City Research Online

City, University of London Institutional Repository

The Commitment Problem of Secured Lending

Daniela Fabbri*
Cass Business School

Anna Maria C. Menichini†
University of Salerno and CSEF

September 2015

Abstract

The paper presents a new theory of trade credit where firms buy inputs on credit from suppliers to restore the benefits of secured bank financing impaired by contract incompleteness. In a setting where investment is endogenous and unobservable to financiers, we show that a bank-secured credit contract is time-inconsistent: Upon being granted credit, the entrepreneur has an incentive to alter the original input combination, jeopardizing the bank's revenues. Anticipating the entrepreneur's opportunism, the bank offers an unsecured credit contract, reducing the surplus from the venture. One way for the entrepreneur to commit to the contract terms is to purchase inputs on credit from the supplier. The supplier observes the input investment and acts as a guarantor that inputs will be purchased as contracted, thus facilitating access to secured bank financing. The commitment role of trade credit still holds in a multi-period extension that investigates the impact of bank relationship lending on secured debt and trade credit. Our model provides novel testable predictions on optimal financial contracts in both one-period and repeated-lending relationships.

Keywords: collateral, commitment, trade credit, bank financing.
JEL classification: G32, G33, K22, L14.

---

*Contact address: Faculty of Finance, Cass Business School, 106 Bunhill Row, London, EC1Y 8TZ, UK. Tel.: +44 (0)20 7040 0966. Fax.: +44 (0)20 7040 8881. E-mail: Daniela.Fabbri.1@city.ac.uk

†Department of Economics and Statistics, University of Salerno, Via Giovanni Paolo II, 84084 Fisciano (SA), Italy. Tel.: +39 089 962174. Fax.: +39 089 963169. E-mail: amenichini@unisa.it

Introduction

Firms procure funds not only from specialized financial intermediaries but also from suppliers, which generally delay payments of inputs. Trade credit is an important source of external financing for firms of all sizes across both developed and developing countries (Petersen and Rajan, 1997; Beck, Demirgüç-Kunt, and Maksimovic, 2008; Giannetti , Burkart, and Ellingsen, 2011) and across both domestic and foreign markets (Auboin and Engeman, 2014; Manova, 2013). Researchers have mostly rationalized and documented the substitution effect of trade credit, arguing that firms rely on trade credit when they are constrained on bank financing (e.g., Calomiris, Himmelberg, and Wachtel, 1995; Petersen and Rajan, 1997; Burkart and Ellingsen, 2004; Love, Preve, and Sarria Allende, 2007; among others). Recent empirical evidence, however, indicates that the complementarity effects of trade credit are also relevant. Giannetti, Burkart, and Ellingsen (2011) show that U.S. firms obtaining credit from suppliers can secure financing from relatively uninformed banks. Garcia-Appendini (2010) documents that small, non-financial firms in the U.S. are more likely to secure bank credit if they have been granted trade credit from their suppliers.[1]

We propose a new theory of trade credit that can be used to explain these stylized facts. Firms buy capital inputs on account to restore the benefits of secured bank financing impaired by the borrower's inability to credibly commit to investment in pledgeable assets. We show that if the investment in a given asset is not contractible, pledging it as collateral fails to increase external financing. This is because collateral introduces a problem of moral hazard in the form of asset substitution (i.e., the entrepreneur has an ex post incentive to alter the input mix), making secured bank credit unfeasible. We show that the entrepreneur can use trade credit to mitigate this problem. It follows that when investment is non-contractible, buying inputs on account facilitates access to secured bank lending.[2]

We construct a one-period model where an entrepreneur produces a good for with uncertain demand. The entrepreneur uses two inputs, capital and labor, whose purchase is entirely financed by external financiers. The inputs have different collateral values. For simplicity, only capital can be pledged to financiers. Being specialized financial intermediaries, banks typically offer the cheapest source of financing. If banks observe the amount of inputs purchased and thus invested, the optimal

---

[1]Cook (1999) documents that accounts payable raise the likelihood of a Russian firm obtaining a bank loan. Alphonse, Ducret, and Severin (2006) document that the more trade credit U.S. firms use, the more indebted they are to banks, more so for firms with short-term banking relationships. Along the same lines, Gama, Mateus, and Teixera (2010) find that the use of payables allows younger and smaller firms in Spain and Portugal to increase the availability of bank financing.

[2]A secured loan is a loan backed by a specific asset whose existence, ownership, and value are known to the lender before signing the contract (i.e., real-estate-based lending). In our paper, we use a slightly wider interpretation, which includes loans backed by a pool of assets, like inventories, that can be seized by the creditor in the event of non-payment.

contract is secured debt. The input combination is tilted towards capital, which is fully pledged as collateral. Collateral gives the bank protection against losses in default, thereby increasing the amount of external financing and the total surplus of the lending relation. However, the value of the collateral is not exogenous - it can be affected by the borrower's input choice. So, if the investment is not observable, upon receiving the bank loan, the entrepreneur has an incentive to alter the original input combination towards the input with the lowest collateral value and higher productivity. This jeopardizes the bank's expected revenues by reducing the liquidation income in case of default. Anticipating that it will not break even, the bank abandons the secured contract, thus causing an efficiency loss.

One way to avoid this is for the entrepreneur to purchase the capital input on credit and pledge it to the supplier in case of default. As the provider of the input, the supplier observes the input investment. Knowing the investment level and having a stake in the default state, he implicitly guarantees purchase of the quantity of inputs specified in the financial contracts, and thus available for liquidation to all creditors, thereby restoring the benefits of secured bank financing.

This commitment effect of trade credit is robust to the possibility of a costly collusive agreement between the entrepreneur and the supplier and to repeated entrepreneur-bank interactions. Specifically, we extend our static baseline model to a multi-period setting to investigate how trade credit and secured debt are affected by relationship lending, i.e., a long-term contract with credit amounts and repayment obligations contingent on some information about the borrower's past behavior (e.g., Boot and Thakor, 1994; 2000). We show that trade credit is still the best way to solve the commitment problem if projects are not too lengthy. We also find that entrepreneurs are more likely to use trade credit than relationship lending when inputs have high collateral value and low-quality information is collected by the relationship lender.

In practice, entrepreneurs largely use secured loans, many of which are sensitive to the commitment problem. Asset-based lending (ABL) is one important source of short-term financing (typically with a three-year maturity) that many firms in the U.S. and Canada use to fund their working capital. In 2002, ABL in the U.S. was $326 billion, almost a quarter of total short-term credit, which increased to $590 billion in 2008.[3] In ABL, the bank lends funds to a firm in exchange for collateral, which generally includes equipment, small machinery, inventory, and accounts receivable. Since the value of the asset pledged as collateral is clearly affected by input purchases that are not easily observable by the bank, ABL is particularly vulnerable to the commitment problem analyzed in this paper. Moreover, the firm likely purchases most of these assets on credit, which is consistent with our theoretical setting. Our model is less suited for situations where the assets pledged as collateral are registered, such as in

---

[3]See Udell (2004) for a detailed description of the characteristics of ABL.

real estate-based lending.[4] However, even in these cases, the 2007-2009 financial crisis demonstrated that there were instances in which creditors were unable to identify ex ante the appropriate value of the collateral underlying mortgage loans and asset-backed securities. This suggests that the non-contractibility of the collateral, a key ingredient of our model, may be commonplace.

Our paper is related to two strands of the literature. The first focuses on the role of collateral in lending, while the second examines the determinants of trade credit use. As for the first strand, collateral is often a key element of lending arrangements. Berger and Udell (1990) and Harhoff and Korting (1998) document that nearly 70% of commercial industrial loans in the U.S., the U.K., and Germany are secured. More recent papers report similar evidence for Spain (Jimenez, Salas, and Saurina, 2006), Germany, the U.K., and France (Qian and Strahan, 2007; Davydenko and Franks, 2008). There are several theoretical reasons for collateral use. First, collateral reduces the lender's losses in case of default (lender risk reduction). Second, it reduces the distortions due to asymmetric information, in situations of adverse selection (Bester, 1985; Chan and Kanatas, 1985; Besanko and Thakor, 1987a, 1987b), moral hazard, such as risk shifting via asset substitution (Jackson and Kronman, 1979; Smith and Warner, 1979), under-investment, or inadequate effort supply (Stulz and Johnson, 1985; Chan and Thakor, 1987; Boot and Thakor, 1994; Inderst and Mueller, 2007) or both adverse selection and moral hazard (as in Boot, Thakor and Udell, 1991). All these papers point to the idea that borrowing not only against returns but also against assets increases the firm's debt capacity. This conclusion is obtained in settings where projects mostly use one input and the entrepreneur pledges outside collateral.

Our paper contributes to the literature by extending the setting to a multi-input technology with inside collateral. By allowing the investment in pledgeable assets and financing to be jointly and endogenously chosen, we obtain new economic insights that reveal the limitations of collateral. Specifically, our conclusion that any secured bank contract is time-inconsistent when investment is unobservable challenges the accepted view that collateral boosts the firm's debt capacity through lender risk reduction. Moreover, in contrast with the risk-shifting literature, our analysis shows that it is the collateral itself that introduces a problem of entrepreneurial opportunism in the form of ex post asset substitution, absent in the unsecured contract.

Two ingredients are crucial to the time inconsistency of the secured contract: a multi-input

---

[4]Real-estate-based lending or loans secured by movable goods (cars, trucks, etc.) have characteristics that depart from our theoretical setting. First, the problem of investment non-observability is not so relevant in this case, since the goods are registered and their actual purchase is accordingly certifiable to the bank. Second, the credit is generally granted directly to the seller of the asset, to the notary (for real assets), or to the leasing company (for movable goods). This implies that the entrepreneur cannot misuse the loan.

technology and the non-observability of the investment. With one input only, the non-contractibility of the investment is immaterial, as the size of the loan can be used to infer the input choice. With investment observability, no commitment problem arises, as the entrepreneur can credibly commit to the ex ante efficient investment. Thus, investment observability is a crucial determinant of a firm's debt capacity. In this respect, we add to the theoretical literature on collateral where asset characteristics like the degree of tangibility (Almeida and Campello, 2007; Rampini and Viswanathan, 2013), the redeployability (Williamson, 1988; Shleifer and Vishny, 1992; Marquez and Yavuz, 2013), the ease in transferring its ownership to creditors in case of distress (Hart and Moore, 1994), or the speed of depreciation (Rajan and Winton, 1995) are important determinants of firms' debt capacity.[5]

Our paper is also related to the literature on trade credit. Some researchers have sought to explain why agents might prefer to borrow from firms rather than from financial intermediaries. The traditional explanation is that trade credit plays a non-financial role, thereby reducing transaction costs (Ferris, 1981), allowing price discrimination between customers with different creditworthiness (Brennan et al., 1988), fostering long-term relationships with customers (Summers and Wilson, 2002), and even providing a warranty for quality when customers cannot observe product characteristics (Long, Malitz, and Ravid, 1993). Financial theories hold that suppliers are at least as good as banks as financial intermediaries. In Biais and Gollier (1997) and Burkart and Ellingsen (2004), this is ascribed to information advantages. Fabbri and Menichini (2010) show that trade credit can be cheaper than bank credit because of the supplier's liquidation advantage.

In this paper, we develop a new theory of trade credit, where firms buy inputs on account to restore the benefits of secured bank financing jeopardized by contract incompleteness. Our paper is most closely related to Biais and Gollier (1997) and Burkart and Ellingsen (2004). Like Burkart and Ellingsen (2004), the supplier has an information advantage that stems from observing input purchases. Like Biais and Gollier (1997), trade credit works as a signaling device and facilitates bank lending. However, collateral plays no role in either Biais and Gollier (1997) or in Burkart and Ellingsen (2004), while it is crucial in our model.

The remainder of the paper is organized as follows. In Section 1, we present the model. In Section 2, we describe the commitment problem that plagues an entrepreneur-bank lending relationship. In Section 3, we show that trade credit can solve the commitment problem and characterize the properties of the optimal financial contract. Section 4 allows for collusion between entrepreneur and supplier. Section 5 considers a multi-period setting and investigates the impact of relationship lending on secured

---

[5]These theoretical predictions are supported by empirical evidence showing that asset tangibility and salability increase debt capacity (e.g., Almeida and Campello, 2007; Benmelech, 2009; Campello and Giambona, 2013).

debt and trade credit. In Section 6, we discuss the robustness of our theoretical setting along several dimensions. We conclude in Section 7. All proofs are in the Appendix.

1   Model setup and assumptions

A risk-neutral entrepreneur has an investment project that uses two inputs: capital (K) and labor (N). Their investment levels are denoted by I_K, I_N. The amount of the input invested is converted into a verifiable state-contingent output, $Y \in \{0, y\}$. Uncertainty affects production through demand (i.e., production is demand-driven). During periods of high demand, the invested inputs generate output $Y = y$ with probability p according to a homothetic, strictly quasi-concave production function, $y = f(I_K, I_N)$. During periods of low demand, there is no output $(Y = 0)$, but inputs are redeployable and can be pledged as collateral to creditors. Inputs are substitutes, but a positive amount of each is essential for production. Cross partial derivatives, f_{NK}, are positive.[6] The characteristics of the technology are common knowledge.

The entrepreneur is a price-taker both in the input market and in the output market. The output price is normalized to 1, and so is the price of the two inputs.[7]

The entrepreneur has no wealth, so he needs external funding from competitive banks $(L_B \geq 0)$ and/or suppliers $(L_S \geq 0)$. We assume that lending is exclusive: the entrepreneur cannot borrow from multiple banks or suppliers.[8] The banks and suppliers play different roles. Banks lend cash. The supplier of labor provides the input, which is fully paid for in cash. The supplier of capital, however, not only sells the input, but can also act as a financier, by delaying the payment of the inputs supplied. Each party is protected by limited liability.

Cost of funds. Banks have an intermediation advantage relative to suppliers: lower cost of raising funds on the market than suppliers $(r_B < r_S)$. This assumption is consistent with the role of banks as specialized financial intermediaries.

Collateral value. Capital inputs are redeployable and can be pledged as collateral to creditors. They cannot be repossessed by the entrepreneur.[9] Financiers are equally good at liquidating the unused capital, whose scrap value in case of default is given by βI_K, with $0 < \beta < 1$.[10] Labor has

---

[6]This is tantamount to saying that having more of a given input increases the marginal product of the other input. This condition is satisfied by the most commonly used production functions (e.g., Cobb-Douglas, CES, and their transformations).

[7]This normalization is without loss of generality since we use a partial equilibrium setting.

[8]In Section 6.2, we discuss the relevance of this assumption in our analysis.

[9]In Section 6.1, we discuss the implications of having the entrepreneur seizing the collateral.

[10]This assumption allows us to highlight the commitment role of trade credit. Giving the supplier a comparative advantage in liquidating the capital input would not alter our qualitative results, as long as this advantage is not too high, i.e., $\beta_S \leq \frac{(1-p)\beta_B r_S}{(r_B - p r_S)}$, where β_S, β_B are the liquidation value of one unit of unused capital input when repossessed

zero collateral value.

Information. In contrast with most theories of financial intermediation that portray banks as informationally superior lenders (e.g., Ramakrishnan and Thakor, 1984), we assume that capital input suppliers have an information advantage vis-à-vis the bank when lending to the entrepreneur. This assumption, frequently interpreted as a natural by-product of the supplier's business, is commonly accepted in the theoretical literature on trade credit (i.e., Biais and Gollier, 1997; Burkart and Ellingsen, 2004), and has empirical support (Giannetti, Burkart, and Ellingsen, 2011). Suppliers are frequently in the same industry as their customers, and they often visit their premises. In our setting, this assumption is even more reasonable, given that the supplier's information advantage concerns the observation of the input purchase. Since they provide the input, suppliers costlessly observe that an input transaction has taken place. Banks do not observe the transaction, and the cost of acquiring this information is assumed to be too high to make observation worthwhile.[11] This asymmetry implies that, while suppliers can condition their lending on the investment, banks cannot.

Contracts. Since there is no output in the bad state, limited liability implies that repayments to banks and suppliers in the bad state are zero. However, financiers can still get the scrap value of unused inputs. The contract between the entrepreneur and the bank thus specifies the loan, L_B, the repayment obligation in the good state, R_B, and the fraction of the collateral obtained in case of default, $\gamma \in [0, 1]$. The contract with the supplier of the capital input specifies the input purchase, I_K, the amount of trade credit, L_S, the repayment obligation in the good state, R_S, and the fraction of the collateral obtained in case of default, $1 - \gamma$. Last, labor is fully paid for when purchased.[12] Thus, the contract between the entrepreneur and his workers specifies the investment in labor, I_N.

The sequence of events is as follows. At $t = 1$, the entrepreneur makes contract offers to competitive banks and suppliers specifying the size of the loans, L_B, L_S, the repayment obligations, R_B, R_S, the share of the collateral that goes to the bank and the supplier in case of default, $\gamma, (1 - \gamma)$, and the amount of capital input to be purchased, I_K. At $t = 2$, banks and suppliers decide whether to accept or reject the contract; if they accept, at $t = 3$, the investment decisions, I_K, I_N, are taken and trade credit is provided to the entrepreneur; at $t = 4$, uncertainty resolves; at $t = 5$, repayments are made.

by the supplier and the bank, respectively. If the above assumption were violated, trade credit would be cheaper than bank credit and therefore strictly preferred. This case has been analyzed in Fabbri and Menichini (2010).

[11]Full non-observability by the bank and full observability by the supplier are not crucial to our analysis. We could still get our results by assuming that both the bank and the supplier partially observe the input purchase, but the supplier has an information advantage over banks.

[12]Labor regulations do not generally allow firms to postpone the payment of salaries to workers.

2 The entrepreneur-bank contract

In this section, we analyze the entrepreneur-bank contract. In Section 2.1, we analyze the benchmark case where the investment is contractible, deriving the well-known optimality result of secured lending, due to the lender's risk reduction. In Section 2.2, we show that when the investment is non-contractible any secured debt contract is time-inconsistent, and the only feasible contract is the unsecured one.

2.1 The benchmark case: Contractible investment

Since bank credit is cheaper than trade credit, in period $t = 1$, the entrepreneur makes a contract offer only to the bank. The amount of inputs and financing are obtained by solving the following optimization problem $(\mathcal{P}^{\mathcal{C}})$:

$$\max_{I_K, I_N, L_B, R_B} p\left[f\left(I_K, I_N\right) - R_B\right] \tag{1}$$

$$\text{s.t. } pR_B + (1-p)\beta I_K \geq L_B r_B, \tag{2}$$

$$L_B \geq I_N + I_K, \tag{3}$$

where (1) gives the entrepreneur's expected profit, (2) is the bank's participation constraint, stating that banks participate in the venture if their expected returns cover at least the opportunity cost of funds, and (3) is the resource constraint, requiring that input purchase cannot exceed available funds.[13] Competition among banks implies that (2) is binding. Solving (2) for R_B and using the binding resource constraint (3), the objective function (1) becomes:

$$pf\left(I_K, I_N\right) - r_B\left(I_K + I_N\right) + (1-p)\beta I_K. \tag{4}$$

The solution to this problem is summarized in Proposition 1.

Proposition 1 *When investment is contractible, the entrepreneur and the bank sign a secured contract (commitment contract, henceforth) with loan $L_B^C = I_K^C + I_N^C$, bank repayment $R_B^C = \frac{1}{p}\left\{\left(I_N^C + I_K^C\right)r_B - (1-p)\beta I_K^C\right\}$ in the good state, and βI_K^C in the bad state, where I_K^C, I_N^C are the investment levels solving the first order conditions (26) and (27) in the Appendix. The entrepreneur gets expected profits $\Pi^C \equiv pf\left(I_K^C, I_N^C\right) - r_B\left(I_K^C + I_N^C\right) + (1-p)\beta I_K^C$.*

Point C in Figure 1 represents the optimal input combination under the commitment contract. The input mix is tilted towards capital. The collateral value makes the actual price of capital equal to

---

[13]The assumption that only creditors can repossess the assets in distress implies that unused inputs enter the bank's participation constraint (2) but not the objective function (1).

$r_B - (1-p)\beta$, which is lower than the price of labor, r_B. In our model, the actual input price depends on both the selling price and the cost of finance (i.e., the cost of the credit for input purchases). Since the selling price is set at 1 for both inputs by assumption, differences in the input prices reflect only differences in the cost of finance. Thus, when a secured contract is signed, the cost of financing the capital input is lower than that of financing labor, the difference being the collateral value of capital. In this case, the two inputs have different actual prices, although they are both financed by the bank and the selling price is the same. In contrast, if both inputs are financed through an unsecured contract, they have the same financing cost, namely r_B, and thus they also have the same actual price.

2.2 Non-contractible investment

The result in Proposition 1 is obtained under the assumption that the entrepreneur can commit to the investment level specified in the bank contract at $t = 1$. However, if the investment is unobservable, then at $t = 3$, once the loan L_B^C has been granted, the entrepreneur can increase his profit by altering the input combination. The entrepreneur re-optimizes by solving programme $\mathcal{P}^{\mathcal{D}}$:

$$\max_{I_K, I_N, R_B} p\left[f\left(I_K, I_N\right) - R_B\right] \tag{5}$$

$$\text{s.t. } R_B = R_B^C, \tag{6}$$

$$L_B^C = I_N + I_K, \tag{7}$$

where constraint (6) requires that the entrepreneur honors his repayment obligation in non-defaulting states (i.e., R_B^C in Proposition 1),[14] while constraint (7) requires that the ex post total input expenditure be equal to the loan obtained in the secured contract (i.e., L_B^C in Proposition 1).

The solution to the above programme is called deviation contract. The input combination under the deviation contract is $I_K^D\left(L_B^C, R_B^C\right) \equiv I_K^D < I_K^C$ and $I_N^D\left(L_B^C, R_B^C\right) \equiv I_N^D > I_N^C$, and the corresponding entrepreneur's expected profits are:

$$\Pi^D \equiv p\left[f\left(I_K^D, I_N^D\right) - R_B^C\right] > \Pi^C. \tag{8}$$

The increase in profits $(\Pi^D > \Pi^C)$ is the result of a distortion in the input combination: the entrepreneur overinvests in labor and underinvests in capital. The incentive to deviate from the original mix arises because the new input combination is chosen after the loan has been granted. Thus, the entrepreneur only cares about meeting his repayment obligation in the good state and he is not concerned with repaying the bank in the bad state. Indeed, no collateral is pledged to the lender

---

[14]Because output is verifiable, any return from production will be claimed by creditors and the entrepreneur will get zero return in the good state if he fails to meet his repayment obligation.

in the bad state (i.e., no capital inputs are in the lender's participation constraint (6)). The lack of collateral increases the actual input price of capital and leaves the price of labor unchanged. As a result, labor becomes relatively cheaper than in the commitment case. The entrepreneur is thus better off reducing his investment in capital and increasing it in labor inputs.

The input combination under the deviation contract characterized above is represented by point D in Figure 1. Point D lies to the right of point C on a higher isoquant and on a flatter isocost than that going through point C. To see why, consider that the slopes of the two isocost lines tangent to isoquants y^C and y^D represent the ex ante and the ex post input price ratios, respectively (i.e., those obtained before and after the bank loan has been received). The ex ante input price ratio is that implied by the secured credit contract (point C). As the contract is secured, the ratio is $r_B / [r_B - (1-p)\beta] > 1$, and hence larger, the higher the collateral value of the capital input. Conversely, since the contract used by the bank to finance the capital input purchase at point D is unsecured, the financing cost of the two inputs is the same and equal to r_B. Therefore, the ex post input price ratio is 1. Since at the new ex post input prices it must still be possible to afford the original contract, the new isocost line has to pass through the initial optimum (point A). By the quasi-concavity of the production function, the new input combination lies on a higher isoquant, and implies a decrease in I_K and an increase in I_N. The difference between the ex ante and the ex post input price ratios is precisely why the entrepreneur can obtain higher profit by choosing an input combination that is different from the ex ante efficient one.
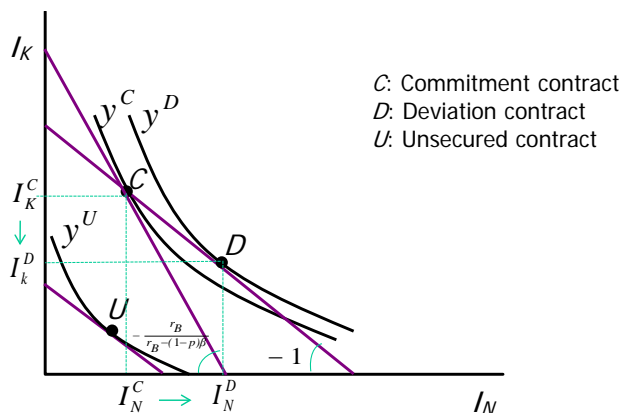
Figure 1: Contractible and non-contractible investment. Points C, D, and U represent the optimal input combination and the production level under the commitment, the deviation and the unsecured contract, respectively. Point D is not an equilibrium contract, since the bank does not break even.

However, point D in Figure 1 is not an equilibrium. Because of the decreased investment in capital inputs, in case of default the entrepreneur fails to meet its obligations. Anticipating that it will not

break even, at the contracting stage the bank will be willing to sign only an unsecured contract with all the repayment obligations paid for in the good state. Setting the collateral equal to zero in the bank participation constraint (2), solving it for R_B and the resource constraint (3) for L_B, the objective function (1) becomes $pf(I_K, I_N) - (I_N + I_K) r_B$, which, compared with the benchmark profits (4), shows the efficiency loss due to the inability of the entrepreneur to pledge collateral. The solution to the maximization problem is described in Proposition 2.

Proposition 2 *When investment is non-contractible, the entrepreneur and the bank sign an unsecured credit contract. The bank lends $L_B^U = I_K^U + I_N^U < L_B^C$ to the entrepreneur, where I_K^U and I_N^U are the investment levels solving (30) and (31) in the Appendix, with $I_K^U < I_K^C$, $I_N^U < I_N^C$. The bank gets repaid only in non-defaulting states: $R_B^U = \frac{1}{p} L_B^U r_B$. The entrepreneur gets expected profits $\Pi^U \equiv pf(I_K^U, I_N^U) - (I_K^U + I_N^U) r_B < \Pi^C$. There is an efficiency loss due to the inability to pledge capital input as collateral.*

Point U in Figure 1 is the optimal input combination when investment is non-contractible. The new isoquant y^U lies below y^C. While the bank is indifferent between points C and U – it gets zero expected profit either way – the entrepreneur's profit is strictly lower at B, because the lower debt capacity implied by the inability to pledge capital inputs as collateral reduces the overall investment and therefore output. The distance between the isoquants, $y^C - y^U$, represents the benefits of collateral. It follows that, to exploit this benefit, the entrepreneur would rather commit to the investment level of the secured credit contract (point C). Notice also that at U the actual price of the two inputs is the same and equal to r_B (as in point D), which implies the same level of capital and labor.[15]

In Proposition 2, we posit that only the unsecured contract (point U) can be an equilibrium outcome. Both the commitment (point C) and any partially secured contract (any point between C and U) are time inconsistent since the entrepreneur retains the incentive to alter the input mix in favor of labor. To get the intuition, consider the relation between the ex ante input price ratio and the ex post one. As long as these two relative prices are different, there will be an incentive to change the input combination, no matter how much of the loan has been secured. Consider first the fully secured contract (point C in Figure 1). The ex ante input price ratio is $r_B / [r_B - (1-p)\beta] > 1$, while the ex post one is $r_B/r_B = 1$. It follows that, once the loan has been granted, labor is actually less expensive (and capital more expensive), thus inducing the entrepreneur to increase the reliance on labor and lower that on capital. The same reasoning applies to any partially secured bank loan (i.e., to any contract where $0 < \gamma < 1$). In this case, the ex ante input price ratio will be $r_B / [r_B - (1-p)\gamma\beta]$,

---

[15]Due to the homotheticity of the production function, the two points U and D lie on the same expansion path.

where $\gamma < 1$, while the ex post one is 1. Since the ex ante price ratio is still greater than the ex post one, again labor becomes cheaper ex post, with an ex post incentive to alter the input combination.

3 The commitment role of trade credit

So far we have shown that when the project needs two inputs and the investment in the pledgeable one is non-contractible, any secured entrepreneur-bank contract (fully or partially) has a problem of input substitution, which causes losses to the bank. The unsecured loan eliminates this problem but also the benefits of collateral.

In this section, we introduce the supplier of the pledgeable input as a second financier. By observing the input transaction, he has a natural information advantage, which the entrepreneur can use to restore his ability to pledge the capital input as a collateral to the bank. In particular, the entrepreneur can sign a partially secured credit contract with the supplier. Observing the input transaction and having a stake in the default state, the supplier guarantees that the input investment is carried out as contracted. This induces the bank to accept a partially secured credit contract as well, mitigating the efficiency loss due to the lack of commitment.[16]

While supplier financing restores the benefits of secured lending, this comes at a cost since trade credit is assumed to be more expensive than bank credit ($r_S > r_B$). To avoid the uninteresting case in which the cost of trade credit is larger than its benefit, we introduce Assumption 1.

Assumption 1 $\frac{r_B}{p} \geq \left(1 - \frac{\gamma}{p}\right) r_S + \left(\frac{\gamma}{p}\right) r_B$, $\forall \gamma \in [0, 1]$.

The left-hand side of Assumption 1 represents the financing cost of the pledgeable input when the entrepreneur does not take trade credit and thus only has access to unsecured bank financing. The right-hand side is the financing cost when the entrepreneur takes trade credit and also has access to secured bank credit. When Assumption 1 holds, the cost of finance under the unsecured bank contract is no less than under any mix of secured trade and bank credit. Thus, taking trade credit is beneficial to the entrepreneur. The cost on the right-hand side of the expression in Assumption 1 is a weighted average of the fund-raising costs of the bank (r_B) and supplier (r_S) with weights that depend on the share of inputs pledged as collateral (γ goes to the bank and $1 - \gamma$ to the supplier) and on the probability of default. When default is very likely and the bank liquidates most of the entrepreneur's inputs (γ close to 1), having access to a secured bank loan through trade credit is enormously beneficial, since it reduces the overall cost of financing the inputs. In the extreme case in

---

[16]For trade credit to work as a commitment device, we must assume that it is too costly for the entrepreneur to resell the inputs purchased on credit on a secondary market.

which $\gamma = 0$, the right-hand side of Assumption 1 reaches its upper bound, r_S. In this case, it can still be optimal to take trade credit if $\frac{r_B}{p} \geq r_S$. This condition only depends on the model's parameter values and turns out to be a sufficient condition for Assumption 1 to be satisfied.

To find the optimal entrepreneur-bank-supplier contract, we proceed in three steps. First, we find the contract for a generic γ. Next, we show that if the amount of trade credit is too small (γ too high), the entrepreneur could still "cheat" at the expense of the bank. Finally, we derive the entrepreneur's profit under deviation and introduce the incentive compatibility condition that ensures that deviation is unprofitable. This allows us to determine the incentive-compatible γ and to fully characterize the optimal contract.

When the bank and supplier can both provide external financing, the optimization problem is given by programme $\mathcal{P}^\gamma$:

$$\max_{L_B, L_S, R_B, R_S I_K, I_N, \gamma} p\left[f\left(I_K, I_N\right) - R_B - R_S\right], \tag{9}$$

$$\text{s.t. } pR_B + (1-p)\gamma\beta I_K \geq L_B r_B, \tag{10}$$

$$pR_S + (1-p)(1-\gamma)\beta I_K \geq L_S r_S, \tag{11}$$

$$L_B + L_S \geq I_N + I_K, \tag{12}$$

$$R_S \geq (1-\gamma)\beta I_K, \tag{13}$$

where (9) denotes the entrepreneur's expected profit, and (10) and (11) are the participation constraints of the bank and the supplier, respectively. Condition (12) is the resource constraint when trade credit is also available, while constraint (13) requires repayments to the supplier be non-decreasing in revenues.[17] Competition among banks and among suppliers implies that (10) and (11) are binding. In order to minimize the reliance on costly trade credit, the entrepreneur would let the supplier act only as a liquidator, pledging him the minimum collateral that guarantees commitment, and setting the repayment in the good state R_S equal to zero. However, this implies that returns be decreasing in revenues, violating constraint (13). Thus, R_S must be set at the minimum level satisfying constraint (13), which is then binding. In turn this implies that the supplier gets a flat contract.

Proposition 3 describes the solution to programme $\mathcal{P}^\gamma$.

Proposition 3 *The entrepreneur-bank-supplier contract has investment $I_K^*(\gamma), I_L^*(\gamma)$, solving (32) and (33) in the Appendix, and displays the following properties:*

---

[17]Constraint (13) is standard in the literature (Innes, 1990). Most observed financial contracts have non-decreasing repayments.

a. *The supplier gets a secured contract with flat repayments across states: an amount $L_S^*(\gamma) = \frac{1}{r_S}(1-\gamma)\beta I_K^*(\gamma)$ is lent in exchange for the right to a share $1 - \gamma$ of the collateral value of the unused inputs $\beta I_K^*(\gamma)$ in the default state and a repayment $R_S^*(\gamma) = (1-\gamma)\beta I_K^*(\gamma)$ in the good state; the share of inputs bought on account $\frac{L_S^*(\gamma)}{I_K^*(\gamma)}$ is decreasing in γ.*

b. *The bank gets a secured contract with increasing repayments: an amount $L_B^*(\gamma) = I_N^*(\gamma) + \left[1 - \frac{1}{r_S}(1-\gamma)\beta\right]I_K^*(\gamma)$ is lent in exchange for the right to a share γ of the collateral value of the unused inputs $\beta I_K^*(\gamma)$ in the default state and a repayment $R_B^*(\gamma) = \frac{1}{p}[L_B^*(\gamma)r_B - (1-p)(1-\gamma)\beta I_K^*(\gamma)] > \gamma\beta I_K^*(\gamma)$ in the good state. $L_B^*(\gamma)$ is increasing in γ.*

c. *Expected profits are increasing in γ and β and are given by:*

$$\Pi^*(\gamma) \equiv pf(I_K^*(\gamma), I_N^*(\gamma)) - (I_K^*(\gamma) + I_N^*(\gamma))r_B + \left[\frac{r_B}{r_S}(1-\gamma) + (\gamma - p)\right]\beta I_K^*(\gamma). \quad (14)$$

d. *Asset tangibility $\frac{I_K^*(\gamma)}{I_N^*(\gamma)}$ is increasing in γ.*

In Proposition 3, we derive the properties of the optimal contract for a generic γ. The parameter γ is the fraction of collateral that goes to the bank and is crucial in our story because it affects both the entrepreneur's profit and his incentive to deviate. Since bank credit is cheaper than trade credit, the higher the γ, the higher the reliance on bank credit, and the higher the entrepreneur's profit (see point c of Proposition 3). Thus the entrepreneur would like to set γ as high as possible. However, the higher the γ, the higher his incentive to deviate from the ex ante contract. The reason is the following. When the entrepreneur decides to deviate, both the bank and the supplier's low state return are jeopardized. However, unlike the bank, prior to the contract execution the supplier observes the change in capital input provision and refuses to sell the inputs on credit, thus implying a lower total loan and the scaling down of production by the entrepreneur. The entrepreneur faces therefore a trade-off when deviating: on one side, the increase in production due to a change in the input mix, and on the other side the fall in production because of the forgone trade credit. It turns out that, by lowering γ, the bank can make the fall in production larger, thus affecting the trade-off and the entrepreneur's incentives to deviate. The issue is then to determine the maximum γ that prevents deviation (i.e., the value of γ that makes the entrepreneur indifferent between a mix of secured bank and trade credit and the deviation contract with forgone trade credit). To this aim, we define the profit from deviation and derive the incentive compatibility constraint that prevents it.

Definition 1 *Define the entrepreneur's expected profit after deviating from the contract specified in Proposition 3 as:*

$$\Pi^F(\gamma) \equiv p\left[f\left(I_K^F, L_B^*(\gamma) - I_K^F\right) - R_B^*(\gamma)\right], \tag{15}$$

where $I_K^F\left(L_B^*(\gamma)\right)$ is the level of capital chosen under deviation and satisfying programme $\mathcal{P}^{\mathcal{F}}$ in the Appendix.

Given all profitable deviations, the optimal entrepreneur-bank-supplier contract that is incentive compatible requires that deviation be less profitable than honoring the ex ante efficient contract. This is ensured by the following incentive compatibility constraint:

$$\Pi^F(\gamma) - \Pi^*(\gamma) \leq 0. \tag{16}$$

Solving programme $\mathcal{P}^\gamma$ under constraint (16) leads to Proposition 4.

Proposition 4 *Under mild conditions, the entrepreneur-bank-supplier contract that prevents deviation has $0 \leq \gamma \leq \gamma^*$, where γ^* satisfies condition (16) with equality. Since the entrepreneur's expected profit $\Pi^*(\gamma)$ is increasing in γ, the entrepreneur will offer the contract with the highest possible γ, i.e., $\gamma = \gamma^*$.*

In Proposition 4, we identify the optimal share of collateral going to the bank as the upper bound of the set of collateral shares that are incentive-compatible for the entrepreneur. Using γ^* in Proposition 3, we fully characterize the optimal three-party contract. Trade credit is used as a commitment device and its amount is the lowest possible that makes commitment credible to the bank.

Point E in Figure 2 depicts the input combination and the level of output corresponding to the entrepreneur-bank-supplier contract implied by Proposition 4. To see why this contract is incentive-compatible, suppose that initially the entrepreneur signs the three-party contract (point E) and then decides to deviate. Upon observing the entrepreneur altering the input mix, the supplier refuses to sell inputs on credit. This implies a decline in external financing, with a subsequent decline in the scale of production that makes deviation costly to the entrepreneur. By construction of the entrepreneur-bank-supplier contract, this decline is such that the entrepreneur is indifferent between sticking to the original contract and deviating. Graphically, the new optimum under deviation is at point D, which lies on the same isoquant as point E and thus involves the same level of output. The vertical distance between the isocost lines intersecting point E and tangent to point D represents the amount of trade credit the entrepreneur has to renounce to make the contract incentive-compatible. This guarantees that point E is the equilibrium outcome.
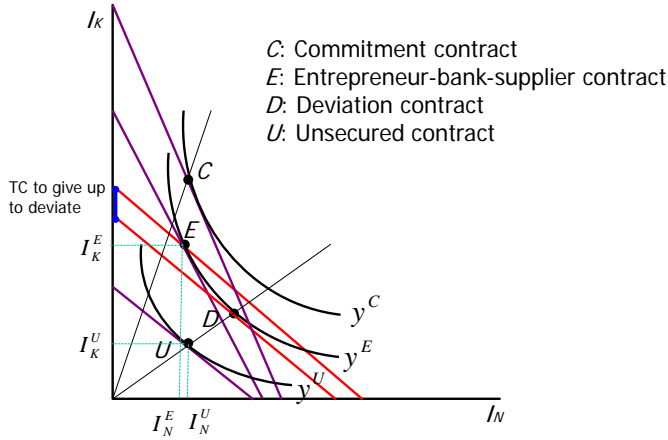
Figure 2: The commitment role of trade credit. Points C, E, and U represent the optimal input combination and the production level under the commitment, the entrepreneur-bank-supplier and the unsecured contract, respectively. Point D represents the production level if the entrepreneur deviates from the entrepreneur-bank-supplier contract. Points D and E lie on the same isoquant.

This discussion implies that whether deviating from the original contract is profitable or not depends on the amount of trade credit the entrepreneur is getting under the original contract, which corresponds to the amount he has to forgo in case of deviation. It follows that the bank can always prevent deviation by reducing its supply of financing (i.e., by reducing γ). By doing so, the bank forces the entrepreneur to give up a larger amount of trade credit when deviating. This reduces production so much as to offset the benefit of deviation. The commitment effect of trade credit rests upon two features of the supplier's contract. First, the supplier provides a share of the capital input on credit. Second, the supplier has the right to a share of its collateral value in case of default. Both conditions have to be satisfied for the entrepreneur to have no incentive to alter the input mix ex post.

Point E lies between point C (commitment contract) and point U (unsecured contract) in Figure 2. The three-party contract does not allow the entrepreneur to achieve point C since the cheaper bank credit is partially replaced by the more expensive trade credit. However, it generates larger profit than the unsecured bank contract. By signaling that the bank loan will be used to purchase the inputs as specified, trade credit facilitates access to secured bank financing.

Notice that the guarantee implicitly offered in our model by the supplier shares some similarities with a standby letter of credit. This letter is issued by a financial institution on behalf of the buyer of some goods to guarantee that the seller will be paid on time and for the correct amount. It works as a credit-enhancement device and facilitates trade. In our story, the supplier also works as a credit enhancement device by guaranteeing that there is enough collateral to repay the bank in case of default, and fosters secured bank lending. Thus, while with a letter of credit it is the bank that provides a guarantee to the seller on behalf of the buyer, in our case it is the supplier that provides

a guarantee to the bank (still on behalf of the buyer). However, besides this similarity, there is an important difference between a letter of credit and the implicit guarantee offered by the supplier in our story. Indeed, while a letter of credit brings credit risk to the underwriting financial institution, which becomes liable in case of buyer insolvency, the supplier in our model does not bear any additional credit risk and eliminates any risk that the bank will not be fully repaid in default.

3.1 Testable predictions

The discussion above allows us to derive testable predictions on the relation between optimal financial contracts and the characteristics of the assets invested into the project. Since trade credit allows entrepreneurs to access secured bank financing, there is a positive relation between the joint use of trade/secured bank credit and the degree of asset pledgeability. Point E in Figure 2 has indeed an input combination more intensive in pledgeable assets than point U. Related empirical literature classifies balance-sheet data on assets into tangibles (with high collateral value as plant, property and equipment) and intangibles (with low collateral value as patents, goodwill and trademarks). If we use tangibility as an empirical proxy for the degree of asset pledgeability, we obtain Prediction 1.

Prediction 1. *Entrepreneurs are more likely to finance investments intensive in tangible assets with both secured bank and trade credit than with unsecured bank credit only.*

The collateral value of an input also depends on its liquidation or scrap value β (i.e., its second-hand market value, also called degree of input redeployability). While the entrepreneur's profits under the three-party contract are increasing in input liquidation value, as shown in Proposition 3, profits under the unsecured contract do not depend on β, because under such a contract there is no collateral pledging. Thus the benefits of trade credit (i.e., the difference between the profits under the commitment and under the unsecured contract) are larger when the input liquidation value is larger. To some extent, the input liquidation value reflects industry characteristics. For example, standardized goods are likely to have greater scrap value than differentiated products or services. We can thus state Prediction 2.

Prediction 2. *Secured bank and trade credit are more likely to be used by entrepreneurs buying standardized inputs than by entrepreneurs buying differentiated inputs or services.*

The reliance on trade credit also depends on the degree of substitutability between inputs. The commitment problem arises because the entrepreneur can change the input combination after the loan has been granted. For trade credit to play a role as a commitment device, inputs need to be at least partial substitutes. With a higher degree of substitutability between inputs, the commitment problem is more severe and trade credit becomes more valuable. In the extreme case of inputs used in fixed

proportions, there is no extra-profit to be gained by changing the input combination. Since there is no incentive to deviate, the entrepreneur can access secured bank financing with no need to use trade credit. Thus, we obtain Prediction 3.

Prediction 3. *Secured bank and trade credit are more likely to be used by entrepreneurs with technologies that have input substitutability than by entrepreneurs with technologies that have inputs used in fixed proportions.*

3.2 A numerical example

In this section, we consider a numerical example to illustrate how trade credit can improve on bank financing when the entrepreneur is unable to commit to the investment level set at the contracting stage with the bank. We use the following generic parameter values consistent with Assumption 1: $Y \in \{0, y\}$, with $y = AI_K^a I_L^b$, where $A = 20$, $p = .5$; $a = b = .4$; $w = \rho = 1$; $r_B = 1$; $r_S = 1.05$; $\beta = 0.7$.

Commitment contract. If the entrepreneur can commit to the investment level set at the contracting stage with the bank, it relies only on bank credit. Under our parameter assumptions, the optimal contract, defined in Proposition 1., implies an investment in capital $I_K^C = 3,728$ and labor $I_L^C = 2,424$. Even if the selling price of the two inputs is the same ($w = \rho = 1$), the actual input price ratio ($r_B / [r_B - (1 - p)\beta]$) is roughly equal to 1.54. With a ratio greater than one, capital is relatively less expensive than labor (because of the collateral pledged in the bad state), which explains why capital investment is larger than labor investment. Input purchase is fully financed by secured bank financing for an amount $L_B^C = 6,152$. The bank gets the repayment $R_B^C = 9,695$ in the good state and the collateral value of capital inputs $C = \beta I_K^C = 2,610$, in the bad state. Production is $y^C = 6,059$ and profits are $\Pi^C = 1,212$.

Deviation contract. Suppose now that the entrepreneur cannot commit to the investment level set at the contracting stage. The bank provides the loan of the commitment contract $L_B^C = 6,152$, expecting returns $R_B^C = 9,695$ and $C = \beta I_K^C = 2,610$, in the good and bad state, respectively. Upon being granted credit, the entrepreneur has an incentive to re-optimize and choose a different input mix. While the entrepreneur will make sure to meet the bank good-state repayment obligation, he is not concerned with repaying the bank in the bad state. This implies that the entrepreneur solves a maximization problem where no collateral is pledged. Therefore, the actual input price ratio is now equal to one, implying an equal amount of capital and labor $I_K^D = I_L^D = 3,076$. With the input price ratio under deviation (called ex post) lower than the ratio under the commitment contract (called ex ante), $1 < 1.54$, capital is more expensive in the deviation contract (and labor cheaper), which explains

why the entrepreneur wants to deviate and reduce the investment in capital below the commitment level. Under the new input choice, both production and profits increase to $y^D = 6,172 > y^C$ and $\Pi^D = 1,324 > \Pi^C$, respectively. While the entrepreneur is clearly better off, the bank is worse off. Because of the lower investment in capital input, the bank return in the bad state (i.e., the capital input liquidation value) is strictly lower than the contracted one: $\beta I_K^D = 2,153 < 2,610 = \beta I_K^C$. Thus, the bank no longer breaks even and the deviation contract is not an equilibrium contract.

Unsecured contract. Anticipating the entrepreneur's deviation, the bank offers the fully unsecured contract described in Proposition 2. This contract implies a smaller loan amount than in the commitment contract $L_B^U = 2,048 < L_B^C = 6,152$, due to the entrepreneur's inability to pledge the collateral as partial repayment of the loan. The bank gets a repayment only in the good state equal to $R_B^U = 4,096$. Since the contract is unsecured, there is no incentive to deviate. Moreover, an equal amount of capital and labor is invested, $I_K^U = I_L^U = 1,024$. Due to the lower loan size, production and profits are strictly lower than in the commitment contract: $y^U = 2,560 < y^C = 6,059$ and $\Pi^U = 512 < \Pi^C = 1,212$. Thus, the unsecured contract eliminates the entrepreneur's incentive to deviate but also the benefits of the collateral. The entrepreneur would be better off if he could credibly commit to the input combination agreed in the commitment contract.

Entrepreneur-bank-supplier contract. To credibly commit to the input mix specified in the commitment contract, the entrepreneur takes trade credit. Since trade credit is more expensive than bank credit, it is optimal to take the minimum amount of trade credit that stops deviation and rely for the rest on the cheaper bank credit. Trade credit is $L_S^* = 374$ and it allows the entrepreneur to access an amount of secured bank financing equal to $L_B^* = 5,654$. Both financiers get a repayment in the good state: $R_B^* = 9,151$, $R_S^* = 392$. In the bad state, the collateral value of capital inputs is shared, with a fraction $1 - \gamma^* = 0.15$ going to the supplier and $\gamma^* = 0.85$ to the bank. With trade and bank credit together, the entrepreneur gets a total loan strictly larger than the one received under the unsecured contract and close to the one under the commitment contract, i.e., $L_B^U = 2,048 < (L_B^* + L_S^*) = 6,028 < L_B^C = 6,152$, with trade credit being only 6% of the total loan. This allows the entrepreneur to invest $I_K^* = 3,641$ and $I_L^* = 2,386$ in capital and labor, respectively, obtaining slightly lower profits than in the commitment contract, but strictly larger profits than in the unsecured contract: $\Pi^C = 1,212 > \Pi^* = 1,193 > \Pi^U = 512$.

4 Entrepreneur-supplier collusion

In Section 3, we argued that trade credit enables the entrepreneur to overcome the problem of commitment with the bank. This is because the supplier will always refuse to extend credit upon observing the entrepreneur's deviation, as he will fail to break even on the new input combination. This might not be true anymore if the contract between the entrepreneur and the supplier were renegotiated to allow the supplier to at least break even. In this section, we extend the model to allow for collusive agreement between the entrepreneur and the supplier.

Suppose that the entrepreneur, the bank, and the supplier have agreed on the contract terms described in Proposition 3, with $\gamma = \gamma^*$. Once the loan from the bank is obtained, the entrepreneur may then seek an agreement with the supplier to alter the input mix at the expense of the bank (i.e., to reduce the investment in capital and increase that in labor). If the supplier is to accept, they must renegotiate the contract terms (i.e., loan size and repayments), so as to enable the supplier to at least break even:[18]

$$pR_S + (1-p)(1-\gamma)\beta I_K \geq L_S(\gamma) r_S. \tag{17}$$

If agreed, the new arrangement allows an increase in overall profits at the expense of the bank. This allows us to define the gross collusion rent and describe its properties in Proposition 5.[19]

Definition 2 *Define $\Pi^{COL}(\gamma)$ as the profit from collusion for a generic γ solving programme $\mathcal{P}^{COL}$ in the Appendix and $\Pi^{COL}(\gamma) - \Pi^*(\gamma)$ as the gross collusion rent, where $\Pi^*(\gamma)$ is the profit from the entrepreneur-bank-supplier contract as defined in Proposition 3 for a generic γ.*

Proposition 5 *The gross collusion rent $\Pi^{COL}(\gamma) - \Pi^*(\gamma)$ is increasing in γ and is zero iff $\gamma = 0$. Thus, any entrepreneur-bank-supplier contract is vulnerable to collusion between the entrepreneur and the supplier at the expense of the bank.*

In Proposition 5, we posit that a collusive agreement to alter the input combination ex post is always profitable for the entrepreneur and the supplier. Thus, the entrepreneur-bank-supplier contract is not collusion-proof. The only way for the bank to stop other parties from colluding is to offer an unsecured contract ($\gamma = 0$). However, reaching a collusive agreement may be costly. This cost may be due to the time and effort spent in writing and enforcing the side contract. It may also be the effect

---

[18]An alternative collusive agreement would keep the loan size fixed ($L_S^*(\gamma^*)$) and renegotiate only the repayments schedule. However, while this would not alter the qualitative properties of the collusion-proof contract, there is no real reason to impose this constraint on the renegotiation. Moreover, renegotiating all the contract terms is profit-maximizing, as it gives the entrepreneur the possibility of reducing its reliance on costly trade credit.

[19]For simplicity we assume that any collusive rent is seized by the entrepreneur. This is without loss of generality, as alternative distributions do not alter our qualitative results.

of reputational concerns.[20] To capture this cost, define $\alpha \in [0,1]$ as the fraction of the profit from collusion that is lost in reaching such an agreement and $[(1 - \alpha)\Pi^{COL}(\gamma) - \Pi^*(\gamma)]$ as the net rent from collusion. This formulation enables us to find an interior γ that stops parties from colluding. In particular, for a sufficiently high α, the bank can always prevent collusion by focusing on contract offers that guarantee a non-positive collusion rent, i.e., those that satisfy the following constraint:

$$(1 - \alpha)\Pi^{COL}(\gamma) - \Pi^*(\gamma) \leq 0. \tag{18}$$

Is the entrepreneur-bank-supplier contract collusion-proof when we measure the collusion rent net of bargaining costs? More specifically, at $\gamma = \gamma^*$ (the value of γ that ensures no unilateral incentive to deviate), is constraint (18) satisfied? The answer depends on the cost of collusion, α. Let us define $\alpha^*(\gamma^*)$ as the value of α at which the collusion-proof constraint (18) is satisfied with equality when $\gamma = \gamma^*$. When $\alpha \geq \alpha^*(\gamma^*)$, the value of γ that solves (18) is greater than γ^*, so that the entrepreneur-bank-supplier contract can also accommodate collusion. When $\alpha < \alpha^*(\gamma^*)$, the value of γ that solves (18) is less than γ^* and the entrepreneur-bank-supplier contract is open to collusion. This amounts to saying that the value of γ that accommodates both unilateral deviation and collusion has to solve the following global collusion proofness constraint:

$$\max\left\{(1 - \alpha)\Pi^{COL}(\gamma), \Pi^F(\gamma)\right\} - \Pi^*(\gamma) \leq 0. \tag{19}$$

This allows us to state the result in Proposition 6 and then to derive Corollary 1.

Proposition 6 *The collusion-proof entrepreneur-bank-supplier contract has $\gamma = \hat{\gamma}(\alpha) \leq \gamma^*$, where $\hat{\gamma}(\alpha)$ satisfies condition (19) with equality. The properties of this contract are those described in Proposition 3, with $\gamma = \hat{\gamma}(\alpha)$.*

Proposition 6 implies that the properties of the collusion-proof contract, and thus the benefits of using trade credit, depend on the cost of collusion. Three scenarios can arise. First, collusion can be so costly that it is never profitable: $\alpha(\gamma^*) \leq \alpha \leq 1$. The rent from collusion is lower than the rent from deviation, so that the global collusion-proof condition (19) coincides with the incentive-compatibility condition (16). This corresponds to the case already analyzed in Section 3: The entrepreneur buys commitment from the supplier through trade credit and takes a partially secured loan from the bank. The share of capital inputs bought on account (through trade credit) is $\beta(1 - \gamma^*)$ and the share paid in cash (through bank credit) is $\beta\gamma^*$. Both shares are independent of α. The equilibrium is at point E in Figure 2 and the input combination is capital intensive.

---

[20]For a discussion of the main issues concerning collusion in contracts, see Tirole (1992).

Second, collusion may be costly, but profitable: $0 < \alpha < \alpha(\gamma^*)$. The rent from collusion is greater than from deviation, so that the global collusion-proofness condition (19) coincides with the collusion proofness condition (18). The lower the cost of collusion, the larger the amount of trade credit necessary to make the contract collusion-proof, and the lesser the bank's participation in the venture. It follows that the lower the cost of collusion, the larger the share of inputs bought on account through trade credit, and the lower the one bought through bank credit. The properties of the optimal contract are those described in Proposition 3, with $\gamma = \hat{\gamma}(\alpha)$. The equilibrium point is located between points E and U in Figure 2, the exact position depending on the cost of collusion. The lower the cost, the further the equilibrium shifts from point E to point U, with the input combination becoming less capital-intensive.

Lastly, collusion may be costless ($\alpha = 0$). The entrepreneur and the supplier can grab the entire surplus from their agreement. In this case, the only contract that enables the bank to break even is the unsecured one (point U in Figure 2). Since the bank does not have a stake in the bad state return any longer, the bank is reimbursed only in the good state. This arrangement removes any incentive to collude.

From Proposition 6 and the above discussion, we can derive Corollary 1.

Corollary 1 *Under the collusion-proof contract, the share of inputs bought by the entrepreneur on credit is decreasing in the cost of collusion α, while the share of inputs paid for in cash, asset tangibility, and expected profits are increasing in α.*

5 Relationship lending and trade credit in a multi-period setting

So far we have shown that when the investment in the capital input is not contractible, any secured bank contract is time-inconsistent and trade credit can be used to solve the problem and give the entrepreneur access to secured bank financing. These results depend on the information advantage of the supplier vis-à-vis the bank: The supplier, as a provider of the input, knows the investment level in capital, the bank does not. This argument could suggest that our conclusions strictly depend on the static nature of the credit relations. In a dynamic setting, the bank could exploit the repeated interaction to set up a long-term relationship, where information about the past investment level can be collected and used to determine future contract terms, possibly incorporating a penalty for misbehavior. Anticipating a future punishment in case of deviation, the entrepreneur would have an incentive to abide by the contract and there might be no need to use trade credit. The threat of a punishment would thus work as an alternative commitment device.

We formally study this possibility by extending the static baseline model to a multi-period setting, similar in spirit to Boot and Thakor (1994), where an intertemporal price-adjustment mechanism is used to reduce efficiency losses from using collateral. In Section 5.1, we assume the information collected by the bank to fully reveal the past investment level (perfect signal). In Section 5.2, we consider instead the case of a noisy signal. We show that trade credit is still the most efficient way to solve the commitment problem, provided that the duration of the project is not too long.

5.1 A perfect signal

Structure of the repeated game. Consider an economy where the entrepreneur has to finance the project defined in Section 1. This project has a duration of $t = T > 1$, that is, it can be repeated for a finite number of periods, T. The entrepreneur enters a long-lasting lending relation with one bank and signs a long-term contract that specifies the contract terms for any $t = 1, ..., T$, contingent on some information about the past investment level. We denote this financing scheme *relationship lending*, because the bank gathers proprietary information through the repeated interaction with the borrower and uses it to set future contract terms.[21]

In each period, the entrepreneur faces the commitment problem described in Section 2. He can invest the amount of inputs specified in the commitment contract (cooperation), gaining the commitment profits Π^C defined in Proposition 1. Alternatively, he can choose a different input mix (deviation) and gain the deviation profits Π^D (equation 8). At the end of each period, the bank obtains a signal revealing with certainty the chosen investment level, which can be used to set the following period's contract terms.[22]

To derive the sequence of profits conditional on each possible decision and on the signal received by the bank, let us start from the last period. At $t = T$, the parties cannot agree on a commitment contract as the entrepreneur surely deviates. Thus, the only contract the parties will agree on is an unsecured contract, whose price depends on the signal received by the bank. In particular, if the information reveals cooperation, they agree on the unsecured contract derived in Proposition 2 and the entrepreneur gets the resulting profits, Π^U. If the information reveals deviation, the bank penalizes the entrepreneur by charging a higher interest rate (punishment contract) and the entrepreneur gets profits $\Pi^P < \Pi^U$.[23] Profits Π^P are defined later.

---

[21]See Boot (2000) for a review of the literature on relationship lending.

[22]Assuming that the entrepreneur can collect this information also in the static setting would not alter any of our results, since the information can only be used with one period lag.

[23]In the last period of the game, the contracts need to be contingent on the information revealed. If not, we end up with the standard result of the repeated games literature that it is not possible to elicit cooperation in a finite horizon model since the parties would unravel the game.

At any time $t < T$, instead, the parties agree on a commitment contract if the signal collected by the bank revealed commitment by the entrepreneur in the previous period, or else on a punishment contract for only one period if the signal revealed deviation, followed by a commitment contract in any successive period, until a deviation signal triggers a punishment contract again.

Cooperation as an optimal strategy. Given the strategy defined above, we have a sequence of T entrepreneur's decisions with associated profits. For simplicity, we assume that the discount factor is zero.[24] In Proposition 7, we posit the condition under which cooperation in all periods weakly dominates deviation in any one period.

Proposition 7 *Under the strategy defined above, cooperation weakly dominates deviation iff:*

$$\Pi^D - \Pi^C \le \Pi^U - \Pi^P. \tag{20}$$

Condition (20) indicates that in any T-period model, the incentive constraint that guarantees cooperation reduces to a comparison between the one-period deviation benefit (left-hand side) and the last period deviation cost (right-hand side). Therefore, the entrepreneur will cooperate if the benefit from deviation is lower than the cost. This crucially depends on the punishment profits Π^P, and thus on the penalty P the entrepreneur has to pay when deviation is detected by the bank.

The next step is to derive the minimum penalty that prevents deviation (i.e., satisfies condition (20) with equality). Suppose that in a generic period the signal reveals that the entrepreneur has deviated. This triggers the punishment contract in the following period. This contract is unsecured as the one defined in Section 3, but with a higher interest rate r'_B that incorporates the penalty P for deviation. The terms of the punishment contract and the corresponding investment levels are thus set to solve the following optimization programme $(\mathcal{P}^{\mathcal{P}})$:

$$\max_{I_K, I_N, L_B, R_B, P} p\left[f\left(I_K, I_N\right) - R_B\right]$$
$$\text{s.t. } pR_B = L_B r'_B \tag{21}$$
$$L_B = I_K + I_N$$

under the additional no-deviation condition (20). The constraints above have the usual meaning, the only novelty being the *punishment* interest rate $r'_B \equiv r_B + \frac{P}{L_B}$ in the individual rationality constraint (21). This interest rate is made of two components: the market interest rate, r_B, plus an extra variable cost calculated as a fraction of the penalty P the entrepreneur has to pay upon deviation on the total loan received, L_B.

---

[24]This is without loss of generality. A positive but not too high discount factor would not alter our qualitative results.

Solving programme $\mathcal{P}^{\mathcal{P}}$, we obtain the entrepreneur's profits under the punishment contract as:[25]

$$\Pi^P \equiv pf\left(I_K^P, I_N^P\right) - L_B^P\left(r_B + \frac{P}{L_B^P}\right) < \Pi^U. \tag{22}$$

The analysis above posits that under a sufficiently high punishment, relationship lending eliminates the entrepreneur's incentive to deviate and provides a viable solution to the commitment problem of secured lending.

Relationship lending versus trade credit. We now compare two alternative ways to solve the commitment problem of secured financing. On the one hand, we have relationship lending with a long-term contract that specifies a sequence of credit amounts and repayment obligations contingent on the borrower's investment history. On the other hand, there is a sequence of single-period trade and bank credit contracts, independent of past project investment and with potentially different suppliers and lenders at each point in time. For brevity, we use the term *trade credit* to identify this three-party contract. If the entrepreneur uses relationship lending, he signs the long-term bank contract that guarantees cooperation in all periods, getting the commitment profits Π^C derived in Proposition 1 for $T - 1$ periods and the profits from the unsecured contract Π^U in the last period. The total profits under relationship lending are thus $(T - 1)\Pi^C + \Pi^U$. If he uses trade credit, he signs the three-party contract defined in Proposition 3 repeatedly for any of the T periods and gets the expected profits $\Pi(\gamma^*)$, renamed here Π^{TC} in each period. Thus, the total profits under trade credit are $T\Pi^{TC}$.

It follows that relationship lending is preferred to trade credit if the following condition is satisfied:

$$(T - 1)\Pi^C + \Pi^U \geq T\Pi^{TC}. \tag{23}$$

When $T = 1$, condition (23) is never satisfied since $\Pi^U < \Pi^{TC}$. This case coincides with the baseline model: when the project lasts for one period, the commitment problem can only be solved by using trade credit. When the game is repeated, cooperation can be achieved and the entrepreneur can gain the commitment profits Π^C for $T - 1$ periods at the cost of getting lower profits Π^U in the last period for his inability to credibly commit. With increasing iterations, the weight of Π^U reduces while that of Π^C increases, so that at some point it becomes more profitable to enter a long-term relationship with the bank rather than relying on trade credit. This discussion is summarized in Proposition 8.

Proposition 8 *Under perfect signal, there is critical duration of the project:*

$$T^* = 1 + \frac{\Pi^{TC} - \Pi^U}{\Pi^C - \Pi^{TC}} > 1, \tag{24}$$

---

[25]Details of the derivation of the minimum penalty P and the punishment profits Π^P are in the Appendix.

such that the entrepreneur is indifferent between relationship lending and trade credit. For $T < T^*$, the entrepreneur strictly prefers trade credit, while for $T \geq T^*$, the entrepreneur weakly prefers relationship lending. The threshold T^* is increasing in the collateral liquidation value β.

The threshold T^* represents the minimum duration of the project that makes relationship lending preferred to trade credit. The term $\Pi^{TC} - \Pi^U$ in (24) is the cost of using relationship lending vis-à-vis trade credit. To sustain cooperation, an unsecured contract must be signed in the last period, so the entrepreneur is giving up Π^{TC} in exchange for Π^U. The term $\Pi^C - \Pi^{TC}$ is instead the benefit of using relationship lending vis-à-vis trade credit in any one of the $(T-1)$ periods of the game. Proposition 8 leads to Prediction 4.

Prediction 4. *Entrepreneurs use trade credit for projects with short duration and relationship lending for projects with long duration.*

Researchers have investigated relationship lending, but they have so far focused on the choice between relationship lending and arm's length debt, rather than trade credit. Interestingly, this evidence goes in the same direction as our prediction by documenting that relationship lending is more likely when the length of the repeated interaction between the bank and the entrepreneur is sufficiently long (Agarwal and Hauswald, 2008).

Proposition 8 also posits that the threshold T^* increases for projects using assets with higher liquidation value (with high β). The intuition is that a higher collateral liquidation value increases the cost of relationship lending more than its benefit (since both Π^C and Π^{TC} increase), making trade credit relatively more attractive to the entrepreneur. This allows us to derive Prediction 5.

Prediction 5. *Entrepreneurs are more likely to use trade credit than relationship lending when the collateral of inputs is higher.*

5.2   A noisy signal

In Section 5.1, we have assumed that the bank collects information revealing with certainty whether the entrepreneur has committed or deviated. In this section, we generalize the model, assuming that the information is noisy and depends on the realization of the state of nature. Upon a low state realization, the value of leftover inputs fully reveals the entrepreneur's behavior: if the entrepreneur has cooperated, the bank's payoff equals the contracted one, while it is lower if he has deviated. Upon a high state realization, the repayment has no information content since the bank's revenues are always sufficient to fully repay the bank, irrespective of the cooperation or deviation decision of the entrepreneur. In this case, the bank receives a signal $\sigma \in \{C, D\}$ that reveals imperfectly whether

the entrepreneur has cooperated or deviated (i.e., whether $\theta = \{C, D\}$, where θ denotes the true entrepreneur's choice). If the entrepreneur has cooperated (i.e., $\theta = C$), the bank gets a signal that reveals it with probability r, while with probability $1 - r$ it gets a false negative (i.e., a signal that the entrepreneur has deviated). We assume that the signal is positively correlated with the entrepreneur's choice (i.e., $\frac{1}{2} < r < 1$). If instead the entrepreneur has deviated (i.e., $\theta = D$), the bank learns it with probability q (it gets a signal $\sigma = D$), while with probability $1 - q$ it gets a false positive (i.e. a signal revealing that the entrepreneur has cooperated). Again, $\frac{1}{2} < q < 1$. Formally:

$$\Pr(\sigma = C, \theta = C) = r$$
$$\Pr(\sigma = D, \theta = C) = 1 - r$$
$$\Pr(\sigma = D, \theta = D) = q$$
$$\Pr(\sigma = C, \theta = D) = 1 - q.$$

Under these assumptions, if the punishment is sufficiently high that cooperation can be sustained in equilibrium, the expected profits that the entrepreneur gets under relationship lending when the project's duration is T are given by:

$$E\Pi|_{C_{(T-1)}} = \frac{\Pi^C + p(1-r)\Pi^P}{1 + p(1-r)} T + \frac{(1 + p(1-r))\Pi^U - p(1-r)\Pi^P - \Pi^C}{(1 + p(1-r))^2} \left[1 - (p(r-1))^T\right]. \quad (25)$$

The derivation of (25) is provided in the Appendix. Comparing the profits under relationship lending (25) and under trade credit ($T\Pi^{TC}$), we get that relationship lending is weakly preferred if $E\Pi|_{C_{(T-1)}} \geq T\Pi^{TC}$. Solving this condition with equality leads to Proposition 9.

Proposition 9 *Under a noisy signal, if $r \geq \max\left\{\frac{1}{2}, \frac{\Pi^C - \Pi^{TC}}{p(\Pi^{TC} - \Pi^P)}\right\}$, there exists a critical duration of the project, $T^{NS}\left(p, r, \Pi^C, \Pi^P, \Pi^U, \Pi^{TC}\right) > T^*$, such that the entrepreneur is indifferent between choosing relationship lending or trade credit. For $T < T^{NS}$, the entrepreneur strictly prefers trade credit, while for $T \geq T^{NS}$, the entrepreneur weakly prefers relationship lending. Moreover, $T^{NS} \to T^*$, as the precision of the signal increases ($r \to 1$).*

Proposition 9 posits that the duration of the project that makes the entrepreneur indifferent between the two alternative financing strategies is longer under the noisy signal than under the perfect signal. The intuition is the following: When the signal is perfect, the bank is fully informed and exploits all the available information in the most efficient way by penalizing the entrepreneur when he has deviated and rewarding him when he has cooperated. When the information is noisy, the bank might penalize or reward the entrepreneur erroneously. This implies that the expected profits under

relationship lending are lower in the case of a noisy signal than in the case of a perfect signal, thereby increasing the critical project duration above which relationship lending is preferred to trade credit.

Proposition 9 also posits that there is a negative relation between the precision of the signal and the threshold level T^{NS}. When the precision of the signal increases, the threshold T^{NS} decreases until reaching T^*, when the signal is perfect. If we assume that the precision of the signal r is positively correlated with the quality/efficiency of the lender assessment procedure, we get Prediction 6.

Prediction 6. *Entrepreneurs are more likely to use relationship lending than trade credit, when the quality of the information collected by the relationship lender is higher (high r).*

Again, although researchers have empirically investigated the choice between relationship lending and arm's length debt (rather than trade credit), their evidence goes in the same direction as our prediction, suggesting a positive relation between the quality of information and relationship lending. For example, Agarwal and Hauswald (2008) use data on new loans granted to small firms by large U.S. financial institutions to document that the better the credit-quality assessment of the bank, the more likely it is that the entrepreneur will apply for relationship lending.

One last remark concerns the degree of generality of our punishment mechanism. We have considered a tit-for-tat mechanism for one period (i.e., upon detecting deviation, the entrepreneur gets the punishment contract only for one period). Afterwards, contracting parties sign a commitment contract again until a new deviation is detected. This punishment mechanism is not crucial for our results. To prove this, we considered an alternative strategy where a deviation signal at $t = i$ triggers a punishment contract at $t = i+1$, followed by a series of unsecured contracts in any successive period up to the final one (for any $t = i+2, ..., T$). It is possible to show that the results are unchanged in the perfect signal case - Proposition 8 holds - while there is no critical duration T^{NS} at which relationship lending becomes preferred when the signal is noisy. In that case, trade credit is the only way to solve the commitment problem for any possible project duration.

5.3 A numerical example

We now illustrate the choice between trade credit and relationship lending with a numerical example. We consider the case of a perfect signal. We use the same parameter assumptions as in the numerical example of the static setting (Section 3.2), where the one-period profits under the commitment, the unsecured, and the three-party contract were $\Pi^C = 1,212$, $\Pi^U = 512$ and $\Pi^{TC} = 1,193$, respectively. Using these profit levels, we first work out the punishment contract that eliminates the entrepreneur's incentive to deviate in a repeated lending relation. We then compare the streams of profits under trade credit and under relationship lending.

Punishment Contract. Once deviation is detected, the entrepreneur and the bank sign the punishment contract defined by programme $(\mathcal{P}^\mathcal{P})$. Under this contract and our parameter values, the bank charges $r'_B = r_B + P/L_B^P = 1,064$ and provides a loan equal to $L_B^P = 1,501$, getting a total repayment in the good state equal to $R_B^P = 3,195$. The investment level is $I_K^P = I_L^P = 751$, and the punishment profits are $\Pi^P = 400$. Anticipating a gain of only $\Pi^P = 400$ after deviation is detected, the entrepreneur will never deviate from the original input combination, since cooperation in any period is more profitable than deviation (i.e., $\Pi^P = 400$ satisfies condition (20) with equality). Thus, with this contract, relationship lending allows the entrepreneur to solve the commitment problem.

Relationship lending versus trade credit. Under a lending relation lasting T periods, the entrepreneur gets the commitment profits $\Pi^C = 1,212$ for $T-1$ periods at the cost of getting the lower unsecured profits $\Pi^U = 512$ in the last period for his inability to credibly commit. Thus, total profits are: $\Pi^C(T-1) + \Pi^U = (T-1)1,212 + 512$. If the entrepreneur uses trade credit instead, he signs repeatedly for T periods the three-party contract derived in Section 3, getting total profits equal to $T\Pi^{TC} = 1,193T$. It follows that whether relationship lending is preferred to trade credit depends on the duration of the project. Substituting the values of Π^C, Π^U and Π^{TC} in condition (24), we find that $T^* = 37$ is the minimum duration of the project that makes relationship lending preferred to trade credit. This threshold implies that the entrepreneur relies on a sequence of secured trade and bank credit contracts to finance projects with a duration up to 36 periods, while he prefers relationship lending for longer projects.

6   Robustness

In this section, we discuss the robustness of our theoretical setting. In Subsection 6.1, we investigate whether alternative bank contracts could solve the commitment problem. In Subsection 6.2, we discuss the assumption of exclusive lending relations. In Subsection 6.3, we examine the role of the supplier as an informed lender. In Subsection 6.4, we discuss the role of information sharing between financiers.

6.1   Can alternative bank contracts solve the commitment problem?

In Section 5, we show that the commitment problem could be overcome in a multi-period lending relationship if the duration of the project is sufficiently long. In this section, we investigate several possibilities as to whether alternative contracts can solve the commitment problem in a static setting.

One possibility could be a loan commitment contract. One strand of the literature shows that loan commitments may solve problems of asymmetric information (Boot, Thakor, and Udell, 1987; Kanatas,

1987; Thakor, 1982; Greenbaum, Kanatas, and Venezia, 1991; Berkovitch and Greenbaum, 1991). Most of these papers view the loan commitment as an insurance against the borrower's deterioration in creditworthiness due to an agency problem. Boot, Thakor, and Udell (1987), for example, show that a pricing structure combining a commitment fee (paid at the commitment's inception) and a lower interest rate allows to mitigate moral hazard problems. The intuition is that the interest rate, which affects the effort choice, is set low enough to induce the entrepreneur to exert high effort, while the initial fee is set so as to allow the bank to break even. Similarly, Boot, Greenbaum and Thakor (1993) show that a loan commitment can resolve an asset-substitution moral hazard problem. Using a loan commitment with a similar repayment structure in our setting does not mitigate the entrepreneur's incentive to deviate. In our model, the incentive problem is due to the wedge between the ex ante and the ex post input price ratio of the secured contract. Setting the bank repayment in the good state sufficiently low would not change this wedge and thus the incentive to deviate would be unaffected. An alternative contract could be the unsecured loan commitment. This contract would remove the incentive problem, since the ex ante and the ex post relative input prices would be equal, but it would also eliminate the need to use a loan commitment contract. The unsecured debt contract analyzed in Section 3 would be equivalent to the loan commitment, with both contracts providing lower profits than the three-party contract with bank and trade credit.

A second possibility could be borrowing instruments from the risk-shifting literature. Smith and Warner (1979), among others, show that some asset substitution problems that induce excess risk-taking can be mitigated by introducing debt covenants. In our context, covenants that specify the input investment levels could solve the commitment problem. In practice, however, covenants do not specifically limit investment policy, possibly because they would be too expensive to enforce; they restrict the firm's financial investment, the disposition of assets, and the firm's merger activity. Alternatively, Green (1984) shows that the risk-shifting problem can be mitigated by resorting to convertible debt. Besides the fact that there is no unanimity on the properties of convertible debt contracts, the theoretical setting and the predictions are very different from ours.[26] In this literature, the asset substitution problem plagues a pure (unsecured) debt contract, owing to conflicts of interest between shareholders and bondholders. In our model, there are no conflicts of interest when the project is financed with an unsecured debt contract. It is the use of collateral that introduces an asset substitution problem into the debt contract. This difference is crucial and makes the convertible debt contract not a viable instrument to solve the commitment problem.

---

[26]For example, Eisdorfer (2009) shows theoretically and provides empirical evidence that convertible debt, in contrast to its perceived role, can induce shareholders' risk-shifting.

So far we have assumed the optimality of the debt contract with all the collateral seized by creditors. This assumption implicitly rules out the possibility of an equity contract, where each party gets a fraction of the return in each state of nature. One may wonder what happens if we allow the entrepreneur and the bank to share the collateral in default. If the investment is contractible, the optimal sharing rule of the collateral is indeterminate. If instead the investment is non-contractible, letting the entrepreneur seize the collateral allows to internalize his incentive problems, removing the incentive to alter the input mix. We have ruled out this possibility on the grounds of realism and efficiency. In practice, the legal system automatically requires that all assets still in place are kept away from the defaulting entrepreneur and used to repay creditors. This regularity has been rationalized by a vast theoretical literature showing that pledging collateral to creditors helps mitigate agency problems. Introducing agency problems into our model would indeed restore the standard arguments for pledging collateral but at the cost of complicating the analysis and drive the reader away from one of the main results of the paper (i.e., that collateral may be useless in lending). We think our setting is the simplest one to clarify this message.

Another possibility is to condition debt financing on the number of employees. This could work as a commitment device if the deviation consisted in cutting the number of employees ex post. However, in our setting the entrepreneur has an incentive to increase the number of employees, and this can be done in any labor market.

6.2 Exclusive lending relations

In Section 1, we assume that the entrepreneur has exclusive lending relationships with the bank and the supplier. What happens if the entrepreneur buys inputs from several suppliers? In this case, the optimal contract implies that each supplier loan is tied closely to the amount of inputs provided by that supplier, not to the total amount of inputs. Thus, each supplier only observes his own fraction of the input purchases. Unless we assume some sort of information-sharing among suppliers, no supplier actually knows whether the ex ante optimal input combination has been purchased. Thus, trade credit no longer solves the commitment problem.

To get this intuition, suppose that the optimal input combination requires 10 units of capital and 8 workers and the liquidation value β is 1 for simplicity. The amount of trade credit that stops deviation is 6, and the bank is willing to finance the rest (4 units of capital plus the 8 workers). If there is only one supplier, our story works. This supplier sells 10 capital inputs, 6 on credit and 4 in cash, and gets 60% of the collateral value of the 10 capital inputs (while the bank gets 40%). But suppose now there are two suppliers: one selling 6 units of capital on credit and having the right to 60% of the collateral

value of the 10 capital inputs in case of default; the other selling 4 inputs in cash. Rather than abiding by the optimal input combination, the entrepreneur could take just the 6 units on credit from the first supplier and use the loan of 4 from the bank to hire workers, rather than buy the remaining 4 units of capital from the second supplier. Both the first supplier (the one selling 6 capital inputs on credit) and the bank would not break-even, since the total collateral value would be lower than expected $(6 < 10)$. Anticipating the entrepreneur's deviation, the bank would never accept a secured contract and the supplier would never provide trade credit.

Similar reasoning applies to the bank lending relationship. With several banks, the aggregate credit obtained from all banks could be larger than the incentive-compatible amount, giving the entrepreneur an incentive to deviate. Thus, exclusivity is also necessary in the bank lending relationship.

6.3 Is the supplier the only informed lender?

In our analysis, we assume that the costly but informed lender is the supplier and the cheap but uninformed financier is the bank. Other interpretations are possible, however. We could interpret the informed lender as a lessor, who receives a fee from the borrower for the use of capital inputs but retains ownership.[27] Although appealing, this interpretation is flawed, in that the lessor fails to solve the entrepreneur's commitment problem. To see why, consider the case analyzed in Section 3, in which the presence of the supplier allows the bank to offer a partially secured contract. Consider a contract for the financing of a given amount of labor units and several units of the capital input, say 50 printers. The contract might state that a certain number of printers (say, 20) are to be financed by a secured contract with the bank, the other 30 leased. Upon receiving the loan from the bank, the entrepreneur has an incentive to reduce the number of printers purchased, say from 20 to 10. The lessor will not stop the entrepreneur from buying fewer printers. For one thing, he does not observe the reduction. And, second, even if he does, he has no incentive to stop the entrepreneur's opportunistic behavior: Remaining the owner of his 30 printers, his return in default is never jeopardized. Anticipating the entrepreneur's deviation, the bank will never propose such an agreement.

Thus, while under leasing a profitable unilateral deviation by the entrepreneur is always possible, under trade credit such deviation is never possible unless the entrepreneur colludes with the supplier. This is precisely why trade credit can guarantee commitment.

---

[27]Eisfeldt and Rampini (2009) model a liquidation advantage of leasing relative to secured lending.

6.4 The role of information sharing between financiers

In Section 3, we show that trade credit can signal the entrepreneur's willingness to abide by the ex ante efficient bank contract. When the bank gets this signal, it decides to sign a secured loan. This result is not based on any sort of information sharing between the bank and the supplier.No information about the amount of trade credit offered or any other feature of the supplier contract needs to be shared with the bank. This is because the entrepreneur's contract offers will reveal all the relevant information to the financiers.

The crucial point here is that both the entrepreneur and the financiers solve the same maximization problem (i.e., programme P^γ under the incentive compatibility constraint (16)). Thus the bank and the supplier can work out the set of sharing rules that prevents the entrepreneur's deviation (and the full contract terms corresponding to each sharing rule in this set), as well as the *optimal* sharing rule together with the amount of capital, labor, bank and trade credit, and financiers' repayments that maximizes the entrepreneur's profit. The bank and supplier will never agree on a contract that does not satisfy constraint (16), which the entrepreneur knows. At the same time, the entrepreneur will never offer contracts originating by sharing rules $(\gamma, 1 - \gamma)$ different from the optimal one (i.e., with $\gamma < \gamma^*$), or that do not add up to one. All these contract offers, although possibly accepted by financiers, would not be profit maximizing for the entrepreneur. It follows that in equilibrium the only contract offered will be the optimal one. The bank will infer from its contract offer how much trade credit is taken by the entrepreneur, as well as the supplier's repayments in both states of nature. In a similar way, the supplier can infer the bank contract terms.

7 Conclusion

We investigate optimal financial contracts when entrepreneurs use a multi-input technology with inside collateral and the investment is unobservable to banks. We show that pledging an asset as collateral does not increase the entrepreneur's borrowing capacity, as collateral introduces a problem of ex post asset substitution. This stops the bank from signing a secured contract, with a subsequent efficiency loss. We argue that the supplier of the capital input is the natural candidate to overcome this problem of contract incompleteness, since he naturally observes whether the investment contracted is actually made. Thus, trade credit conveys relevant investment information to the bank and can be used as a commitment device to restore the benefits of secured bank lending. This commitment effect is robust to the possibility of a collusive agreement between the entrepreneur and the supplier, as well as to repeated entrepreneur-bank interactions.

We thus provide a new theory of trade credit that not only rationalizes recent evidence on a complementarity between trade and bank credit, but also offers several novel testable predictions. Our static analysis predicts that (1) entrepreneurs using both trade credit and secured bank credit invest more intensively in pledgeable assets than entrepreneur using only unsecured bank credit. Moreover, the benefits of secured bank and trade credit are larger for (2) entrepreneurs buying standardized inputs than for those buying differentiated inputs or services, and (3) for entrepreneurs using technologies with input substitutability than for those with fixed-proportions. Our multi-period analysis delivers further predictions on the choice between trade credit and relationship lending. Entrepreneurs are more likely to use trade credit than relationship lending when (4) the duration of the project is short enough; (5) inputs have high collateral value; and (6) low-quality information is collected by the relationship lender. We leave the empirical verification of these predictions to future research.

Proof of Proposition 1. The investment in capital and labor, I_K^C, I_N^C, satisfies the following FOC's:

$$p\frac{\partial f(\cdot)}{\partial I_K} = r_B - (1-p)\beta, \tag{26}$$

$$p\frac{\partial f(\cdot)}{\partial I_N} = r_B, \tag{27}$$

obtained by differentiating the reduced form objective function (4) with respect to I_K and I_N. By the homotheticity of the production function, the optimal input ratio $\left(I_K^C/I_N^C\right)$ is a constant implicitly defined by the input price ratio: $r_B/\left[r_B - (1-p)\beta\right] > 1$. Using I_K^C, I_N^C in constraints (2) and (3), we obtain the optimal bank loan, L_B^C, and the bank repayment, R_B^C. To capture the role of collateral in lending, we carry out a comparative static analysis on β. Using $I_K^C(\beta)$ and $I_N^C(\beta)$ in (26) and (27), and differentiating with respect to β, we obtain $\partial I_K^C/\partial\beta = p(1-p)f_{NN}/\left(f_{KK}f_{NN} - f_{KN}^2\right) > 0$, $\partial I_N^C/\partial\beta = -p(1-p)f_{NK}/\left(f_{KK}f_{NN} - f_{KN}^2\right) > 0$, which shows that I_K^C and I_N^C are both increasing in the resale value of the asset. ∎

Proof of Proposition 2. The solution to this problem proceeds as follows. We first show that the entrepreneur-bank secured credit contract is time-inconsistent, i.e., the entrepreneur has an incentive to ex post alter the input combination at the expense of the bank. The actual input mix chosen once the loan is granted involves a decrease in the investment in the capital input, and hence returns in the default state insufficient to allow the bank to break even. We then show that the bank may prevent losses by signing an unsecured credit contract with a smaller loan and a subsequent efficiency loss.

1. Time inconsistency of the fully secured bank contract. The first step consists in showing that it is profit-maximizing for the entrepreneur to breach the terms of the fully secured contract. Thus, we need to show that the input combination chosen under deviation involves higher profit.

Consider programme $\mathcal{P}^{\mathcal{D}}$ in Section 2.2 faced by an entrepreneur that has obtained a loan L_B^C and must repay R_B^C in the good state. Solving the resource constraint (7) for I_N, we get:

$$I_N\left(L_B^C, I_K\right) = L_B^C - I_K \tag{28}$$

and differentiating the resulting objective function $pf\left(I_K, L_B^C - I_K\right) - pR_B^C$ with respect to I_K

$$p\left(\frac{\partial f(\cdot)}{\partial I_K} + \frac{\partial f(\cdot)}{\partial I_N}\frac{dI_N}{dI_K}\right) = 0. \tag{29}$$

From (28), $dI_N/dI_K = -1$, whence $\left(\partial f(\cdot)/\partial I_K\right)/\left(\partial f(\cdot)/\partial I_N\right) = 1$. The input price ratio is equal to 1, which is lower than input price ratio under the fully secured contract: $r_B/\left[r_B - (1-p)\beta\right]$. Thus, the relative price of capital (labor) increases (decreases). This implies that the optimal input ratio is lower than the input ratio under the secured contract (i.e., it lies on a flatter expansion path). To determine the actual level of the two inputs, and thus the effect on input demands of an increase in the price of input I_K and a decrease in the relative price of I_N while keeping the loan constant, is tantamount to working out the Slutsky compensated demands for inputs I_K and I_N. Since, by the quasi-concavity of the production function, the own-price effect is non-positive, the demand for I_K decreases. Because the loan is constant and equal to L_B^C and the entrepreneur uses only two inputs, the cross-price effect is non-negative, i.e., the demand for input I_N must increase. In particular, solving (29) and using (28), we get: $I_K^D\left(R_B^C, L_B^C\right) < I_K^C, I_N^D\left(R_B^C, L_B^C\right) > I_N^C$, with $I_K^D(\cdot) + I_N^D(\cdot) = L_B^C$.

Last, to determine the effect that deviation has on output, notice that the new optimum lies on an isocost line that crosses the initial optimum (with the same loan L_B^C the entrepreneur can afford the

34

initial input mix) but is flatter than the isocost line of the fully secured contract: the input price ratio is $1 < r_B / [r_B - (1-p)\beta]$. By the convexity of isoquants, this implies a shift on a higher isoquant and thus higher output. Since the cost component of the profit function is the same in the two cases (R_B^C), profits are higher under deviation. This proves that deviating is profit-maximizing for the entrepreneur and that the initial contract is not incentive-compatible (i.e., it is time-inconsistent). However, the decrease in I_K implies that in default the collateral value of the input is insufficient to repay the bank, which does not break even. Anticipating this, the bank agrees to sign only unsecured contracts.

    2.   Determination of the unsecured bank contract. In the unsecured contract, the entrepreneur chooses I_K, I_N, R_B, L_B to maximize (1) subject to the participation constraint (2) with $C = 0$, and to the resource constraint (3). Solving (2) for R_B and using L_B from the resource constraint (3), the objective function (1) becomes $pf(I_K, I_N) - (I_N + I_K) r_B$. The optimal input combination satisfies the following FOC's:

$$p\frac{\partial f(\cdot)}{\partial I_K} = r_B, \tag{30}$$

$$p\frac{\partial f(\cdot)}{\partial I_N} = r_B, \tag{31}$$

which give I_K^U, I_N^U. Notice that again the input price ratio is equal to 1, which implies, by the homotheticity of the production function, that the input combination lies along the same expansion path as the deviation contract. However, relative to the deviation contract, the investment in each input is now lower. This can be seen by considering that the FOC's of the secured contract, (26) and (27), coincide with those of the unsecured one, (30) and (31), when β tends to zero. Thus, evaluating the effect on the investment level and the loan size of switching from the secured to the unsecured contract is tantamount to carrying out a comparative static analysis on β as in the Proof of Proposition 1. Since by Proposition 1, both I_K and I_N are increasing in β, we deduce that $I_K^U < I_K^C$, $I_N^U < I_N^C$ and, by constraint (3), that $I_K^U + I_N^U = L_B^U < L_B^C$. Last, using L_B^U in (2) gives $R_B^U = r_B L_B^U/p$. ∎

    Proof of Proposition 3. Consider programme $\mathcal{P}^\gamma$. Substituting the binding constraints in the objective function gives:

$$\max_{I_K, I_N} EP = pf(I_K, I_N) - r_B\left[I_N + I_K - (1-\gamma)\frac{\beta}{r_S}I_K\right] + [(1-p)\gamma - p(1-\gamma)]\beta I_K,$$

with FOC's:

$$p\frac{\partial f(\cdot)}{\partial I_K} = r_B\left(1 - (1-\gamma)\frac{\beta}{r_S}\right) - \beta(\gamma - p), \tag{32}$$

$$p\frac{\partial f(\cdot)}{\partial I_N} = r_B. \tag{33}$$

Solving (32) and (33), we obtain $I_K^*(\gamma), I_N^*(\gamma)$. Using (32) and (33), we get the new input price ratio as $r_B / [r_B(1 - (1-\gamma)\beta/r_S) + (p-\gamma)\beta]$. By Assumption 1, the ratio is no less than 1 and increasing in γ.[28] It follows that the higher the γ, the higher the input price ratio and the higher the capital-labor ratio, which proves part *d.* of Proposition 3.

    Substituting $I_K^*(\gamma)$ into constraints (11) and (13), we obtain $L_S^*(\gamma) = (1-\gamma)\beta I_K^*(\gamma)/r_S$. The share of inputs bought on credit is equal to $L_S^*/I_K^* = \beta(1-\gamma)/r_S$, and $\partial(L_S^*/I_K^*)/\partial\gamma = -\beta/r_S < 0$, which proves part *a.* of Proposition 3.

---

    [28]For $\gamma = 0$, this is equal to $r_B/r_B - \frac{p}{r_S}\left(\frac{r_B}{p} - r_S\right)\beta \geq 1$ by Assumption 1. For $\gamma = 1$, the input price ratio is equal to its commitment value: $r_B/r_B - (1-p)\beta > 1$.

Using $L_S^*(\gamma)$ and $I_K^*(\gamma)$ in constraints (3) and (11), we get $L_B^*(\gamma) = I_K^*(\gamma) + I_N^*(\gamma) - (1-\gamma)\beta I_K^*(\gamma)/r_S$, $R_B^*(\gamma) = [L_B^*(\gamma) r_B - (1-p)\gamma\beta I_K^*(\gamma)]/p$ with:

$$\frac{\partial L_B^*(\gamma)}{\partial \gamma} = \frac{\partial I_N^*}{\partial \gamma} + \left(1 - \frac{\beta}{r_S}(1-\gamma)\right)\frac{\partial I_K^*}{\partial \gamma} + \frac{\beta}{r_S}I_K^*(\gamma).$$

To determine the sign of $\partial L_B^*(\gamma)/\partial \gamma$, we carry out a comparative static analysis on I_K^* and I_N^*. Using $I_K^*(\gamma), I_N^*(\gamma)$ in (32) and (33), and differentiating with respect to γ, we obtain $\partial I_K^*/\partial \gamma = p\beta[(r_B/r_S) - 1]f_{NN}/(f_{KK}f_{NN} - f_{KN}^2) > 0$, and $\partial I_N^*/\partial \gamma = -p\beta[(r_B/r_S) - 1]f_{NK}/(f_{KK}f_{NN} - f_{KN}^2) > 0$. Substituting the previous two derivatives into $\partial L_B^*(\gamma)/\partial \gamma$, we get:

$$\frac{\partial L_B^*(\gamma)}{\partial \gamma} = \frac{p\beta}{f_{KK}f_{NN} - f_{KN}^2}\left(1 - \frac{r_B}{r_S}\right)\left(f_{NK} - \left(1 - \frac{\beta}{r_S}(1-\gamma)\right)f_{NN}\right) + \frac{\beta}{r_S}I_K^*(\gamma),$$

which, given the assumptions on the technology, is certainly positive. This concludes the proof of part *b.* of Proposition 3.

To prove part *c.* of Proposition 3, consider the expected profits $\Pi^*(\gamma)$ (14). By the envelope theorem, they are increasing in γ: $\partial \Pi^*(\gamma)/\partial \gamma = (1 - r_B/r_S)\beta I_K^*(\gamma) > 0$. Moreover, using Assumption 1, they are also increasing in β: $\partial \Pi^*/\partial \beta = I_K^*[(r_B/p) - r_S(1-\gamma/p) - \gamma(r_B/p)]p/r_S > 0$. ∎

Proof of Proposition 4. To prove this, we need to solve programme $\mathcal{P}^\gamma$ under the no-deviation condition (16). We proceed in three steps. First, we derive the profit under deviation from the commitment contract as a function of γ, $\Pi^F(\gamma)$. Then, we derive the no-deviation condition as a function of γ. Last, we show that there is a unique $\gamma = \gamma^* \in [0,1]$ that solves the no-deviation condition (16).

1. $\Pi^F(\gamma)$ is the maximum profit obtained by solving programme $\mathcal{P}^\mathcal{F}$:

$$\max_{I_K, I_N} p\left[f(I_K, I_N) - R_B\right] \tag{34}$$

$$\text{s.t. } R_B = R_B^*(\gamma) \tag{35}$$

$$I_N = L_B^*(\gamma) - I_K. \tag{36}$$

where $L_B^*(\gamma)$ and $R_B^*(\gamma)$ are the commitment values of the bank loan and the bank good state repayment as defined in Proposition 3. Substituting the binding constraints (35) and (36) into the objective function, and working out the FOC, we get $p\left[\partial f(\cdot)/\partial I_K + (\partial f(\cdot)/\partial I_N)(dI_N/dI_K)\right] = 0$. Using $dI_N/dI_K = -1$ from the resource constraint (36), the latter becomes:

$$\frac{\partial f(\cdot)}{\partial I_K} = \frac{\partial f(\cdot)}{\partial I_N}. \tag{37}$$

Condition (37) along with the resource constraint (36) gives $I_K^F \equiv I_K^F(L_B^*(\gamma))$, $I_N^F \equiv I_N^F(L_B^*(\gamma))$ and $I_K^F = I_N^F$, which, used in the objective function, gives the value function (15) as in Definition 1.

2. Using the previous result, we can express the no-deviation condition (16) as follows:

$$\Pi^F(\gamma) - \Pi^*(\gamma) = p\left[f(I_K^F, L_B^*(\gamma) - I_K^F) - R_B^*(\gamma)\right] - p\left[f(I_K^*, L_B^*(\gamma) - I_K^*) - R_B^* - R_S^*(\gamma)\right] \le 0.$$

3. To prove that there exists a unique $\gamma = \gamma^* \in [0,1]$ that solves the no-deviation condition, we need to show **a.** $\Pi^F(0) - \Pi^*(0) < 0$; **b.** $\Pi^F(1) - \Pi^*(1) > 0$; **c.** $\partial \Pi^*/\partial \gamma, \partial \Pi^F/\partial \gamma > 0$; **d.** $\partial \Pi^F/\partial \gamma > \partial \Pi^*/\partial \gamma$.

a. $\mathbf{\Pi^F(0) - \Pi^*(0) < 0}$. $\Pi^*(0)$ can be worked out by considering that, when $\gamma = 0$, the entrepreneur obtains credit from the bank with an unsecured contract, and from the supplier with a fully secured

contract. $\Pi^F(0)$ instead can be obtained by considering that, by observing the entrepreneur deviating, the supplier stops financing altogether and the entrepreneur obtains only an unsecured credit contract from the bank, whose properties have been discussed in Proposition 2. Under Assumption (1), the profits obtained with the unsecured credit contract ($\Pi^F(0)$) are lower than those obtained under a contract with $\gamma = 0$ and supplier secured financing ($\Pi^*(0)$), which proves our claim.

b. $\Pi^F(1) - \Pi^*(1) > 0$. When $\gamma = 1$, we are in the fully secured bank contract, whose properties have been discussed in Proposition 1. By Proposition 2, the profits under deviation exceed the profits under commitment, which proves our second claim.

c. $\partial \mathbf{\Pi}^*/\partial\gamma, \partial \mathbf{\Pi}^F/\partial\gamma > \mathbf{0}$. That $\partial \Pi^*/\partial\gamma > 0$ has already been proved in the Proof of Proposition (3). To prove that $\partial \Pi^F/\partial\gamma > 0$, differentiate (15) with respect to γ, which by the envelope theorem is equal to:

$$\frac{\partial \Pi^F}{\partial\gamma} = p\left(\frac{\partial f(\cdot)}{\partial I_N^F}\frac{\partial I_N^F}{\partial L_B^*}\frac{\partial L_B^*}{\partial\gamma} - \frac{\partial R_B^*}{\partial\gamma}\right).$$

Using $L_B^*(\gamma)$ and $R_B^*(\gamma)$ from Proposition 3 and constraint (36), the above can be written as

$$\frac{\partial \Pi^F}{\partial\gamma} = p\left[\frac{\partial f(\cdot)}{\partial I_N^F}\frac{\partial L_B^*(\gamma)}{\partial\gamma} - \frac{1}{p}\left(\frac{\partial L_B^*(\gamma)}{\partial\gamma}r_B - (1-p)\beta I_K^*(\gamma) - (1-p)\gamma\beta\frac{\partial I_K^*}{\partial\gamma}\right)\right],$$

where $\partial L_B^*(\gamma)/\partial\gamma = \partial I_N^*/\partial\gamma + (\partial I_K^*/\partial\gamma)(1-(1-\gamma)\beta/r_S) + \beta I_K^*(\gamma)/r_S$. After some manipulations, and using the equality of the marginal productivities under deviation (37), we get:

$$\begin{aligned}\frac{\partial \Pi^F}{\partial\gamma} &= \left(p\frac{\partial f(\cdot)}{\partial I_N^F} - r_B + (1-p)\gamma\beta\right)\frac{\partial I_K^*}{\partial\gamma} + \left(p\frac{\partial f(\cdot)}{\partial I_N^F} - r_B + (1-p)r_S\right)\frac{\beta}{r_S}I_K^*(\gamma) + \\ &\quad - \left(p\frac{\partial f(\cdot)}{\partial I_N^F} - r_B\right)(1-\gamma)\frac{\beta}{r_S}\frac{\partial I_K^*}{\partial\gamma} + \left(p\frac{\partial f(\cdot)}{\partial I_N^F} - r_B\right)\frac{\partial I_N^*}{\partial\gamma}.\end{aligned} \quad (38)$$

The sign of the first term is positive. This can be inferred by considering that by (32) the marginal productivity of capital under commitment is $r_B(1-(1-\gamma)\beta/r_S) - \beta(\gamma - p)$. Under Assumption 1, this is no less than $r_B - \gamma(1-p)\beta$. Since under deviation the reliance on capital decreases, the marginal productivity of capital under deviation increases above its commitment level and is higher than $r_B - \gamma(1-p)\beta$. This in turn implies that the second term is also positive ($r_B - (1-p)r_S < r_B - \gamma(1-p)\beta$).

The sign of the third term can be inferred by considering that the lesser reliance on capital relative to the commitment level increases its marginal productivity above this level (implicitly defined by the right-hand side of (32): $p\partial f(\cdot)/\partial I_K^F > r_B(1-(1-\gamma)\beta/r_S) - \beta(\gamma - p)$). However, the highest possible marginal productivity obtains in the fully unsecured contract (implicitly defined by the right-hand side of (30): r_B. Thus $p\partial f(\cdot)/\partial I_K^F \le r_B$ and the third term in brackets is negative. By the minus sign outside it, the whole third term is positive.

Last, given that by the above argument $p\partial f(\cdot)/\partial I_K^F - r_B \le 0$, and given that $\partial I_N^*/\partial\gamma > 0$, the sign of the fourth term is negative. However, comparing the last two terms of the derivative, a sufficient condition for $\partial \Pi^F/\partial\gamma$ to be positive is that:

$$\left(p\frac{\partial f(\cdot)}{\partial I_K^F} - r_B\right)\left(\frac{\partial I_N^*}{\partial\gamma} - (1-\gamma)\frac{\beta}{r_S}\frac{\partial I_K^*}{\partial\gamma}\right) = \frac{\left(p\frac{\partial f(\cdot)}{\partial I_K^F} - r_B\right)p\beta\left(1-\frac{r_B}{r_S}\right)}{f_{KK}f_{NN} - f_{KN}^2}\left(f_{NK} + (1-\gamma)\frac{\beta}{r_S}f_{NN}\right) \ge 0$$

which holds if $f_{NK} + (1-\gamma)\beta f_{NN}/r_s < 0$. Thus, under mild conditions, profits from deviation (38) are increasing in γ.

d. $\partial \mathbf{\Pi}^F/\partial\gamma > \partial \mathbf{\Pi}^*/\partial\gamma$.

$$\left(p\frac{\partial f(\cdot)}{\partial I_K^F} - r_B\right)\left(\frac{\partial I_N^*}{\partial\gamma} - \frac{\beta}{r_S}(1-\gamma)\frac{\partial I_K^*}{\partial\gamma}\right) + \left(p\frac{\partial f(\cdot)}{\partial I_K^F} - r_B + (1-p)\gamma\beta\right)\frac{\partial I_K^*}{\partial\gamma} + \left(p\frac{\partial f(\cdot)}{\partial I_K^F} - pr_S\right)\frac{\beta}{r_S}I_K^*(\gamma).$$

By the same arguments used to prove that (38) is positive, the first two terms are both positive. We need to determine the sign of the third term. This can be inferred by considering that by (32) $p\partial f(\cdot)/\partial I_K^* = r_B(1-(1-\gamma)\beta/r_S) - (\gamma-p)\beta$. Since under deviation the reliance on capital decreases, the marginal productivity of capital under deviation increases above its commitment level, i.e., $p\partial f(\cdot)/\partial I_K^F \geq p\partial f(\cdot)/\partial I_K^*$. It follows that a sufficient condition for the third term to be positive is $r_B(1-(1-\gamma)\beta/r_S) - (\gamma-p)\beta \geq pr_S$. Rearranging the previous inequality, we get $[(r_B - pr_S)(r_S - \beta) - \gamma\beta(r_S - r_B)]/r_S \geq 0$. Since $(r_S - \beta) > (r_S - r_B)$, a sufficient condition for the inequality to hold is that $(r_B - pr_S) \geq \gamma\beta$, which is a slightly stricter condition than that implied by Assumption 1. ∎

Proof of Proposition 5. The proof proceeds in two steps. First, we derive the profit under collusion as a function of γ, i.e., $\Pi^{COL}(\gamma)$, and the collusion rent as the difference between the profits with collusion and without collusion. Second, we show that the collusion rent is positive for any γ, meaning that any secured entrepreneur-bank-supplier contract is prone to collusion between the entrepreneur and the supplier at the expense of the bank.

1. The ex post optimization programme with entrepreneur-supplier collusion is given by programme $\mathcal{P}^{COL}$:

$$\max_{I_K, I_N, R_B, R_S} p\left[f(I_K, I_N) - R_B - R_S\right]$$

$$\text{s.t. } R_B = R_B^*(\gamma), \tag{39}$$

$$L_S + L_B^*(\gamma) = I_N + I_K, \tag{40}$$

and constraint (17): $pR_S + (1-p)(1-\gamma)\beta I_K \geq L_S r_S$. Constraint (39) requires the bank repayment in the good state be equal to that promised in the commitment contract, i.e., $R_B^*(\gamma)$. The resource constraint (40) requires the total input expenditure be equal to the bank loan obtained in the commitment contract plus the recontracted supplier's loan. Moreover, to guarantee that in the renegotiation the supplier acts as a lender and not as a borrower or a liquidator, we impose the non-decreasing repayment condition (13) that requires that payments to the supplier be non-negative and non-decreasing in returns. Thus, using $R_S = (1-\gamma)\beta I_K$ in the participation constraint (17), we obtain $L_S = (1-\gamma)\beta I_K/r_S$. Replacing it in the resource constraint (40), we get: $(1-\gamma)\beta I_K/r_S + L_B^*(\gamma) \geq I_N + I_K$. Solving this expression for I_N, we get:

$$I_N = \left(\frac{\beta(1-\gamma)}{r_S} - 1\right)I_K + L_B^*(\gamma). \tag{41}$$

Last, using R_B from constraint (39) and I_N from condition (41), the objective function becomes:

$$\max_{I_K} p\left[f\left(I_K, \left(\frac{\beta(1-\gamma)}{r_S} - 1\right)I_K + L_B^*(\gamma)\right) - R_B^*(\gamma) - (1-\gamma)\beta I_K\right].$$

with FOC: $p(\partial f(\cdot)/\partial I_K + (\partial f(\cdot)/\partial I_N)(dI_N/dI_K)) - p\beta(1-\gamma) = 0$. Using $dI_N/dI_K = (1-\gamma)\beta/r_S - 1$ from (41), this becomes:

$$\frac{\partial f(\cdot)}{\partial I_K} - \frac{\partial f(\cdot)}{\partial I_N} = -\left(\frac{\partial f(\cdot)}{\partial I_N} - r_S\right)\frac{\beta(1-\gamma)}{r_S}. \tag{42}$$

Solving for I_K, we get I_K^{COL}, which substituted out in the resource constraint gives $I_N^{COL} = L_B^*(\gamma) - [1-(1-\gamma)\beta/r_S]I_K^{COL}$. Last, using I_K^{COL}, I_N^{COL} in the objective function we obtain the return from collusion:

$$\Pi^{COL}(\gamma) \equiv pf\left(I_K^{COL}, L_B^*(\gamma) - I_K^{COL} + (1-\gamma)\frac{\beta}{r_S}I_K^{COL}\right) - pR_B^*(\gamma) - p(1-\gamma)\beta I_K^{COL}. \tag{43}$$

The difference between the profits under collusion (43) and the profits under commitment for a generic γ (14) gives the gross collusion rent $\Pi^{COL}(\gamma) - \Pi^*(\gamma)$.

2. To prove that any secured three-party contract is prone to collusion, we need to show: **a.** $\Pi^{COL}(0) - \Pi^*(0) = 0$; **b.** $\Pi^{COL}(1) - \Pi^*(1) > 0$; **c.** $\partial \Pi^{COL}/\partial \gamma, \partial \Pi^*/\partial \gamma > 0$; **d.** $\partial \Pi^{COL}/\partial \gamma > \partial \Pi^*/\partial \gamma$.

a. $\Pi^{COL}(0) - \Pi^*(0) = 0$. When $\gamma = 0$, the entrepreneur obtains credit from the bank with an unsecured contract, and from the supplier with a fully secured contract. If the entrepreneur and the supplier collude, they recontract the terms of the credit contract at the expense of the bank. However, since the bank offers only an unsecured contract, there is no collusive agreement that can make the bank worse off. Thus, the new collusive contract is no better than the original commitment contract. It follows that when $\gamma = 0$ the profits under collusion are equal to the profits under commitment, which proves the claim.

b. $\Pi^{COL}(1) - \Pi^*(1) > 0$. To prove this, we can refer to the argument used in the Proof of Proposition (4).

c. $\partial \Pi^*/\partial \gamma, \partial \Pi^{COL}/\partial \gamma > 0$. That $\partial \Pi^*/\partial \gamma > 0$ has already been proved in the Proof of Proposition (3). To prove that $\partial \Pi^{COL}/\partial \gamma > 0$, we differentiate the collusion rent $\Pi^{COL}(\gamma)$, defined in (43), with respect to γ:

$$\frac{\partial \Pi^{COL}(\gamma)}{\partial \gamma} = p\left(\frac{\partial f(\cdot)}{\partial I_N^{COL}}\frac{\partial I_N^{COL}}{\partial L_B^*}\frac{\partial L_B^*}{\partial \gamma} - \frac{\partial R_B^*}{\partial \gamma} + \beta I_K^{COL}\right).$$

Using $L_B^*(\gamma)$ and $R_B^*(\gamma)$ from the commitment problem, and $\partial I_N^{COL}/\partial L_B^* = 1$ from (41), we get:

$$\frac{\partial \Pi^{COL}(\gamma)}{\partial \gamma} = p\left(\frac{\partial f(\cdot)}{\partial I_N^{COL}}\frac{\partial L_B^*}{\partial \gamma} - \frac{1}{p}\left(r_B\frac{\partial L_B^*}{\partial \gamma} - (1-p)\beta I_K^*(\gamma) - (1-p)\gamma\beta\frac{\partial I_K^*}{\partial \gamma}\right) + \beta I_K^{COL}\right),$$

where $\partial L_B^*/\partial \gamma = (1 - (1-\gamma)\beta/r_S)(\partial I_K^*(\gamma)/\partial \gamma) + \beta I_K^*(\gamma)/r_S + \partial I_N^*/\partial \gamma$. Thus:

$$
\begin{aligned}
\frac{\partial \Pi^{COL}(\gamma)}{\partial \gamma} &= \left(p\frac{\partial f(\cdot)}{\partial I_N^{COL}} - r_B\right)\left(\frac{\partial I_N^*}{\partial \gamma} - \frac{1}{r_S}(1-\gamma)\beta\frac{\partial I_K^*}{\partial \gamma}\right) + \left(p\left(\frac{\partial f(\cdot)}{\partial I_N^{COL}} - r_S\right) + r_S - r_B\right)\frac{\beta}{r_S}I_K^*(\gamma) \\
&\quad + \left(p\frac{\partial f(\cdot)}{\partial I_N^{COL}} - r_B + (1-p)\beta\gamma\right)\frac{\partial I_K^*}{\partial \gamma} + p\beta I_K^{COL}.
\end{aligned}
\tag{44}
$$

Notice that $p\partial f(\cdot)/\partial I_N^{COL} < r_B$, given that under deviation the reliance on labor increases, thus reducing the marginal productivity of labor below its commitment level r_B. Since $\partial I_K^*/\partial \gamma > 0$ (by the comparative static analysis in the proof of Proposition 3), the sign of the first term depends on the sign of $\partial I_N^*/\partial \gamma$. A sufficient condition for the first term to be positive is that $f_{NK} + (1-\gamma)\beta f_{NN}/r_S < 0$, which is the condition used in point 3 of the Proof of Proposition 4.

The sign of the second term can be inferred using (42). By Assumption 1, the reliance on labor is never greater than the reliance on capital. This implies that the marginal productivity of labor is no less than the marginal productivity of capital, $\partial f(\cdot)/\partial I_N \geq \partial f(\cdot)/\partial I_K$. Using this result in (42) implies that $\partial f(\cdot)/\partial I_N^{COL} \geq r_S$. Moreover since $r_S > r_B$, we can conclude that the second term is positive.

Finally, notice that in point 3 of the Proof of Proposition 4 we showed that $p\partial f(\cdot)/\partial I_K^D - r_B + (1-p)\beta\gamma > 0$. Comparing this term with the third term of (44), the latter is positive if $\partial f(\cdot)/\partial I_N^{COL} > \partial f(\cdot)/\partial I_K^D$. However, under deviation, by (37), the marginal productivities of the factors are equal $\partial f(\cdot)/\partial I_K^D = \partial f(\cdot)/\partial I_N^D$. This implies that a sufficient condition for the third term to be positive is that $\partial f(\cdot)/\partial I_N^{COL} > \partial f(\cdot)/\partial I_N^D$. This proves that the third term of (44) is positive. Since all the terms of (44) are positive, we deduce that $\partial \Pi^{COL}(\gamma)/\partial \gamma > 0$.

4. $\partial \Pi^{COL}/\partial \gamma > \partial \Pi^*/\partial \gamma > 0$.

$$\left(p\frac{\partial f}{\partial I_N^{COL}} - r_B + (1-p)\beta\gamma\right)\frac{\partial I_K^*}{\partial \gamma} - \left(p\frac{\partial f}{\partial I_N^{COL}} - r_B\right)\left(\frac{\beta(1-\gamma)}{r_S}\frac{\partial I_K^*}{\partial \gamma} - \frac{\partial I_N^*}{\partial \gamma}\right) + \frac{p\beta}{r_S}I_K^*(\gamma)\left(\frac{\partial f}{\partial I_N^{COL}} - r_S\right) + p\beta I_K^{COL}.$$

By the same arguments used in the Proof of Proposition (4), all the terms are positive. ∎

Proof of Proposition 6.

The proof is analogous to that derived for Proposition 4 for a generic γ. In that case the value of γ was determined by the minimum exogenous level of trade credit necessary to generate commitment, γ^*. Here, $\hat{\gamma}(\alpha)$ is obtained by solving the collusion-proofness condition (19) and depends on the cost of collusion α.

To prove that when collusion is costly there exists a contract that is collusion-proof, we use the following line of analysis. By Proposition 5, we know that $\Pi^{COL}(\gamma) < \Pi^*(\gamma)$ for any $\gamma > 0$ and equals zero for $\gamma = 0$. Thus, the commitment contract is open to collusion. Introducing the cost of collusion has the effect of shifting the function $\Pi^{COL}(\gamma)$ downwards, and flattening it: $(1-\alpha)\Pi^{COL}(\gamma)$. Two scenarios can then arise: one in which α is so high that the function $(1-\alpha)\Pi^{COL}(\gamma)$ never intersects $\Pi^*(\gamma)$; and one in which α is not too high and there are values of γ that satisfy constraint (18). Define with $\bar{\alpha}(\gamma)$ the function such that $(1-\alpha)\Pi^{COL}(\gamma)$ and $\Pi^*(\gamma)$ have the same slope, i.e., $(1-\alpha)\partial\Pi^{COL}(\gamma)/\partial\gamma = \partial\Pi^*(\gamma)/\partial\gamma$, $\forall\gamma$, and analyze each scenario separately.

In scenario 1, $\alpha(\gamma) \geq \bar{\alpha}(\gamma)$: the two functions, net collusion profits and commitment profits, never converge, which implies that there is no $\gamma > 0$ that solves constraint (18). It follows that constraint (18) is always slack and therefore trivially satisfied for any γ.

In scenario 2, $\alpha(\gamma) < \bar{\alpha}(\gamma)$. For any such $\alpha(\gamma)$, there exists a value of $\gamma(\alpha)$ that satisfies constraint (18). Suppose that for a particular $\alpha = \tilde{\alpha}$, there exists a $\tilde{\gamma}(\tilde{\alpha})$ that solves (18). If $\tilde{\gamma}(\tilde{\alpha}) \geq \gamma^*$, where γ^* is the value of γ that is incentive-compatible (i.e., that solves (16)), then the incentive compatible contract is already collusion-proof and constraint (18) can be ignored. Conversely, if $\tilde{\gamma}(\tilde{\alpha}) < \gamma^*$, then the no-deviation contract is prone to collusion and the relevant incentive constraint becomes the collusion-proofness constraint (18).

The next step is to find the threshold level of α, among those lower than $\bar{\alpha}(\gamma)$, that allows us to identify the area in which the relevant incentive constraint is the collusion-proofness condition (18) and the area in which the relevant incentive constraint is the no-deviation condition (16). Define $\alpha^*(\gamma^*)$ as the threshold level of α at which constraint (18) is satisfied with equality when $\gamma = \gamma^*$, i.e., $\alpha^*(\gamma^*) = 1 - \Pi^*(\gamma^*)/\Pi^{COL}(\gamma^*) < 1$ (Recall that γ^* is also the value of γ that solves constraint (16). Thus at $\alpha = \alpha^*(\gamma^*)$, $(1-\alpha^*)\Pi^{COL}(\gamma^*) = \Pi^*(\gamma^*) = \Pi^F(\gamma^*)$). It follows that for any $\alpha \geq \alpha^*(\gamma^*)$, constraint (18) is slack and the commitment contract is also collusion-proof. Conversely, for any $\alpha < \alpha^*(\gamma^*)$, constraint (18) is violated and the commitment contract is not collusion proof. To prevent collusion and satisfy (18), the bank has to reduce γ below γ^*. ∎

Proof of Corollary 1. The properties of the collusion-proof contract depend on the cost of collusion, α. Three scenarios can arise. First, collusion can be so costly that it is never profitable: $\alpha(\gamma^*) \leq \alpha \leq 1$. The global collusion-proof condition (19) coincides with the incentive-compatibility condition (16). This corresponds to the case analyzed in Section 3. The properties of the optimal contract are those described in Proposition 3, with $\gamma = \gamma^*$. The share of capital inputs bought on account (through trade credit) is equal to $\beta(1 - \gamma^*)$, and thus independent of α. The share of capital inputs paid for in cash (through bank credit) is equal to $\beta\gamma^*$. Asset tangibility and profits are defined in Proposition 3, with $\gamma = \gamma^*$. Both of them are independent of α.

Second, collusion may be costly but profitable: $0 < \alpha < \alpha(\gamma^*)$. The global collusion-proof condition (19) coincides with the collusion proof condition (18). The properties of the optimal contract are those described in Proposition 3, with $\gamma = \hat{\gamma}(\alpha)$. The lower the cost of collusion, the lower the share of inputs bought through bank credit and the larger the one bought on account through trade credit. Since both asset tangibility and expected profits are increasing in γ by Proposition 3, and γ is increasing in

α, the lower the cost of collusion, the lower the asset tangibility and the expected profits.

Lastly, collusion may be costless ($\alpha = 0$). The entrepreneur and the supplier can grab the entire surplus from their agreement. In this case, the only contract that enables the bank to break even is the unsecured one. No trade credit is used. Asset tangibility is equal to 1 since the investment in capital is equal to the one in labor. Entrepreneurs get the unsecured profits defined in Proposition 2.
∎

Proof of Proposition 7. The proof proceeds as follows. First, we set the condition that guarantees that cooperation in all periods weakly dominates cooperation in all periods but $T - 1$ (no-deviation condition). Then, we consider the case in which deviation occurs at a generic period i. We show that if cooperation in all periods dominates cooperation in all periods but $T - 1$, then it also dominates cooperation in all periods but i. Last, we consider the extreme case in which the entrepreneur deviates whenever he can, i.e., whenever he has access to a commitment contract. We derive the condition for cooperation to dominate and find that this is implied by the no-deviation condition defined above.

1. The entrepreneur's total profits under cooperation in all periods are given by:

$$\Pi|_{C_{(T-1)}} = (T - 1) \Pi^C + \Pi^U. \tag{45}$$

If deviation occurs at time $t = T - 1$, this triggers a punishment by the bank in the last period with a contract U. The sequence of payoffs for the entrepreneur is given by

$$\Pi|_{C_{(T-2)}, D_{T-1}} = (T - 2) \Pi^C + \Pi^D_{T-1} + \Pi^P_T.$$

This strategy is dominated by the cooperation strategy in $t = 1, 2, ...T - 1$ iff:

$$(T - 2) \Pi^C + \Pi^D_{T-1} + \Pi^P_T < (T - 1) \Pi^C + \Pi^U.$$

Since profits are not time-varying, the above condition coincides with (20).

2. Suppose now that deviation occurs at time $t = i$. This triggers a punishment by the bank, which is then followed by a series of commitment contracts again, until $t = T$, where the bank offers the unsecured contract U. The sequence of payoffs is:

$$\Pi|_{C_{(i-1)}, D_i} = \Pi^D + \Pi^P + (T - 3) \Pi^C + \Pi^U. \tag{46}$$

This strategy is weakly dominated by the cooperation strategy in $t = 1, 2, ...T - 1$ if (46) is no higher than (45), i.e., if:

$$2\Pi^C \geq \Pi^D + \Pi^P \tag{47}$$

Since $\Pi^C > \Pi^U$, the above condition is slacker than the no-deviation condition (20).

3. In principle, however, the entrepreneur might deviate each time he has access to a commitment contract, that is to say in every other period, given that upon any deviation there is a punishment. For an odd number of periods the sequence of profits is:

$$\sum_{t=1}^{T} \Pi_t = \frac{T-1}{2}\Pi^D + \frac{T-1}{2}\Pi^P + \Pi^U, \tag{48}$$

while, for an even number of periods:

$$\sum_{t=1}^{T} \Pi_t = \frac{T}{2}\Pi^D + \frac{T}{2}\Pi^P. \tag{49}$$

41

For an odd number of periods, the deviation strategy is weakly dominated by the cooperation strategy in $t = 1, 2, ..., T-1$ iff (48) is no higher than (45), i.e., $\Pi^D + \Pi^P \leq 2\Pi^C$, which is slacker than condition (20). For an even number of periods, the deviation strategy is weakly dominated by the cooperation strategy in $t = 1, 2, ..., T-1$ iff (49) is no higher than (45), i.e., $\Pi^D + \Pi^P \leq 2\Pi^C - \frac{2}{T}\left(\Pi^C - \Pi^U\right)$, which is again slacker than condition (20), given that $(1 - 2/T)\left(\Pi^C - \Pi^U\right) > 0$. ∎

Derivation of the punishment profits Π^P. Consider programme $\mathcal{P}^P$ in Section 5.1. Using the resource constraint in the participation constraint and replacing R_B in the objective function, the latter becomes: $pf\left(I_K, I_N\right) - (I_K + I_N)\,r'_B$. The solutions $I_K^P\left(r'_B\right), I_N^P\left(r'_B\right)$ are obtained by solving the first order conditions: $p\partial f\left(\cdot\right)/\partial I_K = r'_B$, $p\partial f\left(\cdot\right)/\partial I_N = r'_B$. Using the investment levels in the resource constraint, we get the loan size $L_B^P\left(r'_B\right)$. Using this in the individual rationality constraint (21), we get $R_B^P\left(r'_B\right) = \frac{1}{p}r'_B L_B^P$. Notice that, since $r'_B\left(P\right) > r_B$, the investment level in the two inputs falls short of the fully unsecured credit contract, i.e., $I_K^P < I_K^U, I_N^P < I_N^U$, and $L_B^P < L_B^U$. To determine the punishment P necessary to elicit commitment, we replace the expression for profits in the incentive constraint (20), and solve for P, obtaining:

$$P = pf\left(\hat{I}_K, \hat{I}_N\left(L_B^C, R_B^C\right)\right) - pf\left(I_K^C, I_N^C\right) - \left[pf\left(I_K^U, I_N^U\right) - L_B^U r_B\right] + \left[pf\left(I_K^P, I_N^P\right) - r_B L_B^P\right].$$

Proof of Proposition 8. T^* is obtained by solving condition (23). The left-hand side of (23) has the following properties: it is strictly increasing in T, since its derivative is $\left(\Pi^C - \Pi^{TC}\right) > 0$. For $T \to \infty$, it tends to infinity, which implies that condition (23) is satisfied. These properties guarantee that there is a value of T that solves (23) and this value is unique. Moreover,

$$\frac{dT^*}{d\beta} = \left[\frac{\Pi^U(p,r_B)\left(\frac{r_B}{P} - r_S\right)\frac{p\beta}{r_S}}{\left(\Pi^C(p,r_B,\beta) - \Pi^{TC}(p,r_B,r_S,\beta)\right)^2} + \frac{\Pi^{TC}(p,r_B,r_S,\beta)(1-p)}{\left(\Pi^C(p,r_B,\beta) - \Pi^{TC}(p,r_B,r_S,\beta)\right)^2} + \frac{\Pi^C(p,r_B,\beta)\left(1 - \frac{r_B}{r_S}\right)\beta}{\left(\Pi^C(p,r_B,\beta) - \Pi^{TC}(p,r_B,r_S,\beta)\right)^2}\right] I_K^*\left(\gamma\right),$$

which is positive under Assumption 1. ∎

Derivation of the expected profits under relationship lending: equation (25). Suppose to repeat T times the game described in Section 5.1. We can represent this game using the following discrete time process:

1. $X_1 = \Pi^C$.
2. For $t < T - 1$:

$$\begin{aligned}
Pr\left\{X_{t+1} = \Pi^C \mid X_t = \Pi^C\right\} &= x; \\
Pr\left\{X_{t+1} = \Pi^P \mid X_t = \Pi^C\right\} &= 1 - x; \\
Pr\left\{X_{t+1} = \Pi^C \mid X_t = \Pi^P\right\} &= 1.
\end{aligned} \tag{50}$$

3. For $t = T - 1$:

$$\begin{aligned}
Pr\left\{X_{t+1} = \Pi^U \mid X_t = \Pi^C\right\} &= x; \\
Pr\left\{X_{t+1} = \Pi^P \mid X_t = \Pi^C\right\} &= 1 - x; \\
Pr\left\{X_{t+1} = \Pi^U \mid X_t = \Pi^P\right\} &= 1.
\end{aligned} \tag{51}$$

where $x = 1 - p + pr < 1$. The problem is to compute the expected profits under relationship lending: $E\Pi|_{C_{(T-1)}} = E\left[\sum_{t=1}^{T} X_t\right]$. Define $d_t = Pr\left\{X_t = \Pi^C\right\}$, $t < T$. We can rewrite $E\Pi|_{C_{(T-1)}}$ as follows:

$$E\Pi|_{C_{(T-1)}} = \sum_{t=1}^{T-1} E\left[X_t\right] + E\left[X_T\right] = T\Pi^P + \left(\Pi^C - \Pi^P\right)\sum_{t=1}^{T}\{d_t\} + \left(\Pi^U - \Pi^P\right)[1 + (x-1)d_{T-1}] \tag{52}$$

We need to find an expression for d. From (50), d satisfies the following difference equations:

$$d_1 = 1; \ d_{t+1} = 1 + (x-1)d_t.$$

The solution is then:

$$d_t = \frac{1-(x-1)^t}{(2-x)} \tag{53}$$

$$d_{t+1} = \frac{1-(x-1)^{t+1}}{2-x} = \frac{(2-x)+(x-1)[1-(x-1)^t]}{2-x} = 1 + (x-1)\frac{1-(x-1)^t}{2-x}. \tag{54}$$

Therefore, we obtain: $\sum_{t=1}^{T-1} d_t = \sum_{t=1}^{T-1} d_t \frac{1-(x-1)^t}{2-x} = \frac{1}{2-x} = \left\{ T - \frac{1-(x-1)^t}{2-x} \right\}$. Substituting (53) and (54) into (52), we get:

$$E\Pi|_{C_{(T-1)}} = \frac{\Pi^C+(1-x)\Pi^P}{2-x}T + \frac{(2-x)\Pi^U-(1-x)\Pi^P-\Pi^C}{(2-x)^2}\left(1-(x-1)^T\right). \tag{55}$$

Finally, substituting $x = 1 - p + pr$ into (55), we get (25).

Proof of Proposition 9. Using (55), define the profits of relationship lending net of the total profits of trade credit $(T\Pi^{TC})$ as:

$$\frac{\Pi^C+(1-x)\Pi^P-(2-x)\Pi^{TC}}{(2-x)}T + \frac{(2-x)\Pi^U-\Pi^P(1-x)-\Pi^C}{(2-x)^2}[1-(x-1)^T]. \tag{56}$$

The proof proceeds as follows. First, we show that the function (56) takes a negative value if $T = 1$ and tends to infinity if $T \to \infty$. Then, we show that (56) is strictly increasing. These properties together imply that there exists a T^{NS} solving (56) with equality and it is unique. Last, we show that $T^{NS} > T^*$ and $T^{NS} \to T^*$ as $r \to 1$.

1. The form of (56) (as a function of T) is $E\Pi|_{C_{(T-1)}} - T\Pi^{TC} = \alpha T + \beta[1-(x-1)^T]$, where $\alpha = \left[\Pi^C + (1-x)\Pi^P - \Pi^{TC}\right]/(2-x)$ and $\beta = \left[(2-x)\Pi^U - \Pi^P(1-x) - \Pi^C\right]/(2-x)^2$. The function (56) oscillates round the line $Z_T = \alpha T + \beta$, alternately being above and below the line, but converging to the line as the number of periods increases. When $T = 1$, (56) becomes $\Pi^U - \Pi^{TC} < 0$. For $T \to \infty$

$$\lim_{T\to\infty}\left\{E\Pi|_{C_{(T-1)}} - T\Pi^{TC}\right\} = \frac{\Pi^C+(1-x)\Pi^P-(2-x)\Pi^{TC}}{(2-x)}\lim_{T\to\infty}T + \frac{(2-x)\Pi^U-\Pi^P(1-x)-\Pi^C}{(2-x)^2}\lim_{T\to\infty}[1-(x-1)^T].$$

Notice that $\lim_{T\to\infty}T = \infty$ and $\lim_{T\to\infty}[1-(x-1)^T] = 1$ since $-1 < (x-1) < 1$. It follows that $\lim_{T\to\infty}\left\{E\Pi|_{C_{(T-1)}} - T\Pi^{TC}\right\} = \infty$ iff $x \geq 1 - \left(\Pi^C-\Pi^{TC}\right)/\left(\Pi^{TC}-\Pi^P\right)$, i.e., using $x = 1-p+pr$:

$$r \geq 1 - \frac{\Pi^C-\Pi^{TC}}{p(\Pi^{TC}-\Pi^P)}. \tag{57}$$

Thus, under condition (57), the profits of relationship lending net of trade credit profits tend to infinity as the duration of the project tends to infinity.

2. To show that (56) is also strictly increasing in T, take the value of the function (56) in two subsequent periods, i, $i + 1$, and calculate the difference between them:

$$\frac{\Pi^C+(1-x)\Pi^P-(2-x)\Pi^{TC}}{(2-x)} + \frac{(2-x)\Pi^U-\Pi^P(1-x)-\Pi^C}{(2-x)}[(x-1)^i] \tag{58}$$

Under condition (57), the first term of (58) is positive. The second term can be positive or negative depending on i being odd or even. Since $(2-x)\Pi^U - \Pi^P(1-x) - \Pi^C \leq 0$, under (57), the second term of (58) is positive if i is odd and negative if i is even. Moreover, since $\mid x - 1 \mid < 1$, $(x-1)^i$ takes

values in the interval $(-1, +1)$ and gets smaller for increasing values of i. It follows that (58) takes its maximum value when $i = 2$. Thus, (58) is strictly positive at $i = 2$ iff:

$$\frac{\Pi^C + (1-x)\Pi^P - (2-x)\Pi^{TC}}{(2-x)} + \frac{(2-x)\Pi^U - \Pi^P(1-x) - \Pi^C}{(2-x)}[(x-1)^2] > 0.$$

We can rewrite the above condition as $x[\Pi^C + (1-x)\Pi^P] - \Pi^{TC} + (x-1)^2\Pi^U > 0$. Adding and subtracting $(1-x)[\Pi^C + (1-x)\Pi^P] + (1-x)\Pi^{TC}$ and rearranging, this becomes:

$$\left\{\Pi^C + (1-x)\Pi^P - (2-x)\Pi^{TC}\right\} + (1-x)\left\{\Pi^{TC} - \Pi^C + (1-x)(\Pi^U - \Pi^P)\right\} > 0. \qquad (59)$$

Under condition (57), the first term in curly brackets of (59) is positive while the second is negative. Since $0 < x < 1$, it follows that a sufficient condition for (59) to be satisfied is that $\left\{(\Pi^{TC} - \Pi^C) + (1-x)(\Pi^U - \Pi^P)\right\} < 0$, which implies:

$$r \geq 1 - \frac{\Pi^C - \Pi^{TC}}{p(\Pi^U - \Pi^P)}. \qquad (60)$$

Conditions (60) and (57) are equal except for the denominator of the fraction. Since $\Pi^U - \Pi^P < \Pi^{TC} - \Pi^P$, it follows that under (57), condition (60) is always satisfied. Thus, under (57) there exists a T^{NS} solving (56) with equality. This T^{NS} is unique.

3. To show that $T^{NS} > T^*$, take the difference between conditions (23) and (56). Since the term $T\Pi^{TC}$ cancels out, we are left with comparing the profits under relationship lending with perfect signal and with noisy signal. This difference is positive for any model with a generic duration T. Indeed, at any $t < T$ stage of the game, the entrepreneur gets Π^C with probability 1 under perfect signal and a linear combination of Π^C and Π^P under noisy signal. Since $\Pi^C > \Pi^P$, the expected profits under noisy signal are lower than the profits under perfect signal for any $t < T$. In the last period of the game, the entrepreneur gets Π^U under perfect signal and a linear combination of Π^U and Π^P under noisy signal. Since $\Pi^P < \Pi^U$, the expected profits under noisy signal are lower than the profits under perfect signal. It follows that the overall stream of profits under relationship lending with perfect signal is larger than with noisy signal for any generic T repetitions of the game.

Finally, to show that $T^{NS} \to T^*$ as $r \to 1$, it suffices to show that (56) tends to (23) as $r \to 1$, or, alternatively, that (25) tends to (23) as $x \to 1$. By taking the limit of (25) for $x \to 1$:

$$\lim_{x \to 1} \left\{\frac{\Pi^C + (1-x)\Pi^P - (2-x)\Pi^{TC}}{(2-x)}T + \frac{(2-x)\Pi^U - \Pi^P(1-x) - \Pi^C}{(2-x)^2}[1 - (x-1)^T]\right\} = (T-1)\Pi^C + \Pi^U - T\Pi^{TC},$$

which is condition (23). ■

Acknowledgements

References

Agarwal, S., Hauswald, R., 2008. The choice between arm's-length and relationship debt: Evidence from e-loans. WP 2008-10, Federal Reserve Bank of Chicago.

Almeida, H., Campello, M., 2007. Financial constraints, asset tangibility and corporate investment. Review of Financial Studies, 20, 1429-1460.

Alphonse, P., Ducret, J., Severin, E., 2006. When trade credit facilitates access to bank finance: Evidence from U.S. small business data, mimeo.

Auboin, M., Engemann, M., 2014. Testing the trade credit and trade link: Evidence from data on export credit insurance. Review of World Economics, 150, 715-743.

Beck, T., Demirgüç-Kunt, A., Vojislav Maksimovic, V., 2008. Financing patterns around the world: Are small firms different? Journal of Financial Economics, 89, 467-87.

Benmelech E., 2009. Asset salability and debt maturity: Evidence from 19th century American railroads. Review of Financial Studies, 22, 1545-1583.

Berger, A.N., Udell, G.F., 1990. Collateral, loan quality and bank risk. Journal of Monetary Economics, 22, 21–42.

Berkovitch E., Greenbaum S.I., 1991. The loan commitment as an optimal financing contract. Journal of Financial and Quantitative Analysis, 26, 83-95.

Besanko, D., Thakor, A.V., 1987a. Competition, collateral, and sorting equilibria in the credit market. International Economic Review, 28, 671-690.

Besanko, D., Thakor, A.V., 1987b. Competitive equilibrium in the credit market under asymmetric information. Journal of Economic Theory, 42, 167-182.

Bester, H., 1985. Screening vs. rationing in credit markets with imperfect information. American Economic Review, 75, 850-855.

Bias, B., Gollier, C., 1997. Trade credit and credit rationing. Review of Financial Studies, 10, 903-937.

Boot, A.W.A., 2000. Relationship banking: what do we know? Journal of Financial Intermediation, 9, 7–25.

Boot, A.W.A., Greenbaum, S.I., Thakor A.V., 1993. Reputation and discretion in financial contracting. American Economic Review, 83 (5), 1165-1183.

Boot, A.W.A., Thakor A.V., 1994. Moral hazard and secured lending in an infinitely repeated credit market game. International Economic Review, 35, 899-920.

Boot, A.W.A., Thakor A.V., 2000. Can Relationship Lending Survive Competition? Journal of Finance, 55 (2), 679-713.

Boot, A.W.A., Thakor A.V., Udell G.F. 1987. Competition, risk neutrality and loan commitments. Journal of Banking and Finance, 11, 449-471.

Boot, A.W.A., Thakor A.V., Udell G.F., 1991. Secured lending and default risk: equilibrium analysis, policy implications and empirical results. Economic Journal, 101, 458-472.

Brennan, M., Maksimovic, V., Zechner, J., 1988. Vendor financing. Journal of Finance, 43, 1127-1141.

Burkart, M., Ellingsen, T., 2004. In-kind finance: A theory of trade credit. American Economic Review, 94, 569-590.

Calomiris, C.W., Himmelberg, C.P., Wachtel, P., 1995. Commercial paper, corporate finance and the business cycle: A Microeconomic Perspective. Carnegie-Rochester Conference Series on Public Policy 42, 203-250.

Campello, M., Giambona, E., 2013. Real Assets and Capital Structure. Journal of Financial and Quantitative Analysis, 48, 1333-1370.

Chan, Y.S., Kanatas, G., 1985. Asymmetric valuations and the role of collateral in loan agreements. Journal of Money, Credit and Banking, 17, 84–95.

Chan, Y.S., and Anjan V. Thakor (1987), Collateral and competitive equilibria with moral hazard and private information. Journal of Finance, 42, 345–363.

Cook, L.D., 1999. Trade credit and bank finance: Financing small firms in Russia. Journal of Business Venturing, 14, 493-518.

Davydenko S.A., Frank, J.F., 2008. Do bankruptcy codes matter? A study of defaults in France, Germany, and the U.K. Journal of Finance, 63, 565–608.

Eisdorfer, A., 2009. Convertible debt and risk-shifting incentives. Journal of Financial Research, 32, 423–447.

Eisfeldt, A.L., Rampini, A., 2009. Leasing, ability to repossess, and debt capacity. Review of Financial Studies, 22, 1621-1657.

Ferris, J.S., 1981. A transaction theory of trade credit use. Quarterly Journal of Economics, 96, 247-270.

Fabbri, D., Menichini, A.M.C., 2010. Trade credit, collateral liquidation and borrowing constraints. Journal of Financial Economics, 96, 413-432.

Garcia-Appendini, E., 2010. Supplier certification and trade credit, Bocconi University Working Paper.

Gama, A.P., Mateus, C., Teixera, A., 2010. Does trade credit facilitate access to bank finance? An empirical evidence from Portuguese and Spanish small medium size enterprises. International Research Journal of Finance and Economics, 45, 26-45.

Giannetti, M., Burkart, M., Ellingsen, T., 2011. What you sell is what you lend? Explaining trade credit contracts. Review of Financial Studies, 24, 1261-1298.

Green, R.C., 1984. Investment incentives, debt, and warrants. Journal of Financial Economics 13, 115–136.

Greenbaum S.I., Kanatas G., Venezia I., 1991. Loan commitments and the management of uncertain credit demand. Journal of Real Estate Finance and Economics, 4, 353-366.

Harhoff, D., Körting, T. 1998. Lending relationships in Germany: Empirical results from survey data. CEPR Discussion Papers 1917.

Hart, O., Moore, J., 1994. A theory of debt based on the inalienability of human capital. Quarterly Journal of Economics, 109, 841-879.

Inderst, R., Mueller, H., 2007. A lender-based theory of collateral. Journal of Financial Economics, 84, 826-859.

Innes, R., 1990. Limited liability and incentive contracting with ex ante action choices. Journal of Economic Theory, 52, 45–67.

Jackson, T.H., Kronman, A.T., 1979. Secured financing and priorities among creditors. The Yale Law Journal, 88, 1143–1182.

Jimenez, Z.G., Salas, V., Saurina, J., 2006. Determinants of collateral. Journal of Financial Economics, 81, 255–281.

Kanatas G., 1987. Commercial papers, bank reserve requirements, and the informational role of loan commitments. Journal of Banking and Finance, 11, 425-448.

Long, M.S., Malitz, I.B., Ravid, S.A., 1993. Trade credit, quality guarantees, and product marketability. Financial Management, 22, 117–127.

Love, I., Preve, L.A., Sarria-Allende, V., 2007. Trade credit and bank credit: Evidence from recent financial crisis, Journal of Financial Economics, 83, 453–469.

Manova, K., 2013. Credit Constraints, Heterogeneous Firms, and International Trade, Review of Economic Studies, 80, 711-744.

Marquez, R., Yavuz, M.D., 2013. Specialization, productivity and financing constraints. Review of Financial Studies, 26, 2961-2984.

Petersen, M.A., Rajan, R.G., 1997, Trade credit: Theory and evidence. Review of Financial Studies, 10, 661-691.

Qian, J., Strahan, P.E., 2007. How laws and institutions shape financial contracts: The case of bank loans. Journal of Finance, 52, 2803-2834.

Rajan, R., Winton, A., 1995. Covenants and collateral as incentives to monitor. Journal of Finance, 50 (4), 1113-1146.

Ramakrishnan, R.T.S., Anjan V. Thakor, A.V., 1984. Information reliability and a theory of financial intermediation. Review of Economic Studies, 51, 415-432.

Rampini, A., Viswanathan, S., 2013. Collateral and capital structure. Journal of Financial Economics, 109, 466–492.

Shleifer, Andrei, and Robert Vishny (1992), Liquidation values and debt capacity: A market equilibrium approach. Journal of Finance, 47, 1343-1366.

Smith, C.W. Jr., Warner, J.B., 1979. On financial contracting. An analysis of bond covenants. Journal of Financial Economics, 7, 117-161.

Stulz, R.M., Johnson, H., 1985. An analysis of secured debt. Journal of Financial Economics, 4, 501-21.

Summers, B., Wilson, N., 2002. Trade credit terms offered by small firms: Survey evidence and empirical analysis. Journal of Business and Finance Accounting, 29, 317-335.

Udell, J., 2004. Asset-based finance - Proven disciplines for prudent lending, Commercial Finance Association.

Thakor A.V., 1982. Toward a theory of bank loan commitments. Journal of Banking and Finance, 6, 55-83.

Tirole, J., 1992. Collusion and the theory of organizations, in Advances in Economic Theory, vol. 2, ed. J.J. Laffont, Cambridge University Press.

Williamson, O.E., 1988. Corporate finance and corporate governance. Journal of Finance, 43, 567-592.