Measuring systemic risk in the European banking sector: A copula *CoVaR* approach

Emmanouil N. Karimalis[a,b,1], Nikos K. Nomikos[a,*]

[a] *Cass Business School, City, University of London, 106 Bunhill Row, London EC1Y 8TZ, UK.*
[b] *Bank of England, Threadneedle Street, London EC2R 8AH, UK.*

Abstract

We propose a new methodology based on copula functions to estimate *CoVaR*, the *Value-at-Risk* (*VaR*) of the financial system conditional on an institution being under financial distress. Our Copula *CoVaR* approach provides simple, closed-form expressions for various definitions of *CoVaR* for a broad range of copula families and allows the *CoVaR* of an institution to have time-varying exposure to its *VaR*. We extend this approach to estimate other "co-risk" measures such as *Conditional Expected Shortfall* (*CoES*). We focus on a portfolio of large European banks and examine the existence of common market factors triggering systemic risk episodes. Further, we analyse the extent to which bank-specific characteristics such as size, leverage, and equity beta are associated with institutions' contribution to systemic risk and highlight the importance of liquidity risk at the outset of the financial crisis in summer 2007. Finally, we investigate the link between macroeconomy and systemic risk and find that changes in major macroeconomic variables can contribute significantly to systemic risk.

Keywords: Systemic Risk, European Banking, Risk Spillovers, Value-at-Risk, Copulas

JEL classification: G11, G18, G20, G21, G32, G38

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[*]Corresponding author: n.nomikos@city.ac.uk; tel.: +44 (0) 207040 0104.

Email addresses: `Emmanouil.Karimalis@bankofengland.co.uk` (Emmanouil N. Karimalis), `N.Nomikos@city.ac.uk` (Nikos K. Nomikos)

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1. Introduction

The recent financial crisis has highlighted in the most prominent way the importance of prudent monitoring and assessment of systemic risk. Systemic risk can be seen as the adverse consequence, for the financial system and the broader economy, of a financial institution being in financial distress. The failure of large credit institutions can not only threaten the stability of the financial system but also have dramatic effects on the real economy. It is well-documented that conditional correlations between asset returns are much stronger in periods of financial distress (see e.g. Longin and Solnik (2001); Ang and Chen (2002); Jondeau and Rockinger (2006); Chollete et al. (2009), among others) and typically arise from exposure to common shocks, although amplifications of financial shocks are also associated with balance sheet channels and liquidity spirals (see e.g. Brunnermeier (2009); Adrian and Shin (2010)). As a result, losses tend to spread across financial institutions during stress times, amplifying the risk of systemic contagion.

Assessing the level of contribution of the so-called systemically important financial institutions (SIFIs) to systemic risk and designing a regulatory framework capable of ensuring financial stability is the foremost objective of international financial regulatory institutions. The *Value-at-Risk* (*VaR*), the risk measure most widely used by financial institutions, is not capable of capturing the systemic nature of risk since it focuses on the risk of an individual institution when viewed in isolation. As a result, there has been a growing interest in developing alternative risk measures that reflect systemic risk and avoid the shortcomings of *VaR*. For instance, Acharya et al. (2017) measure the propensity of a financial institution to be undercapitalised when the financial system as a whole is undercapitalised, using the systemic expected shortfall (SES). Greenwood et al. (2015) compute bank exposures to system-wide deleveraging and evaluate a variety of interventions to reduce the vulnerability of financial institutions to fire sales. Brownlees and Engle (2012) introduce the SRISK index, the expected capital shortage of a firm conditional on a substantial market decline, as an alternative measure of systemic risk while, Engle et al. (2014) develop an econometric approach to measure the systemic risk of European financial institutions. Billio et al. (2012) propose several econometric measures to capture the connectedness among financial institutions based on principal components analysis and Granger-causality networks. An extensive survey of the main quantitative measures of systemic risk in the literature can be found in Bisias et al. (2012).

An alternative measure of systemic risk is the *Conditional Value-at-Risk* (*CoVaR*) of Adrian and Brunnermeier (2016), which attempts to capture risk spillovers among financial institutions and has attracted a lot of attention by the regulatory and academic communities, especially since the 2007 financial crisis. The general framework of *CoVaR* depends on the conditional distribution of a random variable $R_{s,t}$ representing the returns of the entire financial system at time t given that another financial institution i, represented by a random variable $R_{i,t}$, is in distress. Currently, there are two alternative definitions of $CoVaR$ in the literature. In the original definition by Adrian and Brunnermeier (2016), $CoVaR$ is defined as the conditional distribution of $R_{s,t}$ given that $R_{i,t} = VaR_t^i$, while in the modified definition of $CoVaR$, proposed by Girardi and Ergün (2013), the conditioning event is $R_{i,t} \leq VaR_t^i$. In other words, the former definition represents the *VaR* of the system assuming that institution i is *exactly* at its *VaR* level whereas the latter definition of $CoVaR$ represents the same risk metric assuming that institution i is *at most* at its *VaR* level. The latter definition of $CoVaR$ is arguably very useful. First of all, it considers more severe distress events for institution i that are further in the tail of the loss distribution (below VaR_t^i level) in contrast to the highly selective and over-optimistic scenario $R_{i,t} = VaR_t^i$. Moreover, $CoVaR$ estimates based on $R_{i,t} \leq VaR_t^i$ can be tested for statistical accuracy and independence using modified versions of the standard Kupiec (1995) and Christoffersen (1998) tests, respectively. Finally, and perhaps most importantly, Mainik and Schaanning (2014) show that conditioning on $R_{i,t} \leq VaR_t^i$ has great advantages for dependence modelling.

Our study builds on the $CoVaR$ methodology described above and uses copula functions to estimate $CoVaR$ under both definitions. We derive simple closed-form expressions for a broad range of copula families that allow the modelling of various forms of dependence, while focusing on extreme co-movements of financial system-institution returns, which is, in practice, the main concern of all systemic risk measures. Given the distinctive characteristics of copula families, our modelling approach enables the separation of dependence from marginal distributions providing greater flexibility and eliminating misspecification biases. A dynamic version of the model is also proposed - one that is capable of incorporating time-varying correlation into $CoVaR$ calculations. Through counterexamples, we show that $CoVaR$ measures generated by our modelling approach share the dependence consistency properties found in Mainik and Schaanning (2014). In addition, we extend the Copula $CoVaR$ methodology to other "co-risk" measures and derive expressions for *Conditional Expected Shortfall* (*CoES*) under both definitions. Furthermore, we show that our approach can be easily employed by financial regulators as a useful stress testing tool for assessing the impact of extreme market conditions on the stability of the financial system.[2]

Focusing on a portfolio of large European banks, we measure the contribution of each individual bank to systemic risk using both $CoVaR$ and *CoES* systemic risk metrics. We show that the ordering of systemically important institutions and the magnitude of the

[2]We note that, independently from this study, Hakwa et al. (2015) and Bernardi et al. (2017) also provide expressions for estimating CoVaR using copulas. However, our methodology provides a number of distinct advantages as it permits the use of time-varying correlations, it allows for the calculation of alternative measures of systemic risk, such as *Co-Expected Shortfall (CoES)*, and also allows for the computation of $CoVaR$ as defined both in Adrian and Brunnermeier (2016) and in Girardi and Ergün (2013).

corresponding systemic risk measures are affected by the choice of underlying distributions, but are robust across different systemic risk measures. In a cross-country comparison, we find that banks from Spain and France have, on average, the highest contribution to systemic risk. Moreover, we investigate whether common market factors or institution specific characteristics are important determinants of systemic risk. We show that liquidity risk is an important determinant of systemic risk contribution. The large impact of funding liquidity in the pre-crisis period partly explains the "liquidity spirals" that occurred after the break out of the financial crisis in summer of 2007. Its relative impact has been reduced in the post-crisis period due to the coordinated intervention of the European Central Bank (ECB) and the Federal Reserve in the interbank market. We also find that size and leverage are the most robust determinants of systemic risk contribution concluding that larger and more leveraged financial institutions can be harmful for the overall stability of the financial system. Finally, we investigate the link between systemic risk and macroeconomy and the extent in which changes in key macroeconomic variables contribute to systemic risk. Intuitively, we find that changes in unemployment, industrial production, stock market index and GDP contribute significantly to systemic risk.

The rest of the paper is organised as follows: Section 2 formally defines the *CoVaR* and *CoES* measures and presents the Copula *CoVaR* methodology. Derivation of closed-form expressions both for *CoVaR* and *CoES* systemic risk measures are also presented in this section. Section 3 describes the data we use in the empirical part of this study and Section 4 presents the computation of systemic risk measures. Section 5 reports the results of individual contribution to systemic risk. This section also analyses the determinants of systemic risk and discusses their implications for the stability of the financial system. Section 6 concludes.

2. *CoVaR* Methodology

2.1. Definition of CoVaR

Consider a random variable $R_{i,t}$ that represents the returns of financial institution i at time t $(i = 1, \ldots, N\,;\, t = 1, \ldots, T)$. The *Value-at-Risk* (*VaR*) of the random variable $R_{i,t}$ at the confidence level $\alpha \in (0, 1)$, $VaR^i_{\alpha,t}$, is defined as the α-quantile of the return distribution

$$VaR^i_{a,t} = F^{-1}_{i,t}(\alpha), \tag{1}$$

where $F^{-1}_{i,t}$ is the generalised inverse distribution function of the return distribution $F_{i,t}$, i.e., $F^{-1}_{i,t}(\alpha) := \inf\{r_{i,t} \in \mathbb{R} : F_{i,t}(r_{i,t}) \geq \alpha\}$.[3] Equivalently, Equation (1) can also be writ-

[3]It is common to present downside risk statistics, such as *VaR*, in positive values. In this paper, we do not follow this sign convention and instead maintain the original (negative) sign of the conditional quantile for all downside risk measures reported in the subsequent sections, such as *VaR*, *CoVaR*, Δ*CoVaR*, *CoES* and Δ*CoES*.

ten as

$$Pr(R_{i,t} \leq VaR^i_{a,t}) = \alpha. \tag{2}$$

Two different definitions of *Conditional Value-at-Risk* (*CoVaR*) appear in the literature using different conditioning events. The notation $CoVaR^{=}_{\alpha,\beta,t}$ denotes the original definition, introduced by Adrian and Brunnermeier (2016), representing the β-quantile of the returns of financial system $R_{s,t}$ conditional on $R_{i,t} = VaR^i_{\alpha,t}$, while the notation $CoVaR_{\alpha,\beta,t}$ denotes the alternative definition, proposed by Girardi and Ergün (2013), where the conditioning event is $R_{i,t} \leq VaR^i_{\alpha,t}$. Formally, $CoVaR^{=}_{\alpha,\beta,t}$ and $CoVaR_{\alpha,\beta,t}$ are defined as the β-quantiles of the following conditional distributions

$$Pr(R_{s,t} \leq CoVaR^{=}_{\alpha,\beta,t} | R_{i,t} = VaR^i_{\alpha,t}) = \beta, \tag{3}$$

$$Pr(R_{s,t} \leq CoVaR_{\alpha,\beta,t} | R_{i,t} \leq VaR^i_{\alpha,t}) = \beta, \tag{4}$$

where $s \neq i$. The confidence levels α and β are decided *ex-ante* by the financial regulator. Typical values are 1% or 5%. In most studies a common confidence level for α and β is used, i.e., $\alpha = \beta$, however, working with different confidence levels, i.e., $\alpha \neq \beta$, is also possible.

Adrian and Brunnermeier (2016) employ linear quantile regressions to obtain $CoVaR^{=}_{\alpha,\beta,t}$ estimates. The $CoVaR^{=}_{\alpha,\beta,t}$ estimates derived from this procedure, however, do not have a time-varying exposure to institution's $VaR^i_{\alpha,t}$. On the other hand, Girardi and Ergün (2013) follow a three-step procedure based on univariate GARCH-type models and the bivariate DCC model of Engle (2002) to estimate $CoVaR_{\alpha,\beta,t}$. As a result, time-varying correlation is incorporated into their $CoVaR_{\alpha,\beta,t}$ estimates. Their approach, however, requires numerical integration which can be computationally intensive and time expensive. In addition, the specification of the marginal distribution depends on the choice of the bivariate distribution of $R_{s,t}$ and $R_{i,t}$. In practice, the distributional characteristics of $R_{s,t}$ and $R_{i,t}$ can differ substantially and hence, restricting the marginal specification may introduce misspecification bias in the computation of $CoVaR_{\alpha,\beta,t}$.

2.2. Copula CoVaR Methodology

In this section we show how the *Conditional Value-at-Risk* (*CoVaR*) can be estimated using copula functions. We provide simple analytical expressions for a broad range of copula families for both *CoVaR* definitions. In this respect, our Copula *CoVaR* approach overcomes the burden of numerical integration and also incorporates the time-varying dependence between $R_{s,t}$ and $R_{i,t}$ into the computation of systemic risk measures through the copula parameter(s). Furthermore, Copula *CoVaR* approach provides greater flexibility in the specification of the marginals and the dependence structure (i.e. the marginal specification is not restricted by the choice of the bivariate copula distribution), eliminating in this way potential misspecification bias in the computation of risk measures.

This modelling setting also enables the decomposition of systemic risk into three main components: (a) the dependence structure; (b) the magnitude of dependence and (c) the marginal series. As a result, we can assess the relevant contribution of any of these three components to systemic risk.

The joint distribution function of bivariate random variables (Y, X) is

$$F_{YX}(y, x) = Pr(Y \leq y, X \leq x).$$

The famous theorem of Sklar (1959) gives the connection of marginals and copulas with the joint distribution. Let F_{YX} represent a bivariate cumulative distribution function with marginal distributions F_Y and F_X, then there exists a two dimensional copula cumulative distribution function C on $[0,1]^2$, such that for all $(y, x) \in \overline{\mathbb{R}}^2$ it holds that

$$F_{YX}(y, x) = C(F_Y(y), F_X(x)).$$

For continuous F_Y and F_X, C is uniquely determined by

$$C(u, v) = F_{YX}(F_Y^{-1}(u), F_X^{-1}(v)),$$

where random variables $u = F_Y(y)$ and $v = F_X(x)$ (i.e., obtained by the probability integral transform) are uniformly distributed on $[0, 1]$, while $F_Y^{-1}(u)$ and $F_X^{-1}(v)$ are the generalised inverse distribution functions of the marginals.

It can be shown (Bouyè and Salmon, 2009), that the conditional probability distribution $Pr(Y \leq y|X = x)$ can be expressed in terms of a copula function as

$$Pr(Y \leq y|X = x) = \frac{\partial C(u, v)}{\partial v}. \tag{5}$$

In contrast, the conditional probability distribution $Pr(Y \leq y|X \leq x)$ can be expressed in terms of a copula function as

$$Pr(Y \leq y|X \leq x) = \frac{Pr(Y \leq y, X \leq x)}{Pr(X \leq x)} = \frac{C(F_Y(y), F_X(x))}{F_X(x)} = \frac{C(u, v)}{v}. \tag{6}$$

The class of Archimedean copulas has recently found wide usage in the economics and finance literature due to their simple closed-form cumulative distribution functions and their properties allowing the modelling of the dependence between random variables. [4]

[4]For the various applications of copulas in finance see for example, Kole et al. 2007; Heinen and Valdesogo 2008; Chollete et al. 2009; Min and Czado 2010; Brechmann et al. 2012; Czado et al. 2012; Nikoloulopoulos et al. 2012; Brechmann and Czado 2013; Weiß and Scheffer 2015; Scheffer and Weiß 2017, among others. A review of the literature on copula-based models for economic and financial time series can be found in Patton (2012).

Bivariate Archimedean copulas are defined as

$$C(u,v) = \varphi^{-1}\left[\varphi(u)+\varphi(v)\right],$$

where $\varphi : [0,1] \to [0,\infty)$ is a continuous strictly decreasing convex function such that $\varphi(1)=0$ and φ^{-1} is the inverse of φ. The function φ is called *generator function* of the copula C (see Nelsen (2007), for further details).

We begin with the presentation of $CoVaR^{=}_{\alpha,\beta,t}$ in terms of Archimedean copulas and provide general solutions through their corresponding generator functions.[5] From the general result in Equation (5) we have

$$Pr(Y \le y|X=x) = \frac{\partial C(u,v)}{\partial v} = \frac{\varphi'(v)}{\varphi'(C(u,v))} = \frac{\varphi'(v)}{\varphi'\left(\varphi^{-1}\left[\varphi(u)+\varphi(v)\right]\right)}. \quad (7)$$

Assuming that the above random variables Y and X represent the financial system, $R_{s,t}$, and the returns of institution i, $R_{i,t}$, with distribution functions $F_{s,t}$ and $F_{i,t}$, respectively; the conditional distribution $Pr(R_{s,t} \le CoVaR^{=}_{\alpha,\beta,t}|R_{i,t} = VaR^{i}_{\alpha,t})$ can be equivalently expressed in terms of a copula generator function as follows

$$Pr(R_{s,t} \le CoVaR^{=}_{\alpha,\beta,t}|R_{i,t} = VaR^{i}_{\alpha,t}) = \frac{\varphi'(v)}{\varphi'\left(\varphi^{-1}\left[\varphi(u)+\varphi(v)\right]\right)} = \beta.$$

Solving for u, under the general condition that $\partial/\partial v\, C(u,v)$ is partially invertible in its first argument u, we obtain the copula conditional quantile

$$u^{=} \equiv u = \varphi^{-1}\left[\varphi\left(\varphi'^{-1}\left(\frac{1}{\beta}\varphi'(v)\right)\right) - \varphi(v)\right]. \quad (8)$$

Applying the probability integral transform in Equation (8), we derive an explicit expression for $CoVaR^{=}_{\alpha,\beta,t}$ for a broad range of Archimedean copula functions, that is

$$CoVaR^{=}_{\alpha,\beta,t} = F^{-1}_{s,t}\left(\varphi^{-1}\left[\varphi\left(\varphi'^{-1}\left(\frac{1}{\beta}\varphi'\left(F_{i,t}\left(VaR^{i}_{\alpha,t}\right)\right)\right)\right) - \varphi\left(F_{i,t}\left(VaR^{i}_{\alpha,t}\right)\right)\right]\right), \quad (9)$$

where $F^{-1}_{s,t}$ is the generalised inverse distribution function of $F_{s,t}$. From the definition of *VaR* it holds that $v = F_{i,t}(VaR^{i}_{\alpha,t}) = F_{i,t}(F^{-1}_{i,t}(\alpha)) = \alpha$. Therefore, the expression for

[5] We also derive explicit expressions for $CoVaR^{=}_{\alpha,\beta,t}$ for the elliptical copula families, i.e., Gaussian and Student-t copulas. Due to space limitation we do not report the general expressions for those particular copula families but are available upon request. Unfortunately, there are no explicit solution for $CoVaR_{\alpha,\beta,t}$ for these particular copula families and hence numerical integration is required.

$CoVaR^{=}_{\alpha,\beta,t}$ in Equation (9) can be simplified further as follows

$$CoVaR^{=}_{\alpha,\beta,t} = F^{-1}_{s,t}\left(\varphi^{-1}\left[\varphi\left(\varphi'^{-1}\left(\frac{1}{\beta}\varphi'(\alpha)\right)\right) - \varphi(\alpha)\right]\right). \tag{10}$$

Alternatively, an analytical expression can also be given for $CoVaR_{\alpha,\beta,t}$ for a wide range of Archimedean copula families. Given the general result in Equation (6), the conditional distribution $Pr(R_{s,t} \leq CoVaR_{\alpha,\beta,t}|R_{i,t} \leq VaR^{i}_{\alpha,t})$ can be equivalently written as

$$Pr(R_{s,t} \leq CoVaR_{\alpha,\beta,t}|R_{i,t} \leq VaR^{i}_{\alpha,t}) = \frac{\varphi^{-1}\left[\varphi(u) + \varphi(v)\right]}{v} = \beta. \tag{11}$$

Similarly, from the definition of *VaR* it holds that $v = F_{i,t}(VaR^{i}_{\alpha,t}) = F_{i,t}\left(F^{-1}_{i,t}(\alpha)\right) = \alpha$. Therefore, the expression in Equation (11) can be expressed as

$$\varphi^{-1}\left[\varphi(u) + \varphi(\alpha)\right] = \alpha \cdot \beta. \tag{12}$$

Finally, after solving for u and applying the probability integral transform, under the general condition that $C(u, v)$ is partially invertible in its first argument u, $CoVaR_{\alpha,\beta,t}$ has a general representation for Archimedean copulas, that is

$$u^{\leq} \equiv u = \varphi^{-1}\left[\varphi(\alpha \cdot \beta) - \varphi(\alpha)\right], \tag{13}$$

$$CoVaR_{\alpha,\beta,t} = F^{-1}_{s,t}\left(\varphi^{-1}\left[\varphi(\alpha \cdot \beta) - \varphi(\alpha)\right]\right). \tag{14}$$

The general representation of *CoVaR* in Equation (10) and in Equation (14) implies a constant correlation between $R_{s,t}$ and $R_{i,t}$. However, it is known that the dependence structure between financial asset returns is not constant but rather, time-varying (see e.g. Engle (2002); Patton (2006); Manner and Reznikova (2012), and references therein). Numerous studies have also indicated that the correlation between financial series tends to be more pronounced during downturns than during upturns, a stylised feature that should be considered in the estimation of systemic risk. In this respect, the use of constant correlations may affect the risk estimates and lead to incorrect inferences. We follow the specification proposed by Patton (2006) in order to introduce a dynamic version of the Copula *CoVaR* model and hence incorporate time-varying correlation into *CoVaR* estimation. Patton (2006) proposed observation-driven copula models, for which the time-varying dependence parameter(s) of a copula is a parametric function of transformed lagged data. In Appendix A we derive analytical expressions for $CoVaR^{=}_{\alpha,\beta,t}$ and $CoVaR_{\alpha,\beta,t}$, while in Appendix B we present the time-varying parameter specification for the Clayton, Frank, Gumbel and BB7 copulas, respectively.[6] These copula families are very popular in the literature for modelling the dependence between financial asset returns since they allow for very flexible dependency structures and can capture various forms of tail dependence.

2.3. Extension to CoES

The *CoVaR* concept can be easily adopted for other "co-risk" measures. One of them is the *Conditional Expected Shortfall* (*CoES*). We denote by $CoES^{=}_{\alpha,\beta,t}$ the expected shortfall of the financial system conditional on $R_{i,t} = VaR^{i}_{\alpha,t}$ and similarly by $CoES_{\alpha,\beta,t}$ the expected shortfall of the financial system conditional on $R_{i,t} \leq VaR^{i}_{\alpha,t}$. In this respect, *CoES* estimates can be easily obtained for both definitions within our framework as follows

$$CoES^{=}_{\alpha,\beta,t} = \frac{1}{\beta}\int_{0}^{\beta} CoVaR^{=}_{\alpha,q,t}\, dq, \tag{15}$$

$$CoES_{\alpha,\beta,t} = \frac{1}{\beta}\int_{0}^{\beta} CoVaR_{\alpha,q,t}\, dq, \tag{16}$$

where $CoVaR^{=}_{\alpha,q,t} = Pr(R_{s,t} \leq F^{-1}_{s,t}(q)|R_{i,t} = VaR^{i}_{\alpha,t})$ and $CoVaR_{\alpha,q,t} = Pr(R_{s,t} \leq F^{-1}_{s,t}(q)|R_{i,t} \leq VaR^{i}_{\alpha,t})$.

2.4. Systemic Risk Contributor and Dependence Consistency

Following Adrian and Brunnermeier (2016), we adopt Δ*CoVaR* as a measure of institution i's contribution to systemic risk and also define by $\Delta CoVaR_{\alpha,\beta,t}$ the difference between the *CoVaR* of the financial system conditional on $R_{i,t} \leq VaR^{i}_{\alpha,t}$ and the *CoVaR* of the financial system conditional on $R_{i,t} \leq VaR^{i}_{0.5,t}$ (institution i being *at most* at its median state), that is

$$\Delta CoVaR_{\alpha,\beta,t} = CoVaR_{\alpha,\beta,t} - CoVaR_{0.5,\beta,t}\,.$$

The computation of $CoVaR^{=}_{0.5,\beta,t}$ or $CoVaR_{0.5,\beta,t}$ is straightforward and can be carried out as in the $CoVaR^{=}_{\alpha,\beta,t}$ or $CoVaR_{\alpha,\beta,t}$ case by simply modifying the stress scenario. We also employ Δ*CoES* as a measure of institution i's contribution to systemic risk where the contribution is measured in terms of *CoES*. Therefore, we define

$$\Delta CoES^{=}_{\alpha,\beta,t} = CoES^{=}_{\alpha,\beta,t} - CoES^{=}_{0.5,\beta,t}\,,$$

$$\Delta CoES_{\alpha,\beta,t} = CoES_{\alpha,\beta,t} - CoES_{0.5,\beta,t}\,,$$

where $\Delta CoES^{=}_{\alpha,\beta,t}$ denotes the difference between the *CoES* of the financial system conditional on $R_{i,t} = VaR^{i}_{\alpha,t}$ and the *CoES* of the financial system conditional on $R_{i,t} = VaR^{i}_{0.5,t}$, while $\Delta CoES_{\alpha,\beta,t}$ denotes the same risk metric with stress scenarios being

[6] The $\partial/\partial v\, C(u,v)$ of Gumbel copula is not invertible in its u and hence we cannot derive analytical expressions for $CoVaR^{=}_{\alpha,\beta,t}$.

$R_{i,t} \leq VaR^i_{\alpha,t}$ and $R_{i,t} \leq VaR^i_{0.5,t}$, respectively.

To investigate whether the different representations for measuring contribution to systemic risk, derived within the Copula $CoVaR$ framework, encompass the dependence consistency properties reported in Mainik and Schaanning (2014), we compare $\Delta CoVaR$ estimates for the bivariate distribution with a Clayton copula.[7] Figure 1 presents $\Delta CoVaR^{=}_{\alpha,\beta,t}$ and $\Delta CoVaR_{\alpha,\beta,t}$ measures as a function of the dependence parameter θ for a Clayton copula with Student-t marginals with three degrees of freedom at three different confidence levels, i.e., 1%, 5% and 10%. The behaviour of risk measures in these two models confirms the results in Mainik and Schaanning (2014). Initially, $\Delta CoVaR^{=}_{\alpha,\beta,t}$ increases with respect to the dependence parameter; however, after a certain threshold it counter-intuitively starts to decrease. In other words, $\Delta CoVaR^{=}_{\alpha,\beta,t}$ fails to detect dependence when it becomes more pronounced. On the other hand, $\Delta CoVaR_{\alpha,\beta,t}$ increases with respect to the dependence parameter. Therefore, conditioning on $R_{i,t} \leq VaR^i_{\alpha,t}$ gives a much more consistent response to dependence than conditioning on $R_{i,t} = VaR^i_{\alpha,t}$.

[Insert Figure 1 here]

3. Data

We focus on the STOXX Europe 600 Banks Index that consists of 46 large European banks from 15 European countries, characterised by a large market capitalisation, international activity, cross-country exposure and a representative size in the local market. The STOXX Europe 600 Banks Index is a component of the STOXX Europe 600 Index that represents large, mid and small capitalisation companies across 18 countries of the European region. It is the largest, in terms of market capitalisation, sector index of STOXX Europe 600 Index (€748.5 billion as of June, 2013), which indicates the relative importance and size of the banking sector in Europe. We exclude 4 institutions from the initial sample because the history of their corresponding datasets is narrow and does not cover the time period we want to analyse. Therefore, the resulting sample is formed by a total of 42 European banks, starting on 01/04/2002 and ending on 31/12/2012. This time period provides a good platform to assess the level of contribution of the systemically important financial institutions in Europe to systemic risk since it includes a number of significant events (e.g. the U.S subprime mortgage crisis, the Lehman Brothers collapse, the European sovereign debt crisis etc.). We assign the Q3 2007 - Q4 2012 as the crisis period because the majority

[7] We have also compared $\Delta CoVaR$ for the bivariate distribution with a Frank copula. The dependence consistency properties are in line with the results reported for the bivariate distribution with a Clayton copula. Similar dependence consistency results are obtained when $\Delta CoES$ is employed for the same stochastic models.

of those events occurred within this period of time.[8]

Following Adrian and Brunnermeier (2016) and Lòpez-Espinosa et al. (2012), we work with weekly returns to avoid the non-synchronicity of daily data. Therefore, we obtain weekly equity adjusted prices - to account for capital operations (i.e., splits, dividends etc.) - from the Datastream database and generate weekly log returns. There are 562 weekly returns for each institution in our sample, a list of which can be found in Appendix C. For each bank, an equally-weighted average of the returns of the remaining banks in the sample is used as a proxy for the financial system. This way, the resulting system return portfolios can be considered representative of the European financial system, allowing the study of possible spillover effects between a stressed institution and the financial system. Moreover, this approach rules out any spurious correlation that may be induced by banks that are more heavily represented in the composition of the financial system proxy. For example, HSBC has a total contribution of 20.5% to the composition of the STOXX Europe 600 Banks Index. As a result, if the corresponding index is used as a proxy for the financial system, systemic risk estimates generated conditional on HSBC will be severely affected by the presence and large scale factor of HSBC in the financial system's portfolio proxy.

4. Copula *CoVaR* Estimation

The computation of *CoVaR* or *CoES* requires the estimation of the parameter(s) of the marginal densities and the copula function that captures the dependence between $R_{s,t}$ and $R_{i,t}$. Assume a vector of system and institution returns $\mathbf{R}_t = (R_{s,t}, R_{i,t})'$, $(t = 1, \ldots, T\,; i = 1, \ldots, N)$ where $s \neq i$. Given that a copula function and the marginals are continuous, their joint probability density function can be expressed in terms of the copula density function, $c(\cdot, \cdot\,; \theta_t)$, and the univariate marginal densities, $f_{s,t}(R_{s,t}; \phi_s)$ and $f_{i,t}(R_{i,t}; \phi_i)$, as follows

$$f(R_{s,t}, R_{i,t}) = c(u_t, v_t\,; \theta_t) \cdot f_{s,t}(R_{s,t}\,; \phi_s) \cdot f_{i,t}(R_{i,t}\,; \phi_i), \tag{17}$$

where θ_t denotes the copula parameter while ϕ_s and ϕ_i denote the parameters for the system's and institution i's marginal distributions, respectively. In the above expression $u_t = F_{R_{s,t}}(R_{s,t}\,; \phi_s)$ and $v_t = F_{R_{i,t}}(R_{i,t}\,; \phi_i)$ are the uniformly transformed marginal series. The log-likelihood function of Equation (17) is given by

$$L(\theta_t, \phi_s, \phi_i) = \sum_{t=1}^{T} \left[\log c(u_t, v_t\,; \theta_t) + \log f_{s,t}(R_{s,t}\,; \phi_s) + \log f_{i,t}(R_{i,t}\,; \phi_i)\right]. \tag{18}$$

[8] We believe that it is not a trivial task to distinguish between the 2007-2008 Global financial crisis and the ensuing Euro-crisis as a number of significant, interrelated events took place during the intervening period of time. Thus, we follow the timeline suggested by Alter and Schüer (2012) and extend it to cover the period up to Q4 2012, which corresponds to the end of the sample examined here.

The marginal densities $f_{s,t}(R_{s,t};\, \phi_s)$ and $f_{i,t}(R_{i,t}\,; \phi_i)$ can be conditional densities and the series $R_{s,t}$ and $R_{i,t}$ are usually modelled by a GARCH-type model, whose residuals are treated as *i.i.d* random variables. Under this setting, full maximum likelihood estimates (MLE) can be obtained by maximising Equation (18) with respect to the parameters $(\theta_t, \phi_s, \phi_i)$. In general, the full MLE estimation would be our first choice due to the well-known optimality properties of maximum likelihood. However, the Inference Functions for Margins (IFM) method is usually preferred to full MLE due to its computational tractability and comparable efficiency. The IFM method (see Joe (1997), for further details) is a multi-step optimisation technique. It divides the parameter vector into separate parameters for each marginal distribution and parameters for the copula model. Therefore, one may break up the optimisation problem into two parts. In this study we adopt the IFM method to estimate the parameters of the marginal distributions and copula function and subsequently obtain $CoVaR$ and $CoES$ estimates.

It is well-documented, since the pioneering works of Mandelbrot (1967) and Fama (1965), that asset return distributions are skewed and fat-tailed. Moreover, the volatility of asset returns is not constant; it is mean-reverting and tends to cluster. Another important stylised characteristic of asset returns volatility is that a large negative price shock increases volatility much more than a positive price shock of the same magnitude, which is also known as "leverage-effect". To address these features we assume that the returns of the financial system and of institution i at time t, $\mathbf{R}_t = (R_{s,t}, R_{i,t})'$, follow an AR(1)-GJR-GARCH(1,1) model of Glosten et al. (1993). Therefore for $j \equiv s, i$ and time $t = 1, \ldots, T$ we estimate

$$R_{j,t} = \mu_{j,t} + \varepsilon_{j,t} = \phi_{j,0} + \phi_{j,1} R_{j,t-1} + \sigma_{j,t} z_{j,t}\,, \tag{19}$$

$$\sigma^2_{j,t} = \omega_j + \alpha_j \sigma^2_{j,t-1} + \beta_j \varepsilon^2_{j,t-1} + \xi_j I_{t-1} \varepsilon^2_{j,t-1}\,, \tag{20}$$

where I_{t-1} is an indicator function equal to 1 if $\varepsilon_{j,t-1} < 0$, and 0 otherwise. We assume that the distribution of the innovations $z_{j,t}$ is a white noise process with zero mean, unit variance and a distribution function given by $F_{z_j,t}$. To allow for asymmetry in the marginal distributions, we assume that the distribution of the innovations follows the skewed-t distribution, as introduced in Fernández and Steel (1998). For comparison, we also estimate the time-series models in Equation (19) and in Equation (20) based on the assumption of normal distributed innovations. We denote the cumulative distribution functions of the financial system and institution i's innovations by $u_t \equiv F_{z_s,t}(z_{s,t})$ and $v_t \equiv F_{z_i,t}(z_{i,t})$, respectively. The dependence parameter is then estimated by maximising the log-likelihood function in Equation (18), conditional on the estimated parameters of the marginal series.

In this respect, $CoVaR$ estimates can be obtained by evaluating the analytical expressions derived in section 2.2. Note that the conditional quantiles implied by Equation (8) and in Equation (13) correspond to the conditional quantiles of innovations. To obtain time-varying *CoVaR* measures, we rescale $CoVaR^{=}_{\alpha,\beta,t}$ or $CoVaR_{\alpha,\beta,t}$ estimates with the fitted conditional mean $\mu_{s,t}$ and standard deviation $\sigma_{s,t}$ of $R_{s,t}$, obtained from estimated models in Equation (19) and in Equation (20), that is

$$CoVaR^{=}_{\alpha,\beta,t} = \mu_{s,t} + \sigma_{s,t}\, F^{-1}_{zs,t}(u^{=}_{t}),$$

$$CoVaR_{\alpha,\beta,t} = \mu_{s,t} + \sigma_{s,t}\, F^{-1}_{zs,t}(u^{\leq}_{t}),$$

where $F^{-1}_{zs,t}$ is the generalised inverse of the financial system's innovation distribution function and $u^{=}_{t}$ and $u^{\leq}_{t}$ are the conditional quantiles of the general solutions in Equation (8) and in Equation (13), respectively.[9] Also note that the conditional quantiles in Equation (8) and in Equation (13) correspond to a static model (i.e., θ is constant). However, the dynamic version of the model (i.e., θ_t is time-varying) implies that conditional quantiles also have time-varying exposure to dependence. Therefore, we use the subscript t in $u^{=}_{t}$ and $u^{\leq}_{t}$ to distinguish between the dynamic and static model.

5. Results

5.1. Computing CoVaR and CoES measures

In this section we present results based on the representation of *CoVaR* by Girardi and Ergün (2013). As discussed earlier, under this definition *CoVaR* is dependent consistent measure of systemic risk, and can be statistically evaluated, providing a distinctive opportunity to assess the statistical adequacy of systemic risk models. In our search for the copula model that can sufficiently describe the dependence between financial system and institution returns, we consider four alternative copula functional forms: Clayton, Frank, Gumbel and BB7. We are interested in positive dependence between the variables, as is modeled by each of the listed copulas, but also in different types of tail dependence. For example, the Clayton copula only allows for negative tail dependence and would hence fit best if negative changes in financial system and institution returns are more highly correlated than positive changes. In contrast, the Gumbel copula only allows for positive tail dependence, while the Frank copula does not allow for tail dependence. Finally, the BB7 copula allows for asymmetric upper and lower tail dependence. In practice, *CoVaR* focuses on the joint tail distribution of the financial system-institution pair returns and thus tail dependence is a rather important concept for *CoVaR* computation.

We estimate dynamic $CoVaR_{\alpha,\beta,t}$ and $CoES_{\alpha,\beta,t}$ measures for each institution i. We employ two alternative distributional assumptions for the marginal series: Gaussian and

[9] To obtain time-varying *CoES* measures, the same process as in the computation of *CoVaR* is followed, however, the copula conditional quantiles $u^{=}_{t}$ and $u^{\leq}_{t}$ are obtained from the corresponding expressions in Equation (15) and in Equation (16), respectively.

Skewed-t. The selection of the best-fitting copula model for each system-institution pair is based on the Akaike Information Criterion (AIC) (Akaike, 1974).[10] All risk measures ($VaR^i_{\alpha,t}$, $CoVaR_{\alpha,\beta,t}$, $CoES_{\alpha,\beta,t}$) are computed at the same confidence level, i.e., $\alpha = \beta = 5\%$. We also evaluate $CoVaR_{\alpha,\beta,t}$ estimates for statistical accuracy and independence using modified versions of the standard Kupiec (1995) and Christoffersen (1998) tests (see Girardi and Ergün (2013), for further details on the implementation of the modified tests). Figure 2 shows time-series average $VaR^i_{\alpha,t}$, $CoVaR_{\alpha,\beta,t}$ and $CoES_{\alpha,\beta,t}$ measures, while Figure 3 shows time-varying average Kendall's τ correlations implied by the estimated bivariate copula families, across all financial system-institution pairs with skewed-t marginals. The light blue shaded area in the graphs corresponds to Q3 2007 - Q4 2012 crisis period. It is clear that $CoVaR_{\alpha,\beta,t}$ and $CoES_{\alpha,\beta,t}$ estimates are higher in absolute value during this period. This is partly due to the increasing correlation between financial system-institution returns as shown in Figure 3.

[Insert Figures 2 and 3 here]

Nevertheless, the time-varying correlation results cannot fully support the empirical findings in Longin and Solnik (2001) and Ang and Chen (2002), indicating that conditional correlations between financial asset returns are much stronger in downturns than in upturns. The time-varying Kendall's τ correlations are slightly more pronounced during the crisis period than in the pre-crisis period for most of the pairs; the average value being 0.44 in the pre-crisis period and 0.47 in the crisis period for all pairs under consideration. Figures 2 and 3 indicate also the importance of consistency of systemic risk measures with respect to dependence, particularly during crisis periods. It is clear from the two graphs that high values of Kendall's τ correlations are associated with higher, in absolute value, systemic risk estimates. Therefore, a systemic risk measure that provides an inconsistent response to dependence may fail to detect systemic risk when it is more pronounced, i.e., during periods of financial distress, and thus lead financial system regulators to make inappropriate policy decisions..

Figure 4 displays a cross-section plot of an institution's average $VaR^i_{\alpha,t}$ and its contribution to systemic risk, measured by average $\Delta CoVaR_{\alpha,\beta,t}$. We note that there is a weak relationship between the institution's $VaR^i_{\alpha,t}$ and its $\Delta CoVaR_{\alpha,\beta,t}$ in the cross-section. Similar findings are also reported in Adrian and Brunnermeier (2016) and Girardi and Ergün (2013) leading to the conclusion that regulating the risk of financial institutions in isolation, through institutions' *VaR*, might not be the optimal policy for protecting the

[10] The Bayesian Information Criterion (BIC) of Schwarz (1978) was also employed in the selection procedure for the best-fitting copula model; however, the results remained almost unaffected since both criteria selected the same copula families for the majority of the pairs analysed.

financial sector against systemic risk. Figure 5 plots the time-series average of $VaR^i_{\alpha,t}$ and $\Delta CoVaR_{\alpha,\beta,t}$ over time. It is evident that $VaR^i_{\alpha,t}$ and $\Delta CoVaR_{\alpha,\beta,t}$ measures have a strong relationship in the time series.

[Insert Figures 4 and 5 here]

Adrian and Brunnermeier (2016) report the same strong relationship, while Girardi and Ergün (2013) confirm a weak relationship between these two risk measures in the time series. Given our findings, we conclude that the association between these two measures over time is primarily determined by the different definitions of $\Delta CoVaR$ and not by the different $CoVaR$ definitions.[11]

5.2. Systemic risk contribution

Table 1 ranks the contribution of each individual bank to overall systemic risk, as measured by the time-series average of $\Delta CoVaR_{\alpha,\beta,t}$ and $\Delta CoES_{\alpha,\beta,t}$ estimates, under the assumption of Gaussian and skewed-t marginals, respectively. Table 1 also displays the selected copula functions and the average value of Kendall's τ correlation coefficients implied by the estimated copula parameters of each financial system-institution pair. The Frank copula is the most preferred functional form for describing the dependence between financial system and institution returns and the Gumbel copula is the second most popular choice under the assumption of Gaussian marginals. In contrast, the BB7 copula is the most popular functional form for modelling the dependence under the skewed-t marginals assumption, while the Frank copula is the second most favoured choice. The Clayton copula has not been selected for any of the pairs analysed under both marginal assumptions. It is clear from Table 1 that the distribution assumptions in the marginals affect the selection of the best-fitting copula and hence the overall $CoVaR_{\alpha,\beta,t}$ and $CoES_{\alpha,\beta,t}$ results. Therefore, particular attention should be paid when specifying marginals since the use of inappropriate marginals not only introduces biases directly but also affects systemic risk measures indirectly, through copula parameter estimation or copula misspecification.

[Insert Table 1 here]

The average $\Delta CoVaR_{\alpha,\beta,t}$ and $\Delta CoES_{\alpha,\beta,t}$ estimates with skewed-t marginals are much

[11] This conclusion results from estimating $CoVaR$ under both stress scenarios $R_{i,t} = VaR^i_{\alpha,t}$ and $R_{i,t} \leq VaR^i_{\alpha,t}$ and employing different $\Delta CoVaR$ definitions for three copula models: Clayton, Gumbel and Frank. Numerical integration is used to estimate $CoVaR$ when explicit expressions are not available in our Copula $CoVaR$ framework. The weak relationship between $\Delta CoVaR$ and VaR in the time series is supported only when the definition of $\Delta CoVaR$ used is that of Girardi and Ergün (2013), regardless of different $CoVaR$ definitions.

higher in absolute value than those generated under the assumption of Gaussian marginals. The size differences in systemic risk measures, however, result not only from the alternative marginal assumptions but also from the characteristics of the copula functions that model the dependence for each pair. The dominant copula function when assuming Gaussian marginals is Frank, while BB7 is the most popular copula family under skewed-t marginals. As explained, the Frank copula does not imply tail dependence, while the BB7 copula allows for asymmetric tail dependence. In this regard, the general dependence structure, and especially the dependence structure in extremes, affects substantially the computation of $CoVaR_{\alpha,\beta,t}$ and $CoES_{\alpha,\beta,t}$. This is also confirmed by the implied Kendall's τ estimates reported in Table 1. It is clear from Table 1 that for those copula families that do not imply lower tail dependence, such as the Frank or the Gumbel copula family, the average $\Delta CoVaR_{\alpha,\beta,t}$ and $\Delta CoES_{\alpha,\beta,t}$ estimates are primarily driven by the degree of dependence.

The stronger the dependence between financial system-institution returns the higher the average values of $\Delta CoVaR_{\alpha,\beta,t}$ and $\Delta CoES_{\alpha,\beta,t}$. In contrast, when the dependence between the financial system and an institution's returns is modelled by an asymmetric BB7 copula, the average $\Delta CoVaR_{\alpha,\beta,t}$ and $\Delta CoES_{\alpha,\beta,t}$ estimates are not monotonic functions of Kendall's τ correlation estimates but their values are also affected by the degree of tail dependence. Figure 6 shows the average time-varying upper (λ^U) and lower (λ^L) tail dependence indices estimated from those pairs modelled by a BB7 copula under the assumption of skewed-t marginals. There is clear evidence of asymmetric tail dependence.

[Insert Figure 6 here]

The average value of upper and lower tail dependence indices is 0.45 and 0.50, respectively, leading to the conclusion that joint negative extremes occur more often than joint positive extremes. To investigate further the impact of asymmetries on the tails in the computation of systemic risk metrics, we compute non-parametric (N-P) estimates (an average of non-parametric estimates in Dobrić and Schmid (2005)) for upper (λ^U) and lower (λ^L) tail dependence coefficients and sample Kendall's τ correlation coefficients for each financial system-institution pair of standardised residuals, obtained from the fit of the univariate time-series models in section 5. Table 2 reports average $\Delta CoVaR_{\alpha,\beta,t}$, non-parametric Kendall's τ correlation estimates and non-parametric tail dependence indices for each pair. It is not surprising that banks having high coefficients of lower tail dependence appear among the most systemic financial institutions, indicating in this way the importance of asymmetries in systemic risk modelling.

[Insert Table 2 here]

The ranking of the systemically important financial institutions in Table 1 varies significantly across different marginal distributional assumptions but is more consistent across different systemic risk measures within the same marginal distributional assumptions. For example, Santander bank is ranked as the 2^{nd} most systemic financial institution according to its average contribution to systemic risk, measured by $\Delta CoVaR_{\alpha,\beta,t}$, under the assumption of Gaussian marginals, while it is ranked in the 7^{th} place when skewed-t marginals are assumed instead. Moreover, BNP Paribas is ranked as the 3^{rd} most systemic bank based on its average $\Delta CoES_{\alpha,\beta,t}$ measure under normality, but under the assumption of skewed-t marginals it is ranked in the 26^{th} place. Nevertheless, the hierarchy of systemic banks across $\Delta CoVaR_{\alpha,\beta,t}$ and $\Delta CoES_{\alpha,\beta,t}$ does not differ significantly under the same marginal distribution assumptions, implying that qualitative results depend more on the underlying distribution assumptions in the marginals and dependence structure and less on the systemic risk measures *per se*.[12]

From the ranking results in Table 1 and the market capitalisation values of financial institutions reported in Table 9 in Appendix C, it can also be shown that banks which are large in size with strong cross-country exposure and international activity appear among the most systemic financial institutions under both distribution assumptions. For instance, banks such as BBVA, UBS, Deutsche Bank, Credit Suisse or BNP Paribas, are placed among those institutions. Table 3 displays a cross-country comparison of systemic risk contribution measured by the average $\Delta CoVaR_{\alpha,\beta,t}$ and $\Delta CoES_{\alpha,\beta,t}$ of financial firms belonging to the same country. Financial institutions from France and Spain appear to be the most systemic ones according to their average $\Delta CoVaR_{\alpha,\beta,t}$ and $\Delta CoES_{\alpha,\beta,t}$ estimates under Gaussian and skewed-t marginal distribution assumptions, respectively. In contrast, banks from Portugal, Ireland or Greece are classified among the least systemic financial institutions in our sample.

One may regard this classification as an economic paradox, since banks that belong to those national economies that have suffered the most from the European sovereign debt crisis - and the market value of whose corresponding share prices has declined significantly during the crisis - appear among the least systemic financial institutions in the cross-country comparison. However, banks from these particular countries are typical commercial banks with substantial presence in the local market but limited international activity and cross-country exposure. Therefore, the implied correlation and, more impor-

[12] We also estimated $\Delta CoVaR_{\alpha,\beta,t}$ measures using market-capitalisation weighted returns as oppose to equally-weighted returns. The magnitude of $\Delta CoVaR_{\alpha,\beta,t}$ measures was not substantially different from equally-weighted return $\Delta CoVaR_{\alpha,\beta,t}$ measures; however, as expected, the ordering of systemic institutions was moderately different. Due to space limitation we do not report these results but are available from the authors upon request.

tantly, the dependence in extreme events between these banks and the financial system is typically reduced generating in this way lower in absolute value systemic risk estimates. This is also confirmed by the fact that the Frank copula which does not allow for tail dependence is the preferred copula functional form for most of these particular pairs.

[Insert Table 3 here]

These findings should not be regarded as a weakness of the *CoVaR* model but rather as a merit. According to Brunnermeier et al. (2009), a systemic risk measure should be able to identify the risk to the system by individually "systemically important" institutions, which are highly interconnected and large enough to cause negative spill over effects on others, as well as by small institutions that are "systemic" when acting as parts of a herd. In this respect, the relative size and the interconnectedness of each particular financial institution are factors that should be considered in systemic risk measurement. The *CoVaR* methodology implicitly incorporates institution size and interconnectedness into systemic risk estimation through correlation and dependence on extreme events. In our study, the financial system is represented by components of the STOXX Europe 600 Banks Index, which includes the largest banks in terms of market capitalisation in Europe. It is a portfolio of 42 financial institutions from 15 different European countries. The majority of and the largest in size among these financial institutions come from countries such as Germany, France, Spain, Italy and Great Britain. Therefore, the implied dependence between each of these particular institutions and the financial system is, by construction, stronger due to within-country dependence (e.g increased commonalities for institution returns from same country) and the dependence that arises from their large size and dominant position in the European market. This may partly explain why banks from these particular countries are listed among the most systemic financial institutions in our study. The results in Table 4 support this argument. Table 4 reports average sample Kendall's τ and non-parametric upper (λ^U) and lower (λ^L) tail dependence estimates for each country. It is evident that Kendall's τ correlations and non-parametric tail dependence coefficients are much stronger for these particular countries, implying a stronger dependence and dependence in the tails of the joint distribution and consequently higher, on average, systemic risk estimates.

[Insert Table 4 here]

5.3. Backtesting and Stress testing CoVaR

A well-specified risk model should satisfy the appropriate theoretical statistical properties. Therefore, the proportion of exceedances should approximately equal the confidence level, while the exceedances should not occur in clusters but independently. Table 5 reports the average p-values from the modified Kupiec (1995) and Christoffersen (1998) statistical tests for the unconditional coverage, independence and conditional coverage of $CoVaR_{\alpha,\beta,t}$ estimates under both Gaussian and skewed-t distribution assumptions computed at $\alpha = \beta = 5\%$ level.

[Insert Table 5 here]

The null hypotheses of unconditional and conditional coverage are rejected at the 5% level of significance under the Gaussian assumption. On the other hand, the null hypotheses cannot be rejected at conventional significance levels under the assumption of skewed-t marginals. Thus, it seems that a combination of copula functions - that allow for asymmetries in the tails - with asymmetric marginals is a better candidate for systemic risk modeling. Our test results are in line with the *CoVaR* backtesting results in Girardi and Ergün (2013) and the results in the *VaR* literature that reject the underlying assumption of normality in favour of alternative distributions which allow for asymmetries. We also compare the Copula $\Delta CoVaR$ estimates to those calculated using the methodology of Girardi and Ergün (2013) using both Normal and Student-t models. The approach of Girardi and Ergün is preferred for comparison purposes as it is analytically tractable and enables direct comparisons with our Copula *CoVaR* methodology. We compare the performance of the models graphically, in Figure 7, and statistically using the modified conditional coverage and independence tests, reported in Panel B of Table 5.

[Insert Figure 7 here]

As shown in Figure 7, $\Delta CoVaR$ estimates show strong dependence, the correlation between Copula and Girardi and Ergün $\Delta CoVaR$ being 99.97% for the Normal and 99.95% for non-normal models. Consistent with the copula estimates, the Student-t $\Delta CoVaR$ estimates in Girardi and Ergün pass the conditional and unconditional coverage tests thus providing, nominally at least, similar performance to that of the copula model. However, visual comparison of the estimates indicates that the Copula model provides consistently lower (in absolute terms) values thus leading to more efficient estimates and allocation of capital. This empirical feature, along with the superior analytical tractability of the Copula *CoVaR*, indicate the superiority of the proposed method.

Stress testing exercises are also useful for financial regulators to gauge the potential implications of extreme market conditions for the stability of the financial system as a whole. Before the outset of the financial crisis, financial stability stress tests were largely focused on the implications of system-wide macroeconomic shocks and rarely considered idiosyncratic shocks such as the failure of a single large firm. Recently, there has been a growing interest in such systemic stress testing exercises by central banks and financial regulators. Our modelling framework can be easily employed as part of the tool-kit for financial stability assessment. Stress testing exercises under this framework can simulate scenarios that are absent from historical data or are more likely to occur than historical observation suggests, as well as simulate shocks that reflect permanent structural breaks or temporal dependence breakdowns.

Figure 8 displays a scenario analysis example for HSBC and demonstrates its influence on systemic risk as measured by $CoVaR_{\alpha,\beta,t}$ under certain scenarios. In particular, Figure 8 plots the implied $CoVaR_{\alpha,\beta,t}$ measures generated by the Clayton, Gumbel, Frank and BB7 copulas for $\beta = 0.01$ to 0.80, $\alpha = 0.05$ and the dependence parameter(s) estimated for each particular copula family assuming skewed-t marginals.[13] Therefore, the discrepancies in $CoVaR_{\alpha,\beta,t}$ measures are due to the employment of different copula models and do not arise from marginal specifications. The implied $CoVaR_{\alpha,\beta,t}$ results in Figure 8 have an appealing interpretation. For instance, we are 99% confident, given that HSBC is at most at its 95% *VaR* level, that the financial system will not experience a distress event worse than −16.84% according to the Clayton copula. For the same confidence level, $CoVaR_{\alpha,\beta,t}$ estimates implied by the Gumbel, Frank and BB7 copulas are −15.08%, −13.56% and −16.74%, respectively.[14]

[Insert Figure 8 here]

Given the unique ability of copula functions to enable the separation of dependence from marginal distributions, we are able to quantify the potential effects on the stability of the financial system of risks associated with marginal distribution assumptions or risks related to the dependence structure. For example, a scenario that implies a structural break in the correlation between the financial system and an institution's returns can be analysed by modifying the level of Kendall's τ parameter, while a change in the dependence structure can be studied through alternative copula functional forms. Similarly, a scenario that implies high volatility or severe equity price declines can be examined through alternative marginal specifications. Complex stress test exercises that combine all the above scenarios can also be analysed simultaneously, thus providing a powerful tool for systemic risk assessment.

[13] We could also set the parameters for the Clayton, Gumbel and Frank copulas to a pre-specified value such as the Kendall's τ sample correlation coefficient because there is a one-to-one relationship between these particular one-parameter copula families and Kendall's τ. Such a relationship, however, does not exist for the two-parameter BB7 copula. To maintain the consistency of the implied systemic risk estimates, we use the estimated parameter(s) for each particular copula family instead.

[14] Similar stress testing exercises can also be obtained using $CoES_{\alpha,\beta,t}$ as a measure of systemic risk.

Figure 8 also provides a distinct graphical way to illustrate the importance of tail dependence in systemic risk computation and facilitate the interpretation of the results in Tables 1 and 3. We note in Figure 8 that the Clayton and BB7 copulas, which allow for lower tail dependence, produce much larger in absolute value $CoVaR_{\alpha,\beta,t}$ measures compared to the corresponding measures generated by the Frank or Gumbel copulas, which do not allow for lower tail dependence. As already explained, the average $\Delta CoVaR_{\alpha,\beta,t}$ or $\Delta CoES_{\alpha,\beta,t}$ measures reported in Tables 1 and 3 do not differ in size only due to alternative distribution assumptions in marginals but also due to the different characteristics of the alternative copula functional forms employed. Therefore, copula misspecification may critically affect the systemic risk estimates and therefore dependence modelling should proceed with caution.

5.4. Systemic risk determinants

In this section, we investigate the main drivers of systemic risk in the European banking system. The analysis is split into three main parts. In the first part, we investigate whether there are common market factors explaining an institution's contribution to systemic risk and seek to understand how this relationship is altered in the face of changes in the market environment. We also investigate how and in which direction these factors affect systemic risk. As explained, systemic risk measures can be decomposed within the Copula *CoVaR* framework due to the unique ability of copula functions to enable the separation of dependence from marginal distributions. Thus, *CoVaR* is an increasing nonlinear function of the correlation between the financial system and institution i and of the financial system's volatility. This separation allows us to assess the impact of market factors on these variables and analyse their importance for the stability of the financial system.

Therefore, the dependent variables in our formal empirical work are $\Delta CoVaR_{\alpha,\beta,t}$, Kendall's τ correlations and the financial system's volatility σ_s estimates, obtained in section 5.1.[15] For each set of the dependent variables $y_{i,t}$ we run the following panel regression model

$$\begin{aligned} y_{i,t} = {} & \beta_0 + \beta_1\, Vix_{t-1} + \beta_2\, Liquidity_{t-1} + \beta_3\, \Delta Euribor_{t-1} + \beta_4\, \Delta Slope_{t-1} + \beta_5\, \Delta Credit_{t-1} \\ & + \beta_6\, S\&P_{t-1} + \beta_7\, I_{crisis}\, Vix_{t-1} + \beta_8\, I_{crisis}\, Liquidity_{t-1} + \beta_9\, I_{crisis}\, \Delta Euribor_{t-1} \\ & + \beta_{10}\, I_{crisis}\, \Delta Slope_{t-1} + \beta_{11}\, I_{crisis}\, \Delta Credit_{t-1} + \beta_{12}\, I_{crisis}\, S\&P_{t-1} + \varepsilon_{i,t}\,, \end{aligned} \tag{21}$$

where $y_{i,t}$ denotes the set of $\Delta CoVaR^i_{\alpha,\beta,t}$, Kendall's τ^i_t and financial system's volatility $\sigma^i_{s,t}$ estimates for each financial institution i and week t. The I_{crisis} represents dummy

[15] All results are based on skewed-t marginal distribution assumptions. We also analysed the same relationships based on the results from Gaussian marginals. Moreover, we employed $\Delta CoES_{\alpha,\beta,t}$ as an alternative measure of an institution's contribution to systemic risk. The qualitative results, however, remained unchanged.

variables that take the value of zero in the pre-crisis period and the value of one in the period we designate as the crisis period.[16] In addition, the right-hand side of Equation (21) includes the following market variables:

(i) *Vix*, which is a proxy for the implied volatility in the stock market reported by the Chicago Board Options Exchange (CBOE).

(ii) *Liquidity*, which is a short term "liquidity spread" defined as the difference between the three-month interbank offered rate and the three-month repo rate. This spread is a common proxy for short-term funding liquidity risk. We use the three-month Euribor rate and the three-month Eurepo rate, both reported by the European Banking Federation (EBF).

(iii) Δ*Euribor*, which is the change in the three-month Euribor rate.

(iv) Δ*Slope*, which is the change in the slope of the yield curve, measured by the spread between the German ten-year government bond yield and the German three-month Bubill rate.

(v) Δ*Credit*, which is the change in the credit spread between the ten-year Moody's seasoned BAA-rated corporate bond and the German ten-year government bond.

(vi) *S&P*, which is the S&P 500 Composite Index returns and used as a proxy for equity market returns.

The data have been obtained from Bloomberg and are sampled weekly. Table 6 reports bank fixed-effect panel regression estimates for $\Delta CoVaR_{\alpha,\beta,t}$, Kendall's τ and the financial's system volatility σ_s estimates on the above lagged market variables. Across both sub-periods, the lagged values of the *Vix*, *Liquidity* and Δ*Euribor* variables appear highly significant in explaining the variation in $\Delta CoVaR_{\alpha,\beta,t}$ at conventional significance levels.

In particular, higher lagged values of implied market volatility are associated with more negative $\Delta CoVaR_{\alpha,\beta,t}$ measures in the pre-crisis period. In contrast, the impact of lagged *S&P Return*, Δ*Spread* and Δ*Slope* variables on $\Delta CoVaR_{\alpha,\beta,t}$ does not appear statistically significant in this period (Δ*Slope* is significant only at 10% level).

[16] As an additional robustness test, we separate the 2007-2008 global financial crisis from the more recent Euro-crisis. In particular, following Acharya et al. (2017), we define July 2007 - December 2008 as the period of the global financial crisis. Further, we consider two separate sub-periods for the Euro-crisis. Firstly, the period December 2009 - May 2010 which corresponds to Stage 6 in Alter and Schüer (2012). Secondly, the period December 2009 - 9 September 2012 which covers the period up to the official announcement of the outright monetary transactions (OMT) program by the European Central Bank, the importance of which in bringing an end to the Eurozone crisis is highlighted in Saka et al. (2015). Therefore, crisis dummy variables were used only for these two sub-periods (i.e. July 2007 - December 2008 and December 2009 - May 2010 or December 2009 - 9 September 2010). The results in both cases were qualitatively similar to those presented in Table 6.

[Insert Table 6 here]

The results in Table 6 also highlight the importance of funding liquidity in systemic risk contribution. Banks typically raise short-term funding in the unsecured interbank market or through over-the-counter collateralised repurchase agreements (repos). In times of uncertainty, banks charge higher rates for unsecured loans and thus interbank offered rates increase. The spread between the Euribor and the Eurepo rate measures the difference in interest rates between short-term fundings of different risk. As Figure 9 shows, this spread had shrunk to historical low levels during the pre-crisis period but it began to surge upward during the crisis period. The positive impact of funding liquidity on $\Delta CoVaR_{\alpha,\beta,t}$ in the pre-crisis period is confirmed by the results in Table 6. The coefficient of *Liquidity* in this period is negative and rather significant in magnitude. On average, a 1% increase in *Liquidity*, which indicates a worsening of funding liquidity, contributes almost 13.7% to systemic risk as measured by $\Delta CoVaR_{\alpha,\beta,t}$.

[Insert Figure 9 here]

The results are in line with a large number of theoretical and empirical research papers that associate market declines with liquidity dry-ups to explain the triggering of systemic episodes (see e.g. Brunnermeier (2009); Adrian and Shin (2010); Brunnermeier and Pedersen (2009); Hameed et al. (2010), and references therein). The burst of the crisis in the summer of the 2007, caused two "liquidity spirals". Financial institutions' capital eroded due to the initial decline in asset prices and the increase in the wholesale funding cost. Consequently, both events triggered fire-sales, pushing asset prices further down, and increased the uncertainty in the interbank lending market. As a result, European banks that relied excessively on short-term funding were particularly exposed to a dry-up in liquidity. In this respect, the large size of the pre-crisis liquidity spread coefficient estimate partly explains why the sudden dry-up in liquidity had such a severe impact on the stability of the financial system.

The regression results in Table 6 for the Δ*Euribor* variable are also of great interest. As explained, the Euribor rate represents the unsecured rate at which a large panel of European banks borrow funds from one another. An increase in short-term rates implies a higher borrowing cost for banks. In this respect, banks relying on short-term funding are more vulnerable to liquidity risk. The pre-crisis coefficient estimate of the change in the three-month Euribor rate variable indicates the positive relation between changes in the short-term rates and systemic risk contribution. On average, an increase by 1% in the change of the three-month Euribor rate adds an additional 3.7% to $\Delta CoVaR_{\alpha,\beta,t}$.

In contrast, the signs of almost all estimated coefficients have switched in the crisis period indicating an asymmetric response of market factors to systemic risk in these subperiods. In particular, the coefficient estimates of the *Liquidity* and Δ*Euribor* variables have switched from negative in the pre-crisis period to positive in the crisis period. One of the main reasons behind this behaviour is the coordinated intervention of central banks in both the United States and Europe in response to the freezing up of the interbank market. To alleviate the liquidity crunch, the European Central Bank (ECB) and the Federal Reserve (Fed) reduced the interest rates at which financial institutions borrow from them; they also expanded their balance sheets by broadening the type of collateral that banks could use, and increased the maturity of their loans to the banks (see Giannone et al. (2012), for further details). Figure 10 shows average $CoVaR_{\alpha,\beta,t}$ estimates and a timeline of key events and measures taken by the European Central Bank (ECB) to provide liquidity and restore financial stability over the recent financial crisis. We note in Figure 10 that the highest $CoVaR_{\alpha,\beta,t}$ measures (in absolute value) are reported after the Lehman Brothers collapse in September 2008. Figure 10 also depicts the action taken by the European Central Bank (ECB) in response to the liquidity crunch and the overall financial market turmoil. It can be shown that the systemic risk measures returned to lower levels (in absolute value) while the initial liquidity dry-up in the interbank market calmed down and the short-term interbank rates returned to lower levels, as Figure 9 and Figure 11 display, respectively.

[Insert Figures 10, 11 here]

The overall increase in systemic risk during the crisis period, however, is not only driven by the solvency problems of several Euro-area financial institutions, but also by the sovereign debt crisis of a large number of Eurozone member countries. As Figure 10 suggests, systemic risk estimates reached their highest levels after the collapse of Lehman Brothers in September 2008; however, high values are also associated with the inability of several countries in the Euro-zone to repay or refinance their government debt without the assistance of third parties. As Shambaugh (2012) points out, the euro area faced three interdependent crises, that is, a sovereign debt crisis, a banking crisis and a growth and competitiveness crisis. In this respect, the problems of undercapitalised banks and high sovereign debt are mutually reinforcing, and both are amplified by slow and unequally distributed - among euro area member countries. - growth Therefore, our regression results and the asymmetric response of market factors on systemic risk should be viewed in conjunction with the overall characteristics of the crisis in the Eurozone.

It is also of great interest to investigate the effect of market factors on Kendall's τ correlation estimates and the financial system's volatility σ_s estimates. Kendall's τ correlation estimates are asymmetrically related to lagged values of the *Vix* and *Liquidity* variables, although the magnitude of the asymmetries is not large. Interestingly, liquidity shocks (the widening of liquidity spread) reduce Kendall's τ correlation in the pre-crisis period, while having a positive impact on it in the crisis period. A widening in Δ*Credit* also suggests a decrease in Kendall's τ correlation in both periods. The above market factors also appear significant in explaining the financial system's volatility and demonstrate the same asymmetric behaviour. In the pre-crisis period, an increase in the *Vix*, *Liquidity* or Δ*Credit* variables increases the financial system's volatility and as a consequence the level of systemic risk, while the impact of these factors on the financial system's volatility is the opposite in the post-crisis period. The Δ*Euribor* variable is also asymmetrically related to the financial system's volatility; however, the degree of asymmetry is pretty high between these sub-periods, with the regression coefficients changing from 3.453 to -5.494. This substantial asymmetric response also highlights the impact of the European Central Bank's (ECB) intervention in the interbank market.

In the post-crisis period, an increase in the change of the three-month Euribor rate counterintuitively, suggests a reduction in the financial system's volatility. However, as shown in Figures 10 and 11, the action taken by the European Central Bank (ECB) during the crisis period eventually reduced the level of short-term interest rates and, thus, distorted the positive pre-crisis relationship between the change in short-term rates and the financial system's volatility. From the results in Table 5, it can also be seen that the impact of funding liquidity is primarily transmitted on $\Delta CoVaR_{\alpha,\beta,t}$ through the financial system's volatility and not through Kendall's τ correlation. In other words, the sudden dry-up of liquidity in the pre-crisis period reduced the level of correlation among financial institutions but considerably increased the volatility of the financial system. This can also be confirmed by comparing the estimated coefficients of the *Liquidity* variable with the estimated coefficients of the $\Delta CoVaR_{\alpha,\beta,t}$ and the financial system's volatility variables, which are almost identical in absolute value.

In the second part of our analysis, we investigate how individual characteristics of financial institutions contribute to systemic risk. In this regard, we employ panel regressions and regress quarterly-aggregated $\Delta CoVaR_{\alpha,\beta,t}$ measures on a set of institution-specific variables. In particular, we consider the following panel regression model with fixed effects:

$$\begin{aligned}\Delta CoVaR^i_{\alpha,\beta,t} = \beta_0 &+ \beta_1\, VaR^i_{\alpha,t-k} + \beta_2\, MtB_{i,t-k} + \beta_3\, Size_{i,t-k} + \beta_4\, Leverage_{i,t-k} \\ &+ \beta_5\, Beta_{i,t-1} + \beta_6\, Vol_{i,t-k} + \varepsilon_{i,t}. \end{aligned} \tag{22}$$

where $\Delta CoVaR^i_{\alpha,\beta,t}$ represents the quarterly-aggregated $\Delta CoVaR$ measures for institution i computed from the first stage as described in Section 5.1. In addition, we use the following set of quarterly bank-specific characteristics:

(i) $VaR^i_{\alpha,t-k}$ defined as the quarterly-aggregated *VaR* measures for bank i at quarter

$t-k$, calculated by averaging the corresponding weekly measures within each quarter.

(ii) $MtB_{i,t-k}$ defined as the ratio of the market to book value of total equity for bank i at quarter $t-k$ and used as a proxy for growth opportunities.

(iii) $Size_{i,t-k}$ defined as the log of book value of total equity for bank i at quarter $t-k$.

(iv) $Leverage_{i,t-k}$ defined as the ratio of the total assets to book value of total equity for bank i at quarter $t-k$ and used as a proxy for the solvency of the bank.

(v) $Beta_{i,t-k}$ is the equity market beta for bank i at quarter $t-k$, calculated from weekly equity return data within each quarter.

(vi) $Vol_{i,t-k}$ is the equity return volatility for bank i at quarter $t-k$, calculated from weekly equity return data within each quarter.

The balance-sheet data for each individual bank are obtained from Worldscope database. Table 7 reports results from panel regressions, after controlling for bank fixed-effects and, additionally, allowing for bank and time clustered errors. We report results from three different specifications based on the forecast horizon of explanatory variables: one quarter, one year and two years. Across forecast periods, *Size* and *Leverage* appear to be the most robust determinants of systemic risk. The estimated coefficient of the *Size* variable is negative and highly significant, suggesting that bigger institutions contribute more to systemic risk than smaller institutions.

[Insert Table 7 here]

These findings support the empirical results in Section 5.2. Some of the largest banks in our sample are placed among the most systemic financial institutions based on their average $\Delta CoVaR$ or $\Delta CoES$ measures as reported in Table 1. Furthermore, *Leverage* is negative and significant across all forecasting horizons. As explained, *Leverage* is used as a proxy for the solvency of the financial institution. The negative coefficient estimates of *Leverage* across all forecasting horizons imply that highly leveraged banks contribute more to systemic risk than low leveraged banks. In addition, the *VaR* of each financial institution and equity return volatility are statistically significant at the one quarter horizon, whereas equity beta is statistically significant at the two year horizon. Overall, our results in Table 7 are in line with other studies. Similar to Acharya et al. (2017), Adrian and Brunnermeier (2016) and Girardi and Ergün (2013), we find that size, leverage and equity beta are important determinants of systemic risk. However, we found no statistical support for the hypothesis that the market to book value of total equity ratio is important in explaining institutions' contribution to systemic risk.

In the third part of our analysis, we attempt to shed some light into how major macroeconomic variables contribute to systemic risk. As a result, we employ panel regressions and regress quarterly-aggregated $\Delta CoVaR_{\alpha,\beta,t}$ measures on major country-specific macroeconomic variables. In particular, we consider the following panel regression model with fixed effects:

$$\begin{aligned} \Delta CoVaR^i_{c,\alpha,\beta,t} = \beta_0 &+ \beta_1\ \Delta Unemployment_{c,t-k} + \beta_2\ Inflation_{c,t-k} + \beta_3\ \Delta Share_{c,t-k} \\ &+ \beta_4\ \Delta IndustrialProduction_{c,t-k} + \beta_5\ \Delta Rates_{c,t-1} + \beta_6\ GDPGrowth_{c,t-k} \\ &+ \beta_7\ \Delta CurrentAccount_{c,t-k} + \beta_8\ \Delta Debt/GDP_{c,t-k} + \varepsilon^i_{c,t}. \end{aligned} \tag{23}$$

where $\Delta CoVaR^i_{c,\alpha,\beta,t}$ represents the quarterly-aggregated $\Delta CoVaR$ measures for institution i at its country of domicile c computed from the first stage as described in Section 5.1. In addition, we use the following set of quarterly macro-economic characteristics:

(i) $\Delta Unemployment_{c,t-k}$ is the change in harmonised unemployment rate for country c at quarter $t-k$.

(ii) $Inflation_{c,t-k}$ is the percentage change of consumer price index for country c at quarter $t-k$.

(iii) $\Delta Share_{c,t-k}$ is the growth rate of stock market index for country c at quarter $t-k$.

(iv) $\Delta IndustrialProduction_{c,t-k}$ is the growth rate of industrial production for country c at quarter $t-k$.

(v) $\Delta Rates_{c,t-1}$ is the change in long-term (i.e. 10-year maturity) sovereign yields for country c at quarter $t-k$.

(vi) $GDPGrowth_{c,t-k}$ is the GDP growth rate for country c at quarter $t-k$.

(vii) $\Delta CurrentAccount_{c,t-k}$ is the change in current account (as % of GDP) for country c at quarter $t-k$.

(viii) $\Delta Debt/GDP_{c,t-k}$ is the change in debt to GDP ratio for country c at quarter $t-k$.

The macroeconomic data for each country are obtained from OECD's statistics database.[17] Table 8 reports results from panel regressions, after controlling for bank fixed-effects and, additionally, allowing for bank and time clustered errors. We report results from three different specifications based on the forecast horizon of explanatory variables: one quarter, one year and two years.

[Insert Table 8 here]

[17] For more details see http://stats.oecd.org

Regression results show that an increase in unemployment rate, a decrease in the domestic stock market index, a decrease in industrial production and a decrease in GDP growth, contribute to an increase in the systemic risk next quarter, at the 5% level. Nevertheless, all macroeconomic variables in the analysis are statistically insignificant at 5% for the one and two year horizons.

To our knowledge, there are only a few studies that investigate the link between the macroeconomy and systemic risk, as measured by CoVaR. For example, Lòpez-Espinosa et al. (2012) use macroeconomic variables related to the business cycle, namely, unemployment and interest rates, as control variables in their baseline regression model but do not investigate the extent to which major macroeconomic variables can contribute to systemic risk. The link between systemic risk and macroeconomy has been considered by Nicolo et al. (2011), who use a dynamic factor model to model quarterly time series of macroeconomic indicators of financial and real activity and obtain forecasts of systemic real risk and systemic financial risk. More recently, Buch et al. (2014) analyse the link between banks and the macroeconomy. In particular, the authors investigate how macroeconomic shocks are transmitted to individual banks. In contrast, Giglio et al. (2016) use a large set of systemic risk measures to examine how a buildup of systemic risk in the financial sector increases systemic risk in the real economy and show that systemic risk measures contain useful information regarding the probability of future macroeconomic downturns. Overall, the results from the regression analysis in Table 8 contribute to the literature that investigates the link between systemic risk and macroeconomy. The analysis indicates that movements in certain macroeconomic variables such as unemployment, industrial production, GDP and share index contribute significantly to systemic risk as measured by our Copula $\Delta CoVaR$ estimates.

6. Summary

During the 2007-2008 financial crisis, losses were spread out rapidly across financial institutions, thus affecting the entire financial system. According to Adrian and Brunnermeier (2016), these spillovers were realisations of systemic risk – the risk that the distress of an individual institution, or a group of institutions, will induce financial instability on a broader scale. To capture these spillover effects, Adrian and Brunnermeier (2016) proposed the *Conditional Value-at-Risk* (*CoVaR*). This new measure of systemic risk, attracted quickly the attention of the academic and regulatory communities.

In this study, we propose a new methodology for estimating *CoVaR*, based on copula functions. The proposed methodology circumvents some of the limitations in estimation of the original $CoVaR$ model. In particular, the proposed Copula $CoVaR$ methodology provides simple, explicit expressions for a broad range of copula families, while allowing the *CoVaR* of an institution to have time-varying exposure to its *VaR*. Further, this methodology is extended to estimate other systemic risk measures, such as the *Conditional Expected Shortfall* (*CoES*).

Given the properties of copula functions that enable the separation of dependence from marginal distributions, the model provides a distinct way for quantifying how shocks in the conditional volatilities or dependence structure of financial institutions' assets can affect systemic risk. The Copula *CoVaR* methodology can also facilitate stress testing and sensitivity analysis and thus inform regulators for potential threats to the stability of the financial system. Under certain conditions, the model can be also extended to incorporate additional conditioning scenarios and thus study how a group of financial institutions being in distress, can affect financial stability. Therefore, the proposed Copula *CoVaR* methodology also has great advantages for systemic risk measurement with significant policy implications.

We focus on a portfolio of large European banks and estimate *CoVaR* and *CoES* measures. We illustrate the importance of taking asymmetries into account and highlight the threats to accurate systemic risk measurement posed by misspecification biases in the marginals or the dependence model. We also investigate whether there are common market factors explaining an institution's contribution to systemic risk. In principle, lagged values of the implied market volatility, of funding liquidity, of credit spread and of the change in the three month Euribor rate are significant in explaining $\Delta CoVaR$. They also appear important in explaining the correlation between the financial system and each institution, as well as the financial system's volatility. The asymmetric behaviour of market factors across the pre-crisis and crisis periods, is partly attributed to the coordinated intervention of central banks in response to the financial crisis.

Finally, we investigate the impact of bank-specific and major macroeconomic factors on systemic risk. Across all alternative model specifications considered, size and leverage appears to be most robust bank-specific determinants of systemic risk, implying that bigger and highly leveraged financial institutions can generate large systemic risk externalities. In addition, we find that changes in certain macroeconomic variables such as an increase in unemployment rate, a decrease in the domestic stock market index, a decrease in industrial production and a decrease in GDP growth, contribute to an increase in the systemic risk next quarter.

List of figures

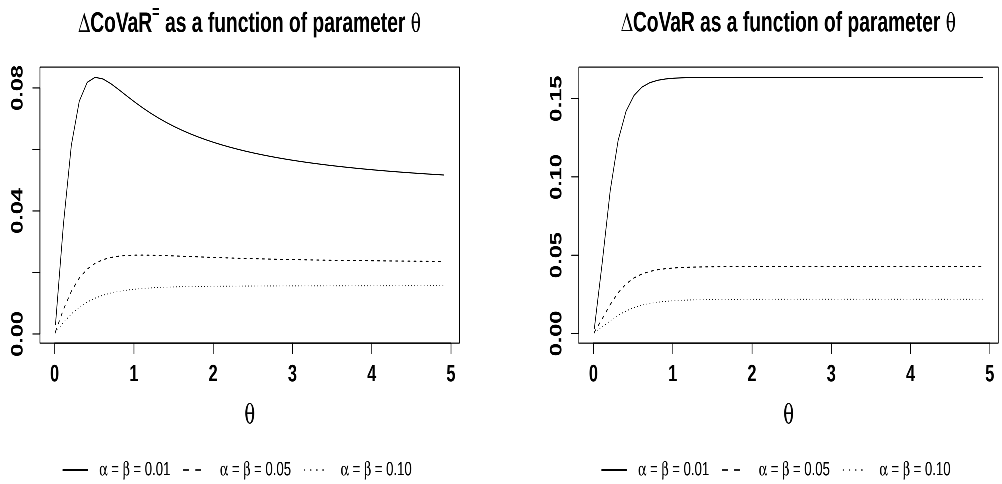

Figure 1: Clayton with Student-t marginals with 3 degrees of freedom: $\Delta CoVaR^{=}_{\alpha,\beta,t}$ and $\Delta CoVaR_{\alpha,\beta,t}$ as a function of θ.

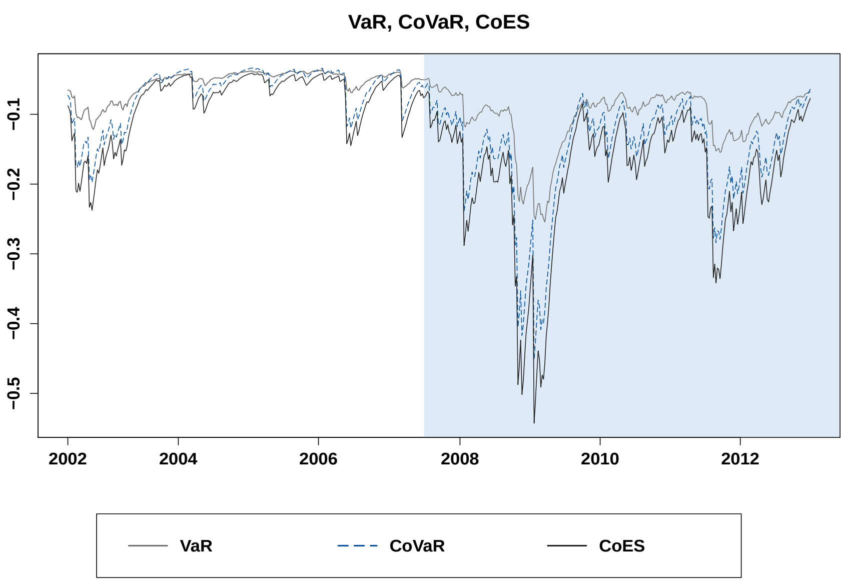

Figure 2: This figure shows time-series average values of weekly $VaR^i_{\alpha,t}$, $CoVaR_{\alpha,\beta,t}$ and $CoES_{\alpha,\beta,t}$ measures across all financial system-institution pairs. All risk measures are generated under the assumption of skewed-t marginals and computed at $\alpha = \beta = 5\%$ level. The light blue shaded area corresponds to Q3 2007 - Q4 2012 crisis period.

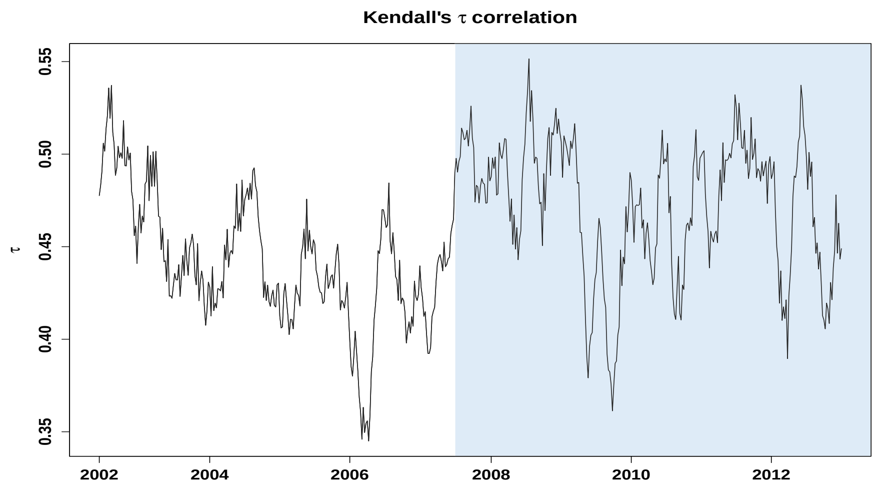

Figure 3: This figure shows time-series average Kendall's τ correlation estimates implied by estimated copula families across all financial system-institution pairs. All risk measures are generated under the assumption of skewed-t marginals and computed at $\alpha = \beta = 5\%$ level. The light blue shaded area corresponds to Q3 2007 - Q4 2012 crisis period.

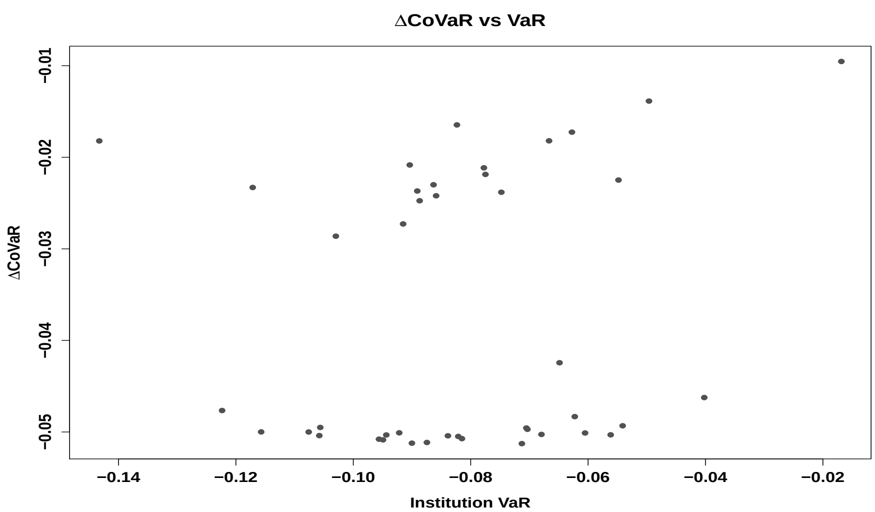

Figure 4: This scatter plot shows the cross-sectional link between the time-series average of the financial institution's risk in isolation, measured by $VaR^i_{\alpha,t}$, and the time-series average contribution to systemic risk, measured by $\Delta CoVaR_{\alpha,\beta,t}$. All risk measures are generated under the assumption of skewed-t marginals and computed at $\alpha = \beta = 5\%$ level.

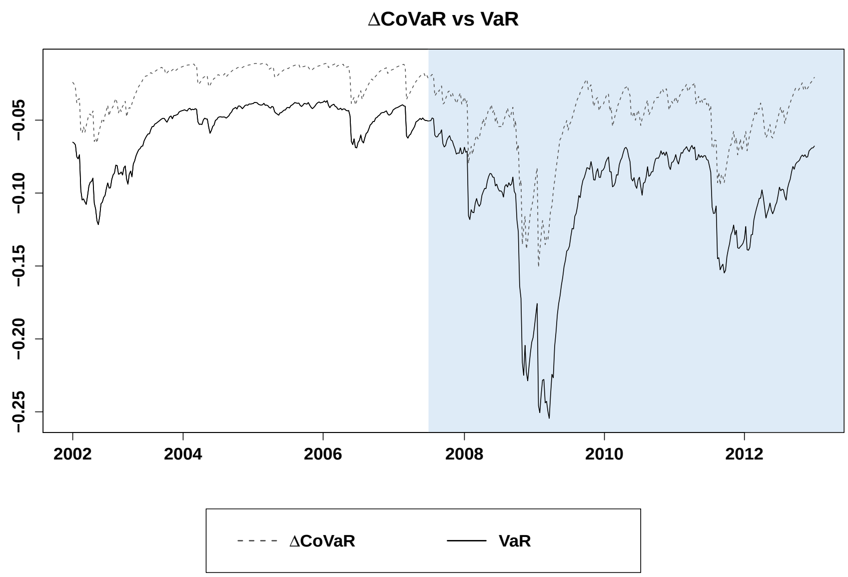

Figure 5: This figure shows the time-series average of weekly $\Delta CoVaR_{\alpha,\beta,t}$ and $VaR^{i}_{\alpha,t}$ measures. All risk measures are generated under the assumption of skewed-t marginals and computed at $\alpha = \beta = 5\%$ level. The light blue shaded area corresponds to Q3 2007 - Q4 2012 crisis period.

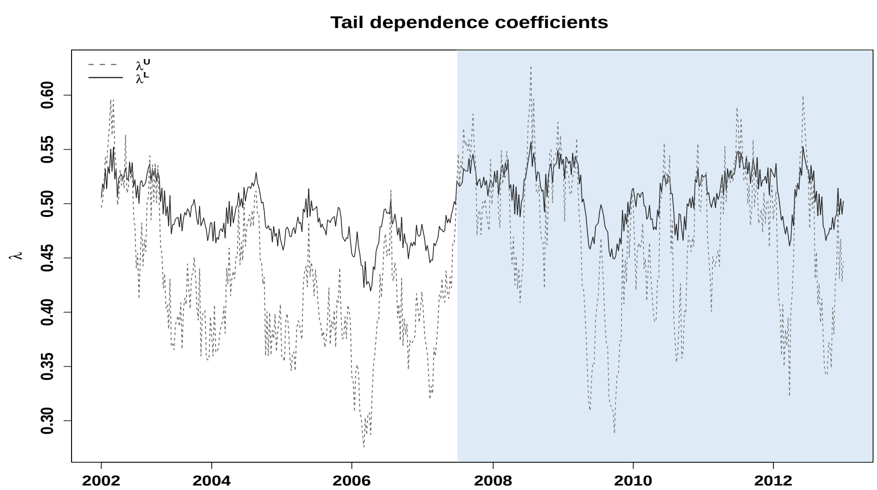

Figure 6: This figure shows time-varying average values of upper (λ^U) and lower (λ^L) tail dependence coefficients implied by BB7 copulas under the assumption of skewed-t marginals. All risk measures are computed at $\alpha = \beta = 5\%$ level. The light blue shaded area corresponds to Q3 2007 - Q4 2012 crisis period.

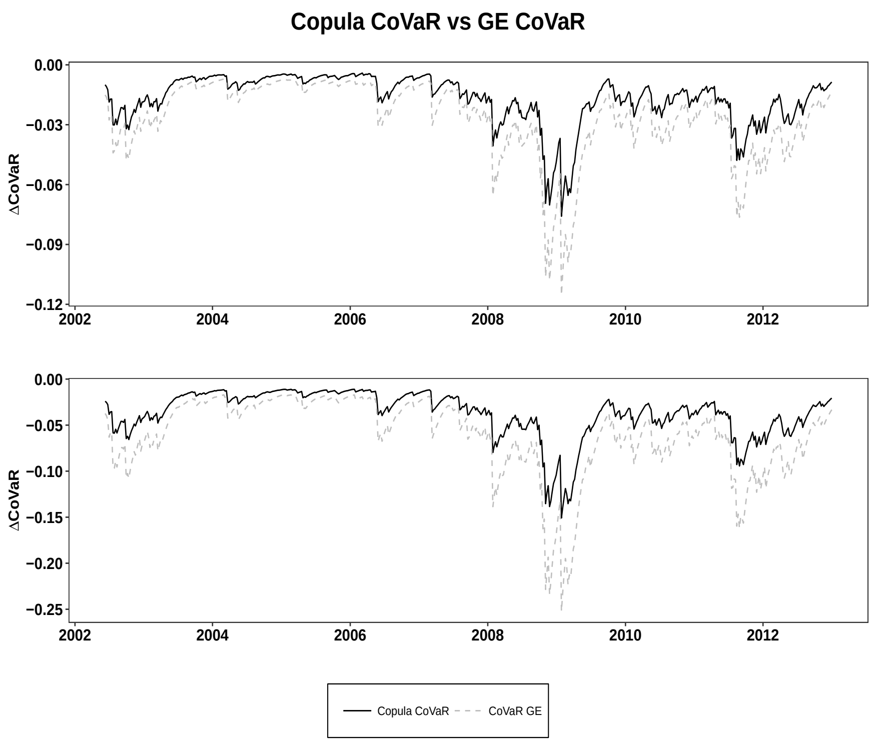

Figure 7: This figure shows the time-series average of weekly $\Delta CoVaR$ estimates for the Copula CoVaR and CoVaR as defined in Girardi and Ergün (2013). The upper panel shows estimates using Normal marginals for the Copula CoVaR model and a Normal bivariate distribution for the CoVaR model in Girardi and Ergün (2013) while the lower panel shows estimates using Skew-t marginals for the Copula CoVaR model and a Student-t bivariate distribution for the CoVaR model in Girardi and Ergün (2013). All risk measures are computed at $\alpha = \beta = 5\%$ level.

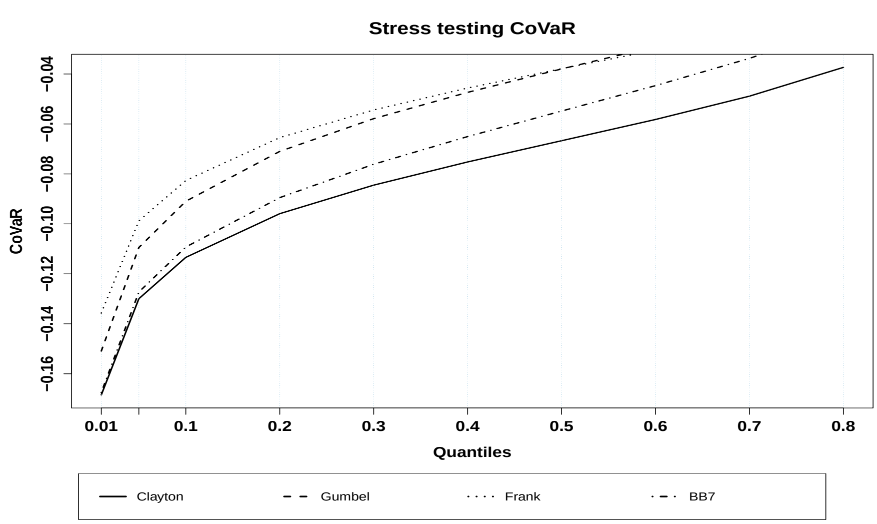

Figure 8: This figure shows the implied $CoVaR_{\alpha,\beta,t}$ estimates of the financial system conditional on HSBC returns generated by the Clayton, Gumbel, Frank and BB7 copulas with skewed-t marginals across different quantile levels ($\beta = 0.01$ to 0.8 and $\alpha = 0.05$). The Kendall's τ sample correlation parameter is equal to 0.51, i.e., $\tau = 0.51$.

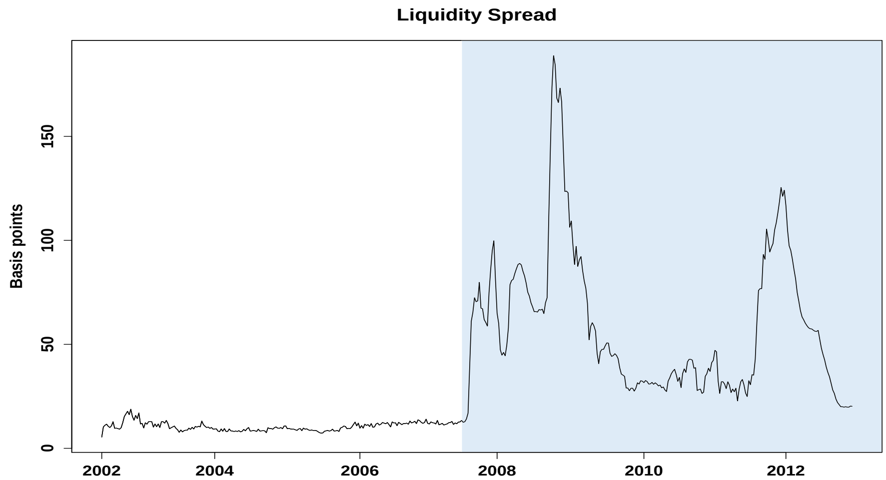

Figure 9: This figure shows the short-term Liquidity Spread between the 3-month Euribor rate and 3-month Eurepo rate measured in basis points. The light blue shaded area corresponds to Q3 2007 - Q4 2012 crisis period.

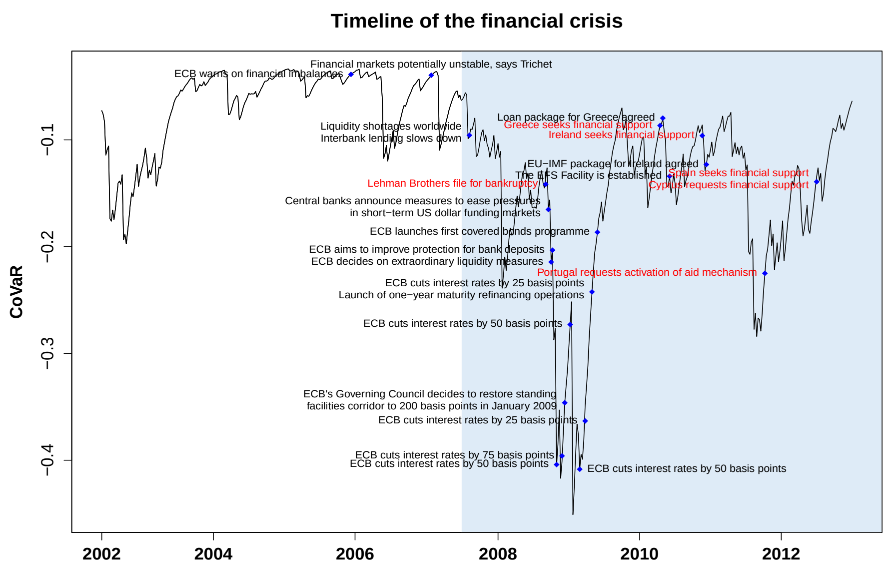

Figure 10: This figure shows average $CoVaR_{\alpha,\beta,t}$ estimates, key events (in red) and measures taken by the European Central Bank (ECB) to provide liquidity to the interbank market and restore financial stability. The light blue shaded area corresponds to Q3 2007 - Q4 2012 crisis period. Source of timeline events: European Central Bank (ECB), www.ecb.europa.eu.

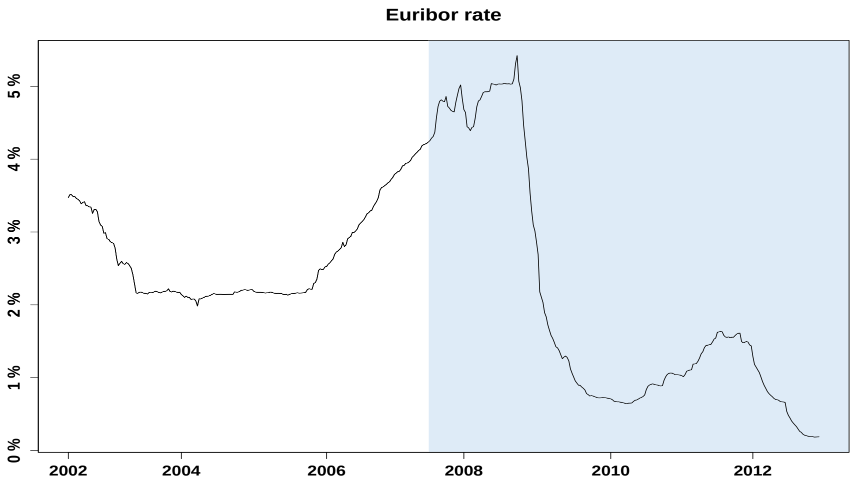

Figure 11: This figure shows the 3-month euro interbank offered rate (Euribor), the interest rate at which euro interbank 3-month deposits are offered by one prime bank to another prime bank within the euro area. The light blue shaded area corresponds to Q3 2007 - Q4 2012 crisis period.

List of tables

Table 1: This table ranks the average contribution to systemic risk for each individual institution.

ΔCoVaR results								ΔCoES results							
Normal Marginals				Skewed-t Marginals				Normal Marginals				Skewed-t Marginals			
Bank	Copula	ΔCoVaR	τ	Bank	Copula	ΔCoVaR	τ	Bank	Copula	ΔCoES	τ	Bank	Copula	ΔCoES	τ
BBVA	Gumbel	-2.741	0.62	POP	BB7	-5.127	0.46	BBVA	Gumbel	-2.549	0.62	POP	BB7	-5.236	0.46
SCH	Gumbel	-2.709	0.61	DBK	BB7	-5.122	0.55	SCH	Gumbel	-2.522	0.61	DBK	BB7	-5.227	0.55
BNP	Gumbel	-2.691	0.60	UBSN	BB7	-5.114	0.53	BNP	Gumbel	-2.507	0.60	UBSN	BB7	-5.226	0.53
CRDA	Gumbel	-2.585	0.57	CSGN	BB7	-5.086	0.51	CRDA	Gumbel	-2.417	0.57	CSGN	BB7	-5.186	0.51
UCG	Gumbel	-2.524	0.55	CRDA	BB7	-5.078	0.53	UCG	Gumbel	-2.365	0.55	CRDA	BB7	-5.181	0.53
SEA	Gumbel	-2.488	0.54	SEA	BB7	-5.073	0.50	SEA	Gumbel	-2.336	0.54	LLOY	BB7	-5.178	0.46
NDA	Gumbel	-2.476	0.54	SCH	BB7	-5.049	0.56	NDA	Gumbel	-2.326	0.54	BSAB	BB7	-5.168	0.38
BARC	Gumbel	-2.450	0.54	BBVA	BB7	-5.041	0.57	BARC	Gumbel	-2.302	0.54	SEA	BB7	-5.160	0.50
LLOY	Gumbel	-2.334	0.50	LLOY	BB7	-5.039	0.46	LLOY	Gumbel	-2.203	0.50	SVK	BB7	-5.155	0.41
SGE	Frank	-1.934	0.64	UCG	BB7	-5.033	0.50	SGE	Frank	-1.746	0.64	SCH	BB7	-5.125	0.56
DBK	Frank	-1.883	0.63	BSAB	BB7	-5.031	0.38	DBK	Frank	-1.700	0.63	UCG	BB7	-5.124	0.50
KB	Frank	-1.802	0.60	NDA	BB7	-5.027	0.50	KB	Frank	-1.626	0.60	BP	BB7	-5.123	0.47
UBSN	Frank	-1.798	0.60	SVK	BB7	-5.011	0.41	UBSN	Frank	-1.623	0.60	DNB	BB7	-5.121	0.38
CSGN	Frank	-1.798	0.59	BP	BB7	-5.009	0.47	CSGN	Frank	-1.622	0.59	NDA	BB7	-5.118	0.50
CBK	Frank	-1.713	0.57	KB	BB7	-5.000	0.52	CBK	Frank	-1.545	0.57	DAB	BB7	-5.111	0.38
ISP	Frank	-1.704	0.56	CBK	BB7	-4.999	0.49	ISP	Frank	-1.537	0.56	BBVA	BB7	-5.110	0.57
BMPS	Frank	-1.664	0.55	DNB	BB7	-4.971	0.38	BMPS	Frank	-1.500	0.55	BARC	BB7	-5.085	0.48
KNF	Frank	-1.660	0.53	DAB	BB7	-4.956	0.38	KNF	Frank	-1.497	0.53	CBK	BB7	-5.078	0.49
MB	Frank	-1.652	0.56	BARC	BB7	-4.951	0.48	MB	Frank	-1.490	0.56	KB	BB7	-5.075	0.52
RBS	Frank	-1.645	0.54	SYD	BB7	-4.932	0.33	RBS	Frank	-1.483	0.54	SYD	BB7	-5.073	0.33
POP	Frank	-1.594	0.53	JYS	BB7	-4.832	0.34	POP	Frank	-1.437	0.53	JYS	BB7	-5.033	0.34
PMI	Frank	-1.587	0.53	ETE	BB7	-4.766	0.35	PMI	Frank	-1.430	0.53	BPSO	BB7	-4.888	0.29
HSBA	Frank	-1.580	0.52	BPSO	BB7	-4.624	0.29	HSBA	Frank	-1.424	0.52	ETE	BB7	-4.887	0.35
BP	Frank	-1.575	0.53	BCV	BB7	-4.244	0.27	BP	Frank	-1.419	0.53	BCV	BB7	-4.582	0.27
SWED	Frank	-1.526	0.50	SGE	Frank	-2.862	0.62	SWED	Frank	-1.374	0.50	SGE	Frank	-2.910	0.62
STAN	Frank	-1.492	0.50	BNP	Frank	-2.728	0.61	STAN	Frank	-1.343	0.50	BNP	Frank	-2.769	0.61
ERS	Frank	-1.464	0.50	ISP	Frank	-2.474	0.53	ERS	Frank	-1.318	0.50	ISP	Frank	-2.507	0.53
SVK	Frank	-1.455	0.48	BMPS	Frank	-2.420	0.53	SVK	Frank	-1.310	0.48	BMPS	Frank	-2.460	0.53
DAB	Frank	-1.383	0.46	MB	Frank	-2.382	0.54	DAB	Frank	-1.244	0.46	MB	Frank	-2.422	0.54
BSAB	Frank	-1.357	0.45	KNF	Frank	-2.368	0.50	BSAB	Frank	-1.220	0.45	KNF	Frank	-2.405	0.50
POH	Frank	-1.321	0.45	RBS	Frank	-2.330	0.51	POH	Frank	-1.188	0.45	RBS	Frank	-2.354	0.51
BKIR	Frank	-1.315	0.44	PMI	Frank	-2.300	0.51	BKIR	Frank	-1.183	0.44	PMI	Frank	-2.335	0.51
DNB	Frank	-1.305	0.44	HSBA	Frank	-2.249	0.51	DNB	Frank	-1.173	0.44	HSBA	Frank	-2.285	0.51
BES	Frank	-1.274	0.42	SWED	Frank	-2.187	0.48	BES	Frank	-1.145	0.42	SWED	Frank	-2.216	0.48
JYS	Frank	-1.260	0.39	STAN	Frank	-2.115	0.48	JYS	Frank	-1.132	0.39	STAN	Frank	-2.144	0.48
SYD	Frank	-1.227	0.41	ERS	Frank	-2.084	0.47	SYD	Frank	-1.102	0.41	ERS	Frank	-2.118	0.47
ETE	Frank	-1.217	0.40	BKIR	Frank	-1.821	0.41	ETE	Frank	-1.093	0.40	BKIR	Frank	-1.845	0.41
BCP	Frank	-1.212	0.40	POH	Frank	-1.820	0.42	BCP	Frank	-1.088	0.40	POH	Frank	-1.840	0.42
BPE	Frank	-1.081	0.37	BES	Frank	-1.725	0.39	BPE	Frank	-0.969	0.37	BES	Frank	-1.744	0.39
BPSO	Frank	-1.016	0.34	BCP	Frank	-1.646	0.37	BPSO	Frank	-0.910	0.34	BCP	Frank	-1.659	0.37
BCV	Frank	-1.011	0.33	BPE	Frank	-1.386	0.33	BCV	Frank	-0.905	0.33	BPE	Frank	-1.399	0.33
VATN	Frank	-0.813	0.28	VATN	Frank	-0.953	0.23	VATN	Frank	-0.726	0.28	VATN	Frank	-0.953	0.23

This table reports average $\Delta CoVaR_{\alpha,\beta,t}$, $\Delta CoES_{\alpha,\beta,t}$ and implied Kendall's τ estimates along with the selected copula families of each financial system-institution pair in our sample under two marginals specifications: Normal and Skewed-t. All risk measures are computed at $\alpha = \beta = 5\%$ level.

Table 2: Dependence and tail dependence estimates.

Bank	ΔCoVaR	τ	λ^L	λ^U
POP	-5.127	0.51	0.46	0.24
DBK	-5.122	0.62	0.65	0.42
UBSN	-5.114	0.57	0.59	0.44
CSGN	-5.086	0.56	0.49	0.41
CRDA	-5.078	0.57	0.46	0.54
SEA	-5.073	0.54	0.62	0.40
SCH	-5.049	0.61	0.59	0.44
BBVA	-5.041	0.63	0.62	0.53
LLOY	-5.039	0.50	0.41	0.42
UCG	-5.033	0.56	0.48	0.28
BSAB	-5.031	0.43	0.42	0.24
NDA	-5.027	0.54	0.48	0.51
SVK	-5.011	0.47	0.30	0.34
BP	-5.009	0.51	0.55	0.29
KB	-5.000	0.57	0.56	0.33
CBK	-4.999	0.55	0.57	0.24
DNB	-4.971	0.42	0.45	0.21
DAB	-4.956	0.43	0.38	0.26
BARC	-4.951	0.54	0.54	0.43
SYD	-4.932	0.37	0.34	0.16
JYS	-4.832	0.37	0.31	0.27
ETE	-4.766	0.38	0.34	0.23
BPSO	-4.624	0.31	0.30	0.11
BCV	-4.244	0.28	0.19	0.21
SGE	-2.862	0.63	0.60	0.50
BNP	-2.728	0.61	0.44	0.52
ISP	-2.474	0.53	0.42	0.33
BMPS	-2.420	0.54	0.50	0.24
MB	-2.382	0.53	0.36	0.19
KNF	-2.368	0.50	0.37	0.26
RBS	-2.330	0.51	0.47	0.41
PMI	-2.300	0.51	0.50	0.24
HSBA	-2.249	0.51	0.33	0.32
SWED	-2.187	0.48	0.46	0.31
STAN	-2.115	0.48	0.36	0.23
ERS	-2.084	0.47	0.38	0.26
BKIR	-1.821	0.41	0.29	0.15
POH	-1.820	0.42	0.20	0.22
BES	-1.725	0.38	0.25	0.18
BCP	-1.646	0.37	0.41	0.00
BPE	-1.386	0.33	0.13	0.17
VATN	-0.953	0.24	0.17	0.12

This table reports average $\Delta CoVaR_{\alpha,\beta,t}$, non-parametric Kendall's τ correlation estimates and non-parametric upper (λ^U) and lower (λ^L) tail dependence estimates (an average of non-parametric estimates in Dobrić and Schmid (2005)) of each financial system-institution pair in our sample. $\Delta CoVaR_{\alpha,\beta,t}$ estimates are obtained under the assumption of skewed-t marginals. All risk measures are computed at $\alpha = \beta = 5\%$ level.

Table 3: This table ranks the average contribution to systemic risk by country.

ΔCoVaR results				ΔCoES results			
Normal Marginals		Skewed-t Marginals		Normal Marginals		Skewed-t Marginals	
Country	ΔCoVaR	Country	ΔCoVaR	Country	ΔCoES	Country	ΔCoES
France	-0.0222	Spain	-0.0506	France	-0.0204	Spain	-0.0516
Spain	-0.0210	Germany	-0.0505	Spain	-0.0193	Germany	-0.0515
Sweden	-0.0199	Belgium	-0.0499	Sweden	-0.0184	Norway	-0.0512
Great Britain	-0.0190	Norway	-0.0497	Great Britain	-0.0175	Belgium	-0.0507
Belgium	-0.0180	Denmark	-0.0490	Belgium	-0.0163	Denmark	-0.0507
Germany	-0.0180	Greece	-0.0476	Germany	-0.0162	Greece	-0.0489
Italy	-0.0160	Sweden	-0.0432	Italy	-0.0145	Sweden	-0.0441
Austria	-0.0146	Swiss	-0.0384	Austria	-0.0132	Swiss	-0.0399
Swiss	-0.0136	Great Britain	-0.0333	Swiss	-0.0122	Great Britain	-0.0341
Finland	-0.0132	France	-0.0326	Finland	-0.0119	France	-0.0332
Ireland	-0.0132	Italy	-0.0320	Ireland	-0.0118	Italy	-0.0328
Norway	-0.0131	Austria	-0.0208	Norway	-0.0117	Austria	-0.0212
Denmark	-0.0129	Ireland	-0.0182	Denmark	-0.0116	Ireland	-0.0185
Portugal	-0.0124	Finland	-0.0182	Portugal	-0.0112	Finland	-0.0184
Greece	-0.0122	Portugal	-0.0168	Greece	-0.0109	Portugal	-0.0170

This table reports average $\Delta CoVaR_{\alpha,\beta,t}$ and $\Delta CoES_{\alpha,\beta,t}$ estimates for each country in our sample under two marginal specifications: Normal and Skewed-t. All risk measures are computed at $\alpha = \beta = 5\%$ level.

Table 4: Dependence and tail dependence estimates by country.

Country	τ	λ^L	λ^U
Germany	0.58	0.61	0.33
Belgium	0.57	0.56	0.33
Spain	0.54	0.52	0.37
France	0.58	0.47	0.46
Sweden	0.51	0.46	0.39
Norway	0.42	0.45	0.21
Great Britain	0.51	0.42	0.36
Italy	0.48	0.41	0.23
Austria	0.47	0.38	0.26
Swiss	0.41	0.36	0.30
Greece	0.38	0.34	0.23
Denmark	0.39	0.34	0.23
Portugal	0.38	0.33	0.09
Ireland	0.41	0.29	0.15
Finland	0.42	0.20	0.22

This table reports average non-parametric Kendall's τ correlation estimates and non-parametric upper (λ^U) and lower (λ^L) tail dependence estimates (an average of non-parametric estimates in Dobrić and Schmid (2005)) for each country in our sample.

Table 5: Statistical test results: Copula CoVaR vs Girardi and Ergün CoVaR

Panel A: Copula $CoVaR_{\alpha,\beta,t}$		
Test	Normal Marginals	Skewed-t Marginals
Unconditional coverage	0.0206	0.3633
Independence	0.5649	0.8802
Conditional coverage	0.0286	0.5410

Panel B: Girardi and Ergün $CoVaR_{\alpha,\beta,t}$		
Test	Normal Marginals	Student-t Marginals
Unconditional coverage	0.0387	0.5283
Independence	0.8529	0.9842
Conditional coverage	0.0855	0.7738

This table reports average p-values of statistical tests for unconditional coverage, independence and conditional coverage for $\Delta CoVaR$ estimates as defined by the copula model (Panel A) and in Girardi and Ergün (2013) (Panel B). All risk estimates are computed at $\alpha = \beta = 5\%$ level.

Table 6: Panel regression results.

Variables	ΔCoVaR	Kendall's τ	Volatility σ_s
Vix_{t-1}	-0.149^{***}	0.004^{***}	0.140^{***}
$Liquidity_{t-1}$	-13.739^{***}	-0.313^{***}	13.997^{***}
$\Delta Euribor_{t-1}$	-3.691^{***}	0.057	3.453^{***}
$\Delta Slope_{t-1}$	0.658^{*}	-0.024	-0.607^{*}
$\Delta Credit_{t-1}$	-0.089	-0.027^{***}	0.101
$S\&P_{t-1}$	-0.021	0.001	0.019
$Vix_{t-1} \cdot I_{crisis}$	0.044^{***}	-0.003^{***}	-0.044^{***}
$Liquidity_{t-1} \cdot I_{crisis}$	11.866^{***}	0.337^{***}	-12.234^{***}
$\Delta Euribor_{t-1} \cdot I_{crisis}$	5.951^{***}	-0.035	-5.494^{**}
$\Delta Slope_{t-1} \cdot I_{crisis}$	-0.301	0.017	0.276
$\Delta Credit_{t-1} \cdot I_{crisis}$	-0.932^{***}	-0.014^{***}	0.947^{***}
$S\&P_{t-1} \cdot I_{crisis}$	0.032	-0.002	-0.029
Adj. R^2	0.770	0.219	0.876

This table displays results from bank fixed-effects panel data methodology (within estimator). The columns ΔCoVaR, Kendall's τ and Volatility report estimated coefficients from regressions of weekly $\Delta CoVaR_{\alpha,\beta,t}$ measures, Kendall's τ correlation estimates and the financial system's volatility σ_s estimates on the same lagged values of market variables: *Vix*, *Liquidity*, Δ*Euribor*, Δ*Slope*, Δ*Credit* and *S&P*. The I_{crisis} is a crisis dummy that takes the value of 0 for the Q2 2002 - Q2 2007 pre-crisis period and 1 for the Q3 2007 - Q4 2012 crisis period. Estimated coefficients for spreads, yield changes, Vix and market returns correspond to percent changes. The results are based on weekly data from Q2 2002 - Q4 2012. All $\Delta CoVaR_{\alpha,\beta,t}$ measures are estimated at 5% level. Kendall's τ correlations are obtained after transforming the time-varying copula parameters for each financial system-institution i pair to theoretical Kendall's τ values. The financial system's volatility σ_s estimates are obtained by a univariate asymmetric AR(1)-GJR-GARCH(1,1) model for each financial system portfolio. Following Thompson (2011), we compute standard errors that cluster by both firm and time. *** denotes significant at 1%, ** denotes significant at 5% and * denotes significant at 10%.

Table 7: Determinants of systemic risk - Individual institution characteristics.

Variable	1-Quarter	1-Year	2-Year
VaR_{t-k}	−0.213 49**	0.128 39*	0.004 43
MtB_{t-k}	0.000 03	−0.010 95*	−0.015 09**
$Size_{t-k}$	−0.015 66***	−0.040 46***	−0.038 94***
$Leverage_{t-k}$	−0.000 74***	−0.001 28***	−0.000 76***
$Beta_{t-k}$	0.008 47	−0.004 08	−0.015 95***
$Volatility_{t-k}$	−1.834 96***	0.436 13*	0.341 99
$Adj.\ R^2$	0.440	0.288	0.325

This table displays results from the bank fixed-effects panel regression methodology (within estimator). The columns report estimated coefficients from regressions of lagged quarterly bank-specific data on quarterly aggregated $\Delta CoVaR_{\alpha,\beta,t}$ measures. The column 1-Quarter correspond to results based on lagged variables equal to one-quarter, while columns 1-Year and 2-Year corresponds to results based on lagged variables equal to one year and two years, respectively. The results are based on quarterly data from Q2 2002 - Q4 2012.All $\Delta CoVaR_{\alpha,\beta,t}$ measures are estimated at 5% level. Following Thompson (2011), we compute standard errors that cluster by both firm and time. *** denotes significant at 1%, ** denotes significant at 5% and * denotes significant at 10%.

Table 8: Determinants of systemic risk - Macroeconomic characteristics.

Variable	1-Quarter	1-Year	2-Year
$\Delta Unemployment_{t-k}$	−0.007 73**	0.002 44	0.005 16
$Inflation_{t-k}$	−0.003 09	−0.006 52	0.005 66
$\Delta Share_{t-k}$	0.001 36***	0.000 52	−0.000 43
$\Delta IndustrialProduction_{t-k}$	0.001 57**	−0.000 60	−0.001 01
$\Delta Rates_{t-k}$	0.003 25	−0.006 44	−0.017 82
$GDPGrowth_{t-k}$	0.003 97**	0.002 51	0.001 92
$\Delta CurrentAccount_{t-k}$	0.000 47	0.000 46	0.000 52
$\Delta Debt/GDP_{t-k}$	0.000 04	0.000 29	−0.000 14
$AdjR^2$	0.467	0.044	0.059

Bank fixed-effects panel regressions (within estimator). The columns report estimated coefficients from regressions of lagged quarterly macroeconomic data on quarterly aggregated $\Delta CoVaR_{\alpha,\beta,t}$ measures. The column 1-Quarter, 1-Year and 2-Year correspond to results based on values lagged by one quarter, one year and two years, respectively. The results are based on quarterly data from Q2 2002 - Q4 2012.All $\Delta CoVaR_{\alpha,\beta,t}$ measures are estimated at 5% level. Following Thompson (2011), we compute standard errors that cluster by both firm and time. *** denotes significant at 1%, ** denotes significant at 5% and * denotes significant at 10%.

Appendix

A. *CoVaR* derivation for Archimedean copulas

A.1. Clayton Copula

The Clayton copula is a member of the Archimedean copula family with dependence parameter $\theta \in (0, \infty)$ and generator function $\varphi = \frac{(u^{-\theta}-1)}{\theta}$. The perfect dependence is observed at $\theta \to \infty$ whereas $\theta \to 0$ implies independence. Clayton copula allows for modelling positive dependence and asymmetric (lower only) tail dependence. The distribution function is given by

$$C(u, v\,; \theta) = (u^{-\theta} + v^{-\theta} - 1)^{-\frac{1}{\theta}}.$$

Following the notation introduced in section 2.2, an explicit expression for $CoVaR^{=}_{\alpha,\beta,t}$ for the Clayton copula can be derived, that is

$$\frac{\partial C(u,v)}{\partial v} = \Big(1 + u^{\theta}(v^{-\theta} - 1)\Big)^{\frac{-(1+\theta)}{\theta}} = \beta. \tag{24}$$

Solving for u and applying the probability integral transform, $CoVaR^{=}_{\alpha,\beta,t}$ is obtained as follows

$$u^{=} \equiv u = \Big(1 + v^{-\theta} \cdot (\beta^{-\frac{\theta}{1+\theta}} - 1)\Big)^{-\frac{1}{\theta}},$$

$$CoVaR^{=}_{\alpha,\beta,t} = F^{-1}_{s,t}\bigg(\Big(1 + \alpha^{-\theta} \cdot (\beta^{-\frac{\theta}{1+\theta}} - 1)\Big)^{-\frac{1}{\theta}}\bigg). \tag{25}$$

Alternatively, using the general expression in equation Equation (11) an explicit expression for $CoVaR_{\alpha,\beta,t}$ for the Clayton copula can be given as follows

$$\frac{C(u,v)}{v} = \beta,$$

$$\Big(u^{-\theta} + v^{-\theta} - 1\Big)^{-\frac{1}{\theta}} = v \cdot \beta.$$

Thus, solving for u and applying the probability integral transform, $CoVaR_{\alpha,\beta,t}$ can be obtained in a closed-form expression, that is

$$u^{\leq} \equiv u = \Big(1 + (v \cdot \beta)^{-\theta} - v^{-\theta}\Big)^{-\frac{1}{\theta}},$$

$$CoVaR_{\alpha,\beta,t} = F^{-1}_{s,t}\bigg(\Big(1 + (\alpha \cdot \beta)^{-\theta} - \alpha^{-\theta}\Big)^{-\frac{1}{\theta}}\bigg). \tag{26}$$

A.2. Frank Copula

This copula is also a member of the Archimedean copula family with dependence parameter $\theta \in (-\infty, \infty)\backslash\{0\}$ and generator function $\varphi = -\ln\left(\frac{e^{-\delta u}-1}{e^{-\delta}-1}\right)$. Frank copula allows for both positive and negative dependence structures, however, it does not imply tail dependence. The distribution function is given by

$$C(u, v\,; \delta) = -\frac{1}{\delta}\ln\Big(\frac{1}{1-e^{-\delta}}\Big[(1-e^{-\delta}) - (1-e^{-\delta u})(1-e^{-\delta v})\Big]\Big).$$

An analytical expression for $CoVaR^{=}_{\alpha,\beta,t}$ for this copula family can be derived as

$$CoVaR^{=}_{\alpha,\beta,t} = F^{-1}_{s,t}\Big(-\frac{1}{\delta}\ln\Big(1-(1-e^{-\delta})\cdot\Big[1+e^{-\delta\alpha}\cdot(\beta^{-1}-1)\Big]^{-1}\Big)\Big). \tag{27}$$

In contrast, an explicit expression for $CoVaR_{\alpha,\beta,t}$ for the Frank copula is given as follows

$$CoVaR_{\alpha,\beta,t} = F^{-1}_{s,t}\Big(-\frac{1}{\delta}\ln\Big[1-\frac{(1-e^{-\delta})-(1-e^{-\delta})(e^{-\delta\beta\alpha})}{(1-e^{-\delta\alpha})}\Big]\Big). \tag{28}$$

A.3. Gumbel Copula

The Gumbel copula with dependence parameter $\theta \in [1, \infty]$ and generator function $\varphi(t) = (-\log t)^{\theta}$ belongs also to the Archimedean copula family. Gumbel copula captures only positive dependence while it allows for asymmetric (upper only) tail dependence. For $\theta = 1$, Gumbel copula implies independence while the perfect positive dependence is observed as $\theta \to \infty$. The distribution function is given by

$$C(u, v\,; \theta) = \exp\Big(-\Big((-\log\, u)^{\theta} + (-\log\, v)^{\theta}\Big)^{\frac{1}{\theta}}\Big).$$

Unfortunately, the $\partial/\partial v\, C(u, v)$ of Gumbel copula is not partial invertible in its first argument u and hence we cannot derive an analytical expression for $CoVaR^{=}_{\alpha,\beta,t}$. However, an analytical expression for $CoVaR_{\alpha,\beta,t}$ can be given as follows

$$CoVaR_{\alpha,\beta,t} = F^{-1}_{s,t}\Big(\exp\Big(-\Big[\big(-\log(\alpha\cdot\beta)\big)^{\theta} - \big(-\log\,\alpha\big)^{\theta}\Big]^{\frac{1}{\theta}}\Big)\Big). \tag{29}$$

A.4. BB7 Copula

The BB7 copula, known as Joe-Clayton copula, is a two-parametric Archimedean copula family with $\theta \geq 1$ and $\delta > 0$. This copula family captures positive dependence while it allows also for asymmetric upper and lower tail dependence. In particular, the δ parameter measures lower tail dependence and the θ parameter measures upper tail dependence. Moreover, the Joe copula is the limiting case of BB7 for $\delta \to 0$ whereas for $\theta = 0$ one obtains the Clayton copula. The distribution function for this copula family is given by

$$C(u, v\,; \theta, \delta) = 1 - \Big(1 - \big[(1-(1-u)^{\theta})^{-\delta} + (1-(1-v)^{\theta})^{-\delta} - 1\big]^{-\frac{1}{\delta}}\Big)^{\frac{1}{\theta}}.$$

Analytical expressions for $CoVaR^{=}_{\alpha,\beta,t}$ and $CoVaR_{\alpha,\beta,t}$ can be obtain from the general solutions in equations Equation (10) and Equation (14) respectively, with

$$\varphi(v\,;\theta,\delta) = [1-(1-v)^{-\theta}]^{-\delta} - 1,$$

$$\varphi^{-1}(v\,;\theta,\delta) = 1-[1-(1+v)^{-1/\delta}]^{1/\theta},$$

$$\varphi^{'}(v\,;\theta,\delta) = -[1-(1-v)^{\theta}]^{-\delta-1}\delta[-(1-v)^{\theta}\theta/(-1+v)].$$

B. Dynamic Copula *CoVaR*

For the Clayton and Gumbel copulas the following parametric representation is proposed

$$\theta_t = \Lambda_1\Big(\omega + \beta\cdot\theta_{t-1} + \alpha\cdot\frac{1}{10}\sum_{j=1}^{10}|u_{t-j}\cdot v_{t-j}|\Big),$$

where $\Lambda_1(x)$ is an appropriate transformation to ensure the parameter always remains in its domain: $\exp(x)$ for Clayton copula and $(\exp(x)+1)$ for the Gumbel. On the other hand, the parameter δ of Frank copula is defined in $[-\infty,\infty]\backslash\{0\}$ at all times. Thus, we employ the following evolution equation for this particular copula family

$$\delta_t = \omega + \beta\cdot\delta_{t-1} + \alpha\cdot\frac{1}{10}\sum_{j=1}^{10}|u_{t-j}\cdot v_{t-j}|,$$

where the evolution of δ_t is constrained to ensure that remains in its domain. For the two-parametric Archimedean BB7 copula a similar parametric representation for each tail dependence coefficient is considered. The BB7 copula is constructed by taking a particular Laplace transformation of Clayton's copula. The BB7 copula distribution is given by

$$C(u,v\,;\theta,\delta) = 1-\Big(1-\big[\big(1-(1-u)^{\theta}\big)^{-\delta} + \big(1-(1-v)^{\theta}\big)^{-\delta} - 1\big]^{-\frac{1}{\delta}}\Big)^{\frac{1}{\theta}},$$

where $\theta = 1/\log_2(2-\tau^U)$, $\delta = -1/\log_2(\tau^L)$ and $\tau^U, \tau^L \in (0,1)$. Therefore, the following evolution equations can be considered for the BB7 copula

$$\tau^U_t = \Lambda_2\Big(\omega_U + \beta_U\cdot\tau^U_{t-1} + \alpha_U\cdot\frac{1}{10}\sum_{j=1}^{10}|u_{t-j}\cdot v_{t-j}|\Big),$$

$$\tau^L_t = \Lambda_2\Big(\omega_L + \beta_L\cdot\tau^L_{t-1} + \alpha_L\cdot\frac{1}{10}\sum_{j=1}^{10}|u_{t-j}\cdot v_{t-j}|\Big),$$

where $\Lambda_2(x) \equiv (1+\exp(-x))^{-1}$ is the logistic transformation, used to keep τ^U and τ^L in $(0,1)$ at all times.

C. European financial institutions

Table 9: List of European financial institutions

Bank	Datastream tickers	Country	Weight (%)	MCap (€ Bil.)
HSBC	HSBA	Great Britain	20.44	142.65
BCO SANTANDER	SCH	Spain	7.33	51.17
UBS	UBSN	Swiss	6.60	46.08
BNP PARIBAS	BNP	France	6.17	43.09
BARCLAYS	BARC	Great Britain	5.37	37.50
BCO BILBAO VIZCAYA ARGENTARIA	BBVA	Spain	4.95	34.56
STANDARD CHARTERED	STAN	Great Britain	4.64	32.38
DEUTSCHE BANK	DBK	Germany	4.43	30.92
LLOYDS BANKING GRP	LLOY	Great Britain	4.34	30.28
CREDIT SUISSE GRP	CSGN	Swiss	4.32	30.12
NORDEA BANK	NDA	Sweden	3.14	21.89
GRP SOCIETE GENERALE	SGE	France	3.00	20.92
UNICREDIT	UCG	Italy	2.82	19.69
INTESA SANPAOLO	ISP	Italy	2.44	17.00
SWEDBANK	SWED	Sweden	2.28	15.88
SVENSKA HANDELSBANKEN A	SVK	Sweden	2.11	14.69
SKANDINAVISKA ENSKILDA BK A	SEA	Sweden	1.59	11.07
DNB	DNB	Norway	1.49	10.38
DANSKE BANK	DAB	Denmark	1.21	8.41
CREDIT AGRICOLE	CRDA	France	1.02	7.15
ROYAL BANK OF SCOTLAND GRP	RBS	Great Britain	1.00	6.95
COMMERZBANK	CBK	Germany	0.89	6.21
KBC GRP	KB	Belgium	0.88	6.13
ERSTE GROUP BANK	ERS	Austria	0.86	6.00
BCO POPULAR ESPANOL	POP	Spain	0.56	3.93
BCO SABADELL	BSAB	Spain	0.55	3.85
NATIXIS	KNF	France	0.40	2.77
BANK OF IRELAND	BKIR	Ireland	0.37	2.55
POHJOLA BANK	POH	Finland	0.33	2.27
MEDIOBANCA	MB	Italy	0.32	2.23
JYSKE BANK	JYS	Denmark	0.29	2.03
BCO POPOLARE	BP	Italy	0.24	1.65
BCA POPOLARE EMILIA ROMAGNA	BPE	Italy	0.23	1.63
BCA MONTE DEI PASCHI DI SIENA	BMPS	Italy	0.20	1.43
BCO ESPIRITO SANTO	BES	Portugal	0.20	1.38
BCO COMERCIAL PORTUGUES	BCP	Portugal	0.19	1.31
NATIONAL BANK OF GREECE	ETE	Greece	0.18	1.29
BCA POPOLARE DI SONDRIO	BPSO	Italy	0.17	1.21
SYDBANK	SYD	Denmark	0.17	1.17
BCA POPOLARE DI MILANO	PMI	Italy	0.15	1.07
BANQUE CANTONALE VAUDOISE	BCV	Swiss	0.15	1.05
VALIANT	VATN	Swiss	0.15	1.03

This table lists the 42 out of 46 in total banks from 15 European countries belonging to STOXX 600 Banks Index and corresponding Datastream tickers, Market Capitalisation values and relative STOXX 600 Banks Index weights as of June, 2013. Source: STOXX Limited (www.stoxx.com).

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