings of age 64 agents one period earlier

♠ contribution rate to balance unfunded, state pension scheme

Table 2: Mean return on risky assets 6% Standard deviation 17.5%
Safe return = 2%; Flat rate Pensions

		Replacement rate of state pension			
		0%	20%	40%	50%
275	Aggregate Wealth (W)	738	530	350	
β=1.0	W/National Income	7.54	5.77	4.12	3.89
perfect	Saving Rate	16%	13%	10%	8.3%
	Risky Assets/Wealth	60%	75%	89%	94%
annuities	Share of Capital Income	33%	29%	23%	20%
	Expected Utility	-113.5	-119.1	-131.8	-141.5
	Equivalent prod gain		-2.4%	-7.2%	-10.4%

Equivalent prod gain is the percent rise in labor productivity needed to generate the same expected utility in a world with no state pensions.

Table 3: Mean return on risky assets 6% Standard deviation 17.5%
Safe return = 2%; Flat rate Pensions

		Replacement rate of state pension			
		0%	20%	40%	50%
264	Aggregate Wealth (W)	804	550	345	
β=0.5	W/National Income	7.98	5.88	4.06	3.27
Semi-perfect	Saving Rate	23%	18%	13%	10%
	Risky Assets/Wealth	60%	78%	93%	96%
annuities	Share of Capital Income	35%	30%	23%	19%
	Expected Utility	-122.0	-122.7	-133.3	-142.4
	Equivalent prod gain		-0.3%	-4.3%	-7.4%

Table 4: Mean return on risky assets 6% Standard deviation 17.5%
Safe return = 2%; Flat rate Pensions

		Replacement rate of state pension			
		0%	20%	40%	50%
250	Aggregate Wealth (W)	915	566	335	
β=0.0	W/National Income	8.68	5.95	3.95	3.12
no	Saving Rate	32%	23%	15%	11%
annuities	Risky Assets/Wealth	59%	82%	95%	97%
	Share of Capital Income	38%	31%	23%	18%
	Expected Utility	-137.9	-126.6	-134.7	-143.2
	Equivalent prod gain		+4.4%	+1.2%	-1.9%

Table 5: Mean return on risky assets 4% Standard deviation 17.5%
Safe return = 2%; Flat rate Pensions

		Replacement rate of state pension			
		0%	20%	40%	50%
185	Aggregate Wealth (W)	596	411	250	
β=1.0	W/National Income	7.23	5.28	3.37	2.56
perfect	Saving Rate	15%	11%	7.5%	6%
annuities	Risky Assets/Wealth	39%	53%	75%	85%
	Share of Capital Income	20%	16%	12%	10%
	Expected Utility	-125	-127.1	-136.5	-144.8
	Equivalent prod gain		-0.8%	-4.1%	-7.1%

Table 6: Mean return on risky assets 4% Standard deviation 17.5%
Safe return = 2%; Flat rate Pensions

		Replacement rate of state pension			
		0%	20%	40%	50%
171	Aggregate Wealth (W)	650	421	238	
β=0.5	W/National Income	7.80	5.37	3.23	2.39
Semi-perfect	Saving Rate	21%	15%	9%	7%
annuities	Risky Assets/Wealth	39%	56%	79%	88%
	Share of Capital Income	22%	17%	12%	9%
	Expected Utility	-136.9	-131.7	-138.0	-145.7
	Equivalent prod gain		+2.0%	-0.4%	-3.1%

Table 7: Mean return on risky assets 4% Standard deviation 17.5%
Safe return = 2%; Flat rate Pensions

		Replacement rate of state pension			
		0%	20%	40%	50%
159	Aggregate Wealth (W)	740	424	225	
β=0.0	W/National Income	8.63	5.38	3.06	2.22
no	Saving Rate	29%	18%	10%	7%
annuities	Risky Assets/Wealth	38%	60%	83%	91%
	Share of Capital Income	24%	17%	11%	8%
	Expected Utility	-159.7	-136.7	-139.3	-146.3
	Equivalent prod gain		+8.1%	+7.1%	+4.5%

Table 8

% of Population Credit Constrained by Age:

A: average rate of return on risky assets = 6%; flat rate pension worth 40% of average income at retirement

Age	20	25	35	80	90
$\beta = 1$	47	29	10	0	0
$\beta = 0.5$	47	29	10	2	15
$\beta = 0$	46	20	10	4	33

B: average rate of return on risky assets = 6%; flat rate pension worth 20% of average income at retirement

Age	20	25	35	80	90
$\beta = 1$	41	25	4	0	0
$\beta = 0.5$	41	23	7	0	0
$\beta = 0$	40	22	8	0	2

Table 9: Mean return on risky assets 6% Standard deviation 17.5%
Safe return = 2%; Final salary related, unfunded pensions

		Replacement rate of state pension			
		0%	20%	40%	50%
315	Aggregate Wealth (W)	804	588	400	
β=0.5	W/National Income	7.98	6.21	4.56	3.76
Semi-perfect	Saving Rate	23%	19%	14%	12%
annuities	Risky Assets/Wealth	60%	75%	91%	95%
	Share of Capital Income	35%	31%	26%	22%
	Expected Utility	-122.0	-128.1	-141.0	-150.0
	Equivalent prod gain		-2.4%	-7.0%	-9.8%

Table 10: Mean return on risky assets 6% Standard deviation 17.5%
Safe return = 2%; Final salary related, unfunded pensions

		Replacement rate of state pension			
		0%	20%	40%	50%
298	Aggregate Wealth (W)	915	612	390	
β=0.0	W/National Income	8.68	6.32	4.44	3.59
no	Saving Rate	32%	24%	16%	13%
annuities	Risky Assets/Wealth	59%	79%	94%	97%
	Share of Capital Income	38%	33%	26%	21%
	Expected Utility	-137.9	-133.6	-143.4	-152.0
	Equivalent prod gain		+1.6%	-1.9%	-4.7%

Table 11

Gains in welfare from improvements in annuity rates: equivalent rise in labor productivity

$$\beta = 0.5 \rightarrow \beta = 1$$

average rate of return on risky assets	replacement rate of state, unfunded pension			
	0%	20%	40%	50%
6%	3.5%	1.5%	0.6%	0.3%
4%	4.4%	1.8%	0.5%	0.3%

Figure 1: Conditional life expectancy

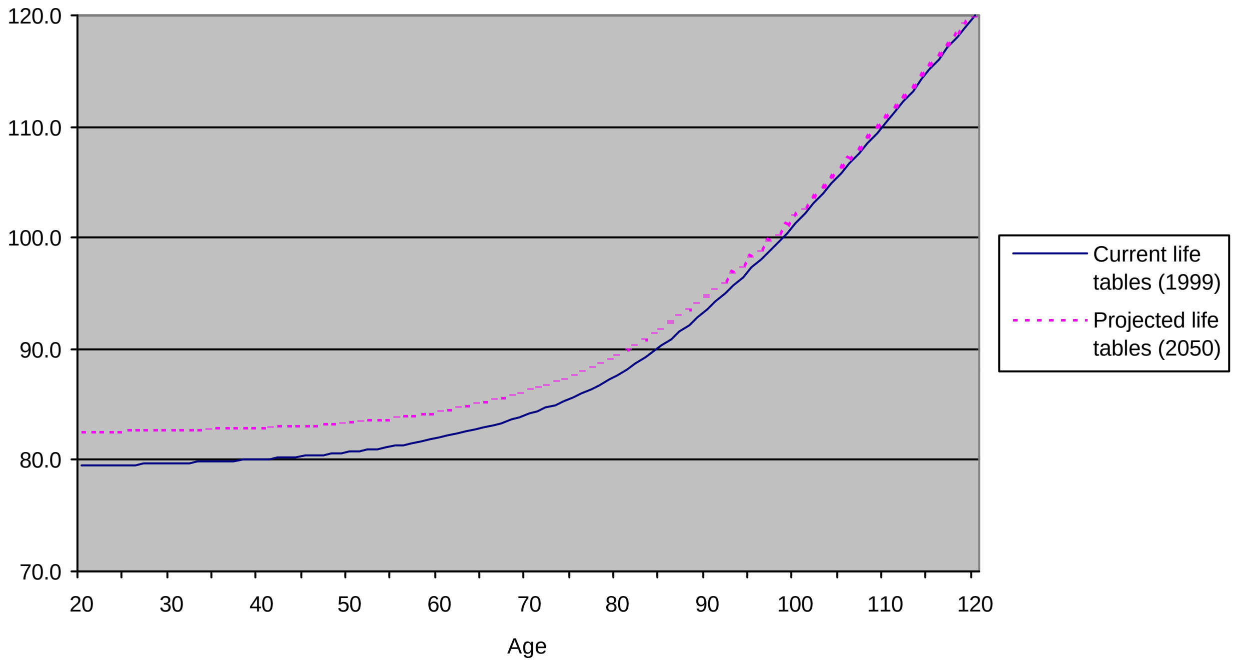

Figure 2: Attrition rates for pensioners - proportion surviving of those alive at 65

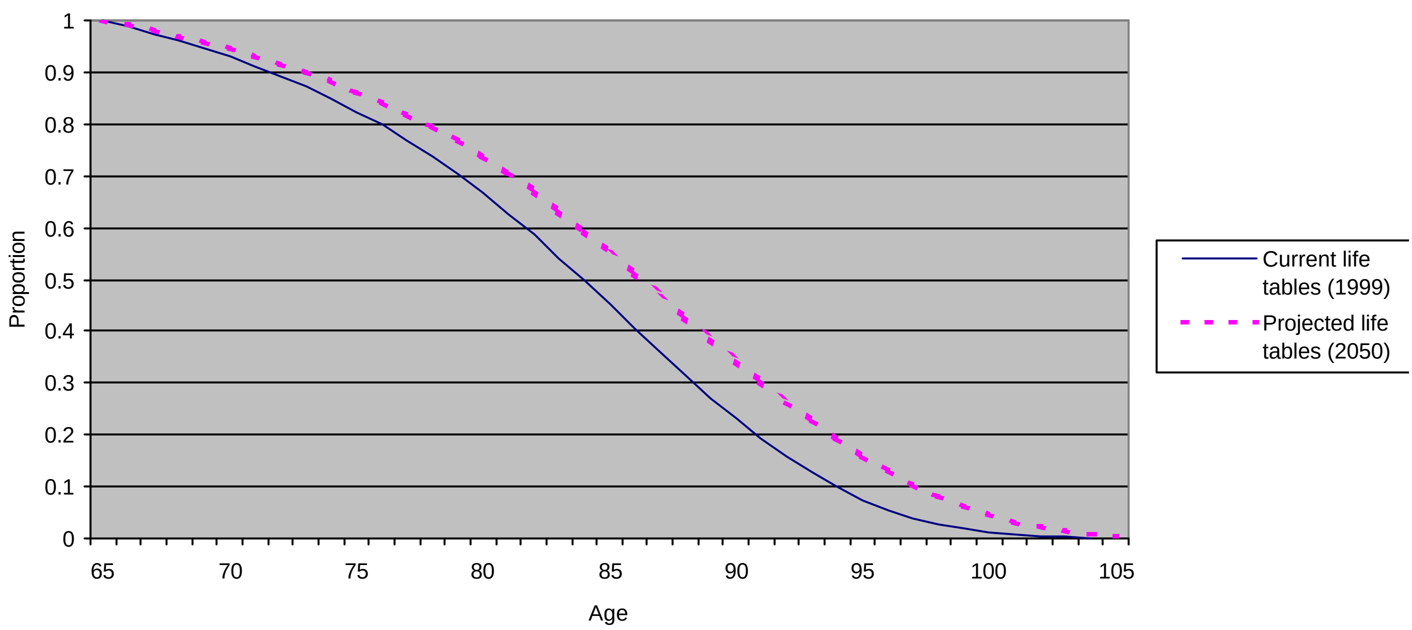

Figure 3: Population structure in steady state and in short run based on Japanes demographic projections

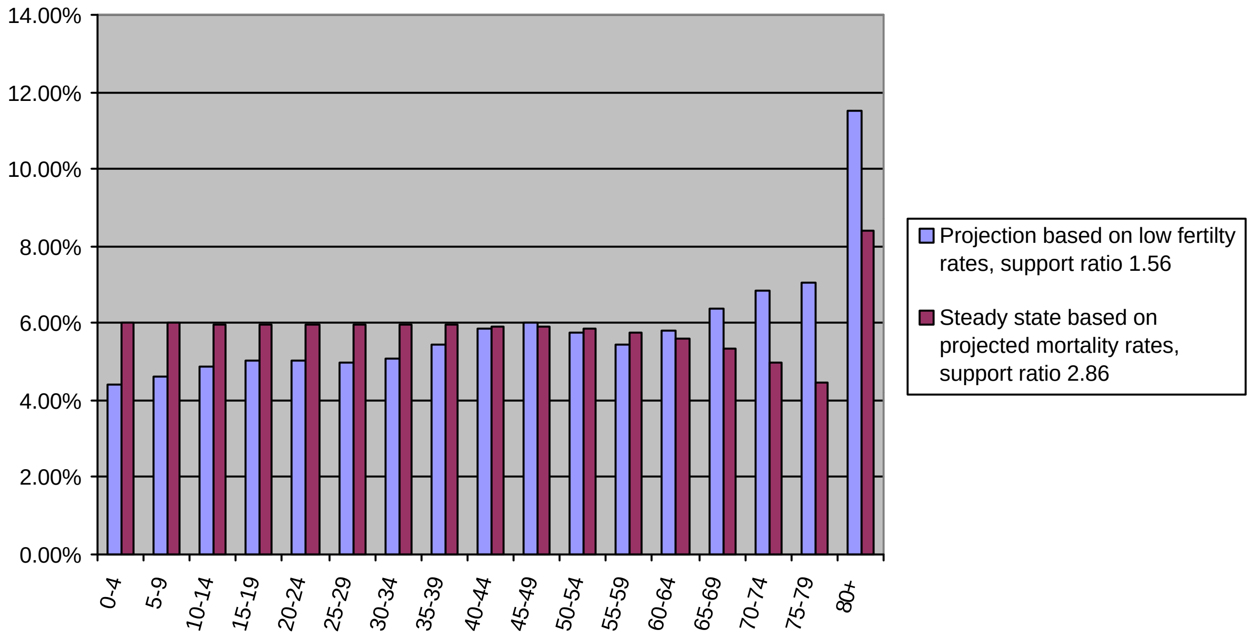

Figure 4: Actual income distribution of Japanese workers (1996) and its lognormal approximation with standard deviation 0.45

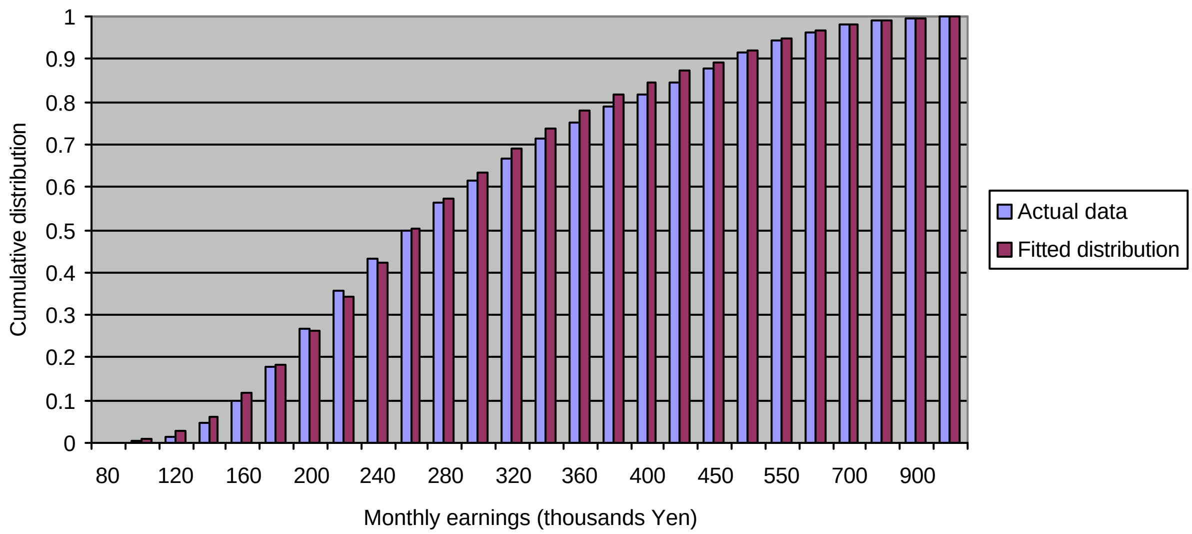

Figure 5: Current population structure and steady state based on current mortality rates

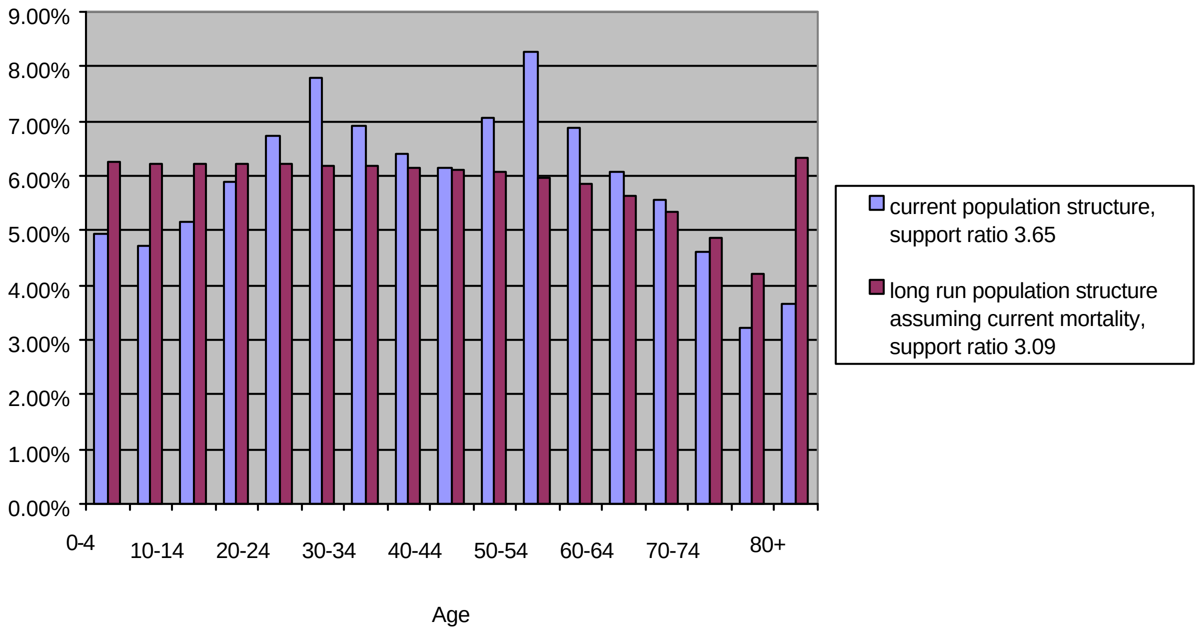

Figure 6: Average life cycle income in steady state assuming current mortality rates and 50% pension generosity

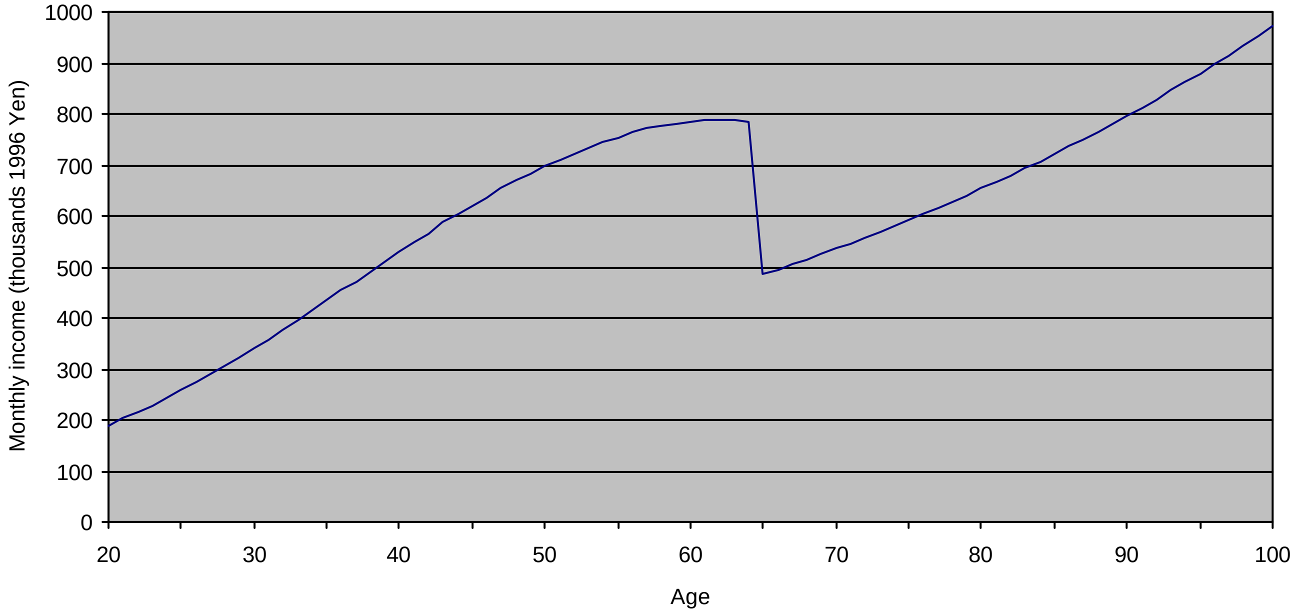

Figure 7: proportion of credit constrained households in steady state assuming current mortality rates, 50% pension, 1960-2000 risky returns with semi-perfect annuities market, time preference -1.5% and elasticity of substitution 0.33

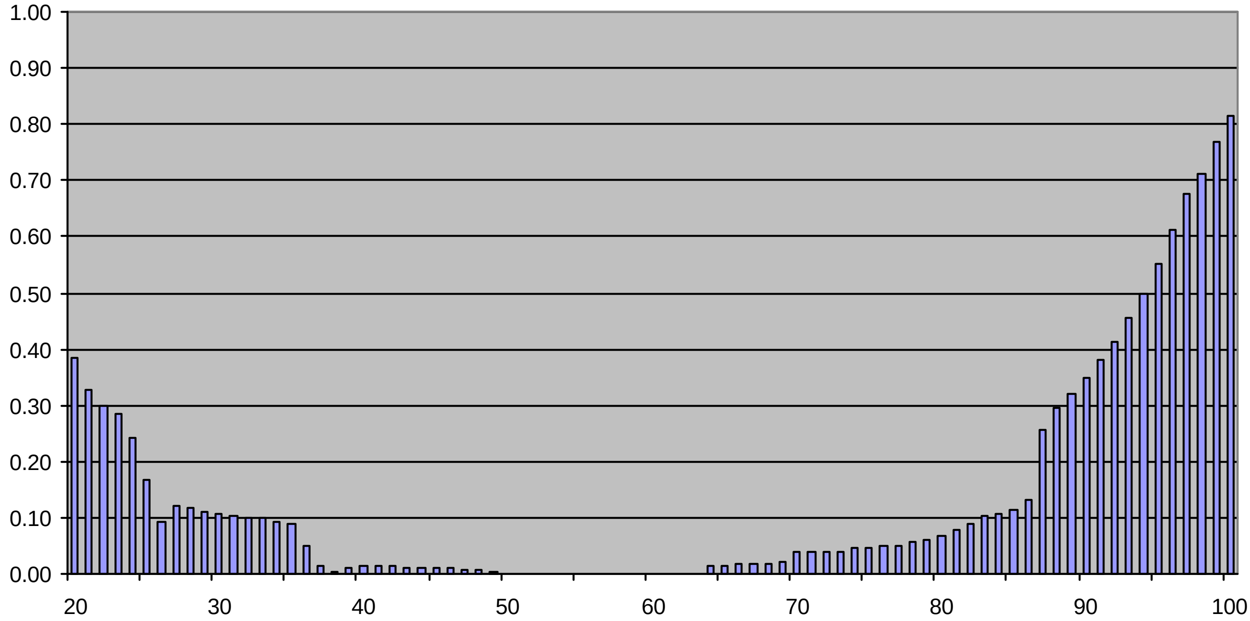

Figure 8: Average level of assets over life cycle in steady state in steady state assuming current mortality rates, 50% pension, 1960-2000 risky returns with semi-perfect annuities market, time preference -1.5% and elasticity of substitution 0.33

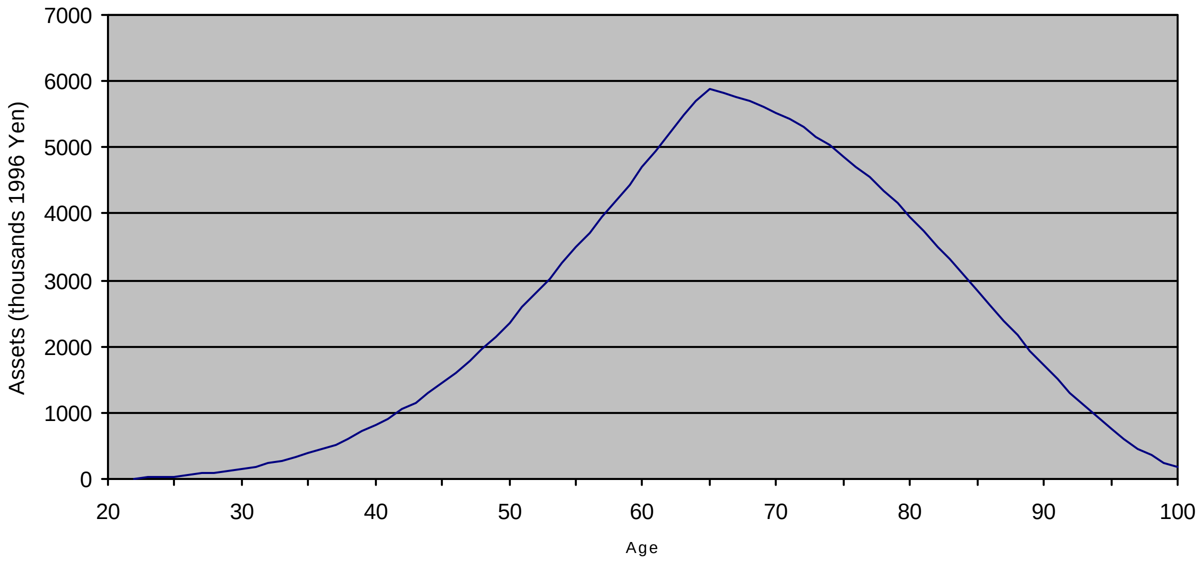

Figure 9: Average age-specific saving rate in steady state assuming current mortality rates, 50% pension, 1960-2000 risky returns with semi-perfect annuities market, time preference -1.5% and elasticity of substitution 0.33

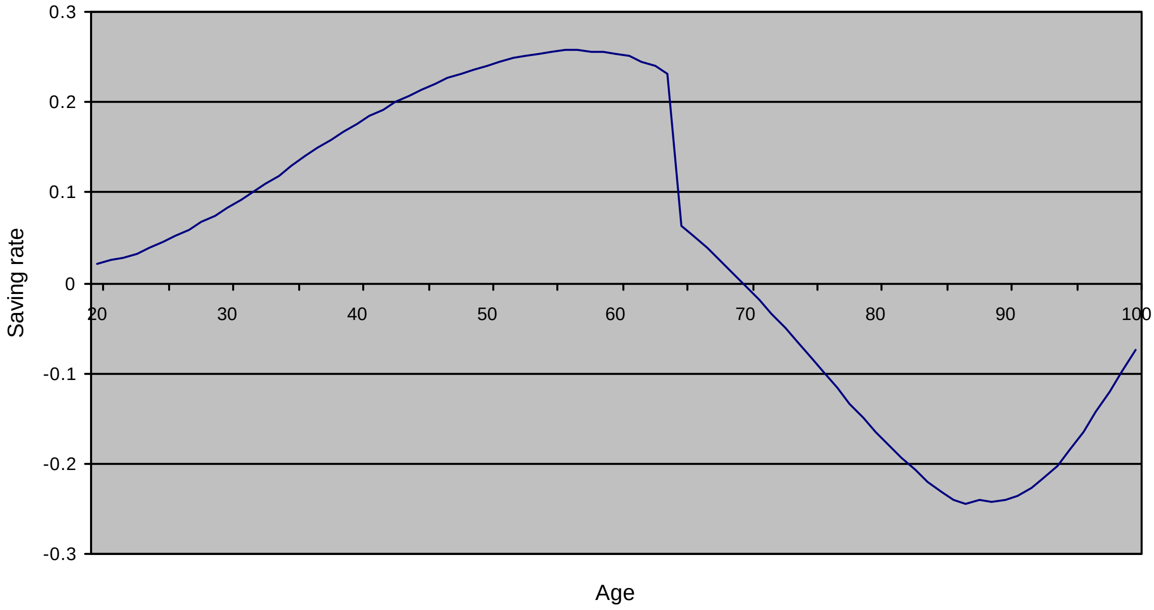

Figure 10: proportion of risky investment over life cycle assuming current mortality rates, 50% pension, 1960-2000 risky returns with semi-perfect annuities market, time preference -1.5% and elasticity of substitution 0.33