# A Stochastic Dynamics Approach for Response Spectrum Analysis of Bilinear Systems Using Time-Dependent Equivalent Linear Properties

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ABSTRACT: A stochastic averaging approach is used in conjunction with non-stationary response spectrum compatible random processes to derive time-varying effective linear properties (ELPs), namely damping ratio and natural frequency, for bilinear hysteretic oscillators subject to seismic excitation specified by an elastic response spectrum. It is shown numerically that the peak response of linear oscillators defined by these ELPs at the time instant for which the variance of either the excitation process or the estimated non-linear response process approximates well the peak inelastic response of a given bilinear oscillator. This is demonstrated by considering a uniformly modulated random process compatible with the Eurocode 8 (EC8) elastic response spectrum to derive ELPs from bilinear oscillators of various properties, and an ensemble of 40 artificial EC8 compatible non-stationary accelerograms to obtain peak responses of the same oscillators in a Monte Carlo based analysis. The reported numerical data suggest that peak response of bilinear hysteretic systems can be reliably estimated by the peak response of surrogate linear systems without the need to undertake non-linear response history analysis for seismic excitation represented by appropriate non-stationary random processes.

## 1. INTRODUCTION

For over six decades, various statistical linearization formulations have been considered to determine the probabilistic attributes of the response of stochastically excited non-linear structural systems (e.g., Roberts and Spanos 2003). These techniques consider surrogate (equivalent) linear oscillators whose effective linear properties (ELPs), are derived by enforcing appropriate response statistics criteria of equivalency alongside with linear random vibrations input-output relationships to approximate the response of non-linear systems. From a structural dynamics viewpoint, these ELPs offer intuitive insights to the non-linear dynamic behaviour of structures since they are related to the concepts of “stiffness” and “damping” which are amenable to a clear physical interpretation. From a practical viewpoint, statistical linearization techniques are widely used as an alternative to computationally demanding Monte Carlo simulations involving non-linear response history analyses (NLRHA), for large ensembles of time-histories compatible with a given stochastic excitation model.

In this regard, Giaralis and Spanos (2010) introduced a statistical linearization-based framework to estimate the peak inelastic response of non-linear systems exposed to earthquake excitations defined by a linear elastic seismic response spectrum without performing NLRHA. This is achieved by considering a stochastic process compatible with a given seismic response spectrum (e.g., a uniform-hazard spectrum) to derive ELPs which depend on both the characteristics of the non-linear structural system and on the response spectrum compatible stochastic process. Next, these ELPs are used together with the given response spectrum for different damping ratios to provide estimates of the peak inelastic response of the non-linear structure. In Giaralis and Spanos (2010) the early statistical linearization formulation of Caughey (1960) has been utilized to derive ELPs defining effective damped linear single-degree-of-freedom (SDOF) oscillators from inelastic oscillators following a bilinear hysteretic restoring force. Further, Giaralis et al. (2011) and Spanos and Giaralis (2013) considered a "dimension reduction" step (see also Kougioumtzoglou and Spanos 2013) in conjunction with higher-order statistical linearization schemes to derive ELPs corresponding to SDOF linear oscillators approximating the displacement and velocity variance of the response of stochastically excited bilinear hysteretic oscillators.

All the above studies considered response-spectrum compatible quasi-stationary stochastic processes compatible with a given seismic response spectrum in the mean sense which does not account for the time-varying amplitude and frequency content of strong ground motions. Further, the linearization techniques yielded deterministically defined scalar ELPs and, therefore, only the average (mean) value of the peak inelastic response could be estimated. Herein, the potential of incorporating a recently developed stochastic averaging-based formulation to derive time-dependent ELPs of bilinear hysteretic oscillators exposed to non-stationary stochastic processes (Kougioumtzoglou and Spanos 2009, Tubaldi and Kougioumatzoglou 2015) to the framework of Giaralis and Spanos (2010) is studied. This consideration enables the use of stochastic models which account for the evolutionary (time-dependent) intensity and frequency content of the input seismic action. Further, it renders possible the estimation of second-order statistics of the peak inelastic values using a given seismic response spectrum since the ELPs derived from the adopted formulation are random variables with known statistical attributes in time (i.e., mean value and standard deviation). The latter can be achieved by representing the seismic input action via non-stationary response spectrum compatible processes. In this work, a uniformly modulated non-stationary process compatible with the elastic response spectrum of the European Sesimic EC8 derived by Giaralis and Spanos (2012) is utilized to exemplify numerically the applicability and the accuracy of the considered formulation within the adopted response-spectrum based framework in a Monte Carlo based context.

## 2. STOCHASTIC AVERAGING TREATMENT OF BILINEAR HYSTERETIC OSCILLATORS

Consider a viscously damped quiescent bilinear hysteretic single-degree-of-freedom (SDOF) oscillator with mass $m$, viscous damping constant $c$, yielding deformation $u_y$, pre-yield stiffness $k$, and post-yield over pre-yield stiffness ratio $a$. Its response to a zero-mean non-stationary seismic ground acceleration stochastic process $a_g(t)$ is governed by the stochastic differential equation given as

$$\ddot{u}(t) + 2\xi_o\omega_o\dot{u}(t) + \frac{f_h\left(u(t),\dot{u}(t)\right)}{m} = -a_g(t). \qquad (1)$$

In the above equation, $u(t)$ denotes the displacement response of the oscillator relative to the ground motion, $\omega_o = (k/m)^{1/2}$ is the pre-yield natural frequency, $\xi_o = c/2\omega_o m$ is the pre-yield critical damping ratio and a dot over a symbol represents differentiation with respect to time $t$. Further, the function $f_h$ represents the oscillator restoring force, sketched in Figure 1, which controls the evolution of the inelastic behavior of the oscillator. The latter function can be mathematically written as

$$f_h\left(u(t),\dot{u}(t)\right) = aku(t) + (1-a)kz(t), \qquad (2)$$

where $z(t)$ is an auxiliary state variable governed by the differential equation

$$\dot{z}(t) = u_y \dot{u}(t)\left[1 - H(\dot{u}(t))H(z(t)-1) - H(-\dot{u}(t))H(-z(t)-1)\right], \tag{3}$$

in which $H(v)$ is the Heaviside step function, assuming the values $H(v)$=1 for $v \geq 0$ and $H(v)$=1 for $v<0$.

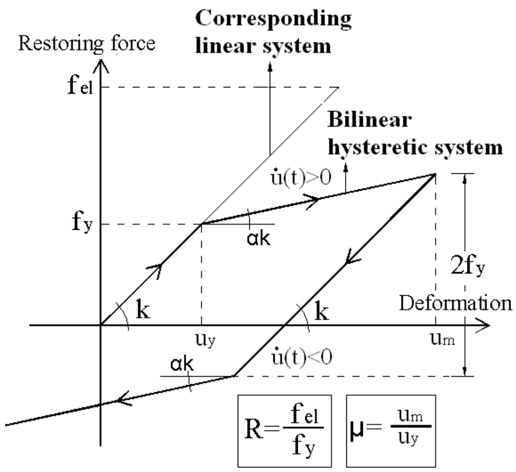


*Figure 1:Bilinear hysteretic restoring force and definitions of the strength reduction factor R and ductility μ.*

Focusing on lightly damped bilinear oscillators (e.g. $\xi$<0.10), it can be assumed that the response process $u(t)$ in Eq. (1) exhibits a pseudo-harmonic behavior described by the system of equations (Caughey 1960)

$$\begin{aligned} u(t) &= A(t)\cos\left[\omega(A)t + \varphi(t)\right] \\ \dot{u}(t) &= -\omega(A)A(t)\sin\left[\omega(A)t + \varphi(t)\right] \end{aligned}, \tag{4}$$

where the response amplitude process, $A(t)$, and the phase, $\varphi(t)$, are slowly varying functions in time and, thus, they can be treated as constant over one cycle of oscillation. Next, manipulating the system of Eqs. (4) yields

$$A^2(t) = u^2(t) + \frac{\dot{u}^2(t)}{\omega^2(A)}. \tag{5}$$

Further, it is viable to define an equivalent linear system (ELS) governed by the equation

$$\ddot{y}(t) + \omega_{eq}^2(A)y(t) + \beta_{eq}(A)\dot{y}(t) = -a_g(t), \tag{6}$$

which corresponds to a linear SDOF oscillator with effective natural frequency and viscous damping properties (ELPs) $\omega_{eq}(A)$ and $\beta_{eq}(A)$, respectively. The above ELPs are functions of the time-dependent non-linear response amplitude $A(t)$ of Eqs. (4) and are expressed as

$$\omega_{eq}^2(A) = a\omega_o^2 + \frac{k_{eq}(A)}{m}, \tag{7}$$

and

$$\beta_{eq}(A) = 2\xi_o\omega_o + \frac{c_{eq}(A)}{m}. \tag{8}$$

The terms $k_{eq}(A)$ and $c_{eq}(A)$ appearing in Eqs. (7) and (8), respectively, correspond to the contributions of the hysteretic part of the response expressed by the function $f_h$ in Eq.(2) to the effective stiffness and viscous damping properties of the ELS of Eq.(6). They are given by (Caughey 1960, Roberts and Spanos 2003)

$$k_{eq}(A) = \frac{(1-a)k}{A}C_h(A) \tag{9}$$

and

$$c_{eq}(A) = \frac{(1-a)k}{A\omega(A)}S_h(A), \tag{10}$$

respectively, where $C_h(A)$ and $S_h(A)$ are obtained by the closed-form expressions

$$C_h(A) = \begin{cases} \frac{A}{\pi}\left(\Lambda - 0.5\sin(2\Lambda)\right); & A > u_y \\ A & ; A \leq u_y \end{cases}, \tag{11}$$

and

$$S_h(A) = \begin{cases} \frac{4u_y}{\pi}\left(1 - \frac{u_y}{A}\right); & A > u_y \\ 0 & ; A \leq u_y \end{cases}, \tag{12}$$

in which $\cos(\Lambda)=1-2u_y/A$. Note that in deriving Eqs. (9) to (12) an error function (difference) between Eqs. (4) and (6), that is, between the governing equations of the hysteretic oscillator and of the ELS, has been defined and minimized in the mean square sense [see Caughey (1960) and Roberts and Spanos (2003) for detailed derivations].

To this end, it is important to emphasize that the herein considered ELPs, $\omega_{eq}(A)$ and $\beta_{eq}(A)$, can be treated as stochastic processes, since they depend on the response amplitude non-stationary stochastic process $A(t)$ of Eq. (6). In this regard, their time-varying statistics such as the mean value and standard deviation can be obtained by applying the mathematical expectation operator $E_A[\cdot]$ with respect to the process $A(t)$. For instance, the time-varying mean values of the ELPs in Eqs. (7) and (8) are given by the expressions

$$\omega_{eq}\left(t\right)=E_A\left[\omega_{eq}\left(A\right)\right], \tag{13}$$

and

$$\beta_{eq}\left(t\right)=E_A\left[\beta_{eq}\left(A\right)\right], \tag{14}$$

respectively. The evaluation of the expectations appearing in Eqs. (13) and (14) necessitates the consideration of an underlying time-varying probability density function (PDF), $f(A,t)$, characterizing the evolutionary statistical attributes of the amplitude process $A(t)$. To this aim, following Kougioumtzoglou and Spanos (2009), it is assumed that $A(t)$ has the time-dependent Rayleigh distribution

$$f\left(A,t\right)=\frac{A\left(t\right)}{\sigma_u^2\left(t\right)}\exp\left(-\frac{A\left(t\right)^2}{2\sigma_u^2\left(t\right)}\right), \tag{15}$$

where $\sigma_u^2\left(t\right)$ is the non-stationary variance of the hysteretic system response process $u(t)$. The choice of the above PDF $f(A,t)$ is motivated by the fact that the non-stationary PDF of the response amplitude of a linear lightly damped SDOF oscillator subject to Gaussian white noise excitation is a time-dependent Rayleigh one of the form of Eq. (15), observing the property $\lim_{t\to\infty} f\left(A,t\right)=\frac{A}{\varsigma^2}\exp\left(-\frac{A^2}{2\varsigma^2}\right)$, where $\varsigma^2$ is the stationary response variance of the SDOF oscillator (Spanos 1979). Furthermore, Kougioumtzoglou and Spanos (2009) showed that the PDF of Eq. (15) is applicable to non-linear oscillators under evolutionary stochastic excitations, as well.

Making use of the PDF of Eq. (15), the time-varying ELPs in Eqs. (7) and (8) become functions of the non-stationary variance of the bilinear oscillator $\sigma_u^2\left(t\right)$, and, therefore, the governing equation of motion of the ELS in Eq. (6) becomes

$$\ddot{y}\left(t\right)+\beta_{eq}\left(\sigma_u^2\left(t\right)\right)\dot{y}\left(t\right)+\omega_{eq}^2\left(\sigma_u^2\left(t\right)\right)y\left(t\right)= -a_g\left(t\right). \tag{16}$$

Next, a combination of deterministic and stochastic averaging yields the following first order stochastic differential equation for the bilinear hysteretic response amplitude (e.g. Kougioumtzoglou and Spanos, 2009)

$$\dot{A}\left(t\right)=K_1\left(A,t\right)+K_2\left(A,t\right)w\left(t\right), \tag{17}$$

where

$$K_1\left(A,t\right)= -\frac{1}{2}\beta_{eq}\left(\sigma_u^2\left(t\right)\right)A\left(t\right)+\frac{\pi G\left(\omega_{eq}\left(\sigma_u^2\left(t\right)\right),t\right)}{2A\omega_{eq}^2\left(\sigma_u^2\left(t\right)\right)}, \tag{18}$$

$$K_2\left(A,t\right)=-\sqrt{\frac{\pi G\left(\omega\left(\sigma_u^2\left(t\right)\right),t\right)}{\omega_{eq}^2\left(\sigma_u^2\left(t\right)\right)}}, \tag{19}$$

and $w(t)$ is a zero mean white noise process of intensity one. In Eqs. (18) and (19), $G(\omega,t)$ is the evolutionary power spectral density function (EPSD) characterizing the acceleration strong ground motion process $a_g(t)$ in the domain of frequencies $\omega$. Further details on the selection of this EPSD for the purposes of this work are discussed in subsequent sections.

The Fokker-Planck equation associated with the stochastic differential equation in Eq. (17) which governs the evolution of the displacement response amplitude PDF $f(A,t)$ is written as (e.g. Soong 1973)

$$\frac{\partial}{\partial t} f\left(A, t+\Delta t \middle| A', t\right) = -\frac{\partial}{\partial A}\left[K_1\left(A,t\right) f\left(A, t+\Delta t \middle| A', t\right)\right] + \ldots \frac{1}{2}\frac{\partial^2}{\partial A^2}\left[K_2^2\left(A,t\right) f\left(A, t+\Delta t \middle| A', t\right)\right] \quad (20)$$

Substituting the Rayleigh PDF of Eq. (15) into Eq. (21) and manipulating yields the following equation for the evolution of the non-linear system response variance (see also Kougioumtzoglou and Spanos (2009) for a detailed derivation)

$$\dot{\sigma}_u^2\left(t\right) = -\beta_{eq}\left(\sigma_u^2\left(t\right)\right)\sigma_u^2\left(t\right) + \frac{\pi G\left(\omega_{eq}\left(\sigma_u^2\left(t\right)\right), t\right)}{\omega_{eq}^2\left(\sigma_u^2\left(t\right)\right)}. \quad (21)$$

The latter equation is a first-order non-linear ordinary differential equation, which can be solved by standard numerical schemes such as the Runge-Kutta. Once the evolution of the non-linear response variance $\sigma_u^2\left(t\right)$ is numerically determined, the non-linear response amplitude PDF of Eq. (15) can be readily determined as well as the statistics of the time-varying ELPs of Eqs. (7) and (8), such as their average value given by Eqs. (13) and (14). These time-dependent ELPs can be used within the response spectrum based probabilistic framework reviewed in the following section to estimate the peak inelastic response of bilinear hysteretic oscillators exposed to a given response spectrum without the need to consider response spectrum compatible accelerograms.

## 3. RESPONSE SPECTRUM FRAMEWORK FOR PEAK INELASTIC RESPONSE ESTIMATION

Consider a bilinear hysteretic oscillator with known properties subject to seismic excitation represented by a given (elastic) response spectrum. It is herein proposed to utilize the stochastic averaging based technique reviewed in the previous section to estimate the peak response of the above oscillator without the need for non-linear response history analyses for ensembles of response spectrum compatible ground motions. This is accomplished by incorporating the aforementioned technique in the probabilistic response spectrum-based framework of Giaralis and Spanos (2010) (see also Spanos and Giaralis 2013). To this aim, a non-stationary stochastic process $\alpha_g(t)$ in Eq. (1) characterized by an EPSD $G(\omega,t)$ in Eqs. (17)-(19) compatible with the given response spectrum is first employed. Next, the statistics of the time-varying ELPs in Eqs. (7) and (8), evaluated at a judicially selected time instant, can be used in conjunction with the considered response spectrum to approximate the peak response of the bilinear hysteretic oscillator. The steps involved in the above framework are delineated in the flowchart of Figure 2. The accuracy of the proposed approach is numerically assessed in the following section.

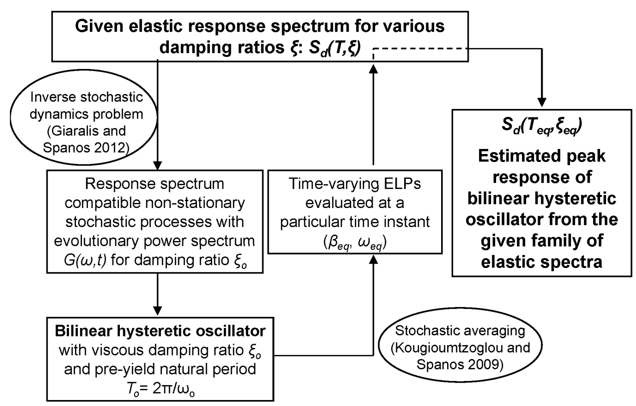


*Figure 2: Proposed probabilistic response spectrum framework for estimating the peak response of bilinear hysteretic oscillators.*

## 4. NUMERICAL ASSESSMENT FOR THE EUROCODE 8 RESPONSE SPECTRUM

The elastic response spectrum of the Eurocode 8 (EC8) seismic code (CEN 2004) is taken as a paradigm to assess the applicability and accuracy of the adopted stochastic averaging approach within the framework of Figure 2. Specifically, the EC8 pseudo-acceleration response spectrum for peak ground acceleration 0.36g (g=981cm/s$^2$), ground type "B" and damping ratio $\xi$= 5% (gray thick curve in Figure 3(a)), is assumed to represent the seismic action. A uniformly modulated non-stationary stochastic process compatible, in the mean sense, with the above EC8 spectrum is used as the ground excitation $a_g(t)$ in Eq. (1). The considered process has been derived as detailed in Giaralis and Spanos (2012) and is defined by means of the EPSD

$$G(t,\omega) = C^2 t^2 \exp(-bt) \times \frac{\omega_g^4 + 4\zeta_g^2\omega^2\omega_g^2}{(\omega_g^2-\omega^2)^2 + 4\zeta_g^2\omega_g^2\omega^2} \frac{\omega^4}{(\omega_f^2-\omega^2)^2 + 4\zeta_f^2\omega_f^2\omega^2}, \quad (22)$$

with $\omega_g$=10.73 rad/s, $\zeta_g$=0.78, $\omega_f$= 2.33 rad/s, $\zeta_f$=0.90, C= 17.76 cm/s$^{2.5}$, and $b$=0.58 s$^{-1}$. The above EPSD is plotted in Figure 3(b) on the time-frequency plane. Further, the median response spectrum of an ensemble of 100 realizations (i.e., non-stationary artificial accelerograms) compatible with the EPSD of Eq.(22) is plotted in Figure 3(a) to illustrate the good level of mean sense compatibility achieved by the underlying stochastic process with the considered EC8 spectrum.

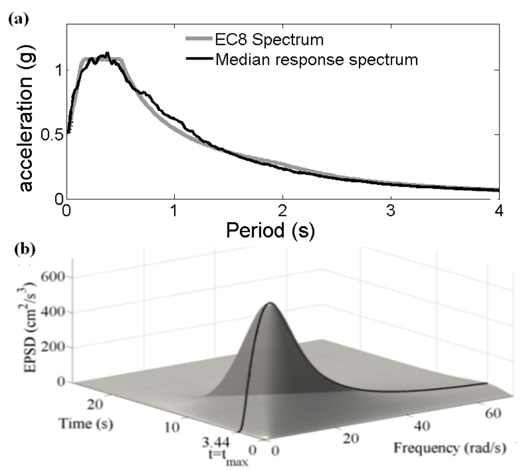


*Figure 3:* (a) *Considered EC8 response spectrum and median response spectrum of 100 realizations compatible with the EPSD of Eq.(23)* plotted in (b).

Next, the EC8 compatible EPSD of Eq. (22) enters Eq. (21) and the latter is numerically solved in conjunction with Eqs. (7), (8), and (15) for bilinear oscillators of various different properties to obtain the variance of the response deflection $\sigma_u^2(t)$, and the time-varying ELPs $\omega_{eq}(A)$ and $\beta_{eq}(A)$. For illustration, Figure 4(a) plots the input/excitation (acceleration) and output/deflection variances normalized by their peak values attained at two different time instants: $t_{in}$ and $t_{out}$, respectively, for a bilinear oscillator with $a$= 0.5, $\omega_o$= 2π rad/s, $\xi_o$= 5% and $u_y$= 6cm. Further, Figure panels 4(b) and (c) plot the mean and the mean ± one standard deviation of the time-varying ELPs for the same bilinear oscillator. As expected, $t_{out}$> $t_{in}$ indicating the existence of an output/input lag. It is also noted that ELPs at time instants close to $t_{out}$ correspond to "softer" equivalent linear systems with higher damping ratios which is in alignment with engineering intuition.

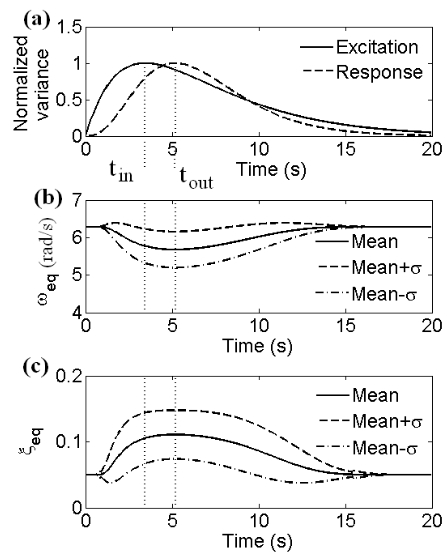


*Figure 4:* (a) *Excitation (acceleration) and response (deflection) variances normalized to their peak values;* (b) and (c) *ELPs for bilinear oscillator with a= 0.5, $\omega_o$= 2πrad/s, $\xi_o$= 5%, and $u_y$= 6cm.*

Moreover, the mean and the mean ± one standard deviation (σ) of ELP values attained at $t_{in}$ and at $t_{out}$ time instants for bilinear oscillator with $\alpha$= 0.5 and $\omega_o$= 2π rad/s are plotted in Figure 5 as functions of the strength reduction factor R defined in Figure 1. As R increases, i.e., as bilinear oscillators with lower yielding strength are considered, the effective natural frequency decreases monotonically yielding softer equivalent linear oscillators, while the effective damping ratio increases to account for the increased energy dissipation through more severe plastic/ hysteretic behaviour of the corresponding nonlinear oscillators. However, the effective damping ratios tend to saturate and even to slightly decrease as the level of yielding increases and more flexible oscillators are considered. Such trends have been observed in the literature in the context of standard statistical linearization techniques assuming stationary input excitation and yielding deterministically defined ELPs (e.g. Giaralis and Spanos 2010 and Spanos and Giaralis 2013). Interestingly, the difference between ELP statistics attained at $t_{in}$ and $t_{out}$ is negligible and, therefore, they can be used interchangeably.

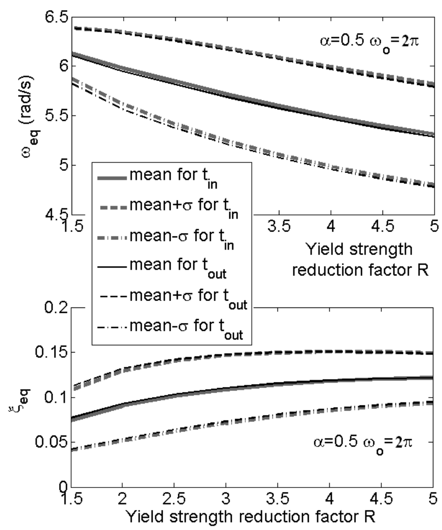


*Figure 5: ELPs for bilinear oscillator with a= 0.5, $\omega_o$= 2π rad/s, and various yielding displacement $u_y$*

Lastly, the peak absolute ductility μ, defined in Figure 1, is plotted versus the strength reduction factor R (dots of various shapes) for different bilinear hysteretic oscillators obtained from standard NRHA (Monte Carlo-MC results). The analysis uses the standard constant acceleration Newmark algorithm and is undertaken for an ensemble of 40 artificial non-stationary accelerograms compatible with the EC8 spectrum of Figure 3(a). These accelerograms have been generated by the wavelet-based stochastic approach of Giaralis and Spanos (2009). Superposed in Figure 6, is the peak response normalized by the yielding deformation $u_y$ of equivalent linear oscillators (curves of various types) whose properties (ELPs) are derived as previously discussed.

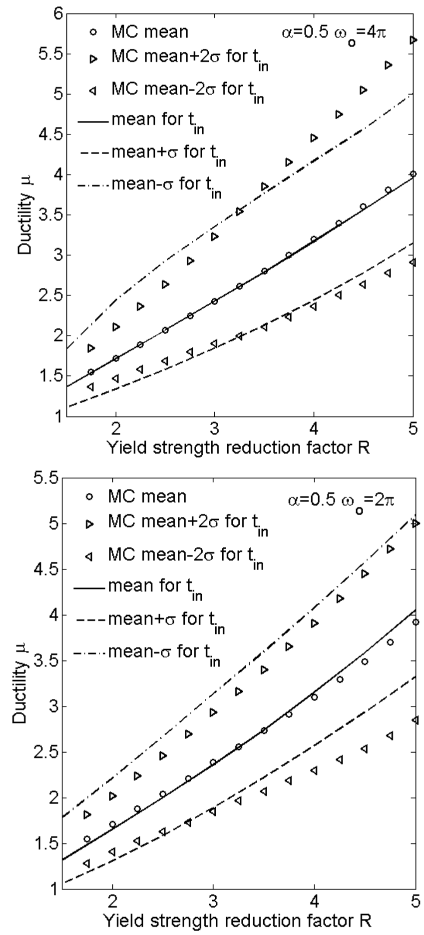


*Figure 6: Peak response of various bilinear hysteretic oscillators and of the corresponding equivalent linear oscillators for 40 EC8 compatible accelerograms.*

It is seen that the average of the peak nonlinear responses is approximated well by the peak responses of the corresponding equivalent linear oscillators. Further, it is observed that the peak response of linear oscillators with ELPs equal to the mean ± one σ (see Fig. 5) approximate reasonably well the mean ± two σ of the peak non-linear response of the bilinear oscillators for the considered suite of spectrum compatible accelerograms. Overall, these results suggest that the adopted stochastic dynamics technique may provide useful estimates of the peak response statistics of bilinear oscillators within the framework of Figure 2.

## 5. CONCLUDING REMARKS

A stochastic averaging approach has been used in conjunction with non-stationary response spectrum compatible random processes to derive time-varying ELPs, for bilinear hysteretic oscillators subject to seismic excitation specified by an elastic response spectrum. It has been numerically demonstrated, by considering a uniformly modulated random process compatible with the EC8 elastic response spectrum, that peak response statistics of the bilinear oscillators can be well approximated by the peak response of equivalent linear oscillators defined by the ELPs evaluated at the time instant when the variance of the input process is maximized. Therefore, the adopted stochastic dynamics approach circumvents the need for considering response spectrum compatible accelerograms, while it allows for the representation of the seismic input action by means of realistic non-stationary stochastic processes. Future extensions of this work will involve the consideration of stochastic processes representing pulse-like ground motions (Lungu and Giaralis 2013) and of multi-degree-of-freedom bilinear structures represented by surrogate linear oscillators, one for each dynamic degree of freedom.

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