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The Austrians and the Arrow Impossibility Theorem

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Version 1: July 2017

Abstract

Some economists in the Austrian tradition interpret the Arrow Impossibility Theorem (AIT) as support for their laissez-faire standpoint. They presume that the AIT cannot be applied in market settings, but in the context of voting, it adds weight to their arguments against the possibility of socialism and planning. The examples of Boettke (2001) and Boettke and Leeson (2002) are taken as examples. It is shown that their stance is founded on a misinterpretation of Arrow, and no compelling case for exempting markets from the AIT is found. The Austrian writers draw on Buchanan's critique of Arrow, which critique is in turn shown to be faulty and unreliable. Finally a possible avenue for future Austrian research is identified, showing that the AIT might not apply to markets.

1. Introduction

Peter Boettke, an economist in the Austrian tradition, interprets the Arrow Impossibility Theorem (AIT) as support for the Austrian *laissez-faire* standpoint. That is, he regards the AIT as condemning planning as inconsistent, unless managed by a dictator, while the market mechanism is left untouched by the theorem. AIT, in this interpretation, therefore fits nicely into the socialist calculation debate, supporting the Austrian contention of the impossibility of socialism. This paper appraises the grounds for this approach. Ultimately, this interpretation finds its roots in work by Buchanan, and the paper also addresses Buchanan's critique of the Arrow theorem. The paper finds that Buchanan's view, and the Austrian work drawing on it, cannot be relied upon to underpin the interpretation that AIT undermines planning while leaving the market unscathed. Nevertheless, a basis does seem to exist for claiming that the market mechanism is exempt from the scope of the AIT critique, and the paper identifies an opportunity for further work in this area.

The next section sets out some occasions in which Boettke has adopted the stance that AIT, correctly interpreted, undermines planning but not the price mechanism, along with the *prima facie* evidence that that was not at all Arrow's view of what he was doing. Section 3 sets out a brief statement of the Arrow theorem. Section 4 appraises the arguments put forward for the Austrian view of the AIT. Section 5 investigates the roots of that approach in the work of James Buchanan. Section 6 sketches an argument for the Austrian view of Arrow which might be attractive for further work. Section 7 concludes.

2. Boettke and Arrow

Peter Boettke writes

> The Arrow theorem … could be reinterpreted as an application of Mises' impossibility thesis to non-market decision-making via democratic voting. In the absence of the price system, actors would confront a set of incoherent signals about how they should orient their behavior … as Arrow demonstrated … majority-rule pairwise voting might not produce the required agreement (a highly formal result which echoes Hayek's discussion of limits of democratically derived agreement in *The Road to Serfdom*). (Boettke, 2001: 60)

Again:

> In the face of market failure, Arrow asked, was there a democratic collective choice mechanism that was capable of allocating scarce resources efficiently. The inability to unambiguously aggregate preferences across voters led to a negative answer. Whatever collective choice mechanism we choose will be imperfect. Collective choice can be efficient but dictatorial, or it can be democratic but inefficient. (Boettke, 2001: 196)

The message is clear: the Arrow theorem is about non-market decision-making, about how we cope in the absence of markets, or when markets fail. "Collective choice" here excludes choice via the price mechanism. In the case of the first passage, we are not referred to any previous literature explaining this interpretation. The second passage is accompanied by several paragraphs of discussion of Arrow's work, but this does not extend to justifying the claim that AIT is about political rather than economic choice: this is simply assumed throughout.

Boettke and Leeson (2002) correctly starts from Arrow's (SCIV: 2) opening statement that a dictator can be rational in the sense that any individual can be rational, "But Arrow wanted to figure out whether the market and voting can make the same claim to rationality" (B&L: 14). For several pages thereafter, the attention is exclusively on voting and makes much of the paradox of voting. This is odd, as the paradox of voting was known long before Arrow – both Condorcet and Charles Dodgson wrote about it. B&L summarise the discussion saying that "Arrow is led to a conclusion that either collective choices are democratic, but irrational, or they are rational, but dictatorial." (B&L: 15). Nothing in the text would lead the reader to imagine that "collective choice" referred to market mechanisms as much as non-market ones. B&L cite Buchanan saying, in a private letter, "Clearly, if we cannot aggregate individual preferences … socialism is impossible" (18). They B&L describe this as an "implication of Arrow's paradox". Later in the paper, B&L present an argument that separating market and non-market mechanisms in the consideration of the AIT in this way is an appropriate stance, and that argument will be considered in a subsequent section. B&L conclude their argument by wondering why Mises and Hayek "did not pick up Arrow's argument … after it appeared in the early 50s to help bolster their claims" (20).

To regard the AIT as undermining planning, while leaving market mechanisms unchallenged, is surprising. It is certainly not what Arrow thought he was doing: "In the following discussion of the consistency of various value judgements as to the mode of social choice, the distinction between voting and the market mechanism will be disregarded, both being regarded as special cases of the more general category of collective social choice" (Arrow, 1963: 5). Right at the start he says that "The methods of voting and the market … are methods of amalgamating the tastes of many individuals in the making of social choices" and ""The formal existence of methods of aggregating individual choices [is] the problem posed in this study" (Arrow, 1963: 2). In the additional chapter, added for the 1963 edition, Arrow comments that "Since the market mechanism does satisfy the

Condition of Independence of Irrelevant Alternatives, it must violate another condition, which is clearly that of Collective Rationality" (Arrow, 1963: 110 n 50). The only reason for saying that if the market mechanism satisfied one condition if must violate on of the others, is that it is subject to the AIT. The 'General Possibility Theorem', or AIT, is summarised in Theorem 2 (Arrow, 1963: 59). Arrow writes: "Theorem 2 shows that … there is no method of voting which will remove the paradox of voting … Similarly, the market mechanism does not create a rational social choice." (Arrow, 1963: 59). Arrow is thus explicit that AIT applies equally to market and non-market settings. Even if Arrow were less clear, regardless of what he thought he was doing, it is the case that the proof of the theorem makes no reference to market or non-market modes of choice, and there is thus a strong default for regarding AIT as equally true of either.

3. The Arrow Theorem[1]

a. The conditions

Arrow's theorem says that no social choice procedure can exist which satisfies the five conditions **O**, **U**, **P**, **D** and **I**. The argument proceeds via a *reductio ad absurdam*: we assume that all the conditions hold and then demonstrate that they lead to a contradiction. Relaxing any of the conditions removes the impossibility of deriving a SWF, but, in general, such SWFs will exhibit perverse features. The conditions are expressed in many different ways in the literature and what follows is based on the editors' introductory essay, "Individual Preferences and Collective Decisions", to Part II of Barry and Hardin (1982: 213-228). The five conditions are as follows:

Condition **O**:   The SWF is a unique **O**rdering of the alternatives facing society based only on individual orderings. An ordering is a consistent ranking. In particular this implies transitivity for both preference and indifference. If society prefers x to y and y to z, then it prefers x to z, and so on. It also requires that the SWF is complete (or connected) – that is, that every possible social state is ranked against every other state: $\forall x,y, x \succ y, y \succ x,$ or $x \sim y$.

Condition **U**:   The social choice rule must have **U**nrestricted domain: it must work for *every logically possible combination* of individual orderings.

Condition **P**:   The social choice rule must be **P**areto-efficient: if one individual prefers x to y and all other individuals either prefer x to y or are indifferent between x and y, then the SWF must prefer x to y.

Condition **D**:   There must not be a **D**ictator, that is, a person whose preference of x for y is always (in every logically conceivable constellation of preferences) the social preference, for any x and y, regardless of the preferences of others.

Condition **I**:   The social ordering of any pair of alternatives x and y is a function solely of the individual orderings of x and y: it is **I**ndependent of irrelevant alternatives – individual orderings of x and z, for example.

b. Proof of the theorem

One further concept is needed: that of *decisiveness*. If a group or individual is decisive over x and y, and prefers x to y, then society prefers x to y, whatever anyone else's preference may be. We also assume that society consists of a finite number of individuals. The proof then proceeds by showing

that (a) if there is an SWF which satisfies conditions **O**, **U**, **P**, and **I** (that is, all except non-dictatorship), then for some constellation of preferences there must be a decisive individual, and (b) if there is a decisive individual then he is a dictator.

Consider any pair of alternatives, x and y, where society prefers x to y. It cannot be the case that everyone in society prefers y to x, by condition **P** (Pareto). There must be a set of decisive individuals. If only one person prefers x then the set only contains one person; if everyone prefers x then the set of all the individuals in society is decisive. Normally, it will be a set of intermediate size, but that is irrelevant. There will thus be a (non-empty) decisive set for each pair of alternatives where society is not indifferent between the two. Consider the set of all of these decisive sets. From the assumption that society consisted of a finite number of individuals, this set of decisive sets must have a smallest member, or a subset of equally large smallest members, in which case we pick any member of this subset. We can show that there must be a possible pattern of preferences for which this smallest set of decisive individuals has only one member.

We will suppose initially that this smallest decisive set, V, has more than one member, and show that this leads to a contradiction. Suppose V is decisive over x and y, and that it (and hence society – because V is decisive) prefers x to y. This must lead to a contradiction. Since it consists of more than one member we can divide it into two parts, one, V_1, consisting of one member and the other, V_2, consisting of all the other members of V. We also give the name V_3 to the set of all the members of society not in V. Condition **U**, unlimited domain, tells us that the SWF must work for any logically possible pattern of preferences. So we can pick any pattern of preferences we like. Suppose the pattern of preferences is that for V_1, $x \succ y \succ z$, for V_2, $z \succ x \succ y$, and for V_3, $y \succ z \succ x$. For ease of reference this pattern is set out in the table below.

Table 1: Preferences of the three sets of agents, V_1, V_2 and V_3

Rank	V_1	V_2	V_3	S	S'
1	X	z	y	x	x
2	Y	x	z	y	}y, z
3	Z	y	x	z	

Now, we know that V is decisive over x and y, so for society $x \succ y$. This is shown in the column headed S. But where does society rank z? Suppose society preferred z to y. Only V_2 prefers z to y – both V_1 and V_3 prefer y to z – so that would make V_2 decisive. But V_2 is one person less than V, the smallest decisive set, so that is not possible. So society either prefers y to z (Column S) or is indifferent between y and z (Column S'). Hence society must prefer x to z, given $x \succ y$ and $y \succcurlyeq z$, by transitivity (condition **O**). These two alternatives, $x \succ y \succ z$, and $x \succ y \sim z$, are shown in columns S and S', respectively, of Table 3. But now V_1 is decisive since both V_2 and V_3 prefer z to x. However, V_1 consists of only one person, so the assumption that the smallest decisive set, V, consisted of more than one person turns out to be self-contradictory. We have shown, therefore, what was required, that there is a pattern of possible preferences such that there is a decisive individual. That completes the first part of the proof.

The second part of the proof shows that a decisive individual is a dictator. Suppose A is decisive for x against y and that he also prefers x to z. Also suppose, invoking condition **U**, that everyone prefers y to z. Condition **P** says that society prefers y to z. If every individual prefers y to z, it would certainly be Pareto-inefficient for the SWF to prefer z to y, or even to be indifferent between them. Hence, $x \succ y \succ z$ for society and so, invoking transitivity, $x \succ z$. But condition **I**, independence of irrelevant alternatives, says that the social choice between x and z is independent of individual preferences over y. Hence, regardless of others' preferences, x is preferred to z by society if and only if A prefers x to z. Hence, if A is decisive for x against y, we can replace y by any other alternative which A finds less

desirable than *x*. Similarly we can replace *x* by any alternative which *A* finds more desirable than *y*. Hence for any possible pair of alternatives, *A*'s preference is decisive: he is a dictator. But the first part of the proof showed that there was a possible set of preferences such that there was decisive individual. Hence, for this pattern of preferences there is a dictator. But this violates condition **D**, of non-dictatorship. The concept of an SWF which simultaneously satisfies all five conditions is therefore inconsistent.

What this shows is that there is a possible pattern of individual preference orderings such that a social ordering derived from them which satisfies the Pareto and Independence conditions must be dictatorial if it is to be consistent.

4.   The argument for treating market and non-market mechanisms differently

Boettke and Leeson (2002) begin their discussion by confirming that ""Arrow assumed away the differences between markets and voting because both were being treated as special cases of the more general case of social choice … In Arrow's analysis, neither the market mechanism nor the voting booth can amalgamate the tastes of many individuals in such a way that we can meaningfully define a social welfare function" (2002: 16). There are some minor misunderstandings in the subsequent discussion. In a diagram of a two-dimensional production possibility frontier, the community indifference curve drawn tangent to it is labelled "Social Welfare Function", and they say that "For Arrow, both market and voting fail to aggregate preferences into a coherent social welfare function" (17). A social welfare function (SWF), in Arrow's account, is the mechanism for aggregating preferences, such as voting or the market mechanism, not the order which emerges from that mechanism. Nevertheless, these are not critical to their account and can be placed on one side.

Boettke and Leeson believe that "Arrow mischaracterised the situation" (17). By "[t]reating the economic problem as a planning problem" Arrow "draws our analytical attention away from how the entrepreneurial market process coordinates the decisions within the market so that they tend to realize the mutual gains from trade." (17) In itself, this is a very weak opening. It says no more than that we should focus on the role of the market process as a coordination mechanism and the gains from trade which it allows to be realised, instead of treating the market as a planning problem. But for this argument to work, we would need much more information. In particular, what is the evidence for the claim that Arrow treats the market process as a planning problem, and exactly how does that vitiate his account? B&L are silent on this. Treating the market as a coordination device which realises gains to trade sounds highly uncontentious. It is for B&L to explain how that would produce a different result from Arrow's. Specifically, how would it remove the market from the field of application of the AIT, while retaining the validity of AIT as far as planning is concerned? At this point, B&L draw on the work of James Buchanan. Buchanan's critique of the Arrow Theorem will be considered in a subsequent section.

The final paragraph of the final substantial section of their article (pp 19-20) spells out the case for considering Arrow's consequences for planning but not for markets. As well as being extremely brief, their argument is cloudy, but it appears to be that a rational SWF function is a precondition for planning but not for markets:

> [s]ince the *ex ante* existence of a meaningful and stable social welfare function is necessary for the operation of market socialism and Arrow demonstrated that no such social welfare function can be arrived at, market socialism is not a viable economic arrangement. Unlike for

market socialism, the existence of a stable social welfare function is *not* necessary for the operation of the market. (19)

Alas, B&L offer no argument to support either of these claims. The claims are in any event rather strange, and suggest some confusion as to what the AIT is about. What it is *not* about is the bare possibility of any particular kind of SWF – market, planning, or some mixture of the two. Rather it is about the possibility of any such SWF issuing in an ordering, that is, a transitive enumeration of the possible states between which we must choose, subject to the conditions of non-dictatorship, independence of irrelevant alternatives, Pareto efficiency, and universal domain. B&L talk of socialism and markets as requiring 'ex ante' a 'meaningful' SWF. But as pointed out before, it is the market or planning mechanism which is the SWF. Perhaps what they mean is the order of alternatives issuing from the SWF mechanism. But it is impossible for an ordering to emerge prior to the institution of the SWF that it will emerge from, so this seems very confused. Perhaps they mean that someone has to be able to write down a description of a SWF before implementing it in the case of planning, while they don't need to in the case of markets. It is difficult to see why, and more difficult to see what the point of the argument is. All this discussion of what is necessary '*ex ante*' really makes little sense. That is not what the Arrow Theorem is about. In the account of B&L, we cannot have planning because it requires *ex ante* a 'meaningful' SWF – presumably one which satisfies the five conditions – and that's impossible from the AIT. Meanwhile, we can have markets because they do not – *ex ante* – require a satisfactory SWF. Well, of course, as far as existence is concerned, we can see markets every day, and planning has taken place on many occasions: in most places the mode of making these social decisions is a mix of the two. So both are *possible*. But what Arrow has shown is that they cannot be satisfactory, if by that we mean, simultaneously satisfying the five conditions. B&L have given us no grounds for believing that the AIT applies any differently to market and non-market SWFs.

In contrast to the "*ex ante*" discussion of planning, B&L claim that "The social welfare function maximization is at best an *ex post* representation of an outcome that the market already achieved … [AIT] merely raises doubts about the appropriateness of a particular *ex post* characterization of the market process" (19-20). It is difficult to interpret these remarks. We are considering two kinds of SWF, market and non-market. Most systems in reality are mixtures of the two, but it is useful to consider them as poles of a spectrum. They are modes of aggregating individual preferences to generate an order of the social alternatives between which we must choose. They are not before or after anything; in particular, they are not *ex ante* plans for anything or *ex post* characterisations of anything. (They are thing which B&L wish to plan for, the thing which they wish to characterise.) But we can ask whether *this* SWF, say, the market, or *this other* SWF, say planning, satisfies our criteria for a socially desirable SWF. Arrow's answer in both cases is *no*. B&L have given us no cogent reason for varying Arrow's opinion. This does not necessarily mean that no such reason exists. A subsequent section will explore one possibility.

5. **Buchanan's response to the Arrow Theorem**

B&L draw on Buchanan's (1954a: 17-19) criticisms of the Arrow Theorem, and Boettke (private communication) underlines the necessity of consulting Buchanan's paper to understand the point that the Arrow theorem may be applied to planning but not markets. This section addresses Buchanan's critique and the subsequent use made of it[2]. I argue that no weight should be attached to Buchanan's analysis of the AIT as it is characterised by misunderstanding and misrepresentation, and in general a failure to engage with what Arrow is saying.

Buchanan starts the paper with some very strong statements as to what it achieves. His approach "reveals a weakness in the formal analysis itself and demonstrates that some of the more significant implications drawn from his [Arrow's] analysis are inappropriate" (1954a: 114). His conclusions are that the paradox of voting is a feature, not a bug, and more importantly, that

> the voting process is fundamentally different from the market when the two are considered as decision-making processes rather than as bases for deriving social welfare functions. Here it will be demonstrated that the market does produce consistent choices and that the market does not belong in the category of collective choice at all. (114)

Buchanan begins the analysis by repeating Arrow's terminology, contained in Arrow's Definition 4 (Arrow, 1963: 23): "*By a* social welfare function *will be meant a process or rule, for each set of individual orderings … for alternative social states … states a corresponding social ordering of alternative states*." This terminology, Buchanan believes, is "singularly unfortunate" and "the source of the confusion" in Arrow. What confusion? That "between the definition of the social welfare function and the actual *processes* of choice: voting and the market" (Buchanan, 1954a: 114-115). But the confusion on this is one entirely manufactured by Buchanan: the SWF *is* the "actual process of choice". By defining one, you necessarily define the other. Despite Arrow having said this in the clearest possible terms, Buchanan finds it necessary to introduce a distinction, such that these terms will no longer mean what Arrow meant by them. He immediately draws the consequence of his terminological adjustment: "As will be shown in this paper, the decision-making *process* may produce consistent choice, even though the *rule* which *states* the social ordering from the individual values may not exist" (Buchanan, 1954a: 115). So Buchanan's protest is that the SWF, the market or planning process, should be considered both as a rule or programme for doing something, and as the implementation of that rule in some sequence of actions. But if the process which takes as inputs the individual orderings and produces as output the social ordering, if that process is regular, lawful, then a rule or law exists, whether we happen to know what it is or not. The process is the rule or law in action. There cannot be lawful decision-making process for which the rule may not exist.

Buchanan summarises the Conditions and sets out Arrow's conclusions. He then adds "But Arrow extends the argument to say that these ordinary decision-making mechanisms do not allow rational social choice. Now this is a horse of quite a different color, with which the Arrow argument should not legitimately concern itself at all" (Buchanan, 1954a: 115). This is a very strange comment to make, as, of course, it is the rationality of the social ordering emerging from any SWF whatever, which is at the *heart* of the problem Arrow is addressing. Why on Earth should Arrow not concern himself with it? The reason soon becomes clear. He is failing to understand the term *rationality* in the very precise and minimal sense which Arrow unambiguously gives it. This is not from ignorance. On other occasions Buchanan treats rationality in exactly the same way as Arrow does. In his paper on "Individual Choice in Voting and the Market" which appeared in the *JPE* later the same year, he refers in the text to "rationality in individual behavior" and explains in a footnote that "Rationality in individual behavior is defined in the normal manner, that is, the individual is able to rank alternatives, and such ranking is transitive" (Buchanan, 1954b: 341 and n 23). But here he takes issue with Arrow for referring to *rationality* and makes it mean something opaque and mystical.

Section II of the paper is on "The Concept of Social Rationality". It starts with the claim that "It is difficult to know exactly what is meant by "rational social choice" in the Arrow analysis" (Buchanan, 1954a: 116). This is surprising as the definition of rationality is simple and stated unambiguously at an early stage (Arrow, 1963: 2-3). It is that decision-making is consistent, and that implies transitivity. So what is Buchanan getting at? "The mere introduction of the idea of social rationality suggests the fundamental philosophical issues involved. Rationality or irrationality as an attribute of the social group implies the imputation to that group of an organic existence apart from that of its

individual components." This is wholly untrue. We are not discussing the rationality of the social group as an organic entity, we are discussing the ordering of alternative states which has emerged from the SWF, and asking whether it is consistent, whether indeed it is an ordering. These philosophical musings are utterly misplaced. Their function is to undermine Arrow without engaging with what Arrow says: "if the idea of acceptable social welfare functions and of social or collective rationality is completely divorced from the decision-making processes of the group, what is there left of the Arrow analysis?" (Buchanan, 1954a: 118). Indeed. If we are compelled to consider the decision-making processes of society without asking whether they can be relied upon to generate an ordering, then nothing *is* left of the AIT.

Undermining everything he has said up to this point, Buchanan now admits that our ordinary understanding of rationality can be applied, but does so as if he is passing on to another aspect of Arrow's argument:

> It is still possible to test these processes for consistency; but consistency or rationality in this sense must … be defined in terms of satisfying "the condition of rationality, as we ordinarily understand it.' [Arrow, 1963: 3] This implies only that choices can be made (are connected) and that the choices are transitive." (Buchanan, 1954a: 118)

Buchanan complains of the lack of the "time dimension" in Arrow's presentation (118-9). He says that the violation of transitivity in the SWF, and the consequent paradox of voting, far from being a bug, is a feature. That A is preferred to B, B to C, and C to A, allows provisional, temporary decisions to be made, rather than the definite and irreversible decision which would be taken in its absence. The latter would be a dictatorship of the majority. So the intransitivity allows the debate to continue, and further steps towards consensus to be taken. This is a confusion. In the Arrow Theorem, the choice emerging from the SWF is a ranking of states of the world that we can achieve today by some possible set of actions. There is nothing to say that we will make corresponding choices tomorrow, when new knowledge is available, as to the costs and benefits of the states then available, and as to our own preferences regarding the choices then to be made. That is as true whether the choice emerging from the SWF is deficient in some respect – violating rationality or any of the other conditions – or not. JMB is not comparing like with like. The question we are to address is not what will keep the issue from final settlement, but what the properties will be of the choice we make today. We should prefer a rational choice to an irrational one, that is an ordering to a list of alternatives that lacks transitivity.

Section IV "Collective Choice and Free Markets" (Buchanan, 1954a: 121-122) deals specifically with the claim that the Arrow theorem cannot be applied to market settings. Buchanan bases his analysis on a misreading of Arrow. A SWF depends on translating "individual values into social building blocks. If these values consist only of individual orderings of social states … this step cannot be taken" (Buchanan, 1954a: 122). This, he says is "clearly revealed" in Arrow's statement that

> "The relation of known preference or indifference is clearly transitive, but it is not connected since, for example, *it does not tell us* how the individual compares two social alternatives, one of which yields him more of one commodity than the second, while the second yields him more of a second commodity than the first." (Arrow, 1963: 61; Buchanan's italics).

But this is a complete misreading of Arrow. The "known preference relation" is something that Arrow introduces to deal with the case of imposing certain conditions on the individual preference scales. These conditions are known as "the individualistic assumptions". Arrow has set out the AIT in the preceding chapters and now wishes to explore the effect of relaxing the conditions in various ways. In this chapter he relaxes the universal domain condition, and places a restriction on the

individual rankings of alternatives. The "individualist assumptions" are that (a) the individual only discriminates between the alternatives on the criteria of commodities he receives (including leisure and saving) in each; and (b) in any comparison of two states, in which he has more of one commodity in one and the same of every other commodity in both, then he prefers the former. Arrow shows that the known preference relation is transitive but incomplete, as we do not know whether the individual prefers a state with more of one commodity to one with more of another, everything else held constant. The relation of known preference is a quasi-ordering. Arrow then shows that AIT still applies. So known preference is a special case. Its incompleteness tells us nothing about the completeness of the individual's true preference scale. If Buchanan were to refer to the individual orderings considered in the rest of Arrow's book, which do not have this restriction, then his argument would be reversed: because they are orderings, that is, both complete (ie connected) and transitive, the step of going from individual orderings to a social ordering is possible.

Unfortunately, in this section Buchanan gives no further argument as to why we should consider the market exempt from the AIT. His final remark summarises his helplessness: "the market does not call upon individuals to make a decision collectively at all. This being the case, market choice is just as consistent as, and no more consistent than, the individual choice of which it is composed." But this is simply to assert without argument that Arrow is wrong. On the contrary, when individuals act in their own interests in the market place, they without knowing or willing it make a collective decision. The allocation of resources which emerges is the collective decision which they have made. Is this this decision consistent – "as consistent as … the individual choice of which it is composed"? Arrow says no: even if the individual decisions are guaranteed to be an ordering, the social choice might not be. Buchanan has given us no reason to doubt that.

It is worth concluding this section by reminding readers that Arrow was able to comment on Buchanan's paper in his commentary appended to the 1963 edition of his book. He says that the criticisms of the AIT, specifically including Buchanan's "are based on misunderstandings of my position and indeed of the full implications of the critics' own positive views" (Arrow, 1963: 103).

> Where Bergson, Little and I seek in varying ways to explicate the notion of social welfare in operational terms, Buchanan's positivism is more extreme. Choice is only individual; the very concept of social welfare is inadmissible, and my use of the term "collective rationality" (by which I meant that social choices corresponding to any given set of individual orderings were so interrelated as to satisfy the definition of an ordering) was strongly attacked on the ground that only individuals can be rational. Nevertheless, Buchanan and Tullock [in *The Calculus of Consent*] do put great stress on the selection of a constitution as the central step in developing a social choice mechanism. (Arrow, 1963: 107-8)

The point being that a constitution is a set of restrictions on what we decide to regard as legitimate modes of social choice.

This section has found that no weight can be attached to Buchanan's analysis when we interpret Arrow's theorem. Nevertheless, this does not mean that there is no possible account which allows a separate treatment of market and non-market SWFs. A subsequent section sketches one possible approach.

6. A possible challenge to some of Arrow's conclusions

Market-favourable (for example, Austrian) writers, and market-sceptical (socialist, or in American English, 'liberal') writers, should both find the Arrow theorem disquieting. The first group, because the AIT says that markets must violate what seem to be natural and minimal requirements for a satisfactory mode of social choice, and the second group, because planning suffers a similar fate. It is of interest to ask how the Arrow Theorem might be adapted or challenged in order to relieve the distress of one group or the other. The present paper is concerned with Austrian interpretations of Arrow, so we will focus on that, though with an occasional remark about the socialist side. I stress that this is entirely speculative: I am suggesting an avenue which Austrians may wish to explore. I am not making any positive assertions.

If the goal is to look for weaknesses in Arrow's argument so far as the market mechanism is concerned, a promising starting point would lie in the assumption of ordinal preferences: the proof of the AIT depends entirely on ordinal preferences. As soon as any element of cardinality is admitted, the proof fails. Does leave us with any wiggle room? At first blush this might not seem attractive. Is it not the case that market transactions can only ever express ordinal preferences, while voting systems – such as Borda count systems – can express intensity as well as rank? If so the AIT applies necessarily to market modes of social choice, but not necessarily to non-market modes, if the latter impound anything more than purely ordinal preferences. The natural conclusion would be that the AIT thus says exactly the opposite of what the Austrians claim: market methods of preference aggregation cannot work for all possible constellations of preferences, while no such statement can be made about non-market methods. This warrants closer inspection. The insights of previous Austrian writers provide assistance.

In view of Austrian insistence on subjectivity, it is worth reminding ourselves that Austrian thought is built on an understanding of *subjective* use-value, or utility, and *objective* exchange-value, or prices. For Mises, in *Socialism. An Economic and Sociological Analysis*, "[i]n an exchange economy, the objective exchange value of commodities becomes the unit of calculation … The subjective valuation of one individual is not directly comparable with the subjective valuation of others. It only becomes so as an exchange value arising from the interplay of subjective valuations" (Mises, 1951: 115; a very similar formulation appears in Mises, 1935: 97). Later in the same work he refers to the necessity, for economic calculation, of "an objective recognizable unit of value which would enable economic calculations to be made" (Mises, 1951: 135; again a similar formulation appears in Mises, 1935: 116).

A consequence of this is that these subjective utilities are essentially ordinal – the individual is able to compare states, and to prefer one to the other, or be indifferent between them, but nothing can be said about intensity: about how much the individual prefers A to B, or about interpersonal comparison: about how much utility individual 1 derives from a state relative to individual 2. But the prices which emerge from the interplay of individuals trading in the market place are objective: that is, they are cardinal numbers[3] which express not only an ordinal ranking, but intensity and interpersonal comparisons as well. Joseph Salerno has supplied a Postscript to Mises's economic calculation essay in which he restates Mises's argument. He explains that

> What is needed, then, to produce the cardinal numbers necessary for computing the costs and benefits of production processes is what Mises calls the "intellectual division of labor" which emerges when private property owners are at liberty to exchange goods and services against money according to their individual value judgments and price appraisements … In their consumer roles, all people make monetary bids for the existing stocks of final goods according to their subjective valuations, leading to the emergence of objective monetary

exchange ratios which relate the values of all consumer goods to one another … There thus comes into being the market's monetary price structure, a genuinely "social" phenomenon in which every unit of exchangeable goods and services is assigned a socially significant cardinal number. (Salerno, 1990: 52-3)

This understanding, that objective exchange values are socially significant cardinal numbers will be of the greatest importance when we ask about the negative appraisal of market mechanisms implied by the AIT. Arrow repeatedly states that the analysis is based on the assumption of ordinal utilities. At the outset he says

Even if, for some reason, we should admit the measurability of utility for an individual, there still remains the question of aggregating the individual utilities. At best, it is contended that, for an individual, his utility function is uniquely determined up to a linear transformation; we must still choose one out of the infinite family of indicators to represent the individual, and the values of the aggregate (say a sum) are dependent on how the choice is made for each individual. In general, there seems to be no method intrinsic to utility measurement which will make the choices compatible. It requires a definite value judgment not derivable from individual sensations to make the utilities of different individuals dimensionally compatible and still a further value judgment to aggregate them according to any particular mathematical formula … it seems to make no sense to add the utility of one individual, a psychic magnitude in his mind with the utility of another individual … We will therefore assume throughout this book that the behavior of an individual in making choices is describable by means of a preference scale without any cardinal significance, either individual or interpersonal. (Arrow, 1963: 10-11)

The assumption of cardinal utilities, unattractive as it is, for the reasons Arrow states, would therefore immediately invalidate the AIT, as the proof depends entirely on the assumption of ordinality. An important footnote here is provide by Amartya Sen: "the Arrow impossibility result can be readily extended to the use of individual cardinal utility functions (rather than individual orderings) as the arguments of collective choice rules" (Sen, 1970: 123; this result is demonstrated on pp 124-5 and 129-130). So when we talk about cardinality, we need to bear in mind that that cardinality must be sufficient to provide interpersonal comparison, if any challenge to the AIT is to be launched. Let us consider how such a challenge might be posed.

The market mechanism is an information processing and communication device. It receives information from transactions which provide signals as to the benefits and costs of some activity. This information is aggregated into prices which have the function of signalling to market participants the costs and benefits to society of an activity the agent might engage in. Let us consider the information which the market receives from agents. This information consists of many observations of prices and quantities transacted. Each price actually realised is a representation of the purchaser's preferences – the information that the market is able to process is that the agent attaches at least as much utility to the unit of the activity as to the money price. The market never receives the naked preferences, but always a representation of those preferences in money terms: the market processes not preferences per se, but preferences in the form of demands. Preferences are ordinal, yet the demand, the representation of those preferences transmitted to the market is cardinal; and the translation from ordinal, subjective utility to a cardinal, objective representation in money terms is carried out by the individual market participant himself. The information which the market receives from a transaction is not only cardinal in the sense that it embodies information on intensity of preferences – it also embodies interpersonal comparison. £10.51 is not only greater than £7.32, we know how much greater it is: £3.19. If one agent values an item at £5.00 and

another at £6.00, and there is one unit to allocate, the market performs an interpersonal comparison and awards that unit to the purchaser willing to pay the higher price. This says little about the underlying utility enjoyed by the two individuals – it might be the case that the poorer agent, or the one with the more pressing needs elsewhere, who can only afford to offer £5.00, might derive a greater increment in pleasure by consuming the item. But the budgets or endowments of the agents and the constellation of needs that they experience yield a socially valid, or socially significant, comparison of their utilities. The preferences which act as inputs are those not the naked, ordinal preferences of the agents, but those preferences subject to a cardinal representation by comparison with money prices. If the logic of this argument, which seems to flow naturally from Austrian analysis, were accepted, this would challenge the application of the AIT to market processes. Given that the inputs to the market SWF are cardinal, the AIT can no longer be proved. There is no longer any reason why the output of the calculation device that the market constitutes should not be an ordering, and at the same time satisfy the four other conditions. In particular, the proof would fail because the preferences shown in Table 1, above, would now be cardinal numbers. Cyclical social preferences would now be impossible: the worst that could happen is social indifference between the three outcomes. But there is nothing irrational about that, and it would be impossible to prove that there would be a single decisive individual for some possible constellation of preferences.

I think this says enough to indicate a possible line for future enquiry. That inquiry would face a number of challenges. Firstly, it would have to explain what was wrong with Arrow's statement (1963: 110 n 50) that the market mechanism violates the rationality requirement, which "violation is precisely the well-known intersection of community indifference curves". Secondly, and perhaps linked to the above, the fact that the AIT does not apply, does not necessarily mean that we cannot prove the violation of one more of the five conditions. For example, a Borda count is a method of voting which represents (though in what might be considered a rather crude way) both intensity of the voter's feelings, and the social significance to be attached to his feelings. So, arguably, the AIT for the reasons set out above, cannot be proved. Nevertheless, it can certainly be shown that the Borda count method violates the independence of irrelevant alternatives condition. And lastly, if the market is exempt from the AIT, then so also will be any voting method which attaches a cardinal representation to the voter's preferences. The Borda count method has just been cited as an example. That one still fell foul of one of the five conditions. But there may be other forms of voting which get round this problem. For instance, it might be possible to set up quasi-markets for a range of issues, and allow voters to allocate a limited budget of votes to the questions that they felt most strongly about. There is indeed more to say about all these points, but they would take us beyond the scope of this paper.

7. Conclusion

This paper has considered the treatment of Arrow by the Austrian school of economics, focusing on the work of Peter Boettke. It has found that Austrian attempts to exempt market processes from the negative conclusions of the Arrow Impossibility Theorem are unjustified. Moreover, references by Austrian writers to Buchanan's critique of Arrow were not cogent: Buchanan's response rested largely on misunderstanding and misrepresentation of what Arrow had done. Despite these negative conclusions, a possible avenue for future work by Austrian writers was offered, holding out the prospect of erecting a defence of market forces against the AIT which might work.

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[1] This section draws on Chapter 3 of my PhD thesis (Denis, 2001: 52-75).

[2] Arrow introduces some confusion on this by referring to the wrong paper by Buchanan in the second edition of *Social Choice and Individual Values*. In the 1963 edition Arrow adds a concluding chapter addressing the discussion of the AIT since the original publication of his book in 1951. He says (1963: 103) that "[a] long series of distinguished critics have [sic] argued, in one form or another, that the problem of social choice has been incorrectly posed in this book." A footnote (1963: 103 n32) refers us, *inter alia*, to J.M. Buchanan, "Individual Choice in Voting and the Market," *Journal of Political Economy*, Vol 62 , August, 1954, pp 334-43. However, this paper by Buchanan makes no direct criticism of Arrow's book, which is mentioned twice, in footnotes, with no hint that there is anything wrong with it. It seems that this is just an error on Arrow's part, and he meant the April 1954 paper, "Social Choice, Democracy, and Free Markets". The latter paper is indeed dedicated to refuting the AIT.