ON BLOCKS OF STRONGLY P-SOLVABLE GROUPS

Radha Kessar, Markus Linckelmann

Abstract. We prove that a block of a finite strongly p-solvable group G with defect group P is Morita equivalent to its corresponding block of $N_G(Z(J(P)))$ via a bimodule with endo-permutation source.

Let p be a prime. Following [8, Ch. 6, §5] a finite group G is called *strongly p-solvable* if G is p-solvable and either $p \geq 5$ or $p = 3$ and $Sl_2(3)$ is not involved in G. If G is a strongly p-solvable finite group and P a Sylow-p-subgroup of G, then G is p-constrained and p-stable (cf. [8, Ch. 8, §1]), and hence $G = N_G(Z(J(P)))O_{p'}(G)$ by a theorem of Glauberman (cf. [8, Ch. 8, Theorem 2.11]). The purpose of this paper is to show that this theorem, which is used in the proof of the p-nilpotency theorem of Glauberman and Thompson, has a generalisation to blocks of strongly p-solvable groups. We denote by $\mathcal{O}$ a complete discrete valuation ring having an algebraically closed residue field $k = \mathcal{O}/J(\mathcal{O})$ of characteristic p. We state our Theorem and explain the notation below.

Theorem. *Let G be a strongly p-solvable finite group, let b be a block of $\mathcal{O}G$ and let P a defect group of b. Set $N = N_G(Z(J(P)))$ and denote by c the Brauer correspondent of b in $\mathcal{O}N$. Then the Brauer categories $\mathcal{F}_{G,b}$ and $\mathcal{F}_{N,c}$ are equal and there is an indecomposable $\mathcal{O}Gb$-$\mathcal{O}Nc$-bimodule M with the following properties.*

(i) As $\mathcal{O}(G \times N)$-module, M has vertex $\Delta P = \{(u,u)\}_{u \in P}$ and an endo-permutation $\mathcal{O}\Delta P$-module W as source.

(ii) The bimodule M and its $\mathcal{O}$-dual M^* induce a Morita equivalence between the block algebras $\mathcal{O}Gb$ and $\mathcal{O}Nc$.

The structure of the source algebras of blocks of p-solvable finite groups has been completely determined by Puig. Once the stated equality of Brauer categories is established (using Glauberman's aforementioned theorem), the rest the proof of the Theorem consists of showing that the reduction techniques used in Puig's work "commute" with taking normalisers of $Z(J(P))$.

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With the notation of the Theorem, the block c of $\mathcal{O}N$ is the unique block satisfying $\mathrm{Br}_P(c) = \mathrm{Br}_P(b)$; this makes sense as $C_G(P) \subseteq N$. Fix a block e_P of $kC_G(P)$ such that $\mathrm{Br}_P(b)e_P = e_P$; that is, (P, e_P) is a maximal b-Brauer pair and a maximal c-Brauer pair.

By [1], for any subgroup Q of P there is a unique block e_Q of $kC_G(Q)$ such that $(Q, e_Q) \subseteq (P, e_P)$. Denote by $\mathcal{F}_{G,b}$ the category whose objects are the subgroups of P and whose set of morphisms from a subgroup Q of P to another subgroup R of P is the set of group homomorphisms $\varphi : Q \to R$ for which there exists an element $x \in G$ satisfying $\varphi(u) = xux^{-1}$ for all $u \in Q$ and ${}^x(Q, e_Q) \subseteq (R, e_R)$. Thus the automorphism group of a subgroup Q of P as object of the category $\mathcal{F}_{G,b}$ is canonically isomorphic to $N_G(Q, e_Q)/C_G(Q)$. By Alperin's fusion theorem, the category $\mathcal{F}_{G,b}$ is completely determined by the structure of P and the groups $N_G(Q, e_Q)/C_G(Q)$ where either $Q = P$ or (Q, e_Q) is an essential b-Brauer pair (cf. [18, §48]). Note that $O_p(G) \subseteq Q$ whenever the pair (Q, e_Q) is essential. The category $\mathcal{F}_{N,c}$ is defined similarly. Since $O_p(N)$ contains $Z(J(P))$ and $C_N(Q) = C_G(Q)$ for any subgroup Q of P containing $Z(J(P))$ we have an inclusion of categories $\mathcal{F}_{N,c} \subseteq \mathcal{F}_{G,b}$.

If b is the principal block of $\mathcal{O}G$, then e_Q is the principal block of $kC_G(Q)$, for any subgroup Q of P. Thus the above condition ${}^x(Q, e_Q) \subseteq (R, e_R)$ is equivalent to ${}^xQ \subseteq R$. Therefore, if b is the principal block of $\mathcal{O}G$, we write $\mathcal{F}_G$ instead of $\mathcal{F}_{G,b}$.

In general, the definition of $\mathcal{F}_{G,b}$ depends on the choice of a maximal b-Brauer pair, but since all maximal b-Brauer pairs are G-conjugate, it is easy to see that $\mathcal{F}_{G,b}$ is unique up to isomorphism of categories. Note that we always have $\mathcal{F}_P \subseteq \mathcal{F}_{G,b}$.

Let P be a finite p-group. An $\mathcal{O}$-free $\mathcal{O}P$-module U is called an *endo-permutation module* if $\mathrm{End}_{\mathcal{O}}(U)$ has a P-stable $\mathcal{O}$-basis with respect to the action of P by conjugation. This concept is due to Dade [5, 6]. It is shown in [5], that an endo-permutation $\mathcal{O}P$-module U has at most one isomorphism class of indecomposable direct summands of vertex P. Since the restriction to any subgroup Q of P of an endo-permutation $\mathcal{O}P$-module U is an endo-permutation $\mathcal{O}Q$-module, it follows that if U has an indecomposable direct summand with vertex P, then $\mathrm{Res}^P_Q(U)$ has exactly one isomorphism class of indecomposable direct summands with vertex Q for any subgroup Q of P. We will say that U is *G-stable* if the indecomposable summands with vertex Q of $\mathrm{Res}^P_Q(U)$ are isomorphic to those of $\mathrm{Res}_\varphi(U)$ for any subgroup Q of P and any group homomorphism $\varphi : Q \to P$ for which there is an element $x \in G$ satisfying $\varphi(u) = xux^{-1}$ for all $u \in Q$. Here $\mathrm{Res}_\varphi(U)$ is the endo-permutation $\mathcal{O}Q$-module obtained from restricting U via φ; that is, with $u \in Q$ acting as $\varphi(u)$ on U. In the terminology of [17] this is the same as saying that the interior P-algebra $\mathrm{End}_{\mathcal{O}}(U)$ contains G-fusion. We state without proof two obvious Lemmas needed in the proof of the Theorem.

Lemma 1. *Let G, H be finite groups and let P be a p-subgroup of both G and H. Let U, V be endo-permutation $\mathcal{O}P$-modules having direct summands of vertex P.*

(i) If U is G-stable, then $\mathrm{Res}^P_Q(U)$ is G-stable for any subgroup Q of P.

(ii) If U is G-stable and V is H-stable, then the $\mathcal{O}(P \times P)$ -module $U \underset{\mathcal{O}}{\otimes} V$ is $G \times H$-stable.

Lemma 2. *Let G be a finite group and let P be a p-subgroup of G. Let U be an endo-permutation $\mathcal{O}P$-module having an indecomposable direct summand V of vertex P. If U is G-stable then V is G-stable.*

We quote the following well-known result on the local structure of certain blocks of p-constrained groups; recall that a finite group G is *p-constrained* if $C_G(Q) \subseteq O_{p',p}(G)$, where Q is a Sylow-p-subgroup of $O_{p',p}(G)$.

Proposition 3. *([9, 3.1, 3.4]) Let G be a finite p-constrained group and let b be a G-stable block of $\mathcal{O}O_{p'}(G)$. Then b is still a block of $\mathcal{O}G$, any Sylow-p-subgroup P of G is a defect group of b as block of $\mathcal{O}G$, and for any p-subgroup Q of G, the central idempotent $\mathrm{Br}_Q(b)$ is a block of $kC_G(Q)$.*

The last statement in Proposition 3 means that b is of principal type, which in turn implies that $\mathcal{F}_{G,b} = \mathcal{F}_G$. We will need Puig's algebra theoretic formulation of Fong's reduction [7], as outlined in [14].

Proposition 4. *(cf. [9, 3.1(i)]) Let G be a finite p-solvable group and let b be a block of $\mathcal{O}G$. There is a subgroup H of G containing $O_{p'}(G)$ and an H-stable block e of $\mathcal{O}O_{p'}(H)$ such that*

$$\mathcal{O}Gb \cong \mathrm{Ind}_H^G(\mathcal{O}He)$$

as interior G-algebras. Moreover, every Sylow-p-subgroup of H is then a defect group of e and b as blocks of $\mathcal{O}H$ and $\mathcal{O}G$, respectively.

Proposition 5. *(cf. [10, 4.4]) Let G be a finite p-solvable group, let b be a G-stable block of $\mathcal{O}O_{p'}(G)$ and set $S = \mathcal{O}O_{p'}(G)b$. Denote by $\hat{G}$ the $\mathcal{O}^\times$-group opposite to that defined by the action of G on S, and set $\hat{L} = \hat{G}/N$, where any element y of N is identified to its canonical image (y, yb) in $\hat{G}$. There is a unique algebra isomorphism*

$$\mathcal{O}Gb \cong S \underset{\mathcal{O}}{\otimes} \mathcal{O}_*\hat{L}$$

mapping xb to $s \otimes \overline{(x,s)}$, where $x \in G$ and $s \in S^\times$ such that $(x,s) \in \hat{G}$ and where $\overline{(x,s)}$ is the canonical image of (x,s) in $\hat{L}$.

We show next, that this reduction is compatible with taking normalisers of $Z(J(P))$.

Proposition 6. *Let G be a finite p-solvable group, let b be a block of $\mathcal{O}G$, let H be a subgroup of G containing $O_{p'}(G)$ and let e be an H-stable block of $\mathcal{O}O_{p'}(H)$ such that $\mathcal{O}Gb \cong \mathrm{Ind}_H^G(\mathcal{O}He)$. Let P be a Sylow-p-subgroup of H and let Z be a subgroup of P such that $N_G(P)$ is contained in $N_G(Z)$. Denote by c and f the blocks of $\mathcal{O}N_G(Z)$ and $\mathcal{O}N_H(Z)$ which are the Brauer correspondents of b and e, respectively. Then f is an $N_H(Z)$-stable block of $\mathcal{O}O_{p'}(N_H(Z))$ and we have an isomorphism of interior $N_G(Z)$-algebras*

$$\mathcal{O}N_G(Z)c \cong \mathrm{Ind}_{N_H(Z)}^{N_G(Z)}(\mathcal{O}N_H(Z)f)\ .$$

Proof. By [10, 4.2], we have $O_{p'}(N_H(Z)) = O_{p'}(C_H(Z)) = O_{p'}(H) \cap C_H(Z)$. By Proposition 3, $\mathrm{Br}_Z(e)$ is a block of $kC_H(Z)$. Since e is H-stable and every block of $kN_H(Z)$ is contained in $kC_H(Z)$, it follows that $\mathrm{Br}_Z(e)$ is a block of $kN_H(Z)$. As $e \in \mathcal{O}O_{p'}(H)$, it follows that $\mathrm{Br}_Z(e) \in kO_{p'}(N_H(Z))$. Thus $\mathrm{Br}_Z(e)$ lifts to a unique $N_H(Z)$-stable block f of $\mathcal{O}O_{p'}(N_H(Z))$, which is the Brauer correspondent of e. If $x \in N_G(Z) - N_H(Z)$, then ${}^x\mathrm{Br}_Z(e)\mathrm{Br}_Z(e) = \mathrm{Br}_Z({}^xe \cdot e) = 0$, because ${}^xe \cdot e = 0$ as $\mathcal{O}Gb \cong \mathrm{Ind}_H^G(\mathcal{O}He)$. Thus ${}^xf \cdot f = 0$. Therefore, $\mathrm{Ind}_{N_H(Z)}^{N_G(Z)}(\mathcal{O}N_H(Z)f) \cong \mathcal{O}N_G(Z)\mathrm{Tr}_{N_H(Z)}^{N_G(Z)}(f)$; in particular, the algebras $\mathcal{O}N_H(Z)f$ and $\mathcal{O}N_G(Z)\mathrm{Tr}_{N_H(Z)}^{N_G(Z)}(f)$ have isomorphic centers. As f is primitive in $Z(\mathcal{O}N_H(Z))$, it follows that $\mathrm{Tr}_{N_H(Z)}^{N_G(Z)}(f)$ is primitive in $Z(\mathcal{O}N_G(Z))$. But then $c = \mathrm{Tr}_{N_H(Z)}^{N_G(Z)}(f)$, which implies the result. □

Proof of the Theorem. By Proposition 4, there is a subgroup H of G containing $O_{p'}(G)$ and an H-stable block e of $\mathcal{O}O_{p'}(H)$ such that $\mathcal{O}Gb \cong \mathrm{Ind}_H^G(\mathcal{O}He)$. By general properties of algebra induction (cf. [15]), the blocks $\mathcal{O}Gb$ and $\mathcal{O}He$ have isomorphic source algebras. In view of Proposition 6, we may therefore assume that $G = H$ and $b = e$, and then c is an N-stable block of $\mathcal{O}O_{p'}(N)$. Set $S = \mathcal{O}O_{p'}(G)b$. Then S is a matrix algebra of rank n^2 prime to p, on which G acts. We consider the subgroup

$$\hat{G} = \{(x, s) | x \in G, s \in S^\times \ \text{ such that } \ {}^xt = sts^{-1} \ \text{ for all } \ t \in S\}$$

of $G \times S^\times$, endowed with the group homomorphism $\mathcal{O}^\times \to \hat{G}$ mapping $\lambda \in \mathcal{O}^\times$ to $(1_G, \lambda^{-1}1_S)$; that is, $\hat{G}$ is the $\mathcal{O}^\times$ -group opposite to that defined by the action of G on S (cf. [10] for more details on this terminology). By the Skolem-Noether theorem, every algebra automorphism of S is inner, and thus, for any $x \in G$, there is $s_x \in S^\times$ such that $(x, s_x) \in \hat{G}$. Equivalently, the canonical projection $\hat{G} \to G$ onto the first component is surjective. We may choose the s_x in such a way that $\det(s_x) = 1$ for all $x \in G$, and then $s_x s_y = \lambda_{x,y} s_{xy}$ for any $x, y \in G$ and some n^{th} roots of unity $\lambda_{x,y}$ in $\mathcal{O}^\times$ (all this is well-known; see e. g. [10]). Note that this means that the 2-cocycle λ representing the central extension $\hat{G}$ of G by $\mathcal{O}^\times$ has actually values in the canonical image of $k^\times$ in $\mathcal{O}^\times$ (this will be relevant when we apply [16, (e)] below, which is formulated for $k^\times$-groups). There is a unique group homomorphism $\sigma : P \to S^\times$ such that $\det(\sigma(y)) = 1$ for all $y \in P$ and such that $(y, \sigma(y)) \in \hat{G}$; in other words, the action of P on S "lifts canonically". Since S is a matrix algebra, we may write $S = \mathrm{End}_{\mathcal{O}}(U)$ for some free $\mathcal{O}$-module U of rank n. Then U becomes an $\mathcal{O}P$-module via the group homomorphism σ. In fact, U becomes an endo-permutation $\mathcal{O}P$-module, because S is a direct summand of the group algebra $\mathcal{O}O_{p'}(G)$, which has $O_{p'}(G)$ as P-stable $\mathcal{O}$-basis, and thus S has a P-stable $\mathcal{O}$-basis.

We observe next that U is G-stable. If Q is a subgroup of P and $x \in G$ such that ${}^xQ \subseteq P$, denote by $\varphi : Q \cong {}^xQ$ the isomorphism sending $y \in Q$ to ${}^xy$. By the uniqueness of σ we have $\sigma({}^xy) = {}^x\sigma(y)$ for all $y \in Q$; indeed, both sides have determinant 1 and act by conjugation as ${}^xy$ on S. Choose $s \in S^\times$ such that $(x, s) \in \hat{G}$, or equivalently, such that ${}^xt = sts^{-1}$ for all $t \in S$. Then the map sending $u \in U$ to $s(u)$ is an isomorphism $\mathrm{Res}_\varphi(U) \cong \mathrm{Res}^P_Q(U)$; in particular, U is G-stable.

We identify $O_{p'}(G)$ to its canonical image $\{(y, yb) \mid y \in O_{p'}(G)\}$ in $\hat{G}$, and set $\hat{L} = \hat{G}/O_{p'}(G)$. Thus $\hat{L}$ is a central $\mathcal{O}^\times$-extension of the group $L = G/O_{p'}(G)$.

Analogously, set $T = \mathcal{O}O_{p'}(N)c$. This is a matrix algebra over $\mathcal{O}$ of rank m^2 prime to p on which N acts. Set

$$\tilde{N} = \{(y, t) \in N \times T^\times | {}^y(t') = tt't^{-1} \text{ for all } t' \in T\}$$

endowed with the group homomorphism $\mathcal{O}^\times \to \tilde{N}$ sending $\lambda \in \mathcal{O}^\times$ to $(1_N, \lambda^{-1}1_T)$ For any $y \in N$, denote by t_y an element in $T^\times$ such that $(y, t_y) \in \tilde{N}$ and such that $\det(t_y) = 1$. Denote by $\tau : P \to T^\times$ the unique group homomorphism such that $(u, \tau(u)) \in \tilde{N}$ and such that $\det(\tau(u)) = 1$ for all $u \in P$. Write $T = \mathrm{End}_{\mathcal{O}}(V)$ for some free $\mathcal{O}$-module V of rank m. As before, V becomes an endopermutation $\mathcal{O}P$-module via τ, and V is H-stable by the uniqueness of τ.

Similarly, identify $O_{p'}(N)$ to its canonical image in $\tilde{N}$ and set $\tilde{L} = \tilde{N}/O_{p'}(N)$.

Let W be an indecomposable direct summand of $U^* \underset{\mathcal{O}}{\otimes} V$, viewed as $\mathcal{O}\Delta P$-module, such that W has vertex ΔP. It follows from Lemma 1 and Lemma 2 that W is $G \times H$-stable.

By Glauberman's theorem [8, Ch. 8, Theorem 2.11], we have $G = NO_{p'}(G)$, Since, by Proposition 4, both b and c are of principal type, we have the equality of the Brauer categories $\mathcal{F}_{G,b} = \mathcal{F}_{N,c}$. By [10, 4.2], we have $O_{p'}(G) \cap N = O_{p'}(N)$. Thus the inclusion $N \subseteq G$ induces a group isomorphism $N/O_{p'}(N) \cong G/O_{p'}(G) = L$. Identify $N/O_{p'}(N)$ to L through this isomorphism. The crucial step is to show that there is an isomorphism of $\mathcal{O}^\times$-groups

$$\hat{L} \cong \tilde{L}$$

which induces the identity on the canonical quotients L of $\hat{L}$, $\tilde{L}$, and which preserves the canonical images of P in $\hat{L}$ and $\tilde{L}$ elementwise. Before we prove the existence of such a group isomorphism, let us show how this concludes the proof of the Theorem.

Identifying $\mathcal{O}_*\tilde{L}$ and $\mathcal{O}_*\hat{L}$ through the algebra isomorphism induced by this group isomorphism, together with Proposition 5, shows that we have isomorphisms of interior P-algebras

$$\mathcal{O}Gb \cong S \underset{\mathcal{O}}{\otimes} \mathcal{O}_*\hat{L} \text{ and } \mathcal{O}Nc \cong T \underset{\mathcal{O}}{\otimes} \mathcal{O}_*\hat{L} \;.$$

If we consider $U \underset{k}{\otimes} V^*$ as S-T-bimodule, then through the above algebra isomorphisms, the bimodule $M = U \underset{\mathcal{O}}{\otimes} \mathcal{O}_*\hat{L} \underset{\mathcal{O}}{\otimes} V^*$ has vertex ΔP and W as source. Clearly M and its dual induce a Morita equivalence between $\mathcal{O}Gb$ and $\mathcal{O}Nc$.

It remains to prove the existence of a group isomorphism $\hat{L} \cong \tilde{L}$ as claimed. By the above remarks the central extensions $\hat{L}$, $\tilde{L}$ of L by $\mathcal{O}^\times$ can be represented by 2-cocycles with values in the canonical image of $k^\times$ in $\mathcal{O}^\times$, and hence we may assume that $\mathcal{O} = k$. We will show, that there is an isomorphism of $k^\times$-groups $\tilde{N} \cong \hat{N}$ preserving the canonical images of P and of $O_{p'}(N)$ and inducing the identity on the canonical quotients N of $\hat{N}$, $\tilde{N}$. Since $O_{p'}(N)$ and $Z = Z(J(P))$ are normal subgroups of coprime order in N, they commute, and thus the Brauer construction with respect to Z applied to S yields $S(Z) = T$; in particular, $\mathrm{Br}_Z(b) = c$. By Puig's version [16, (e)] of Dade's splitting theorem on fusion for endo-permutation modules (applied to Z instead of P), there is a group homomorphism $f : N_{S^\times}(\sigma(Z)) \to T^\times$ which extends the group homomorphism $(S^Z)^\times \to T^\times$ induced by Br_Z and which satisfies $f(s)\mathrm{Br}_Z(s')f(s^{-1}) = \mathrm{Br}_Z(ss's^{-1})$ for all $s \in N_{S^\times}(\sigma(Z))$ and all $s' \in S^Z$. The latter condition implies that if $(x, s) \in \hat{N}$ then $(x, f(s)) \in \tilde{N}$, and the map sending (x, s) to $(x, f(s))$ is in fact a $k^\times$-group isomorphism inducing the identity on N. Now τ and $f \circ \sigma$ are two group homomorphisms from P to $T^\times$ lifting the action of P on T, hence they are equal, and therefore the above isomorphism $\hat{N} \cong \tilde{N}$ preserves the canonical images of P elementwise. Finally, since f extends Br_Z we get for any $x \in O_{p'}(N)$ that $f(xb) = \mathrm{Br}_Z(xb) = xc$, and so the isomorphism $\hat{N} \cong \tilde{N}$ preserves the canonical images of $O_{p'}(N)$. Taking the quotients by these images yields a $k^\times$-isomorphism $\hat{L} \cong \tilde{L}$ with the required properties. □

Remarks. (1) If G is a strongly p-solvable finite group then $SL_2(p)$ is not isomorphic to a subquotient of G, and hence every block of $\mathcal{O}G$ is $SL_2(p)$-*free* in the sense of [12, 1.2]. By [12, Theorem 1.4] the equality $\mathcal{F}_{G,b} = \mathcal{F}_{N,c}$ holds more generally for any block b of a finite group whenever the block b is $SL_2(p)$-free.

(2) If b is the principal block of a strongly p-solvable finite group G, then c is the principal block of N and the inclusion $N \subseteq G$ induces an algebra ismomorphism $\mathcal{O}Nc \cong \mathcal{O}Gb$. This in turn is equivalent to Glauberman's Theorem [8, Ch. 8, Theorem 2.11] because the kernel of the principal block b of $\mathcal{O}G$ is well-known to be $O_{p'}(G)$.

(3) Glauberman's Theorem [8, Ch. 8, Theorem 2.11] is used in the proof of the p-nilpotency Theorem of Glauberman and Thompson, stating that for an odd prime p a finite group G is p-nilpotent if and only if $N = N_G(Z(J(P))$ is p-nilpotent, where here P is a Sylow-p-subgroup of G. The latter has a block theoretic version as well; cf. [11].

(4) A. Watanabe [19] pointed out that the statements and proofs remain valid with $Z(J(P))$ replaced by any normal subgroup Z of P with the property that $N_G(Z)$ controls fusion in b.

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Radha Kessar, Markus Linckelmann
Department of Mathematical Sciences
Meston Building
Aberdeen, AB24 3UE
U.K.