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# Why the deferred annuity makes sense

an application of hyperbolic discounting to the annuity puzzle

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# Outline

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# Motivation

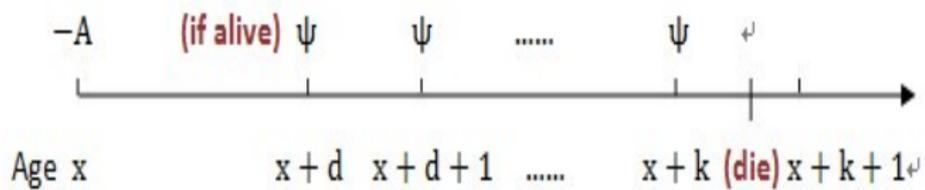
- Over the past few decades, traditional DB pension plan has gradually lost its dominance in private pension sectors and DC pension plan has become increasingly popular. Under the DC pension plan, member would not receive lifelong guarantees at retirement, instead, they can choose to take a lump sum, make periodic withdrawals or invest in an annuity.
- Although various scholars around the world have proved that purchasing an annuity can assure retirees of higher retirement incomes for the rest of their lives, the empirical data has long reflected that retirees are reluctant to convert any retirement savings into annuities. This is called the "Annuity Puzzle".

# Motivation

- Over the past few decades, traditional DB pension plan has gradually lost its dominance in private pension sectors and DC pension plan has become increasingly popular. Under the DC pension plan, member would not receive lifelong guarantees at retirement, instead, they can choose to take a lump sum, make periodic withdrawals or invest in an annuity.
- Although various scholars around the world have proved that purchasing an annuity can assure retirees of higher retirement incomes for the rest of their lives, the empirical data has long reflected that retirees are reluctant to convert any retirement savings into annuities. This is called the "Annuity Puzzle".

# Background

What is an annuity/deferred annuity?



# Background

Since Yaari (1965), large literature offers many possible reasons to explained the annuity puzzle:

## Rational factors:

- Unattractive annuity price: Brown and Warshawsky (2001)
- Bequest motive: Friedman and Warshawsky (1990), Vidal-Melia and Lejarraga-Garcia (2006), Lockwood (2012)
- Existence of social security and private DB pension plans: Dushi and Webb (2004), Butler et al. (2016)
- Healthcare expenditure shocks: Sinclair and Smetters (2004)
- Intra-family mortality sharing: Brown and Poterba (2000)

## Behavioral factors:

- Cumulative prospect theory: Hu and Scott (2007)
- Time inconsistent preference: Schreiber and Weber (2015)
- Framing effect: Brown et al. (2008)
- Others include poor financial education of retirees and regret aversions: (Cannon and Tonks, 2008)

# Objectives

- a. Can we use the hyperbolic discount model to explain the low demand of immediate annuities at the point of retirement?
- b. Are pensioners at 65 years old interested in purchasing a Retirement Age Deferred Annuity (RADA)?
- c. Would people at working age have an interest in buying a Working Age Deferred Annuity (WADA)?
- d. How would working-age members respond to a question asking them to decide today whether to buy an immediate annuity at retirement?

# Introduction to Hyperbolic Discount Model

## Three Anomalies:

- **Decreasing Impatience**

Q1: Choose between: (A1), one apple today; (B1), two apples tomorrow

Q2: choose between: (A2), one apple in one year; (B2), two apples in one year and one day

- **The Absolute Magnitude Effect**

Q: What compensations people need if following benefits are delayed for 3-month?

(1) A dinner worth \$15

(2) A trip to San Francisco worth \$250

(3) A good used car worth \$3000

- **The Gain-Loss asymmetry**

$(10, 0) \sim (21, 1)$  vs  $(-10, 0) \sim (-15, 1)$

$(100, 0) \sim (157, 1)$  vs  $(-100, 0) \sim (-133, 1)$

# Introduction to Hyperbolic Discount Model



$$V(c_0, c_1, \dots, c_T) = \sum_{t=0}^T \delta(t)v(c_t)$$

# Introduction to Hyperbolic Discount Model



$$V(c_0, c_1, \dots, c_T) = \sum_{t=0}^T \delta(t)v(c_t)$$

- Discount functions

- Proportional Discount Model (Herrnstein, 1981):

$$\delta(t) = (1 + \alpha t)^{-1} \quad \text{with } \alpha > 0$$

- Power Discount Model (Harvey, 1986):

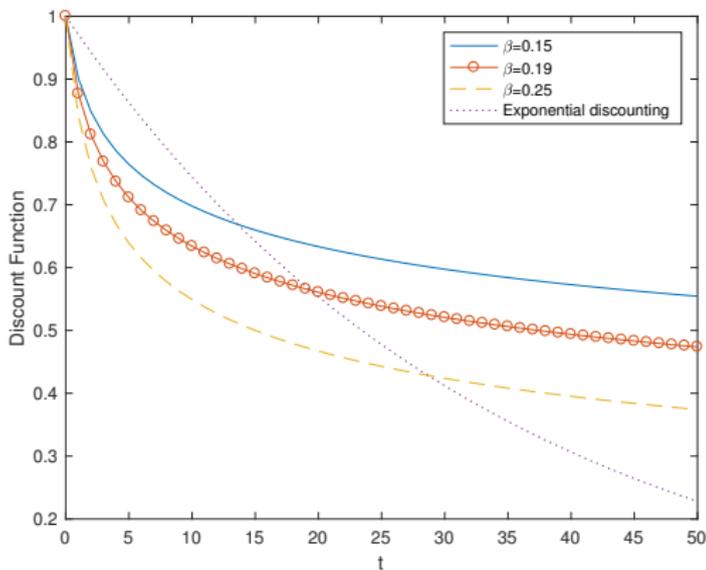
$$\delta(t) = (1 + t)^{-\beta} \quad \text{with } \beta > 0$$

- General Hyperbolic Discount Model (Loewenstein and Prelec, 1992):

$$\delta(t) = (1 + \alpha t)^{-\frac{\beta}{\alpha}} \quad \text{with } \alpha > 0, \beta > 0$$

# Introduction to Hyperbolic Discount Model

- A plot of Discount function,  $\delta(t)$ , against time,  $t$ .



# Perceived Annuity Values and Reservation Prices

## a. Immediate annuities for retirees

Consider a retiree at age  $x$  ( $x \geq 65$ ) who needs to make a decision on whether to spend a lump sum amount  $A$  to purchase an immediate annuity which pays  $\psi$  per annum in advance. Let  ${}_t p_x$  denote the probability that an  $x$ -year-old person can survive for  $t$  years and the maximum attainable age is set to be 120. The overall value of this investment for the  $x$ -year-old is:

$$V_1(x) = v(-A) + \sum_{i=x}^{119} (\delta(i-x) \times {}_{i-x} p_x \times v(\psi)) \quad (1)$$

# Perceived Annuity Values and Reservation Prices

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$$V_1(x) = v(-A) + \sum_{i=x}^{119} (\delta(i-x) \times {}_{i-x} p_x \times v(\psi)) \quad (1)$$

## b. RADA for retirees

Consider a 65-year-old pensioner ( $x = 65$ ) who has just retired. By investing the pension lump sum amount  $A$  in a  $d$ -year deferred annuity, the pensioner is entitled to a lifelong guaranteed annual income of  $\psi$  in  $d$  years. The perceived value of this RADA at the time of purchase is:

$$V_2(d) = v(-A) + \sum_{i=65+d}^{119} (\delta(i-65) \times {}_{i-65} p_{65} \times v(\psi)) \quad (2)$$

# Perceived Annuity Values and Reservation Prices

## c. WADA for working age individuals

An individual at age  $x$  ( $25 \leq x \leq 64$ ) considers investing in a WADA which provides annual incomes of  $\psi$  once the annuitant survives the retirement age 65. The overall perceived value of this investment at the time of purchase is:

$$V_3(x) = v(-A) + \sum_{i=65}^{119} (\delta(i-x) \times {}_{i-x}p_x \times v(\psi)) \quad (3)$$

# Perceived Annuity Values and Reservation Prices

## c. WADA for working age individuals

An individual at age  $x$  ( $25 \leq x \leq 64$ ) considers investing in a WADA which provides annual incomes of  $\psi$  once the annuitant survives the retirement age 65. The overall perceived value of this investment at the time of purchase is:

$$V_3(x) = v(-A) + \sum_{i=65}^{119} (\delta(i-x) \times {}_{i-x}p_x \times v(\psi)) \quad (3)$$

## d. Decision on purchasing an immediate annuity at retirement for working age individuals

A pension scheme member at working age ( $25 \leq x \leq 64$ ) is asked to make a decision in advance on whether to choose a pension lump sum  $A$  at age 65 or choose a corresponding fair annuity starting at the same age. The overall perceived value of this annuity investment when making the decision is:

$$V_4(x) = \delta(65-x) \times {}_{65-x}p_x \times v(-A) + \sum_{i=65}^{119} (\delta(i-x) \times {}_{i-x}p_x \times v(\psi)) \quad (4)$$

# Perceived Annuity Values and Reservation Prices

How to determine if an annuity is attractive?

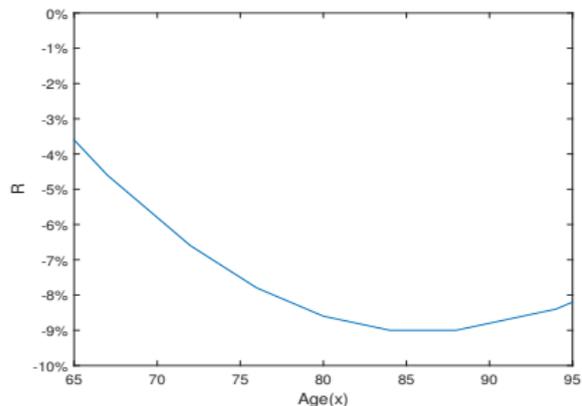
$$R = \frac{\text{Reservation price} - \text{Actuarially fair price}}{\text{Actuarially fair price}}$$

if  $R > 0$ , an annuity is attractive.

if  $R < 0$ , an annuity is unattractive.

# Basic Results

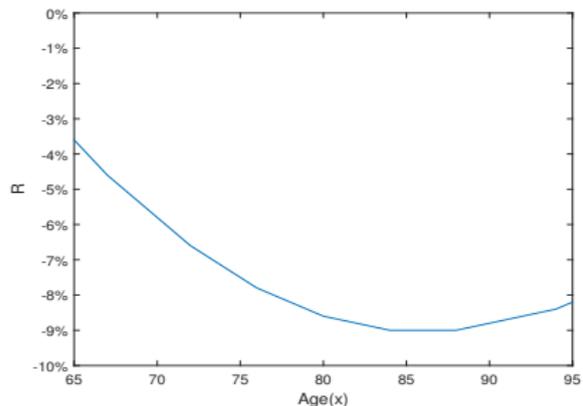
a.



- Immediate annuities are generally not attractive to purchase for retirees
- Immediate annuities starting at around age 85 is the least attractive

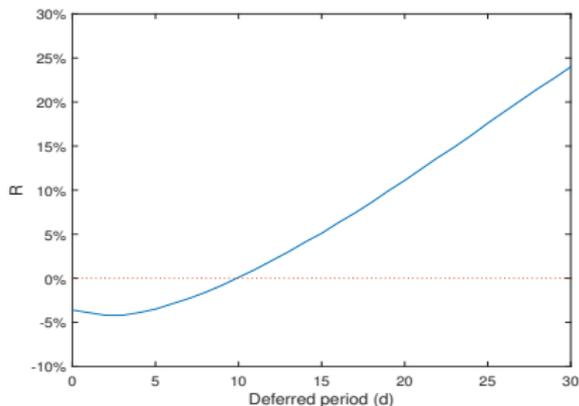
# Basic Results

a.



- Immediate annuities are generally not attractive to purchase for retirees
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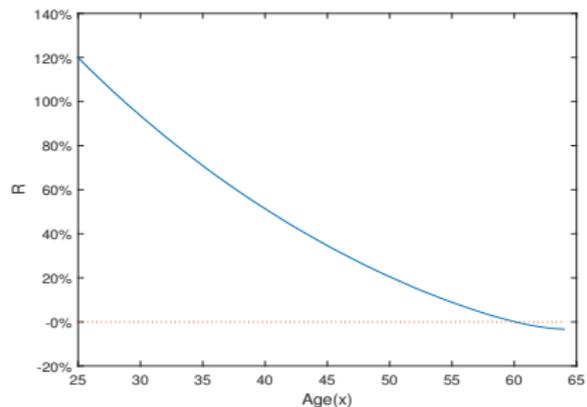
b.



- Annuities that are deferred for more than 10 years are perceived to be attractive for a 65-year-old retiree.
- The attractiveness of deferred annuities is increasing with the length of the deferred period

# Basic Results

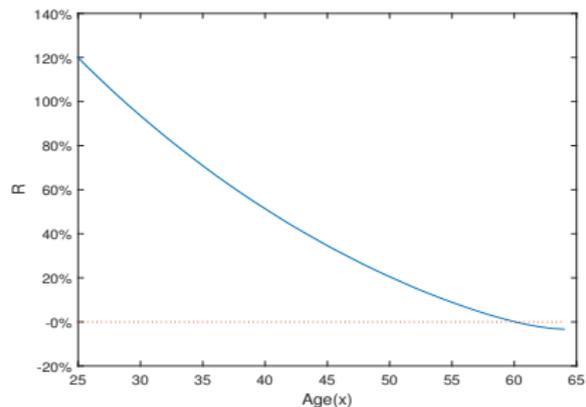
C.



- People at working age generally find retirement annuities attractive to purchase
- A negative relationship between age and the attractiveness of WADA

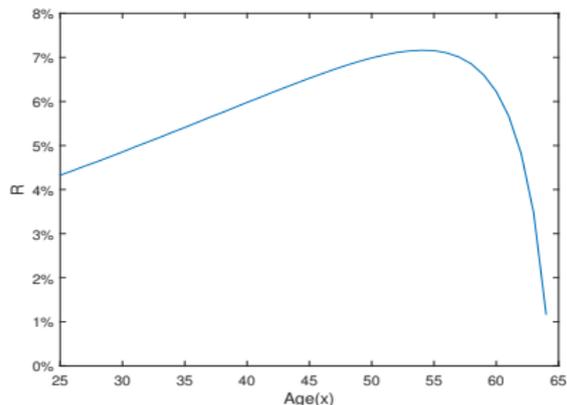
# Basic Results

c.



- People at working age generally find retirement annuities attractive to purchase
- A negative relationship between age and the attractiveness of WADA

d.



- For individuals above age 55, the attractiveness of annuities declines sharply with age
- Policy makers who want to promote annuitisation can ask individuals to make annuitisation decisions 10 years before retirement

tex.sx

# Sensitivity Analysis

## Major results from Sensitivity Analysis:

- Power discount rate sensitivity:
  - Retirees with a greater level of impatience are less likely to purchase annuity products
- Income level sensitivity:
  - Wealthy people who can afford an annuity with higher annual incomes are willing to pay a lower-than-market price, while poor people are willing to pay a much higher-than-market price for annuities.
  - The conclusion that longer-term deferred annuities are more attractive is robust for people with different levels of retirement savings.
- Mortality rate sensitivity:
  - People with longer life expectancies are more interested in purchasing annuity products

# Conclusions

- Time inconsistent preference is one of the behavioral obstacles that stop retirees from converting their DC account balances into annuities at retirement.
- Hyperbolic discounters tend to find deferred annuities, both WADA and RADA, attractive; and the attractiveness is increasing with the deferred period.
- To promote the purchase of annuities among retirees and release the burden from social benefit claiming, governments are advised to introduce a pre-commitment device asking people to make annuitisation decisions 10 years before retirement

# Sensitivity analysis of the Relative Price Difference ( $R$ ) in Scenario a and Scenario b

$R$	Age of first annuity payment				
	65	70	75	80	85
<b>Scenario a</b>					
HB baseline	-3.60%	-5.84%	-7.51%	-8.57%	-8.99%
HB sensitivity analysis					
Lower interest rate ( $r = 1\%$ )	-20.00%	-19.32%	-18.33%	-17.01%	-15.39%
Higher interest rate ( $r = 5\%$ )	13.59%	8.08%	3.48%	-0.11%	-2.66%
Less impatience ( $\beta = 0.15$ )	4.82%	1.64%	-0.97%	-2.95%	-4.28%
Greater impatience ( $\beta = 0.25$ )	-14.76%	-15.83%	-16.33%	-16.21%	-15.45%
Lower income level ( $\psi = 0.0721$ )	34.08%	30.30%	27.06%	24.29%	21.86%
Higher income level ( $\psi = 3$ )	-15.81%	-17.55%	-18.72%	-19.22%	-18.99%
Lighter mortality rates (S2PFL)	-1.94%	-4.57%	-6.65%	-8.11%	-8.89%
Greater mortality rates (SPML03)	-4.65%	-6.63%	-8.03%	-8.82%	-9.00%
<b>Scenario b</b>					
HB baseline	-3.60%	-3.50%	0.09%	5.15%	11.10%
HB sensitivity analysis					
Lower interest rate ( $r = 1\%$ )	-20.00%	-25.03%	-27.35%	-28.88%	-30.22%
Higher interest rate ( $r = 5\%$ )	13.59%	21.95%	35.74%	53.29%	74.58%
Less impatience ( $\beta = 0.15$ )	4.82%	7.22%	12.47%	19.15%	26.80%
Greater impatience ( $\beta = 0.25$ )	-14.76%	-17.54%	-15.94%	-12.83%	-8.87%
Lower income level ( $\psi = 0.0721$ )	34.08%	37.28%	42.40%	49.56%	58.06%
Higher income level ( $\psi = 3$ )	-15.81%	-16.71%	-13.61%	-9.25%	-4.11%
Lighter mortality rates (S2PFL)	-1.94%	-1.27%	2.61%	7.92%	14.15%
Greater mortality rates (SPML03)	-4.65%	-4.98%	-1.65%	3.17%	8.80%

# Sensitivity analysis of the Relative Price Difference ( $R$ ) in Scenario c and Scenario d

$R$	Age of decision making				
	25	35	45	55	65
<b>Scenario c</b>					
HB baseline	119.85%	70.90%	34.63%	8.88%	-3.60%
HB sensitivity analysis					
Lower interest rate ( $r = 1\%$ )	-16.72%	-21.24%	-24.52%	-25.72%	-20.00%
Higher interest rate ( $r = 5\%$ )	459.09%	258.56%	133.05%	55.51%	-13.59%
Less impatience ( $\beta = 0.15$ )	158.52%	99.11%	54.99%	23.17%	4.82%
Greater impatience ( $\beta = 0.25$ )	72.42%	35.90%	9.01%	-9.46%	-14.76%
Lower income level ( $\psi = 0.0721$ )	212.74%	143.11%	91.51%	54.89%	34.08%
Higher income level ( $\psi = 3$ )	89.74%	47.50%	16.20%	-6.02%	-15.81%
Lighter mortality rates (S2PFL)	127.53%	76.68%	38.95%	12.01%	-1.94%
Greater mortality rates (SPML03)	125.75%	75.34%	37.96%	11.29%	-4.65%
<b>Scenario d</b>					
HB baseline	4.32%	5.41%	6.52%	7.15%	-3.60%
HB sensitivity analysis					
Lower interest rate ( $r = 1\%$ )	-13.42%	-12.51%	-11.59%	-11.07%	-20.00%
Higher interest rate ( $r = 5\%$ )	22.93%	24.21%	25.52%	26.26%	13.59%
Less impatience ( $\beta = 0.15$ )	40.37%	39.12%	36.96%	32.21%	4.82%
Greater impatience ( $\beta = 0.25$ )	37.28%	35.26%	31.82%	24.40%	-14.76%
Lower income level ( $\psi = 0.0721$ )	45.31%	46.85%	48.44%	49.33%	34.08%
Higher income level ( $\psi = 3$ )	-8.97%	-8.03%	-7.07%	-6.53%	-15.81%
Lighter mortality rates (S2PFL)	7.65%	8.68%	9.69%	10.04%	-1.94%
Greater mortality rates (SPML03)	6.84%	7.89%	8.92%	9.34%	-4.65%

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