



## City Research Online

### City, University of London Institutional Repository

---

**Citation:** Zhang, Y., Sun, Z., van Zuijlen, A. & van Bussel, G. (2017). Numerical simulation of transitional flow on a wind turbine airfoil with RANS-based transition model. *Journal of Turbulence*, 18(9), pp. 879-898. doi: 10.1080/14685248.2017.1334908

This is the accepted version of the paper.

This version of the publication may differ from the final published version.

---

**Permanent repository link:** <https://openaccess.city.ac.uk/id/eprint/18490/>

**Link to published version:** <https://doi.org/10.1080/14685248.2017.1334908>

**Copyright:** City Research Online aims to make research outputs of City, University of London available to a wider audience. Copyright and Moral Rights remain with the author(s) and/or copyright holders. URLs from City Research Online may be freely distributed and linked to.

**Reuse:** Copies of full items can be used for personal research or study, educational, or not-for-profit purposes without prior permission or charge. Provided that the authors, title and full bibliographic details are credited, a hyperlink and/or URL is given for the original metadata page and the content is not changed in any way.

---

---

---

City Research Online:

<http://openaccess.city.ac.uk/>

[publications@city.ac.uk](mailto:publications@city.ac.uk)

---

1 Numerical simulation of transitional flow on a wind  
2 turbine airfoil with RANS-based transition model

3 Ye Zhang<sup>a</sup>, Zhengzhong Sun<sup>b,\*</sup>, Alexander van Zuijlen<sup>a</sup>, Gerard van Bussel<sup>a</sup>

4 <sup>a</sup>*DUWIND, Faculty of Aerospace Engineering, TUDelft, Kluyverweg 1, 2629HS, Delft,*  
5 *The Netherlands*

6 <sup>b</sup>*City University of London, Northampton Square, London, EC1V 0HB, UK*

---

7 **Abstract**

8 This paper presents a numerical investigation of transitional flow on the  
9 wind turbine airfoil DU91-W2-250 with chord-based Reynolds number  $Re_c =$   
10  $1.0 \times 10^6$ . The RANS-based transition model using laminar kinetic energy  
11 concept, namely the  $k - k_L - \omega$  model, is employed to resolve the boundary  
12 layer transition. Some ambiguities for this model are discussed and it is  
13 further implemented into OpenFOAM-2.1.1. The  $k - k_L - \omega$  model is first  
14 validated through the chosen wind turbine airfoil at the angle of attack (AoA)  
15 of  $6.24^\circ$  against wind tunnel measurement, where lift and drag coefficients,  
16 surface pressure distribution and transition location are compared. In order  
17 to reveal the transitional flow on the airfoil, the mean boundary layer profiles  
18 in three zones, namely the laminar, transitional and fully turbulent regimes,  
19 are investigated. Observation of flow at the transition location identifies the  
20 laminar separation bubble. The AoA effect on boundary layer transition over  
21 wind turbine airfoil is also studied. Increasing the AoA from  $-3^\circ$  to  $10^\circ$ , the  
22 laminar separation bubble moves upstream and reduces in size, which is in  
23 close agreement with wind tunnel measurement.

24 *Keywords:* Boundary layer transition, Laminar separation bubble, Wind  
25 turbine aerodynamics, CFD, RANS modeling, Laminar kinetic energy

---

26 **1. Introduction**

27 At present, wind turbines are being up-scaled towards 10-20 MW in off-  
28 shore wind farms. The power increase gives rise to larger rotor blades, which  
29 are apparently more costly and more flexible. Therefore, detailed flow in-  
30 vestigations over such large blades are needed to ensure operations. One

---

\*Corresponding author  
Preprint submitted to Journal of Turbulence  
Email address: Zhengzhong.Sun@city.ac.uk (Zhengzhong Sun)

31 particular phenomenon that plays a key role in blade performance is the  
32 laminar-turbulence transition (LTT). The LTT is not only crucial in aero-  
33 dynamic characteristics of wind turbine airfoil, but also in forming laminar  
34 separation bubble (LSB). The LSB is very sensitive to flow perturbation and  
35 it may burst during the blade rotation. Consequently, it could cause the  
36 double-stall phenomenon, which decreases the wind turbine performance sig-  
37 nificantly [1]. As a result, accurate LTT prediction is of great importance for  
38 the aerodynamic design and analysis of wind turbine blade, and it is aimed  
39 as the first objective in the present work.

40 Benefiting from the rapid development of flow simulation methodology,  
41 transition has been extensively investigated by Computational Fluid Dynam-  
42 ics (CFD) methods. The Direct Numerical Simulation (DNS) and the Large  
43 Eddy Simulation (LES) have delivered promising results in transition simu-  
44 lations [2][3]. However, the expensive computational hours due to high grid  
45 resolution and unsteady simulation are still deterring their widespread appli-  
46 cation. On the other hand, the Reynolds Averaged Navier-Stokes (RANS)-  
47 based turbulent flow modeling is still the workhorse in the aerodynamic re-  
48 lated simulations, as it is able to provide reasonably good results for attached  
49 flow and flow with minor separation under small or moderate requirements of  
50 computation resources. Therefore, it would be very useful to accurately pre-  
51 dict transitional flow using RANS models. One of the most widely adopted  
52 approaches [4] for transition prediction in general-purpose CFD methods is  
53 the concept of intermittency, which is used to blend together laminar and tur-  
54 bulent flow regimes. The transport equation of the intermittency factor  $\gamma$  is  
55 numerically solved to predict transition. The main drawback of this approach  
56 is that it needs non-local information, for example the integral thickness of  
57 the boundary layer and the state of flow beyond boundary layer [5]. The  
58 intermittency concept in transition prediction has been further improved by  
59 Menter et al [6] in order to eliminate the non-local information. An additional  
60 transport equation of the transition onset Reynolds number  $Re_{\theta t}$ , a function  
61 of the boundary layer momentum thickness, is formulated. This model shows  
62 very promising prediction for 2D and 3D configurations, but the empirical  
63 correlations used in this model are proprietary [7]. A complete review on  
64 RANS-based transition modeling can be found in several articles [5][8][9].  
65 The present introduction does not aim to provide a thorough review of all  
66 the relevant methods for transition simulation. Instead, emphasis is placed  
67 on the recently proposed RANS-based transition model using the laminar  
68 kinetic energy ( $k_L$ ) concept, namely the  $k - k_L - \omega$  transition model, which

69 enables transition modeling without any empirical input or pre-knowledge of  
70 the flow.

71 The concept of laminar kinetic energy in boundary layer transition was  
72 originally proposed by Mayle[10] to address the transition-induced aerody-  
73 namic and heat transfer problems in gas turbine engines. But, the original  
74 model containing  $k_L$  is not a single-point model and requires pre-knowledge  
75 of the flow field. The true single-point transition model using laminar kinetic  
76 energy was actually proposed later by Walters & Leylek [11], and it contains  
77 three transport equations for turbulent kinetic energy, laminar kinetic energy  
78 ( $k_L$ ) and turbulent dissipation ( $\epsilon$ ), namely the  $k - k_L - \epsilon$  transition model.  
79 The equation of turbulent dissipation was shortly replaced by that of specific  
80 dissipation rate ( $\omega$ ) by Walters & Leylek [12] and becomes the  $k - k_L - \omega$   
81 transition model. The  $k - k_L - \omega$  model was later improved by Walters &  
82 Cokljat [13] in order to include shear-sheltering concept as transition initia-  
83 tion. The Walters-Cokljat  $k - k_L - \omega$  model receives attention quickly and  
84 was validated with transitional flat plate test cases by Fürst [14], who states  
85 that there are some errors or probable typos for the  $k - k_L - \omega$  model in the  
86 original paper [13].

87 The Walters-Cokljat  $k - k_L - \omega$  model has been evaluated through several  
88 types of flow. In the flat plate transition cases, comparison was carried  
89 out against the ERCOFTAC T3 database [13][14][15], where several free-  
90 stream turbulence levels and pressure gradients are concerned. Since the  
91 model was originally proposed to address transition-induced heat transfer  
92 problem, transition in cascade was also validated in gas turbine applications  
93 [11][13][16][17][18]. Transition on the Aerospatiale airfoil is the third flow  
94 type for validation. Laminar separation bubble was claimed to be present  
95 at the transition location [19], however, no detailed analysis of transition  
96 process and the laminar separation bubble were provided. Therefore, the  
97 second objective of the present work is to perform detailed analysis of the  
98 transitional flow over the wind turbine airfoil.

99 Different from airfoils in gas turbine and aeronautical applications, wind  
100 turbine dedicated airfoils have distinctive features, such as much larger thick-  
101 ness in the inboard part of the blade. However, wind turbine airfoils have  
102 not been extensively simulated through this transition model. The transition  
103 cases that are publicly available are summarized in Table 1, where the free  
104 stream turbulence levels and the flow Reynolds numbers are also included.  
105 Figure 1 illustrates the range of turbulent intensity and Reynolds number for  
106 all the listed simulations. In the present paper, the investigation of transi-

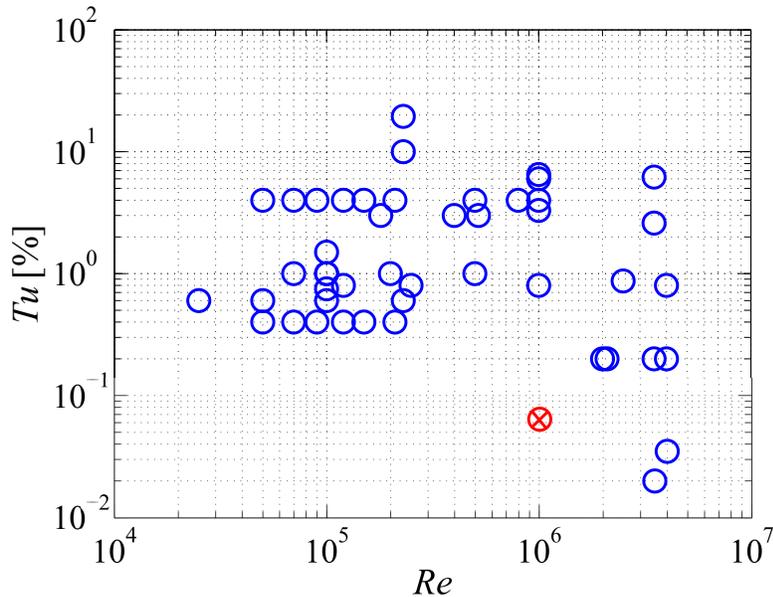


Figure 1: Turbulence intensity and  $Re$  number in the summarized transition simulations using  $k - k_L - \omega$  model

107 tional flow over wind turbine airfoil under the condition of  $Re_c = 1.0 \times 10^6$   
 108 and  $Tu = 0.06\%$  extends the current knowledge in this area.

109 To summarize, the present work envisages to carry out transition simu-  
 110 lation using the  $k - k_L - \omega$  model for the DU91-W2-250 wind turbine airfoil  
 111 with chord based Reynolds number of  $1.0 \times 10^6$ , and to investigate the lam-  
 112 inar separation bubble on airfoil surface and its response for different angles  
 113 of attack. The DU91-W2-250 airfoil is chosen because an extensive wind  
 114 tunnel measurement database is available, allowing comparison of surface  
 115 pressure distribution, coefficients of lift and drag and the transition loca-  
 116 tion. The open-source CFD package OpenFOAM is used as flow solver. The  
 117 paper is organized as following: the  $k - k_L - \omega$  transition model is first  
 118 briefly introduced, followed by the numerical aspects including flow domain  
 119 discretization and grid convergence study. In the results section, the airfoil  
 120 model is validated at AoA of  $6.24^\circ$ . The AoA is afterwards varied in the  
 121 range of  $-3^\circ \sim 10^\circ$  so as to reveal the change of laminar separation bub-  
 122 ble. Conclusions are finally drawn from the observations and analysis of the  
 123 resolved transition flow.

Table 1: Summary of boundary layer transition cases with  $k - k_L - \omega$  model addressed in the literature

Transition cases		$Tu$	$Re$	
Walters&Leylek (2004)[11]	ZPG flat plate	0.02%	3,500,000	
		0.2%	3,500,000	
		2.6%	2,000,000	
		6.2%	2,000,000	
	Turbine cascade	0.6%	230,000	
		10%	230,000	
19.5%		230,000		
Walters&Leylek (2005)[12]	Highly loaded compressor-like flat plate	1.2%		
		6.4%		
Walters&Cokljat (2008)[13]	ZPG flat plate ERCOFTAC T3A-	0.87%	2,500,000	
		T3A	3.3%	1,000,000
		T3B	6.5%	1,000,000
	ZPG flat plate ERCOFTAC T3C2	T3C2	3.0%	520,000
		T3C3	3.0%	400,000
		T3C4	3.0%	180,000
		T3C5	4.0%	800,000
		VPI cascade	10%	23,000
	19.5%		23,000	
	VKI cascade	0.8%	1,000,000	
		4.0%	1,000,000	
		6.0%	1,000,000	
		1.0%	500,000	
		4.0%	500,000	
A-airfoil $AoA = 13.3^\circ$	0.2%	2,000,000		
S809 airfoil 0-20 degree	0.2%	2,000,000		
Sanders et al. (2011)[16][17]	Lightly loaded turbine blade	0.75%	100,000	
		1%	100,000	
		1.5%	100,000	
	Highly loaded turbine blade	1%	100,000	
		0.6%	25,000	
		0.6%	50,000	
		0.6%	100,000	
Clare Turner (2012)[15]	ZPG flat plate			
	Valeo-CD airfoil	-*	160,000	
Furst (2013)[14]	ZPG flat plate ERCOFTAC T3A-	0.91%	3,000,000	
		T3A	3.3%	3,000,000
		T3B	9.43%	3,000,000
		T3C2	3.5%	2,000,000
Pacciani et al. (2011)[18]	T106C low speed	0.4%	50,000	
			70,000	
			90,000	
			120,000	
			150,000	
		210,000		
	T106C high speed	4%	50,000	
			70,000	
			90,000	
			120,000	
			150,000	
		210,000		
T108 high speed	0.8%	$1.2 \times 10^5$		
		$2.5 \times 10^5$		
Medina& Early (2014)[20]	Flat Plate	1%	$0.7 \times 10^5$	
			$2.0 \times 10^5$	
		0.035%	$4 \times 10^6$	
Accordi & de Lemos (2015)[19]	A-airfoil	0.8%	$4 \times 10^6$	
		0.2%	$4 \times 10^6$	

124 **2. Methodology**

125 *2.1. Laminar kinetic energy and effective turbulent length scale*

126

127 In the framework of  $k - k_L - \omega$  transition model, the streamwise velocity  
 128 fluctuation component  $u'$  accounts for nearly entire fluctuations of kinetic  
 129 energy in the laminar region. It is thus named the laminar kinetic energy  $k_L$   
 130 by Mayle and Schulz[10]. The growth of  $k_L$  is explained through the “splat  
 131 mechanism” by Volino[21], in which the negative wall-normal fluctuation  
 132 component  $v'$  in free stream eddies entrains high momentum fluid from the  
 133 outer region closer to the wall and this momentum transfer results in the  
 134 streamwise fluctuation component  $u'$ . The “splat mechanism” illustrated by  
 135 Walters & Leyelek [12] is shown in Figure 3. The turbulent energy spectrum  
 136 is divided into large scale eddies and small scale ones. The former initiates  
 137 “splat” and gives rise to laminar kinetic energy, whereas the latter generates  
 138 typical turbulence. In order to cut off the eddy size in the  $k - k_L - \omega$  transition  
 139 model, an effective turbulent length scale  $\lambda_{eff}$  is used.

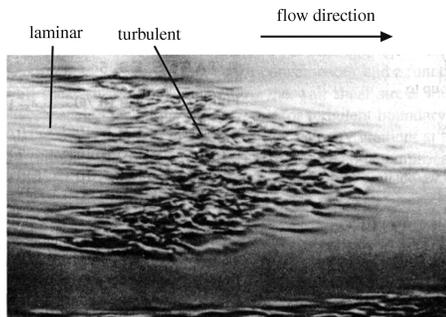


Figure 2: Laminar to turbulence transition over flat plate[22].

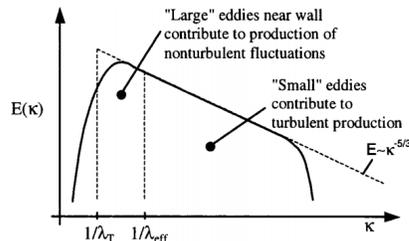


Figure 3: The “splat mechanism” for production of laminar kinetic energy[12].

140 *2.2. The  $k - k_L - \omega$  Transition Model*

141 The present  $k - k_L - \omega$  transition model is based on the low- $Re$   $k - \omega$  shear  
 142 stress transport (SST) eddy-viscosity model. Different from the other RANS-  
 143 based transition models, such as  $\gamma - Re_\theta - SST$ , the advantage of the present  
 144 model is the elimination of intermittency factor, which is a semi-empirical  
 145 parameter that bridges the pre-transitional and turbulent boundary layer and  
 146 enforces transition onset[11]. The  $k - k_L - \omega$  model is a three-equation model,

147 the transport equation of  $k_L$  is added to model the low frequency velocity  
 148 fluctuations. The transport equations for the turbulent kinetic energy  $k_T$ , the  
 149 laminar kinetic energy  $k_L$  and the specific dissipation rate  $\omega$  in incompressible  
 150 form are represented below:

$$\begin{aligned} \frac{Dk_T}{Dt} = & \underbrace{P_{k_T}}_{\text{production}} + \underbrace{R_{BP} + R_{NAT}}_{\text{bypass and natural transition}} - \underbrace{\omega k_T}_{\text{destruction}} - \underbrace{D_T}_{\text{anisotropic dissipation}} \\ & + \underbrace{\frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\alpha_T}{\sigma_k} \right) \frac{\partial k_T}{\partial x_j} \right]}_{\text{diffusion}} \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{Dk_L}{Dt} = & \underbrace{P_{k_L}}_{\text{production}} - \underbrace{R_{BP} - R_{NAT}}_{\text{bypass and natural transition}} - \underbrace{D_L}_{\text{anisotropic dissipation}} + \underbrace{\frac{\partial}{\partial x_j} \left( \nu \frac{\partial k_L}{\partial x_j} \right)}_{\text{diffusion}} \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{D\omega}{Dt} = & \underbrace{C_{\omega 1} \frac{\omega}{k_T} P_{k_T}}_{\text{production}} + \underbrace{\left( \frac{C_{\omega R}}{f_W} - 1 \right) \frac{\omega}{k_T} (R_{BP} + R_{NAT})}_{\text{bypass and natural transition}} - \underbrace{C_{\omega 2} f_W^2 \omega^2}_{\text{destruction}} \\ & + \underbrace{C_{\omega 3} f_{\omega} \alpha_T f_W^2 \frac{\sqrt{k_T}}{d^3}}_{\text{boundary layer wake correction}} + \underbrace{\frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\alpha_T}{\sigma_{\omega}} \right) \frac{\partial \omega}{\partial x_j} \right]}_{\text{diffusion}} \end{aligned} \quad (3)$$

151 Note that the turbulent kinetic energy  $k_T$  is produced by the small-scale  
 152 eddy and can be modeled through the main strain as  $P_{k_T} = \nu_{T,s} S^2$ , whereas  
 153 the laminar kinetic energy  $k_L$  is produced by  $P_{k_L} = \nu_{T,l} S^2$ , which is assumed  
 154 to be generated by large-scale near-wall fluctuations [11]. The small-scale  
 155 eddy viscosity  $\nu_{T,s}$  and the large-scale turbulence viscosity  $\nu_{T,l}$  are defined  
 156 as:

$$\nu_{T,s} = f_{\nu} f_{INT} C_{\mu} \sqrt{k_{T,s}} \lambda_{eff} \quad (4)$$

$$\nu_{T,l} = \min \left\{ f_{\tau,l} C_{11} \left( \frac{\Omega \lambda_{eff}^2}{\nu} \right) \sqrt{k_{T,l}} \lambda_{eff} + \beta_{TS} C_{12} Re_{\Omega} d^2 \Omega, \frac{0.5 * (k_L + k_{T,l})}{S} \right\} \quad (5)$$

In Equation 4, the effective small-scale turbulence is calculated by

$$k_{T,s} = f_{SS} f_W k_T \quad (6)$$

where  $f_W$  is the damping function which relates the effective turbulent length scale  $\lambda_{eff} = \min(C_\lambda d, \lambda_T)$  and turbulent length scale  $\lambda_T = \frac{\sqrt{k_T}}{\omega}$ .

$$f_W = \left( \frac{\lambda_{eff}}{\lambda_T} \right)^{\frac{2}{3}} \quad (7)$$

Note that the damping function used here includes the exponent 2/3, as suggested in paper [14] and [12].

The viscous wall effect is included in the  $f_\nu$  term, which is

$$f_\nu = 1 - \exp\left(-\frac{\sqrt{Re_T}}{A_\nu}\right) \quad (8)$$

where the effective turbulence Reynolds number is calculated by

$$Re_T = \frac{f_W^2 k_T}{\nu \omega} \quad (9)$$

In addition, the shear-sheltering effect [23] is included in the  $f_{SS}$  term:

$$f_{SS} = \exp\left[-\left(\frac{C_{SS}\nu\Omega}{k_T}\right)^2\right] \quad (10)$$

In order to satisfy the realizability constraint, the turbulence viscosity coefficient  $C_\mu$  is following Shih [24]:

$$C_\mu = \frac{1}{A_0 + A_s\left(\frac{S}{\omega}\right)} \quad (11)$$

In Equation 4 the term  $f_{SS}$  representing the intermittency effect on the turbulence production is

$$f_{INT} = \min\left(\frac{k_T}{C_{INT} k_{TOT}}, 1\right) \quad (12)$$

Note that the present expression is based on the corrected form by Fürst [14].

Regarding the large-scale turbulence viscosity in Equation 5, the relations are:

$$Re_\Omega = \frac{d^2\Omega}{\nu} \quad (13)$$

$$\beta_{TS} = 1 - \exp\left[-\frac{\max(Re_\Omega - C_{TS,crit}, 0)^2}{A_{TS}}\right] \quad (14)$$

$$f_{\tau,l} = 1 - \exp\left(-C_{\tau,l} \frac{k_{T,l}}{\lambda_{eff}^2 \Omega^2}\right) \quad (15)$$

The dissipation terms should balance the diffusion terms in the laminar sub-layer, which yields Equation 1 and 2:

$$D_T = 2\nu \frac{\partial\sqrt{k_T}}{\partial x_j} \frac{\partial\sqrt{k_T}}{\partial x_j} \quad (16)$$

$$D_L = 2\nu \frac{\partial\sqrt{k_L}}{\partial x_j} \frac{\partial\sqrt{k_L}}{\partial x_j} \quad (17)$$

The bypass transition term  $R_{BP}$  and natural transition term  $R_{NAT}$  in the transport equations are modeled as:

$$R_{BP} = C_R \beta_{BP} k_L \omega / f_W \quad (18)$$

$$R_{NAT} = C_{R,NAT} \beta_{NAT} k_L \Omega \quad (19)$$

where

$$\beta_{BP} = 1 - \exp\left(-\frac{\phi_{BP}}{A_{BP}}\right) \quad (20)$$

$$\phi_{BP} = \max\left[\left(\frac{k_T}{\nu\Omega} - C_{BP,crit}\right), 0\right] \quad (21)$$

$$\beta_{NAT} = 1 - \exp\left(-\frac{\phi_{NAT}}{A_{NAT}}\right) \quad (22)$$

$$\phi_{NAT} = \max\left[\left(Re_\Omega - \frac{C_{NAT,crit}}{f_{NAT,crit}}\right), 0\right] \quad (23)$$

$$f_{NAT,crit} = 1 - \exp\left(-C_{NC} \frac{\sqrt{k_L d}}{\nu}\right) \quad (24)$$

Table 2: The constants in the  $k - k_L - \omega$  transition model

$A_0 = 4.04$	$C_{INT} = 0.75$	$C_{\omega_1} = 0.44$	$A_s = 2.12$
$C_{TS,crit} = 1000$	$C_{\omega_2} = 0.92$	$A_\nu = 6.75$	$C_{R,NAT} = 0.02$
$C_{\omega_3} = 0.3$	$A_{BP} = 0.6$	$C_{11} = 3.4 \times 10^{-6}$	$C_{\omega R} = 1.5$
$A_{NAT} = 200$	$C_{12} = 1 \times 10^{-10}$	$C_\lambda = 2.495$	$A_{TS} = 200$
$C_R = 0.12$	$C_{\mu,std} = 0.09$	$C_{BP,crit} = 1.2$	$C_{NAT,crit} = 1250$
$C_{\tau,l} = 4360$	$C_{NC} = 0.1$	$C_{SS} = 1.5$	$\sigma_k = 1, \sigma_\omega = 1.17$

157 All the constants appeared in the model are summarized in Table 2. A  
 158 thorough description of their physical meanings is available from the original  
 159 paper[13] and they are also expressed in Table 3.

### 160 2.3. Case setup and grid independence study

161 The wind turbine airfoil of interest is the DU91-W2-250 with 25% $c$  thick-  
 162 ness. It is a widely used airfoil for the inboard part of commercial wind  
 163 turbine blades [25][26]. The airfoil has a blunt trailing edge with thickness  
 164 of 0.2% $c$ . Structured  $O$ -type grid is generated around the airfoil surface, see  
 165 Figure 4. The outer boundary of the simulation domain extends 100 chord  
 166 length from the airfoil’s aerodynamic centre ( $\frac{1}{4}c$ ) so as to minimize the far-  
 167 field boundary effect. The first wall-normal grid distance from the airfoil  
 168 surface is small enough to ensure the dimensionless wall distance  $y^+ < 1$ ,  
 169 such that the viscous sublayer of the turbulent boundary layer can be re-  
 170 solved. The requirement of  $y^+ < 1$  is essential in use of  $k - k_L - \omega$  model  
 171 [13]. A stretching ratio of 1.1 for near-wall grid is applied to smoothly in-  
 172 crease the size of the grid cells in the wall-normal direction. As transition  
 173 takes place across a very short distance, the number of nodes along airfoil  
 174 surface should be fine enough ( $\sim 0.003c$ ) to capture transition and to resolve  
 175 the laminar separation bubble.

176 The SIMPLE algorithm [27] is used to decouple the pressure and ve-  
 177 locity of the steady-state incompressible Navier-Stokes equations. Second-  
 178 order discretization scheme is chosen for both the convection and diffusion  
 179 terms. The total variation diminishing limited linear differencing schemes  
 180 with Sweby limiter are applied for velocity and turbulence quantities. All  
 181 the residuals converge to a magnitude less than  $10^{-4}$  after  $10^4$  iterations.  
 182 Meanwhile, the lift and drag coefficients also converge. The boundary con-

Table 3: Physical meanings of the quantities in the  $k - k_L - \omega$  transition model.

Name	Meaning
$D_L$	laminar kinetic energy dissipation
$D_T$	turbulent kinetic energy dissipation
$P_{k_L}$	laminar kinetic energy production
$P_{k_T}$	turbulent kinetic energy production
$R_{BP}$	bypass transition production
$R_{NAT}$	natural transition production
$Re_T$	turbulence Reynolds number
$Re_\Omega$	vorticity-based Reynolds number
$S$	magnitude of mean strain rate tensor
$\Omega$	magnitude of mean rotation rate tensor
$\alpha_T$	effective diffusivity for turbulent quantities
$\beta_{BP}$	bypass transition threshold function
$\beta_{NAT}$	natural transition threshold function
$\beta_{TS}$	Tollmien-Schlichting threshold function
$\lambda_T$	turbulent length scale
$\lambda_{eff}$	effective turbulent length scale
$\nu$	molecular kinematic viscosity
$\nu_{T,l}$	turbulent kinematic viscosity of large scale eddy
$\nu_{T,s}$	turbulent kinematic viscosity of small scale eddy
$\omega$	specific dissipation rate
$\phi_{BP}$	model bypass transition parameter
$\phi_{NAT}$	model natural transition parameter
$d$	wall distance
$f_W$	inviscid near-wall damping function
$f_\nu$	viscous damping function
$f_\omega$	boundary layer wake term damping function
$f_{INT}$	intermittency damping function
$f_{SS}$	shear-sheltering damping function
$f_{\tau,l}$	time-scale damping function
$k_T$	turbulent kinetic energy
$k_{T,l}$	effective "large-scale" turbulent kinetic energy
$k_{T,s}$	effective small-scale turbulent kinetic energy
$k_{TOT}$	total fluctuation kinetic energy, $k_T + k_L$

183 dition at the inlet is specified as Dirichlet-type condition with fixed value  
184 for the velocity and turbulent intensity, while Neumann boundary condition  
185 with zero gradient is set at the outlet boundary. A non-slip wall condition is  
186 applied at the airfoil surface. Free-stream turbulence is specified through the  
187 turbulence intensity  $Tu$  and its length scale  $l$ . In order to facilitate proper  
188 comparison with experiment, the choice of  $Tu$  follows that in the wind tunnel  
189 measurement carried out with  $Tu = 0.06\%$ . The turbulent length scale is  
190 estimated to be  $l = 1mm$ , corresponding to the  $1mm$  diameter of the wire  
191 mesh in the wind tunnel settling chamber. The inlet boundary condition  
192 including velocity and turbulent parameters is summarized in Table 4.

193 Four grid densities as listed in Table 5 are investigated to check grid  
194 independence as well as to examine the capability in transition identification

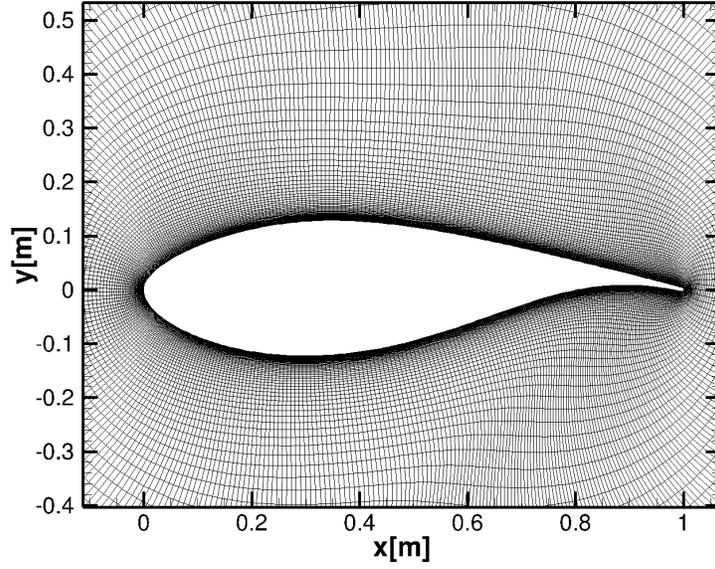


Figure 4: Grid around the DU91-W2-250 airfoil.

Table 4: Inlet boundary condition in the simulation.

Name	Quantity
$\alpha$	$6.24^\circ$
$Re_c$	$1.0 \times 10^6$
$k_T$	$1.152 \times 10^{-4} \text{ m}^2/\text{s}^2$
$\omega^1$	$10.73 \text{ s}^{-1}$
$Tu$	0.06%
$\nu_T/\nu$	0.73

195 at  $Re_c = 1.0 \times 10^6$  and  $AoA = 6.24^\circ$ . The maximum  $y^+$  along the airfoil  
 196 surface is also included in Table 5. The distributions of pressure coefficient  
 197 using the four grids are shown in Figure 5. It is apparent that transition,  
 198 which is represented by the kink in the  $C_p$  curve, is not captured by Grid  
 199 A and B. The  $C_p$  curves from Grid C and D overlap, thus grid independent  
 200 solution is obtained by Grid C. Since the 2D computation is not so expensive,  
 201 Grid D with node size of  $851 \times 387 \times 2$  is adopted for the present simulations.

Table 5: Grid configurations used in grid independence study

Case	Nodes distribution	$y^+$	Total cells
A	$151 \times 68 \times 2$	$<2$	20,536
B	$302 \times 137 \times 2$	$<1$	82,748
C	$602 \times 274 \times 2$	$<0.5$	329,896
D	$851 \times 387 \times 2$	$<0.3$	658,674

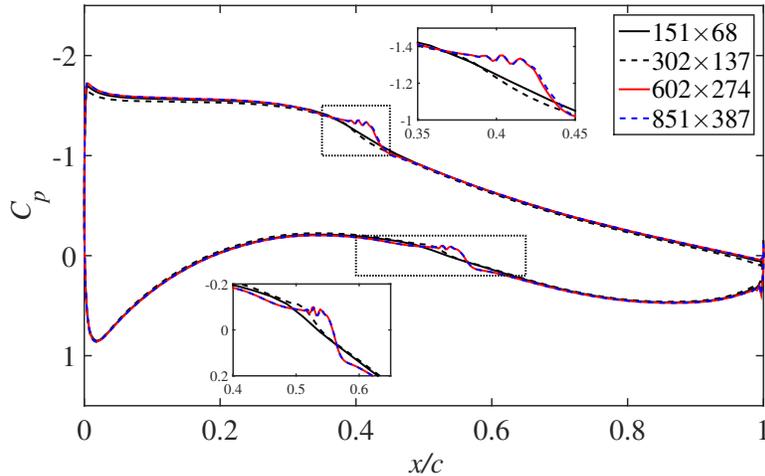


Figure 5: Mesh resolution study of the pressure coefficient  $C_p$

### 202 3. Results and discussions

203 In this section, simulation result at  $\alpha = 6.24^\circ$  is studied comprehensively.  
 204 The boundary layer transitions resulted from a range of AoAs are later inves-  
 205 tigated, aiming to reveal the effect from AoA. Finally, the effects of  $k - k_L - \omega$

206 transition model on the integral aerodynamic characteristics, including  $C_L$   
 207 and  $C_D$ , are discussed.

208 *3.1. Transition at  $\alpha = 6.24^\circ$  with  $Re = 1.0 \times 10^6$*

209 Flow validation is first performed for the case of AoA  $\alpha = 6.24^\circ$  with  
 210  $Re = 1.0 \times 10^6$  through lift and drag coefficients and pressure distribution.  
 211 The transition result is also analyzed in detail so as to reveal the transition  
 212 process resolved by the model and the role of laminar separation bubble in  
 213 transition.

214 *3.1.1. Comparison with experiment.*

Table 6: Comparison of  $C_l$  and  $C_d$  at  $Re = 1.0 \times 10^6$

	$k - k_L - \omega$	$k - \omega$ SST	Experiment
$C_l$	1.2362	1.1095	1.133
$C_d$	0.0146	0.0226	0.0121
Transition at upper surface ( $x/c$ )	0.36~0.42	-	0.43
Transition at lower surface ( $x/c$ )	0.48~0.56	-	0.53

215 The wind tunnel measurement database for the DU-W2-250 airfoil allows  
 216 comparison of surface pressure distribution, lift and drag coefficients, as well  
 217 as transition location. The pressure distributions along the upper and lower  
 218 surfaces are compared in Figure 6, where the result of  $k - \omega - SST$  model  
 219 is also included. Note that the simulation using  $k - \omega - SST$  model is  
 220 carried out with the same grid (Grid D). Both models exhibit reasonably  
 221 good performance in surface pressure prediction. Since the lift coefficient is  
 222 mainly determined by the pressure over airfoil,  $C_L$  for both models are within  
 223 10% difference, see Table 6.

224 The boundary layer transition is represented through the kink in the curve  
 225 of pressure distribution returned by  $k - k_L - \omega$  model at  $x/c \approx 0.4$  on the  
 226 suction side and  $x/c \approx 0.5$  on the pressure side. The pressure undulation  
 227 associated with transition is perhaps caused by the unsteady nature of the  
 228 laminar separation bubble, which will be discussed in Section 3.1.2. The  
 229 transition locations on the upper and lower surfaces at  $\alpha = 6.24^\circ$  are also  
 230 listed in Table 6. Note that the transition locations in present simulation

231 are represented through the streamwise extension of the laminar separation  
 232 bubble, which is the distance between the separation point of laminar bound-  
 233 ary layer and the reattachment point of turbulent boundary layer. It can be  
 234 found that the reattachment point agrees with the wind tunnel measure-  
 235 ment. In contrast, no such pressure kink is present in the pressure curves of  
 236  $k - \omega$  SST model, which simulates the fully turbulent boundary layer.

237 The drag coefficient  $C_D$  is more sensitive to laminar turbulence transi-  
 238 tion. Because the turbulent boundary layer produces larger friction than the  
 239 laminar boundary layer, failure in transition prediction will result in signifi-  
 240 cant discrepancy in  $C_D$ . Strikingly different  $C_f$  parameters are predicted by  
 241 the two models, see Figure 7. Because the  $k - \omega - SST$  model is not able  
 242 to model transition, larger  $c_f$  is predicted in the portion before transition on  
 243 both surfaces, resulting in a drag coefficient 86% larger than that in the wind  
 244 tunnel measurement. The  $k - k_L - \omega$  model apparently has better accuracy in  
 245  $C_D$ , only 20% larger. The drag coefficient for both models are also compared  
 246 in Table 6.

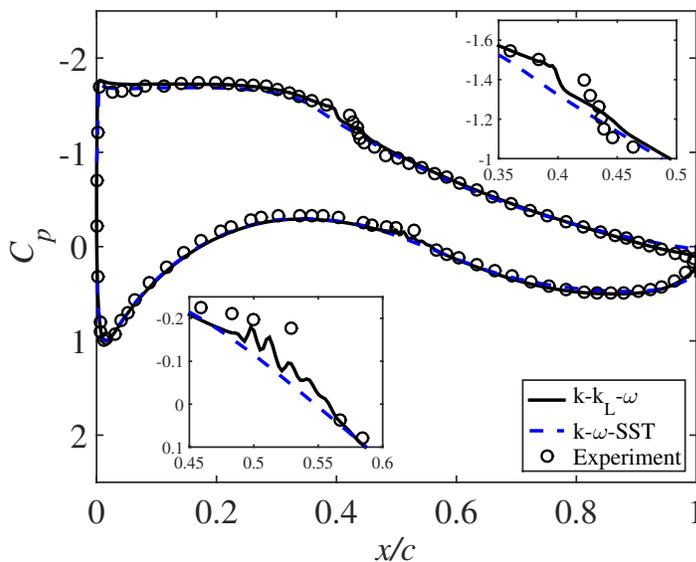


Figure 6: Pressure coefficient  $C_p$  distributions along airfoil surfaces

247 *3.1.2. Transition on the airfoil*

248 **The laminar separation bubble**

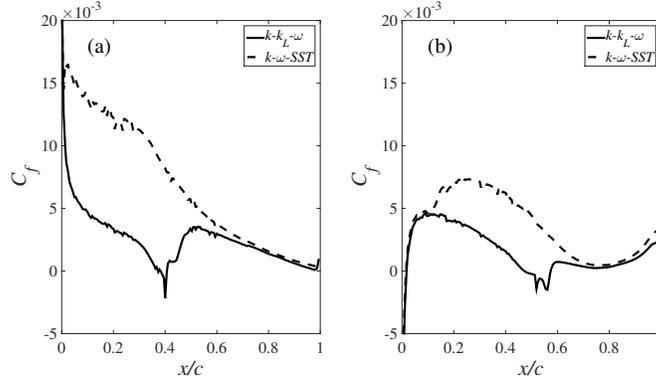


Figure 7: Skin friction coefficient  $c_f$  distributions along airfoil surface. Upper surface  $c_f$  (left), lower surface  $c_f$  (right)

249 The negative values of  $C_f$  inside the transition region in Figure 7 suggest  
 250 that flow recirculation takes place with boundary layer transition. Since  
 251 the result of the  $k - \omega - SST$  model is also included. The higher values  
 252 of  $C_f$  before transition again suggests that transition is not resolved by the  
 253  $k - \omega - SST$  model. The two transition regions containing separation bubbles  
 254 on the upper and lower surfaces are enlarged in Figure 8. Both separation  
 255 bubbles are in fact tiny in size. The one on the upper surface is centered at  
 256 about  $x/c = 0.39$  with length of  $0.06c$  and height less than  $0.001c$ , while the  
 257 other one on the lower surface is centered more downstream at  $0.51c$  with  
 258 longer length of  $0.08c$  and smaller height of  $0.0002c$ .

### 259 **Boundary layer evolution**

260 Visualization of the boundary layer evolution is useful in understanding  
 261 the transition process. Three typical boundary layer profiles in laminar,  
 262 transitional and turbulent stages on the upper surface are therefore plotted  
 263 respectively in Figure 9. Note that the velocity magnitude  $U_t$  in the profiles  
 264 is the tangential velocity component along the wall-normal direction. The  
 265 turbulent boundary layer profiles predicted by the  $k - \omega - SST$  model at  
 266 the same locations are also included and used as a reference of turbulent  
 267 boundary layer.

268 The boundary layer is of laminar type with thickness  $\delta_{kkl} = 1.87mm$  at  
 269  $x/c = 0.20$ , corresponding to a local Reynolds number  $Re_l = 240,000$ . The  
 270 local Reynolds number is defined as  $Re = \frac{U_t l}{\nu}$ , where  $l$  is the surface distance  
 271 between stagnation point and the local position. This profile is less full than  
 272 the turbulent one, whose thickness is  $\delta_{k\omega} = 4.05mm$ .

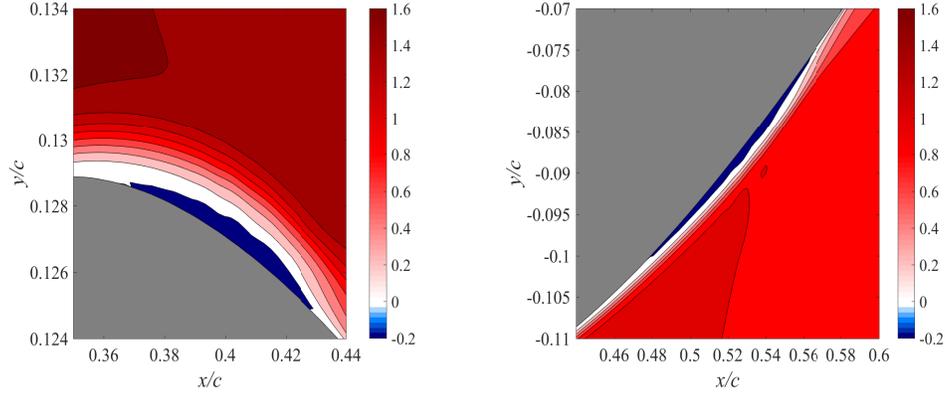


Figure 8: The contours of streamwise velocity component on the airfoil upper surface (left) and lower surface (right).

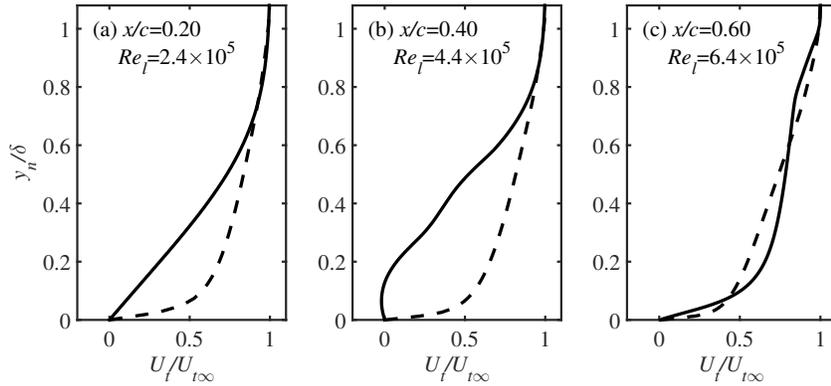


Figure 9: Boundary layer evolution along upper surface: (a) laminar; (b) transition; (c) turbulent.  $\delta$  is the boundary layer thickness, which is determined by using  $0.99U_{t\infty}$ . The solid profile (—) is the boundary layer from  $k - k_L - \omega$  model, the dashed profile (- -) is the boundary layer profile from  $k - \omega$  SST model

273 In the transition region at  $x/c = 0.40$  and  $Re_l = 440,000$ , velocity deficit  
 274 is present due to the presence of separation bubble at the immediate vicinity  
 275 of the wall. The boundary layer thickness is  $\delta_{kkl} = 3.35mm$  and  $\delta_{k\omega} =$   
 276  $7.85mm$  for the  $k - k_L - \omega$  and  $k - \omega$  SST models, respectively. Further  
 277 downstream at  $x/c = 0.60$  and  $Re_l = 640,000$ , a typical turbulent boundary  
 278 layer profile ( $\delta_{kkl} = 6.43mm$  and  $\delta_{k\omega} = 14.75mm$ ) is obtained. The laminar  
 279 and turbulent boundary layer profiles at  $x/c = 0.20$  and  $0.60$  respectively are  
 280 further compared in wall-unit, see Figure 10. The linear viscous sublayer at  
 281  $x/c = 0.2$  extends up to  $y^+ \sim 30$ , whereas, the turbulent profile has a log  
 282 portion between  $y^+ = 40 \sim 110$  and the viscous sublayer is also well resolved,  
 283 which extends till  $y^+ \sim 20$ .

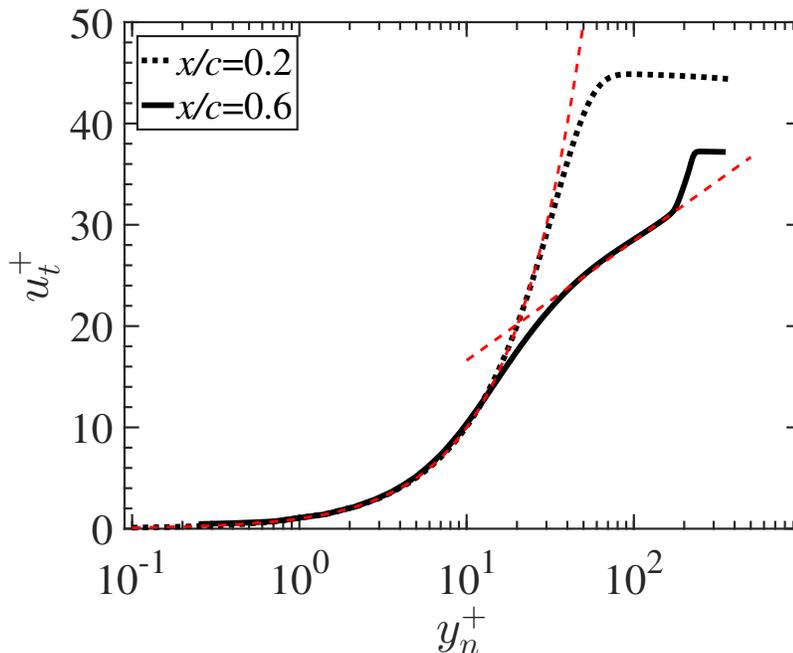


Figure 10: Laminar and turbulent boundary layers in wall unit on the upper surface predicted by  $k - k_L - \omega$  model.

### 284 3.1.3. Laminar kinetic energy and turbulent kinetic energy

285 The transition process is also featured with the evolution of laminar  
 286 kinetic energy and turbulent kinetic energy. According to the theory of  
 287  $k - k_L - \omega$  model,  $k_L$  dominates the laminar region, where  $k_T$  should be zero.

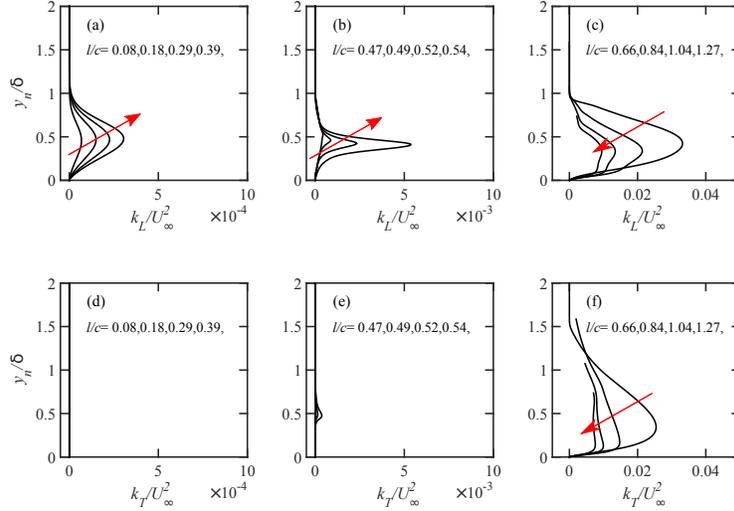


Figure 11: Evolution of laminar kinetic energy  $k_L$  and turbulent kinetic energy  $k_T$  on the upper surface in laminar region (a)(d), transition region (b)(e) and turbulent region (c)(f). The arrow indicates the increase of  $l/c$ , where  $l$  is the arc length along the upper surface.

288 Following the onset of transition,  $k_T$  starts to increase in the transitional  
 289 part, representing the generation of turbulence. Evolutions of  $k_L$  and  $k_T$   
 290 in the laminar, transitional and turbulent regions are shown in Figure 11. The  
 291 magnitude of  $k_L$  increases linearly in the laminar region while no  $k_T$  is present  
 292 in this part. In the transitional region (see Figure 11(b)),  $k_L$  is subject to  
 293 exponential growth, and  $k_T$  begins to appear, although its intensity is still  
 294 much smaller than  $k_L$ . In the turbulent region,  $k_L$  and  $k_T$  grow initially to  
 295 a maximum magnitude of  $0.035U_\infty^2$  and  $0.025U_\infty^2$  respectively. The intensity  
 296 burst for both is later followed by a decay close to the trailing edge, see Fig-  
 297 ure 11(c). The two quantities on the lower surface have similar evolution,  
 298 thus they are not shown here for conciseness.

### 299 3.2. Angle of attack effect on transition

300 In order to study the capability of  $k - k_L - \omega$  transition model to predict  
 301 the location of transitional laminar separation bubble for a range of angle  
 302 of attack. Five angles of attack ranging from  $-3^\circ$  to  $10^\circ$  are simulated.  
 303 These angles of attack are chosen in the linear regime because the RANS  
 304 simulation is known to predict accurate results. The transition locations

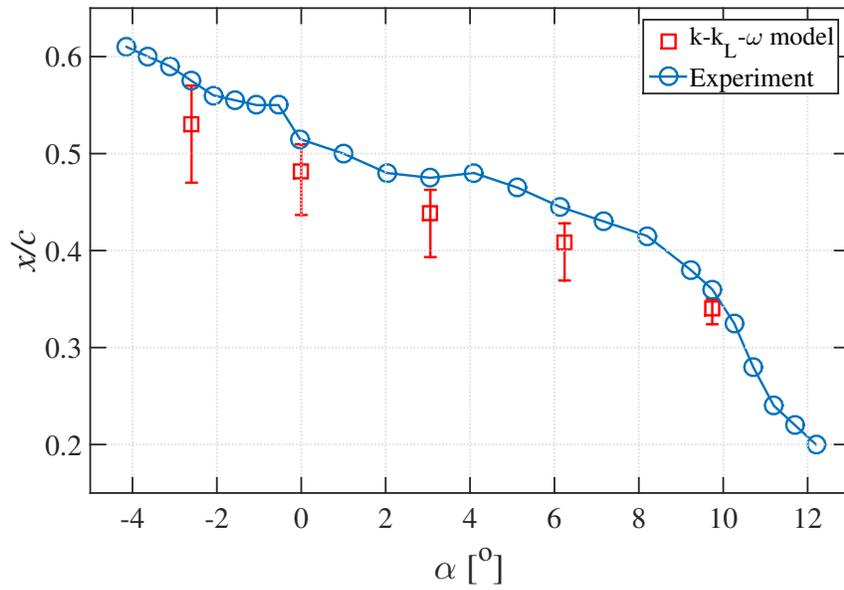
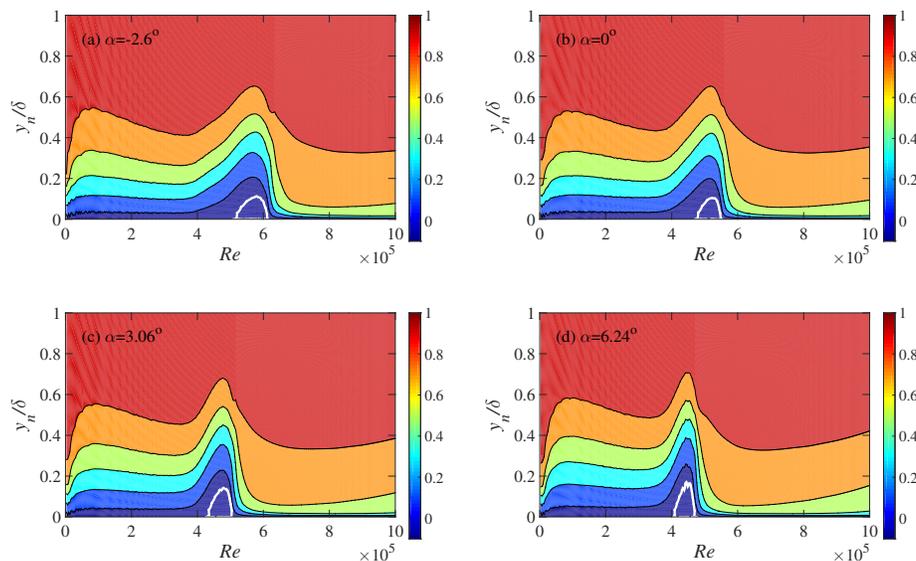


Figure 12: Comparison of transition location on the upper surface between  $k - k_L - \omega$  prediction and TU Delft wind tunnel measurement, the red square represents the center of the recirculating flow in the laminar separation bubble.

305 are first compared with experiment in Figure 12. The transition location  
 306 predicted by present simulations is again represented through the start and  
 307 end points of the laminar separation bubble.

308 The airfoil model for low turbulence wind tunnel measurement is of high  
 309 surface finish to ensure natural transition. According to the procedure of us-  
 310 ing microphone in the wind tunnel measurement for transition detection, the  
 311 transition location is based on the first location along airfoil where pressure  
 312 fluctuation intensity is amplified. In the present simulations, the end point of  
 313 the separation bubble is close to the measured transition location, although  
 314 the offset grows slightly when AoA is larger than  $3^\circ$ . Some of the behaviors  
 315 exhibited by the laminar separation bubble, such as the upstream motion  
 316 and the size reduction, can already be observed in Figure 12, but they will  
 317 be discussed in more detail through the boundary layer velocity contours and  
 318 evolution of boundary layer profiles.



[options]class

Figure 13: Contour of tangential velocity  $U_t/U_{t\infty}$  on the upper surface at different angles of attack,  $U_{t\infty}$  is the local “free stream” velocity.

319 The contours of tangential velocity  $U_t$  for  $\alpha = -2.6^\circ, 0^\circ, 3.06^\circ$  and  $6.24^\circ$   
 320 are shown in Figure 13. The laminar separation bubble is highlighted through  
 321 the dividing contour isoline with value  $U_t = 0$ . In order to reveal the size of

Table 7: Corresponding Reynolds number of the separation bubble

$AoA$	$Re_l$ at the starting point of separation bubble	$Re_l$ at the end point of separation bubble
$-2.6^\circ$	518,000	614,000
$0^\circ$	477,000	551,000
$3.06^\circ$	431,000	505,000
$6.24^\circ$	407,000	471,000
$9.74^\circ$	365,000	370,000

322 separation bubble relative to the boundary layer, the wall-normal distance  
 323 is scaled with the local boundary layer thickness. The separation bubble  
 324 exhibits slight growth in height:  $h = 0.1\delta$  at  $\alpha = -2.6^\circ$ , while  $h = 0.2\delta$  when  
 325  $\alpha = 6.24^\circ$ . The length of separation bubble becomes smaller, which means  
 326 turbulent boundary layer reattaches within a shorter distance when the angle  
 327 of attack is higher. The bubble length reduces abruptly when  $\alpha$  increases to  
 328  $9.74^\circ$ , suggesting a much shorter transition process at larger angle of attack.  
 329 Due to the tiny separation bubble at  $\alpha = 9.74^\circ$ , its contour plot is not shown.  
 330 The corresponding Reynolds number  $Re_l$  of the start and end points of the  
 331 separation bubble at the five angles of attack are summarized in Table 7.

332 The evolutions of boundary layer profile for the same angles of attack are  
 333 further visualized in Figure 14. This type of transition visualization provides  
 334 another perspective in addition to the contour plots. The laminar separation  
 335 bubble is highlighted through the connection of the points where tangential  
 336 velocity magnitude is zero. In the pre-transition region, all the boundary  
 337 layer profiles feature the typical laminar type and the velocity gradient in the  
 338 near wall region is relatively small, which explains the smaller  $C_f$ . Once the  
 339 separation bubble is produced, the transitional boundary layer deviates from  
 340 the upstream laminar profile and velocity deficit can be observed right above  
 341 the reversed flow. After a short recovery distance of about  $0.1c$ , the profile  
 342 in the post-transition boundary layers features typical turbulent boundary  
 343 layer.

### 344 3.3. Transition effects on airfoil polar

345 As shown in Section 3.1.1, the  $k - k_L - \omega$  transition model delivers good  
 346 results in predicting aerodynamic characteristics of the DU91-W2-250 airfoil  
 347 at  $\alpha = 6.24^\circ$ . Significant improvement of drag force prediction has been

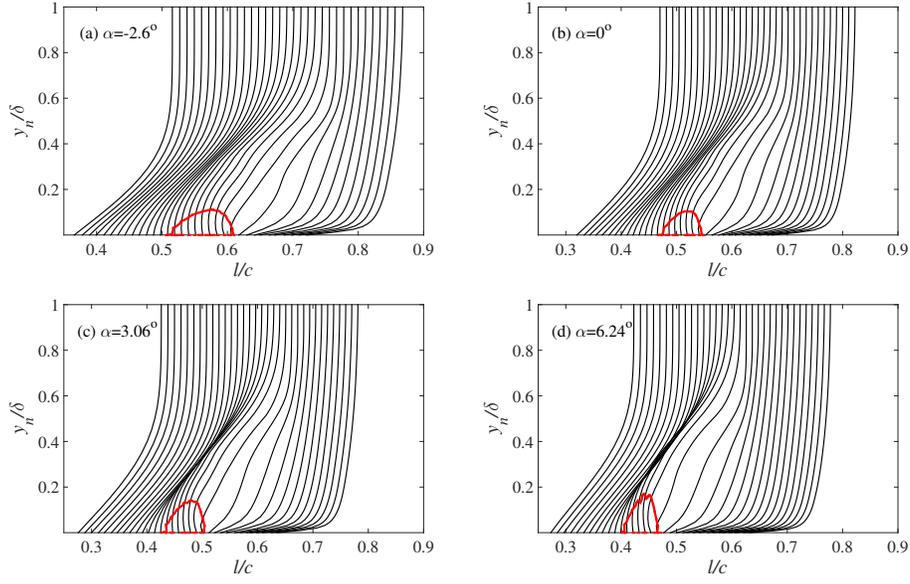


Figure 14: The evolution of boundary layers for different angles of attack. The solid line indicates the laminar separation bubble.

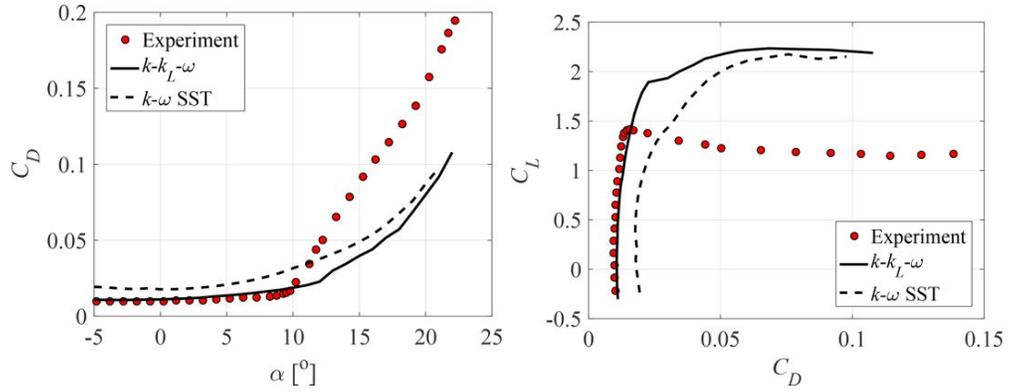


Figure 15: Transition effects on airfoil polars of  $C_D$  and  $C_L/C_D$

348 observed from  $k-k_L-\omega$  model in comparison to the  $k-\omega-SST$  model. The  
 349 performance of this transition model is further investigated and evaluated  
 350 by extending the AOA to a wider range, namely  $\alpha = -5^\circ \sim 23^\circ$ . Figure  
 351 15 presents the results of airfoil drag  $C_D$  and  $C_L/C_D$  polar. In the linear  
 352 regime, the drag force by the transition model  $k-k_L-\omega$  is in agreement  
 353 with the experiment, however notable over-prediction is found in the results  
 354 of the  $k-\omega-SST$  model. This observation is consistent with the results  
 355 in Section 3.1.1 for AoA =  $6.24^\circ$ , and it indicates that in the linear regime  
 356 CFD simulation with transition modeling is necessary in order to predict  $C_D$   
 357 and  $C_L/C_D$  accurately. When  $\alpha > 10^\circ$ , due to the large trailing edge flow  
 358 separation, both RANS models fail to offer good result. Delayed detached  
 359 eddy simulation (DDES) is recommended for such highly separated flow.

#### 360 4. Conclusions

361 The RANS-based three-equation  $k-k_L-\omega$  transition model has been  
 362 successfully applied to simulate the boundary layer transition on the DU91-  
 363 W2-250 wind turbine airfoil at a range of angles of attack. Validation was  
 364 performed for the case of AoA  $\alpha = 6.24^\circ$ . Comparison with wind tunnel mea-  
 365 surement demonstrates its accuracy in predicting transition and other quan-  
 366 tities including pressure distribution, lift and drag coefficients. Detailed anal-  
 367 ysis of boundary layer transition at  $\alpha = 6.24^\circ$  shows the laminar separation  
 368 bubble on both airfoil surfaces, which is closely associated with transition.  
 369 The evolution of boundary layer across transition is studied by evaluating the  
 370 velocity profiles at three typical stages: laminar boundary layer, transitional  
 371 boundary layer and fully turbulent boundary layer. The variation of  $k_L$  and  
 372  $k_T$  across transition are also analyzed. Investigation on the flow field at a  
 373 range of angles of attack clearly indicates that transition moves upstream  
 374 with the increase of AoA. Regarding the accurate predictions of  $C_D$  and  
 375  $C_L/C_D$  for DU91-W2-250 airfoil in the linear regime ( $-3^\circ < \text{AoA} < 10^\circ$ ),  
 376 transition model is required and recommended in RANS simulation. This  
 377 model is inaccurate when large trailing edge separation occurs at  $\text{AoA} > 10^\circ$ .  
 378 More advanced modeling methodology, such as DDES, is recommended for  
 379 flow with massive separation.

#### 380 Acknowledgment

381 The authors would like to acknowledge Ir.Nando Timmers from TU Delft  
 382 for the discussion on the experiment. Meanwhile, the financial support of

383 China Scholarship Council is also gratefully acknowledged. Finally, we ac-  
384 knowledge the OpenFOAM community and code developers.

## 385 **References**

- 386 [1] P. Devinant, T. Laverne, J. Hureau, Experimental study of wind-turbine  
387 airfoil aerodynamics in high turbulence, *Journal of Wind Engineering*  
388 and *Industrial Aerodynamics* 90 (6) (2002) 689–707.
- 389 [2] A. Monokrousos, L. Brandt, P. Schlatter, D. S. Henningson, Dns and  
390 les of estimation and control of transition in boundary layers subject to  
391 free-stream turbulence, *International Journal of Heat and Fluid Flow*  
392 29 (3) (2008) 841–855.
- 393 [3] S. Lardeau, M. Leschziner, T. Zaki, Large eddy simulation of transitional  
394 separated flow over a flat plate and a compressor blade, *Flow, turbulence*  
395 and *combustion* 88 (1-2) (2012) 19–44.
- 396 [4] R. Langtry, F. Menter, Transition modeling for general cfd applications  
397 in aeronautics, *AIAA paper 522 (2005)* (2005) 14.
- 398 [5] D. Di Pasquale, A. Rona, S. Garrett, A selective review of cfd transition  
399 models, *AIIA Paper (2009-3812)*.
- 400 [6] F. Menter, T. Esch, S. Kubacki, Transition modelling based on local  
401 variables, in: *5th International Symposium on Turbulence Modeling*  
402 and *Measurements*, Mallorca, Spain, 2002.
- 403 [7] G. Cheng, R. Nichols, K. D. Neroorkar, P. G. Radhamony, Valida-  
404 tion and assessment of turbulence transition models, in: *47th AIAA*  
405 *Aerospace Sciences Meeting and Exhibit*, Orlando, FL, No. AIAA-2009-  
406 1141, 2009.
- 407 [8] R. B. Langtry, A correlation-based transition model using local variables  
408 for unstructured parallelized cfd codes, Ph.D. thesis, Univ. of Stuttgart  
409 (2006).
- 410 [9] T. Praisner, J. Clark, Predicting transition in turbomachinery part i: A  
411 review and new model development, *Journal of Turbomachinery* 129 (1)  
412 (2007) 1–13.

- 413 [10] R. Mayle, A. Schulz, The path to predicting bypass transition, in:  
414 ASME 1996 International Gas Turbine and Aeroengine Congress and  
415 Exhibition, American Society of Mechanical Engineers, 1996, pp.  
416 V001T01A065–V001T01A065.
- 417 [11] D. K. Walters, J. H. Leylek, A new model for boundary-layer transi-  
418 tion using a single-point rans approach, in: ASME 2002 International  
419 Mechanical Engineering Congress and Exposition, American Society of  
420 Mechanical Engineers, 2002, pp. 67–79.
- 421 [12] D. K. Walters, J. H. Leylek, Computational fluid dynamics study of  
422 wake-induced transition on a compressor-like flat plate, *Journal of Tur-  
423 bomachinery* 127 (1) (2005) 52–63.
- 424 [13] D. K. Walters, D. Cokljat, A three-equation eddy-viscosity model for  
425 reynolds-averaged navier–stokes simulations of transitional flow, *Journal  
426 of Fluids Engineering* 130 (12) (2008) 121401.
- 427 [14] J. Fürst, J. Příhoda, P. Straka, Numerical simulation of transitional  
428 flows, *Computing* 95 (1) (2013) 163–182.
- 429 [15] C. Turner, Laminar kinetic energy modelling for improved laminar-  
430 turbulent transition prediction, Ph.D. thesis, The University of Manch-  
431 ester (2012).
- 432 [16] D. D. Sanders, W. F. OBrien, R. Sondergaard, M. D. Polanka, D. C.  
433 Rabe, Predicting separation and transitional flow in turbine blades at  
434 low reynolds numberspart i: Development of prediction methodology,  
435 *Journal of Turbomachinery* 133 (3) (2011) 031011.
- 436 [17] D. D. Sanders, W. F. OBrien, R. Sondergaard, M. D. Polanka, D. C.  
437 Rabe, Predicting separation and transitional flow in turbine blades at  
438 low reynolds numberspart ii: the application to a highly separated tur-  
439 bine blade cascade geometry, *Journal of Turbomachinery* 133 (3) (2011)  
440 031012.
- 441 [18] R. Pacciani, M. Marconcini, A. Fadai-Ghotbi, S. Lardeau, M. A.  
442 Leschziner, Calculation of high-lift cascades in low pressure turbine con-  
443 ditions using a three-equation model, *Journal of Turbomachinery* 133 (3)  
444 (2011) 031016.

- 445 [19] I. A. Accordi, M. J. de Lemos, Single-point transition modeling using  
446 the laminar kinetic energy concept, *International Journal of Heat and*  
447 *Mass Transfer* 89 (2015) 1095–1109.
- 448 [20] H. Medina, J. Early, Modelling transition due to backward-facing  
449 steps using the laminar kinetic energy concept, *European Journal of*  
450 *Mechanics-B/Fluids* 44 (2014) 60–68.
- 451 [21] R. J. Volino, A new model for free-stream turbulence effects on bound-  
452 ary layers, in: *ASME 1997 International Gas Turbine and Aeroengine*  
453 *Congress and Exhibition*, American Society of Mechanical Engineers,  
454 1997, pp. V003T09A015–V003T09A015.
- 455 [22] B. R. Munson, D. F. Young, T. H. Okiishi, *Fundamentals of fluid me-*  
456 *chanics*.
- 457 [23] R. G. Jacobs, P. A. Durbin, Shear sheltering and the continuous spec-  
458 trum of the orr–sommerfeld equation, *Physics of Fluids (1994-present)*  
459 10 (8) (1998) 2006–2011.
- 460 [24] T.-H. Shih, W. W. Liou, A. Shabbir, Z. Yang, J. Zhu, A new k- eddy  
461 viscosity model for high reynolds number turbulent flows, *Computers &*  
462 *Fluids* 24 (3) (1995) 227–238.
- 463 [25] W. Timmer, R. Van Rooij, Summary of the delft university wind turbine  
464 dedicated airfoils, in: *ASME 2003 Wind Energy Symposium*, American  
465 Society of Mechanical Engineers, 2003, pp. 11–21.
- 466 [26] R. Van Rooij, W. Timmer, Roughness sensitivity considerations for thick  
467 rotor blade airfoils, in: *ASME 2003 Wind Energy Symposium*, American  
468 Society of Mechanical Engineers, 2003, pp. 22–31.
- 469 [27] S. V. Patankar, D. B. Spalding, A calculation procedure for heat, mass  
470 and momentum transfer in three-dimensional parabolic flows, *Internation-*  
471 *al Journal of Heat and Mass Transfer* 15 (10) (1972) 1787–1806.