Dependence in Credit Default Swap and Equity Markets: Dynamic Copula with Markov-Switching

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Abstract

Theoretical credit risk models à la Merton (1974) predict a non-linear negative link between a firm's default likelihood and asset value. This motivates us to propose a flexible empirical Markov-switching bivariate copula that allows for distinct time-varying dependence between credit default swap (CDS) spreads and equity prices in "crisis" and "tranquil" periods. The model identifies high dependence regimes that coincide with the recent credit crunch and the European sovereign debt crises, and is supported by in-sample goodness of fit criteria versus nested copula models that impose within-regime constant dependence or no regime-switching. Value at Risk forecasts to set day-ahead trading limits for hedging CDS-equity portfolios reveal the economic relevance of the model from the viewpoint of both regulatory and asymmetric piecewise linear loss functions.

JEL classifications: C13; C41; G21; G28.

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1 Introduction

Appropriately modeling the dependence structure of credit portfolios and systematic risk factors is important for risk managers in order to set trading limits and for traders in order to hedge the market risk of their credit positions and for pricing credit derivatives. In particular, the use of models that acknowledge shifts in the relationship between financial institutions' credit exposures and the underlying equity market can be beneficial towards the design of more adequate regulatory frameworks and reduce systemic risks during stressed market conditions. Merton's (1974) theory suggests a link between credit derivative prices and equity prices. Firm-value structural models originating from Merton's theoretical framework rest on the fundamental asset value process, namely, a firm default likelihood and the price of its debt are functions of the firm's asset value and the level of debt. As asset value and its volatility are latent, the implementation of structural credit risk models for publicly-traded firms relies on the observable equity return and a volatility proxy, while the credit default swap (CDS) spread can be taken as a measure of firm default risk.

CDS spreads can be argued to provide more reliable signals on the default riskiness of corporate borrowers than bond spreads as bond prices are often distorted by tax and liquidity issues. The perception of the CDS premium as a rather "direct" measure of default risk together with the rapid development of the CDS market have spurred an enthusiastic debate over the determinants of CDS spreads and, in particular, their sensitivity to structural factors such as equity returns and volatility, macrovariables, firm-specific balance sheet information and credit ratings.[1] Norden and Weber (2009) investigate the link between changes in CDS spreads and stock returns, while Madan and Unal (2000), Blanco et al. (2005), and Zhang et al. (2009) also consider stock return volatility. Ericsson et al. (2004) find that volatility and leverage alone explain a substantial proportion of the variation in CDS premia. Yu (2006) is the first to document shifts between "turbulent" and "calm" regimes in the dynamics of CDS spreads. A common denominator to the above studies is that they focus on the determinants of single-name CDS spreads. The launch of broad-based CDS indices in 2001 by JP Morgan and Morgan Stanley marks a new era in credit derivatives trading by offering more liquidity, tradeability and transparency; unlike single-name CDSs that are traded over the counter, CDS indices are highly standardized and actively traded in the open market. However, research into the dependence structure dynamics between CDS index spreads and equity market indicators is still sparse. Bystrom (2008) finds that stock returns and stock market volatility are able to explain most of the variation

[1] CDS contracts are designed to protect bondholders against default of the reference entity in a way similar to traditional insurance policies. The CDS market has been criticized, however, as providing a false sense of security to debt holders that contributed to the 2008 Financial Crisis and the Greek debt crisis. In response to this critique, the 2009 Dodd-Frank Act required the Commodity Futures Trading Commission (CFTC) to regulate swaps. In the European sovereign bond market the Collective Action Clause (CAC), which allows the majority of bondholders to agree to a debt restructuring, was introduced to provide an additional layer of protection for bonds issued by Eurozone members. In practice, the CAC introduces a degree of uncertainty in the size of the payout to CDS holders since the payout will decrease if the post-restructuring bond value falls. This extra dimension of risk is discussed in the Wall Street Journal article by Charles Forelle (see http://www.wsj.com/articles).

in iTraxx CDS spreads. Using Markov-switching regressions, Alexander and Kaeck (2008) show that the determinants of CDS index spreads are regime-specific; implied volatility is strongly related to CDS spreads in the high volatility regime while stock returns play a bigger role in the tranquil regime.

While all of the aforementioned empirical studies implicitly rely on the conventional Pearson's correlation ρ as (linear) dependence measure, firm structural models inspired from Merton (1974) suggest that the marginal effect of a fall in equity value is non-constant (as linear approaches would predict) but instead driven by firm fundamentals such as leverage. Using an extension of Merton's model with realized volatility and jumps, Zhang et al. (2009) provide evidence that the strength of the relation between credit risk and equity value depends on the firm's credit rating. Empirical studies have consistently suggested that credit spread predictions obtained from Merton-type structural credit risk models underestimate historical credit spreads; *e.g.*, Jones et al. (1984), and Eom et al. (2004). This may partly stem from the fact that the actual dependence structure of debt with equity has complex features that linear correlation models fail to capture. Recent work supports this conjecture. Hull et al. (2004) show that theoretical CDS spreads implied from Merton's model using equity value and volatility as inputs are non-linearly related to historical CDS spreads. Using adaptive non-parametric regressions, Giammarino and Barrieu (2009) provide evidence that the relationship between iTraxx Europe CDS index returns and two systematic factors, Euro Stoxx 50 returns and changes in the VStoxx 50 volatility index, suffered structural changes between 2004 and 2008.

Our paper extends recent research on the non-linear relation between credit spreads and tradeable systematic risk factors by adopting copulae which represent a very versatile framework to estimate multivariate distributions. The main appeal of the copula framework is that it facilitates separate modeling of the marginal distributions and the dependence and thus, a variety of dependence structures can be captured with more flexibility and parsimony than in competing frameworks (*e.g.*, multivariate GARCH). Patton (2006) introduces conditional or *dynamic* copulae to capture time-varying dependence structures which represent an important improvement upon *static* copulae. The dynamic copula framework is extended by Christoffersen et al. (2012) in order to accommodate asymmetries and trends in time-varying cross-market dependence. Far less attention has been paid to the possibility of regime-switching (RS) effects; to our knowledge, the only empirical investigations that do so are those by Chollete et al. (2009), Okimoto (2008) and Rodriguez (2007) for international equity markets, Garcia and Tsafack (2011) for bond and equity markets, and Stöber and Czado (2013) for foreign exchange markets. Existing RS copula models have the limitation of assuming within-regime constant dependence given that a latent economic or financial state could linger on for years.

This paper provides both methodological and empirical contributions to the literature. On the former, we propose flexible *Markov-switching dynamic* (autoregressive) copulae which capture asymmetry in the form of high or 'crisis' dependence and low or 'tranquil' dependence. Our models generalize existing Markov-switching static copulae by allowing for distinct mean-reversion in dependence within each regime. We seek to provide empirical insights on the dependence structure between credit and equity markets; namely, we jointly model the European credit market, proxied by the iTraxx CDS index and the underlying systematic equity return factor proxied by the Stoxx index in the aggregate European stock market, and two sectors that had been at the 'epicenter' of the late 2000s financial crisis: Automotive and Subordinated Financial. We carry out an in-sample statistical comparison of various copula models and make inferences on cross-market dependence at the center and tails of the bivariate distribution. Given that CDS indices have become a very important instrument for risk hedging and arbitrage trading and therefore, a key component of institutional investors' portfolios, we assess the relevance of the proposed Markov-switching dynamic copulae in the context of CDS-equity portfolios from a risk management perspective. The economic merit of the competing copula is assessed through a Value at Risk (VaR) forecasting exercise to set daily trading limits for CDS-equity portfolios. These portfolios can be rationalized in light of the theoretical (*i.e.*, Merton's (1974) model) and well-documented negative dependence between corporate credit default risk and equity value; thus, for instance, the CDS index can be included in a well-diversified stock portfolio to hedge equity market risk.

We document various sudden changes in the dependence structure of CDS-equity markets over the period from September 2005 to March 2011. The transitions to the *high* dependence regime largely reflect the onset of the automotive industry and energy crises in 2005, the credit crunch in 2007 and the European sovereign debt crises in 2009. The signal-to-noise ratio for the identification of the dependence regimes is higher at sectoral than marketwide level. *Markov-switching dynamic* copula models are supported over nested copulae not only by conventional in-sample statistical criteria but also by loss functions that measure the accuracy of out-of-sample VaR forecasts for CDS-equity portfolios. Using both regulatory loss functions and the quantile-tailored 'tick' loss function, the VaR simulation highlights the economic relevance of the proposed copula by showing that it suggests more cautious 1-day-ahead trading limits. A mismatch is documented between in-sample statistical fit and economic value of predictability regarding the choice of specific copula function; log-likelihood values and Akaike Information Criteria support the Student's t copula but lower average losses are associated to the VaR forecasts from the asymmetrically-tailed Gumbel copula.

One of the reforms put forward by the Basel Committee on Banking Supervision (2011) is about strengthening capital requirements for credit exposures arising from banks credit derivatives such as CDS positions, and introducing stressed-VaR capital requirements for the trading book. Our study suggests that copula models that explicitly parameterize sudden shifts in the dynamic (mean-reverting) dependence structure between credit exposures and the equity market facilitate more conservative downside-risk measures. Thus, the proposed copula framework is useful towards the Basel macroprudential goal of making the banking sector more resilient through appropriate stress testing and systemic risk measurement.

The rest of this paper is organized as follows. Section 2 outlines the methodology and Section 3 describes the CS and equity market data. Section 4 discusses the inferences on CDS-equity dependence, and provides an in-sample statistical comparison of various copula formulations together with an economic comparison based on Value at Risk forecasts. Section 5 concludes with a summary and directions for further research.

2 Modeling framework

Our interest is to demonstrate that jointly modeling the behavior of daily changes in CDS and equity prices using a flexible copula approach that accommodates time-variation in a crisis regime and in a normal regime is statistically and economically meaningful. This section presents the Markov-switching copula model proposed as a generalization of existing copula models.

2.1 Marginal processes and copula functions

The daily returns or logarithmic changes in equity prices and CDS spreads, both denoted r_t, are modeled as

$$r_t = a_0 + \sum_{i=1}^{p} a_i r_{t-i} + \sum_{j=1}^{q} b_j \varepsilon_{t-j} + \varepsilon_t, \tag{1}$$

$$\sigma_t^2 = c_0 + c_1 \sigma_{t-1}^2 + d_1 \varepsilon_{t-1}^2, \tag{2}$$

where the filtered returns $x_t = \varepsilon_t / \sigma_t$, $t = 1, ..., T$, are assumed $x_t \overset{i.i.d.}{\sim} skT(0, 1; \nu, \zeta)$, with $\nu > 2$ and ζ denoting the degrees of freedom (dof) and asymmetry parameters of the distribution proposed by Hansen (1994) which nests the Student's t ($\zeta = 0$) and the Gaussian ($\zeta = 0$, $\nu \to \infty$) distribution. The parameters of this ARMA(p, q)-GARCH(1,1)-skT model are estimated by maximum likelihood (ML). We fix at one the lag orders in the conditional variance equation (2) as this is the most-widely used specification to capture the conditional heteroskedasticity of financial asset returns; see Giacomini and Komunjer (2005), Jondeau and Rockinger (2006), Kuester et al. (2006) and Chollete et al. (2009) inter alia. The optimal AR and MA lag order combination, p and q up to $1, 2, \ldots, 5$ days, is selected using the Akaike Information Criterion (AIC). Henceforth, we will employ the subscript n to distinguish the two return processes under study.

The joint bivariate probability density function (abbreviated to *pdf*, hereafter) of the filtered CDS and equity returns can be formulated in a copula framework as follows

$$\boldsymbol{f}(x_1, x_2) = \boldsymbol{c}(u_1, u_2) \times \prod_{n=1}^{2} f_n(x_n), \tag{3}$$

where $\{x_n \coloneqq \frac{\varepsilon_{t,n}}{\sigma_{t,n}}\}, t = 1, ..., T$, denotes the vector of filtered returns for each of the two processes $n = 1, 2$,

$\boldsymbol{c}(\cdot)$ is a parametric copula function, and $f_n(x_n), n = 1, 2$ are the univariate *pdf*s (or *margins*) of two $Uniform(0, 1)$ variables obtained through the probability integral transform, $u_n = F_n(x_n)$, with $F_n(\cdot)$ the skewed Student's t cumulative distribution function. Let $(\boldsymbol{\phi}_1, \boldsymbol{\phi}_2)'$ and $\boldsymbol{\theta}$ denote the parameters of the margins and the copula, respectively. By conveniently decomposing the log-likelihood function as the log-likelihood of the margins and the log-likelihood of the copula, the parameters can be conveniently estimated in two stages; first, estimate the parameters of the two margins and, second, the copula parameters conditional on the margins. Formally, the log-likelihood function can be expressed as

$$\begin{aligned}\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\phi}) &= \sum_{n=1}^{2} \sum_{t=1}^{T} \log f_{n,t}(x_{n,t}; \boldsymbol{\phi}_n) + \sum_{t=1}^{T} \log \boldsymbol{c}(u_{1,t}, u_{2,t}; \boldsymbol{\theta}) \\ &= \sum_{n=1}^{2} \mathcal{L}_n(\boldsymbol{\phi}_n) + \mathcal{L}_{\boldsymbol{C}}(\boldsymbol{\phi}_1, \boldsymbol{\phi}_2; \boldsymbol{\theta}), \end{aligned} \tag{4}$$

and the corresponding two-step ML estimator of the copula parameters is asymptotically normal and consistent but not efficient. Simulations in Joe (2005) and Patton (2006) suggest that the efficiency loss is generally small in practice. The computational advantage of the two-step ML estimation approach makes it especially convenient for the comparison of copula with the same set of univariate margins. For more details on copula theory and financial applications, see Nelsen (2006), Cherubini et al. (2004) and Patton (2013).

The bivariate copula functions $c(\cdot)$ considered in this paper are elliptical/symmetric (*e.g.,* Student's t) or Archimedean (*e.g.,* Gumbel). Unlike the elliptical Gaussian copula which is solely parameterized by the linear Pearson's correlation ρ, the Student's t copula can capture extreme return comovements (or clustering) via ρ and the dof parameter ν; the smaller ν, the more prominent the tail dependence or clustering of extreme returns. Archimedean copulae can capture asymmetric tail dependence. Gumbel copula describes upper tail dependence but by 180° rotation it models the lower tail. A summary of these copula functions is provided in the on-line addendum (Section A) to the paper; for further details, see Cherubini et al. (2004).

2.2 Dynamic copula model

In a dynamic context the dependence structure is modeled conditionally and so the implied *rank correlation* and *tail dependence* measures are time-varying. Patton (2006) sets the foundations for time-varying copulae by proving Sklar's theorem for conditional distributions, and proposes the ARMA$(1, m)$ dependence structure

$$\gamma_t = \Lambda(\omega + \varphi\gamma_{t-1} + \psi\Gamma_t), \tag{5}$$

which permits mean-reversion in γ_t, the dependence measure of interest. The underlying constant copula parameters are collected in the vector $\boldsymbol{\theta} = (\omega, \varphi, \psi)'$ and the forcing variable Γ_t is defined as

$$\Gamma_t = \begin{cases} \frac{1}{m}\sum_{j=1}^{m} F_1^{-1}\left(u_{1,t-j}\right) F_2^{-1}\left(u_{2,t-j}\right) & \textit{elliptical} \\ \frac{1}{m}\sum_{j=1}^{m} \mid u_{1,t-j} - u_{2,t-j} \mid & \textit{Archimedean} \end{cases} \tag{6}$$

where $F_n^{-1}(u_{n,t})$, $n = 1, 2$ is the inverse *cdf* of the margins. As in Patton (2006), we use $m = 10$; Γ_t captures any variation in dependence and concordance for elliptical and Archimedean copulae respectively. In the context of elliptical copulae, the pertinent dependence measure is the conventional *correlation*, $\gamma_t = \rho_t$, and $\Lambda(y) = (1 - e^{-y})(1 + e^{-y})^{-1}$ is the modified logistic transformation to ensure $\rho_t \in (-1, 1)$. In the Gumbel copula $\gamma_t = \eta_t$ and $\Lambda(y) = 1 + e^y$ to ensure $\eta_t \in (1\infty)$. The estimated γ_t can be mapped into the time-varying *rank correlation* and *tail dependence* measures, τ_t and λ_t; see formulae in the on-line addendum (Section A).

The dynamic conditional correlation (DCC) model of Engle (2002) inspired the copula formulation

$$\begin{aligned} \boldsymbol{Q}_t &= (1 - \varphi - \psi)\,\bar{\boldsymbol{Q}} + \varphi \boldsymbol{Q}_{t-1} + \psi \boldsymbol{\epsilon}_{t-1} \cdot \boldsymbol{\epsilon}'_{t-1}, \quad \varphi + \psi < 1;\ \varphi, \psi \in (0, 1)\,, \\ \boldsymbol{R}_t &= \widetilde{\boldsymbol{Q}}_t^{-1} \boldsymbol{Q}_t \widetilde{\boldsymbol{Q}}_t^{-1}, \end{aligned} \tag{7}$$

where $\bar{\boldsymbol{Q}}$ is the 2×2 unconditional covariance of $\boldsymbol{\epsilon}_t = (\epsilon_{1,t}, \epsilon_{2,t})'$ estimated as $\bar{\boldsymbol{Q}} = T^{-1}\sum_{t=1}^{T} \boldsymbol{\epsilon}_t \boldsymbol{\epsilon}'_t$ with $\epsilon_{n,t} \equiv F_n^{-1}(u_{1n,t})$, $n = 1, 2$; $\mathbf{Q}_t$ is the conditional covariance matrix, $\widetilde{\mathbf{Q}}_t$ is a diagonal matrix with elements the square root of $diag(\mathbf{Q}_t)$, and $\mathbf{R}_t$ is a correlation matrix with off-diagonal element ρ_t.

ARMA and DCC copula have in common the "autoregressive" dependence structure (via φ and ψ), and the nesting of *static* copulae under the restriction $\varphi = \psi = 0$. However, the DCC copula is not straightforward to apply to non-elliptical copulae whereas it can be easily extended to multivariate contexts which is rather challenging with the ARMA copula; see Manner and Reznikova (2012) for further discussion.

2.3 Regime-switching copula model

We propose flexible *Markov-switching dynamic* copula models with two dependence states, *low* or 'tranquil' versus *high* or 'crisis', and time-variation (mean-reverting dynamics) in each. The within-regime dynamics aspect distinguishes them from typical regime-switching models where a *static* copula function governs each regime; the latter approach is unrealistic in finance because a given state may linger for months or years.

So as to outline our regime-switching copula framework, let S_t be a state variable that dictates the regime at time t. The joint *pdf* of the filtered returns x_{1t} and x_{2t} conditional on being in regime s is given by

$$\boldsymbol{f}\left(x_{1t}, x_{2t} \mid I_{t-1}; S_t = s\right) = \boldsymbol{c}^{S_t}\left(u_{1t}, u_{2t} \mid \boldsymbol{\theta}^{S_t}\right) \times \prod_{n=1}^{2} f_n\left(x_n\right), \tag{8}$$

with $s \in \{H, L\}$; H denotes the *high* dependence regime and L the *low* dependence regime, $\boldsymbol{c}_t^{S_t}$ the regime-switching copula function, and I_{t-1} the available information set at time $t-1$. The state variable S_t follows an order-one Markov chain parameterized by the transition probability matrix

$$\boldsymbol{\pi} = \begin{pmatrix} \pi_{HH} & \pi_{HL} \\ \pi_{LH} & \pi_{LL} \end{pmatrix}, \tag{9}$$

where π_{HH} is the probability of being in the *high* dependence regime at time t conditional on being in the same regime at $t-1$; π_{LL} is similarly defined for the *low* dependence regime.

First, we propose a regime-switching ARMA (RS-ARMA) copula formulation with dependence structure

$$\gamma_t^{S_t} = \Lambda\left(\omega^{S_t} + \varphi^{S_t}\gamma_{t-1}^{S_{t-1}} + \psi^{S_t}\Gamma_t\right), \tag{10}$$

where Γ_t and $\Lambda(\cdot)$ are as defined in Section 2.2. When the underlying copula function is, say, a Student's t, the unknown parameter vector is $\boldsymbol{\theta} = (\omega_H, \omega_L, \varphi_H, \varphi_L, \psi_H, \psi_L, \pi_{HH}, \pi_{LL}; \nu_H, \nu_L)'$. We call this an RS-ARMA copula model to distinguish it from nested RS models with constant within-regime dependence structure.

The Markov-switching dynamic framework suggested allows for the dof parameter to depend on the Markovian state, that is, ν^{S_t}, which accommodates the possibility of Gaussian (no-tail) dependence when $\nu^{S_t} \to \infty$ in one regime and tail dependence in the other. Extant studies typically model the "tranquil" dependence regime using Gaussian copula and the "crisis" dependence regime using non-Gaussian copula; see, *e.g.* Rodriguez (2007), Okimoto (2008), Chollete et al. (2009) and Garcia and Tsafack (2011). Our framework can be generalized further by allowing the underlying copula function to switch from elliptical to Archimedean across regimes, and the marginal distributions to switch across regimes too; we do not pursue these extensions here to avoid complex models that would entail a more challenging numerical maximization of the likelihood function. The main goal of our paper is to demonstrate that extending existing regime-switching copula models to accommodate distinct within-regime dynamic dependence (i.e., distinct degrees of mean reversion in dependence) is economically relevant from a risk management perspective.

Second, in a similar spirit we propose a regime-switching DCC (RS-DCC) copula formulation where each regime is governed by a distinct DCC type copula

$$\mathbf{Q}_t^{S_t} = \left(1 - \varphi^{S_t} - \psi^{S_t}\right) \bar{\mathbf{Q}} + \varphi^{S_t} \mathbf{Q}_{t-1}^{S_{t-1}} + \psi^{S_t} \epsilon_{t-1} \cdot \epsilon_{t-1}', \quad \varphi^{S_t} + \psi^{S_t} < 1;\ \varphi^{S_t}, \psi^{S_t} \in (0,1) \tag{11}$$
$$\mathbf{R}_t^{S_t} = \left(\widetilde{\mathbf{Q}}_t^{S_t}\right)^{-1} \mathbf{Q}_t \left(\widetilde{\mathbf{Q}}_t^{S_t}\right)^{-1},$$

with $\mathbf{Q}_t^{S_t}$ the auxiliary matrix driving the dependence. When the RS-DCC is formulated upon, say, the Student's t copula function, the unknown parameter vector is $\boldsymbol{\theta} = (\varphi_H, \varphi_L, \psi_H, \psi_L, \pi_{HH}, \pi_{LL}; \nu_H, \nu_L)'$.

Both the RS-ARMA and RS-DCC formulations portray a switching "autoregressive" dependence structure; namely, the degree of mean-reversion in dependence and its long-run equilibrium level (or attractor) are regime-specific. They nest simpler copula models. If there is no regime-switching ($\gamma_t^{S_t}=\gamma_t$) they collapse to the ARMA and DCC copulae. If there is no within-regime time variation ($\gamma_t^{S_t}=\gamma^{S_t}$) the conventional RS copula emerges. Finally, if there is no time variation at all ($\gamma_t^{S_t} = \gamma$) they both become the static copula.

Estimation of the RS copula parameters requires inferences on the probabilistic evolution of the state variable S_t. Probability estimates based on information up to time t are called "filtered probabilities" and those based on full-sample information are "smoothed probabilities". Our estimation approach builds on Hamilton's (1989) *filtering* algorithm and Kim's (1994) *smoothing* algorithm; see Appendix A for details.

3 Data and marginal distributions

The data are daily closing CDS spread quotes at 5-year maturity from *Bloomberg* on the iTraxx Europe, iTraxx Europe Autos, and iTraxx Europe Subordinated Financials indices, and daily closing prices on the Dow Jones Stoxx Europe 600 index, the Stoxx Europe 600 Financials index and the Stoxx Europe 600 Automobiles & Parts index from September 9, 2005 to March 11, 2011 ($T = 1,380$ days). We focus on the cost of insuring against default on automotive companies' debt as this sector was badly hit by the recent financial crisis; see crisis timeline in Appendix B, and further details on the sample in the on-line addendum (Section C).

Like Cathcart et al. (2013), Alexander and Kaeck (2008), and Bystrom (2008) we model the log change in the CDS indices which represents the "return" from speculating that the cost of default protection will change; see also Markit (2010). Figure 1 plots daily index levels (Panel A) and log changes (Panel B).

[Insert Figure 1 around here]

The plots illustrate that, as expected according to theory, CDS indices and equity indices move in opposite directions. September 2007 marks the start of a steady downward trend in equity prices, attaining the lowest level in 2009, coupled with a steady rise in default risk premiums. Table 1 provides summary statistics.

[Insert Table 1 around here]

CDS Financial has the highest mean return. Both Figure 1 and Table 1 reveal that CDS indices are more volatile than equity indices; CDS Auto is by and large the most volatile. The Jarque-Bera test confirms the stylized non-Gaussianity of daily returns. The Ljung-Box (LB) test and Engle's ARCH LM test suggest, respectively, serial dependence and heteroskedasticity. Both the Pearson's product-moment correlation ρ and Kendall's rank correlation τ confirm the stylized negative association between CDS returns and equity returns, in line with the Merton (1974) model prediction that growth in firm value reduces the probability of default. See the on-line addendum (Section B) for a formal definition of these dependence measures. The weakest correlation is observed in the automotive sector. As detailed in Section C of the on-line addendum, the Stoxx Europe 600 Automobiles & Parts index contains 9 out of the 10 companies in the iTraxx Auto index which are relatively large and well-established car manufacturers with high credit ratings. Our finding of a relatively weak correlation between the CDS and equity market in the automotive sector aligns well with the evidence in Zhang et al. (2009) which suggests that the credit spreads of low credit rating firms respond more dramatically to deteriorating equity market conditions. This is consistent with the wisdom from extant credit risk structural models. As dictated by the Merton (1974) model, for instance, the returns of debt claims and stocks should be correlated, especially, for high default risk levels. This is because the value of debt becomes more sensitive to changes in asset value the higher the probability of the firm's financial distress. In this sense, the relatively weaker CDS-equity correlation observed in the Auto sector is neither surprising in the light of existing theory nor a new empirical finding.

Table 2 reports ML estimates for the conditional marginal distributions of the daily log index changes.

[Insert Table 2 around here]

The dof parameter ν of the skT density is relatively small and suggests that CDS index returns have fatter tails than Stoxx index returns, while the asymmetry parameter ζ is significant in three cases: CDS Financial, equity Europe and equity Auto.[2] As diagnostic checks for the margins, in Panel B of the table we report p-values of the serial independence LB test on the first four moments of the estimated probability integral transformations, $(\hat{u}_t - \bar{u})^j$, $j \in \{1, 2, 3, 4\}$ where $\hat{u}_t = F(\hat{x}_t)$, $t = 1, ..., T$, and the Kolmogorov-Smirnov (KS)

[2]The sum of the (G)ARCH coefficients reported in Table 2 is indistinguishable from unity in all six cases (CDS and equity returns) suggesting that the model is not covariance stationary. In this scenario, the variance forecasts may increase without bound with the horizon, and the forecast uncertainty increases also without bound. However, in the present application the horizon is a very short one-day ahead and hence, this non-stationarity effect ought to be negligible. More generally, Kleibergen and Van Dijk (1993) theoretically show that even in the case of a non-stationary conditional variance process under certain conditions the probability of a decreasing variance in the next period may exceed the probability of an increasing variance in which case the shocks to the variance are not likely to persist for long.

test for the null hypothesis that the transforms are $Uniform(0,1)$ or, equivalently, that the standardized innovations are skT distributed. Following Genest and Rémillard (2008) and Patton (2013) the test p-values are obtained by bootstrap simulation to account for parameter estimation error. Artificial (bootstrap) series of CDS and equity returns, $\{(r^*_{t,CDS})\}_{t=1}^T$ and $\{(r^*_{t,equity})\}_{t=1}^T$, are constructed by putting together the independent errors randomly drawn with replacement from the empirical distribution of filtered returns or residuals,$\{\hat{x}_t\}$, and the ARMA-GARCH parameters estimated from the actual data. The marginal models are re-estimated on each of the M simulated time-series ($M = 1,000$ replications) which facilitates the empirical distribution of the test statistics. The validity of this semi-parametric bootstrap approach for goodness-of-fit testing is established by Genest and Rémillard (2008). The simulated p-values indicate that the filtered returns are i.i.d. and skT distributed. We also assessed spillover effects through eq. (1) augmented with cross-variable lags; that is, $r_t^{equity} = a_0 + \sum_{i=1}^p a_i^{equity} r_{t-i}^{equity} + \sum_{i=1}^k a_i^{CDS} r_{t-i}^{CDS} + \sum_{j=1}^q b_j \varepsilon_{t-j} + \varepsilon_t$ to assess spillovers from the CDS to the equity market and a counterpart equation for r_t^{CDS} to assess the reverse effect. The results provided in the on-line addendum (Section D) suggest no spillovers and thus, they further validate eqs. (1)–(2) as reasonably good models of the conditional marginal distributions.

4 Empirical analysis

4.1 In-sample fit of static, dynamic and regime-switching copulae

This section begins with a preliminary discussion of the ability of competing formulations of the Student's t and Gumbel copula to predict in-sample the dependence between CDS and equity markets. We discuss the 90° anticlockwise-rotated Gumbel copula which focuses on the "adverse" tail for the negatively correlated pair of variables at hand (*i.e.*, rising CDS and falling equity); unreported goodness-of-fit measures of the 270° rotated Gumbel copula that captures the "favorable" tail are clearly inferior which represents evidence of asymmetric dependence. We also considered the 90° anticlockwise rotated symmetrized Joe-Clayton (SJC) copula and observe that its fit is generally inferior and so we do not discuss it further; the results are provided in on-line addendum (Section E). Since the findings from the (RS-)ARMA and (RS-)DCC formulations are similar, to preserve space in the manuscript the estimation results for the latter are also gathered in the on-line addendum (Section F). Table 3 shows the copula models' AIC and maximized log-likelihood (LL).

[Insert Table 3 around here]

Student's t copulae, which account for tail dependence in a symmetric way, attain lower AIC and higher LL than the competing asymmetric Gumbel copula, irrespective of the formulation employed. On this basis and to ease the exposition, a large part of the subsequent discussion in this section focuses on inferences from the Student's t copula. The *dynamic* formulation (ARMA or DCC) clearly provides better in-sample fit than the *static* formulation, irrespective of the underlying copula function employed. The *regime-switching* (RS) feature also enhances the static copula's ability to describe the dependence structure of CDS-equity markets. However, allowing for distinct Student's t copulae in each regime by making the dof parameter regime-dependent, ν^{S_t}, entails no improvement as can be seen from comparing the goodness-of-fit of the RS copula (static and ARMA) and their parsimonious versions which restrict $\nu^{S_t} = \nu$. Overall, accommodating time-varying dependence structure within each regime leads to the lowest AIC and largest LL. The best data fit is achieved by the parsimonious RS-ARMA Student's t model, eq. (10), with parameter vector $\boldsymbol{\theta} = (\omega_H, \omega_L, \varphi, \psi, \pi_{HH}, \pi_{LL}; \nu)'$ such that the dof parameter and mean-reversion in dependence pattern are identical across regimes but the long-run equilibrium (or "attractor") is regime-specific. In order to simplify the exposition, hereafter we focus on this parsimonious RS-ARMA formulation.

Table 4 reports parameter estimates of competing Student's t copula formulations. The reported standard errors for the copula parameters are based on the "sandwich form" asymptotic covariance matrix that takes into account the estimation error from two-step ML; detailed formulae are provided in Patton (2013; p.922).

[Insert Table 4 around here]

The correlation parameter ρ of the static copula suggests significantly negative dependence for all pairs; in line with the Merton (1974) theory, a firm's likelihood of default is a decreasing function of asset value proxied by the market value of its equity. Moreover, the significant parameters φ and ψ in the dynamic ARMA formulations confirm that the dependence structure is indeed time-varying. Turning attention to the RS-ARMA model reported in Table 4, the estimated probabilities π_{HH} and π_{LL} plausibly suggest longer duration for the "tranquil" or *low* dependence regime. The statistical significance of the dependence regimes can be tested by means of a LR test for the null hypothesis $H_0 : \omega_H = \omega_L$; under this restriction the model becomes an ARMA copula. A similar test for the null hypothesis $H_0 : \rho_H = \rho_L$ is deployed with the conventional RS copula; under this restriction this copula becomes the *static* copula. The 'nuisance parameter' problem (unidentified parameters under H_0) invalidates standard asymptotic theory for these LR tests; however, the p-values are very small, all below 0.008, providing *prima facie* evidence of regime-switching.

Figure 2 plots the smoothed probabilities of the *high* dependence regime in the RS-ARMA copula.

[Insert Figure 2 around here]

In both the Auto and Financial sectors, the dependence between CDS and equity markets enters a *high* or "crash" regime by late 2007 reflecting the onset of the credit crunch, and lingers on for about a year.

Although the global credit crisis originated from the huge losses of subprime CDS investment in the financial sector, the automotive industry was badly by hit various iliquidity shocks, a sharp fall in consumer confidence and soaring oil prices. After a short pause, both sectors re-enter the *high* dependence regime by late 2009 when the European sovereign debt crisis erupts. These findings confirm the break date set *a priori* by Fung et al. (2008) in their analysis of the US CDS-equity market dependence where a linear vector autoregressive (VAR) model is estimated separately in each of the two sub-periods. A notable advantage of our modeling framework, relative to the *ex post* (break date) selection approach in Fung et al. (2008), is its forecasting applicability given that that the switching mechanism is endogenized. Moreover, a linear VAR model is not be able to capture tail dependence, neither symmetric nor asymmetric, which is another useful feature of the proposed copula model as discussed below in the context of Figure 3.

Four transitions into the *high* dependence regime are identified in Figure 2 for the Auto and Financial CDS-equity pairs. One in 2005 roughly coinciding with the downgrade of two systemically relevant firms in the auto industry (Ford and GM), another in 2006 reflecting the deterioration of the US housing market, a third one in 2007 reflecting the credit crunch, and a fourth entry in 2009 concurrent with the Greek debt crisis. The less successful regime identification achieved in the market wide CDS-equity models reflects that the signal-to-noise ratio is reduced by pooling entities from various sectors with different transition timings.

The corresponding Kendall's *rank correlation* τ and *tail dependence* λ measures are plotted in Figure 3.

[Insert Figure 3 around here]

Several observations can be made. First, the dependence structure is time-varying as suggested earlier by the significance of the parameters driving the (RS-)ARMA dynamics. Second, the RS and RS-ARMA copulae reveal notable upward shifts in dependence between CDS and equity markets at plausible time points. For instance, the sudden downgrade by S&P's of two important car manufacturers, GM and Ford, from BBB to BB in May 2005 and to B in December 2005 led to a dramatic increase in dependence for the Auto CDS-equity pair which the non-regime-switching ARMA copula tends to smooth out. Crude oil prices reached historically high levels of over $77 per barrel in July 2006 which pushed the Auto CDS-equity dependence to a *high* regime; again this pattern is better captured by the RS-ARMA copula. For the Financial CDS-equity pair, the most dramatic increase in dependence roughly occurs in late 2009 when a credit rating downgrade from A- to BBB+ is announced by the Standard & Poor's credit rating agency for Greece.

We can see evidence of two *tail dependence* regimes which reflect the presence of two distinct CDS-equity bivariate distributions pertaining, respectively, to "normal" and "crisis" episodes. The finding is quite intuitive since it is well-known that due, for instance, to latent market sentiments of panic and fear, the likelihood of joint extreme events in the CDS market and equity market is stronger in crises than in normal periods; tail

CDS-equity dependence exacerbates during periods of market stress. While the tail dependence estimates may seem small, they are broadly aligned with those in Garcia and Tsafack (2011) for European equity-bond pairs and with those in Jondeau and Rockinger (2006) for cross-country equity market pairs. The *high* tail dependence regime is most apparent for the Financial CDS-equity pair which confirms that financial firms are particularly sensitive to extreme bad news in crisis. Regardless of the level of tail dependence, the graphs endorse the RS-ARMA copula as very useful for capturing sudden shifts in tail dependence and confirm the biases arising from the use of non-regime-switching copula models; namely, by smoothing the degree of dependence, these models tend to overestimate the dependence in normal periods and underestimate it during crises. The upshot is that employing an implausible model of market dependence that does not permit distinct regimes or that constrains the within-regime dependence structure to be static can be costly.

4.2 Out-of-sample copula forecasts for risk management

The economic value of the proposed *regime-switching dynamic* copulae is demonstrated via a Monte Carlo simulation to set 1-day-ahead Value at Risk (VaR) trading limits for portfolios of equity and CDS instruments. Since the 1996 Market Risk Amendment (MRA) to the Basel Accord, the VaR measure has played a central role in regulatory capital assessments and remains one of the most common portfolio risk control tools in banks and insurance firms. The MRA stipulates that banks should internally compute VaR on a daily basis for backtesting purposes although regulators require 10-day-ahead VaR to be reported for establishing the minimum capital requirement, possibly to mitigate the costs of too frequent monitoring.

The 1-day-ahead VaR is an α-quantile prediction of the future portfolio profit and loss (P/L) distribution. It provides a measure of the maximum future losses over a time span $[t, t+1]$, which can be formalized as

$$\mathrm{P}\left[R_{t+1} \leqslant VaR_{t+1}^{\alpha} | I_t\right] = \alpha, \tag{12}$$

where R_{t+1} denotes the portfolio return on day $t+1$, and I_t is the information set available on day t. The nominal coverage $0 < \alpha < 1$ is typically set at 0.01 or 0.05 for long trading positions (*i.e.*, left tail) meaning that the risk manager seeks a high degree of statistical confidence, 99% and 95%, respectively, that the portfolio loss on trading day $t+1$ will not exceed the VaR extracted from information up to day t.

VaR can be estimated using various methods such as non-parametric (simulation), semi-parametric (CAViaR), fully parametric (location-scale) and optimal combinations thereof; see, *e.g.*, Kuester et al. (2006) and Fuertes and Olmo (2013). Large banks and financial institutions utilize multivariate VaR models for capturing the asset dependence structure of their trading portfolios. We carry out a Monte Carlo copula-based simulation to obtain the VaR forecasts. Using the copula parameters estimated with data up to time

t we obtain a one-day ahead forecast of the copula dependence structure which forms the basis to simulate a sample of cross-dependent $Uniform(0,1)$ variables; for details see Cherubini et al. (2004). The latter are transformed into samples of cross-dependent skT innovations via the inverse *skewed Student's t* density and used together with the marginal models' parameter estimates from the actual data up to time t to simulate daily CDS and equity returns. The day-ahead equally-weighted portfolio return is calculated as $r^*_{t+1} = 0.5r^*_{t+1,CDS} + 0.5r^*_{t+1,equity}$. This approach is repeated $J = 100,000$ times to construct the day-ahead P/L distribution $\{r^*_{t+1,j}\}_{j=1}^{J}$. from which the quantile of interest (*i.e.*, day-ahead VaR forecast) is obtained.

Various backtesting methods are available for assessing the accuracy of VaR forecasts according to the sequence of *hits*, also called exceedances or exceptions, formally

$$H_{t+1} = \begin{cases} 1 & \text{if } R_{t+1} < VaR^{\alpha}_{t+1} \\ 0 & \text{otherwise} \end{cases}, \; t = 1, 2, ..., T_1, \tag{13}$$

where T_1 is the size of the out-of-sample (or evaluation) period. Thus an exception occurs on day $t+1$ when the *ex post* portfolio loss is larger than the maximum loss anticipated according to the VaR model.

The *unconditional coverage* test of Kupiec (1995; UC) is designed to assess whether the expected *hit* rate is equal to the nominal coverage rate, namely, the hypotheses are $H_0 : \mathbb{E}(H_{t+1}) = \alpha$ versus $H_A : \mathbb{E}(H_{t+1}) \neq \alpha$. Since the random variable H_{t+1} is binomial, the expected probability of observing N exceptions over an T_1 trading days is $(1-\alpha)^{T_1-N}\,\alpha^N$ under H_0. The corresponding likelihood ratio statistic is

$$LR_{UC} = -2\ln\left(\frac{(1-\alpha)^{T_1-N}\,\alpha^N}{(1-\hat{\alpha})^{T_1-N}\,\hat{\alpha}^N}\right) \overset{asy}{\sim} \chi^2_1. \tag{14}$$

where $\hat{\alpha} = \frac{N}{T_1}$ is the observed *hit* rate. A weakness of this test is its unconditional nature, *i.e.* it only "counts" hits but disregards how clustered they are. A well-specified risk management model should efficiently exploit all the available information I_t so that VaR exceptions are unpredictable, *i.e.*$\mathbb{E}(H_{t+1}|I_t) = \mathbb{E}(H_{t+1}) = \alpha$.

The *conditional coverage* test of Christoffersen (1998; CC) overcomes this drawback. Its aim is to assess whether the correct out-of-sample VaR specification property, $\mathbb{E}(H_{t+1}|I_t) = \alpha$ is met. An implication of this property is that H_{t+1} should be *iid* binomial with mean α. Hence, this is essentially a test of the joint hypothesis of correct unconditional coverage and independence of the hits via the LR statistic

$$LR_{CC} = LR_{UC} + LR_{Ind} = -2\ln\left(\frac{(1-\alpha)^{T_1-N}\,\alpha^N}{(1-\hat{\pi}_{01})^{n_{00}}\,\hat{\pi}_{01}^{n_{01}}\,(1-\hat{\pi}_{11})^{n_{10}}\,\hat{\pi}_{11}^{n_{11}}}\right) \overset{asy}{\sim} \chi^2_2, \tag{15}$$

where n_{10} denotes the number of transitions or instances when an exception occurred on day t and not on day $t-1$ and $\hat{\pi}_{10} = \frac{n_{10}}{n_{10}+n_{11}}$ is the estimated probability of having an exception on day t conditional on not having an exception on day $t-1$. Thus the test can detect if the probability of observing an exception, under the assumption of independence, is equal to α which amounts to testing that $\pi_{01} = \pi_{11} = \alpha$.

However, the condition of correct VaR specification $\mathbb{E}(H_{t+1}|I_t) = \alpha$ is stronger than what the Christoffersen (1998) CC test can detect. The out-of-sample hits H_{t+1} should be uncorrelated with *any* variable in I_t, meaning that H_{t+1} should be a completely unpredictable process. The Christoffersen (1998) test can only detect autocorrelation of order one because it is built upon a first-order Markov chain assumption for the hits.

The *dynamic quantile* for conditional coverage developed by Engle and Manganelli (2004; DQ) is able to address this shortcoming. This is essentially a Wald test for the overall significance of a linear probability model $\boldsymbol{H} - \alpha\mathbf{1} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ with $\boldsymbol{H} = (H_{t+1})$, $\mathbf{1}$ a vector of ones, $\boldsymbol{X} = \left(H_t,\ ...,H_{t-k},\ VaR_{t+1}^{\alpha}\right)'$ the regressor vector, and $\boldsymbol{\beta} = (\beta_1, ..., \beta_{k+2})'$ the corresponding slope coefficients; $\boldsymbol{H} - \alpha\mathbf{1}$ is the demeaned hit variable. The null hypothesis is $H_0 : \boldsymbol{\beta} = 0$ and it can be tested using the Wald type test statistic

$$DQ = \frac{\hat{\boldsymbol{\beta}}'\boldsymbol{X}'\boldsymbol{X}\hat{\boldsymbol{\beta}}}{\alpha(1-\alpha)} \overset{asy}{\sim} \chi^2_{k+2}, \tag{16}$$

where k is a plausible maximum lag for the hit variable. Following Kuester et al. (2006), we employ $k = 4$.

One drawback of these widely-used backtesting approaches is that they do not provide a ranking of VaR models. According to the requirements of the Basel Committee on Banking Supervision, the magnitude as well as the number of exceptions are a matter of regulatory concern. The quadratic loss function suggested by Lopez (1998) takes into account both aspects by adding a penalty based on the size of the exceptions

$$L_{t+1}^{Q} = \begin{cases} 1 + \left(R_{t+1} - VaR_{t+1}^{\alpha}\right)^2 & \text{if } R_{t+1} < VaR_{t+1}^{\alpha} \\ 0 & \text{otherwise} \end{cases}, \tag{17}$$

and thus, larger tail losses get a disproportionately heavier penalty. However, the above loss function can be subject to the criticism that squared monetary returns lack financial intuition. Blanco and Ihle (1999) suggest focusing on the relative size of exceptions (percentage) via the loss function

$$L_{t+1}^{\%} = \begin{cases} \frac{R_{t+1} - VaR_{t+1}^{\alpha}}{VaR_{t+1}^{\alpha}} \times 100 & \text{if } R_{t+1} < VaR_{t+1}^{\alpha} \\ 0 & \text{otherwise} \end{cases}. \tag{18}$$

The average losses $\overline{L}^{Q} = \frac{1}{T_1}\sum_{t=1}^{T_1} L_{t+1}^{Q}$ and $\overline{L}^{\%} = \frac{1}{T_1}\sum_{t=1}^{T_1} L_{t+1}^{\%}$ contain additional information on how good the VaR model is for predicting tail behavior of the portfolio P/L distribution. Therefore, they can rank those VaR models that pass the initial backtesting stage according to their potential cost to the risk manager. A weakness of these regulatory loss functions is that they tend to select very conservative VaR models because if $VaR_t = -\infty$, $\forall t \Rightarrow L_t = 0$, $\forall t$. To sidestep this problem, we adopt the 'tick' loss function

$$L_{t+1}^{tick} = \left(\alpha - 1_{\left\{R_{t+1} - VaR_{t+1}^{\alpha} < 0\right\}}\right)\left(R_{t+1} - VaR_{t+1}^{\alpha}\right), \tag{19}$$

that is implicit in quantile regression theory and quantile forecasting problems; see Giacomini and Komunjer (2005), Gneiting (2011) and Fuertes and Olmo (2013). It asymmetrically penalizes negative exceedances or downside risk underpredictions, *i.e.* $R_{t+1} < VaR_{t+1}^{\alpha}$, more heavily with weight $(1-\alpha)$ than positive exceedances or overpredictions, *i.e.* $R_{t+1} > VaR_{t+1}^{\alpha}$, with weight α. This loss function is 'optimal' for quantile forecasting because the expected loss is minimized under the true quantile.

The out-of-sample or holdout period for the VaR forecast evaluation is March 11, 2009 to March 11, 2011 ($T_1 = 511$ days). The simulation exercise to compute the VaR forecasts is deployed sequentially over a rolling window of fixed-length ($T_0 = 869$ days). The first estimation window runs from September 10, 2005 to March 10, 2009 and the corresponding VaR_{t+1}^{α} forecast is for March 11, 2009. Table 5 summarizes the 1% VaR forecasts obtained from various formulations of the Student's t copula and 90° rotated Gumbel. The counterpart results for the 5% VaR are reported in the on-line addendum (Section G).

[Insert Table 5 around here]

In regards to backtesting, the p-values of the LR_{UC}, LR_{CC} and DQ tests clearly that for the Student's t copula the most reliable VaR forecasts are those from the RS-ARMA formulation. The superior performance of the latter is confirmed by the average regulatory and 'tick' loss functions, particularly, in the CDS-equity Auto and Financial portfolios; for both portfolios, the largest reduction in average out-of-sample losses relative to the static copula is achieved by the RS-ARMA formulation which also improves upon the conventional RS copula. VaR forecasts obtained from dynamic copula that additionally allow for regime-switching behavior (*i.e.*, RS-ARMA) adapt faster and more effectively to changing market conditions.

Next we turn attention to the Gumbel copula which also captures tail dependence but in an asymmetric manner. In the dynamic ARMA formulation, the average level of tail dependence λ_t inferred from Gumbel copula is about 0.25 for the six CDS-equity pairs and is strongly significant in each case whereas the tail dependence suggested by the Student's t copula is very low (order of magnitude 10^{-3}). This contrast is likely to have an impact on the VaR forecasts; namely, Gumbel copula can be expected to yield more conservative VaR forecasts than Student's t copula. Indeed, like-for-like comparisons reveal that the Gumbel copula leads to a more reliable risk management model than the Student's t copula. Consistently across all three portfolios, the DQ test does not reject the null hypothesis of correct 1% VaR model specification based on the

Gumbel copula, irrespective of whether the formulation is purely static, RS static, ARMA, or RS-ARMA. In line with our expectations based on the tail dependence estimates, out-of-sample VaR forecasts from Gumbel copula are more conservative (*i.e.*, larger expected losses) than those from Student's t copula; the empirical coverage of the Gumbel-based VaRs are always below those of the Student's t VaRs. Relatedly, the average out-of-sample regulatory losses in excess of VaR lessen in the Gumbel-based formulations, although this is not the case according to the less conservative "tick" loss function. Thus, viewed through the lens of regulators, relaxing the assumption of symmetric tail dependence between CDS and equity markets (by adopting the Gumbel copula function) can help reducing systemic risks, which has welfare implications, by leading to more cautious VaRs. Our findings reinforce those in Okimoto (2008) where it is shown that international (U.S. and U.K.) equity models that ignore asymmetric tail-dependence in bear (or crisis) markets lead to over-optimistic VaR forecasts that underestimate the true risk of portfolio losses adverse market conditions. We further observe that when the underlying copula function is Gumbel the superiority of the RS-ARMA formulation versus static, dynamic and conventional RS formulations remains unchallenged, according to the average portfolio losses. Overall, the VaR analysis endorses the regime-switching dynamic copula models proposed which suggests that allowing not only for latent "crisis" and "tranquil" regimes of dependence but also for within-regime time variation in the dependence structure can be economically beneficial.

5 Summary and concluding remarks

A thorough understanding of the dependence between financial markets is crucial to risk managers for obtaining reliable Value at Risk (VaR) measures and to regulators and policymakers for designing stress-testing frameworks that enhance the stability of financial institutions and financial systems as a whole. This paper studies the joint behavior of credit default swap (CDS) and equity markets from September 2005 to March 2011. We propose flexible copula models with "normal" and "crisis" regimes which are obtained by allowing the dependence parameters corresponding to a given Markovian state to vary over time.

The proposed copula reveals significant negative co-movement between CDS and stock index returns in line which predictions from the theoretical credit risk model of Merton (1974). It also confirms that the dependence structure is time-varying and non-linear. Significant regime-switching dependence is revealed not only in the central part of the bivariate distributions but also in the tails; namely, *low* and *high* dependence periods alternate over time. The latter broadly coincide with the automotive crisis, the subprime mortgage crisis and the European sovereign debt crises. The findings suggest that during periods of stress, the systematic factor proxied by equity market returns plays a stronger role as driver of corporate defaults. An out-of-sample Value-at-Risk (VaR) forecasting exercise suggests that neglecting regime-switching effects leads to underestimation of the maximum potential losses of CDS-equity portfolios. Furthermore, relaxing the assumption of static within-regime dependence improves the accuracy of out-of-sample VaR forecasts and produces smaller average regulatory losses. Lastly, the asymmetric Gumbel copula which focuses on the adverse tail of the bivariate CDS-equity distribution leads to more conservative day-ahead VaR predictions.

The bivariate copula models here proposed could be extended to a trivariate setting as this would allow capturing, for instance, the joint interactions between credit risk in the banking sector, and credit risk and equity market risk in the automobiles sector with implications for asset management and hedging. Moreover, as the oil price is an important crisis indicator, another possible extension is to exploit the price of oil as an exogenous driver of the dependence regimes in the auto sector and/or to include it as exogenous regressor in the marginals. We leave these extensions as directions for further research.

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APPENDIX A. Estimation of regime-switching copula parameters

Using the Hamilton (1989) filtering algorithm, we first obtain the filtered probability of unobservable regime S_t given the available information set, $\mathrm{P}[S_t = s \mid I_t]$, via a two-step iterated process for $t = 1, \ldots, T$ starting from an initial value at $t = 0$. Next, we adopt the Kim (1994) algorithm to obtain the *smoothed probabilities,* $\mathrm{P}[S_t = s \mid I_T]$, or probabilities of each regime given the full sample information set I_T, starting from the last filtered probability at $t = T$ as initial value and iterating backwards from $t = T - 1$ to $t = 1$.

The *filtering* algorithm involves the following two sequential steps:

1. Inference about the current state given the past values of the observed variable

$$\mathrm{P}[S_t = s \mid I_{t-1}] = \sum_{k=0}^{1} \mathrm{P}[S_t = s \mid S_{t-1} = k, I_{t-1}]\, \mathrm{P}[S_{t-1} = k \mid I_{t-1}], \tag{20}$$

   where $s = \{0, 1\}$ denotes regimes $\{H,L\}$, respectively; $I_t = [u_{1,t}, u_{2,t}, I_{t-1}]$ is the time t information set; and $\mathrm{P}[S_t = s \mid S_{t-1} = k, I_{t-1}]$ are the entries of the transition probability matrix π.

2. Filtering of S_t in order to generate future forecasts on the prevailing state

$$\mathrm{P}[S_t = s \mid I_t] = \frac{c_t^{S_t}(u_{1t}, u_{2t} \mid S_t = s, I_{t-1})\, \mathrm{P}[S_t = s \mid I_{t-1}]}{\sum_{k=0}^{1} c_t(u_{1t}, u_{2t} \mid S_t = k, I_{t-1})\, \mathrm{P}[S_t = k \mid I_{t-1}]}, \tag{21}$$

   where $c_t^{S_t}(\cdot)$ is the *pdf* of RS-ARMA or RS-DCC copula models.

The initial regime probabilities $\mathrm{P}[S_0 = s|I_0]$, are set at the unconditional probabilities of business regimes according to the NBER recessions ($s = 0$) and expansions ($s = 1$), and the probabilities of the two dependence regimes in the first period can be obtained via eq. (21). The full set of filtered probabilities of the two unobserved regimes can then be obtained by iterating the above steps 1. and 2. until the end of the sample period.

The total log-likelihood depends on all the data and can be decomposed into a part that contains the marginal densities and a part that contains the density of the RS copula, that is

$$\sum_{n=1}^{2} \sum_{t=1}^{T} \log f_{n,t}(x_{n,t}; \phi_n) + \sum_{t=1}^{T} \log \left(\sum_{S_t=1}^{2} c_t^{S_t}(u_{1,t}, u_{2,t} \mid S_t, I_{t-1})\, \mathrm{P}[S_t \mid I_{t-1}] \right). \tag{22}$$

The parameters are estimated by two-step maximum likelihood (ML).

Once the parameters are estimated and all filtered probabilities $\mathrm{P}[S_t = s \mid I_t]$, $s \in \{0, 1\}$ and $t =$

$1, 2, \ldots, T$, are derived, we obtain the *smoothed probabilities*

$$\mathrm{P}\left[S_t = s \mid I_T\right] = \sum_{k=0}^{1} \frac{\mathrm{P}\left[S_{t+1} = s \mid S_t = k, I_t\right] \mathrm{P}\left[S_t = s \mid I_t\right] \mathrm{P}\left[S_{t+1} = k \mid I_T\right]}{\sum_{j=0}^{1} \mathrm{P}\left[S_{t+1} = s \mid S_t = k, I_t\right] \mathrm{P}\left[S_t = k \mid I_t\right]}, \; t = T-1, T-2, ..., 1, \quad (23)$$

starting from $\mathrm{P}\left[S_T = s \mid I_T\right]$ and iterating backwards for $t = T-1, T-2, ..., 1$. A threshold value of 0.5 is adopted for the smoothed probabilities to identify the dependence regimes. If $\mathrm{P}\left[S_t = 0 \mid I_T\right] > 0.5$, the dependence process is identified as being in the high or "crisis" regime. If $\mathrm{P}\left[S_t = 0 \mid I_T\right] \leq 0.5$, it is identified as being in the low or "normal" regime.

APPENDIX B. Timeline of late 2000s financial crisis

Credit Crunch

- **July 10, 2007**: S&P announces it may cut ratings on $12bn of subprime debt.
- **August 9, 2007**: ECB pumps 95bn euros into the banking system to improve liquidity.
- **October 1, 2007**: UBS announces $3.4bn losses from sub-prime related investments.
- **October 30, 2007**: Merrill Lynch unveils $7.9bn exposure to bad debt.
- **January 19, 2008**: World Bank predicts slowdown of global economic growth in 2008.
- **January 21, 2008**: Global stock markets suffer their largest fall since September 2001.
- **February 17, 2008**: UK government nationalizes Northern Rock.
- **March 17, 2008**: Wall Street's 5th largest bank, Bear Stearns, is acquired by JP Morgan Chase.
- **April 8, 2008**: IMF warns that potential losses from the credit crunch could reach $1tn.
- **September 7, 2008**: Large US mortgage lenders Fannie Mae and Freddie Mac are nationalized.
- **September 15, 2008**: Lehman Brothers files for Chapter 11 bankruptcy protection.
- **September 16, 2008**: US Fed announces $85bn rescue package for AIG.
- **September 17, 2008**: Lloyds TSB announces takeover of largest British mortgage lender HBOS.
- **October 13, 2008**: UK government announces £37bn injection to RBS, Lloyds TSB and HBOS.
- **November 6, 2008**: Bank of England cuts base interest rate to lowest level since 1955.

Energy Crisis

- **March 5, 2005**: Crude oil prices rose to new highs above $50 per barrel (bbl).
- **September 2005**: US hurricane Katrina pushes gasoline prices to a record high.
- **August 11, 2005**: Crude oil prices broke the psychological barrier of $60 bbl.
- **July 13, 2006**: Israeli attacks on Lebanon pushed crude oil prices to historical highs above $78.40 bbl.
- **October 19, 2007**: US light crude rose to $90.02 bbl.
- **March 5, 2008**: OPEC accused the US of economic "mismanagement" responsible for oil prices.
- **March 12, 2008**: Oil prices surged above $110 bbl.

Automotive Industry Crisis

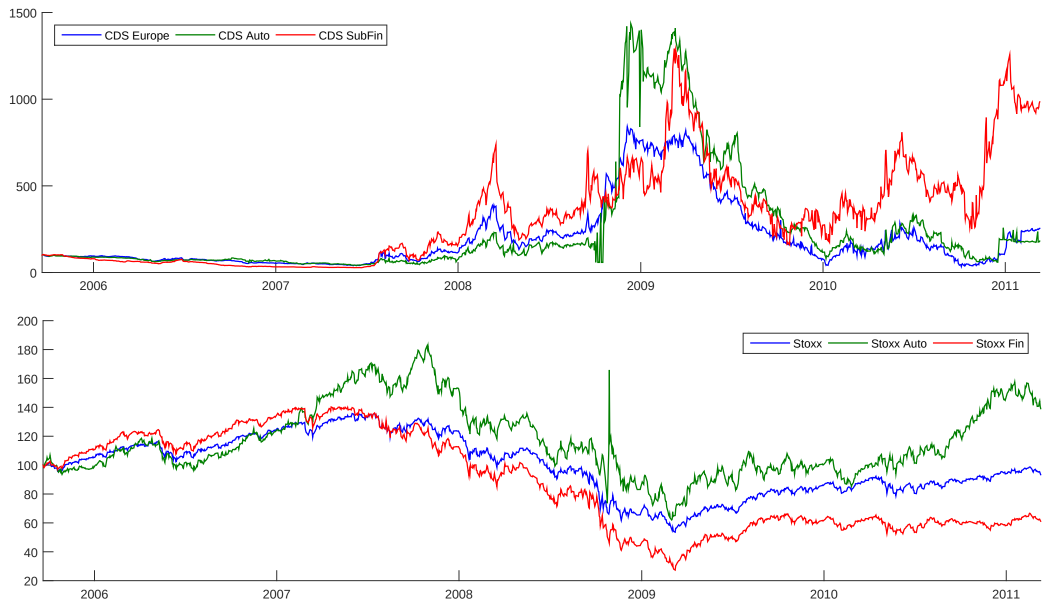
- **May 5, 2005**: S&P cut the debt ratings of GM and Ford to "junk" status.
- **February 12, 2008**: GM announced its operating loss was $2bn.
- **October 7, 2008**: SEAT cut production at its Martorell plant by 5%.
- **November 20, 2008**: PSA Peugeot Citroen predicts sales volumes would fall by at least 10% in 2009, following a 17% drop in the current quarter.
- **November 23, 2008**: Jaguar Land Rover was seeking a $1.5bn loan from the government.
- **December 11, 2008**: The Swedish government injected $3.5bn to rescue its troubled auto markers, Volvo and Saab.
- **December 19, 2008**: US government said it would use up to $17.4bn to help the big three US carmakers, General Motors, Ford and Chrysler.
- **December 20, 2008**: GM and Chrysler receive CA$4bn government loans from Canada and the province of Ontario.
- **January 8, 2009**: Nissan UK announced it was to shed 1200 jobs from its factories in North East England.
- **January 22, 2009**: Fiat announces a 19% drop in revenues for 2008 Q3.
- **February 11, 2009**: PSA Peugeot Citroen announced it would cut 11,000 jobs world wide.
- **February 12, 2009**: Renault announces a 78% drop in profits for 2008.
- **April 22, 2009**: GM admits it will default on a $1bn bond debt payment due in June.
- **April 30, 2009:** Chrysler files for Chapter 11 bankruptcy protection.
- **June 1, 2009:** GM files for Chapter 11 bankruptcy protection.

European sovereign debt crisis

- **October 10, 2008**: Fitch downgrades Iceland Sovereign debt from A+ to BBB-.
- **December 8, 2009**: Fitch ratings agency downgraded Greece's credit rating from A- to BBB+.
- **April 23, 2010**: Greek PM calls for Eurozone-IMF rescue package. FTSE falls more than 600p.
- **May 18, 2010**: Greece gets first bailout of $18bn from EFSF, IMF and bilateral loans
- **November 29, 2010**: Ireland receives $113bn bailout from EU, IMF and EFSF
- **January 5, 2010**: S&P downgrades Iceland's rating to junk grade.

Sources: news.bbc.co.uk; www.reuters.com; www.bloomberg.com.

Panel A: Daily levels



Panel B: Daily logarithmic changes (%)

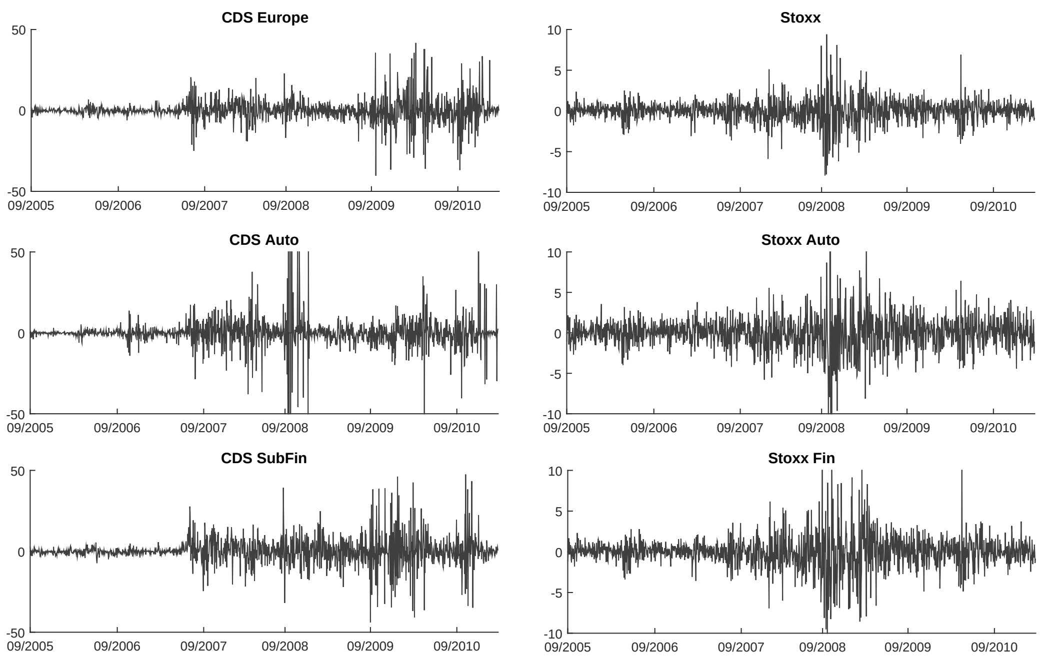


Figure 1: Evolution of CDS and Equity Indices: Panel A plots daily levels of equity market indices (Stoxx) and matched CDS indices (iTraxx) in three sectors: Europe, Auto, Financials. All daily time-series are appropriately normalized to start at 100. Panel B plots daily logarithmic changes. Details on the data and sources are provided in Section 3 of the paper.

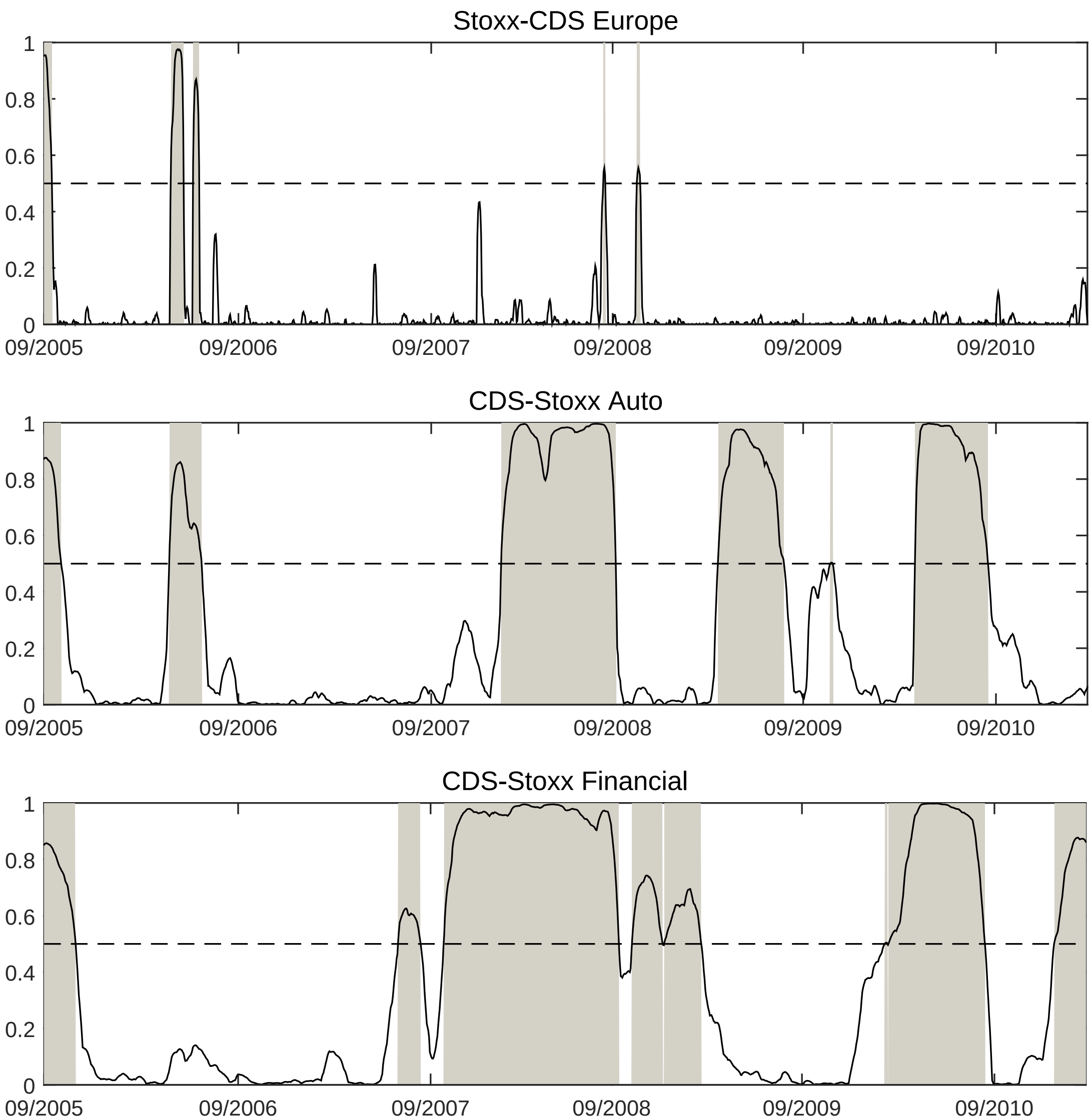


Figure 2: Smoothed Probability of High Dependence Regime: The graphs show the smoothed probability of *high* dependence regime inferred from the RS-ARMA Student's t copula model for pairs of CDS-equity indices. Days when the smoothed probability exceeds 0.5 (fall below 0.5) are shaded in grey (white) to indicate that they pertain to the high or 'crisis' (low or 'normal') dependence regime according to the model.

Panel A. Kendall's rank correlation (τ) — Panel B. Tail dependence (λ)

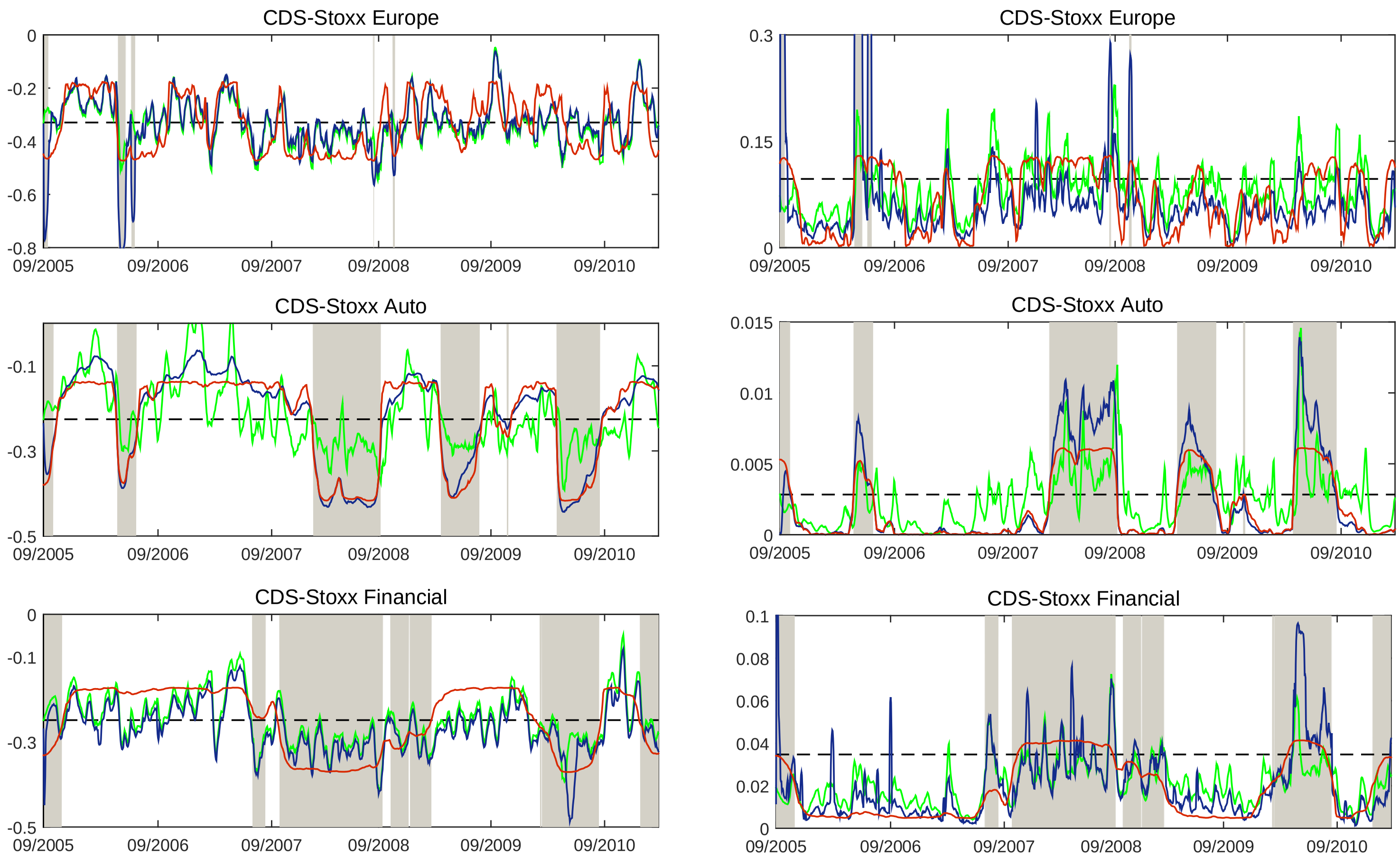


Figure 3: Copula dependence measures: Left-column graphs plot the *rank correlation* (Kendall's τ) measure and right-column graphs plot the *tail dependence* (λ) measure for pairs of CDS-equity indices in three sectors: overall Europe, Auto and Financials. Dashed line is static copula, red line is regime-switching static (RS) copula, green line is dynamic (ARMA) copula, and blue line is regime-switching dynamic (RS-ARMA) copula; all formulations are based on the Student's t copula function. For the RS and RS-ARMA copula models, the rank correlation measure plotted is the weighted average $\tau_t^H p_t^H + \tau_t^L p_t^L$, and the tail dependence measure is $\lambda_t^H p_t^H + \lambda_t^L p_t^L$ with weights given by the smoothed probability of each regime, *i.e.* $p_t^s = \mathrm{P}\left[S_t = s \mid I_T\right]$, $s \in \{H, L\}$; see Appendix A in the paper for details. Grey shaded areas represent days that pertain to the high or 'crisis' dependence regime.

Table 1: Descriptive Statistics for Daily Logarithmic Changes in CDS and Equity Indices

	CDS indices (iTraxx)			Equity indices (Stoxx)		
	Europe	Auto	Financial	Europe	Auto	Financial
Panel A. Individual descriptive measures and test p-values						
Mean	0.068	0.043	0.166	-0.005	0.024	-0.036
Median	-0.193	-0.021	-0.057	0.067	0.057	0.019
Maximum	41.745	199.212	47.500	9.410	40.817	14.666
Minimum	-40.297	-177.407	-43.987	-7.930	-35.427	-10.179
Std. Dev.	7.074	13.327	8.266	1.419	2.699	2.042
Skewness	0.394	2.610	0.343	-0.053	3.192	0.303
Kurtosis	11.520	106.320	10.418	9.741	97.591	10.081
Observations	1381	1381	1381	1381	1381	1381
Jarque-Bera test	0.001	0.001	0.001	0.001	0.001	0.001
Ljung-Box(10) test	0.000	0.000	0.008	0.000	0.000	0.000
ARCH(10) test	0.000	0.000	0.000	0.000	0.000	0.000
Panel B. Pairwise correlation measures						
Pearson's correlation ρ						
Stoxx (matched sector)	-0.366	-0.157	-0.345	-	-	-
Kendall's rank correlation τ						
Stoxx (matched sector)	-0.352	-0.256	-0.274	-	-	-

The table presents summary statistics of the daily logarithmic changes in percentage of iTraxx credit default swap (CDS) indices and Stoxx equity indices. The three test entries report p-values; Jarque-Bera test for the null hypothesis of normality, Ljung-Box test for the null hypothesis of no autocorrelation up to 10 days, and ARCH test for the null hypothesis of no volatility clustering up to 10 days. The reported correlations between CDS and equity are for matched Stoxx Europe 600 marketwide, Auto or Subordinated Financial indices; for instance, the correlation value -0.157 pertains to the iTraxx Auto and Stoxx Auto pair, and so forth.

Table 2: Estimation Results for ARMA-GARCH-skT Marginal Models

	CDS indices (iTraxx)			Equity indices (Stoxx)		
	Europe	Auto	Financial	Europe	Auto	Financial
Panel A. Parameter estimates (standard errors)						
			Conditional mean			
Intercept	-0.166^{**}	-0.143^{**}	-0.300^{**}	0.062^{**}	0.107^{**}	0.052^{*}
	(0.049)	(0.049)	(0.054)	(0.025)	(0.040)	(0.028)
AR1	0.079^{**}	-	-	-	-	-
	(0.032)	-	-	-	-	-
			Conditional variance			
Intercept	0.075^{**}	0.080^{*}	0.070^{**}	0.017^{**}	0.058^{**}	0.019^{**}
	(0.029)	(0.047)	(0.034)	(0.007)	(0.022)	(0.009)
ARCH1	0.171^{**}	0.216^{**}	0.167^{**}	0.112^{**}	0.115^{**}	0.113^{**}
	(0.023)	(0.028)	(0.021)	(0.021)	(0.021)	(0.021)
GARCH1	0.829^{**}	0.784^{**}	0.833^{**}	0.881^{**}	0.876^{**}	0.887^{**}
	(0.026)	(0.036)	(0.023)	(0.019)	(0.018)	(0.019)
ν	5.005^{**}	3.524^{**}	4.806^{**}	8.959^{**}	8.024^{**}	7.685^{**}
	(0.485)	(0.213)	(0.436)	(2.090)	(1.949)	(1.528)
ζ	0.030	-0.029	-0.051^{**}	-0.112^{**}	-0.005	-0.047^{*}
	(0.026)	(0.020)	(0.023)	(0.034)	(0.008)	(0.028)
Panel B. Bootstrap p-values of goodness-of-fit tests						
Ljung-Box(10) test:						
1^{st} Moment	0.6521	0.6222	0.6215	0.6341	0.6721	0.6645
2^{nd} Moment	0.6658	0.6469	0.6447	0.6371	0.6596	0.6785
3^{rd} Moment	0.7013	0.6882	0.6865	0.6385	0.6703	0.6846
4^{th} Moment	0.7120	0.7033	0.6979	0.6354	0.6921	0.6933
KS test:	1.000	0.9702	0.9213	0.8420	1.000	0.9954

Panel A reports the parameters of the conditional mean and variance eqs. (1)-(2) estimated with daily logarithmic return data ($T = 1380$ observations); ν and ζ are the degrees-of-freedom parameter and asymmetry parameter of Hansen's (1994) skewed Student's t distribution. Estimation is by maximum likelihood (ML) and standard errors are reported in parentheses. ** and * denote significance at the 5% and 10% levels, respectively. Panel B reports bootstrap p-values for diagnostic tests. The Ljung-Box (LB) test is deployed on the first four moments of the probability integral transforms, $(u_t - \bar{u})^j$, $j = \{1, 2, 3, 4\}$, to assess the null hypothesis of no autocorrelation up to lag order 10. The Kolmogorov-Smirnov (KS) test is deployed on u_t to asses the null hypothesis that the probability integral transform is $Uniform(0,1)$ or, equivalently, that the underlying filtered return series x_t is skewed Student's t distributed. The p-values are based on $M = 1000$ replications of the observed equity and CDS returns according to a semi-parametric bootstrap approach, as described in Section 3 of the paper, to account for estimation error.

Table 3: Goodness-of-Fit of CDS-Equity Market Copulae

		CDS-Equity indices		
Copula formulation	Goodness-of-fit	Europe	Auto	Financial
Panel A. Static				
Student's t (Static)	AIC	*−379.413*	*−167.542*	−214.191
	LL	*191.707*	*85.771*	109.095
Gumbel (Static)	AIC	−362.281	−166.509	*−218.837*
	LL	182.140	84.255	*110.418*
Panel B. Regime-switching Static Copula				
Student's t	AIC	-402.518	-191.128	-221.746
$(\rho^{S_t}; \nu^{S_t})$	LL	*207.259*	*101.564*	*116.746*
Student's t	AIC	*−404.514*	*−193.126*	*−223.487*
(ρ^{S_t})	LL	207.257	101.563	116.744
Gumbel	AIC	−384.918	−187.505	−223.458
(η^{S_t})	LL	196.459	97.752	115.729
Panel C. Dynamic Copula				
Student's t (ARMA)	AIC	*−405.568*	*−185.540*	*−228.668*
	LL	*206.784*	*96.770*	*118.334*
Gumbel (ARMA)	AIC	−382.284	−180.170	−223.250
	LL	194.142	93.083	114.625
Panel D. Regime-switching Dynamic Copula (RS-ARMA)				
Student's t	AIC	-405.459	-186.365	-221.958
$(\omega^{S_t}, \varphi^{S_t}, \psi^{S_t}; \nu^{S_t})$	LL	212.729	***103.182***	120.978
Student's t	AIC	-407.564	-187.09	-223.879
$(\omega^{S_t}, \varphi^{S_t}, \psi^{S_t})$	LL	212.782	102.545	120.939
Student's t	AIC	***-414.242***	***-192.315***	***-230.319***
(ω^{S_t})	LL	***214.121***	103.158	***122.160***
Gumbel	AIC	-381.486	-183.304	-219.821
$(\omega^{S_t}, \varphi^{S_t}, \psi^{S_t})$	LL	198.743	99.201	117.912
Gumbel	AIC	−392.916	−188.819	−226.447
(ω^{S_t})	LL	202.458	100.410	119.224

This table reports goodness-of-fit measures of Student's t and Gumbel copulae in competing static, dynamic and regime-switching formulations for paired CDS (iTraxx) indices and equity (Stoxx) indices in three sectors: Europe, Auto, Financials. AIC is the Akaike information Criterion and LL is the maximized log-likelihood. For RS and RS-ARMA copula we indicate in parenthesis below the name in the first column the parameters that depend on the latent Markovian state S_t; for details, see section 2.3. The results are based on the 90° anticlockwise-rotated Gumbel copula to capture the 'adverse' tail of the negatively correlated CDS-equity returns which here represents increasing credit default risk and decreasing equity value. For each return pair, italic font denotes the best copula formulation (as dictated by the largest LL or lowest AIC) in Panels A-D; bold italic font denotes the best copula overall.

Table 4: Estimation Results for Static, Dynamic and Regime-Switching Student's t Copulae of CDS-Equity Indices

	Static		RS					ARMA			
Sector	ρ	ν	ρ_H	ρ_L	π_{HH}	π_{LL}	ν	ω	φ	ψ	ν
Europe	-0.495^{**}	8.943^{**}	-0.679^{**}	-0.273^{*}	0.969^{**}	0.966^{**}	27.804	-0.087	1.809^{**}	0.211^{*}	10.550^{**}
	(0.029)	(2.379)	(0.054)	(0.165)	(0.048)	(0.079)	(44.334)	(0.134)	(0.428)	(0.119)	(2.422)
Auto	-0.347^{**}	22.025^{*}	-0.610^{**}	-0.214^{**}	0.981^{**}	0.992^{**}	29.246	-0.017	1.941^{**}	0.105^{*}	22.010^{*}
	(0.033)	(12.938)	(0.092)	(0.078)	(0.181)	(0.112)	(29.201)	(0.027)	(0.165)	(0.061)	(12.988)
Financial	-0.381^{**}	11.513^{**}	-0.551^{**}	-0.267^{**}	0.991^{**}	0.995^{**}	18.553	-0.002	2.030^{**}	0.082^{*}	14.316^{**}
	(0.031)	(5.250)	(0.086)	(0.085)	(0.132)	(0.061)	(14.499)	(0.042)	(0.203)	(0.045)	(7.123)

	RS-ARMA						
Sector	ω_H	ω_L	φ	ψ	π_{HH}	π_{LL}	ν
Europe	-1.839^{**}	-0.076	1.831^{**}	0.190^{**}	0.900^{*}	0.996^{**}	12.858^{*}
	(0.936)	(0.120)	(0.366)	(0.111)	(0.168)	(0.010)	(7.061)
Auto	-0.194^{**}	-0.002	2.026^{**}	0.021	0.979^{**}	0.993^{**}	31.794
	(0.087)	(0.021)	(0.115)	(0.041)	(0.040)	(0.009)	(25.974)
Financial	-1.548^{**}	-0.004	2.021^{**}	0.078^{*}	0.901^{**}	0.998^{**}	14.695
	(0.711)	(0.054)	(0.291)	(0.071)	(0.185)	(0.008)	(9.707)

This table gives the two-stage ML estimation results of Student's t copula models in static, regime-switching static (RS), dynamic (ARMA), and RS-ARMA formulations. The subscript H (L) indicates the *high* or 'crisis' (*low* or 'normal') dependence regime. The parameter π_{HH} (π_{LL}) captures the probability of staying in the *high* (*low*) dependence regime. Numbers in parentheses are standard errors computed with a 'sandwich form' asymptotic covariance matrix that accounts for the estimation error that arises from conditioning the log-likelihood for the copula parameter estimates (second stage) on the parameter estimates of the margins (first stage); see Patton (2013; p.922) for details. ** and * denote significance at the 5% and 10% levels, respectively.

Table 5: Value at Risk Forecasts for CDS-Equity Porfolios at 1% Nominal Coverage

	Student's t				Gumbel			
	Static	**RS**	**ARMA**	**RS-ARMA**	**Static**	**RS**	**ARMA**	**RS-ARMA**
				CDS-equity Europe				
Total Exception	2.74%	1.96%	2.35%	1.96%	1.37%	1.37%	1.37%	1.37%
LR_{UC} test	0.001	0.055	0.009	0.055	0.426	0.426	0.426	0.426
LR_{CC} test	0.003	0.127	0.024	0.127	0.652	0.652	0.652	0.652
DQ test	0.000	0.253	0.019	0.106	0.929	0.937	0.930	0.931
Quadratic loss	1.041	0.606	0.835	0.768	0.278	0.273	0.275	0.271
% benefit vs static	–	**41.77%**	19.74%	26.23%	–	1.45%	1.08%	**2.18%**
Percentage loss	2.46%	1.24%	2.05%	2.00%	0.62%	0.61%	0.61%	0.60%
% benefit vs static	–	**49.66%**	16.81%	18.75%	–	0.89%	1.29%	**2.51%**
Tick loss	2820	2896	2943	2614	3577	3582	3569	3534
% benefit vs static	–	-2.70%	-4.36%	**7.30%**	–	-0.13%	0.23%	**1.22%**
				CDS-equity Auto				
Total Exception	2.35%	1.76%	1.96%	1.37%	0.98%	0.98%	0.98%	0.98%
LR_{UC} test	0.009	0.118	0.055	0.426	0.961	0.961	0.961	0.961
LR_{CC} test	0.024	0.247	0.127	0.652	0.941	0.941	0.941	0.941
DQ test	0.002	0.076	0.187	0.425	0.879	0.904	0.889	0.936
Quadratic loss	0.806	0.631	0.342	0.340	0.307	0.305	0.305	0.305
% benefit vs static	–	21.72%	57.56%	**57.78%**	–	0.53%	**0.74%**	0.71%
Percentage loss	2.11%	1.58%	1.10%	1.03%	0.48%	0.46%	0.46%	0.46%
% benefit vs static	–	25.46%	48.01%	**51.26%**	–	4.08%	3.13%	**4.87%**
Tick loss	3710	3651	3646	3550	4220	4197	4215	4050
% benefit vs static	–	1.59%	1.73%	**4.31%**	–	0.56%	0.13%	**4.02%**
				CDS-equity Financial				
Total Exception	1.96%	1.76%	1.57%	1.57%	1.37%	1.37%	1.37%	1.17%
LR_{UC} test	0.055	0.118	0.235	0.235	0.426	0.426	0.426	0.700
LR_{CC} test	0.127	0.247	0.428	0.428	0.652	0.652	0.652	0.854
DQ test	0.014	0.031	0.515	0.122	0.113	0.110	0.104	0.112
Quadratic loss	0.839	0.680	0.743	0.513	0.292	0.289	0.289	0.282
% benefit vs static	–	19.05%	11.52%	**38.95%**	–	1.07%	1.22%	**3.44%**
Percentage loss	1.32%	1.27%	1.15%	0.87%	0.52%	0.52%	0.51%	0.51%
% benefit vs static	–	3.46%	12.44%	**34.17%**	–	0.58%	1.32%	**1.88%**
Tick loss	2717	2827	2899	2504	3326	3283	3324	3252
% benefit vs static	–	-4.05%	-6.70%	**7.84%**	–	1.29%	0.06%	**2.22%**

The table summarizes the performance of day ahead out-of-sample 1% VaR forecasts based on four distinct formulations — static, regime-switching static (RS), dynamic (ARMA) and regime-switching dynamic (RS-ARMA) — of the Student's t copula and Gumbel copula for portfolios of CDS (iTraxx) indices and underlying equity (Stoxx) indices. Total exceptions are percentage of days in the two-year out-of-sample period when the actual portfolio loss exceeds the VaR forecast. p-values are reported for the unconditional coverage test proposed by Kupiec (1995; UC) , the conditional coverage test of Christoffersen (1998; CC) and the dynamic quantile test of Engle and Manganelli (2004; DQ). The quadratic, percentage, and tick losses shown are average losses calculated using equations (17), (18) and (19), respectively. The results are based on the 90° anticlockwise-rotated Gumbel copula to capture the 'adverse' tail of the negatively correlated CDS-equity returns which here represents increasing credit default risk and decreasing equity value. Bold indicates the best (loss-minimizing) model for each loss function.