# AN APPLICATION OF COMPUTER AIDED DESIGN

# AND COMPUTER AIDED DRAUGHTING TECHNIQUES TO GEARING

—— BY ——

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# CONTAINS PULLOUTS

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## ABSTRACT

The thesis describes the development of a computer-aided design and computer-aided draughting package of industrial gearboxes for transmitting power between shafts having parallel major axes. The package mainly consists of the following functions:

(a) Determination of gear tooth numbers from the properties of continued and conjugate fractions
(b) Generation of gear teeth by a rack-type cutter
(c) Calculation of gear tooth profile and production of gear tooth grid for finite element stress analysis
(d) Detailed strength analysis and design of a spur or helical, external or internal, standard or modified, single- or double- reduction gear set based on the ISO gear standards
(e) Shaft design and analysis
(f) Bearing selection and analysis
(g) Production of the working drawings of the main gearbox components.

This package works on the IBM PC and its compatible machines, operates in an interactive mode, combines the experience of the designer with the power of the computer, and produces substantial savings in time and cost over conventional design methods. Design examples illustrating this package are given. In addition, the maximum tooth root bending stresses have been studied by using finite element techniques and the results have been compared with those using the ISO gear standards.

## Symbols

For the purposes of this thesis the following symbols apply.

| Symbol | Designation | Units |
|---|---|---|
| $a$ | centre distance | mm |
| $b$ | facewidth | mm |
| $d_1$ | reference diameter of pinion | mm |
| $h_{ao}$ | addendum of basic rack | mm |
| $m$ | module | mm |
| $m_n$ | normal module | mm |
| $n_1$ | rotational speed of pinion | $min^{-1}$ |
| $p_{ao}$ | tip radius of basic rack | mm |
| $r$ | reference radius | mm |
| $r_1$ | reference radius of pinion | mm |
| $r_2$ | reference radius of gear | mm |
| $r_b$ | base radius | mm |
| $r_{b1}$ | base radius of pinion | mm |
| $r_{b2}$ | base radius of gear | mm |
| $u$ | gear ratio | - |
| $x$ | addendum modification coefficient | - |
| $x_n$ | normal addendum modification coefficient | - |
| $z$ | tooth number | - |
| $z_1$ | tooth number on pinion | - |
| $z_2$ | tooth number on gear | - |
| $F_t$ | nominal tangential force at reference circle | N |
| $K$ | resultant load factor | - |
| $K_A$ | application factor | - |
| $K_{F\alpha}$ | transverse load factor for bending stress | - |
| $K_{F\beta}$ | face load factor for bending stress | - |

| Symbol | Designation | Units |
|---|---|---|
| $K_{H\alpha}$ | transverse load factor for contact stress | - |
| $K_{H\beta}$ | face load factor for contact stress | - |
| $K_V$ | dynamic factor | - |
| $K_\alpha$ | transverse load distribution factor | - |
| $K_\beta$ | face load distribution factor | - |
| $K_\sigma$ | resultant factor for bending stress | - |
| $K_\tau$ | resultant factor for torsional stress | - |
| $S_F$ | actual safety factor for bending stress (against breakage) | - |
| $S_{Fmin}$ | minimum demanded safety factor for bending stress (against breakage) | - |
| $S_H$ | actual safety factor for contact stress | - |
| $S_{Hmin}$ | minimum demanded safety factor for contact stress | - |
| $T_1$ | pinion torque | N.m |
| $Y_F$ | tooth form factor for bending stress | - |
| $Y_{NT}$ | life factor of a test specimen | - |
| $Y_S$ | stress correction factor for bending stress | - |
| $Y_{ST}$ | stress correction factor of a test specimen = 2.0 | - |
| $Y_{Rrelt}$ | relative surface condition factor for bending stress | - |
| $Y_X$ | size factor for bending stress | - |
| $Y_\beta$ | helix angle factor for bending stress | - |
| $Y_{\delta relt}$ | relative sensitivity factor for bending stress | - |
| Z | resultant factor for contact stress | - |
| $Z_E$ | elasticity factor for contact stress | - |
| $Z_H$ | zone factor for Hertzian pressure at pitch point for contact stress | - |
| $Z_L$ | lubricant factor for contact stress | - |
| $Z_N$ | life factor for contact stress | - |

| Symbol | Designation | Units |
|---|---|---|
| $Z_R$ | roughness factor for contact stress | - |
| $Z_V$ | speed factor for contact stress | - |
| $Z_W$ | work hardening factor for contact stress | - |
| $Z_X$ | size factor for contact stress | - |
| $Z_\beta$ | helix angle factor for contact stress | - |
| $Z_\varepsilon$ | contact ratio factor for contact stress | - |
| $\alpha$ | pressure angle at reference cylinder | rad |
| $\alpha_n$ | normal pressure angle at reference cylinder | rad |
| $\alpha_t$ | transverse pressure angle at reference cylinder | rad |
| $\alpha_t'$ | transverse pressure angle at pitch cylinder | rad |
| $\beta$ | helix angle at reference cylinder | rad |
| $\eta$ | mechanical efficiency | - |
| $\psi_a$ | facewidth factor (FW/CD) | - |
| $\psi_d$ | facewidth factor (FW/reference diameter) | - |
| $\psi_\sigma$ | equivalent factor for bending stress | - |
| $\psi_\tau$ | equivalent factor for torsional stress | - |
| $\sigma_{-1}$ | bending endurance limit for symmetrical cycle | $N/mm^2$ |
| $\sigma_a$ | bending stress amplitude | $N/mm^2$ |
| $\sigma_{FO}$ | tooth root bending stress | $N/mm^2$ |
| $\sigma_{Flim}$ | endurance limit for bending stress | $N/mm^2$ |
| $\sigma_{Hlim}$ | endurance limit for contact stress | $N/mm^2$ |
| $\sigma_m$ | mean bending stress | $N/mm^2$ |
| $\tau_{-1}$ | torsional endurance limit for symmetrical cycle | $N/mm^2$ |
| $\tau_a$ | torsional stress amplitude | $N/mm^2$ |
| $\tau_m$ | mean torsional stress | $N/mm^2$ |

# Chapter 1

# INTRODUCTION

## 1.1 Computers in Engineering

Digital computers have revolutionized engineering practice, just as they have other aspects of modern life. Computers have become an absolutely essential tool in all phase of engineering, from research and development to marketing and management. The engineer of a generation ago would probably not even recognize many of today's routine engineering activities that rely heavily on digital computers. Even after four decades the computer revolution shows no sign of slowing. New discoveries and improvements in computer-technology continue at such a pace that many products and techniques in the computer field become obsolete within a few years time.

The power of computers in most applications can be traced to two fundamental features: their capacity to store and display large amounts of information and their ability to process this information at a very rapid rate. Information storage and display are important in many engineering activities. Today's engineer relies on computer-based library reference services, computer data bases, and computer scheduling, accounting and inventory control services. The ability of the computer to tackle large and complex calculations has also had a major impact on engineering practice, particularly in research, development, and design activities. Today the engineer routinely uses the computer to solve problems that would have been unthinkable even as recently as a decade ago.

Moreover the microelectronics revolution has stimulated the development of powerful personal microcomputers. Complete computer systems are now relatively inexpensive. Along with being low-cost, this equipment is also more reliable and simpler to use than its computing ancestors. The ready availability of these devices is enabling a large number of engineers to become familiar with computing techniques and to become aware of their potential for effective design.

## 1.2 Design Process in Engineering

The design process involves applications of technology for the transformation of resources, to create a product that will satisfy a need in society. The product must perform its function in the most efficient and economic manner within the various constraints that may be imposed. It is the ability to design an efficient product that sets apart the successful designer.

As a general rule, there are three main types of design work, each of which requires a different level of intellectual ability and creativity.

Primarily, there is the adaptation of existing designs which require only minor modification, often in the dimensions of the product. This type of work represents a large proportion of the total design work undertaken. The designer engaged entirely in this type of work needs only the basic technical skills and will not appreciate the nature of the design process unless other work is undertaken.

Secondly, there is development design, which uses an existing design only as a basis. The technical work involved can be considerable and the final product may be very different from the original item. In certain cases the use of an existing design may make the solution more difficult to obtain, and the level of ability and time spent by the designer may be considerable. This is sometimes called adaptive design.

Finally, the most demanding work concerns the design of a new product that has no precedent. This needs a high level of ability and relatively few designers will be engaged in this type of activity. It is sometimes referred to as creative design.

The design problem gives the designer the opportunity to utilise and develop his creative skills to their full extent. In order to select a technique or idea that will provide a solution to the problem, the designer requires both practical experience and a depth of knowledge. He must then be able to relate widely different elements and recognise their mutual adaptability. Ultimately, a truly creative solution will only emerge if the designer can apply open-ended thinking and can also appreciate alternative solutions and strategies.

The overall design process usually has five stages of development:

Stage (1) Identifying the Problem:
The first step involves identifying a problem, or a requirement of society. However good the final product there must be a need or a market for it. A full specification of this need must be obtained in order to avoid extra work at a later stage. Limitations and requirements of the design must be identified, and stated in terms of constraints and criteria.

Stage (2) Feasibility Study:

Having identified a need for the product, the next stage is to conduct a basic feasibility study. It must be physically possible to produce the product, it must be economically viable and it must be acceptable. If the alternative solutions cannot be physically realised, the project will not proceed unless the problems concerned are overcome by appropriate prior development work.

Stage (3) Preliminary Design:

At this stage, having defined the need or the problem and ascertained that a solution may be feasible, it becomes necessary to select the best approach to the problem. Where several alternatives exist they should all be examined in an unbiased manner, with equal thoroughness. It may be found at this stage that a solution which appeared possible earlier can now not be achieved. At the conclusion of the preliminary design phase it must be decided which approach to the final solution is to be examined further and is most likely to be adopted. It should be remembered that this choice is not irreversible and results obtained later may require that this approach is abandoned and an alternative is developed.

This stage uses combinations of the three fundamental design methods described as follows:

(a) The Iterative Design Method:

This method is frequently used, especially in the early stages of design. The process consists of employing intuition and experience, from which a set of rules, drawn from experience or direct design, to produce a preliminary solution. The resulting design is then analysed to determine if it meets the specified design constraints. If it fails to do so, then it is modified on the basis of further information obtained from the analysis, and reanalysed until the design can meet the specification, or until the designer is convinced that this design is not feasible within the given constraints.

The iterative design process, illustrated in flow chart form in Figure 1.1, therefore contains three main activities:

(1) Initial Design

(2) Analysis

(3) Redesign.

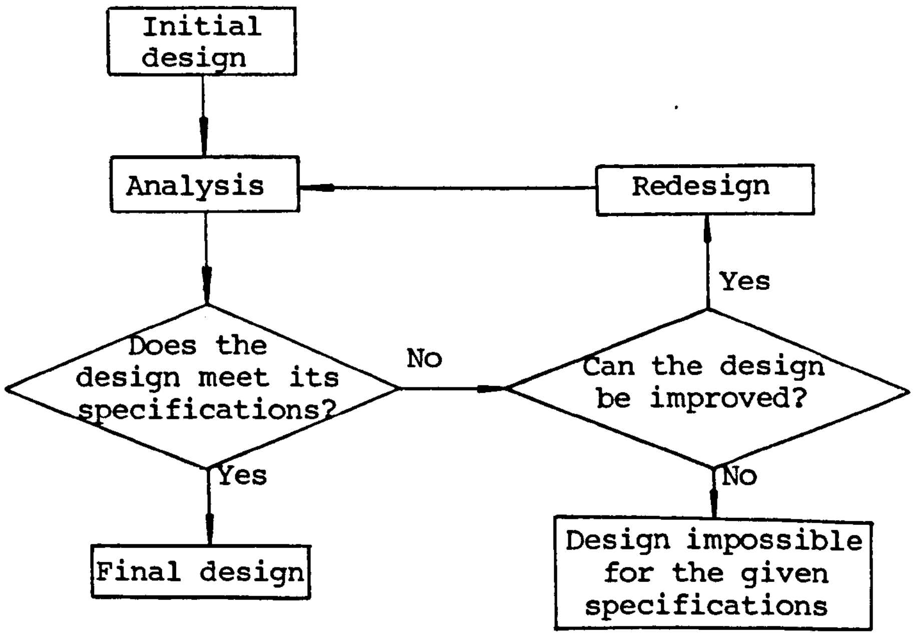



Figure 1.1 The iterative design process

(b) The Direct Design Method:

The iterative method uses the results of a design analysis as a base for redesign decisions. The direct design method uses the design analysis as a method for effecting the design; in other words, a design is produced directly by analysis. This process implies that a design can be defined by a parametric representation, and that interactions between parameters are known.

(c) The Design Selection Method:

A design process may yield a number of solutions which all satisfy given constraints. This is applicable to the situation where the design constraints themselves contain one or more variables. In this situation the selection of the final design, given the performance characteristics for each design, is largely a matter for human judgement by compromising performance as it is affected by design constraints.

Stage (4) Detailed Design:

An approach to the solution has been chosen and a full detailed design is now prepared. All components and systems are fully specified and all inservice requirements should be defined. A complete set of production drawings is also prepared. At this stage less alternatives are available to the designer and a significant economic commitment is usually required. Any major revisions, or the decision to abandon an approach entirely, should be made before progressing to the next stage. If not, considerable investment will be wasted. It may be that events outside the designer's control may cause the product to become unacceptable at a later stage. This possibility should be minimised by constant feedback and continual re-evaluation of the problem identification and feasibility studies.

Stage (5) Production:

At this stage, having finalised and approved the detailed design, the product is constructed. This will involve close co-operation between the designer and the production engineer.

One of the clearest and simplest analyses of the stages in a design study can be summarised as shown in Figure 1.2.

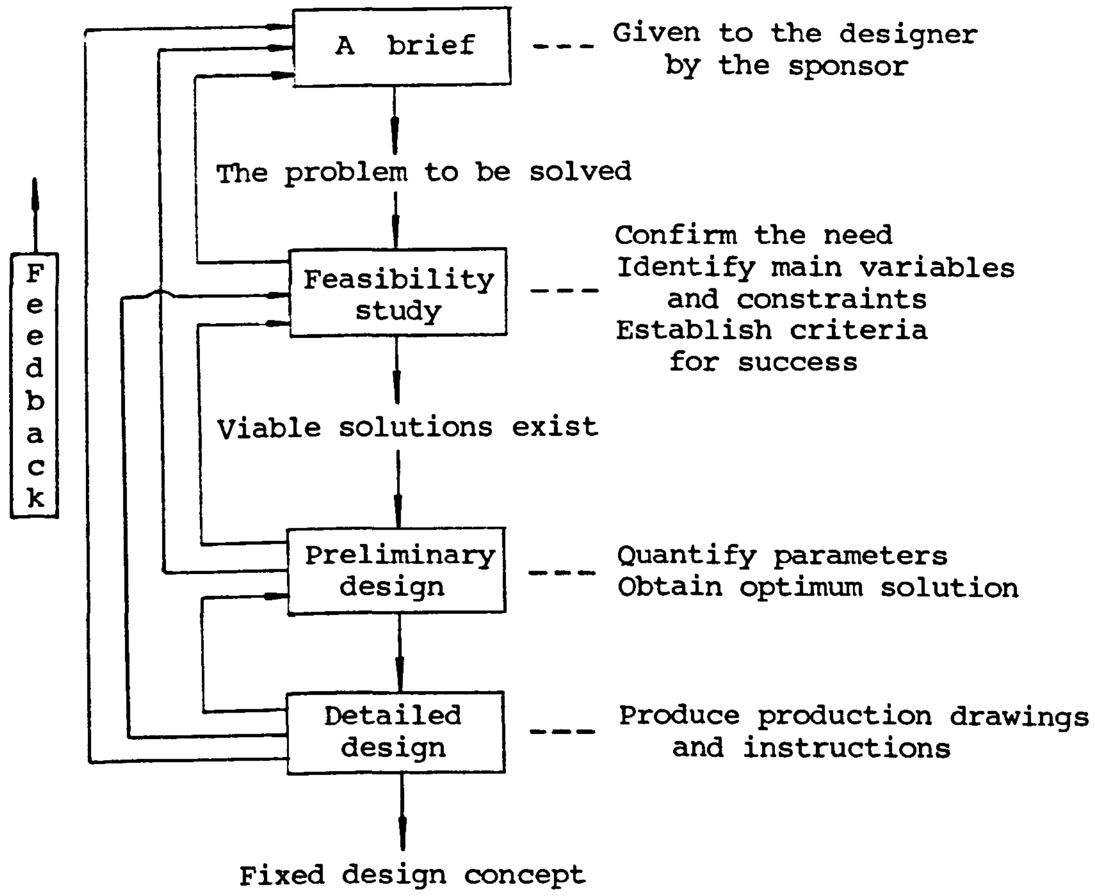



Figure 1.2 Flowchart showing stages in a design process

## 1.3 Computer-Aided Design

The meaning of 'Computer-Aided Design' (CAD) has changed several times in its past twenty years or so of history. For some time, CAD was almost synonymous with finite element structural analysis. Later, the emphasis shifted to computer-aided draughting (most commercially available CAD systems are actually draughting systems). More recently, CAD has been associated with the design of three-dimensional objects, (this is typical in many branches of mechanical engineering). In [26], CAD is considered as a discipline that provides the required know-how in computer hardware and software, in systems analysis and in engineering methodology for specifying, designing, implementing, introducing and using computer based systems for design purposes.

To be brief, CAD can simply be defined as a technique in which the designer and machine are blended into a problem-solving team, intimately coupling the best characteristics of each. The result of this combination works better than either the designer or the computer would work alone, and by using a multi-discipline approach it offers the advantage of integrated team-work.

CAD implies by definition that the computer is not used when the designer is most effective, and vice versa. It is, therefore, the marriage of the characteristics of each which is so important in CAD. These characteristics affect the design of a CAD system in the following areas:

(a) Design Construction Logic:
The use of experience combined with judgement is a necessary ingredient of the design process. The design construction must therefore be controlled by the designer. This means that the designer must have the flexibility to work on various parts of the design at any time and in any sequence, and be able to follow his own intuitive design logic rather than a stylised computer logic.

(b) Information Handling:
Information is required from the specification before the preliminary design stage can proceed. Similarly, when the

detailed design is complete, information must in turn be output to enable the design to be manufactured. The human brain is able to store information in an intuitively ordered manner, but its storage capacity is limited, and the information is not all retained as time passes. By contrast, computers have no ability to organise data intuitively but have large permanent storage capacities. Information storage should therefore be carried out by the computer under the direction of the designer.

The output of manufacturing information, from the detailed design stage of the design process, usually involves the production of drawings. This is a time-consuming, laborious and error prone process when carried out manually but is quite suitable for execution by the computer. It is therefore desirable to allow the computer to generate as much production information as possible, so freeing the designer from repetitious work at all stages of the design process.

(c) Modification:

Design descriptive information must frequently be modified to make correction of errors, to make design changes, and to produce new designs from previous ones. The computer has the ability to detect those design errors which are systematically definable; whereas man can exercise an intuitive approach to error detection. For example, the computer can calculate the torque capacity of a shaft, whilst a designer can tell from experience and judgement that the shaft is too small.

The automatic correction of errors is generally difficult for a computer. It should therefore be left to the designer to monitor corrections of errors and any other design changes which may be needed.

(d) Analysis:

A computer is very good at performing those analytical calculations of a numerical analysis nature which the designer finds time-consuming and tedious. As much as possible of the numerical analysis involved in the design should be done by the computer, leaving the designer free to make decisions based on the results of this and his own intuitive analysis.

The stages in the design process and the possible uses of a computer are illustrated in Table 1.1 [66].

Table 1.1

Potential for Computer Utilisation in a Design Process

| Design Stage | Computer Potential |
|---|---|
| Recognition of a need and problem definition | Desirable, but few applications |
| Feasibility study | Limited application |
| Preliminary design | Certain specific applications and use of specially prepared programs or packages |
| Detailed design and documentation | Preparation of standard specification and draughting |
| Communicating the design | Computer graphics |
| Production | CAM and NC |
| Feedback | Storage on files of subsequent results and reports |

CAD is not a specific term, it includes all aspects of the design process including draughting work. The present trend in CAD is away from systems performing only numerical calculations, and requiring a large computer memory, towards the analysis of complex problems requiring many hours of program development

[66]. CAD is being increasingly adopted by architects, shoemakers, surgeons, industrial designers, etc. It is only a matter of time until CAD becomes a standard tool in all design offices, progressing in tandem with a steady adaptation of the work procedures there. Not only the industrial nations but also the developing countries are beginning to realize that CAD will be an essential constituent of practically all industrial enterprises in the near future [26].

## 1.4 This Thesis

Industrial gearboxes are commonly used in mechanical systems such as in machine tools, cranes and automotive applications and represent an important area of mechanical engineering manufacture. The design of a gear set is a reasonably difficult problem which involves the satisfaction of many design constraints. It is the job of the designer to specify the major geometrical features of a gear set in order to achieve known requirements. This design process is iterative and time-consuming by hand. Currently more exacting analytical techniques are being advocated to establish more reliable rating information and hence a reduction in factors of safety. Inevitably the associated calculations become more complex and the volume of data required increased. These make gearing design a design work which is especially suited to computers.

The application of a computer to the design of gearboxes would offer many significant advantages:

(1) The designer is freed from the tedious calculations.
(2) The design process is sped up greatly.

(3) The calculations can be based on more accurate theory and do not have to be over-simplified in the interest of speed.

(4) The calculations can cover a wide range of alternative designs very quickly.

(5) The calculations can be performed iteratively to examine the sensitivity of performance to a large number of design dimensions and other design parameters such as material, accuracy, etc., thus allowing better optimisation of design.

The application of computer techniques to gearing design has to date been a fairly well explored field. Allan and Forgie [6] show the feasibility of integrating design functions using computers, citing gear train as an example. Cockerham and Waite [16] present a computer-aided procedure for the design of a 20$^{\circ}$ pressure angle spur or helical gear train based on strength and wear calculations outlined in BS436:1940 [10]. The program varies the module, face width, and gear ratio to obtain an acceptable design, in which no account has been taken of non-standard or internal gears. Tucker [82] and Estrin [32] look more closely at the gear parameters, such as addendum ratio and pressure angle, and outline procedures for varying a standard gear mesh to obtain a more favourable gear set. Savage, Coy and Townsend [69] establish an optimal design procedure for standard spur gear pairs. The procedure utilizes standard gear geometry and optimizes the number of pinion teeth to obtain the minimum centre distance for a given application of specified speed reduction and input pinion torque while preventing failure in root failure, pitting failure and scoring in accordance with AMGA standard [2, 3, 4]. Dale [18] describes how computer graphics is being applied to the design of gear trains, which are used in special-purpose machine tools. The problem to be solved is: Given a fixed input drive and a number of fixed output-drive spindles, how can the spindles be driven by the input using the minimum number of connecting gears in the train. Cheah, Murray and Wilkinson [15] developed a computer package to assist in the design and layout of multi-layer spur and helical gear trains. The package assumes that all gears mesh at standard centre distances and that the gear teeth are of standard proportions with zero profile shift. Walton and Taylor [74, 84, 85] present computer programs for the design of some components of a gearbox. Emphasis is given to computer aided design of spur and helical gear trains based on BS436:1940. MTIRA (Machine Tool Industry Research Association) [57, 65] produces a suite of programs (GEARS-2), each of which only concerns certain aspects of gear design, e.g. gear load capacity calculation (BS436:1940), major manufacturing dimension calculation, and tooth thickness dimension calculation. McGoldrick and Bruce [54] develop an interactive program for the computer-aided design of a gear train in multi-spindle drilling heads. The gear train is used to divert power from a single input to multiple outputs. Mehta, Pandey and Misra [49] present a methodology for optimum kinematic design of machine tool gearboxes, using a multiple criterion optimising approach. Ardayfio, Watson and Eberhadt [7] describe the features of a computer aided package for bevel and worm gears and outline the method of coding standard design data and charts into computer software. Pratusevich et al [64] introduces a computing program for calculating the mechanical strength of spur gears on machine tools. Hundal [38] lists a 90 lines of BASIC program for determining the design quantities of standard spur gears according to AGMA standard and shows how a repetitive design process can be simplified by using a computer.

In these computer programs, some concentrate only on specialised aspects of gear design, while others claim to have taken a more integrated approach to the subject. Invariably each package will contain some special facility or feature. To justify the development of a new gear design package based on systematic comparison of functional and other merits with existing packages, is difficult due to the different programming approaches adopted in addition to the problems of acquiring the various packages. The author has endeavoured to produce a package capable of expansion to cover a much wider range of design functions than already exists.

This approach has been used to develop a suite of modular programs organised into a package for the design of single-reduction and double-reduction gear units. The main functions and features of the package are as follows:

(1) Determination of tooth numbers which satisfy a particular gear ratio within a specified tolerance from the properties of continued fractions and conjugate fractions.

(2) Generation of gear teeth by a rack-type cutter, which may be derived from different national standard or nonstandard basic racks.

(3) Calculation and drawing of gear tooth profile, which may be generated by the rack cutter of different national standards.

(4) Production of gear tooth grid, generation of the node number and coordinates of each node, and determination of the node numbers of the load and maximum bending stress

positions for the gear tooth stress analysis using finite element methods.

(5) Detailed geometric calculations of a spur or helical, external or internal, standard or nonstandard gear pair, which provide the designer with the considerable information necessary for strength analysis, manufacturing, and inspection such as pitch diameters, contact ratios, pressure angles at the highest points of single tooth pair contact, base tangent lengths and diameters over pins, and information on undercutting, interference and topping.

(6) Detailed strength analysis and design for a spur or helical, external or internal, standard or modified, single reduction or double reduction gear train based on the international gear rating standard ISO/DIS6336 [39]. In design situations, many alternative solutions, if required, will be presented to the designer by adopting different facewidth factors and helix angles (for helical gears). Moreover, when a double reduction gearbox is employed, the designer may also choose different gear ratio split methods on the basis of different requirements of application. Therefore, much more alternative design solutions may be obtained.

(7) Design and analysis of a stepped shaft supported by two bearings for combined bending and torsional strength, fatigue strength, rigidity, and vibration.

(8) Selection and analysis of a roller contact bearing based on basic dynamic capacity and basic static capacity.

(9) Production of the working drawings of the main gearbox components such as gears and shafts.

(10) The design programs operate in an interactive mode. The

interactive features of the program permit a designer to be involved in crucial stages of the design process, provide considerable flexibility with the designer to apply his experience and skill to the problem solving, and enable him to quickly explore several alternative design conditions in order to achieve a more optimal design solution.

(11) The design programs are implemented on the IBM personal computer and its compatible machines with CGA, EGA, or Hercules display adapter as these computers are relatively cheap to buy and therefore more readily accessible to a designer.

(12) The design programs can present the designer with far more information than is normally required and the documentation for the design is also well prepared by the computer output.

(13) The package is not only useful as a practical design tool in industry but also helpful as a teaching resource for instruction in the gear design process in education.

The author has also studied the maximum tooth root bending stresses using finite element method and compared the results with those using the ISO gear standard.

# Chapter 2

# COMPUTING SYSTEM AND SOFTWARE DEVELOPMENT

## 2.1 System Requirements

The computer software described in the thesis is broadly divided into two types of programs, namely design programs and draughting programs. The draughting programs will be described in detail in Chapter 10. The hardware used by the author in preparing the design programs covers three different kinds of computer systems that are widely used across all industries, education, and in the home. These micros are

(1) IBM PC/AT compatible microcomputer which has 1 MB of memory, a 20 or 40 Mb hard disk, a mathematical co-processor, a Hercules display adapter, and a dot matrix printer;

(2) IBM Personal Computer which has 640 KB of memory, a 20 Mb hard disk, a math co-processor, an Enhanced Graphics Adapter, and IBM Graphics Printer;

(3) Amstrad PC1512 computer which has 512 KB of memory, two disk drives with 360 KB capacity, no math co-processor, a Colour Graphics Adapter, and a dot matrix printer.

The operating system used for developing the design programs on these microcomputers is PC-DOS (MS-DOS) version 3.2. However, allowing for minor hardware differences, the design program should be valid for any IBM compatible hardware with a suitable operating system.

## 2.2 Choice of Language for Implementation

### 2.2.1 High-level Programming Language

Programming in the early days of computing was tedious in the extreme. Programmers required a detailed knowledge of the instructions, registers, and other aspects of the central processing unit (CPU) of the computer for which they were writing code. In the 1950s it became increasingly apparent that this form of programming was highly inconvenient although it did enable the CPU to be used in a very efficient way.

These difficulties spurred a team led by John Backus of IBM to develop the first ever high-level language, Fortran. The language was indeed simple to learn, as it was possible to write mathematical formulae almost as they are usually written in mathematical texts. (in fact, the name Fortran is a contraction of Formula Translation.) This enabled working programs to be written faster than before, for only a small loss in efficiency. We thus see that Fortran was an innovation, as the first high-level language.

But Fortran was revolutionary as well as innovatory. Programmers were relieved of the tedious burden of using machine language, and were able to concentrate more on the problem in hand. Perhaps more important, however, was the fact that computers became accessible to any scientist or engineer willing to devote a little effort to acquiring a working knowledge of Fortran; no longer was it necessary to be an expert on computers to be able to write application programs.

After thirty years' existence, Fortran is far from being the only programming language available on most computers. Nowadays there are over 150 programming languages in use throughout the world for a variety of applications include scientific and engineering computation, simulation and modeling, process control, business and accounting, and so on. The most widely used programming languages at present are FORTRAN, Pascal, C and BASIC [23].

The choice of which programming language should be used to create a particular program is of crucial importance, not only because of possible changes of program, but also because of possible changeovers to other computer systems. High-level programming languages are not machine specific. Programs written in them (source program) are converted into machine language instructions by compilers or interpreters (program translators). When changing from one computer to another, the user does not, therefore, have to change the source program, but only the compiler or interpreter. A programming language is also chosen on how well suited the programming language features are matched to the work that the particular program has to accomplish. In the course of time new languages have been developed, and where they were demonstrably more suitable for a particular type of application they have been adopted in preference to Fortran for that purpose. Fortran's superiority has always been in the area of numerical, scientific, engineering, and technical applications, and there is still no significant competitor in these fields.

The Fortran language has twice been standardized in the framework of ANSI and the International Standards Organization (ISO), in 1966 and 1978. Following the publication of these two standards, known as Fortran 66 and Fortran 77, the technical committee responsible for their development, X3J3, has been working for a decade on the draft of a new standard, for the moment called Fortran 8x, and published in 1987 [53]. Its intention is that following a period of public comment and further work to take the comment into account, the final standard be published in about 1989, just allowing for the x in Fortran 8x to have the value 8.

Fortran is a compiler language; the source program written by the programmer must be translated (compiled) into machine language instructions by the compiler. Although the draft Standard for Fortran 8x has now been published and the final standard might be published in 1989, history has shown the software industry in general to be very slow to adopt new standards. It is unlikely that a Fortran 8x compiler will be available in the near future.

The gearbox design programs of the package are written in Fortran 77. The compiler used for translating (compiling) the programs is the Microsoft (MS) Fortran Optimizing Compiler (Version 4.01) developed and marketed by the Microsoft Corporation of Bellevue, Washington [55, 56]. The MS-Fortran compiler is specifically designed for compiling a Fortran 77 source program on the personal computer. Several versions of this compiler are in use, with Version 4.01 (copyright 1987) being the latest by this time.

The Microsoft FORTRAN Optimizing Compiler V4.01 for the MS-DOS operating system provides all of the features required by the ANSI X3.9-1978 FORTRAN standard, plus many powerful extensions designed to optimize FORTRAN in the microcomputer environment. The Microsoft FORTRAN Optimizing Compiler generates fast, efficient native code. The library provided with the compiler includes code for fast real arithmetic if an 8087/80287 coprocessor is used, or it can provide for software emulation of 8087/80287 operations for systems without a coprocessor. This FORTRAN compiler also includes interlanguage calling support, which allows the user to link Microsoft FORTRAN programs with 8086 assembly-language programs and Microsoft C and Pascal programs.

### 2.2.2 Assembly Language

Programs are often judged by their display quality and visual design alone. Most application programs take pains to make themselves attractive and easy to use. They do not simply display one line after another, scrolling old lines off the top of the screen; they display their messages and results in specific areas of the screen. They also control the color and brightness of what they display, and often they assign special meanings to certain keys. In general, control of the display screen, like most other computer operations, can be done in three ways:

(1) By using the high-level programming language services;
(2) By using the ANSI driver services;
(3) By using the ROM-BIOS video services.

Generally speaking, we always use the high-level programming language services to control the screen as long as they can accomplish what we want to do. For example, if we want to clear the entire screen when using BASIC programming language, we may use the CLS command. However, high-level programming languages are not designed to include every possible function that we might need while programming — though admittedly, some are better than others. Many high-level programming languages (for example, Fortran, Pascal and C) does not support good screen handling. So, if we need to get control of the display screen and if we aren't using a language such as BASIC that provides the services we need, we have to use other services.

In the beginning, DOS did not provide adequate video services. But starting with version 2.00, it became possible to perform the needed screen work through the DOS services enhanced with the ANSI driver program, also known as ANSI.SYS. This program uses a set of commands that, when translated, will perform just about anything the screen is capable of doing. In general, the ANSI driver is only active when we deliberately introduce it into DOS through the CONFIG.SYS file that DOS loads during the start-up operation. The specific command in the CONFIG.SYS file that is used to activate the ANSI.SYS driver is :

DEVICE = ANSI.SYS

When ANSI.SYS is installed, it recognizes certain character sequences as being command sequences. All ANSI.SYS command sequences begin with an escape character whose ASCII value is 27. ANSI.SYS command sequences are not displayed on the screen. ANSI.SYS can carry out four types of commands: erase all or part of the display screen, control cursor position, set display modes and attributes (color, underscore, blinking, etc.), and reassign character strings to individual keys on the keyboard [71]. By using the ANSI driver, for example, the following subroutine written in Fortran 77 is used to clear the entire screen and set the cursor to the home position.

```
SUBROUTINE CLS
PRINT'(1X,2A)',CHAR(27),'[2J'
END
```

From a programmer's point of view, the ANSI driver looks quite different. Use of the driver presents a programmer with two main benefits, both of which can be quite important. For programmers who do not have the skills and tools necessary to build assembly-language interfaces into the ROM-BIOS services, the ANSI driver makes the most crucial BIOS-type services available to any programming language. Furthermore, it can be a great benefit to programmers who want to write programs that are not tied to the PC family, but instead will work on any DOS computer using the ANSI driver.

Despite these apparent advantages, the use of the ANSI driver commands in our programs is not a good idea. For one thing, it requires that the ANSI driver be installed in any computer that our programs are used on, which adds considerably to the instructions that we have to prepare to accompany the programs. It is difficult enough trying to explain the setup and use of our programs to both novices and experts, without adding extra layers of complexity, such as the explanation of how to install the ANSI driver.

A further argument against the use of the ANSI driver is that it is not available under all circumstances. For example, IBM's windowing system, Topview, does not support the features of the ANSI driver, so programs that require the driver cannot be used with Topview. This may well turn out to be true with other windowing environments as well.

But most important of all is the fact that, compared to the BIOS service, the ANSI driver is pathetically slow in generating full screen output. Unless there is very little screen output to be displayed, the ANSI driver is just too slow to be satisfactory.

The ROM-BIOS — an acronym for Basic Input/Output System — is the part of ROM that is in active use all the time the computer is at work. The role of the ROM-BIOS is to provide the fundamental services that are needed for the operation of the computer. For the most part, the BIOS controls the computer's peripheral devices, such as the display screen, keyboard, and disk drives. The ROM-BIOS is a collection of machine-language routines. Conceptually, the ROM-BIOS services are sandwiched between the hardware and the high-level languages (including the operating system). In effect, this means that the BIOS works in two directions in a two-sided process. One side receives requests from programs to perform the standard BIOS input/output services. These services are invoked by our programs with a combination of an interrupt number (which indicates the subject of the service request, such as video services) and service number (which indicates the specific service to be performed). The other side of the BIOS communicates with the computer's hardware devices (display screen, disk drives, etc.), using whatever detailed command codes each device requires. This side of the BIOS also handles any hardware interrupts that a device generates to get attention. For example, whenever we press a key, the keyboard generates an interrupt to let the BIOS know. There are twelve ROM-BIOS interrupts in all. Most of the interrupts are tied to a group of subservices that actually do the work. The video services interrupt 16 (hex 10) has sixteen subservices that provide nearly all the services that are needed to generate display-screen output, control the cursor, and manipulate screen information [59]. In order to make direct use of the ROM-BIOS services from our programs, we need to create an assembly-language interface routine to link our programming language to the ROM-BIOS. For example, the following assembly module may be used to erase the entire display and position the cursor at the home position.

```
CODE    SEGMENT 'CODE'
        ASSUME  CS:CODE
        PUBLIC  CLS
CLS     PROC    FAR
        PUSH    BP              ;save caller's frame pointer
        MOV     BP,SP           ;save stack pointer
        MOV     AH,0FH          ;get current video mode
        INT     10H             ;request video service
        MOV     AH,00H          ;set video mode
        INT     10H             ;request video service
        MOV     SP,BP           ;restore stack pointer
        POP     BP              ;restore caller's frame pointer
        RET                     ;return to caller
CLS     ENDP
CODE    ENDS
        END
```

Assembly language and high-level language each have their own benefits and drawbacks. Compared with high-level language, assembly language requires more work to write because it requires more lines of program code to accomplish the same end, and it is more error-prone because it involves lots of niggling details. One important drawback of assembly language is that it requires more expertise to write than most high-level languages. However, there are important advantages to it as well: assembly language programs are usually smaller and run faster (Table 2.1 shows the size and speed comparison between of the two subroutines described above), because we use our skills to find efficient ways to perform each step, while high-level languages generally carry out their work in a plodding unimaginative way. Also, using assembly language we can tell the computer to do anything it's

Table 2.1

Size and Speed Comparison between of CLS.FOR and CLS.ASM

| | CLS.FOR | CLS.ASM |
|---|---|---|
| Size of Object File (Byte) | 400 | 66 |
| Time Spent Executing 100 Times (Sec.) | 0.87 | 0.49 |

capable of doing because assembly language provides us with full access to the ROM-BIOS and DOS services, while high-level languages normally don't give us a way of performing all the tricks that the computer can do. Broadly speaking, we can say that high-level languages let us tap into 90 percent of the computer' skills, while assembly language lets us use 100 percent, if we're clever enough. This illustrates an important point in the creation of professional-quality programs: often the best programming is done primarily in a high-level language (such as Fortran or Pascal), with assembly language used as a simple and expedient means to go beyond the limits of the high-level language.

By using the ROM-BIOS and DOS services through the assembly language, many general-purpose subroutines are developed by the author. They will be applied to the package to enhance its performance. The following is general descriptions of some subroutines.

| | |
|---|---|
| BEEP | Beep |
| CGA | Set video mode for Colour/Graphics Adapter |
| CHKKBD | Check whether a key has been pressed or not |
| CLS | Clear the entire screen and return the cursor to home position |
| CLSWD | Clear a specified window (part of the screen) |
| DOSVER | Get DOS version number |
| FREESP | Get disk free space |
| HOME | Return the cursor to home position |
| KEYIN | Returns the equivalent ASCII code if a key is pressed at the keyboard |
| LINEH | Draw a horizontal line |

LINEV Draw a vertical line

LOCATE Move the cursor to a given line and column of the screen

LOCERA Erase characters from a given position to the end of line and return the cursor to the given position

MONO Set video mode for Monochrome Adapter

PROMPT Display characters with certain attribute such as blinking reverse video

RCURXY Report current cursor position

PRTSC Print screen

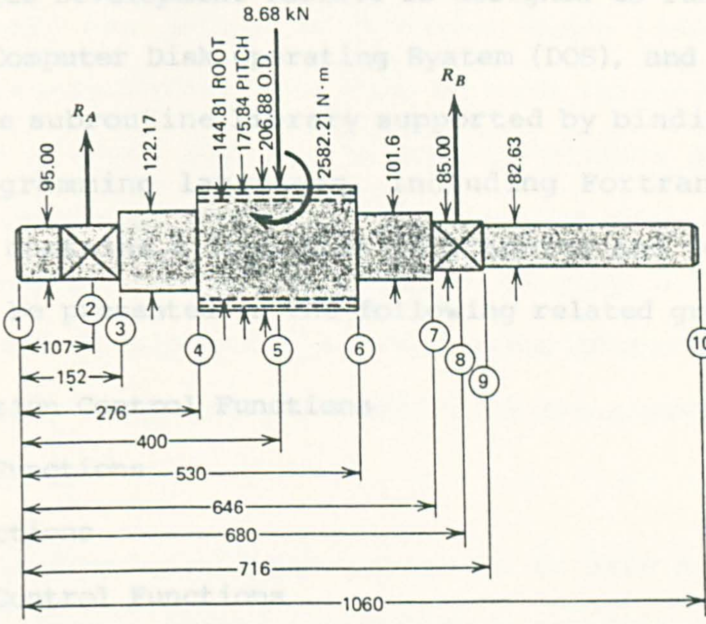
RIGHT Move the cursor right one column

SBOX Draw a box of single-line type

### 2.2.3 Graphics Package

Computer graphics is the most versatile and most powerful means of communication between a computer and a human being. When the results of computation are presented using graphics, complex data is easier to digest and analysed, and relationships become immediately evident. Integrating graphics into data analysis can have positive effect on design quality of product. Compare the two data presentation formats shown in Figure 2.1. The graphic representation of the data will quickly get the attention of the designer, who will locate the critical positions of the shaft.

At present there are many graphics packages available, for a number of computer systems and graphics devices, written in different languages and serving many different purposes. Some are for highly specialised applications, while others are written for varied uses but are specific to the computer systems on/which they are designed to run. The graphics package used for developing the gearbox design programs is the IBM Personal




A shaft with integral worm, dimensions in millimeters.

| DEFLECTIONS (MM) AT ALL NODES ALONG THE SHAFT | | | | |
|---|---|---|---|---|
| .3800E-02[01] | .0000E+00[02] | -.1603E-02[03] | -.6125E-02[04] | -.1086E-01[05] |
| -.1194E-01[06] | -.3792E-02[07] | .1164E-08[08] | .4132E-02[09] | .4361E-01[10] |

| SLOPES (RAD) AT ALL NODES ALONG THE SHAFT | | | | |
|---|---|---|---|---|
| -.3552E-04[01] | -.3552E-04[02] | -.3584E-04[03] | -.3739E-04[04] | -.3909E-04[05] |
| .1686E-04[06] | .1050E-03[07] | .1148E-03[08] | .1148E-03[09] | .1148E-03[10] |

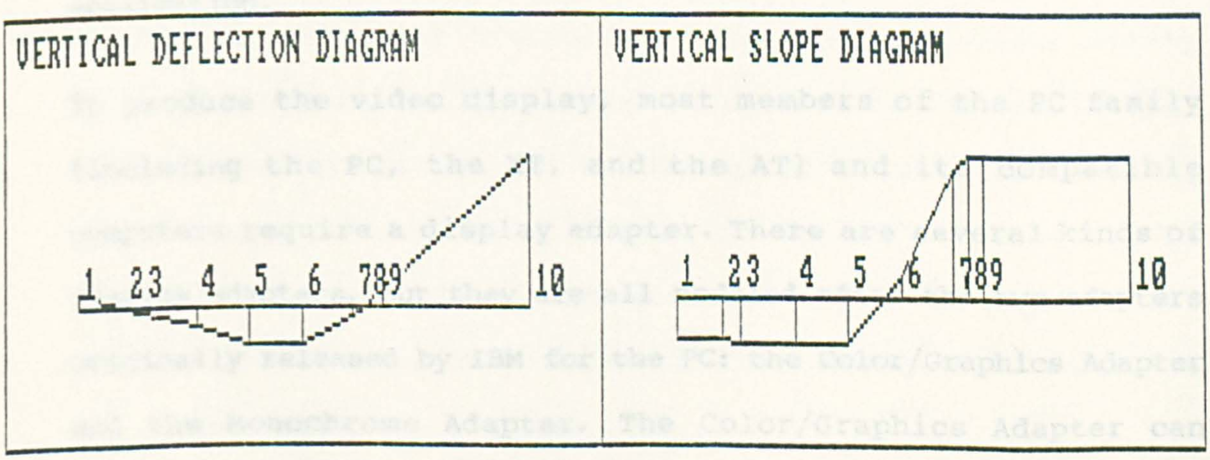



Figure 2.1 Two presentations of the same data

Computer Graphics Development Toolkit [34].

The Graphics Development Toolkit is designed to run under the IBM Personal Computer Disk Operating System (DOS), and implemented as a linkable subroutine library supported by bindings to several major programming languages, including Fortran, Pascal, and BASIC. It contains a long list of graphics and text functions, which may be presented in the following related groups:

Workstation Control Functions

Paging Functions

Pel Functions

Cursor Control Functions

Graphic Functions

Alpha Text Functions

Input Functions

Error Handling

The Graphics Development Toolkit is also device-independent. Therefore, we can direct our application program output to any supported workstation (an input and/or output graphics device such as displays and printers) without having to modify the application.

To produce the video display, most members of the PC family (including the PC, the XT, and the AT) and its compatible computers require a display adapter. There are several kinds of display adapters, but they are all modeled after the two adapters originally released by IBM for the PC: the Color/Graphics Adapter and the Monochrome Adapter. The Color/Graphics Adapter can operate in both text and graphics modes to produce both drawings and characters in several formats and colors. By contrast, the

Monochrome Adapter can operate only in text mode, using a stored set of ASCII alphanumeric and graphics characters and displaying them in only one color. To overcome these limitations, some hardware manufacturers have come up with variations of the IBM Monochrome Adapter, such as the popular Hercules display adapter, which successfully combines the graphics (but not the color) capabilities of the Color/Graphics Adapter with the higher-quality text display of the Monochrome Adapter. The Enhanced Graphics Adapter can create graphics on the monochrome screen in a similar way.

The type of display adapter that might be used has an important effect on program design. The Graphics Development Toolkit only supports Color Graphics Adapter and Enhanced Graphics Adapter, but does not support Hercules display adapter. To make up this weakness, a memory resident utility has been introduced, which simulates Color Graphics Adapter with Hercules Monochrome Adapter, and allows the package to be able to run in graphics mode using Hercules Monochrome Adapter.

## 2.3 Selection of Computer Operation Mode

In general, there are two modes of computer operation, namely batch mode and interactive mode. Batch mode is the traditional way of using a computer. For any design problem where computer aid is being considered it should be the first method to be examined, as in many cases it will be found that it is perfectly adequate and that the more sophisticated techniques of usage are not necessary.

An essential feature of batch program running is that the control of the program is outside the control of the user. Once the computer starts processing a set of data it continues to completion or until some error situation occurs when it terminates. The user has no method of correcting such errors during that run. He must wait until he receives his output, alter his input and run the program again from the beginning.

Interactive mode allows the user to communicate directly with a computer, usually send data or instruction via the keyboard and receive output from the display screen or printer. By these means, the user and the computer can work interactively, modifying and improving the design and correcting errors without having to wait for the final results to be presented to him. This mode is an essential part of a computer-aided design practice.

An analysis of mechanical engineering products reveals that they are made from basic entities such as structures and machine elements. The machine elements can be divided into gears, shafts, bearings, seals, connecting rods and other similar components. In addition, basic situations will exist, such as stress concentration, vibration, lubrication and contact stress. Properties of materials are involved implicitly in all the basic entities and situations. Each entity and situation is a design segment and the designer when synthesizing a possible solution will include a number of these segments which will interact with each other. Therefore, the most efficient way of using the computer in the mechanical engineering design process would appear to be to allow the designer and computer to interact in a conversational mode to evolve an acceptable design by providing the designer with conversational programs and a personal computer system or a personal console having on-line remote access to a large time-sharing multi-access computer system, the computer providing the results of analysis on-line and the designer making decisions and choices based on judgment from the computer's answers. In this way the user is able to interrogate more data and analyse more trial runs, so giving him much more information on which he himself can make the design decisions.

The gearbox design programs developed operate in an interactive mode. Figure 2.2 indicates an outline of the whole gearbox design and draughting package when it is linked with a computer. It will be noticed that the computer is essentially used to complement the designer. At the end of each activity, the designer is able to study the design results before processing to the next stage or re-running the programs if he is not satisfied with the results. In this way, the designer is given considerable freedom to reach an acceptable solution using his ability and experience. The designer's skill at making important design judgements is fully employed while time-consuming and unproductive iterative calculations are left to the computer. A more automated approach involving less interaction with the designer would in general be unsatisfactory for dealing with such design problems.

## 2.4 Program Structure

The design part of the package comprises several programs for the various design steps. Each program can be run separately and used for any design application, but in its intended role, the individual design programs would be controlled by a master program. The modular program structure provides the user with clearly defined options and also aids program development and maintenance. Figures from 2.3 to 2.6 show the main menu and three sub-menus of the design part respectively.

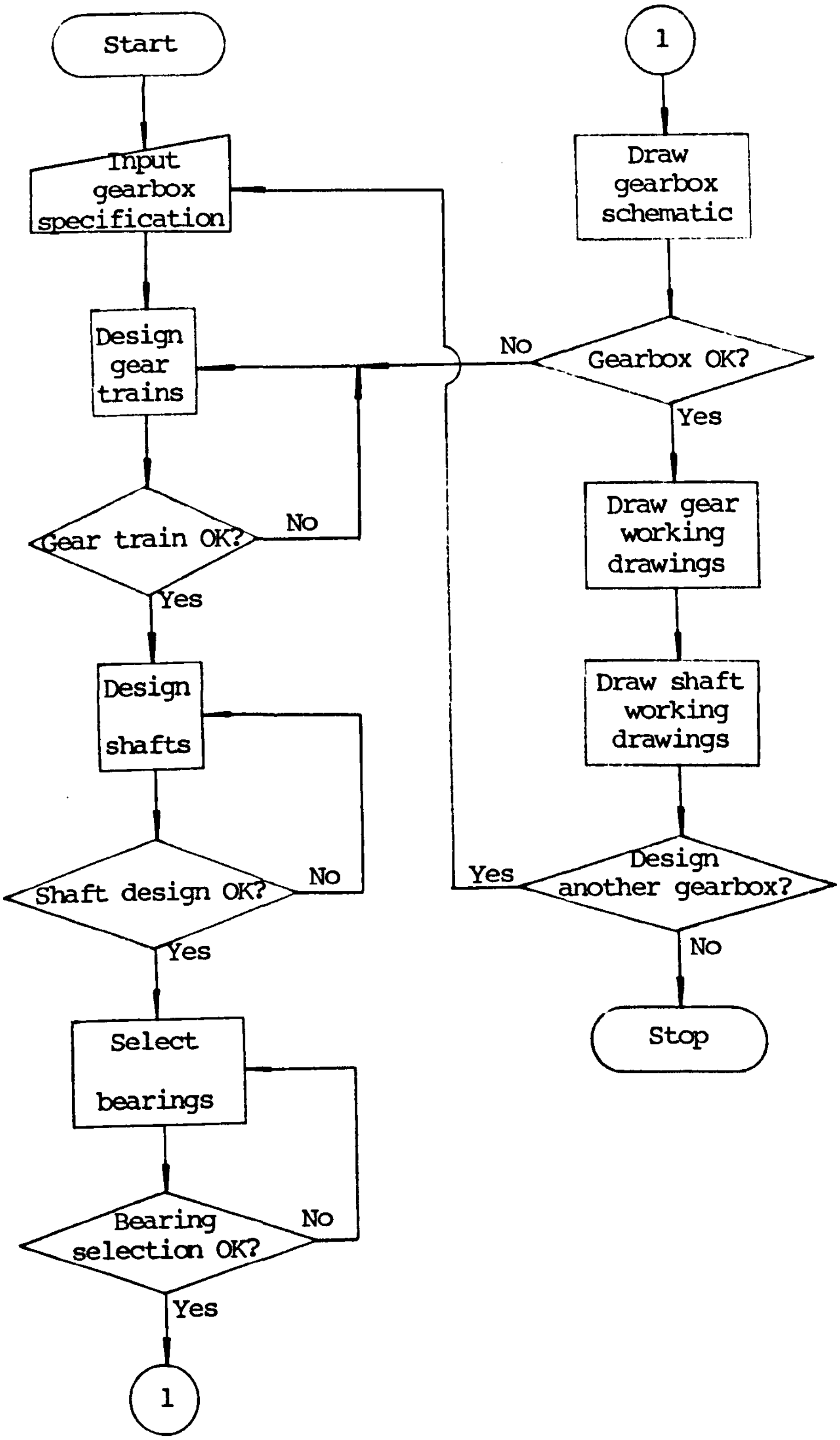



Figure 2.2 Flowchart for gearbox computer package

```
COMPUTER  AIDED  GEARBOX  DESIGN  MENU

GEAR TRAIN DESIGN.................. 1
SHAFT DESIGN....................... 2
BEARING SELECTION.................. 3
QUIT............................... 4

PLEASE CHOOSE THE CODE NUMBER
```

Figure 2.3 Main menu of the gearbox design programs

```
COMPUTER  AIDED  GEAR  DESIGN  MENU

FIND TOOTH NUMBERS FOR SPECIFIED GEAR RATIO........1
GENERATE GEAR TOOTH...............................2
DRAW GEAR TOOTH PROFILE...........................3
PRODUCE FINITE ELEMENT GRID OF GEAR TOOTH.........4
CALCULATE GEOMETRICAL SIZES OF GEAR PAIR..........5
CHECK CONTACT AND BENDING STRENGTHS OF GEAR SET....6
DESIGN GEAR TRAIN.................................7
QUIT..............................................8

PLEASE CHOOSE THE CODE NUMBER
```

Figure 2.4 Sub-menu 1 of the gearbox design programs

```
COMPUTER  AIDED  SHAFT  DESIGN  MENU

CHECK SHAFT........................1
DESIGN SHAFT.......................2
QUIT...............................3

PLEASE CHOOSE THE CODE NUMBER
```

Figure 2.5 Sub-menu 2 of the gearbox design programs

```
COMPUTER AIDED BEARING SELECTION MENU

CHECK BEARING...................... 1
SELECT BEARING..................... 2
QUIT............................... 3

PLEASE CHOOSE THE CODE NUMBER
```

Figure 2.6 Sub-menu 3 of the gearbox design programs

When this computer software runs, input data is typed on the keyboard and output appears on the screen. Both input data and output results will also be recorded in computer files, which may be printed as a permanent record of the design process.

The design programs have many inbuilt user features, which are designed to make the programs as foolproof as possible and to give clear unambiguous instructions to the user. These features are as follows:

(1) An input data language easily interpreted by the designer is employed.

(2) Inputting data takes the form of making a choice wherever possible.

(3) Making a choice by typing a single key without having to press <RETURN>.

(4) There are a number of safeguards made available against wrong entries. For example, the input routine will ignore any alphabetic entry when the numeric entry is expected. Thus accidental typing errors are prevented from causing program failure.

(5) Detection and warning of inconsistent data and data which could cause program failure. For instance, in the bearing selection program the entry of a bore diameter less than zero or not a standard value will result in BEEP or the response

    NO BEARING HAS A XXX MM BORE DIAMETER. THE STANDARD BORE DIAMETERS NEAR TO THE REQUIRED VALUE ARE XXX AND XXX MM. PLEASE INPUT DATA AGAIN.

    Where technical restrictions are known, either through experience or theory, these have been written into the program.

(6) The input data can be corrected by using the editing facility of the program.

(7) The output is clearly labelled and self-explanatory.

(8) Whenever the user is unsatisfied with the design results, he can modify any of the original parameters without having to terminate the design process and re-enter all items of data so that he is able to see the effect of the change immediately.

(9) Optional displaying of data and/or results such as the detail information on gear pair design.

(10) Optional printing of graphical output such as shaft slope and deflection diagrams.

As the software is fully interactive, all the user needs to do is to sit in front of a computer, start the programs and then input data in reply to requests displayed on the screen in front of him. Ideally, the user should not need to know any more about the programs than what he is told by means of the screen. However, it is reasonable for the program writer to assume that the user will have a general knowledge related to gear design, shaft design, and bearing selection. Because it is so quick to input the data to a problem the user is encouraged to seek alternative solutions.

# Chapter 3

## TOOTH NUMBER CALCULATIONS

### 3.1 Introduction

In most problems in gear design, the velocity ratio has a tolerance of a few per cent, and in those cases there is rarely any difficulty in selecting numbers of teeth with an acceptable ratio. On the other hand, it is sometimes necessary, in the setting-up of machine tools for screw cutting, spiral milling, gear cutting, and so on, to discover at least one combination of 'change gears' with a velocity ratio that departs from the specified figure by no more than 1 part in 100,000 or so.

So close an approximation in a single pair of gears would usually demand that each of them should have several hundreds of teeth with the disadvantage of low load capacity in relation to diameters and width. This difficulty is avoided by using a two-stage train and then any velocity ratio up to about 8 to 1 can be closely approximated without using any gear with more than 100 teeth. Where exceptionally fine tolerance is imposed, a three-stage train may be considered, as it offers a much larger number of approximating ratios.

The general problem is to discover a fraction which gives the required ratio, such that the numerator and denominator can be factorized. The terms are then rearranged and multiplied by trial until suitable combinations are found. It is always possible to find a fraction which will give, if not the exact ratio, a result to any desired degree of approximation, provided that a sufficiently large number of pairs of gears can be used.

A great deal of work has been done in investigating means of solving problems of this type with very fine tolerance. Tables of change-gear combinations are published by some machine-tool manufacturer. That issued by the Pfauter Company, for example, comprises more than 200 pages listing upwards of 26,000 combinations covering ratios between 0.1 and 1.0. The greatest departure from any ratio that might be specified is about 20 in 100,000 for ratios near 0.1, 3 in 100,000 for ratios around 0.5, and 1 in 100,000 for ratios of nearly 1.0, but will in many cases be less [50]. These combinations probably satisfy all practical machine-tool requirements, but they do not exhaust all the possibilities. If a special case demands a closer approximation than tabulated combinations afford, or if a table of change-gears is not available, recourse to other methods becomes necessary. McComb and Matson [48] list five methods, all of which involve cut-and-try procedures. Spotts [73] describes a sixth cut-and-try technique. Orthwein [63] develops a microcomputer program, which is founded upon the theorem that a denumerable infinity of rational numbers can be found between any two real numbers whose difference is greater than zero.

The following sections show that some properties of continued fractions and conjugate fractions will greatly contribute to the solution of the problem, and offers a direct means for finding the required number of teeth on each gear. A numerical method is outlined and three examples of its use terminate the discussion.

## 3.2 Continued Fractions

An expression of the form

$$b_1 + \cfrac{1}{b_2 + \cfrac{1}{b_3 + \cfrac{1}{b_4 + \cdots}}} \tag{3.1}$$

is called a continued fraction [61]. A much more convenient way of writing (3.1) is

$$[b_1, b_2, b_3, b_4, \ldots] \tag{3.2}$$

In (3.2) the numbers $b_1, b_2, b_3, b_4, \ldots$ are called the terms or the partial quotients of this continued fraction. The number of terms may be finite or infinite. If the first term $b_1$ is an arbitrary integer and the terms $b_2, b_3, b_4, \ldots$ are positive integers, we call the continued fraction a simple continued faction. If a simple continued fraction has only a finite number of terms $b_1, b_2, b_3, b_4, \ldots b_n$, it is said to be a finite simple continued fraction. Otherwise, it is called an infinite simple continued fraction.

In the notation introduced, a finite simple continued fraction may be denoted by $[b_1, b_2, b_3, b_4, \ldots b_n]$. If $1 <= i <= n$, we call

$$c_i = \frac{p_i}{q_i} = [b_1, b_2, b_3, b_4, \ldots, b_{i-1}, b_i] \tag{3.3}$$

the ith convergent to $[b_1, b_2, b_3, b_4, \ldots b_n]$. By straightforward calculation it is easy to find that

$$c_1 = \frac{p_1}{q_1} = [b_1] = b_1 = \frac{b_1}{1} \tag{3.4}$$

where $p_1 = b_1$, $q_1 = 1$.

$$c_2 = \frac{p_2}{q_2} = [b_1, b_2] = b_1 + \frac{1}{b_2} = \frac{b_2 b_1 + 1}{b_2} \tag{3.5}$$

where $p_2 = b_2 b_1 + 1$, $q_2 = b_2$.

$$c_3 = \frac{p_3}{q_3} = [b_1, b_2, b_3] = b_1 + \cfrac{1}{b_2 + \cfrac{1}{b_3}}$$

$$= \frac{b_3(b_2 b_1 + 1) + b_1}{b_3(b_2) + 1} = \frac{b_3 p_2 + p_1}{b_3 q_2 + q_1} \tag{3.6}$$

where $p_3 = b_3 p_2 + p_1$, $q_3 = b_3 q_2 + q_1$.

$$c_4 = \frac{p_4}{q_4} = [b_1, b_2, b_3, b_4] = b_1 + \cfrac{1}{b_2 + \cfrac{1}{b_3 + \cfrac{1}{b_4}}}$$

$$= \frac{b_4(b_3 b_2 b_1 + b_3 + b_1) + (b_2 b_1 + 1)}{b_4(b_3 b_2 + 1) + b_2} = \frac{b_4 p_3 + p_2}{b_4 q_3 + q_2} \tag{3.7}$$

where $p_4 = b_4 p_3 + p_2$, $q_4 = b_4 q_3 + q_2$.

Continued fractions have many interesting properties [35, 61], among which the following are of importance to gear applications:

(a) Theorem 1: Any rational number can be represented by a finite simple continued fraction.

(b) Theorem 2: The numerator $p_i$ and the denominator $q_i$ of the ith convergent $c_i$ of the continued fraction $[b_1,b_2,b_3,b_4,\ldots,b_n]$ satisfy the equations

$$p_i = b_i p_{i-1} + p_{i-2}$$
$$q_i = b_i q_{i-1} + q_{i-2} \qquad (i=3,4,5,\ldots,n) \qquad (3.8)$$

with the initial values

$$p_1 = b_1 \qquad q_1 = 1 \qquad (3.9)$$

$$p_2 = b_2 b_1 + 1 \qquad q_2 = b_2 \qquad (3.10)$$

(c) Theorem 3: The odd convergents $c_{2i-1}$ of a simple continued fraction increase strictly with i while the even convergents $c_{2i}$ decrease strictly, and every odd convergent is less than any even convergent. Moreover, each convergent $c_i$, $i>=3$, lies between the two preceding convergents, and the value C of the continued fraction is greater than that of any of its odd convergents and less than that of any of its even convergents (except that it is equal to the last convergent $c_n$, whether this be odd or even) (see Figure 3.1), that is

$$c_1<c_3<c_5<\ldots<c_{2i-1}<\ldots< C = c_n <\ldots<c_{2i}<\ldots<c_6<c_4<c_2 \qquad (3.11)$$

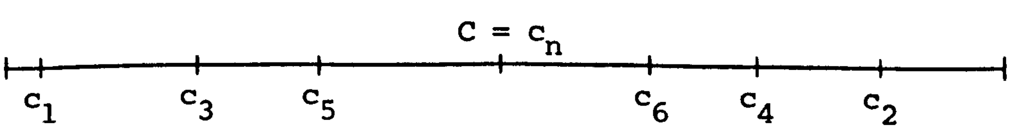



Figure 3.1 Convergents of a simple continued fraction

(d) Theorem 4: Each convergent is nearer to the value **C** of a simple continued fraction than is the preceding convergent, that is

$$ABS(C-c_i) < ABS(C-c_{i-1}) \qquad (i>=2) \qquad (3.12)$$

or

$$ABS(C-\frac{p_i}{q_i}) < ABS(C-\frac{p_{i-1}}{q_{i-1}}) \qquad (i>=2) \qquad (3.13)$$

(e) Theorem 5: Every convergent of a simple continued fraction is in its lowest terms, that is, $p_i$ and $q_i$ have no common divisors other than +1 or -1.

(f) Theorem 6: If $p_i = b_i p_{i-1}+p_{i-2}$ and $q_i = b_i q_{i-1}+q_{i-2}$ are defined as in Theorem 2, then

$$p_i q_{i-1}-p_{i-1}q_i = (-1)^i \qquad (i>=2) \qquad (3.14)$$

---

Nomenclature

| | |
|---|---|
| ABS(x) | Absolute value of **x** |
| INT(x) | The largest integer not greater than **x** |
| MAX(i,j) | Integer maximum |
| MOD(i,j) | Integer remainder |
| NO | Array containing the number of the solutions obtained on the left and right side of the required gear ratio **R** |
| KF | Array containing all the primes less than 200 |
| NF | The number of the prime factors of **n** |
| LF | Array containing **NF** prime factors of **n** |

### 3.3 Conjugate Fractions

Two fractions **a/b** and **c/d** are described as conjugate to each other if

$$ad-bc = \pm 1 \qquad (3.15)$$

Conjugate fractions have the following important properties [50]:

(a) Theorem 7: Between two conjugate fractions **a/b** and **c/d** there cannot be a fraction with smaller numerators or denominators.

(b) Theorem 8: Between two conjugate fractions **a/b** and **c/d** the fraction **(a+c)/(b+d)** has the smallest numerator and denominator and is conjugate to both **a/b** and **c/d**.

(c) Theorem 9: For all positive values of **K** the value of the fraction **(aK+c)/(bK+d)** lies between **a/b** and **c/d** and the fraction is conjugate to **a/b**.

(d) Theorem 10: For all positive values of **K** the value of the fraction **(aK-c)/(bK-d)** lies beyond the range **a/b** to **c/d** and the fraction is conjugate to **a/b**.

### 3.4 Statement of Problem

The required ratio **R** usually may be a rational number or an irrational number. If **R** is rational, we can always find a rational fraction $p_i/q_i$ as close as we please to **R** (Theorem 1 and 4); in other words, if **T** is the tolerance on **R**, however small, we can always find relatively prime integers $p_i$, $q_i$, such that $ABS(R-p_i/q_i) <= T$. When **i** equals to **n**, the difference between **R** and $p_i/q_i$ will reach zero (Theorem 3). If **R** is irrational, for example, which is calculated from a formula that includes a trigonometrical function, it means that R cannot be expressed as the ratio of two integers, however large they may be. It is then necessary to find a rational fraction $p_i/q_i$ which gives an acceptable approximation to the theoretical value of R. As the difference between R and $p_i/q_i$ is impossible to be zero when R is an irrational number, in what follows, we shall always suppose that the required gear ratio R is a rational number.

In practice, there is always a preferred limited value M to the number of teeth, therefore both numerator and denominator of the fraction suitable for a gear train must be less than or equal to their limited value $M^s$ (where s is the stage number of the gear train) (Theorem 5) and resolvable into acceptable tooth numbers for a compound gear train.

To sum up, the problem to be solved may be stated as: find a rational fraction $p_i/q_i$ which is close enough to the required ratio R with an error not greater than the tolerance T, and whose numerator $p_i$ and denominator $q_i$ are both within the limits of 1 to $M^s$ and can be suitably factorized into acceptable tooth numbers.

### 3.5 Method Description

The method described below is based on a combination of the properties of continued fractions and conjugate fractions, and consists of three steps:

Step 1: deriving a series of fractions ($p_1/q_1$, $p_2/q_2$, $p_3/q_3$, ...) from continued fraction theories until the maximum numerator or denominator less than or equal to its limit $M^s$ is obtained;

Step 2: expanding the last two fractions (which are redefined as $P_1/Q_1$ and $P_2/Q_2$) obtained into a series of conjugate fractions to the specified maximum numerator or denominator $M^s$;

Step 3: testing the fractions thus obtained until the nearest fractions having suitable factors on each side of R are located. The nearer of these is the best possible approximation within the limits of the gear train.

## 3.6 Program Description

The whole computer program TOOTHN may be divided into three subprograms relative to three steps of the method. Figure 3.2 is a flowchart of the main program. Input data to program TOOTHN is

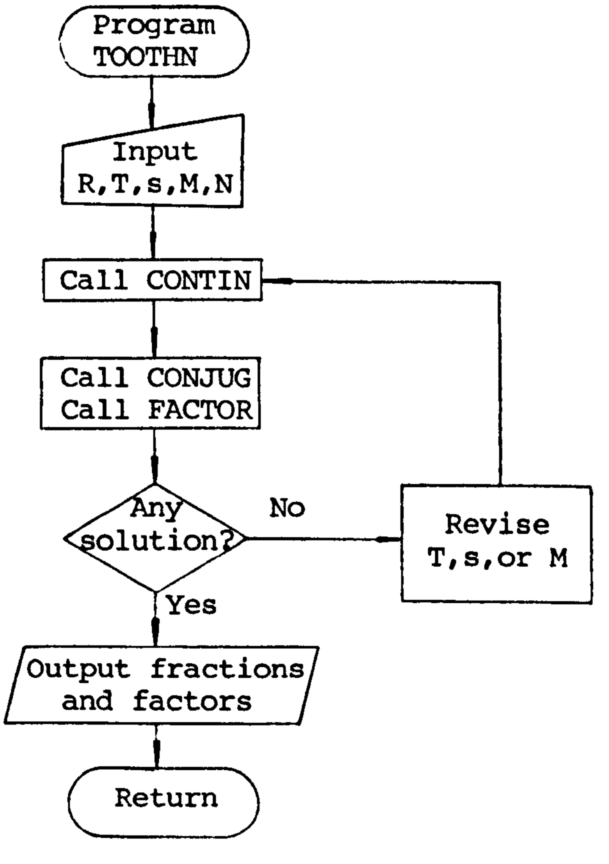



Figure 3.2 Flowchart for the program TOOTHN

the required rational gear ratio **R**, its tolerance **T**, the desired stage number **s** of the gear train, the specified maximum tooth number **M**, and the required number **N** of the solutions on each side of **R**.

The subprogram CONTIN produces the last two fractions **P1/Q1** and **P2/Q2**, in which the maximum of the numerators and denominators is less than or equal to the limit $M^s$. The search for **P1/Q1** and **P2/Q2** is begun by defining quantities $a_1$, $b_1$, $p_1$, and $q_1$ as

$$a_1 = R \qquad b_1 = INT(a_1)$$
$$p_1 = b_1 \qquad q_1 = 1 \tag{3.16}$$

a) If $a_1 = b_1$, the process stops because **R** is an integer.

b) If $a_1 \neq b_1$, quantities $a_2$, $b_2$, $p_2$, and $q_2$ are then defined according to

$$a_2 = \frac{1}{a_1 - b_1} \qquad b_2 = INT(a_2)$$
$$p_2 = b_2 p_1 + 1 \qquad q_2 = b_2 \tag{3.17}$$

a) If $a_2 = b_2$, $P_1/Q_1 = p_1/q_1$ and $P_2/Q_2 = p_2/q_2 = R$.

b) If $MAX(p_2, q_2) > M^s$ and $ABS(R - p_2/q_2) > T$ (see Figure 3.3), there exists no solution to the given condition (Theorem 6 and 7) and we have to revise **T**, **s**, or **M**.

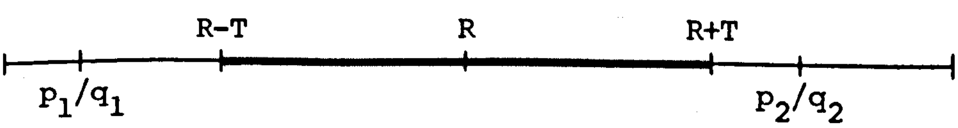



Figure 3.3 Location of $p_1/q_1$ and $p_2/q_2$ without solution

c) If $MAX(p_2,q_2) > M^s$ and $ABS(R-p_2/q_2) <= T$ (see Figure 3.4), we must determine the fraction $p_2/q_2$ which is conjugate to $p_1/q_1$ and meets the condition of $MAX(p_2,q_2) <= M^s$ by the relations

$$q_2 = M^s \quad (R<1) \qquad \text{or} \qquad q_2 = INT(\frac{M^s-1}{p_1}) \quad (R>1)$$

$$p_2 = \frac{p_1 q_2+1}{q_1} \tag{3.18}$$

Then we redefine the fraction nearer to R as P2/Q2 and the other as P1/Q1. In case ABS(R-P2/Q2) > T, T, s, or M must be revised because of no solution, otherwise we get into the subprogram CONJUG.

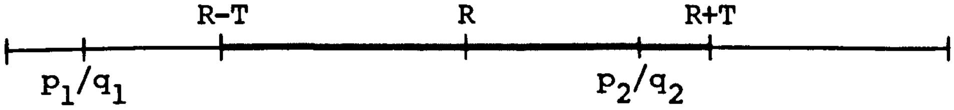



Figure 3.4 Location of $p_2/q_2$ within the tolerance T on R

d) If $a_2 \neq b_2$ and $MAX(p_2,q_2) <= M^s$, the process continues until

$$a_i = b_i \qquad \text{or} \qquad MAX(p_i,q_i) > M^s \tag{3.19}$$

where $a_i$, $b_i$, $p_i$, and $q_i$ are defined as (Theorem 2)

$$a_i = \frac{1}{a_{i-1}-b_{i-1}} \qquad b_i = INT(a_i)$$

$$p_i = b_i p_{i-1}+p_{i-2} \qquad q_i = b_i q_{i-1}+q_{i-2} \qquad (i>=3) \tag{3.20}$$

a)If the process stops when $a_i = b_i$, $P_1/Q_1 = p_{i-1}/q_{i-1}$ and $P_2/Q_2 = p_i/q_i = R$.

b)If the process stops and $ABS(R-p_i/q_i) > T$ when $MAX(p_i,q_i) > M^s$, it implies that there is no solution to the given condition. Otherwise the last two fractions when $MAX(p_i,q_i) <= M^s$ are just P1/Q1 and P2/Q2. The flowchart of subprogram CONTIN is shown in Figure 3.5.

From the properties of continued fractions we know that the two fractions P1/Q1 and P2/Q2 thus obtained have the following characteristics:

(1) P2/Q2 is nearer to the desired ratio R than is P1/Q1 (Theorem 4);

(2) If P2/Q2 is not equal to R, one of the two fractions must be greater than R and the other less than R (Theorem 3);

(3) Both P1/Q1 and P2/Q2 are in their lowest terms (Theorem 5);

(4) P1/Q1 and P2/Q2 are conjugate to each other (Theorem 6).

Using the subprogram CONJUG (see Figure 3.6), a series of conjugate fractions can be built up between and beyond the two conjugate fractions P1/Q1 and P2/Q2 to the specified maximum numerator or denominator $M^s$ by a simple process derived from the properties of conjugate fractions.

For simplicity, we first suppose P2/Q2 is greater than P1/Q1. From Theorem 3, 8 and 9 the nearest fraction A/B less than R and conjugate to P2/Q2 between P1/Q1 and P2/Q2 (i.e. internal expansion, see Figure 3.7) will be

$$\frac{A}{B} = \frac{KP2+P1}{KQ2+Q1} \qquad (3.21)$$

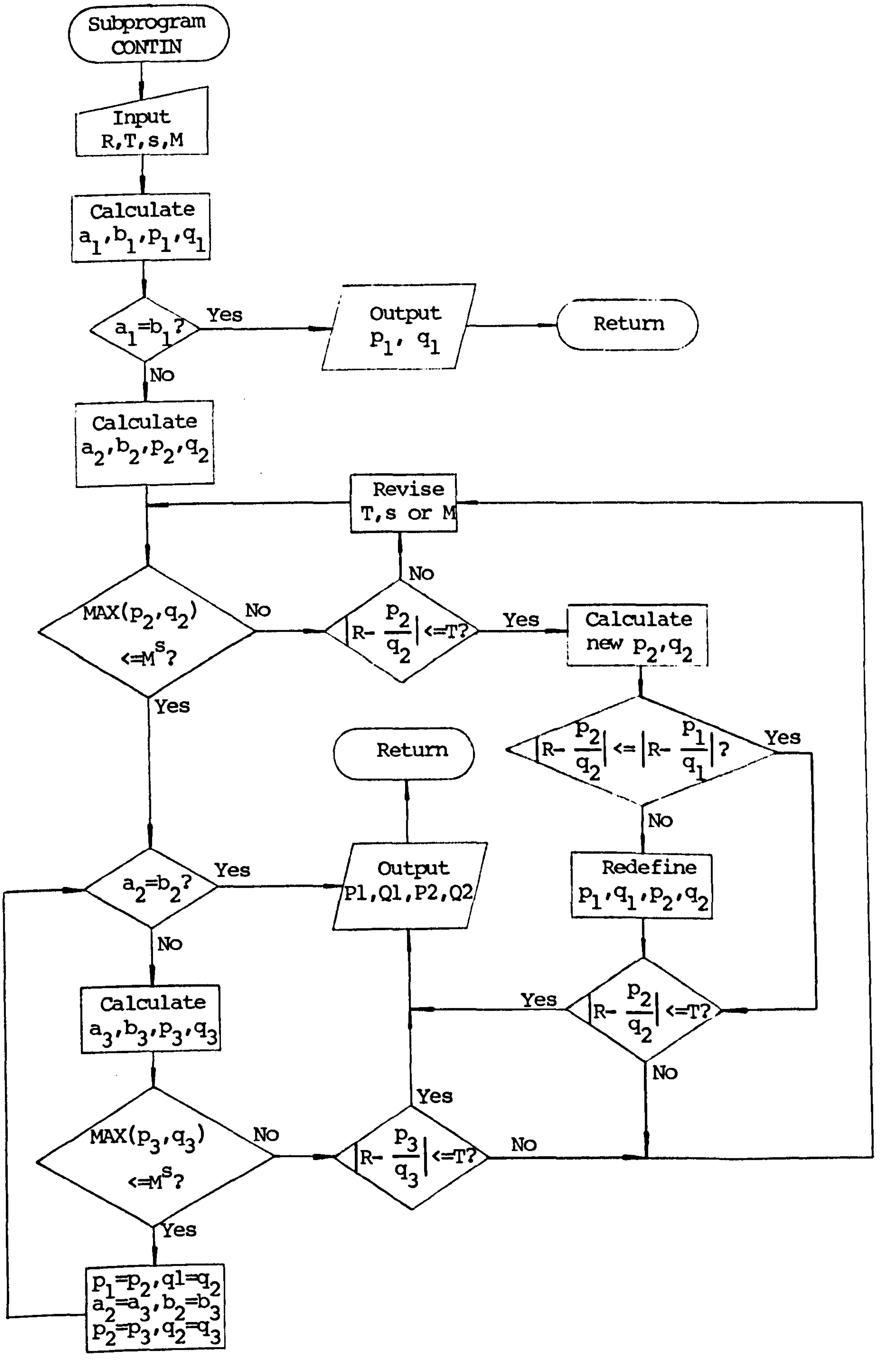



Figure 3.5 Flowchart for the subprogram CONTIN

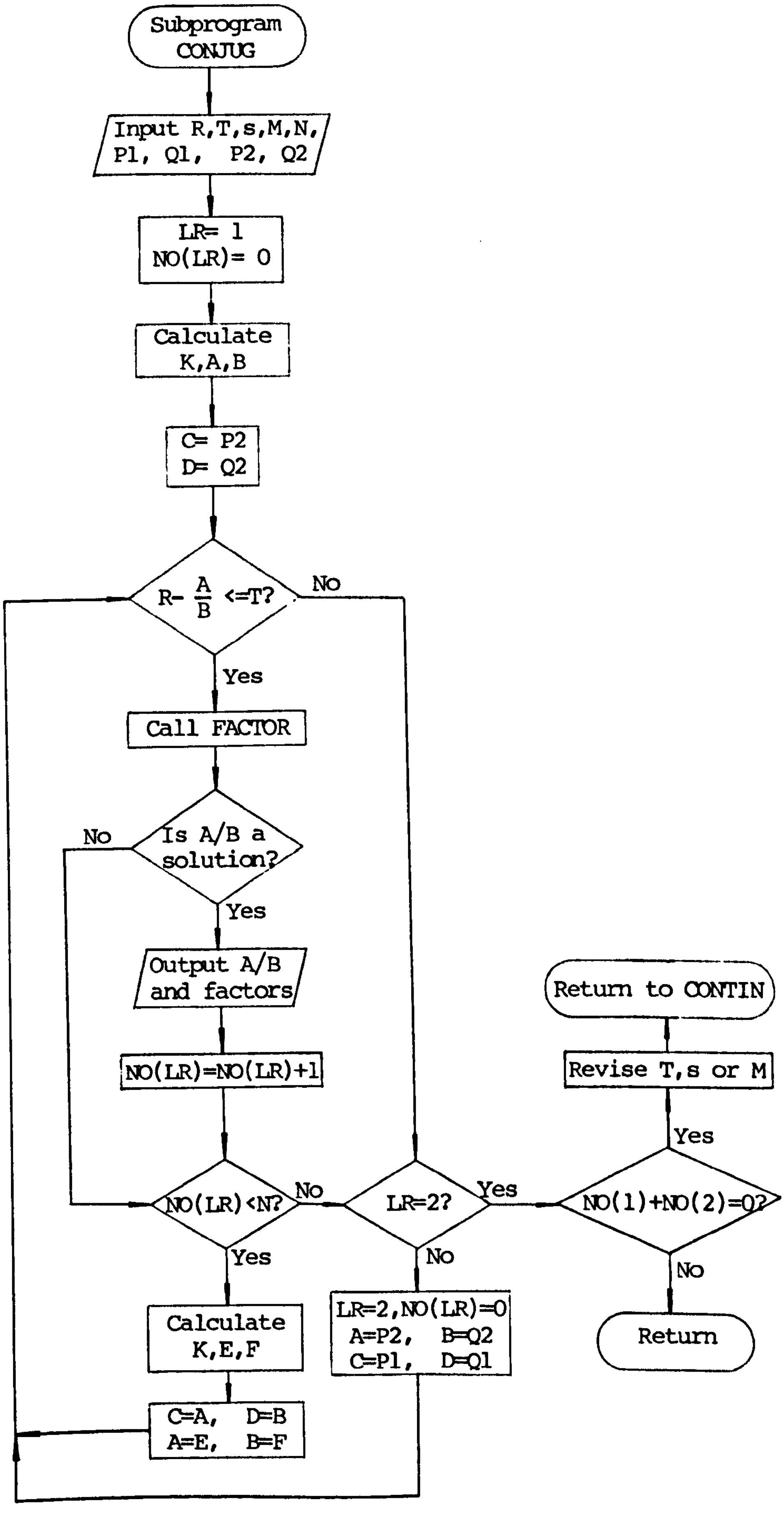



Figure 3.6 Flowchart for the subprogram CONJUG

where

$$K = INT(\frac{M^S - MAX(P1,Q1)}{MAX(P2,Q2)}) \qquad (3.22)$$

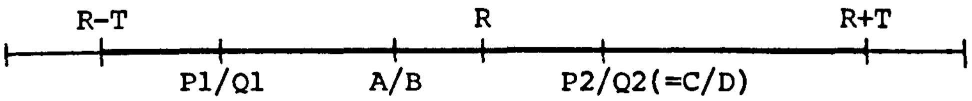



Figure 3.7 Internal expansion between P1/Q1 and P2/Q2

If **K > 0, A/B** is nearer to **R** than is **P1/Q1.** If **K = 0, A/B** is equal to **P1/Q1,** which implies that **MAX(P1+P2, Q1+Q2)** > $M^S$, there is no fraction between **P1/Q1** and **P2/Q2** having both numerator and denominator within the limits of 1 to $M^S$ (Theorem 8), and **P1/Q1** is the nearest fraction less than **R.**

For the further expansion, let **C/D** be equal to **P2/Q2** conjugate to **A/B.** Then the fraction **A/B** is tested by

$$ABS(R-A/B) \quad <= T \qquad (3.23)$$

If the inequality is not true, it means that no solution exists on the left side of **R.** If the inequality holds true, the fraction **A/B** is inspected by the subprogram FACTOR for whether it can be suitably factorized or not. If it can, the first nearest fraction less than **R** is obtained. If it cannot or the required number **N** of the solution on each side of **R** is greater than 1, the further expansion to **A/B** and **C/D** will be carried out. As no solution exists between **A/B** and **C/D,** another nearest fraction less than **R** and conjugate to **A/B** beyond the range **A/B** to **C/D** (i.e. external expansion, see Figure 3.8) should be searched. This fraction **E/F** may be obtained from (Theorem 10)

$$\frac{E}{F} = \frac{KA-C}{KB-D} \tag{3.24}$$

where

$$K = INT\left(\frac{M^S+MAX(C,D)}{MAX(A,B)}\right) \tag{3.25}$$

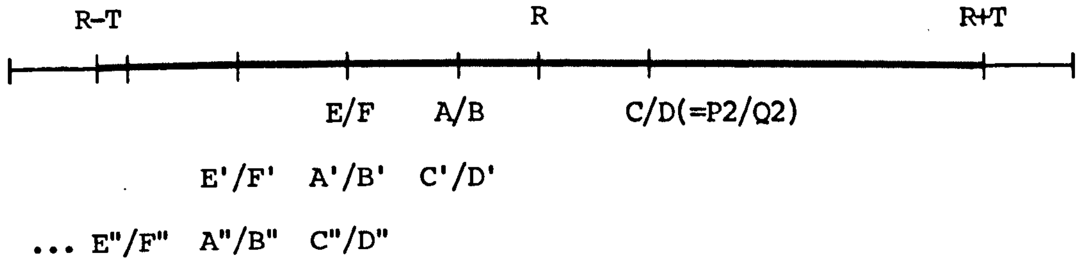



Figure 3.8 External expansion on the left side beyond A/B to C/D

For the convenience of the further external expansion, the original fraction **A/B** and its new conjugate fraction **E/F** are redefined as **C/D** and **A/B** respectively. Then the new fraction **A/B** is tested by the inequality (3.23) and the subprogram FACTOR.

In the same way as stated above, all the new nearest fractions **A/B** less than **R** and conjugate to the old fraction **A/B** can be determined and tested by repetition until the inequality (3.23) is not true or the number of the solution obtained on the left side of **R** is equal to the required number **N**.

After that, we redefine the two conjugate fractions **P2/Q2** and **P1/Q1** obtained from the subprogram CONTIN as **A/B** and **C/D** respectively (see Figure 3.9), and search for the nearest fractions greater than **R** and conjugate to **A/B** beyond the range **A/B** to **C/D** and test them by the inequality (3.23) and the subprogram FACTOR using the same method as described above. In this way, all the possible solutions required on each side of **R** will be found. If there is not any solution to the given condition, we must revise **T**, **s**, or **M**. It is always possible to find a fraction which will give, if not the exact ratio, a result to any desired degree of approximation, provided that a sufficiently large number of pairs of gears or a large tooth number can be used.

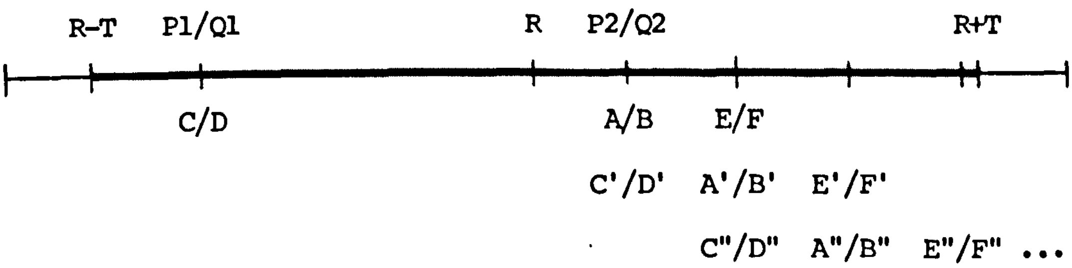



Figure 3.9 External expansion on the right side beyond A/B to C/D

When **P2/Q2** is less than **P1/Q1**, the procedure remarked earlier still holds, but searching for the best possible solution will start from the right side of **R**.

The determination of the factorization of a number into prime factors is achieved by the subprogram FACTOR (see Figure 3.10). The procedure adopted consists in trying out all the lowest primes as possible divisors of the given number **n**. When a prime factor **p** has been found, one can write **n = pm** and determine the factorization of the smallest number **m**. The work is limited by the remarks that if a number is composite it must have a factor not exceeding $\sqrt{n}$, so that only primes $p <= \sqrt{n}$ need be divided into **n**. Another useful observation is that when the smallest prime factor **p** of **n** is found to be greater than $\sqrt[3]{n}$, the other

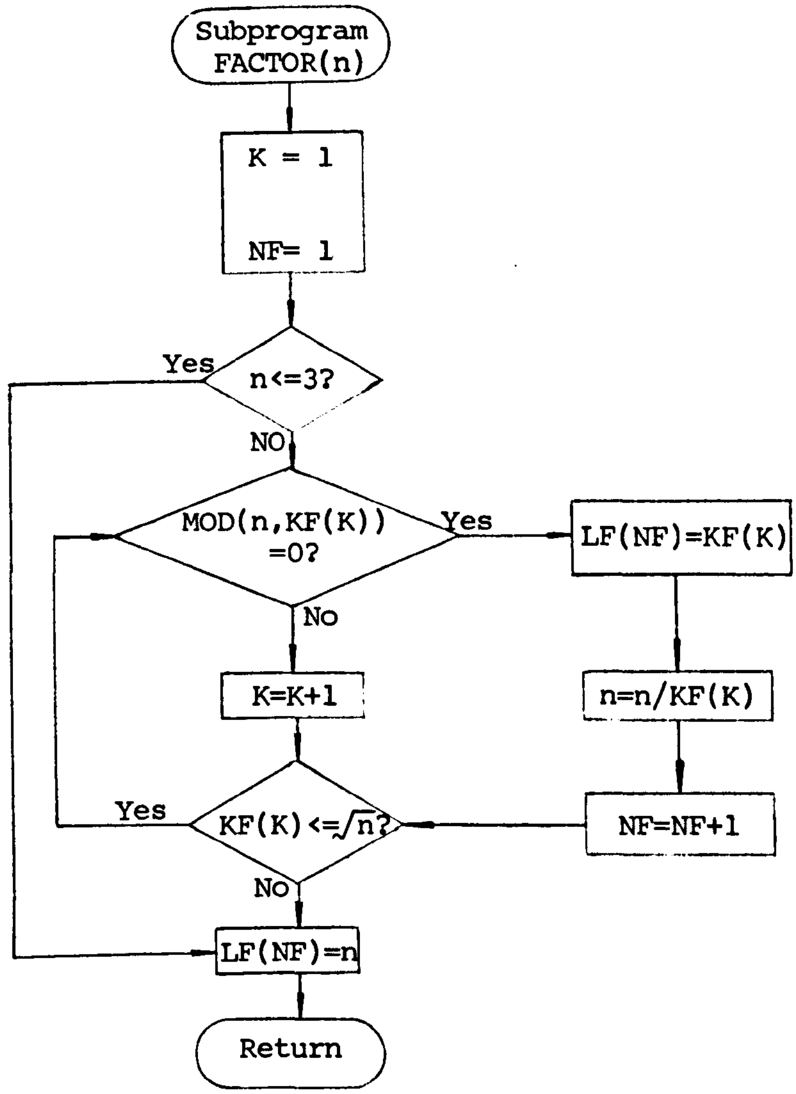



Figure 3.10 Flowchart for the subprogram FACTOR

factor **m** in **n** = **pm** must be a prime [62]. Thus if **m** = **ab** were composite, both **a** and **b** would exceed $\sqrt[3]{n}$, and one would obtain the contradiction

$$n = pab > \sqrt[3]{n}\sqrt[3]{n}\sqrt[3]{n} = n.$$

In order to simplify the program, the inequality $p > \sqrt[3]{n}$ will be expressed as $p > \sqrt{n/p}$ $(= \sqrt{m})$.

Whether a fraction **A**/**B** holds for the problem depends upon whether the prime factors of **A** and **B** can be composed into acceptable gear-tooth numbers. If it is possible, the fraction **A/B** and the prime factors of **A** and **B** are presented, otherwise, for instance, the largest prime factor of **A** or **B** is greater than the limited value **M** of the tooth number, the fraction will be eliminated.

## 3.7 Examples and Discussion

Example 1:

In this example problem we will find the numbers of teeth in each gear in a two-stage gear train which is to have an overall speed ratio of 2.94643 ± 0.0001. There should be at most 200 teeth on each gear.

Input data is **R** = 2.94643, **T** = 0.0001, **s** = 2, **M** = 200 and **N** = 10. Twenty gear ratios are displayed in the second column of Table 3.1 in the form returned to the main program by FACTOR. Among them, the former ten are greater than the required gear ratio **R** and the latter ten are less than **R**. Column Three is the rewritten form of these gear ratios after the factors in numerator **A** and denominator **B** have been grouped in pairs. Column four is the errors of the gear ratios. The computation time required for solving this problem on AMSTRAD PC1512 personal computer approximates 6 seconds.

Table 3.2 derived from [63] is the solutions to the same problem, which may be compared to the results presented in Table 3.1. Comparison shows that the maximum error in Table 3.1 is much less than that in Table 3.2 and the method described above gives the best possible approximation to the required gear ratio within the limits of the gear train.

Table 3.1

| Tooth Combinations to Provide a Gear Ratio of 2.94643 $\pm$ 0.0001 (Stage No.= 2, Tooth No.<= 200 and Desired No. of Solutions = 20) | | | |
|---|---|---|---|
| Ratio Number | Program Output Ratio | Gear Tooth Numbers | Error($\times 10^6$) |
| 1 | $\frac{137 \cdot 163}{11 \cdot 13 \cdot 53}$ | $\frac{137}{53}\ \frac{163}{143}$ | 0.9 |
| 2 | $\frac{13^2 \cdot 193}{2 \cdot 3^3 \cdot 5 \cdot 41}$ | $\frac{169}{90}\ \frac{193}{123}$ | 1.8 |
| 3 | $\frac{2^2 \cdot 47 \cdot 67}{3^2 \cdot 5^2 \cdot 19}$ | $\frac{94}{57}\ \frac{134}{75}$ | 2.7 |
| 4 | $\frac{151 \cdot 157}{2 \cdot 3^3 \cdot 149}$ | $\frac{151}{54}\ \frac{157}{149}$ | 3.0 |
| 5 | $\frac{79 \cdot 149}{5 \cdot 17 \cdot 47}$ | $\frac{79}{47}\ \frac{149}{85}$ | 3.0 |
| 6 | $\frac{7 \cdot 23 \cdot 137}{2 \cdot 19 \cdot 197}$ | $\frac{137}{38}\ \frac{161}{197}$ | 3.3 |
| 7 | $\frac{2 \cdot 3 \cdot 31 \cdot 173}{67 \cdot 163}$ | $\frac{173}{67}\ \frac{186}{163}$ | 3.5 |
| 8 | $\frac{2^2 \cdot 19 \cdot 131}{31 \cdot 109}$ | $\frac{76}{31}\ \frac{131}{109}$ | 3.9 |
| 9 | $\frac{191 \cdot 199}{2^2 \cdot 3 \cdot 5^2 \cdot 43}$ | $\frac{191}{100}\ \frac{199}{129}$ | 4.1 |
| 10 | $\frac{109 \cdot 163}{2 \cdot 3^3 \cdot 5 \cdot 67}$ | $\frac{109}{67}\ \frac{163}{90}$ | 4.5 |

Table 3.1, continued

| Tooth Combinations to Provide a Gear Ratio of 2.94643 $\pm$ 0.0001 (Stage No.= 2, Tooth No.<= 200 and Desired No. of Solutions = 20) | | | |
|---|---|---|---|
| Ratio Number | Program Output Ratio | Gear Tooth Numbers | Error($x10^6$) |
| 11 | $\frac{3 \cdot 5 \cdot 11}{2^3 \cdot 7}$ | $\frac{33}{21} \frac{45}{24}$ | -1.4 |
| 12 | $\frac{2^2 \cdot 43 \cdot 157}{3 \cdot 5 \cdot 13 \cdot 47}$ | $\frac{157}{65} \frac{172}{141}$ | -3.4 |
| 13 | $\frac{2^3 \cdot 7^2 \cdot 47}{13^2 \cdot 37}$ | $\frac{98}{37} \frac{188}{169}$ | -4.3 |
| 14 | $\frac{2 \cdot 83 \cdot 109}{3 \cdot 23 \cdot 89}$ | $\frac{109}{69} \frac{166}{89}$ | -4.3 |
| 15 | $\frac{2^2 \cdot 19 \cdot 199}{3 \cdot 29 \cdot 59}$ | $\frac{76}{59} \frac{199}{87}$ | -4.9 |
| 16 | $\frac{2 \cdot 37 \cdot 191}{3^2 \cdot 13 \cdot 41}$ | $\frac{74}{39} \frac{191}{123}$ | -5.2 |
| 17 | $\frac{151 \cdot 173}{2 \cdot 11 \cdot 13 \cdot 31}$ | $\frac{151}{62} \frac{173}{143}$ | -5.5 |
| 18 | $\frac{7 \cdot 13 \cdot 139}{3^4 \cdot 53}$ | $\frac{91}{53} \frac{139}{81}$ | -5.6 |
| 19 | $\frac{97 \cdot 127}{37 \cdot 113}$ | $\frac{97}{37} \frac{127}{113}$ | -5.7 |
| 20 | $\frac{2 \cdot 59 \cdot 103}{3 \cdot 5^3 \cdot 11}$ | $\frac{103}{55} \frac{118}{75}$ | -5.8 |

Table 3.2

**Tooth Combinations to Provide a Gear Ratio Of 2.94643 ± 0.0001 (18 ≤ N ≤ 200)**

| Ratio Number | Program Output Ratio | Gear Tooth Numbers | Error (x $10^6$) |
|---|---|---|---|
| 1 | $\frac{3(5)11}{(2^3)(7)}$ | $\frac{45}{24}\ \frac{33}{21}$ | 1 |
| 2 | $\frac{(2^4)(79)}{3(11)13}$ | $\frac{32}{22}\ \frac{79}{39}$ | 43 |
| 3 | $\frac{29(127)}{2(5^4)}$ | $\frac{29}{25}\ \frac{127}{50}$ | 30 |
| 4 | $\frac{2(7^2)23}{(3^2)\ 5(17)}$ | $\frac{28}{20}\ \frac{28}{18}\ \frac{46}{34}$ | 25 |
| 5 | $\frac{2(31)47}{23(43)}$ | $\frac{47}{23}\ \frac{62}{43}$ | 19 |
| 6 | $\frac{(2^6)\ 61}{(5^2)\ 53}$ | $\frac{64}{53}\ \frac{61}{25}$ | 15 |
| 7 | $\frac{3(5)11}{23(7)}$ | $\frac{33}{23}\ \frac{20}{28}$ | 14 |
| 8 | $\frac{137(163)}{11(13)53}$ | $\frac{163}{143}\ \frac{137}{53}$ | 1 |
| 9 | $\frac{(2^2)19(131)}{31(109)}$ | $\frac{76}{31}\ \frac{131}{109}$ | 4 |
| 10 | $\frac{(2^2)7(167)}{3\ (23^2)}$ | $\frac{28}{23}\ \frac{167}{69}$ | 10 |
| 11 | $\frac{2(41)53}{(5^2)\ 59}$ | $\frac{53}{25}\ \frac{82}{59}$ | 11 |
| 12 | $\frac{37(113)}{3(11)43}$ | $\frac{37}{33}\ \frac{113}{43}$ | 11 |
| 13 | $\frac{2(19)97}{(3^2)\ 139}$ | $\frac{194}{139}\ \frac{38}{18}$ | 13 |
| 14 | $\frac{2(17)89}{13(79)}$ | $\frac{68}{26}\ \frac{89}{79}$ | 16 |
| 15 | $\frac{2(13^2)\ 7}{11(73)}$ | $\frac{28}{22}\ \frac{169}{73}$ | 21 |
| 16 | $\frac{31(71)}{(3^2)\ 83}$ | $\frac{62}{18}\ \frac{71}{83}$ | 22 |
| 17 | $\frac{17(191)}{2(19)29}$ | $\frac{34}{38}\ \frac{191}{58}$ | 31 |
| 18 | $\frac{(3^4)\ 53}{31(47)}$ | $\frac{81}{47}\ \frac{53}{31}$ | 35 |
| 19 | $\frac{7(173)}{3(137)}$ | $\frac{42}{18}\ \frac{173}{137}$ | 42 |
| 20 | $\frac{(2^2)\ 3(17^2)}{11(107)}$ | $\frac{24}{22}\ \frac{289}{107}$ | 44 |

Example 2:

Find a single pair of gears which provides a velocity ratio of $\sin 45^{\circ} = 0.70711 \pm 0.00005$ with tooth-numbers not greater than 100.

Input data is R = 0.70711, T = 0.0001, s = 1, M = 100 and N = 2. The results are presented in Table 3.3. The computation time is only 0.5 second. From the program output, we know that there exists no suitable fraction greater than the required velocity ratio R and only one solution exists under the given conditions although we hope to obtain four results.

Table 3.3

| Tooth Combinations to Provide a Gear Ratio of 0.70711 $\pm$ 0.00005 (Stage No.= 1, Tooth No.<= 100 and Desired No. of Solutions = 4) | | | |
|---|---|---|---|
| Ratio Number | Program Output Ratio | Gear Tooth Numbers | Error($\times 10^{5}$) |
| 1 | $\frac{70}{99}$ | $\frac{70}{99}$ | -3.9 |

Example 3:

Find alternative two-stage trains which provide a gear ratio of $\sqrt{5} = 2.236068 \pm 0.000001$ without using any gear with more than 120 teeth.

Input Data to the program is R = 2.236068, T = 0.000001, s = 2, M = 120, and N = 2. The program output is shown in Table 3.4. The computation time is about 1 second.

Table 3.4

| Tooth Combinations to Provide a Gear Ratio of 2.236068 $\pm$ 0.000001 (Stage No.= 2, Tooth No.<= 120 and Desired No. of Solutions = 4) | | | |
|---|---|---|---|
| Ratio Number | Program Output Ratio | Gear Tooth Numbers | Error($\times 10^7$) |
| 1 | $\frac{2^2 \cdot 5 \cdot 17 \cdot 19}{3^3 \cdot 107}$ | $\frac{68}{27} \frac{95}{107}$ | -1.6 |
| 2 | $\frac{3^3 \cdot 107}{2^2 \cdot 17 \cdot 19}$ | $\frac{27}{19} \frac{107}{68}$ | 1.1 |
| 3 | $\frac{2^3 \cdot 7^2 \cdot 13}{43 \cdot 53}$ | $\frac{56}{43} \frac{91}{53}$ | 4.5 |

# Chapter 4

## GEAR TOOTH GENERATION

### 4.1 Introduction

Intelligent design of any manufactured product demands some knowledge of the manufacturing process. A necessary preliminary to the study of the detail design of tooth forms is an understanding of the principles underlying the methods by which the teeth will ultimately be produced. It might be assumed that if a tooth is designed, for example, to have an involute profile of certain basic dimensions, the manner in which the tooth form is actually produced is no concern of the designer. Such an assumption, commonly enough encountered, gives a good deal of trouble to those responsible for the production and performance of gears.

The contact surfaces of most practical gearing systems are of involute form. This satisfies the fundamental geometrical requirements imposed by the need for continuous contact and a constant velocity ratio. Because an involute tooth is superficially simple, it does not follow that any involute profile can easily be produced. Furthermore, a tooth profile is not limited to the portion which actually comes into engagement, but extends, as the clearance curve, down to the bottom of the tooth space. The form of the clearance curve is determined by the shape of the cutter in conjunction with the method of production employed, and not only does it influence the strength of the tooth, but it may also limit or interfere with the working portion of the profile.

In modern practice, nearly all involute gear teeth are cut or finished by one or other of a number of 'generating processes'.

This chapter will describe a computer-aided graphics program which portrays the process of two-dimensional (plane) generation of a gear tooth from a rack-type cutter. It is valuable to the understanding of the principles of gear tooth generation, the analysis of the influence of the gear parameters on the tooth forms, and the study of the tooth forms of non-standard and different national gear standards.

## 4.2 Generation of Involute Gear Teeth by Rack-form Cutter

The principle of gear tooth generation is most easily understood by considering the use of a rack-form cutter as a means of producing straight spur teeth.

Figure 4.1 shows the front view of a cutter having a series of identical teeth, equally-spaced in straight-line formation, the space between two adjacent teeth being defined by **ABCDEFGH**. Of the portions of this profile, **AB** and **GH** are concerned only with cutting a clearance space at the bottom of the tooth space. The portions **CD**, **DE** and **EF** do not normally touch the gear blank at all. The parts represented by the straight lines **BC** and **FG** are the edges that generate the working parts of tooth profiles.

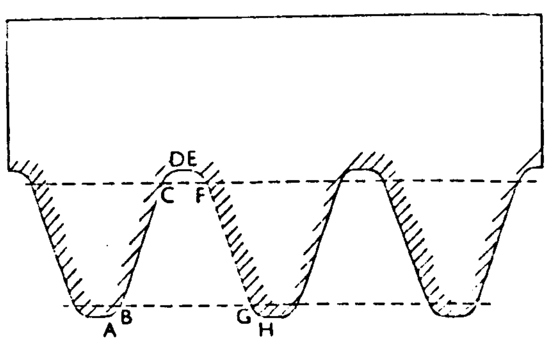



Figure 4.1 Rack-form gear-generating cutter

The arrangement of such a cutter relative to the gear blank is illustrated in Figure 4.2. The cutter **A**, which may be adjusted radially with respect to the axis of the work, is reciprocated in order that its edges may sweep out the surface of the teeth of the imaginary rack forming the basis of the design of the tooth profile of the blank **B**. In addition to this reciprocation, the cutter is advanced in the direction of the pitch line **XPX**, and at the same time the work is rotated about its axis **O** at a speed such that its pitch circle through **P** has the same linear velocity as that of the rack, i.e. the pitch circle of the blank and the pitch line of the rack roll together. For such a process to be continuous the length of the rack cutter would require to be somewhat longer than the pitch circumference of the work; since this is usually impracticable, the cutter is withdrawn from the work after it has advanced a distance equal to one pitch (or an integral number of pitches) and returned to its starting-point, the blank meantime remaining stationary. This cycle is repeated until all the teeth are cut.

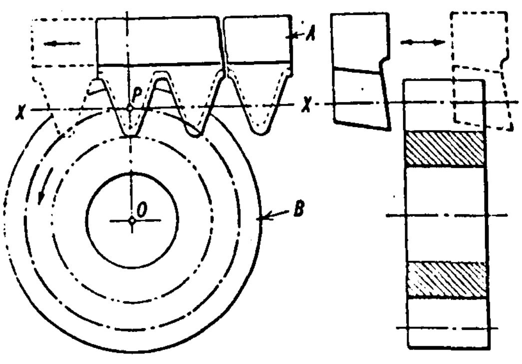



Figure 4.2 Generation of spur gear by rack-form cutter

In principle, the cutter is so set in relation to the gear blank that the distance of its tip line from the centre **O** of the blank is equal to the root radius of the gear to be generated. In practice, it may be impossible to cut the whole depth of tooth in one revolution of the blank; successive revolutions are made with increasing depth of penetration of the cutter until the full depth is attained.

Although the flanks of the teeth of the cutter are straight, those of the teeth generated in the blank are involutes owing to the combination of the lateral displacement of the cutter and the rotation of the blank whilst cutting proceeds. This combination of motions is called 'generation' or 'rolling'. It is characterised by uniform linear velocity of the rack accompanied by uniform angular velocity of the blank.

A brief study of what happens when a rack-form cutter is displaced along the line of centres (i.e. radially) introduces a point of wide application in tooth design. Consider first what may be regarded as the normal position of a cutter in relation to a gear blank, as shown in Figure 4.3 (a), in which the axis of

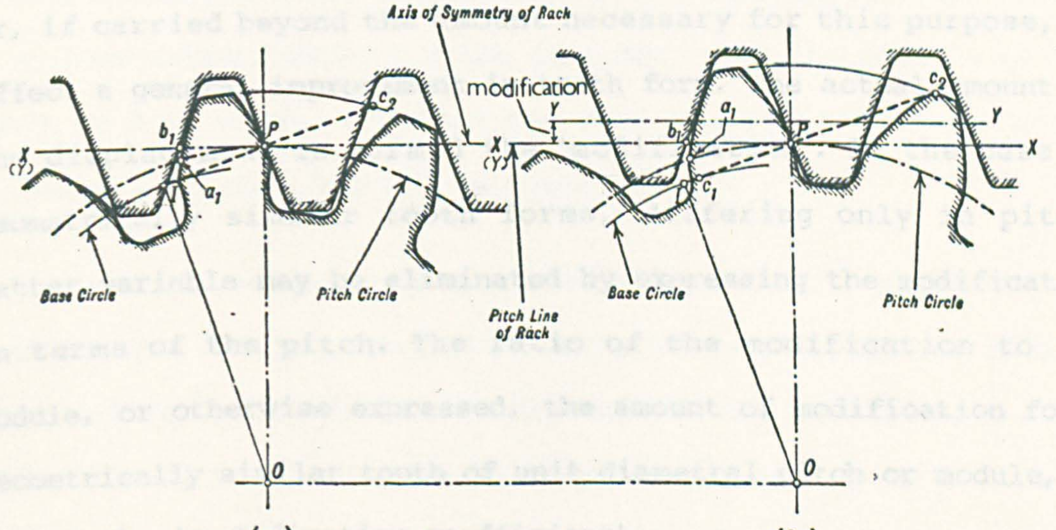



Figure 4.3 Displacement of a rack-form cutter

symmetry YY of the cutter teeth, i.e. the line along which the tooth thickness is equal to the space width, coincides with the pitch line XPX. The thickness of the generated gear teeth $Pa_1$ measured round the pitch circle will then be one-half the pitch, or $Pb_1$, and the addenda of the cutter and the finished gear will be equal.

Now let the axis of symmetry YY of the cutter be moved farther away from the centre of the gear blank, as represented by Figure 4.3 (b). Since the pitch of the cutter remains the same, the circular pitch of the gear, and hence its pitch diameter, will not be altered, and the pitch line of the cutter will still pass through the same pitch point P.

The profiles of the teeth generated in the gear blank will still be involutes to the same base circle, but a different portion of the involute, farther removed from the base circle, is used.

When the axis of symmetry of the rack-form cutter is deliberately shifted during generation, the gear is said to be 'modified'. The purpose of modification may be either to eliminate undercutting or, if carried beyond the amount necessary for this purpose, to effect a general improvement in tooth form. The actual amount of the displacement is termed the 'modification'. In the case of geometrically similar tooth forms, differing only in pitch, the latter variable may be eliminated by expressing the modification in terms of the pitch. The ratio of the modification to the module, or otherwise expressed, the amount of modification for a geometrically similar tooth of unit diametral pitch or module, is termed the 'modification coefficient'.

The teeth generated by any one cutter have a form determined not only by the tooth-form of the cutter and the modification coefficient but by the diameter of the gear blank and the number of teeth generated in the gear. The variety of tooth forms is endless, but the working part of every profile is part of an 'involute'. All involutes generated by any one cutter have a common 'base pitch', and any two of the gears will mesh correctly together at the designed centre distance. The wide range of 'intermeshable' gears that may be produced by a single generating cutter is one of the great practical advantages of gear-generating processes.

## 4.3 Determination of Tooth Profile by Graphical Generation Method

The cutting of gears implies the generation of surfaces. By way of illustration, however, the generation of one plane curve or profile from another may be described, since the process frequently has to be carried out graphically in gear design problems.

Referring to Figure 4.2, suppose that a gear having any pitch circle of centre **O** and pitch point **P** is required to mesh with a rack-form cutter having a pitch line **XPX**, that the profile of the teeth of **A** is known, and that the form of the teeth of **B** (which is shown only as a 'blank') is required.

The graphical solution by generation consists in given to the profile of **A** the property of marking or cutting the surface of the blank **B**; by tracing on or cutting away the surface of **B** along the profile of a tooth of **A** in a number of successive positions of the pitch line and the pitch circle during their relative rolling motion, a tooth profile conjugate to the teeth of **A** results.

The required relative motion of the generating and generated members may be produced in a variety of ways. The actual motions consist of the lateral displacement of **A** and the rotation of **B** about the centre **O**; this corresponds to rolling of the pitch line of **A** and the pitch circle of **B** at the point **P**, but the same relative motion takes place if the pitch circle of **B** is fixed and the pitch line of **A** is rolled round it. This is illustrated in Figure 4.4, in which the pitch circle of **B** has been divided into equal arcs by points **P**, **1**, **2**, **3**, **4**, etc., the pitch line of **A** being similarly divided into line segments of the same length.

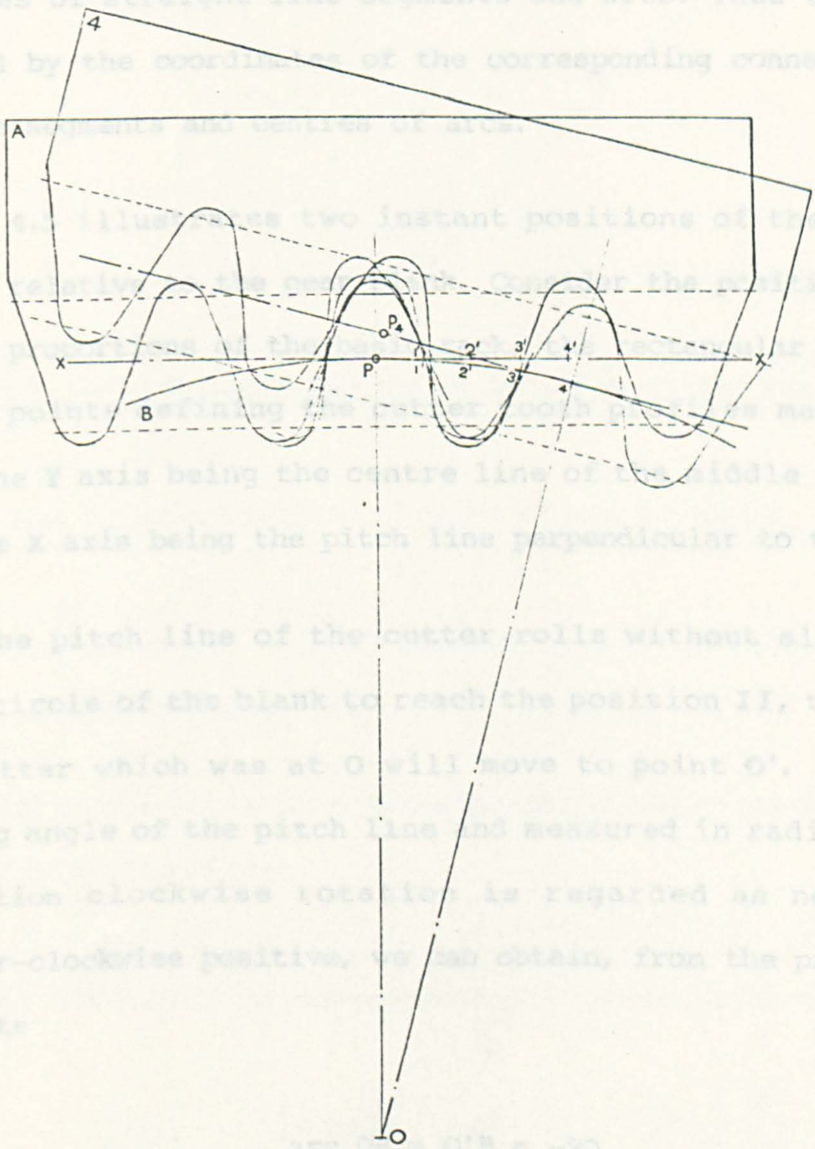



Figure 4.4 Rolling motion between rack-form cutter and gear Blank

When A rolls over B to bring the points 4 and 4' together, the tooth profile of A will be in the position 4. Repeating this process a sufficient number of times gives a corresponding number of positions of the generating tooth profile. The tooth profile of B is the envelope of all the successive positions of the profile of the generating tooth.

## 4.4 Calculation of Positions of the Cutter Tooth Profiles

In general, the tooth profiles of the rack-form cutter consist of a series of straight line segments and arcs. Thus they may be defined by the coordinates of the corresponding connected points of line segments and centres of arcs.

Figure 4.5 illustrates two instant positions of the rack-form cutter relative to the gear blank. Consider the position I. Based on the proportions of the basic rack, the rectangular coordinates of the points defining the cutter tooth profiles may be found, with the Y axis being the centre line of the middle tooth space and the X axis being the pitch line perpendicular to the Y axis.

When the pitch line of the cutter rolls without slip over the pitch circle of the blank to reach the position II, the point on the cutter which was at O will move to point O'. If Q is the rolling angle of the pitch line and measured in radians, and by convention clockwise rotation is regarded as negative and counter-clockwise positive, we can obtain, from the properties of involute

$$\text{arc } OB = O'B = -RQ \qquad (4.1)$$

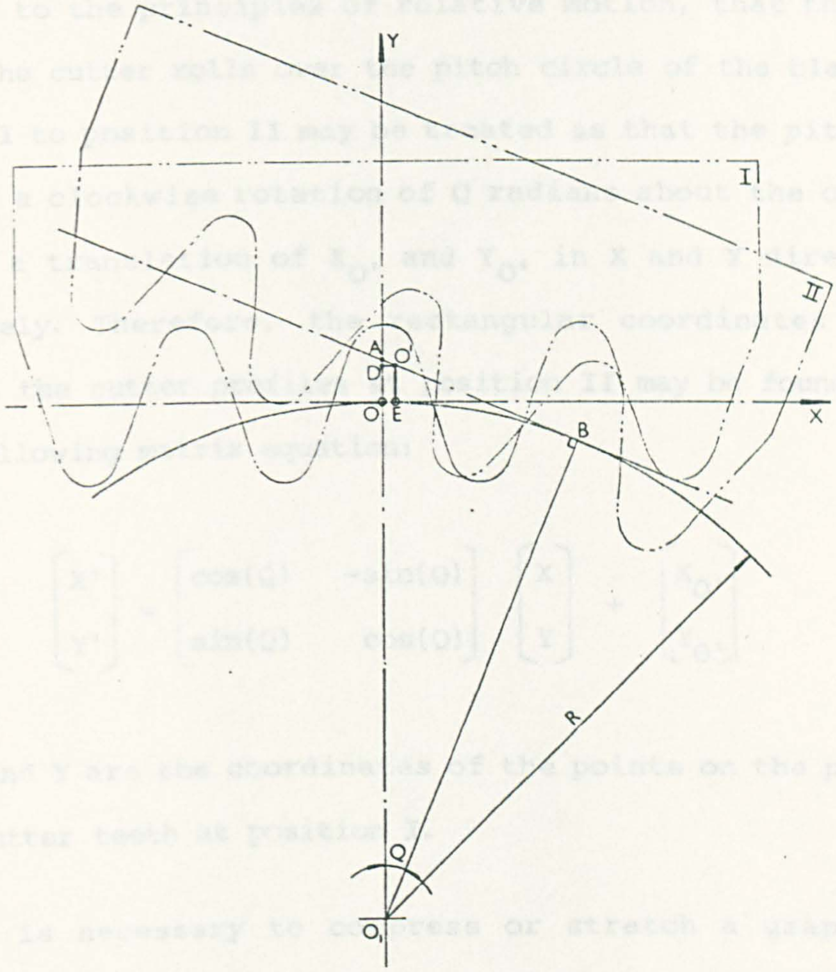



Figure 4.5 Two relative positions of rack-form cutter and gear blank

The $X_{O'}$ and $Y_{O'}$ coordinates of point O' are given by

$$X_{O'} = O'D = AO'\cos(Q) = (AB-O'B)\cos(Q) = R[Q-\tan(Q)]\cos(Q)$$

$$= R[Q\cos(Q)-\sin(Q)] \tag{4.2}$$

$$Y_{O'} = O'E = OD = AO_1-R-AD = R/\cos(Q)-R+R[Q-\tan(Q)]\sin(Q)$$

$$= R[Q\sin(Q)+\cos(Q)-1] \tag{4.3}$$

In matrix notation, the above equations can be written as

$$\begin{bmatrix} X_{O'} \\ Y_{O'} \end{bmatrix} = R \begin{bmatrix} \cos(Q) & -\sin(Q) & 0 \\ \sin(Q) & \cos(Q) & 1 \end{bmatrix} \begin{bmatrix} Q \\ 1 \\ -1 \end{bmatrix} \tag{4.4}$$

According to the principles of relative motion, that the pitch line of the cutter rolls over the pitch circle of the blank from position I to position II may be treated as that the pitch line undergoes a clockwise rotation of Q radians about the origin O and then a translation of $X_{O'}$ and $Y_{O'}$ in X and Y directions, respectively. Therefore, the rectangular coordinates of the points on the cutter profiles at position II may be found by use of the following matrix equation:

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} \cos(Q) & -\sin(Q) \\ \sin(Q) & \cos(Q) \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} X_{O'} \\ Y_{O'} \end{bmatrix} \tag{4.5}$$

where X and Y are the coordinates of the points on the profiles of the cutter teeth at position I.

Often it is necessary to compress or stretch a graph being displayed or drawn. To that end, the coordinates of the points on the cutter profiles are scaled by specifying the scaling factors ($S_x$, $S_y$) so that the new positions of these points will have the coordinates ($S_x X'$, $S_y Y'$). In matrix notation, the scaling for these points can be written as

$$\begin{bmatrix} X'' \\ Y'' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} X' \\ Y' \end{bmatrix} \tag{4.6}$$

For the special case of homogeneous scaling, $S_x = S_y = S$, Eq. (4.6) can be simplified as

$$\begin{bmatrix} X'' \\ Y'' \end{bmatrix} = S \begin{bmatrix} X' \\ Y' \end{bmatrix} \tag{4.7}$$

## 4.5 Program Description

The program GTGENE developed computes the successive positions of the tooth profiles as the rack-form cutter is rolled around the gear blank in convenient angular steps. The cutter may be specified in either of the forms below:

(a) as a straight-sided basic rack;

(b) as a straight-sided basic rack but with a protuberance tip, i.e. a secondary flank angle, grinding allowance and undercut allowance (see Figure 4.6).

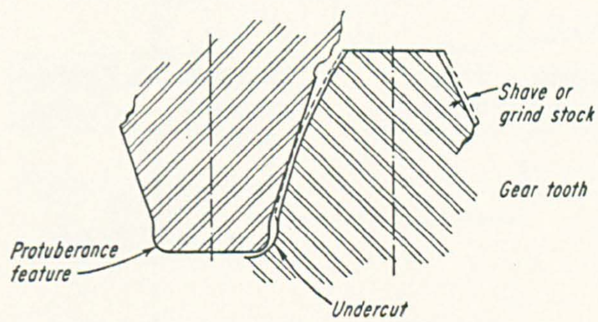



Figure 4.6 Rack-form cutter with protuberance

The parameters of the gear blank, selected by the user, consist of the module, the tooth number, the modification coefficient, and the addendum shortening coefficient.

The output obtained from the program shows the gear tooth profiles defined by the envelope of the successive profiles of the cutter tooth spaces, together with the cutter, the tip circle, pitch circle, and base circle of the gear.

The drawing can illustrate:

(a) the generating process of gear tooth profiles from a rack-form cutter;

(b) the form of the root fillet and any undercutting that may occur;

(c) the effect of the parameters of the cutter, such as the pressure angle, tip radius, protuberance,and modification, on the profiles of the generated gear;

(d) the difference of the gear tooth profiles generated with different gear standards.

The flowchart of two-dimensional generation of gear tooth profiles from a rack-form cutter is shown in Figure 4.7. Two examples are presented in Figure 4.8 and 4.9 respectively.

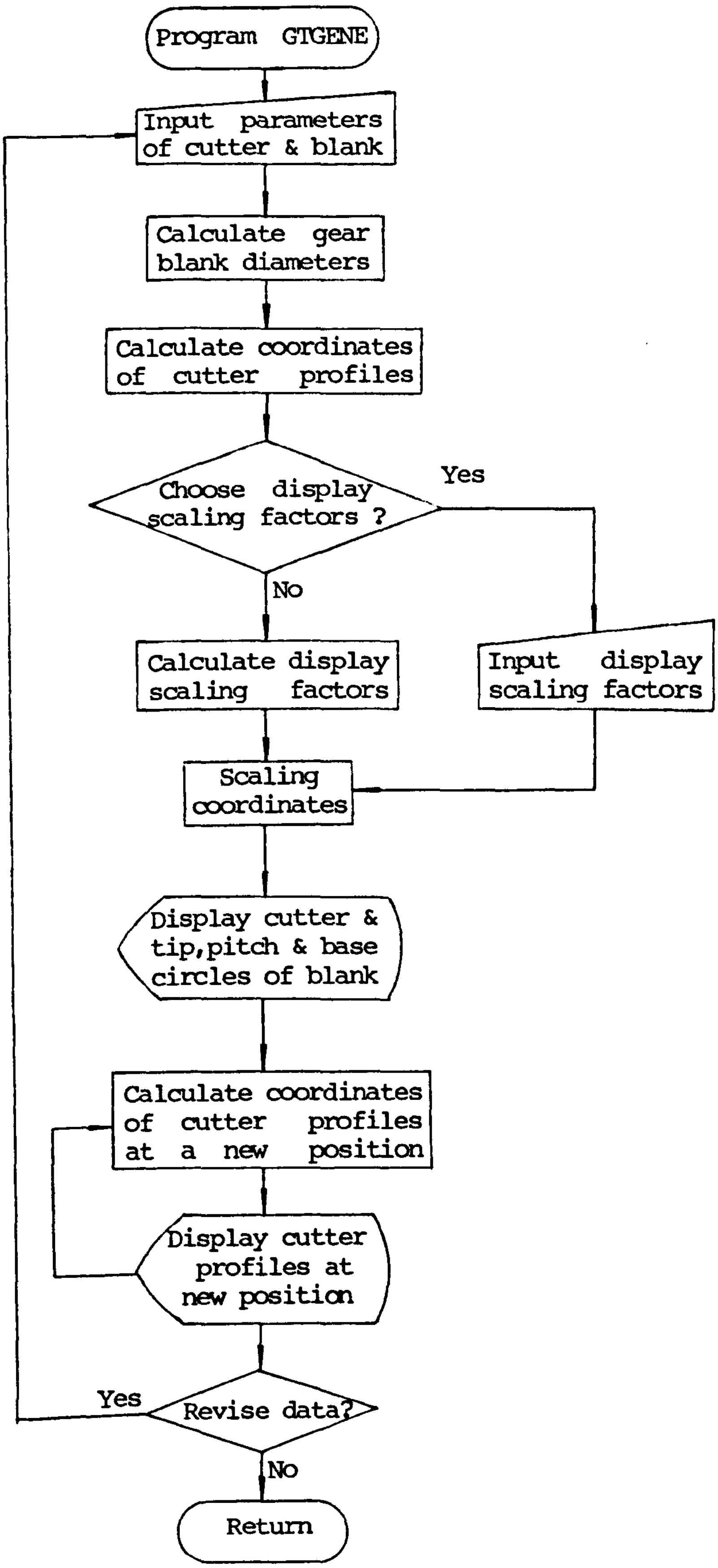



Figure 4.7 Flowchart of gear tooth generation

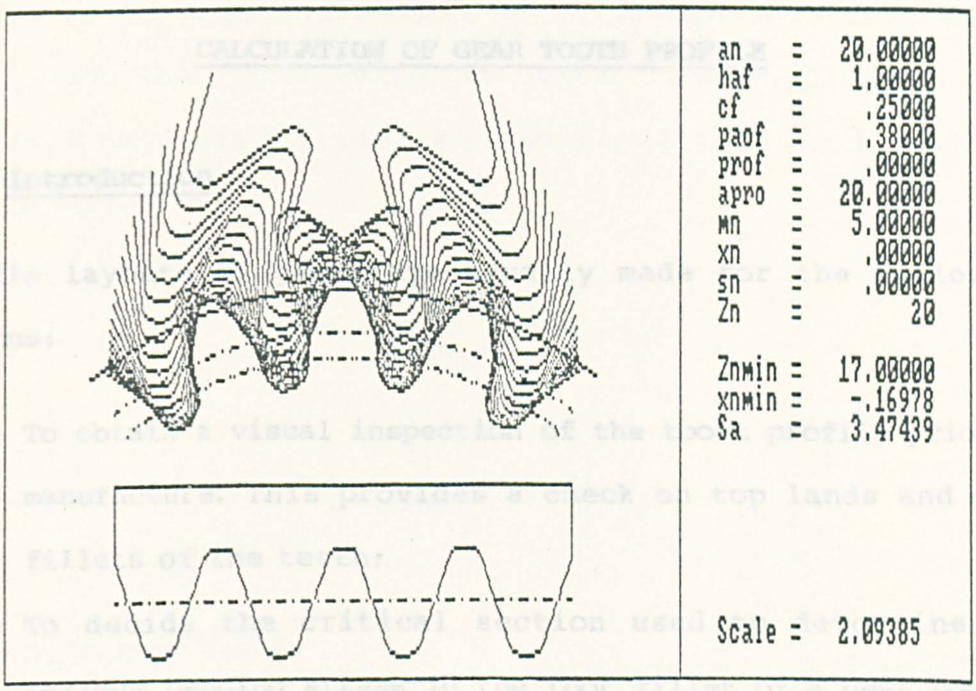



Figure 4.8 Gear tooth generation from a rack-form cutter

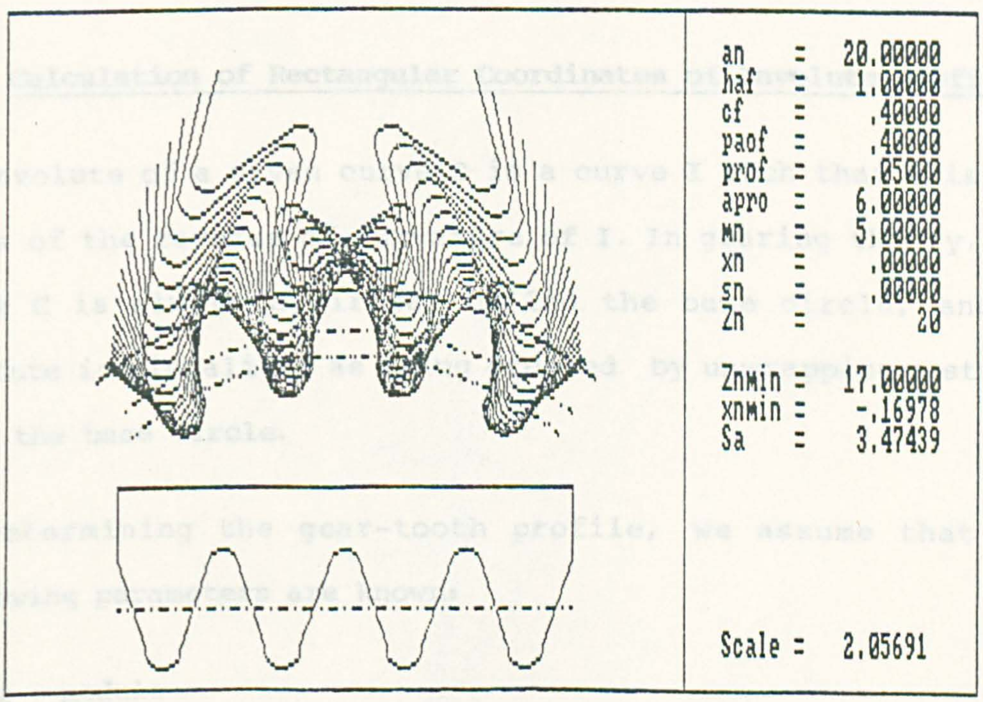



Figure 4.9 Gear tooth generation from a rack-form cutter with a protuberance tip

# Chapter 5

# CALCULATION OF GEAR TOOTH PROFILE

## 5.1 Introduction

Profile layouts of gears are usually made for the following reasons:

(1) To obtain a visual inspection of the tooth profile prior to manufacture. This provides a check on top lands and root fillets of the teeth;

(2) To decide the critical section used to determine the maximum bending stress in the root fillet of a gear tooth;

(3) To provide the rectangular coordinates of the tooth profiles for the calculation of the gear-tooth stresses using the finite element method.

## 5.2 Calculation of Rectangular Coordinates of Involute Profile

An involute of a given curve C is a curve I such that C is the locus of the centres of curvature of I. In gearing theory, the curve C is always a circle, called the base circle, and an involute is visualised as being created by unwrapping a string from the base circle.

In determining the gear-tooth profile, we assume that the following parameters are known:

$m$ = module

$z$ = tooth number

$\alpha$ = pressure angle at reference cylinder

$x$ = addendum modification coefficient

When these values are known, the rectangular coordinates of a point **P** having a pressure angle $\alpha_p$ on an involute curve may be found, with the **Y** axis being the centre line of the gear tooth and the **X** axis being a radius perpendicular to the **Y** axis (see Figure 5.1):

$$X = \frac{r_b}{\cos(\alpha_p)}\sin(\phi-\theta_p) \tag{5.1}$$

$$Y = \frac{r_b}{\cos(\alpha_p)}\cos(\phi-\theta_p) \tag{5.2}$$

where $r_b$ = base radius

$$r_b = \tfrac{1}{2}mz\cos(\alpha) \tag{5.3}$$

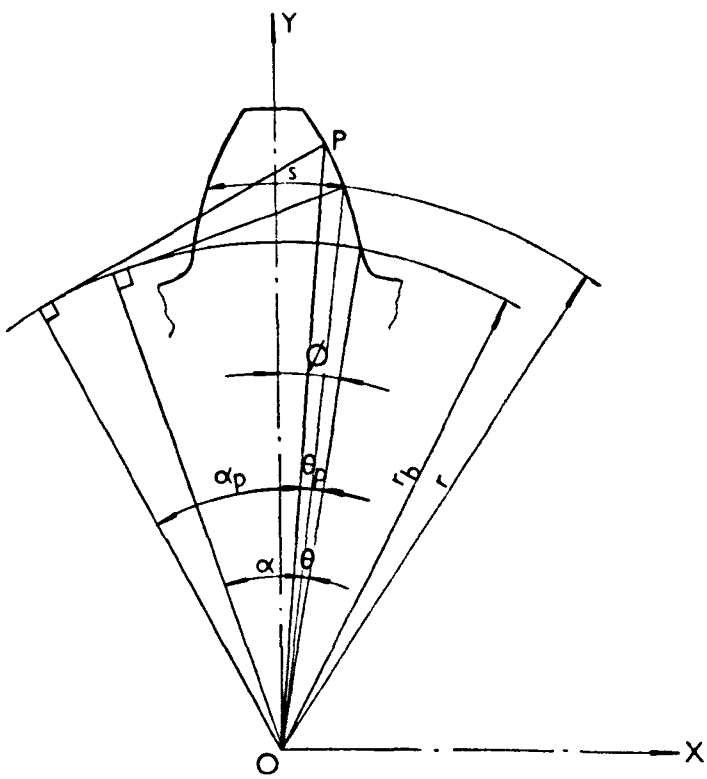



Figure 5.1 Rectangular coordinates of involute profile

$\theta_p = \tan(\alpha_p)-\alpha_p$ = involute polar angle of the point P

$\phi$ = angle between the gear-tooth centre line and the radius to the origin of the involute

$$\phi = \frac{s}{2r} + \theta = \frac{\pi m/2+2xm\tan(\alpha)}{mz} + \tan(\alpha)-\alpha = \frac{\pi+4x\tan(\alpha)}{2z} + \tan(\alpha)-\alpha \quad (5.4)$$

where s = circular thickness

r = reference radius

$\theta$ = involute polar angle of the reference point

## 5.3 Calculation of the Pressure Angle at the Starting Point of Involute Profile

As a rule when gears are produced in large quantities, a generating process such as hobbing or shaping is used. Kinematically the action of a hob is similar to that of a rack cutter. Extensions must be provided on the ends of the teeth to cut the clearance at the base of the teeth of the gear. A typical form of rack cutter teeth is shown in Figure 5.2.

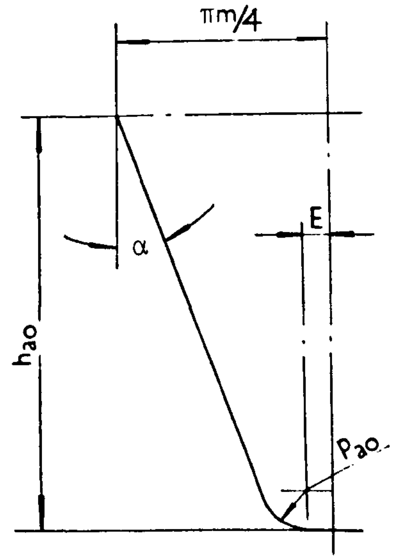



Figure 5.2 Dimensions of a typical basic rack

$$E = \pi m/4-(h_{ao}-p_{ao})\tan(\alpha)-p_{ao}\sec(\alpha)$$

Conjugate action between involute teeth takes place on the line of action and can have a maximum value of $N_1N_2$. $N_1$ and $N_2$ are called the interference points of the gears (Figure 5.3). When the number of teeth in a gear is small, the addendum of the rack or hob extends inside the interference point during the cutting operation. This results in a loss of a portion of the involute adjacent to the base circle, which materially shortens the duration of contact between a mating pair of teeth.

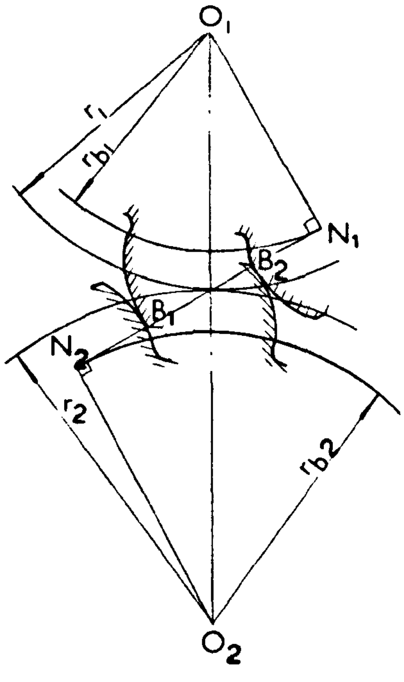



Figure 5.3 Interference points of gears

It is usually assumed that undercutting is due to the straight portion of the cutter tooth, which terminates at K in Figure 5.4, and that the extension for clearance purposes beyond this point has little effect on the undercutting. Large-scale layouts of the cutting process have shown that this is a reasonably good assumption. Hence in Figure 5.5, if point K passes through interference point N, the tooth will not be undercut and the tooth profile outside the base circle will be involute. When

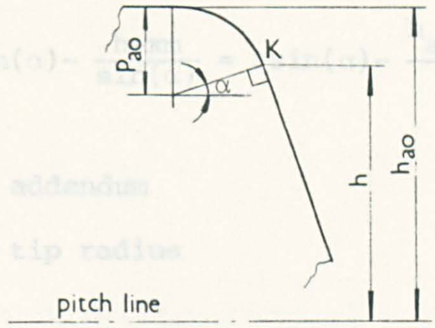



Figure 5.4 Terminal point K of the straight portion of basic rack profile

$$h = h_{ao}-p_{ao}[1-\sin(\alpha)]$$

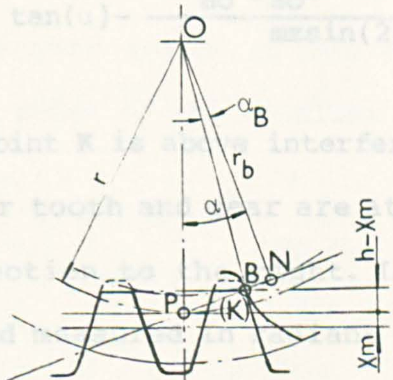



Figure 5.5 Pressure angle at the starting point of involute profile without undercutting

point **K** is below interference point **N**, suppose it is located at point **B** on the line of action, the tooth profile outside point **B** will be involute, but the tooth profile inside point **B** will be noninvolute, named as fillet curve. The point **B** is defined as the starting point of the involute profile on gear tooth. The angle $\alpha_B$ is defined as the pressure angle at the starting point **B** of the involute profile. From Figure 5.5, we have

$$\tan(\alpha_B) = BN/r_b \tag{5.5}$$

Since

$$BN = PN-PB = r\sin(\alpha) - \frac{h-xm}{\sin(\alpha)} = r\sin(\alpha) - \frac{h_{ao}-p_{ao}[1-\sin(\alpha)]-xm}{\sin(\alpha)}$$

where $h_{ao}$ = cutter addendum

$p_{ao}$ = cutter tip radius

and

$$r_b = r\cos(\alpha) = \frac{mz}{2}\cos(\alpha)$$

hence

$$\tan(\alpha_B) = \tan(\alpha) - \frac{4\{h_{ao}-p_{ao}[1-\sin(\alpha)]-xm\}}{mz\sin(2\alpha)} \qquad (5.6)$$

The situation when point **K** is above interference point **N** is shown in Figure 5.6. Cutter tooth and gear are at the point of separating upon further motion to the right. Let $\gamma$ be the rotating angle of the gear and measured in radians when the cutter tooth moves horizontally from position **I** to position **II**, then we find

$$\text{arc } NN' = r_b\gamma = r\gamma\cos(\alpha)$$

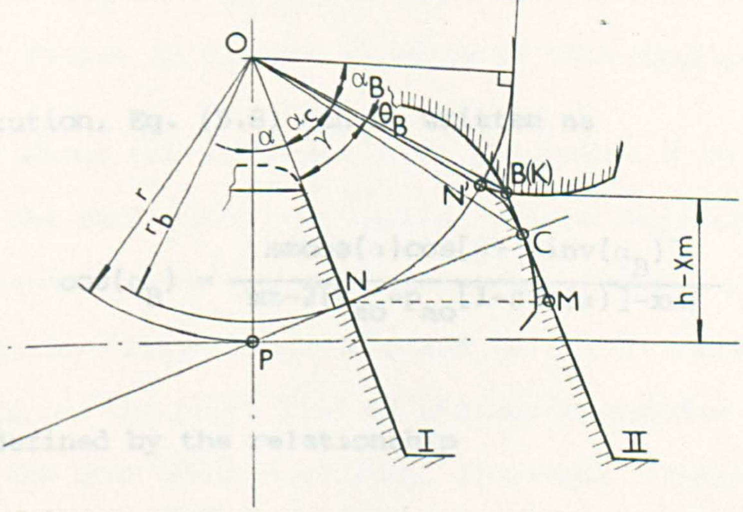



Figure 5.6 Pressure angle at the starting point of involute profile with undercutting

The translation of the cutter tooth is given by

$$NM = r\gamma$$

Thus we obtain

$$NC = NM\cos(\alpha) = r\gamma\cos(\alpha) = \text{arc } NN' \qquad (5.7)$$

Eq. (5.7) indicates that if we take $r_b$ as the base circle and **N'** as the origin point to draw another involute, it must pass through the point **C**. Therefore, the point **N'** is located at the left bottom of the cutter edge **BM**, the portion **N'B** of the involute profile on gear tooth is cut off, and the point **B** becomes the starting point of the involute profile. As

$$\cos(\alpha_B) = \frac{r_b}{OB} \qquad (5.8)$$

and

$$OB = \frac{r-(h-xm)}{\cos(\alpha+\gamma+\theta_B)}$$

by substitution, Eq. (5.8) can be written as

$$\cos(\alpha_B) = \frac{mz\cos(\alpha)\cos[\alpha+\gamma+inv(\alpha_B)]}{mz-2\{h_{ao}-p_{ao}[1-\sin(\alpha)]-xm\}} \qquad (5.9)$$

If OB is defined by the relationship

$$OB = \frac{NC}{\sin(\gamma+\theta_B)} = \frac{r\gamma\cos(\alpha)}{\sin[\gamma+inv(\alpha_B)]}$$

by substituting the value of **OB** from the above equation into Eq. (5.8) and rearranging it we get

$$\gamma \cos(\alpha_B) = \sin[\gamma + inv(\alpha_B)] \tag{5.10}$$

Thus the pressure angle $\alpha_B$ at the starting point of the involute profile on gear tooth may be determined by using Eqs. (5.9) and (5.10), which will be in the following range

$$0 < \alpha_B < \alpha_C < \gamma < \text{arc}\ \tan[\frac{4\{h_{ao} - p_{ao}[1-\sin(\alpha)] - xm\}}{mz\sin(2\alpha)} - \tan(\alpha)]$$

## 5.4 Calculation of Rectangular Coordinates of Tooth Fillet

In theory, each point on a rack cutter creates its own trochoid on the gear to be produced, and the final tooth profile of the gear is the envelope of all the individual trochoids of the points on the cutter edge. In practice, only one trochoid is of interest, namely, the trochoid for the centre **C** of the circle at the corner of the rack cutter. At each point of this rack trochoid we imagine a circle having its centre at the point. The trochoidal fillet is then an envelope of this family of circles.

Figure 5.7 shows the situation when the centre **C** of the circular corner of the rack cutter is located at the nearest position to the gear centre. Draw the radius to point **C** which we call the **Y'** axis. Draw the radius perpendicular to the **Y'** axis and call it the **X'** axis. As the pitch line of the cutter rotates on the pitch circle of the gear without sliding, the angle $\phi$ between the gear-tooth centre line and the **Y'** axis may be found by use of the following equation:

$$\phi = \frac{l}{r} = \frac{\pi m/2+2(h_{ao}-p_{ao})\tan(\alpha)+2p_{ao}\sec(\alpha)}{mz} \qquad (5.11)$$

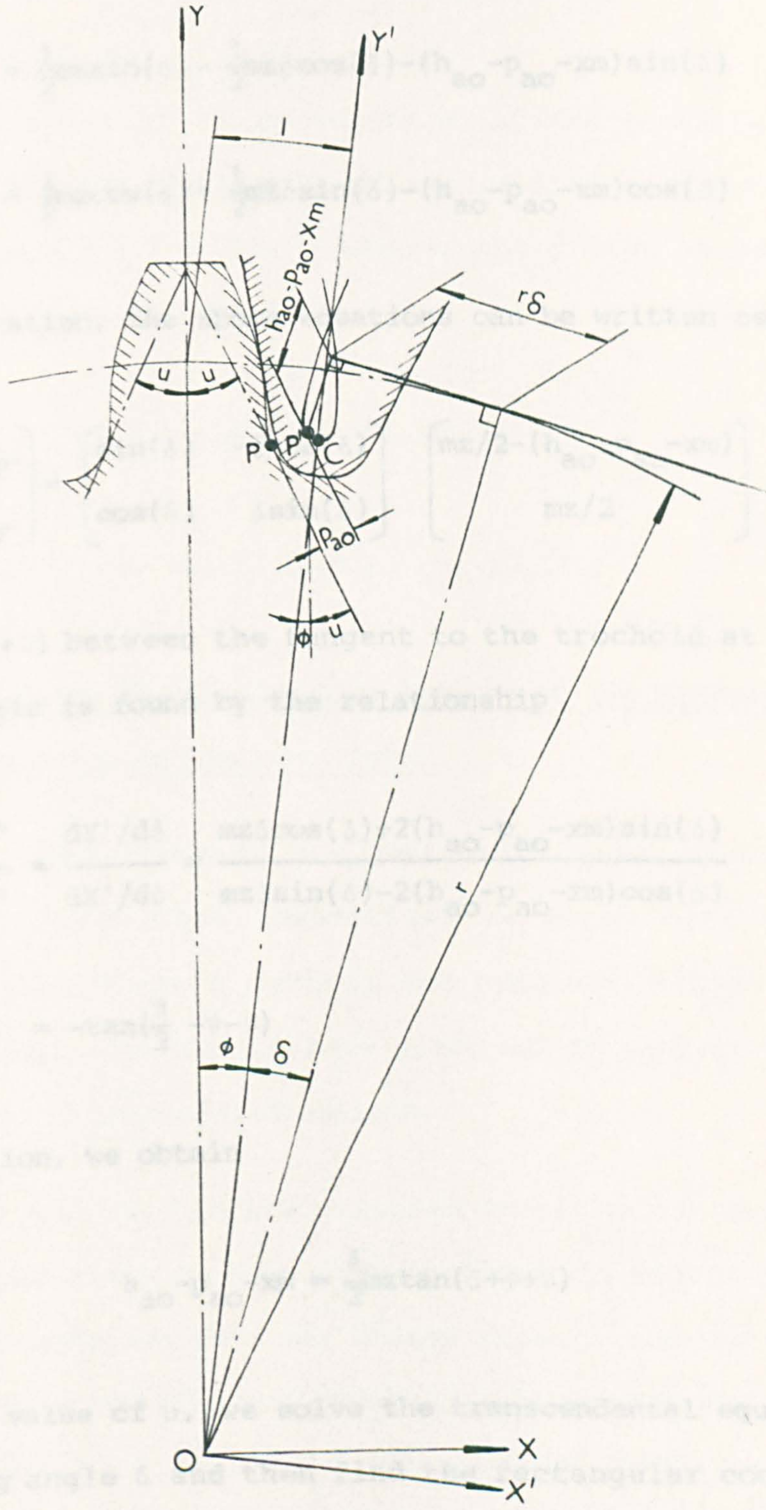



**Figure 5.7 Rectangular coordinates of tooth fillet**

$$l = \pi m/2-E = \pi m/4+(h_{ao}-p_{ao})\tan(\alpha)+p_{ao}\sec(\alpha)$$

If $\delta$ is defined as the rotating angle of the cutter on the pitch circle, the rectangular coordinates of a point P' on the trochoid of the centre of the cutter corner are given by

$$X'_{P'} = \frac{1}{2}mz\sin(\delta) - \frac{1}{2}mz\delta\cos(\delta) - (h_{ao} - p_{ao} - xm)\sin(\delta) \qquad (5.12)$$

$$Y'_{P'} = \frac{1}{2}mz\cos(\delta) + \frac{1}{2}mZ\delta\sin(\delta) - (h_{ao} - p_{ao} - xm)\cos(\delta) \qquad (5.13)$$

In matrix notation, the above equations can be written as

$$\begin{bmatrix} X'_{P'} \\ Y'_{P'} \end{bmatrix} = \begin{bmatrix} \sin(\delta) & -\delta\cos(\delta) \\ \cos(\delta) & \delta\sin(\delta) \end{bmatrix} \begin{bmatrix} mz/2-(h_{ao}-p_{ao}-xm) \\ mz/2 \end{bmatrix} \qquad (5.14)$$

The angle $(\phi+\mu)$ between the tangent to the trochoid at point P' and the Y' axis is found by the relationship

$$\frac{dY'}{dX'} = \frac{dY'/d\delta}{dX'/d\delta} = \frac{mz\delta\cos(\delta)+2(h_{ao}-p_{ao}-xm)\sin(\delta)}{mz\delta\sin(\delta)-2(h_{ao}-p_{ao}-xm)\cos(\delta)}$$

$$= -\tan(\frac{\pi}{2} - \phi - \mu) \qquad (5.15)$$

By simplication, we obtain

$$h_{ao} - p_{ao} - xm = \frac{\delta}{2}mz\tan(\delta+\phi+\mu) \qquad (5.16)$$

For a given value of $\mu$, we solve the transcendental equation for the rotating angle $\delta$ and then find the rectangular coordinates of a point P on the trochoid fillet. This point lies on the normal to the trochoid of the centre of the cutter corner at a distance $p_{ao}$ from the point P' of the trochoid of the centre of the cutter corner.

$$\begin{bmatrix} X'_P \\ Y'_P \end{bmatrix} = \begin{bmatrix} X'_{P'} \\ Y'_{P'} \end{bmatrix} - p_{ao} \begin{bmatrix} \cos(\phi+\mu) \\ \sin(\phi+\mu) \end{bmatrix} \quad (5.17)$$

To find the rectangular coordinates of the point **P** on the trochoid fillet with the gear-tooth centre line as the **Y** axis and a radius perpendicular to the **Y** axis as the **X** axis, we have

$$\begin{bmatrix} X_P \\ Y_P \end{bmatrix} = \begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} X'_P \\ Y'_P \end{bmatrix} \quad (5.18)$$

where $X'_P$ and $Y'_P$ are calculated by Eq. (5.17).

## 5.5 <u>Calculation of Rectangular Coordinates of the Maximum Bending Stress Position on the Gear Tooth</u>

Estimation of root stresses in a gear tooth is so complex due to the peculiar shape that theoretical attempts have not been successful; photoelastic methods and computed finite element methods are more suitable and have given agreed results which are now incorporated into specifications.

The maximum bending tensile stresses occur below the working section of the flank in the root as shown in Figure 5.8; they give crack propagation from any stress raisers such as machining marks or surface defects and eventually break off the complete tooth. The level of stress is not greatly influenced by the position of the acting force on the flank. The maximum stress position in ISO gear standard, which will be described in Section 6.2, is assumed to occur where a tangent at $30^{\circ}$ to a radial line

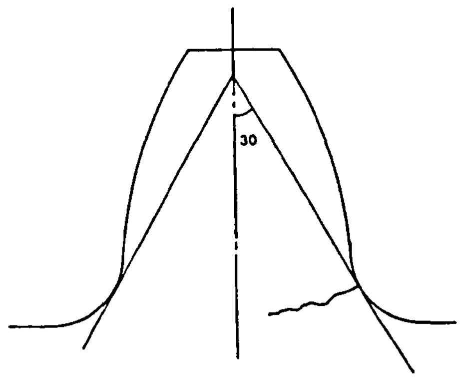



Figure 5.8 Maximum root tensile stress position

meets the tooth [39]. In such case, we take the angle $\mu$ as $30^{\circ}$ and then solve the transcendental Eq. (5.16) in $\delta$. With the value of $\delta$, we calculate the rectangular coordinates (X',Y') of the $30^{\circ}$ tangent point by using Eq. (5.17). Finally, Eq. (5.18) is solved for the rectangular coordinates (X,Y).

## 5.6 Program Description

Based on the theory as stated above, the program GTPROF is developed, which may be used to

(a) draw the profile of a single gear tooth or a whole gear to any required scale;

(b) provide the user with the information on the top land width, whether undercutting occurs, and the minimum modification coefficient for no undercutting to occur;

(c) determine the sizes of the critical section assumed by the ISO gear standard;

(d) supply the user with the rectangular coordinates of the tooth profiles;

(e) build up the gear tooth grid comprised of quadrilateral elements, generate the node number and coordinates of every node, and specify the node numbers of the load and maximum stress nodes for the gear tooth stress analysis using finite element method.

Two samples are presented in Figures 5.9 and 5.10.

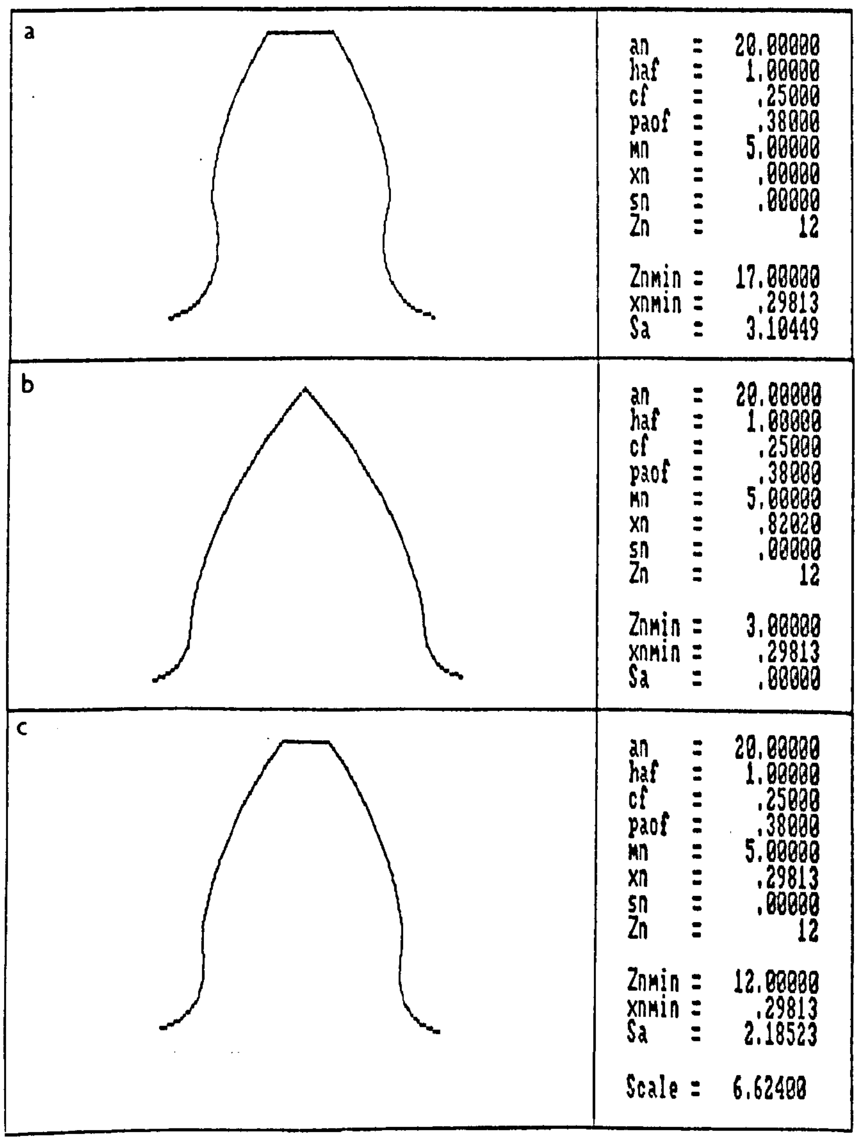
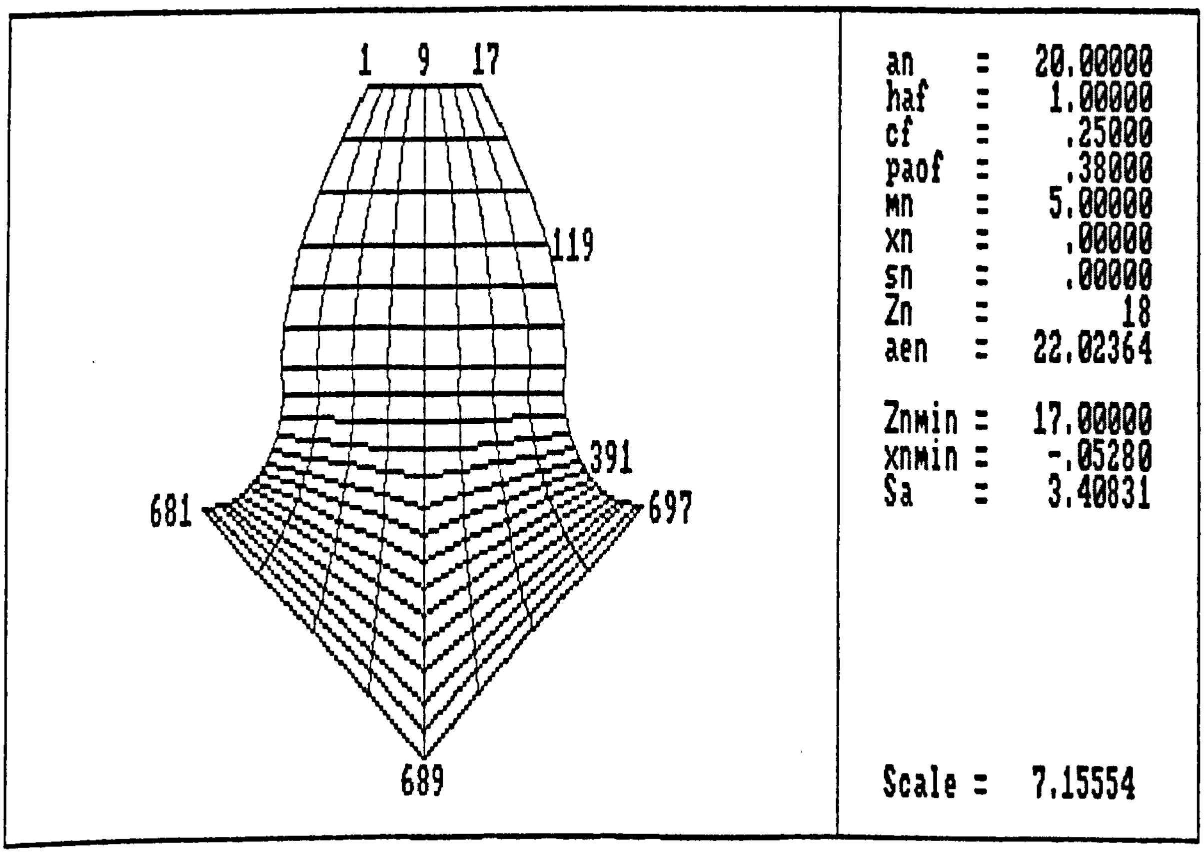



Figure 5.9 Gear tooth drawing from the program GTPROF

(a) Undercut tooth (b) Pointed tooth (c) Normal tooth




| | |
|---|---|
| NORMAL PRESSURE ANGLE AT THE LOAD POINT(DEG.)........ | 22.0236 |
| ANGLE BETWEEN THE LOAD AND THE X AXIS (DEG.)......... | 197.3225 |
| NORMAL PRESSURE ANGLE AT THE TOOTH TIP(DEG.)......... | 32.2505 |
| DIAMETER AT THE LOAD POINT(MM)...................... | 91.2294 |
| THICKNESS OF VIRTUAL TOOTH AT CRITICAL SECTION(MM)... | 9.5314 |
| BENDING MOMENT ARM(MM)............................... | 4.8707 |
| TOTAL NUMBER OF ELEMENTS............................ | 160 |
| LARGEST NODE NUMBER................................. | 697 |
| NODE NUMBER AT THE LOAD POINT....................... | 119 |
| NODE NUMBER AT THE CRITICAL POINT................... | 391 |

-- NODE NUMBERS AND COORDINATES OF PLOTTING POINTS --

| | | |
|---|---|---|
| 1 | -1.7038 | 49.9710 |
| 3 | -1.2779 | 49.9710 |
| 5 | -.8519 | 49.9710 |
| 7 | -.4260 | 49.9710 |
| 9 | .0000 | 49.9710 |
| 11 | .4260 | 49.9710 |
| 13 | .8519 | 49.9710 |
| 15 | 1.2779 | 49.9710 |
| 17 | 1.7038 | 49.9710 |
| 35 | -2.5164 | 48.4677 |
| 37 | -1.8873 | 48.4677 |
| 39 | -1.2582 | 48.4677 |
| 41 | -.6291 | 48.4677 |
| 43 | .0000 | 48.4677 |
| . | . | . |
| . | . | . |
| . | . | . |
| 689 | .0000 | 31.2500 |
| 691 | 1.6822 | 32.9778 |
| 693 | 3.3644 | 34.7057 |
| 695 | 5.0467 | 36.4335 |
| 697 | 6.7289 | 38.1613 |

Figure 5.10 Gear tooth grid and information from the program GTPROF

# Chapter 6

# GEAR TRAIN DESIGN

## 6.1 Introduction

The complete design of a gear set comprises the strength design, geometric calculation and force analysis. Strength design is intended to find the tooth numbers, modules, centre distances, facewidths and materials to suit specified gear ratios, service lives, power requirements and safety factors. Geometric calculation determines the diameters, proportions of the mating elements and also characteristics of meshing — contact ratio, etc. Force analysis concerns the finding of the forces acting upon the shafts necessary for subsequent calculations of shafts. Gear design is a lengthy process. Using manual means of calculation even an experienced designer might take hours to analysis a single gear pair. Design costs become more critical when gear trains must be designed and rated to precise gear standards and meet some special requirements. This chapter shows how far microcomputers can assist in the design of gear train and the use of the gear design programs developed results in dramatically reducing the designer's workload as well as improving the designs.

## 6.2 Strength Design Theory and Approach

### 6.2.1 Gear Rating Standards

#### 6.2.1.1 National Standards

A number of gear standards exist, varying widely in complexity.

The principal gear standards in use are the old British standard BS436:1940 [10], the American Gear Manufacturer's Association standard AGMA170.01 [1], the German standard DIN3990 [19, 28, 29, 30, 31], the corresponding international standard ISO/DIS6336 [39], and the new British standard BS436:1986 [11].

The 'old British' approach to gear rating can best be described as the 'averaged experience' method, whereby gear manufacturers and users collaborate to provide extremely empirical rules of thumb based on operating experience. Permissible loads are specified for 'typical' manufacturing accuracies of a given class with 'typical' loading cycles and corrections for speed, etc., follow a standard law regardless of system dynamics, methods of manufacture, etc. This standard offers a relatively conservative rating compared with later methods.

In complete contrast DIN3990 (and ISO/DIS6336 which is derived from DIN3990 and the new British standard which is derived from ISO/DIS6336) is firmly based on much research work and attempts to make exact calculations for what is going on from measured or estimated figures. This standard does not work on 'typical' figures for, say, misalignment but uses estimated or actual figures to predict the load increases. It is a very much more complex standard than previous standards because every factor is investigated (in theory) exactly. It is no longer sufficient to specify 'average commercial quality' of manufacture but precise details of helix, pitch, mounting, and profile errors are needed for the calculations. The resulting amount of work involved in designing a gear is very much greater than that necessary for the older methods and involves asking a large number of questions to which the answers are not available. Criticisms can be, and are, levelled against the DIN approach because although much of the detailed work is very good and thorough, some of the assumptions are very imprecise or naive; in particular the approach to system dynamics is very weak and should not be used.

The AGMA170.01 standard is a compromise approach which has attempted rather more than the old standards but has fallen short of asking all the embarrassing questions about exact levels of gear accuracy and system dynamics so that it should still be classified as an 'industrial averaged' approach. This leaves it essentially as an imprecise approach [72].

#### 6.2.1.2 Effects of Standards on Design

It would seem at first sight that it does not matter very much which standards are used for design as any standard will produce a gearbox that is unlikely to fail. There are likely to be severe repercussions on costs with the use of the more precise DIN approach since it highlights sensitive points in the design.

An initial design, assessed according to traditional standards, has a fixed load capacity which allows for 'typical' errors and dynamics. There is no advantage in detailed investigation or control of any single factor since it will not affect the allowable rating of the gearset. In contrast the more detailed investigations associated with the DIN approach will emphasize particular points which control the load carrying capacity. This detailed analysis is typical of design and development in airplanes because the economic aspects of excessive weight justify the high initial development costs.

The overall effect of using a more precise standard is that much more time and effort must go into initial design work but that the resulting gearbox is likely to be more economical because it is smaller. This will, in time, give a competitive advantage to the manufacturers who can use their technical superiority to produce smaller gearboxes.

#### 6.2.1.3 ISO Approach

An International Standard is the result of an agreement between the member bodies of ISO. A first important step towards an International Standard takes the form of a draft proposal (DP) — this is circulated for study within an ISO technical committee. When substantial support of a DP has been obtained within the technical committee, the document is sent to the central secretariat for processing as a draft International Standard (DIS). The DIS is submitted to all ISO member bodies for voting, if 75 percent of the votes cast are in favour of the DIS, it is sent to ISO council for acceptance as an International Standard.

The first ISO proposals for gear rating formula were drawn up in 1977, which took ISO/TC60/WG6 some 16 years to finish. Three years later, the Draft Proposals ISO/DP6336 were submitted to the technical committee for study. In 1983, they became the Draft International Standard ISO/DIS6336.

In this standard, the stress levels in the tooth flank and in the tooth root are calculated and compared with basic permissible stress levels derived from tests on simple test specimens. It has a very comprehensive list of factors which are selected from a wide range pertaining to the application and used to modify stress levels. Even though a designer has no intention of using this standard it is worthwhile having a list of the standard factors as a check list or reminder of what may influence a particular design.

In ISO approach, gears may be rated for three primary modes of tooth failure, namely surface fatigue failure (pitting) due to higher Hertzian contact stresses than the surface can stand, bending fatigue failure (root cracking) owing to repeated stresses above the endurance limit of the gear teeth in beam bending, and scuffing which occurs when the oil film breaks down to allow metal to metal contact.

The detailed contact stress calculations are tedious and are mainly concerned with corrections to be applied to the stress due to variations from the arbitrarily selected conditions which are pitch point contact, low helix angle, infinite life, viscosity at $50^oC$ of 100 $mm^2/s$ (100 centistokes), pitch line velocity of 10 m/s, peak to valley roughness of 3 microns (equivalent to 20 microinches C.L.A. roughly) and hard teeth.

Root strength calculations follow a similar pattern for standard conditions of $20^o$ pressure angle, standard $1.25m_n$ dedendum profile ($m_n$ is the normal module), low helix angle, $3x10^6$ cycle life, low root surface roughness, 5 mm module gears.

Scuffing calculations are rather more open to argument since the underlying mechanisms are not fully understood and the behavior of oils, the corresponding coefficients of friction, and the local temperatures are all rather hypothetical. This area is so uncertain and the properties of extreme pressure oils on real surfaces are so difficult to measure that running experience under similar condition is essential.

Although the ISO specification is better than its 'averaged' predecessors in that it is more precise, it should be treated with caution and it is essential that intelligence is used in its application. It is far too easy for a design office to work on the assumption that gears will be fully corrected and expertly bedded at full load when the manufacturing side is making 'commercial' quality gears.

Certain of the ISO factors such as roughness factor and lubrication factor for contact stresses are allowed to exceed 1.0 in the specification but common sense suggests that these be limited to 1.0 when designing. The ISO approach to stressing, like all other standards, ignores the effects of tip relief on stresses due to effective shortening of the length of the line of contact.

Currently the general tendency to design gears is to adopt the ISO standard. For instance, the Chinese standard GB3480-83 was formulated on the basis of the ISO6336 in 1983 and the new BS436 published in 1986 follows the principles developed by the ISO. The gear design program described in this chapter uses the ISO approach. The safety factors of a gear set are evaluated on the basis of two alternative modes of tooth failure, i.e. pitting and root cracking, ignoring the scuffing. The theoretical model is modified to take account of the actual operating conditions. The resulting calculated stresses are compared with a basic permissible fatigue stress, obtained from fatigue tests of selected gear materials.

### 6.2.2 Detailed Analysis of Gear Strength

As the life of a pair of gears may be limited by the endurance of either member, the permissible load for any specified life expectation is the lowest of four quantities corresponding to:

(a) the resistance of the pinion to surface stress;

(b) the resistance of the gear to surface stress;

(c) the resistance of the pinion to bending stress;

(d) the resistance of the gear to bending stress.

For an existing gear pair, actual safety factors for surface contact stress and tooth root bending stress in accordance with ISO gear standard are calculated from the following formulae [39]:

For surface strength

$$S_H = \frac{\sigma_{Hlim} Z_N Z_L Z_R Z_V Z_W Z_X}{Z_H Z_E Z_\varepsilon Z_\beta \sqrt{\frac{F_t(u+1)}{d_1 bu} K_A K_V K_{H\beta} K_{H\alpha}}} >= S_{Hmin} \qquad (6.1)$$

For bending strength

$$S_F = \frac{\sigma_{Flim} Y_{ST} Y_{NT} Y_{\delta relT} Y_{RrelT} Y_X}{\frac{F_t}{bm_n} Y_F Y_S Y_\beta K_A K_V K_{F\beta} K_{F\alpha}} >= S_{Fmin} \qquad (6.2)$$

In Eqs. (6.1) and (6.2) the general influence factors are, in the order in which they should be applied:

$K_A$ = application factor. Allows for dynamics of load system to convert nominal external loads to dynamic actual loads.

$K_V$ = Dynamic factor. Allows for increase of loading due to internal dynamic increasing tooth loads.

$K_\beta$ = Face Load distribution factor. Accounts for variation of loading over facewidth of tooth due to helix errors, misalignments, gear distortion and case distortion.

$K_\alpha$ = Transverse load distribution factor. Accounts for uneven load distribution between successive teeth due to pitch and profile errors preventing equal load sharing. The limit for this occurs when all the load is taken by one tooth as with spur gears.

As a rough guide $K_V$ and $K_\beta$ should be kept below 1.5 for efficient design; $K_\alpha$ should be kept down to below 1.2 or the stressing advantage of helical gears are being lost. A value of $K_A$ above 1.5 suggests that detailed cumulative fatigue life estimates are needed.

Once the actual maximum local tooth load has been determined using the above factors the individual stressing calculations can be carried out and should be made for each gear in a set. The safety factor for the set is the lowest value obtained. Using this approach, therefore, any pair of mating gears can be analysed for safety factor, a straightforward, although laborious procedure.

Present tendencies are to automate gear design by using standard computer routines to carry out all the detailed work of calculation; this approach saves time but has the disadvantage that it is not immediately obvious from the end results what the important factors are in a given design. To overcome this, it is advisable to print out the intermediate factors such as the load distribution factors and the dynamic (internal) factor to see where possible improvements can be made. The alternative is to try a series of possible errors, support stiffnesses etc., to see how they influence the end result.

In the program GCHECK developed for the precise strength analysis of a gear pair, all the values of the factors and endurance limits are either calculated from the relevant equations or obtained by interpolation from the relevant graphs that have been digitised and stored in a data file. Using this program the complete strength analysis of a gear pair can be achieved in a second. The detailed results of analysis presented to the designer include not only the actual safety factors against pitting and breakage, but also all the values of various factors, geometric dimensions, and meshing characteristics so that the designer may judge the design quality of the gear pair from the information provided and make proper modifications by using his ability and experience. Besides, because the input data and detailed results will be saved onto disk, the designer has permanent records of his analysis. Appendix A shows an example of the use of the program GCHECK.

### 6.2.3 Estimation of Major Dimensions for Specified Requirements

The precise analysis of a gearset to determine the factor of safety guarding against the pitting and root cracking, or to determine the reliability corresponding to a specified life in ISO approach needs all the gear parameters known such as pitch circle diameter, module, tooth number and facewidth, therefore, ISO approach can only be used as a checking calculation for an existing pair of mating gears.

In a design situation, where the initial conditions usually involve a knowledge of

(a) the maximum sustained torque $T_1$ transmitted by the pinion;

(b) the speed of the pinion $n_1$ under the loading condition;

(c) the gear ratio u;

(d) the expected service life;

(e) the minimum demanded safety factors for contact stress and bending stress,

then the task becomes more difficult and outside the scope of ISO gear standards. In order to ensure that all the given requirements can be met and the actual safety factors acceptable, an iterative approach is necessary.

In general, the load capacity of a pair of gears is limited by the ability of the materials to withstand the repeated surface stress induced by repeated application of the load to each tooth. Bending stresses are also set up but, except in some casehardened steel gears, these do not usually limit the load to which the gears may be safely subjected. Hence, it is not necessary in the early stages of design to consider bending stresses.

It has been shown by elementary analysis based on the Hertz formula for stress induced in curved contacting surfaces, that, for a given maximum surface stress, the load per unit width of a pair of cylindrical gears of specified tooth form is proportional to the diameter of the gear.

To obtain a preliminary estimate of the size of gears required to carry a given load, we define the facewidth factor $\psi_d$ as

$$\psi_d = b/d_1 \qquad (6.3)$$

By substituting the following relationship

$$F_t = 2000T_1/d_1 \tag{6.4}$$

in Eq. (6.1), using Eq. (6.3), and some rearranging we get

$$d_1 >= \sqrt[3]{\frac{2000T_1(u+1)}{\psi_d u} K_A K_V K_{H\beta} K_{H\alpha} \left(\frac{Z_H Z_E Z_\varepsilon Z_\beta S_{Hmin}}{\sigma_{Hlim} Z_N Z_L Z_V Z_R Z_W Z_X}\right)^2} \tag{6.5}$$

The reference diameter $d_1$ of pinion is related to the centre distance a as follows:

$$d_1 = \frac{2a}{u+1} \tag{6.6}$$

If we designate the facewidth factor $\psi_a$ as

$$\psi_a = b/a \tag{6.7}$$

$\psi_d$ and $\psi_a$ are related by the equation

$$\psi_d = \frac{\psi_a(u+1)}{2} \tag{6.8}$$

With Eqs. (6.6) and (6.8), Eq. (6.5) can be written in the other useful form

$$a >= (u+1)\sqrt[3]{\frac{500T_1}{\psi_a u} K_A K_V K_{H\beta} K_{H\alpha} \left(\frac{Z_H Z_E Z_\varepsilon Z_\beta S_{Hmin}}{\sigma_{Hlim} Z_N Z_L Z_V Z_R Z_W Z_X}\right)^2} \tag{6.9}$$

The value of $\psi_a$ may be specified on the basis of the following considerations.

In the gears under consideration the facewidth factor varies within a wide range: $\psi_a$ = 0.1-1.2.

In gearboxes with shifting gears or trains of gears where it is important to decrease their sizes along axles, the facewidth factor should be chosen within $\psi_a$ = 0.1-0.2.

For closed-type medium-speed gear systems of medium power $\psi_a$ is taken to equal 0.3-0.6 (the most frequently used is 0.4).

In open-type gear drives where the accuracy of assembly and rigidity of the bearings is not high $\psi_a$ should be <= 0.3.

The greater the transmitted power, the more rigid the reduction gear housing and the more accurately the gear is designed, the larger the value $\psi_a$ should be [22, 44].

For a more convenient assembly of the gear, after the facewidth of gears has been found on the basis of the chosen $\psi_a$ and the computed centre distance a, the pinion facewidth is sometimes increased by 5-10mm.

Gearsets having facewidth factors greater than the upper limit of the recommended range are quite likely to have a nonuniform distribution of the load across the face of the tooth because of the torsional deflection of the gear and shaft, because of the machining inaccuracies, and because of the necessity of maintaining very accurate and rigid bearing mountings. When the facewidth factor is less than its lower limit, a larger gear is needed to carry the larger load per unit length of facewidth. Large gears require more space in the gear enclosure and make the finished machine bigger and more expensive. Large gears are more expensive to manufacture because they require larger machines to generate the teeth, and these machines usually have a slower production rate. However, many other considerations arise in design that may dictate a facewidth factor outside the recommended range.

The final gear size has to be obtained using iteration because the $K_V$, $K_{H\beta}$, $K_{H\alpha}$, $Z_L$, $Z_V$, and $Z_R$ factors depend, directly or indirectly, on the reference diameter of pinion $d_1$ or the centre distance **a**. When the major dimension of a gearset has been estimated from Eq. (6.5) or Eq. (6.9), all the corresponding geometrical parameters must be determined. The actual safety factors are then calculated from the Eqs. (6.1) and (6.2). If the actual safety factors are not satisfactory, the main parameters should be modified so that the gear pair possesses an adequate power transmission capacity. Thus it can be seen that an iterative approach is essential to designing a suitable (hopefully, near optimal) pair of gears for a given application using the ISO standard.

## 6.3 Geometric Calculation

The detailed geometric calculations may be carried out by using the program GTGEOM, which provides the designer with the following information necessary for precise strength analysis, manufacturing and inspection of a gear set:

(1) Angular dimensions such as transverse pressure angle, operating pressure angle and base helix angle;

(2) Linear dimensions such as addenda, root, tip and reference

diameters;

(3) Miscellaneous dimensions such as equivalent number of teeth, transverse and total contact ratios and pressure angles at the highest points of single tooth pair contact;

(4) Inspection dimensions such as base tangent lengths and diameters over pins;

(5) Information on undercutting, interference and topping.

## 6.4 Design of Gears Operating on Specified Centre Distances

A frequently arising design problem in toothed gearing is to determine the manufacturing details of gears to mesh at a desired centre distance. There is no rigorous way of solving this problem, because usually there is no unique solution in a single case. The following are several methods employed in the gear design package.

### 6.4.1 Adjusting the Helix Angle to Obtain a Specified Centre Distance

From the equation of centre distance for a standard helical gear train

$$a = \frac{m_n(z_1+z_2)}{2\cos(\beta)} \tag{6.10}$$

we know that the centre distance a varies with the helix angle $\beta$. Therefore, upon condition that the given centre distance a is greater than the value of $m_n(z_1+z_2)/2$, the helix angle $\beta$ required to meet the given centre distance a may conveniently be determined by means of Eq. (6.11)

$$\beta = \text{arc}\cos\left[\frac{m_n(z_1+z_2)}{2a}\right] \tag{6.11}$$

However, a further complication in finding the required helix angle may arise in a modified helical gear train, in which a transcendental function has to be involved.

According to the equation of centre distance for modified gears

$$a' = a\frac{\cos(\alpha_t)}{\cos(\alpha_t')} = \frac{m_n(z_1+z_2)\cos(\alpha_t)}{2\cos(\beta)\cos(\alpha_t')} \tag{6.12}$$

the helix angle $\beta$ required to achieve the desired centre distance a' will be given by

$$\beta = \text{arc}\cos\left[\frac{m_n(z_1+z_2)\cos(\alpha_t)}{2a'\cos(\alpha_t')}\right] \tag{6.13}$$

provided that the specified centre distance a' is greater than the value of $0.5m_n(z_1+z_2)\cos(\alpha_t)/\cos(\alpha_t')$ when the helix angle $\beta$ equals to zero, where

$$\alpha_t = \text{arc}\tan\left[\frac{\tan(\alpha_n)}{\cos(\beta)}\right] \tag{6.14}$$

and

$$\text{inv}(\alpha_t') = \frac{2(x_{n1}+x_{n2})}{z_1+z_2}\tan(\alpha_n)+\text{inv}(\alpha_t) \tag{6.15}$$

It should be noted from Eqs. (6.14) and (6.15) that Eq. (6.13) in fact is a transcendental one as the transverse pressure angle $\alpha_t$ and the working pressure angle $\alpha_t'$ are both a function of the helix angle $\beta$.

There is always a limit to the helix angle $\beta$ as its value has great influence upon the transmission characteristic. If it is too small, the merit of helical gears will be unable to be brought into full play; on the other hand, if it is too large, the end-thrust is likely to be too high so that the sizes of the bearings used to support the gear shafts are increased greatly and the efficiency of gears becomes lower. Therefore the helix angle $\beta$ must be kept within certain limits. For single-helical gears, a helix angle $\beta$ greater than $8^{\circ}$ and less than $20^{\circ}$ is usually satisfactory [37]. For double-helical gears, which have the advantage that, if one gear of a pair is free to float axially, it sets itself so that the tooth load is equally divided between the two halves of the facewidth, a helix angle $\beta$ greater than $25^{\circ}$ and less than $40^{\circ}$ is generally acceptable [75]. If the helix angle $\beta$ determined from Eqs. (6.11) and (13) goes beyond these limits or spur gears are to be used, the following alternatives may be considered.

Example 1:

A 19-tooth single-helical pinion having a modification coefficient of 0.36 drives a 99-tooth gear having a modification coefficient of -0.04 on a parallel shaft. The pinion has a normal module of 2.5mm, a normal pressure angle of $20^{\circ}$, and a helix angle of $15^{\circ}$. If it is to be wished that the gearset operates on a centre distance of 150mm, find the required value of helix angle.

Solution:

Using Eqs. (6.14) and (6.15) under existing conditions we get

$$\alpha_t = \text{arc tan}[\frac{\tan(20^o)}{\cos(15^o)}] = 20.6469^o$$

and

$$\text{inv}(\alpha_t') = \frac{2(0.36-0.04)}{19+99}\tan(20^o)+\text{inv}(20.6469^o) = 0.018427$$

Solving this transcendental equation, we will find

$$\alpha_t' = 21.412^o$$

Introducing the value of $\alpha_t$ and $\alpha_t'$ into Eq. (6.12), we finally obtain

$$a' = \frac{2.5(19+99)\cos(20.6469^o)}{2\cos(15^o)\cos(21.412^o)} = 153.489 \text{ mm}$$

which means that the original centre distance a' doesn't agree with the specified value, so we must try some other helix angles. After solving Eq. (6.13) in which a' is replaced by the specified centre distance of 150mm, we find that the helix angle $\beta$ which meets the requirements equals to $8.695^o$. Since the helix angle thus obtained is within the range of $8^o$ to $20^o$, the solution is valid for the problem.

### 6.4.2 Selecting the Suitable Combined Modification Coefficients to Achieve a Desired Centre Distance

By a direct calculation, the necessary combined modification coefficients $\Sigma x_n$ at a predetermined centre distance **a'** may be established by transposing Eqs. (6.12) and (6.15). Thus if a gear train which would nominally work at **a** is required to work at the arbitrary centre distance **a'**, the order of calculation is

$$\alpha_t' = \text{arc } \cos\left[\frac{a\cos(\alpha_t)}{a'}\right] = \text{arc } \cos\left[\frac{m_n(z_1+z_2)\cos(\alpha_t)}{2a'\cos(\beta)}\right] \qquad (6.16)$$

$$\Sigma x_n = x_{n1}+x_{n2} = \frac{(z_1+z_2)[\text{inv}(\alpha_t')-\text{inv}(\alpha_t)]}{2\tan(\alpha_n)} \qquad (6.17)$$

where $\alpha_t$ is still obtained from Eq. (6.14).

The combined modification coefficients $\Sigma x_n$ must then be distributed between the pinion and gear. The proportion allocated to each will depend upon the total amount to be divided, the gear ratio, and the type of gear application. The values of $x_{n1}$ and $x_{n2}$, suggested by ISO [36], may be defined by

$$x_{n1} = \lambda\frac{z_2-z_1}{z_2+z_1} + \Sigma x_n\frac{z_1}{z_2+z_1} \qquad (6.18)$$

or

$$x_{n1} = \lambda\frac{u-1}{u+1} + \Sigma x_n\frac{1}{u+1} \qquad (6.19)$$

and

$$x_{n2} = \Sigma x_n - x_{n1} \qquad (6.20)$$

where $\lambda = 0.5 - 0.75$ for a speed reducing gear train or $\lambda = 0$ for a speed increasing gear train, and gear ratio

$$u = z_2/z_1 \tag{6.21}$$

Whether the modification coefficients thus obtained may be acceptable depends whether they may avoid undercutting and maintain a reasonable crest width. If either $x_{n1}$ or $x_{n2}$ is unable to do so, we must seek some other methods.

Example 2:

By using nonstandard single-helical gearset, is it possible to mate a 29-tooth pinion with an 88-tooth gear at a centre distance of 250mm? The pinion has a normal module of 4mm, and an $18^{\circ}$ helix angle. The normal pressure angle is $20^{\circ}$.

Solution:

According to Eq. (6.10), the centre distance under the given conditions is only

$$a = \frac{4(29+88)}{2\cos(18^{\circ})} = 246.042 \text{ mm}$$

Suppose method 1 is applied to this problem, the required helix angle $\beta$ to meet the desired centre distance, according to Eq. (6.11), would need $20.6097^{\circ}$, which is outside the scope as previously stated. If, however, modified helical gears may be used, the combined modification coefficients required, from Eqs. (6.16) and (6.17), will be 1.0411. When the constant $\lambda$ in Eq. (6.18) is taken as 0.625, the modification coefficients of the pinion and gear will be

$$x_{n1} = 0.625 \cdot \frac{88-29}{88+29} + 1.0411 \cdot \frac{29}{88+29} = 0.5732$$

and

$$x_{n2} = 1.0411-0.5732 = 0.4679$$

### 6.4.3 Altering the Tooth Sum to Suit a Given Centre Distance

For most power-transmitting gears there is a tolerance, expressed or implied, on velocity ratio in the design stage. This opens up possibilities for the gears to be designed for a special centre distance.

For standard gears, by transposing Eq. (6.10), we get

$$z_1+z_2 = \frac{2a\cos(\beta)}{m_n} \tag{6.22}$$

For modified gears, by transposing Eq. (6.11), we have

$$z_1+z_2 = \frac{2a'\cos(\beta)\cos(\alpha_t')}{m_n\cos(\alpha_t)} \tag{6.23}$$

where the working pressure angle $\alpha_t'$ is still determined from Eq. (6.15), which implies that Eq. (6.23) is also transcendental.

Using Eq. (6.22) or (6.23), the theoretical tooth sum required to meet the given centre distance may be found. However, whether the desired centre distance can be eventually achieved without any error depends upon whether the theoretical tooth sum thus obtained is an integer and whether the tooth numbers in pinion and gear may be selected so that the error in gear ratio is within the given tolerance. If so, a workable gear design is obtained. Otherwise the tooth numbers in pinion and gear which more nearly meet the specified centre distance under the given conditions will be found, and then methods 1 or 2 may be considered in order to make gears fit the predetermined centre distance.

This method may not be successful, however, if the velocity ratio has only very fine tolerances, and so every such case should be very critically examined in order to find out whether the restriction is in fact so severe as was at first stated.

Example 3:

A 35-tooth helical pinion paired with a 118-tooth gear on a parallel shaft has a normal module of 6mm, a normal pressure angle of $20^{\circ}$, and a helix angle of $9.3668^{\circ}$. How many teeth must the pinion and gear have if the specified centre distance is 450mm? Hold a tolerance of 0.025 on gear ratio.

Solution::

Under existing conditions, the gear ratio and the centre distance, according to Eqs. (6.21) and (6.10), are

$$u = 118/35 = 3.3714$$

and

$$a = 6(35+118)/[2\cos(9.3668^{\circ})] = 465.2027 \text{ mm}$$

It shows that the original centre distance does not agree with the desired one. Since

$$\frac{m_n(z_1+z_2)}{2} = \frac{6(35+118)}{2} = 459 > 450$$

the first method cannot not be applied to this problem. Using the second method, the required combined modification coefficients $\Sigma x_n$ will be -2.1863, which is outside the acceptable range. Thus we have to try the third method.

The required tooth sum to meet the specified centre distance may be calculated by using Eq. (6.22)

$$z_1+z_2 = \frac{2 \cdot 450\cos(9.3668^\circ)}{6} = 148$$

from which we have

$$z_1 = \frac{148}{1+u}$$

As

$$3.3714-0.025 = 3.364 <= u <= 3.3714+0.025 = 3.3964$$

which is derived from the given conditions, we obtain

$$33.664 <= z_1 <= 34.051$$

Therefore $z_1$ should be taken as 34. The following is a check calculation of centre distance.

$$a = 6(34+114)/[2\cos(9.3668^\circ)] = 450 \text{ mm}$$

The flowchart of the design of gears operating on a specified centre distance is shown in Figure 6.1.

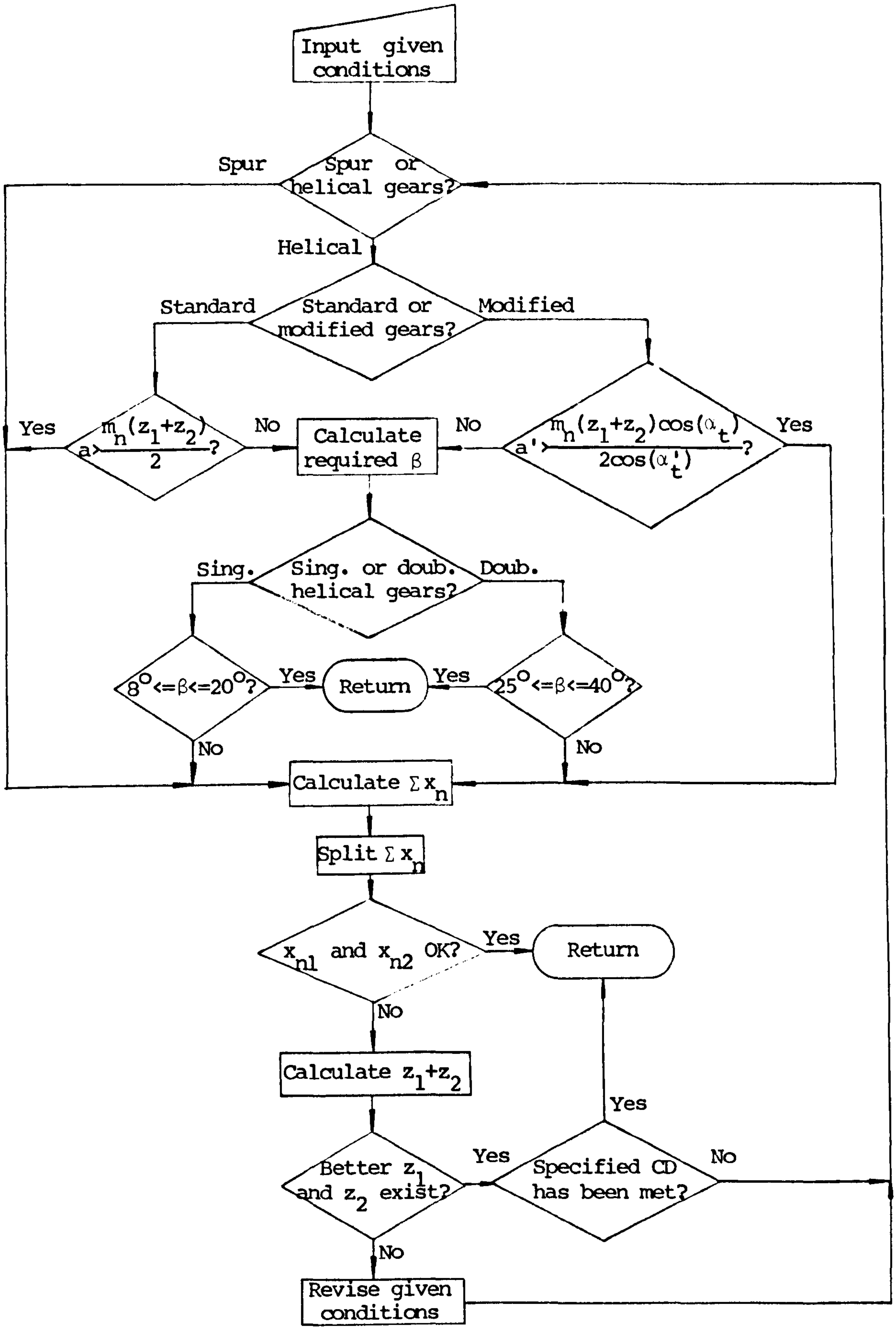



Figure 6.1 Flowchart of design for specified centre distances

## 6.5 Optimum Distribution of Gear Ratio for Double Reduction Gearing

The simplest reduction gearing is comprised of a pair of gears and is employed for a wide range of powers (up to 50,000 h.p.). Purely kinematic considerations impose no serious restriction on the gear ratio for any type of gear. Considerations of cost and space, on the other hand, set limits that would be hard to calculate rigorously in any particular case, but that can nevertheless be expressed as approximate figures for guidance in design. In ordinary spur or helical gear practice, it is usually reasonable to handle ratios from 1:1 to about 8:1 in a single reduction [24]. Where larger ratios are needed, it is customary to reduce the speed in two or more steps. For any one problem of this kind, there may be several alternative solutions. Between these discrimination may be possible only by considering particular requirements such as weight, size and lubrication.

### 6.5.1 General Considerations of Gear Ratio Distribution

In general, the distribution of the overall gear ratio can play a very important part on how easy or hard it is to meet the requirements of the particular application. In this section, the distribution approaches of gear ratio for double reduction gear trains are based on the following considerations:

(a) The first reduction gearset has the same actual safety factor as the second reduction;

(b) The gearboxes have lightest weights;

(c) The gearboxes are of minimum sizes;

(d) The gearboxes employ standardised centre distances;

(e) The commonest lubrication method — Splash (or oil-bath) lubrication may be employed.

On the basis of these considerations, ten types of distribution methods of gear ratio are developed for designing double-reduction gearing operating on various requirements and an example shows their uses.

### 6.5.2 Basic Equations for Equal Contact Strength Requirements

In double-reduction gear train, the actual reliability corresponding to a specified life and the actual factor of safety guarding against the tooth surface failure (pitting) depend on that of either the first-reduction or second-reduction gear pair, whichever is the lower. Consequently, the economical ideal is to select gear materials and dimensions so that the actual safety factors on contact stress for both the first-reduction and second-reduction gear pairs coincide with the minimum demanded safety factor, that is

$$S_{Hh} = S_{Hs} = S_{Hmin} \tag{6.24}$$

where and in what follows, suffixes **h** and **s** refer to the first reduction (high-speed) and second reduction (slow-speed) gear pairs respectively.

The ISO approach for obtaining the safety factor on contact stress is now applied to a double reduction gear system. By combining Eqs. (6.1), (6.3) and (6.4), we find the actual safety factors on contact stress for the first reduction and second reduction to be, respectively,

$$S_{Hh} = \frac{\sigma_{Hlimh} Z_h}{\sqrt{\frac{2000 T_{1h}(u_h+1)}{\psi_{dh} d_{1h}^3 u_h} K_h}} \tag{6.25}$$

and

$$S_{Hs} = \frac{\sigma_{Hlims} Z_s}{\sqrt{\frac{2000 T_{1s}(u_s+1)}{\psi_{ds} d_{1s}^3 u_s} K_s}} \tag{6.26}$$

where **K** = resultant load factor

$$K = K_A K_V K_{H\beta} K_{H\alpha} \tag{6.27}$$

**Z** = resultant factor for contact stress

$$Z = \frac{Z_N Z_L Z_R Z_V Z_W Z_X}{Z_H Z_E Z_\varepsilon Z_\beta} \tag{6.28}$$

If the mechanical efficiency of the first reduction assembly is designated as $\eta_h$, we have

$$T_{1s} = u_h T_{1h} \eta_h \tag{6.29}$$

After some rearranging and the use of Eqs. (6.25), (6.26), and (6.29), Eq. (6.24) becomes

$$\frac{\psi_{ds}}{\psi_{dh}} \left(\frac{d_{1s}}{d_{1h}}\right)^3 \frac{(u_h+1)u_s}{(u_s+1)u_h^2} \frac{(\sigma_{Hlims} Z_s)^2/K_s}{(\sigma_{Hlimh} Z_h)^2 \eta_h/K_h} = 1 \tag{6.30}$$

Substituting $u_s = u/u_h$ in Eq. (6.30) gives

$$\frac{\psi_{ds}}{\psi_{dh}} \left(\frac{d_{1s}}{d_{1h}}\right)^3 \frac{(u_h+1)u}{(u+u_h)u_h^2} C = 1 \tag{6.31}$$

where

$$C = \frac{(\sigma_{Hlims} Z_s)^2/K_s}{(\sigma_{Hlimh} Z_h)^2 \eta_h/K_h} \tag{6.32}$$

The more useful form of Eq. (6.31) can be achieved by using Eqs. (6.6) and (6.8)

$$\frac{\psi_{as}}{\psi_{ah}} \left(\frac{a_s}{a_h}\right)^3 \left(\frac{u_h+1}{u+u_h}\right)^3 uC = 1 \tag{6.33}$$

With the following equations

$$a_h = \frac{d_{2h}}{2}\left(1+\frac{1}{u_h}\right) \tag{6.34}$$

$$a_s = \frac{d_{2s}}{2}\left(1+\frac{1}{u_s}\right) \tag{6.35}$$

Eq. (6.33) can now be written as

$$\frac{\psi_{as}}{\psi_{ah}} \left(\frac{d_{2s}}{d_{2h}}\right)^3 \left(\frac{u_h^3}{u^2}\right) C = 1 \tag{6.36}$$

Eqs. (6.31), (6.33) and (6.36) give the conditions which must be met if the two pairs of gears in a double reduction gear system are to be of equal actual safety factors for contact stress.

### 6.5.3 Basic Equation for Lightest Weight

Nowadays weight is a critical factor in gear design. Weight reduction usually means volume reduction, which in turn lowers cost of materials, manufacturing, and shipping. The method presented here pinpoints the gear sizes and ratios that will permit the lightest possible design — while still meeting the contact strength requirements of the application.

Generally speaking, the weight of a gear train is fairly proportional to the solid rotor volume ($bd^2$) of the individual gears in the train. This may seem an oversimplication — but a specific volume of material is necessary to carry a load at a particular reduction ratio.

After some rearranging and the use of Eq. (6.4), the relationship between the overall gear dimensions, the gear ratio and the torque may be obtained from Eq. (6.1)

$$bd_1^2 = 2000T_1 S_H^2 \frac{K}{(\sigma_{Hlim} Z)^2}\left(\frac{u+1}{u}\right) \tag{6.37}$$

For simplicity, let

$$A = 2000T_1 S_H^2 \tag{6.38}$$

and

$$B = \frac{(\sigma_{Hlim} Z)^2}{K} \tag{6.39}$$

Then, by substitution, we obtain

$$bd_1^2 = \frac{A}{B}\left(\frac{u+1}{u}\right) \tag{6.40}$$

The solid rotor volume of a gear is $V = \pi bd^2/4$. Hence Eq. (6.40) is a function of the solid rotor volume of the input pinion. The weight of a gear system is approximately proportional to the sum of the solid rotor volumes of its gears; therefore a term similar to Eq. (6.40) can be written for the mating gear. For the gear: $d_2 = d_1 u$. Therefore

$$bd_2^2 = bd_1^2 u^2 = \frac{A}{B} u(u+1) \tag{6.41}$$

If this pinion and gear are the only two gears in the gear system, as with the single reduction, the total weight will be a function of their sum:

$$\Sigma bd^2 = bd_1^2 + bd_2^2 = \frac{A}{B}\left(1 + \frac{1}{u} + u + u^2\right) \tag{6.42}$$

For a double reduction gear train we have

$$\Sigma bd^2 = \frac{A_h}{B_h}\left(1 + \frac{1}{u_h} + u_h + u_h^2\right) + \frac{A_s}{B_s}\left(1 + \frac{1}{u_s} + u_s + u_s^2\right) \tag{6.43}$$

From Eqs. (6.24) and (6.29), we find

$$A_s = u_h \eta_h A_h \tag{6.44}$$

By combining Eq. (6.44) and $u_s = u/u_h$:

$$\Sigma bd^2 = \frac{A_h}{B_h}(1+ \frac{1}{u_h} +u_h+u_h^2)+ \frac{A_h\eta_h}{B_s}(u+ \frac{u^2}{u_h} +u_h+ \frac{u_h^2}{u}) \qquad (6.45)$$

However, there is only one value for $u_h$ for any value u that will provide the least weight. To determine the optimum first-reduction gear ratio, we assume that the factor B of each gear mesh is a constant [44, 75]. Then the first derivation of Eq. (6.45) is taken with respect to the input ratio, $u_h$, and the resulting equation is set equal to zero. Thus

$$\frac{d(\Sigma bd^2)}{d(u_h)} = \frac{A_h}{B_h}(2u_h+1- \frac{1}{u_h^2}) + \frac{A_h\eta_h}{B_s}(\frac{2u_h}{u} +1- \frac{u^2}{u_h^2}) = 0 \qquad (6.46)$$

By simplication and rearrangement, we get

$$2(C+ \frac{1}{u})u_h^3+(C+1)u_h^2-(C+u^2) = 0 \qquad (6.47)$$

Solving for $u_h$, we obtain

$$u_h = \sqrt[3]{\frac{C+u^2}{4(C+1/u)} - \frac{(C+1)^3}{216(C+1/u)^3} + \frac{(C+u^2)}{4(C+1/u)}\sqrt{1- \frac{(C+1)^3}{27(C+1/u)^2(C+u^2)}}} +$$

$$\sqrt[3]{\frac{C+u^2}{4(C+1/u)} - \frac{(C+1)^3}{216(C+1/u)^3} - \frac{(C+u^2)}{4(C+1/u)}\sqrt{1- \frac{(C+1)^3}{27(C+1/u)^2(C+u^2)}}} -$$

$$\frac{C+1}{6(C+1/u)} \qquad (6.48)$$

Since

$$\frac{(C+1)^3}{27(C+1/u)^2(C+u^2)} = \frac{C^3+3C^2+3C+1}{27[C^3+(2/u+u^2)C^2+(1/u^2+2u)C+1]} < 1$$

we can write, from the approximation of square root ($\sqrt{1+x} \doteq 1+x/2$ if ABS(x) < 1),

$$\sqrt{1-\frac{(C+1)^3}{27(C+1/u)^2(C+u^2)}} = 1-\frac{(C+1)^3}{54(C+1/u)^2(C+u^2)} \qquad (6.49)$$

By substituting the value in Eq. (6.49) into Eq. (6.48) we find the optimum first-reduction gear ratio to be

$$u_h = \sqrt[3]{\frac{C+u^2}{2(C+1/u)} - \frac{(C+1)^3}{108(C+1/u)^3}} - \frac{C+1}{6(C+1/u)} \qquad (6.50)$$

### 6.5.4 Basic Equation for Minimum Size

In general, the most desirable gear set is the smallest one that will perform the required job. Smaller gears are easier to make, run more smoothly due to smaller inertial loads and pitch line velocities, and are less expensive. In this section an optimal design technique to obtain compact double-reduction gearboxes will be presented. The design objective is to minimize the longitudinal dimension L of a gear train.

Referring to Figure 6.2 we find the longitudinal dimension L to be

$$L = \frac{d_{1h}}{2} + a_h + a_s + \frac{d_{2s}}{2} \qquad (6.51)$$

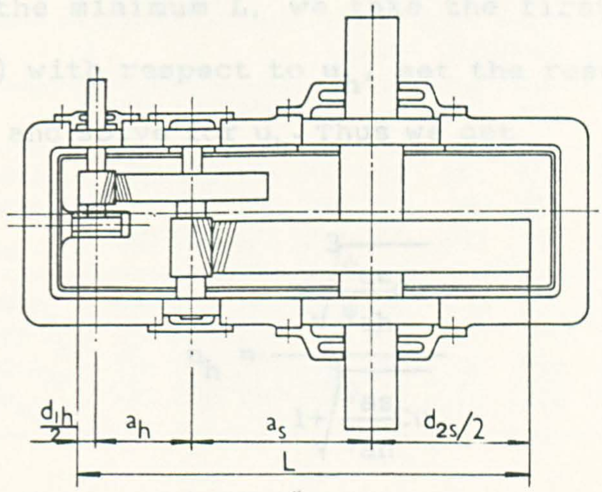



Figure 6.2 Longitudinal dimension L of double-reduction gearing

With Eqs. (6.6) and (6.35), Eq. (6.51) can be written as

$$L = \frac{2+u_h}{1+u_h}a_h + \frac{1+2u_s}{1+u_s}a_s \tag{6.52}$$

If we substitute $\mathbf{b} = \psi_{\mathbf{a}}\mathbf{a}$ and $\mathbf{d_1} = \mathbf{2a/(u+1)}$ in Eq. (6.40) and solve for **a**, we find

$$a = (1+u)\sqrt[3]{\frac{A}{4B\psi_a u}} \tag{6.53}$$

Using Eq. (6.53), we can now write Eq. (6.52) in the form

$$L = (2+u_h)\sqrt[3]{\frac{A_h}{4B_h\psi_{ah}u_h}} + (1+2u_s)\sqrt[3]{\frac{A_s}{4B_s\psi_{as}u_s}}$$

$$= \sqrt[3]{\frac{A_h}{4B_h\psi_{ah}}}\left(\frac{2}{\sqrt[3]{u_h}} + \sqrt[3]{u_h^2}\right) + \sqrt[3]{\frac{A_h\eta_h}{4B_s\psi_{as}}}\left(2\sqrt[3]{\frac{u^2}{u_h}} + \sqrt[3]{u_h^2/u}\right) \tag{6.54}$$

In order to find the value of the first reduction gear ratio that will provide the minimum L, we take the first derivative of equation (6.54) with respect to $u_h$, set the resulting equation equal to zero, and solve for $u_h$. Thus we get

$$u_h = \frac{u + \sqrt[3]{\frac{\psi_{as}}{\psi_{ah}} Cu}}{1 + \sqrt[3]{\frac{\psi_{as}}{\psi_{ah}} Cu}} \qquad (6.55)$$

## 6.5.5 Basic Equation for Standardised Centre Distances

The main parameters of certain types of gearboxes have been standardised in some countries. For instance, Soviet and Chinese standards specify a number of centre distances, facewidth factors, modules, gear ratios, and the helix angles for single-, double-, and triple-reduction gear units. On some occasions a gear train must operate on the centre distances which match these standardised values. In designing a double-reduction gearing of this character, it is essential to select the individual gear ratio properly so that two pairs of gears may have the same actual safety factors.

Table 6.1 show the standardised centre distances of cylindrical gear reducers derived from the Chinese standard JB1130-70 [44]. In this standard the facewidth factors of all the gears are taken to equal 0.4.

Analysing the data in Table 1 we notice that the ratio, $a_s/a_h$, between the centre distances of high-speed and slow-speed gear trains is about 1.5. To obtain an expression of gear ratio split

Table 6.1

Centre Distance a (JB1130-70)

| Type | Stage | Centre Distance (mm) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S.R. | a | 100 | 150 | 200 | 250 | 300 | 350 | 400 | 450 | 500 | 600 | 700 |
| D.R. | High-speed $a_h$ | 100 | 150 | 175 | 200 | 250 | 250 | 300 | 350 | 400 | 450 | 500 |
| | Slow-speed $a_s$ | 150 | 200 | 250 | 300 | 350 | 400 | 450 | 500 | 600 | 700 | 800 |
| | $a=a_h+a_s$ | 250 | 350 | 425 | 500 | 600 | 650 | 750 | 850 | 1000 | 1150 | 1300 |
| T.R. | High-speed $a_h$ | 100 | 150 | 150 | 175 | 200 | 250 | 250 | 300 | 350 | | |
| | Intermediate $a_m$ | 150 | 200 | 250 | 250 | 300 | 350 | 400 | 450 | 500 | | |
| | Slow-speed $a_s$ | 250 | 300 | 350 | 400 | 450 | 500 | 600 | 700 | 800 | | |
| | $a=a_h+a_m+a_s$ | 500 | 650 | 750 | 825 | 950 | 1100 | 1250 | 1450 | 1650 | | |

for employing the standardised centre distances, we solve Eq. (6.33) for the first-reduction gear ratio $u_h$. Thus

$$u_h = \frac{u - \frac{a_s}{a_h}\sqrt[3]{\frac{\psi_{as}}{\psi_{ah}}uC}}{\frac{a_s}{a_h}\sqrt[3]{\frac{\psi_{as}}{\psi_{ah}}uC} - 1} \qquad (6.56)$$

By substituting $a_s/a_h = 1.5$ and $\psi_{as}/\psi_{ah} = 1$ in Eq. (6.56) we get

$$u_h = \frac{u-1.5\sqrt[3]{uC}}{1.5\sqrt[3]{uC} - 1} \qquad (6.57)$$

### 6.5.6 Basic Equation for Splash Lubrication

All gears, regardless of type and material, will gain in life expectancy by being properly lubricated. The lubrication of gears should therefore be as much a part of the design requirements for successful operation as is the calculation of contact stresses or bending stresses. The methods by which lubricant is applied to gear teeth will vary with type of gear, speed in terms of pitch-line velocity, surface finish, hardness of the material, and combination of materials, and may be classified under the following headings: (a)by hand, (b)drip feed, (c)splash, (d)spray with natural cooling, (e)spray with artificial cooling.

Splash lubrication by dipping gears is the commonest method employed in industrial speed reducers [83]. It is satisfactory for tooth speeds up to the order of about 12m/s, but much higher speeds are used for special drives where the running time is comparatively short and some increase in churning loss is less troublesome than a spray lubrication system. Figure 6.3 illustrates a typical splash-lubricated double-reduction gearing, in which both the first-reduction and second-reduction gears are dipping into the lubricant reservoir.

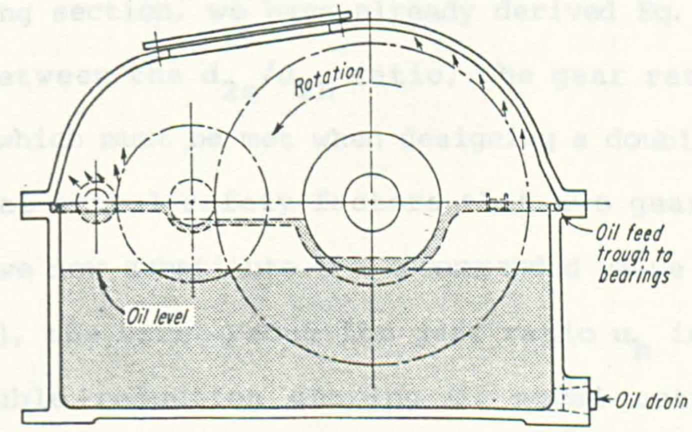



Figure 6.3 Splash-lubricated double-reduction gearing

To assure adequate and satisfactory lubrication, the oil level must be taken into consideration. Too high an oil level may result in increase in resistance to the gear rotation and excessive heat through oil churning. Therefore, as the peripheral velocity of the gear rises, the depth of immersion should be decreased. The recommended depth of immersion for the high-speed gear is about 0.7 of the whole depth but not less than 10mm. The slow-speed gear should not be immersed by more than 100mm. If the two gearsets cannot both be lubricated by dipping gears, it is often satisfactory for the slow-speed gearset to do so, and for the other to be oiled by a special device or by a pinion, a ring, etc. [20].

In the splash-lubricated double-reduction gearing where a gear of every pair is dipping into a sump, the depth of immersion for the slow-speed gear has a minimum value, which depends on the pitch diameters $d_{2h}$, $d_{2s}$ of two gears. The greater the ratio of $d_{2s}$ to $d_{2h}$, the greater the depth of immersion for the slow-speed gear. For good splash lubrication the ratio $d_{2s}/d_{2h}$ is usually kept in the range 1.0 to 1.3 [75].

In the preceding section, we have already derived Eq. (6.36), the relationship between the $d_{2s}/d_{2h}$ ratio, the gear ratio and the $\psi_{as}/\psi_{ah}$ ratio which must be met when designing a double-reduction gearing of equal actual safety factors that two gearsets have. Therefore, if we now substitute the recommended value of $d_{2s}/d_{2h}$ into Eq. (6.36), the first-reduction gear ratio $u_h$ in a splash-lubricated double-reduction gearing of equal actual safety factors may be obtained from the equation

$$u_h = \frac{1}{1.0\text{–}1.3}\sqrt[3]{\frac{u^2}{C\psi_{as}/\psi_{ah}}} \qquad (6.58)$$

### 6.5.7 Ten Methods to Distribute Gear Ratio for Double-Reduction Gearing

Schematic diagrams of the most widespread types of double-reduction gear systems are shown in Figure 6.4. In designing these gear systems, it is essential to select individual gear ratio and facewidth factor ratio properly in order to meet the requirements of particular application. On the basis of the considerations described above ten methods are presented for determining the optimum first-reduction gear ratio and corresponding facewidth factor ratio.

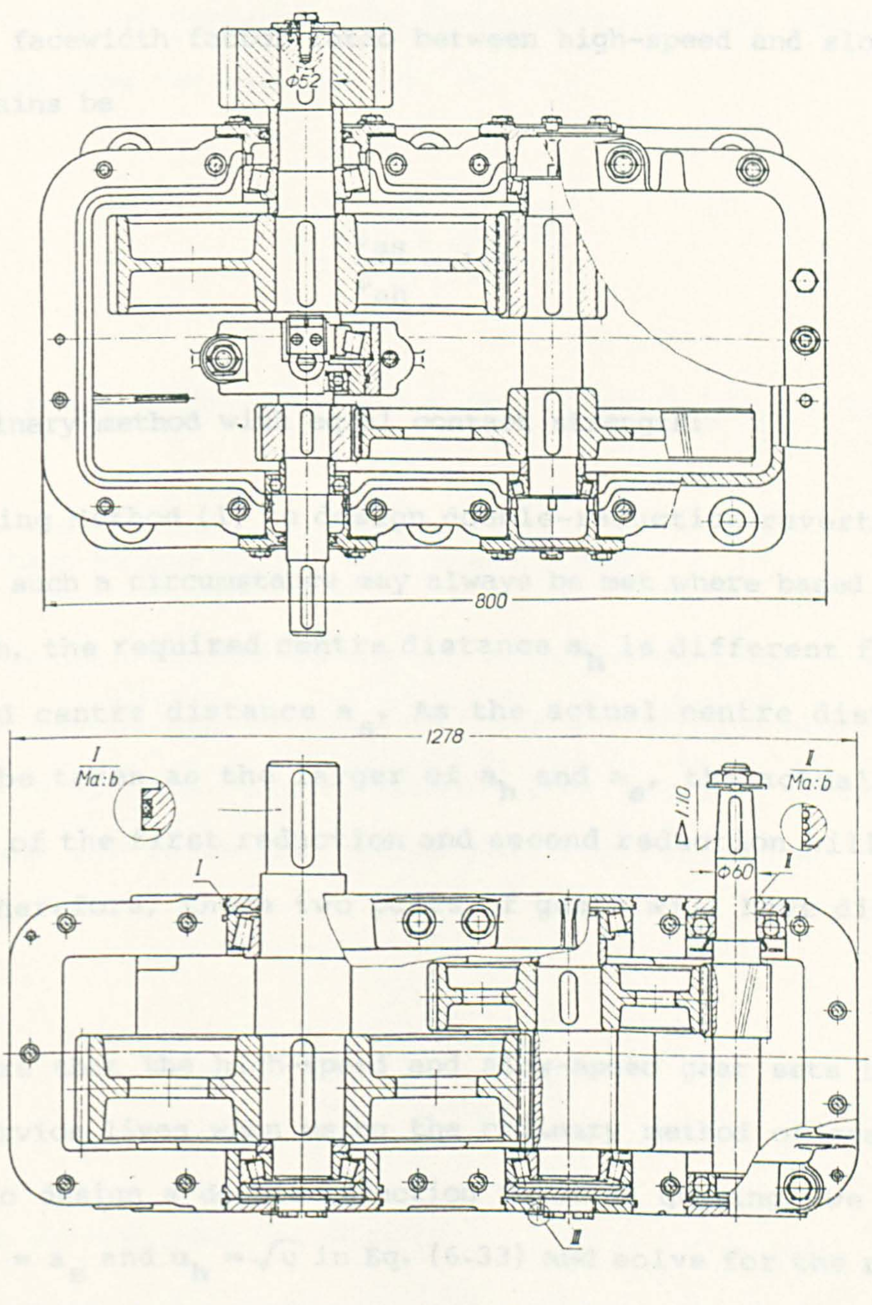



Figure 6.4 Typical double-reduction gear systems

### (A) Double-Reduction Reverted Gearing

(1) Ordinary methods:

It is suggested in [44] that usually the high-speed gear ratio may be determined from the following equation

$$u_h = \sqrt{u} \tag{6.59}$$

and the facewidth factor ratio between high-speed and slow-speed gear trains be

$$\frac{\psi_{as}}{\psi_{ah}} = 1.5 \tag{6.60}$$

(2) Ordinary method with equal contact strength:

When using Method (1) to design double-reduction reverted gear trains, such a circumstance may always be met where based on gear strength, the required centre distance $a_h$ is different from the required centre distance $a_s$. As the actual centre distance a should be taken as the larger of $a_h$ and $a_s$, the actual safety factors of the first reduction and second reduction will not be same. Therefore, these two pairs of gears will have different lives.

To assure that the high-speed and slow-speed gear sets have the same service lives when using the ordinary method of gear ratio split to design a double-reduction reverted gearing, we substitute $a_h = a_s$ and $u_h = \sqrt{u}$ in Eq. (6.33) and solve for the ratio of the facewidth factors. Thus we find

$$\frac{\psi_{as}}{\psi_{ah}} = \sqrt{u}/C = u_h/C \qquad (6.61)$$

(3) Equal contact strength method:

In some cases, for example, where there are external restrictions on dimensions, it is desirable to use the recommended value of facewidth factor ratio as in Eq. (6.60). Then to achieve the same life for the two gearsets, the first-reduction gear ratio may be determined from the equation

$$u_h = \frac{u - \sqrt[3]{1.5uC}}{\sqrt[3]{1.5uC} - 1} \qquad (6.62)$$

which is found by substituting $a_s/a_h = 1$ and $\psi_{as}/\psi_{ah} = 1.5$ in Eq. (6.56).

(4) Lightest weight method with equal lives:

In designing a double-reduction reverted gearing having the lightest weight and two pairs of gears with the same service lives, the optimum first-reduction gear ratio is given by Eq. (6.50) and the facewidth factor ratio is calculated from the equation

$$\frac{\psi_{as}}{\psi_{ah}} = \frac{(u+u_h)^3}{uC(u_h+1)^3} \qquad (6.63)$$

which is derived from Eq. (6.33).

**(B) Double-Reduction Developed Gearing**

(5) Ordinary method:

Where there are no external restrictions on dimensions, and the splash lubrication is to be employed, it is economical for double-reduction developed gear trains to use higher velocity ratios in the higher speed gears. The recommended gear ratio of high-speed pair is calculated by the expression [44]

$$u_h = \sqrt{(1.2\text{-}1.3)u} \qquad (6.64)$$

and ratio of facewidth factor is usually taken as

$$\frac{\psi_{as}}{\psi_{ah}} = 1 \qquad (6.65)$$

(6) Equal contact strength and facewidth factor method to employ splash lubrication:

Based on the analysis in Section 6.5.6 the first-reduction gear ratio to meet this particular requirement is found from the following equation

$$u_h = \frac{1}{1.0\text{-}1.3}\sqrt[3]{u^2/c} \qquad (6.66)$$

(7) Lightest weight method with equal facewidth factor:

To design the double-reduction gearing with lightest weight, the first-reduction gear ratio should be determined from Eq. (6.50). The ratio of facewidth factors in this method is taken as 1.

(8) Lightest weight method with equal contact strength using splash lubrication:

In this circumstance, the optimum first-reduction gear ratio is still calculated from Eq. (6.50). But the ratio of facewidth factors between two pairs of gears should meet the following relationship, according to the basic equation (6.36) of equal contact strength

$$\frac{\psi_{as}}{\psi_{ah}} = \frac{u^2}{C[(1.0\text{-}1.3)u_h]^3} \tag{6.67}$$

(9) Minimum size method with equal facewidth factor:

By substituting the value in Eq. (6.65) into Eq. (6.55), we find the first-reduction gear ratio to be

$$u_h = \frac{u + \sqrt[3]{Cu}}{1 + \sqrt[3]{Cu}} \tag{6.68}$$

(10) Standardised centre distance method with equal contact strength

In this method the first-reduction gear ratio is obtained from Eq. (6.57) and the facewidth factors employed in both high-speed and slow-speed gear trains are 0.4.

### 6.5.8 Example

The double-reduction reverted gear system for a general-purpose industrial application has the following requirements:

| | | | |
|---|---|---|---|
| Input torque: | 155.8735N.m | Input speed: | 485rpm |
| Overall gear ratio: | 11.353 | Working life: | 10years |
| Working days: | 292days/year | Duty: | 16hours/day |

Load characteristic of

| | | | |
|---|---|---|---|
| power source: | uniform | driven machine: | heavy shock |

Minimum demanded safety factor: 1.0

Assume that the given conditions for the present example are as follows:

| | Pinion | Gear |
|---|---|---|
| Material: | structural steel | alloy steel |
| Heat treatment: | normalizing | hardening and tempering |
| Surface hardness: | 230HB | 210HB |
| Manufacturing accuracy: | 6 | 6 |

Gear type: helical Helix angle: $8^{o}$

Tooth number on pinion for

the high-speed stage: 40 the slow-speed stage: 35

Facewidth factor for the first-reduction gearset: 0.1

The design results obtained from different gear ratio split methods are shown in Table 6.2.

Table 6.2

Gear Design Results by Using Different Gear Ratio Split Methods

| Method | $a$ | $m_n$ | $z_1$ | $z_2$ | $\beta$ | $x_{n1}$ | $x_{n2}$ | $\psi_a$ | $S_{H1}$ | $S_{H2}$ | Vol($m^3$) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (1) | 530 | 6.0 | 40 | 135 | 8.0000 | 0.0 | 0.0 | 0.10 | 1.17 | 1.75 | 0.0305 |
| | 530 | 7.0 | 34 | 114 | 12.2949 | 0.0 | 0.0 | 0.15 | 1.07 | 1.61 | 0.0459 |
| (2) | 442 | 5.0 | 40 | 135 | 8.0000 | 0.0 | 0.0 | 0.10 | 1.05 | 1.57 | 0.0176 |
| | 442 | 5.5 | 35 | 118 | 17.7577 | 0.0 | 0.0 | 0.16 | 1.05 | 1.60 | 0.0283 |
| (3) | 508 | 5.5 | 40 | 143 | 8.0000 | 0.0 | 0.0 | 0.10 | 1.15 | 1.72 | 0.0274 |
| | 508 | 7.0 | 34 | 108 | 12.0489 | 0.0 | 0.0 | 0.15 | 1.11 | 1.68 | 0.0394 |
| (4) | 421 | 4.5 | 38 | 145 | 12.0952 | 0.0 | 0.0 | 0.10 | 1.01 | 1.53 | 0.0158 |
| | 421 | 6.0 | 35 | 104 | 8.0000 | 0.0 | 0.0 | 0.19 | 1.01 | 1.52 | 0.0280 |

## 6.6 Description of Gear Design Program

The gear design program developed can be used to design spur or helical, external or internal, standard or modified, single or double reduction gear trains based on the international gear standard ISO/DIS6336. The recommended minimum demanded safety factors for contact stress and bending stress have been contained in the program and will be presented to the designer in due course.

The preset facewidth factors are 0.1, 0.2, 0.3, 0.4, 0.5, and 0.6, helix angles $8^{\circ}$, $12^{\circ}$, $16^{\circ}$ and $20^{\circ}$. Therefore many alternative design solutions will be presented for the designer to choose for each gear system. The actual number of design solutions which may be obtained in theory depends on the type of the gearing to be designed and whether the facewidth factor and helix angle are specified by the designer, and are shown in Table 6.3. However, some of the solutions may not be acceptable because of the large dynamic factor, load distribution factor, etc. In order to speed up the design process, these unacceptable design solutions should be eliminated. Thus the ultimate number of

Table 6.3

Theoretical Number of Solutions Obtained from Gear Design Program

| Gear Type | Design Conditions | | | |
|---|---|---|---|---|
| | $\psi_a$ Fixed<br>$\beta$ Fixed | $\psi_a$ Fixed<br>$\beta$ Not Fixed | $\psi_a$ Not Fixed<br>$\beta$ Fixed | $\psi_a$ Not Fixed<br>$\beta$ Not Fixed |
| Spur | 1 | | 6 | |
| Helical | 1 | 4 | 6 | 24 |

design solutions is uncertain. But sometimes the designer may be interested in surveying the effect of facewidth factor and helix angle on gear size, safety factors, and performance. Therefore whether these unpractical solutions will be eliminated or not is decided by the designer.

When designing double-reduction gearing, the designer may select the first-reduction gear ratio he prefers or choose one of the gear ratio split method described in Section 6.5 on the basis of the particular requirements of the application. However the program is designed to courage the designer to try on different gear ratio split methods so that a more optimal design solution may be achieved.

Input data to the gear design program is divided into two categories, namely design specifications and given conditions. Design specifications usually consist of the following data:

(1) Load characteristics of power source and driven machine;
(2) Expected service life of gearbox;
(3) Gearbox input torque;
(4) Gearbox input speed;
(5) Overall gear ratio and its tolerance;
(6) Stage number of gearbox;
(7) Minimum demanded safety factors for contact stress and bending stress.

Given conditions are comprised of the following information:

(1) Gear materials;
(2) Gear manufacturing accuracy;
(3) Gear arrangement;
(4) Gear type;

(5) Parameters of basic rack;

(6) Pinion tooth number;

(7) Helix angle;

(8) Addendum modification;

(9) Facewidth factor;

(10) Centre distance (if specified).

Design specifications are supplied by the user of gearbox, therefore they should usually keep unchanged in the course of design. By contrast, given conditions are selected by the designer, and they are usually changed from time to time in order that the design specifications for the application may be achieved satisfactorily.

The output, when the design process terminates, is stored in the files called GSPEGIV and GRESULT on the disk. It provides the designer with the detailed documents as follows

(1) Design specifications and given conditions;

(2) Gear tooth proportions and information on contact ratio, undercutting, interference, etc.;

(3) Surface and fillet roughness grades, tolerances on gear tooth, lubricant viscosity, bearing span, and pinion offset distances;

(4) Information on gear strength such as actual safety factors, load factors;

(5) Gear forces.

Figure 6.5 shows a simplified flow chart of the gear train design program. The entire gear design process mainly consists of three stages namely data input, preliminary design, and strength checking.

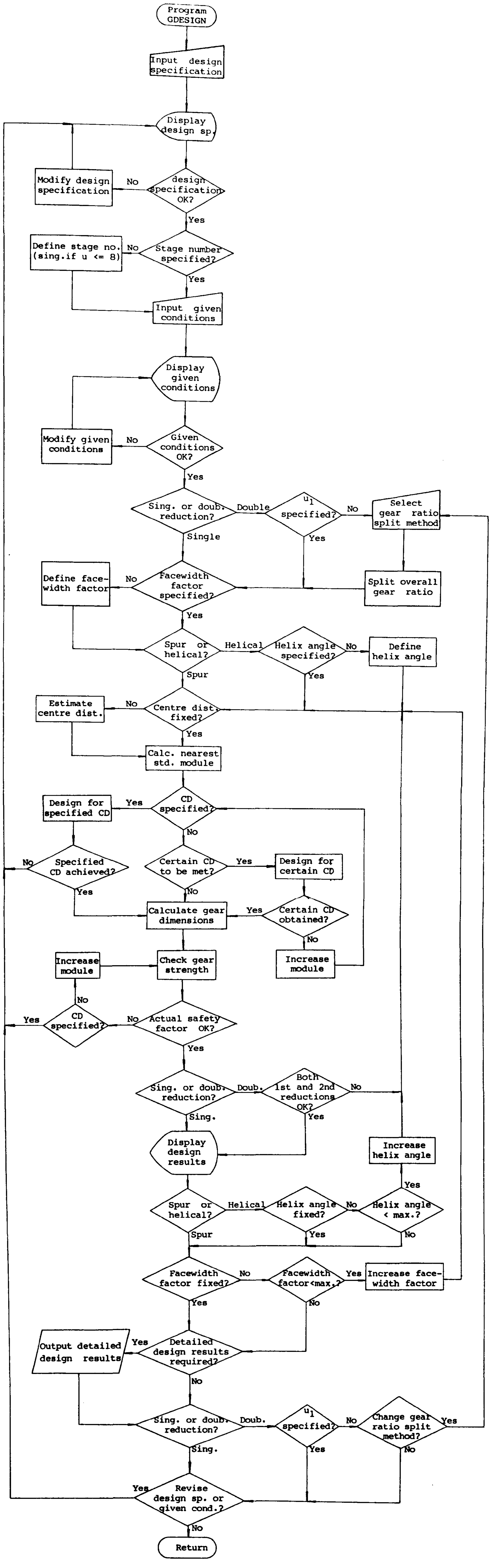



**Figure 6.5 Schematic flowchart of gear train design**

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The data input starts by asking the designer for the gearbox specifications. When the designer has specified his data, he is given an opportunity to check it and to correct it. If the stage number of gearbox is not specified in the specifications, it will be determined by the program. Then the designer is asked to input the given conditions for each gear pair in the gearbox and he also has a chance to check the data input and to correct any errors. After that, the gear ratio split method will be selected by the designer based on the particular requirements if a double-reduction gearbox is to be used and the first-reduction gear ratio is not specified. Otherwise we get into the next design stage.

In the predesign stage, we first estimate the required major dimension — centre distance if it is not specified. Then the required nearest standard module is computed. If the centre distance has been specified, we employ the methods described in Section 6.4 to achieve it. If unsuccessful, the designer has to return to the data input stage to modify the data. If successful, the gear tooth proportions are calculated by calling the program GTGEOM and all the other data necessary for strength checking, such as roughness grades, gear tooth tolerances, bearing span, pinion offset distance, and lubricant viscosity, are determined.

With the relevant data obtained from the previous two stages, the detailed strength analysis may now be carried out by using the program GCHECK. The computer first finds the values of various factors. And then substituting these factor values together with other information into Eqs. (6.1) and (6.2) gives actual safety factors for contact stress and bending stress. These safety factors are then compared with minimum demanded values as to acceptability. If they are satisfactory, a copy of detailed design documents may be obtained. Otherwise the required standard module is increased, and the preliminary design and strength checking stages are repeated until satisfactory results are achieved. If the centre distance is specified and the actual safety factors obtained are not satisfactory, the designer has to get back into the data input stage to modify the input data due to no solution to the original problem.

When a double-reduction gearbox is employed, it will be designed pair by pair by the repeated use of the last two stages of the design process. After one design solution is obtained, the design process will starts again with a new higher facewidth factor or helix angle if the facewidth factor is not specified or the helix angle may be changed. Therefore the program can provide many alternative design solutions for the designer. Moreover, when the designer has tried one of the gear ratio split methods in designing a double-reduction gearbox, he may test other gear ratio split methods. Thus up to 240 solutions may be obtained for the same design problem. If the design results presented are not satisfactory, the designer can readily obtain other solutions by modifying any given conditions such as gear materials, gear types, tooth numbers, or specifications such as expected service life, gear ratio tolerance, stage number of gearbox, without re-inputting the unchanged data.

# Chapter 7

## APPLICATION OF FINITE ELEMENT TECHNIQUES TO GEAR TOOTH STRESS ANALYSIS

### 7.1 Introduction

One of the primary causes of gear tooth failures is the presence of large tensile stresses in the root fillets of loaded gear teeth. These stresses tend to reduce overall gear life and can result in catastrophic tooth failure under peak loading conditions.

Many attempts have been made by earlier investigators to relate tensile fillet stresses observed in statically loaded gear teeth to the geometric appearance of the tooth. As early as 1893, Wilfred Lewis [42] applied elementary beam theory to tooth profiles by inscribing a parabola to represent a beam of uniform strength. As a result of the uniform strength assumption the Lewis formula was unable to deal effectively with abrupt changes in tooth section that occur in the tooth fillet region. Furthermore, a radial component of load exists that tends to modify the stress produced by the applied bending moment. In 1938, Merritt [52] added a term to the Lewis formula that accounted for the radial component of load which was assumed to act along the vertical centreline of the tooth. At this point neither the Lewis nor the Merritt formula was able to take into account abrupt changes in tooth section.

The introduction of the photoelastic technique by Timoshenko and Baud[78] gave investigators the first real opportunity to examine in detail the stress raising effect of gear tooth fillets. Dolan and Broghamer [21] studied the subject in depth and introduced a combined stress correction and stress concentration factor to be used in conjunction with the Lewis stress formula. This factor related the increase in observed fillet stress over the nominal bending stress to load height, fillet radius and pressure angle. In 1955, Jacobson [40] analyzed three photoelastic tooth models with different pitch circles, addendums, and dedendums.

In 1962, Aida and Terauchi [5] studied bending stresses using stress functions and classic elasticity theories. In 1973, the stress data obtained from the Jacobson analysis were matched by Wilcox and Coleman [86] using finite element techniques. More recently, Cardou and Tordion [12] studied the stresses using complex variables. Their results confirm the results obtained using finite element techniques.

Regarding the use of the finite element method itself in gear stress analyses, there have been a number of notable achievements during the past decade. For example, Chabert, Dang Tran, and Mathis [13] have used finite element techniques to examine the stress distribution across the root section. Tobe, Kato, and Inoue [80, 81], and Winter and Hirt [87] have studied root stresses using finite element methods. Oda, Nagamura, and Aoki [60] have examined the effect of rim thickness on the root stresses using the finite element method. Finally, Chang, Huston, and Coy [14] have presented finite element stress analysis results for a variety of loading conditions, support conditions, fillet radii, and rim thickness.

In view of this, the objectives of the research effort of this chapter are to (a) show how the method of finite elements can be applied to the determination of the exact stress distribution in the fillet regions of gear teeth; (b)determine the maximum surface stresses in the tensile fillet on spur gear for the loading on the highest point of single tooth pair contact using finite element method; and (c) compare the results with those of the ISO gear standard.

## 7.2 Finite Element Method

The finite element technique is a very powerful and new approach made possible by advances in computers and computer-aided design methods in recent years. The technique is now widely used for the analysis of many engineering problems involving static, dynamic, and thermal stressing of structures. The theory is based on an elastic structure or continuum being represented by many discrete components or elements interconnected at a finite number of nodal points situated on the element boundaries. The displacements of these nodal points are the basic unknown parameters of the problem.

The load-displacement equations for the whole structure are

$$K\delta = P \tag{7.1}$$

where **K** = structure stiffness matrix

$\delta$ = vector of (unknown) nodal displacements

**P** = vector of applied nodal loads.

If sufficient boundary conditions are specified for $\delta$, the above equations can be solved to obtain the displacements for each node of the structure. From these nodal displacements, the strains and stresses within each element can be computed.

A variety of finite element programs exist. In the present analysis a finite element work was done using LUSAS [43]. LUSAS is a general purpose engineering analysis system which incorporates facilities for linear and nonlinear static stress analysis, step by step dynamic analysis, and steady and transient field (thermal) analysis. The system is based on the finite element displacement method of analysis and contains a comprehensive range of elements and solution procedures for the analysis of most types of engineering structure. The element library includes elements for the analysis of trusses, frames, grillages, membrane structures, plates, thick and thin shells, axi-symmetric solids and general solids. Several types of element may be used to idealise different parts of a structure providing nodal freedoms match.

The system contains a wide range of both linear and nonlinear material types (constitutive models) which cover most engineering materials. The support node conditions may be restrained, restrained with a prescribed displacement, spring or free. The load types available are similarly wide ranging and include point loads, constant body forces, centrifugal forces, surface pressures, temperature and uniformly distributed loads.

The method of solving the load-deflection equations in LUSAS is the frontal techniques, to which random access techniques have been incorporated. This solution method is regarded as the most efficient and advanced that can be used for solving large numbers of equations with large frontwidths (akin to bandwidths), as encountered in finite element analysis. The technique is also efficient for small problems with only a few elements.

## 7.3 Point of Application of Load for Maximum Bending Stress

Consider the teeth $A_1$ and $A_2$ of the driving pinion shown in Figure 7.1 engaging with the teeth $B_1$ and $B_2$ of the gear, and having a contact ratio between 1 and 2. In position (a), the tooth $A_1$ is shown making contact at the tip $C_2$. This point of application would produce the maximum stress at the root of the tooth if the whole load were carried by this tooth. If pitch and profile were exact, however, the teeth $A_2$ and $B_2$ would at this instant make contact at $C_3$ distant one base pitch from $C_2$, and would thus carry part of the load; the maximum stress will thus be induced when the entire load is carried by one pair of teeth, and when the point of contact is as high up the tooth profile as possible.

This occurs at the instant shown in Figure 7.1(b) when the pair of teeth $A_2B_2$ are just coming into engagement. On this basis, the point of contact for maximum stress in the tooth $A_1$ is at $C_4$, distant one base pitch from the commencement of the path of

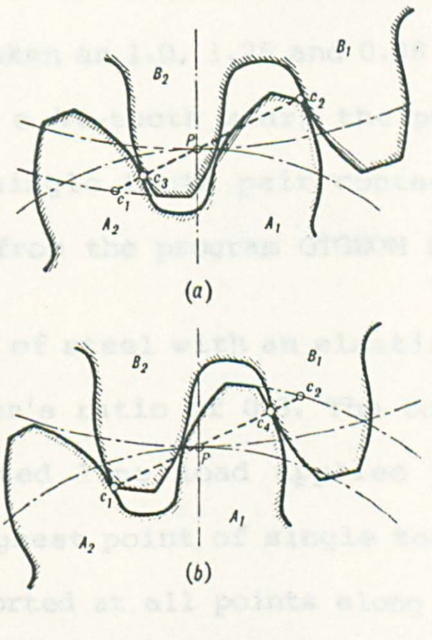


Figure 7.1 Point of contact for maximum bending stress

contact at $C_1$, and defined as the highest point of single tooth pair contact. Similarly, Figure 7.1 (a) represents the instant of maximum stress in gear tooth $B_2$.

The comparative strength of a given pinion tooth will thus vary with the diameter of the gear with which it engages, since this affects the position of extremity of the path of contact and hence the position of the point of application of load for maximum stress.

In the International Gear Standard ISO/DIS6336, the maximum root bending stress of a gear tooth is assumed to occur when a load is applied to the highest point of single tooth pair contact.

## 7.4 Selection of Tooth Model

The gear tooth used in the following stress analysis has a module of 5mm (reference diameter divided by the number of teeth). It is a member of an 18-tooth pinion with the reference diameter thus being 90mm. The tooth profiles are involute curves and the pressure angle is $20^o$. The addendum, dedendum and fillet radius coefficients are taken as 1.0, 1.25 and 0.38 respectively. If the pinion mates with a 54-tooth gear, the pressure angle at the highest point of single tooth pair contact will be $22.0236^o$, which is obtained from the program GTGEOM (refer to Section 6.3).

The pinion is made of steel with an elastic modulus of $2.06 x 10^5$ $N/mm^2$ and a Poisson's ratio of 0.3. The tooth is loaded with a 400N/mm concentrated line load applied normal to the tooth boundary at the highest point of single tooth pair contact. The hub or rim is supported at all points along the boundary as shown in Figure 7.2.

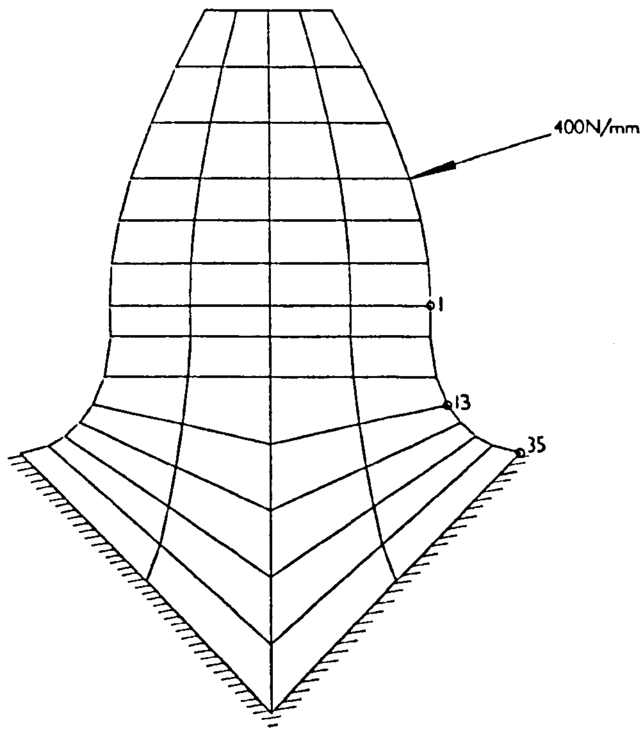


Figure 7.2 Tooth model for finite element stress analysis

## 7.5 Determination of the Maximum Tooth Root Bending Stress Based on the ISO Gear Standard

In ISO/DIS6336 the maximum root bending stress of a gear tooth is determined by the equation

$$\sigma_{FO} = \frac{F_t}{bm_n} Y_F Y_S Y_\beta \qquad (7.2)$$

which is situated where a tangent at 30° to a radial line meets the tooth. Using the program GCHECK the maximum bending stress on the tooth model is found to be

$$\sigma_{FO} = \frac{400\cos(20^\circ)}{5} \cdot 1.6340 \cdot 1.7933 \cdot 1.0 = 220 \text{ N/mm}^2$$

## 7.6 Gear Tooth Geometry and Finite Element Grid Description

The art of finite element analysis lies in the development of a suitable idealisation of a structure to provide the required results. The mesh of elements must not be too fine to make the preparation of data, computer time and interpretation of results expensive, but must not be too coarse to make the accuracy of the results unacceptable.

To develop a suitable idealisation, we needs to have some knowledge of the likely distribution of stresses in the structure, and the fineness of mesh required to provide results of acceptable accuracy at required points in the structure. It is not always possible to know the likely stress distribution in a structure before an analysis and it may be necessary to run a pilot coarse mesh idealisation in the first instance. The mesh can then be refined or modified in the areas of interest, to obtain acceptable results.

The first requirement of the gear tooth stress analysis using the finite element method is the representation of the tooth shape by a network made up of finite elements. Points on the boundary of the network must have a one to one correspondence with points on the boundary of the tooth shape being analyzed. The location of interior points of the network are less restricted but in general, reflect the shape of the tooth boundary.

There are many different types of element available in the LUSAS element library for modelling a structure. In the following analysis the finite element network is comprised of quadrilateral plane membrane elements with 8 nodes and inscribed within the boundaries of a tooth profile corresponding to generation with a hob. At the boundary of the fillet region the mesh of elements is made finer so that the accuracy of the maximum stresses at root surface obtained may be satisfactory. In the interior and top of the tooth where stress gradients are generally lower than in the fillet region the grid is made coaser. Figure 7.3 shows one of the finite element grids applied to the gear tooth stress analysis, which may be produced by using the program GTPROF described in Chapter 5.

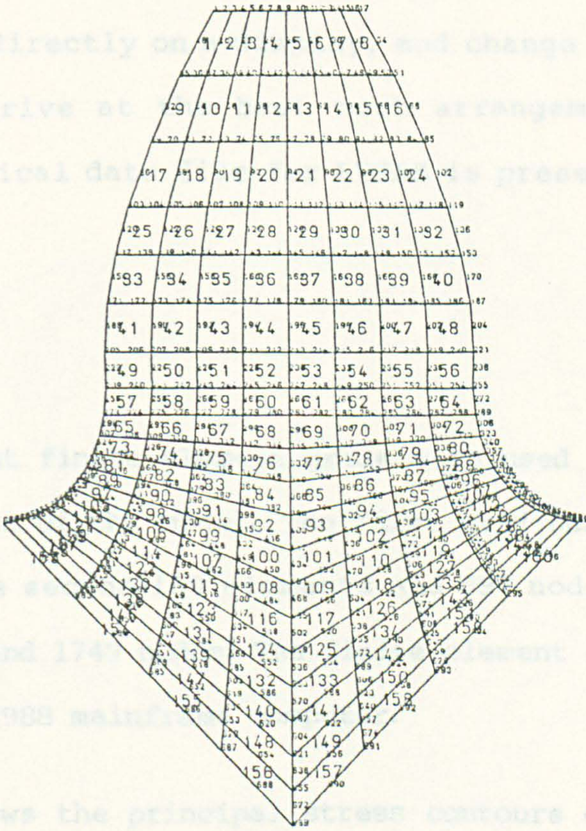

Figure 7.3 Finite element grid for gear tooth

## 7.7 Data Preparation

Once we had decided on the material properties, the loading cases, the boundary conditions, the types of element and the element mesh for the analysis of gear-tooth bending stresses, descriptive data has to be prepared for LUSAS. When this task is performed manually, the user might spend many hours drawing meshes in a drawing office and measuring the coordinates of each nodal point. It is extremely tedious and time-consuming and consequently very expensive. Automating this process can greatly enhance the use of the finite element technique.

By using the program GTPROF the data generation for the finite element analysis of gear tooth may be readily fulfilled because the user is able to see the element connections and position of each element directly on a display, and change a mesh instantaneously to arrive at the best mesh arrangement to suit the problem. A typical data file for LUSAS is presented in Appendix B.

## 7.8 Results

Three different finite element grids were used in the gear-tooth stress analysis of the model. The first grid had 52 elements and 243 nodes, the second 160 elements and 697 nodes, and the third 416 elements and 1749 nodes. The finite element analysis work was done on a ICL2988 mainframe computer.

Figure 7.4 shows the principal stress contours of the gear tooth represented by these meshes. Table 7.1 and Figure 7.5 give the principal surface stresses at different nodal points along the tensile tooth fillet. Point 1, 13, and 35 are the starting point of involute portion, the maximum root stress point assumed in the ISO gear standard, and the starting point of tooth fillet on the tensile tooth surface respectively as shown in Figure 7.2.

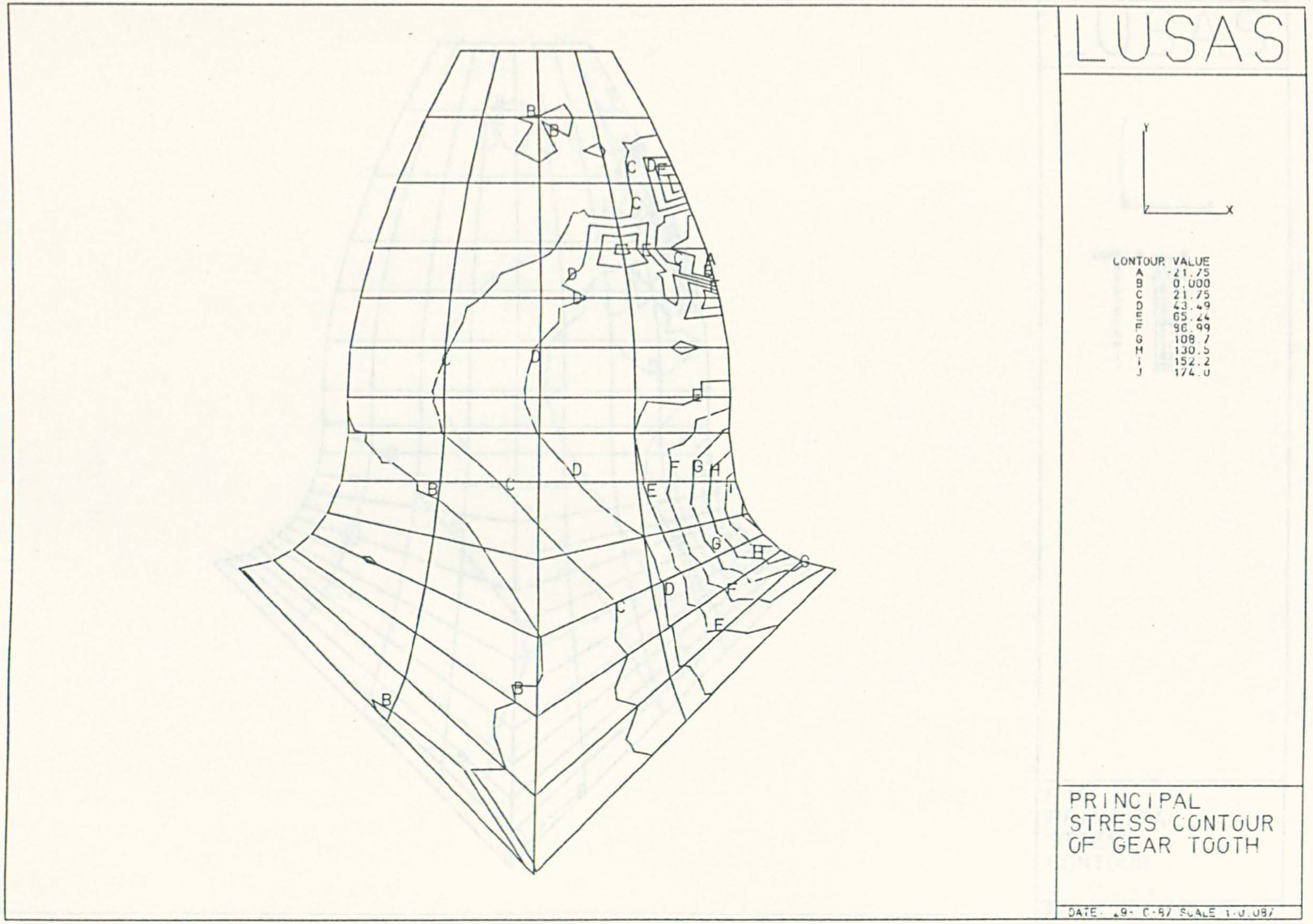


Figure 7.4 Principal stress contours of gear tooth (with 52 elements)

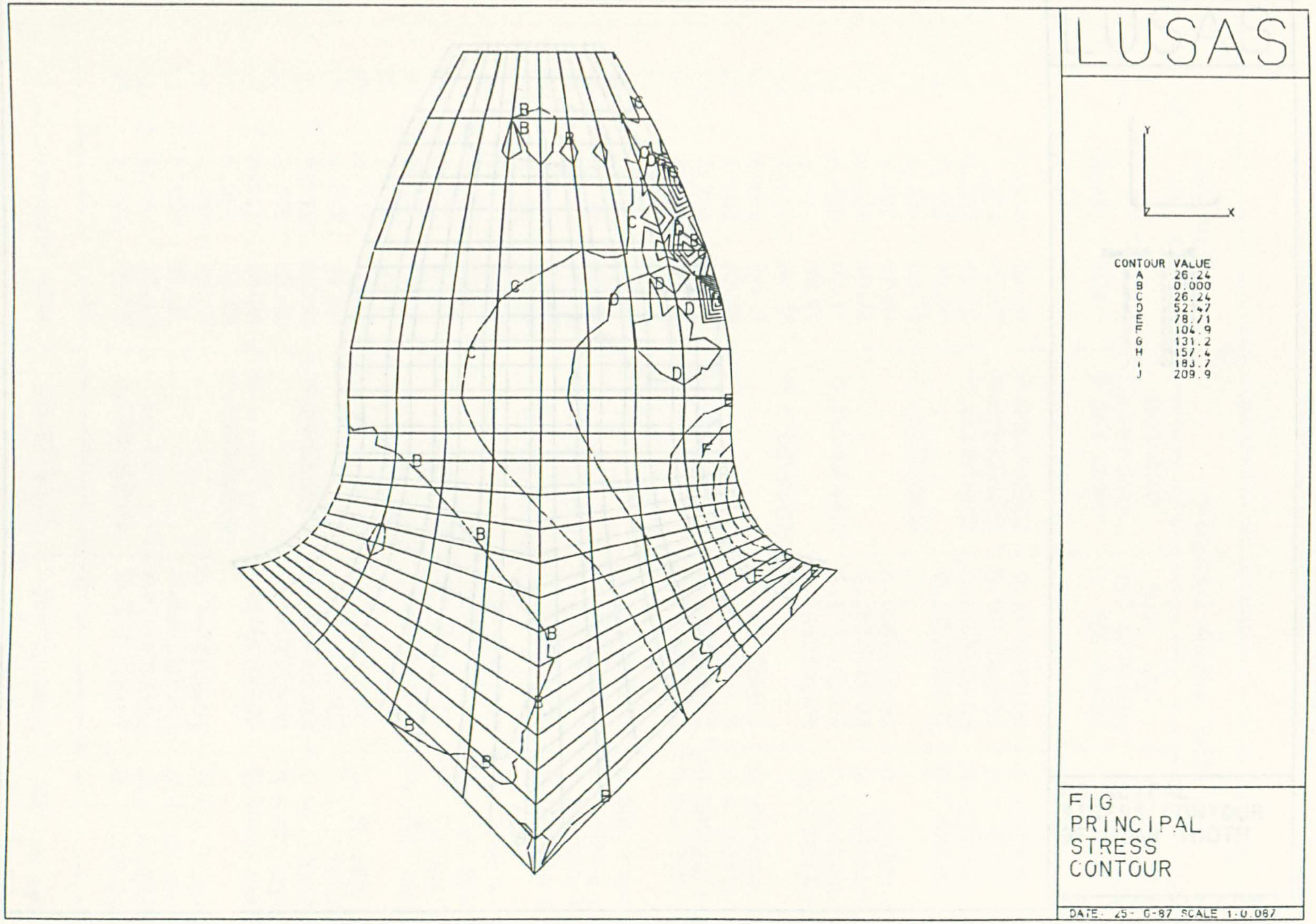


Figure 7.4 Principal stress contours of gear tooth (with 160 elements)

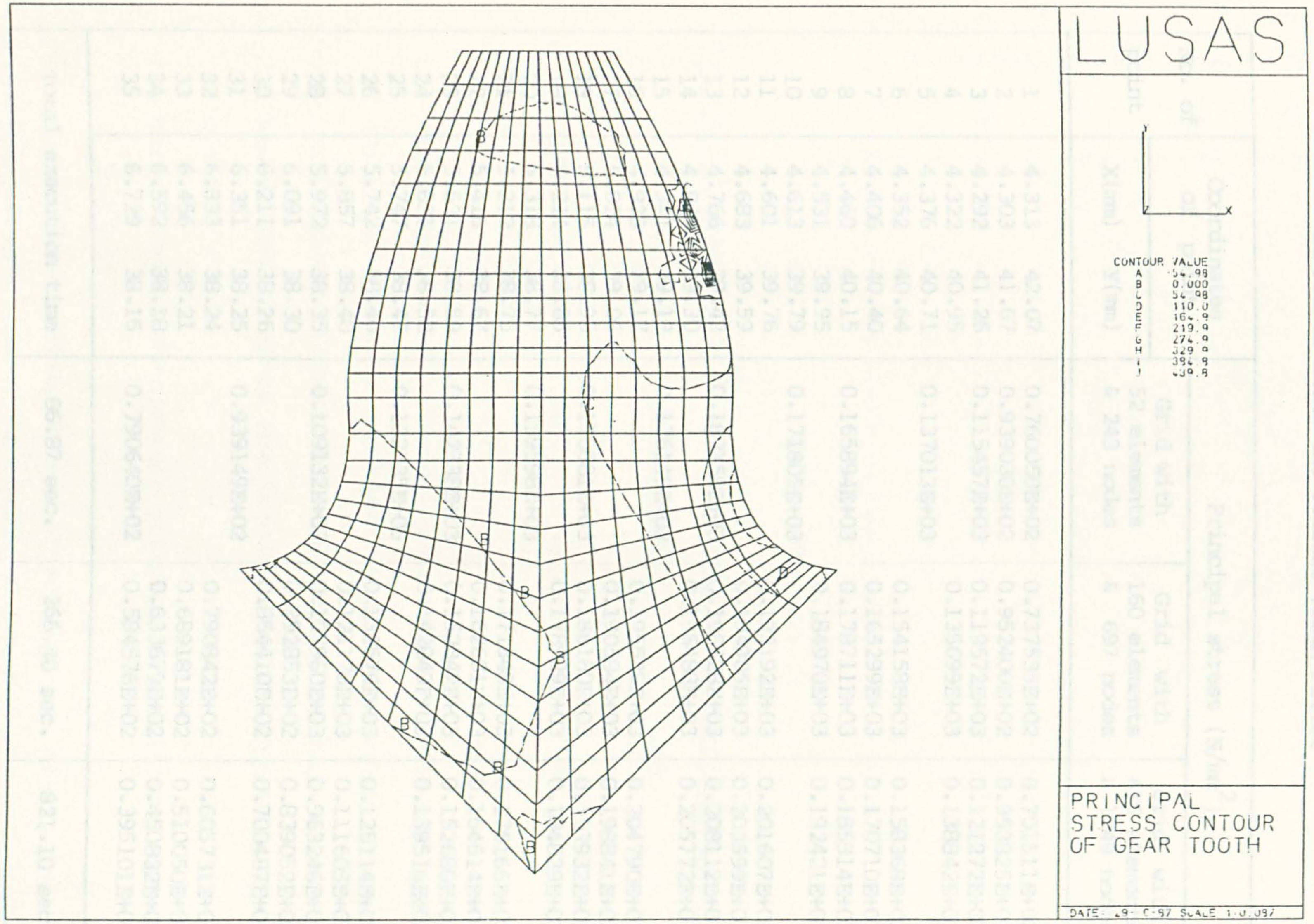


Figure 7.4 Principal stress contours of gear tooth (with 416 elements)

Table 7.1

Results of FEM Calculation for Tooth Surface Stress

at Points along the Tensile Tooth Fillet

| No. of point | Coordinates of point | | Principal stress ($N/mm^2$) | | |
|---|---|---|---|---|---|
| | X(mm) | Y(mm) | Grid with 52 elements & 243 nodes | Grid with 160 elements & 697 nodes | Grid with 416 elements & 1749 nodes |
| 1 | 4.313 | 42.07 | 0.760058E+02 | 0.737535E+02 | 0.701511E+02 |
| 2 | 4.303 | 41.67 | 0.939030E+02 | 0.952400E+02 | 0.953825E+02 |
| 3 | 4.292 | 41.26 | 0.115457E+03 | 0.119572E+03 | 0.121272E+03 |
| 4 | 4.322 | 40.95 | | 0.135699E+03 | 0.138842E+03 |
| 5 | 4.376 | 40.71 | 0.137013E+03 | | |
| 6 | 4.352 | 40.64 | | 0.154158E+03 | 0.158368E+03 |
| 7 | 4.406 | 40.40 | | 0.165299E+03 | 0.170710E+03 |
| 8 | 4.460 | 40.15 | 0.165894E+03 | 0.178711E+03 | 0.185314E+03 |
| 9 | 4.531 | 39.95 | | 0.184970E+03 | 0.192421E+03 |
| 10 | 4.613 | 39.79 | 0.171805E+03 | | |
| 11 | 4.601 | 39.76 | | 0.193192E+03 | 0.201607E+03 |
| 12 | 4.683 | 39.59 | | 0.194905E+03 | 0.203599E+03 |
| 13 | 4.766 | 39.42 | 0.182158E+03 | 0.198528E+03 | 0.208112E+03 |
| 14 | 4.849 | 39.30 | | 0.196488E+03 | 0.205772E+03 |
| 15 | 4.941 | 39.19 | 0.176119E+03 | | |
| 16 | 4.933 | 39.17 | | 0.195538E+03 | 0.204790E+03 |
| 17 | 5.024 | 39.06 | | 0.190584E+03 | 0.198841E+03 |
| 18 | 5.115 | 38.95 | 0.170881E+03 | 0.186160E+03 | 0.193932E+03 |
| 19 | 5.214 | 38.85 | | 0.178929E+03 | 0.184829E+03 |
| 20 | 5.318 | 38.77 | 0.159598E+03 | | |
| 21 | 5.312 | 38.75 | | 0.171348E+03 | 0.176166E+03 |
| 22 | 5.416 | 38.67 | | 0.162601E+03 | 0.164614E+03 |
| 23 | 5.521 | 38.59 | 0.142338E+03 | 0.152342E+03 | 0.152680E+03 |
| 24 | 5.631 | 38.52 | | 0.142849E+03 | 0.139516E+03 |
| 25 | 5.747 | 38.47 | 0.132335E+03 | | |
| 26 | 5.742 | 38.46 | | 0.130576E+03 | 0.125114E+03 |
| 27 | 5.857 | 38.40 | | 0.121170E+03 | 0.111605E+03 |
| 28 | 5.972 | 38.35 | 0.109132E+03 | 0.107960E+03 | 0.963246E+02 |
| 29 | 6.091 | 38.30 | | 0.992853E+02 | 0.839052E+02 |
| 30 | 6.211 | 38.26 | | 0.864410E+02 | 0.700487E+02 |
| 31 | 6.351 | 38.25 | 0.939149E+02 | | |
| 32 | 6.333 | 38.24 | | 0.790842E+02 | 0.605731E+02 |
| 33 | 6.456 | 38.21 | | 0.689181E+02 | 0.510050E+02 |
| 34 | 6.592 | 38.18 | | 0.633679E+02 | 0.450832E+02 |
| 35 | 6.729 | 38.16 | 0.790640E+02 | 0.584576E+02 | 0.395101E+02 |
| Total execution time | | | 86.87 sec. | 266.40 sec. | 821.10 sec. |

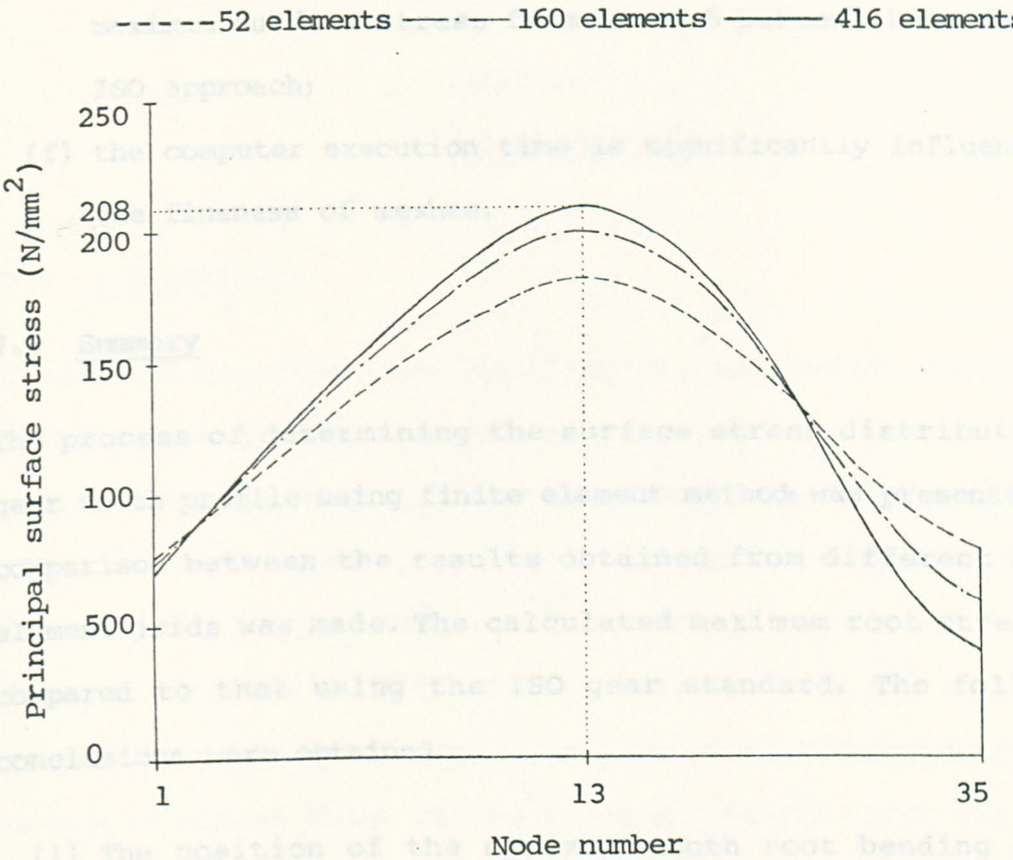


Figure 7.5 Results of FEM calculation for tooth surface stress at points along the tensile tooth fillet

An examination of the numerical values associated with these results shows that

(a) expect for local stress concentrations immediately beneath the load, the maximum stresses occur at the root surfaces;

(b) the fineness of mesh, for the range of meshes considered, has no effect upon the location of the maximum fillet stress;

(c) the maximum surface stress position is consistent with that of ISO/DIS6336;

(d) the maximum surface stress increases with the fineness of mesh;

(e) the finite element analysis using the finest mesh gives the maximum surface stress from about 5 percent less than the ISO approach;

(f) the computer execution time is significantly influenced by the fineness of meshes.

### 7.9 Summary

The process of determining the surface stress distribution on gear tooth profile using finite element methods was presented. The comparison between the results obtained from different finite element grids was made. The calculated maximum root stress was compared to that using the ISO gear standard. The following conclusions were obtained.

(1) The position of the maximum tooth root bending stress determined by finite element methods agreed with the assumption in ISO/DIS6336;

(2) The maximum bending stress obtained from the present finite element analysis is only 5 percent less than the value calculated using the ISO gear standard. The difference between these two values will further decrease as the mesh of elements is made finer;

(3) The computing time required to provide results of acceptable accuracy using finite element methods in gear-tooth stress analysis is so much and the memory so big that at the present time this technique can not be used with a microcomputer in an interactive mode.

# Chapter 8

## SHAFT DESIGN

### 8.1 Introduction

Shafts are important machine elements supported by bearings and transmitting power between rotating members such as gears, pulleys, and sprockets. Shafts are usually designed and made in the form of stepped cylindrical bars (with various diameters along their length) and less frequently — with a constant diameter. Shafts with a constant diameter are the easiest to produce. However, such a cross-section does not correspond to the nature of stress which varies along the length; such shafts are also unacceptable because of considerations of setting-up and maintenance, since they complicate the process of assembly and disassembly and also make more difficult the fastening of parts fitted to them and especially the bearings on shafts. For these reasons shafts made in the form of smooth cylindrical bars are seldom used in mechanical engineering practice. The cross sections of shafts may be solid or hollow. The hollow types of shafts weigh considerably less than the solid shafts of comparable strength but are somewhat more expensive [17].

Power transmitting shafts may be subjected to bending, tension, compression, or torsional loads in three dimensions at different points along their lengths. These loads induce normal bending stresses and shear stresses across the shafts. The steps machined on the shaft to locate and secure the rotating members cause stress-concentration which increase the local stresses. Other stress-concentrations may be caused by keyways and holes which are commonly present on most shafts. The design of stepped power transmission shafts consists primarily of the determination of the correct shaft diameters to ensure satisfactory static and fatigue strength, rigidity, and vibrational stability.

At present there is no generally accepted code but there are a number of design methods as discussed by Shigley [70]. The interactive computer program developed and presented in this chapter enables the user to design stepped shafts of solid circular cross-sections within a few minutes according to the maximum shear stress theory and provides graphical output on axial force, bending moment, torsional moment, slope and deflection curves to evaluate the performance of the design.

## 8.2 Design Theory

### 8.2.1 Determination of Loading

As with many other machine or structural elements, the first stage in designing a shaft is to determine the loading, both in magnitude and direction, to which the shaft is subjected. This may be due to the forces from the rotating elements mounted upon the shaft, the weights of the shafts and fitted-on parts, etc. These forces usually are of three dimensions. Their major components, for example, the gear forces, i.e. tangential, radial, can be easily evaluated with the aid of the transmitted torque and pitch circle diameter. Helical gears have an axial component due to the axial thrust produced. The force vectors are dependent upon the helix hand, the rotational direction of the gear and whether it is the driver or driven member of a pair. In many cases the effect of the weight of the shaft and that of the fitted-on parts may be neglected. We may also neglect (without excessive error) the magnitude of moment of friction forces in the bearings. Since the nature of load distribution over the surfaces is frequently not known, the design load of a shaft is frequently assumed to be concentrated.

### 8.2.2 Preliminary Physical Design

The physical design is to determine the shaft geometry. The shaft should be given such a form that it can satisfy the requirements of production, convenience of assembly, etc. The diameters of all segments along the length of the shaft can only be found based on the strength and stiffness requirements. Before it the dimensions along the length of shaft need to be determined.

The entire length of the shaft depends on the number of fitted-on parts and their width, the location of the bearings and their design, etc. To reduce the weight and cost of these parts the length of the shaft should be reduced as much as possible; this in turn tends to decrease the diameter of the designed part. The length of each carrying section of a stepped shaft depends on the length of the hub of the fitted-on part; if possible, the length of the hub should somewhat exceed that of the respective carrying section on the shaft to relieve the surfaces of the shaft from excessive surface pressures.

The positioning of fitted-on parts will frequently be determined from physical factors not directly associated with the shaft system under consideration. The bearing supports should be positioned as close to gears as possible with a view to obtaining the greatest transverse rigidity. In any event, any shaft system should be designed to be as compact as possible, whilst still satisfying physical and functional requirements.

### 8.2.3 Materials for Shafting

One cannot determine the shaft diameters without considering the material from which the shaft is to be manufactured. The materials employed for shafts should possess: sufficiently high strength characteristics, a low sensitivity to stress concentration, the ability to withstand heat and case-hardening treatment to lessen the effect of stress concentration and increase the wear resistance of journals, and good machinability. The precise properties required of the material will depend upon the particular application, but in general, shafts need to be produced from a tough ductile material, and the greater its mechanical strength, the smaller will be the required shaft size. Since rigidity is also frequently a requirement, a material having a high elastic modulus is also a desirable feature. Consequently, shafts will invariably be produced from steel of one kind or another. For shafts requiring only low strength, carbon steels may be quite suitable, since they are cheaper than alloy steels and relatively easy to machine. A typical carbon steel suitable for shaft design is steel 45 [20, 45]. Adequate heat treatment can impart to it high mechanical properties. In order to obtain minimum diameters higher strength shafts are made from alloy steels of various grades treated by heat and case-hardening methods. However, high cost of these steels and also their increased sensitivity to stress concentration reduces the sphere of their application. Besides, the high mechanical properties of alloy steel can not always be utilised to the best advantage since the small shaft diameter obtained in design often fails to guarantee the required rigidity. Therefore, if stiffness is the deciding factor in design, there is no point in using an alloy steel in place of carbon steel, because all of them have about the same value of modulus of elasticity.

### 8.2.4 Design for Combined Stresses

Once the loading conditions have been determined, the positions of the bearings and the various elements mounted on the shaft established, and the suitable material selected, the preliminary determination of the shaft diameters required for strength can be carried out with the aid of formulae from the theory of the strength of materials.

First, the bending moment diagram is to be determined. As the forces acting on a shaft are usually not co-planar, it will be necessary to resolve the forces into two mutually perpendicular planes, and hence to determine the bending moment diagrams for these two planes (usually referred to as the horizontal and vertical bending moment diagrams, since these are the planes normally used); the resultant bending moment diagram can then be easily obtained by geometrical summation of the bending moments:

$$M = \sqrt{M_h^2 + M_v^2} \tag{8.1}$$

The most frequent case is that where the shaft is loaded simultaneously with the torque **T** and the bending moment **M**. Sometimes separate sections of shafts may be additionally loaded in tension or compression. The tensile or compressive stresses caused by these forces are, as a rule, insignificant as compared to the bending and torsional stresses, so they can be neglected.

The reduced moments $M_{red}$ are usually found from the 3rd theory of strength (i.e. the maximum shear stress theory) in accordance with which

$$M_{red} = \sqrt{M^2 + T^2} \quad (8.2)$$

if we disregard the difference in the stress change conditions due to bending and torsion. If it is taken into account, then

$$M_{red} = \sqrt{M^2 + (\alpha T)^2} \quad (8.3)$$

where $\alpha$ = correction factor whose value is determined when specifying allowable stresses [20, 45].

The diameters of a shaft in the design sections may then be found from the strength condition

$$\sigma_{ca} = \frac{M_{red}}{W} <= [\sigma]_b \quad (8.4)$$

For a round solid shaft the section modulus in bending is equal to

$$W = \frac{\pi d^3}{32} \quad (8.5)$$

The values of allowable stresses $[\sigma]_b$ are different for different materials and depend on the operating conditions. Here we differentiate:

Service I, when the stresses caused by load remain constant in magnitude and sign;

Service II, when the stresses change in a pulsating cycle;

Service III, when the nature of change in stress is a symmetric cycle.

The values of allowable stresses for carbon and alloy steels according to conditions of operation correspond roughly to the following ratios [20]:

$$[\sigma]_{bI}:[\sigma]_{bII}:[\sigma]_{bIII} = 3.8:1.7:1 \tag{8.6}$$

in this case

$$[\sigma]_{bI} \doteq 0.33\sigma_b \tag{8.7}$$

where $\sigma_b$ = tensile ultimate strength.

In general, the bending stresses acting on a shaft change in a symmetrical cycle. Therefore, $[\sigma]_b$ is selected according to service III.

When the torsional stresses change in a pulsating cycle the values of **T** are decreased by introducing the reduction factor $\alpha = \frac{[\sigma]_{bIII}}{[\sigma]_{bII}} = 0.59 < 1$ in order to assess correctly the role of these stresses. If the torsional stresses change in accordance with a symmetric cycle, then $\alpha = 1$.

Thus the minimum diameter of a solid circular shaft in the design cross-section may be found based on the acting reduced bending moment from the following formula:

$$d_{min} = \sqrt[3]{\frac{32M_{red}}{\pi[\sigma]_{bIII}}} = \sqrt[3]{\frac{32\sqrt{M^2 + (\alpha T)^2}}{\pi[\sigma]_{bIII}}} \tag{8.8}$$

The formula (8.8) can be used to determine the minimum shaft diameters for various cross-sections along the entire length. If the design cross-sections of a shaft is intended to support parts secured by keys, it should be increased in diameters (by 3% to one key and by 7% to double keys) to compensate for the weakening of the shaft [45].

### 8.2.5 Further Physical Design

Having estimated the minimum diameter in each cross-section of the shaft, the variation in diameter along the shaft length can then be determined, taking due regard of the physical or other requirements for any of the components which are to fit on the shaft at various points, and the appropriate key sizes, splines, etc.

Abrupt changes of cross-section can cause stress concentrations which are most unwelcome with the nature of load typical for shafts. To decrease local stresses due to abrupt changes of cross-section, fillets, the rounded-off places where two cross-sections of different diameter meet on a shaft, should have arcs of as large radii as possible ($r > 0.1d$) (see Figure 8.1) [20]. However, from the practical viewpoints, large radii make it difficult to mount such other components as gears, pulleys, bearings, etc., because of radius interference. Often, the bore

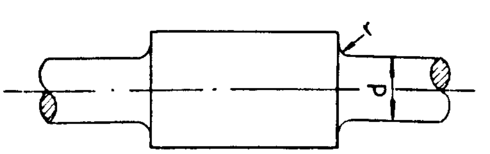

Figure 8.1 Fillet radius of a shaft

of such other components has to be chamfered to clear radii at the point where a shaft changes diameter (see Figure 8.2). On the basis of assembly requirements the fillet radius should be taken somewhat smaller than the radius describing the corresponding section of the component to be fitted ($r < R$) (see Figure 8.3).

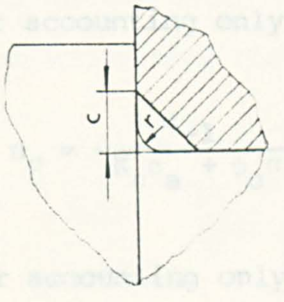

Figure 8.2 Chamfering of component mounted on a shaft

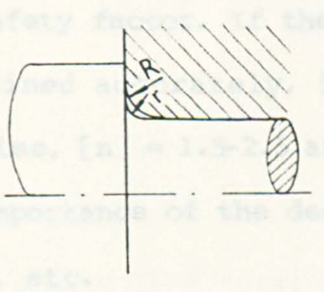

Figure 8.3 Assembly requirement for fillet radius on a shaft

### 8.2.6 Checking for Fatigue Strength

As fatigue may be the important criterion as far as strength is concerned, the proportions of a shaft found after preliminary calculations and structural designing are subjected to fatigue strength calculations to determine the design safety factors in the dangerous cross-sections (which may be found from a consideration of the variations in the shaft diameters, the resultant bending moment diagram and the torque transmitted). For this purpose the following formula will be used [46]:

$$n = \frac{n_\sigma n_\tau}{\sqrt{n_\sigma^2 + n_\tau^2}} >= [n] \qquad (8.9)$$

where n = design safety factor

$n_\sigma$ = safety factor accounting only for bending

$$n_\sigma = \frac{\sigma_{-1}}{K_\sigma \sigma_a + \psi_\sigma \sigma_m} \qquad (8.10)$$

$n_\tau$ = safety factor accounting only for torsion

$$n_\tau = \frac{\tau_{-1}}{K_\tau \tau_a + \psi_\tau \tau_m} \qquad (8.11)$$

[n] = allowable safety factor. If the loads and stresses have been determined accurately, [n] may be taken as 1.3-1.5. Otherwise, [n] = 1.5-2.5 and over depending on the degree of importance of the design, the accuracy of the design data, etc.

If necessary the design is altered after which the values of the safety factors are calculated again. By consecutive approximation we can establish the required proportion between the design and allowable values of the safety factors.

### 8.2.7 Checking for Static Strength

The instantaneous heavy overload applied to a shaft may result in the plastic deformation of the shaft. The calculation of the static strength is used to check the capacity of the shaft to resist the plastic deformation under overloading. The strength condition is [46]

$$n_s = \frac{n_{s\sigma} n_{s\tau}}{\sqrt{n_{s\sigma}^2 + n_{s\tau}^2}} >= [n_s] \qquad (8.12)$$

where $n_s$ = design safety factor based on yield strength

$n_{s\sigma}$ = safety factor accounting only for bending

$$n_{s\sigma} = \frac{\sigma_s}{\sigma_{max}} = \frac{\sigma_s}{\frac{M_{max}}{W} + \frac{P_{max}}{A}} \qquad (8.13)$$

$n_{s\tau}$ = safety factor accounting only for torsion

$$n_{s\tau} = \frac{\tau_s}{\tau_{max}} = \frac{\tau_s}{\frac{T_{max}}{W_T}} \qquad (8.14)$$

where $\sigma_s, \tau_s$ = bending and torsional yield strength respectively

$\sigma_{max}, \tau_{max}$ = maximum bending and torsional stresses respectively

$M_{max}, T_{max}$ = maximum bending and torsional moments at critical cross-section respectively

$W, W_T$ = section modulus in bending and torsion of critical cross-section respectively

$P_{max}$ = maximum axial force at critical cross-section

A = area of critical cross-section

$[n_s]$ = allowable safety factor based on yield strength:

= 1.2-1.4 for shaft made from high ductile steel

= 1.4-1.8 for shaft made from medium ductile steel

= 1.8-2.0 for shaft made from low ductile steel

= 2.0-3.0 for shaft made from cast iron

### 8.2.8 Checking for Rigidity

An adequate rigidity of shafts is very important for the normal operation of every unit and of the entire machine. Excessive flexure of these components can hinder the proper interaction of the conjugate parts mounted on the shafts and lead to unfavourable operating conditions of bearings. In some designs it is very important to limit the torsional deflection of shafts. For example, at high angles of twist of a splined shaft, the splines assume the form of spirals which tend to displace axially the movable toothed wheels mounted on the shaft. This unfavourably affects their meshing.

Rigidity is calculated for the purpose of determining the deflections of shafts and the slopes in definite cross-sections. The calculations may be done by the methods outlined in the theory of strength of materials.

As a rule, it is extremely difficult to find these magnitudes accurately, due to the effect of the rigidity of the bearing housing of a shaft, clearances, local forms of the shaft, etc. For this reason, design for rigidity is usually arbitrary. The degree of rigidity can be assessed only after comparing the design values with the allowable values of deflection and slope obtained from observations of systems operating satisfactorily. The latter circumstance has led to the employment by various branches of mechanical engineering of their own standards which are established in accordance with that factor which is definitive for a given design. The standards shown in Table 8.1 are widely used in mechanical engineering [46].

Table 8.1

Allowable Deflections and Slopes of Shafts

| Type of Shaft | Allowable Deflection $[y_{max}]$mm | Type of Fitted-on Part | Allowable Slope $[\theta_{max}]$rad |
|---|---|---|---|
| General purpose Shaft | $(.0003-.0005)L$ | Sliding Bearing | .001 |
| Shaft with High rigidity | $.0002L$ | Radial Ball Bearing | .005 |
| Electric Motor Shaft | $.1\delta$ | Self-Aligning Bearing | .05 |
| Shaft Carrying Gear | $(.01-.03)m_n$ | Cylindrical Bearing | .0025 |
| Shaft Carrying Worm Gear | $(.02-.05)m_{t2}$ | Taper Roller Bearing | .0016 |
| | | Cross-Section at gear | .001-.002 |

Notes: $L$ = span of shaft; $\delta$ = air gap of motor;
$m_n$ = normal module of gear; $m_{t2}$ = transverse module of worm gear

In general the deflection depends on the magnitude of applied load and its location on the span. If the calculated deflections exceed permissible values, the tentatively designed shaft must be stiffened either by increasing the shaft diameters, by decreasing the bearing centres, or possibly both.

In calculating shafts for torsional deflection their stiffness should be evaluated by the angle of twist $\phi$. For a smooth shaft of constant diameter or for a cylindrical section of a shaft

$$\phi = \frac{TL}{GJ} \tag{8.15}$$

where $T$ = torque operating within the calculated section of the shaft of length $L$

$G$ = shear modulus of elasticity

$J$ = polar moment of inertia of the cross-section.

The allowable angle of twist in a shaft depends upon the service for which it is intended. For general-purpose transmission shafts the angle of twist should be limited to (0.5-1.0) deg per metre of length. For machine shafts the angular twist should not exceed

(0.25-0.5) deg per linear metre [46].

### 8.2.9 Vibration and Critical Speeds of Shafts

It is a commonly recognized fact that at certain speeds a rotating shaft becomes dynamically unstable, and that at these speeds excessive and even dangerous deflections may occur. The speeds at which a shaft becomes dynamically unstable are called the critical speeds and correspond to the speeds at which the number of natural vibrations, or natural frequency, equals the number of revolutions per minute. The operating speed of a shaft should be well removed from the neighborhood of any of the series of speeds at which these extreme deformations and vibrations may occur. In general, if the operating speed of any shaft is removed at least 20 per cent from any critical speed, there will be no vibration troubles.

When the total deflection of the shaft at each supported disk is known as indicated in Figure 8.4, the lowest, or fundamental, critical speed of the shaft stated in terms of the disk weights and the deflections is given by the expression [22, 79]

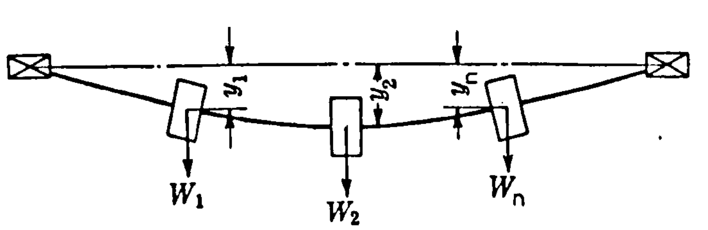


Figure 8.4 Shaft in deflected position

This is a diagrammatic arrangement showing loads $W_1$, $W_2$, and $W_n$ at points along the shaft where the static deflections are respectively $y_1$, $y_2$, and $y_n$.

$$N_{cr} = \frac{30}{\pi}\sqrt{\frac{g(W_1y_1+W_2y_2+\ldots+W_ny_n)}{W_1y_1^2+W_2y_2^2+\ldots+W_ny_n^2}} \tag{8.16}$$

where $g$ = acceleration due to gravity

$W_1, W_2$ = weights of the disks

$y_1, y_2$ = deflections at the respective disks.

or

$$N_{cr} = \frac{30}{\pi}\sqrt{\frac{g(\Sigma Wy)}{\Sigma Wy^2}} \tag{8.17}$$

where $\Sigma \mathbf{Wy}$ represents the sum of all the **Wy** terms and $\Sigma \mathbf{Wy}^2$ represents the sum of all the $\mathbf{Wy}^2$ terms.

If it is desired to account for the weight of the shaft itself, we may divide the shaft into parts, compute the weight of each part, consider the weight of each part as a force acting through its centre of gravity; and proceed as for any group of concentrated loads.

## 8.3 Shaft Model

Figure 8.5 shows a stepped shaft model used in the computer-aided shaft design program. It has solid circular cross-sections, overhanging ends and **n** nodes. This model is supported by two bearings at **a** and **b**, and loaded by **j** concentrated forces along Y axis, **k** axial forces along X axis, **l** bending moments, and **m** twisting moments. The directions chosen for the X, Y and Z axes are the clues to the sign conventions for the forces and moments. $\mathbf{F_1}$ and $\mathbf{P_1}$ are negative because they act in the negative Y and X

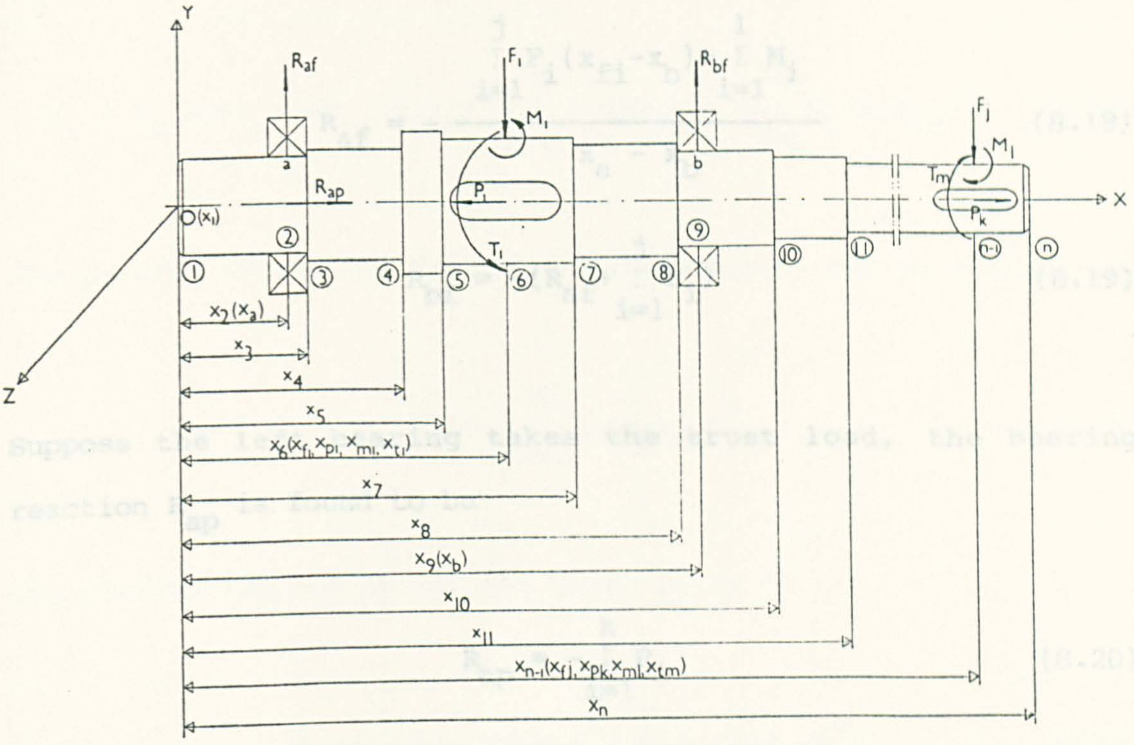


Figure 8.5 Stepped shaft model

directions respectively. $M_1$ and $T_1$ are positive because they conform to the right-hand rule.

This shaft model has keyways and shoulder fillets. Besides these, it may also have other kinds of stress concentration raisers such as grooves, transverse holes, straight-sided splines, involute splines, screw threads, worm threads, and various fits.

In order to simplify the data input and the program structure, every point where the shaft changes its diameter, the load is applied, or the dangerous cross-section might be located, must be specified by a node as indicated in Figure 8.5.

## 8.4 Design Approach

The bearing reactions $R_{af}$ and $R_{bf}$ for the shaft model shown in Figure 8.5 are given by

$$R_{af} = - \frac{\sum_{i=1}^{j} F_i(x_{fi}-x_b) + \sum_{i=1}^{l} M_i}{x_a - x_b} \tag{8.18}$$

$$R_{bf} = -(R_{af} + \sum_{i=1}^{j} F_i) \tag{8.19}$$

Suppose the left bearing takes the trust load, the bearing reaction $R_{ap}$ is found to be

$$R_{ap} = - \sum_{i=1}^{k} P_i \tag{8.20}$$

The general expression for the bending moments, twisting moments, and axial forces at all points along the length of the shaft may be achieved by introduction of so-called singularity or half-range function [58]

$$f_n(x) = \langle x-a \rangle^n \tag{8.21}$$

The two singularity functions for the cases of n = 0 and n = 1 are defined as follows:

$$f_0(x) = \langle x-a \rangle^0 = \begin{cases} 0 & (x<a) \\ 1 & (x>a) \end{cases} \tag{8.22}$$

$$f_1(x) = \langle x-a \rangle^1 = \begin{cases} 0 & (x<a) \\ x-a & (x>=a) \end{cases} \tag{8.23}$$

Through use of Eqs. (8.22) and (8.23), the general equation for the bending moment at any position along the shaft length is found to be

$$M_x = R_{af}\langle x-x_a\rangle^1 + R_{bf}\langle x-x_b\rangle^1 + \sum_{i=1}^{j} F_i\langle x-x_{fi}\rangle^1 - \sum_{i=1}^{1} M_i\langle x-x_{mi}\rangle^0 \qquad (8.24)$$

The general expression for the twisting moment may be written as

$$T_x = -\sum_{i=1}^{m} T_i\langle x-x_{ti}\rangle^0 \qquad (8.25)$$

The general expression for the axial force is

$$P_x = -\left(R_{ap}\langle x-x_a\rangle^0 + \sum_{i=1}^{k} P_i\langle x-x_{pi}\rangle^0\right) \qquad (8.26)$$

The bending moment, twisting moment, and axial force diagrams may then be established from Eqs. (8.24), (8.25), and (8.26). It is evident that when a bending moment, a twisting moment, or an axial force acts on a shaft the corresponding diagram will exhibit an abrupt jump or discontinuity at the point where the load is applied. For ease in treating problems involving resultant bending moments, reduced moments, slopes, and deflections, we compute the bending moments, twisting moments, axial forces for both sides at each node of the shaft. To achieve this, we redefine the singularity function

$$f_0(x) = \langle x-a\rangle^0 = \begin{cases} 0 & (x<=a) \\ 1 & (x>a) \end{cases} \qquad (8.27)$$

Then by using the subscript **n** for node **n** and the subscript **l** for left side, the bending moment, the twisting moment, and the axial force of the left side at node **n** may be represented by the three following equations respectively

$$M_{nl} = R_{af}\langle x_n - x_a\rangle^1 + R_{bf}\langle x_n - x_b\rangle^1 + \sum_{i=1}^{j} F_i\langle x_n - x_{fi}\rangle^1 - \sum_{i=1}^{l} M_i\langle x_n - x_{mi}\rangle^0 \tag{8.28}$$

$$T_{nl} = -\sum_{i=1}^{m} T_i\langle x_n - x_{ti}\rangle^0 \tag{8.29}$$

$$P_{nl} = -\left(R_{ap}\langle x_n - x_a\rangle^0 + \sum_{i=1}^{k} P_i\langle x_n - x_{pi}\rangle^0\right) \tag{8.30}$$

In a similar manner, to obtain the general expressions for the bending moment, twisting moment, and axial force of the right side at node **n**, we may define the singularity function $\langle x-a\rangle^0$ as

$$f_0(x) = \langle x-a\rangle^0 = \begin{cases} 0 & (x<a) \\ 1 & (x>=a) \end{cases} \tag{8.31}$$

Thus the values obtained from Eqs. (8.28), (8.29), and (8.30) will turn into $M_{nr}$, $T_{nr}$, and $P_{nr}$, i.e. the bending moment, the twisting moment, and the axial force of the right side at node **n**.

If the forces acting on the shaft are not co-planar, the horizontal and vertical bending moments for both sides of each node need to be calculated. Then the resultant and reduced moments are found from Eqs. (8.1) and (8.3).

With the reduced moments at hand, under design circumstance, we are now able to determine the minimum diameters of all segments along the shaft length from Eq. (8.7). The reasonable values of these diameters depend upon the physical requirements of the entire shaft.

When the dimensions of the shaft are known, the bending moment values can be divided by the the product **EI** to get a set of **M/EI** values, where **E** is the modulus of elasticity of the shaft material and **I** the moment of inertia of the cross-section. Since a stepped shaft has various diameters, which means it has various moments of inertia, we must divide the bending moments at the section of change by two **EI**'s, thus

$$\left(\frac{M}{EI}\right)_{nl} = \frac{M_{nl}}{EI_{nl}} \qquad \text{and} \qquad \left(\frac{M}{EI}\right)_{nr} = \frac{M_{nr}}{EI_{nr}}$$

where $\left(\frac{M}{EI}\right)_{nl}, \left(\frac{M}{EI}\right)_{nr}$ = values of ME/I at the left and right sides of node n respectively

$I_{nl}, I_{nr}$ = moments of inertia of the left and right sections respectively.

As a result, there is a break in the **M/EI** diagram at the section where there is a break in the shaft diameter.

Having obtained the **M/EI** values, the corresponding slopes and deflections at nodes along the length of the shaft could have been obtained by the numerical integration of the well-known deflection equation

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \qquad (8.32)$$

First integration yields the slopes:

$$\theta_n = \int_{x_1}^{x_n} \left(\frac{M}{EI}\right)dx + \theta_o \qquad (8.33)$$

where $\theta_o$ = starting value of the slope.

If we designate the integral as $\theta_n^*$, Eq. (8.33) can be expressed as

$$\theta_n = \theta_n^* + \theta_o \tag{8.34}$$

Because of the nature of stepped shafts, **M/EI** is a piecewise linear function of **X**. This means that the first integration of **M/EI** can be carried out using a trapezoidal rule, and the result is exact.

Based on the trapezoidal rule, $\theta_n^*$ is found to be

$$\theta_n^* = \begin{cases} 0 & (n=1) \\ \sum_{i=2}^{n} \left[ \left(\frac{M}{EI}\right)_{(i-1)r} + \left(\frac{M}{EI}\right)_{il} \right] \frac{x_i - x_{i-1}}{2} & (2 <= n <= N) \end{cases} \tag{8.35}$$

The second integration of **M/EI** provides the deflections:

$$y_n = \int_{x_1}^{x_n} \theta_n dx + y_o = \int_{x_1}^{x_n} \theta_n^* dx + \theta_o(x_n - x_1) + y_o \tag{8.36}$$

where $y_o$ = starting value of the deflection.

The second integration is of a piecewise quadratic function. Therefore the repeated use of the trapezoidal rule is inexact. To derive the deflection function from the slope, Simpson's rule [47] may be used. Simpson's rule is based on fitting a parabola to three of the points of the function to be integrated.

Let

$$y_n = y_n^* + \theta_o(x_n - x_1) + y_o \tag{8.37}$$

then according to Simpson's rule we get

$$y_n^* = \int_{x_1}^{x_n} \theta_n^* dx = \begin{cases} 0 & (n=1) \\ \sum_{i=2}^{n} (\theta_{i-1}^* + 4\theta_{i-1/2}^* + \theta_i^*)\frac{x_i - x_{i-1}}{6} & (2 <= n <= N) \end{cases} \tag{8.38}$$

where

$$\theta_{i-1/2}^* = \theta_{i-1}^* + [(\frac{M}{EI})_{(i-1)r} + (\frac{M}{EI})_{(i-1/2)l}]\frac{x_{i-1/2} - x_{i-1}}{2}$$

$$= \theta_{i-1}^* + [3(\frac{M}{EI})_{(i-1)r} + (\frac{M}{EI})_{il}]\frac{x_i - x_{i-1}}{8} \tag{8.39}$$

The starting values of $\theta_o$ and $y_o$ may be obtained from the conditions that $y = 0$ at $x = x_a$ and $x = x_b$. From Eq. (8.37) we have

$$\theta_o = \frac{y_b^* - y_a^*}{x_a - x_b} \tag{8.40}$$

$$y_o = \frac{x_b y_a^* - x_a y_b^*}{x_a - x_b} \tag{8.41}$$

If the forces act on the shaft in three dimensions, as it usually is, it is necessary to find the deflections at nodes in both vertical and horizontal planes. Then the resultant deflections at these nodes may be obtained by geometrical summation of these deflections.

## 8.5 Program Description

The shaft design program developed is composed of several subprograms for the various design steps. Each subprogram can be used separately for any design application. For example, the subprogram DEFLSHA which analyses stepped shafts for the bending deflections and slopes is applicable to all cases of stepped cylindrical shafts subjected to concentrated forces and moments. In this computer program, shafts are designed on the basis of combined bending and torsional strength, and checked for fatigue strength, static strength, rigidity, and vibration requirements. A schematic flow chart of the shaft design program is indicated in Figure 8.6.

When this program is operated, first the designer is asked to input data describing the shaft geometry, which consists of total node number, x coordinate of each node measured from the left end of the shaft, node numbers for bearings, and shaft diameters if a 'check' option in the menu chosen. Having defined the shaft to be analysed, the shaft shape is displayed on the monitor. This valuable feature means that the input data are quickly verified. Should a modification to the input data subsequently be required then this can be achieved quickly and easily using the editing facility of the program. The editing facility may also be used to modify a shaft shape to produce a desired design result.

Then the loading conditions of the shaft, the shaft material and its mechanical properties are to be specified. After that, the design (or checking) calculation starts.

The program computes the bearing reactions, the bending moments, the resultant bending moments, the torsional moments, the reduced

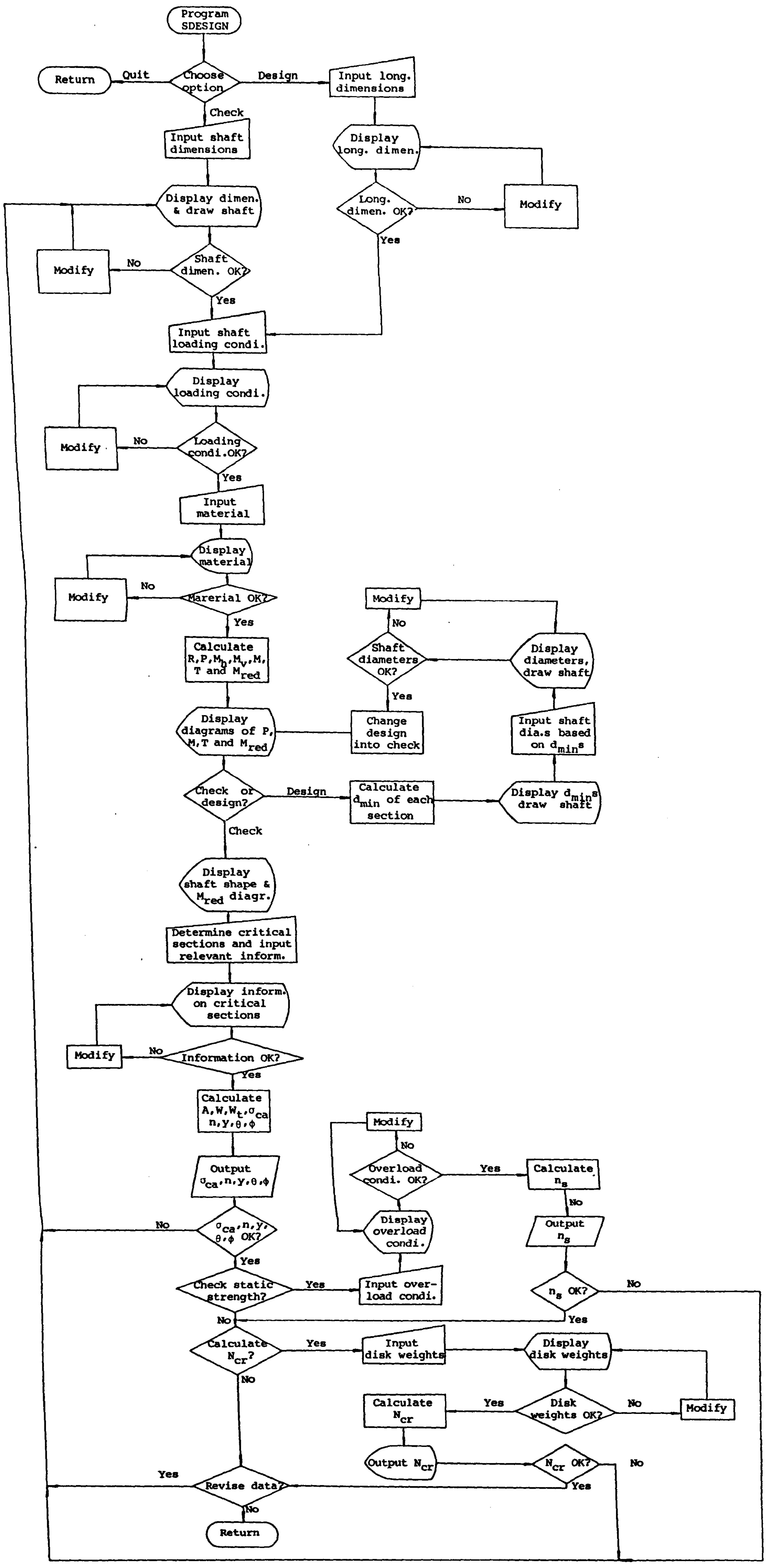


Figure 8.6 Shaft design flowchart

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moments, and the axial forces using the methods described in Sections8.2 and 8.4, and the user is presented with the corresponding diagrams. If the shaft is to be designed, the minimum diameters obtained from combined stresses are displayed on the screen, and their final values suitable for the shaft structure are decided by the designer.

Although the dimensions of the shaft and its material are now known, the checking calculations of the shaft still cannot be carried out because of the lack of adequate information on critical cross-sections. To help the designer locate the critical cross-sections the shaft shape and the reduced moment diagram are displayed again and may also be printed by the dumping facility in the program. The information on critical cross-sections contains the machining method of surface such as grinding, finishing and roughing, the constructional characteristic of surface such as fillet, keyway and groove, and the minimum demanded safety factor.

Once these information is provided by the designer, the combined stresses at critical cross-sections are computed from Eq. (8.4) and the safety factors for fatigue strength from Eq. (8.9). The deflections and twist angles are found next, using the methods in Section 8.4

When these computations have been fulfilled, the combined stresses, safety factors at all designated critical cross-sections, and the deflections, slopes, and angles of twist at all nodes are displayed on the screen. If the strength and stiffness requirements are not met or the designer is not satisfied with the results, he may return to edit menu to change any data such as shaft diameters and material until a satisfactory result achieved.

The checking for static strength is not absolutely necessary to some shafts. If it is required the designer must specify the overload conditions. The calculation of the critical speed of the shaft is also carried out at the request of the designer. If the designer gives the weights of the parts fitted onto the shaft, the shaft weight may be considered or neglected. However, if the designer does not specify the weights of fitted-on parts, it means that the shaft supports no parts except its own weight and its critical speed needs to be obtained. Whether the results are satisfactory or not, the designer is always able to modify the input data and compare the design results until he exits the program.

Before the program terminates, the computer saves the reactions at bearings and corresponding shaft diameters in a data file, which are later needed for bearing selection program.

An example of how the program SDESIGN is used to check the shaft carrying a worm gear for strength and rigidity is given in Appendix C.

## Chapter 9

## BEARING SELECTION

### 9.1 Introduction

In Chapter 8, the design of shafts for gear transmission systems was studied from the standpoint of strength and rigidity, and throughout the discussions it was assumed that such shafts were supported by bearings. At that time, however, shaft design was considered somewhat in isolation of the supporting bearings, but in a real design the bearings need to be considered an integral part of the system.

Bearings may be defined as machine elements which give the desired restraint to the applied external load, ideally with the minimum amount of friction. They may be divided into two major groups, namely

(a) Sliding bearings, and

(b) Rolling contact bearings.

Phelan uses the term sliding bearings to describe all bearings that do not involve the use of balls or rollers as load supporting elements, and rolling contact bearings as those which depend upon contact between rolling elements to give the desired restraint [25].

Sliding bearings are used extensively where full hydrodynamic lubrication can be obtained, specially at high speeds where they are less noisy than rolling contact bearings. Externally pressurised sliding bearings overcome the disadvantages associated with metal to metal contact during starting or at low speeds, but require external supply systems such as pumps, filters, etc.

If high starting torques are anticipated, rolling contact bearings may be preferable because of the inherently lower starting friction associated with the rolling elements. Further, such bearings are readily available as standard components, and it is necessary only for the designer to select the appropriate type and size from a bearing catalogue. Rolling contact bearings generally require less attention to lubrication than sliding bearings, particularly if prepacked grease lubrication is used, and failure of the lubrication is not likely to be as catastrophic as it might be for a sliding or journal bearing. Rolling contact bearings usually give some warning of failure by becoming noisier, whereas sliding bearings may fail suddenly and without warning. Many types of rolling contact bearings are capable of withstanding combinations of radial and axial loads, whereas sliding bearings are usually designed to withstand one type of loading only. Since the bearing clearance in rolling contact bearings is much less than that required in journal bearings, it is possible to obtain more accurate alignment and rigidity, a factor of particular importance for applications such as geared drives and machine tools. The ratio of length to diameter for rolling contact bearings makes them suitable where axial length is at a premium.

From these few brief comments it may be concluded that it is much more suitable to use rolling contact bearings than sliding bearings in gear transmissions taking into account reliability, cost, maintenance, convenience, etc.

## 9.2 Bearing Selection Procedure

### 9.2.1 Force Analysis

The prime consideration with regard to bearing selection concerns the magnitude and direction of the external load, so that the first task will always be that of determining the bearing forces. Before making a force analysis, tentative locations for bearings must be selected; sometimes, these positions may have to be changed when the numerical values for the loads are excessive. Often, the general scheme of the assembly may dictate where the bearings are likely to be placed. For a gear reduction box, for instance, the bearings will most likely be positioned on the sides of the housing for convenience and compactness when the gears are selected. Calculations for bearing loads are merely simple mechanics problems. For simplicity the reactions at each bearing location are usually considered as acting through the centre line of the bearings in all cases [17]. The reaction values are determined in the course of the design of shafts.

### 9.2.2 Selection of Bearing Types

Rolling contact bearings may be classified in a number of different ways. They may be self-aligning or nonself-aligning. They may be for radial applications, trust applications, or a combination of radial and axial use. Typical types are (1) ball and (2) roller, which is suggested by Harris [25].

Since different bearing types have different load carrying capacities, a bearing must be selected which is suitable for carrying the particular type of load. Of course, there will usually be several types of bearings which meet the requirements;

so a decision on the kind of bearing must be made.

A methodical procedure for the correct selection of ball and roller bearings is difficult to establish, because it depends so much upon the particular application, and many factors are involved. However, if a sufficiently comprehensive knowledge of the characteristics of the different bearing types is obtained, the selection of the bearing type is relatively simple. It is suggested that for small high speed applications ball bearings are generally most suitable, but that for large and heavily loaded applications, roller bearings provide the only satisfactory answer. Where permanent or fluctuating mis-alignment occurs between shaft and bearing housing a self-aligning bearing is desirable, whilst a completely free axial displacement of the shaft necessitates the use of a cylindrical roller bearing. Deep groove ball bearings frequently offer the best solution for applications involving relatively heavy axial or combined loading at high speeds.

### 9.2.3 Selection of Bearing Sizes

Having decided the loading conditions at a bearing position and the type of bearing to be used, the required size of bearing can then be calculated. The size of bearing selected for an application is usually influenced by the size of shaft required (for strength and rigidity considerations) and by the available space. Also, the bearing must have a high enough load rating to provide an acceptable life.

#### 9.2.3.1 Determination of Bearing Life

The International Standards Organisation (I.S.O.) has given bearing life close attention and a recommendation for ball bearings has received wide acceptance; this states that the life of a ball bearing is inversely proportional to $(\text{load})^3$. This same organisation has proposed life formulae for roller bearings in which life is inversely proportional to $(\text{load})^{10/3}$. The 'life' of a bearing is defined either in terms of the hours of rotation at a certain speed or in terms of the number of revolutions that 90 per cent of a group of identical bearings will reach or exceed when tested under identical conditions [67].

The life required from the bearing will depend upon the purpose for which it is to be used. Table 9.1 [41] may be used as a guide when more specific information is not available.

Table 9.1

Representative Bearing Design Lives

| Type of Application | Design Life (thousands of hours) |
|---|---|
| Instruments and apparatus for infrequent use | 0.1 - 0.5 |
| Machines intermittently used, where service interruption is of minor importance | 4 - 8 |
| Machines intermittently used, where reliability is of great importance | 8 - 14 |
| Machines for 8-hour service, but not every day | 14 - 20 |
| Machines for 8-hour service, every working day | 20 - 30 |
| Machines for continuous 24-hour service | 50 - 60 |
| Machines for continuous 24-hour service where reliability is of extreme importance | 100 - 200 |

#### 9.2.3.2 Calculation of Dynamic Capacity

To obtain the rating life of a bearing operating at certain conditions, a basic dynamic capacity **C**, published by bearing manufacturers, has to be used. The basic dynamic capacity **C** of a bearing is defined as the constant purely radial load in a radial bearing (or constant purely thrust load in a thrust bearing) that can be carried for a minimum life of 1,000,000 revolutions (which is equivalent to 500 hours of operation at 33.3 rev/min); the minimum life in the definition is that life which 90 per cent of the bearings of a group will reach or exceed.

Then if **P** is the load acting on a bearing, the rating life of the bearing will be [45]

$$L = \left(\frac{C}{P}\right)^{\varepsilon} \quad \text{million revolutions} \qquad (9.1)$$

where $\varepsilon = 3$ for a ball bearing

$\varepsilon = 10/3$ for a roller bearing.

Usually a machine will not be assembled with only one bearing. The importance of knowing the probable survival of a group of bearings should be examined. Assume that the probability of any single bearing failure is independent of the others in the same machine. If the machine is assembled with a total of **n** bearings, each having the same reliability **R**, then the reliability of the group must be

$$R_n = R^n$$

Suppose we have a gear-reduction unit consisting of six bearings, all loaded so that the rating lives are equal. Since the reliability of each bearing is 90 percent, the reliability of all the bearings in the assembly is

$$R_6 = 0.90^6 = 0.5314$$

This points up the need to select bearings having reliabilities greater than 90 percent. For this purpose, we introduce a reliability factor $f_\phi$ which is defined as

$$f_\phi = C_R/C$$

where $C_R$ is the basic dynamic capacity corresponding to a reliability of R. Typical values of reliability factor are listed in Table 9.2 [45]. Thus, the rating life $L_R$ of a bearing with a reliability of R is given by

$$L_R = \left(\frac{C_R}{P}\right)^\varepsilon = \left(\frac{f_\phi C}{P}\right)^\varepsilon \quad \text{million revolutions} \qquad (9.2)$$

Table 9.2

Ball Bearing Reliability Factor $f_{\phi b}$ and Roller Bearing Reliability Factor $f_{\phi r}$

| R | .50 | .60 | .70 | .75 | .80 | .85 | .9 | .92 | .94 | .95 | .96 | .97 | .98 | .99 | .995 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $f_{\phi b}$ | 1.533 | 1.425 | 1.308 | 1.245 | 1.175 | 1.095 | 1 | .954 | .901 | .870 | .837 | .798 | .752 | .692 | .652 |
| $f_{\phi r}$ | 1.411 | 1.325 | 1.238 | 1.190 | 1.136 | 1.074 | 1 | .964 | .922 | .898 | .871 | .840 | .803 | .754 | .720 |

However, because we generally consider life in terms of so many hours at so many revolutions per minute, we convert the equation above into

$$L_{hR} = \frac{10^6}{60n}\left(\frac{f_\phi C}{P}\right)^\varepsilon \quad \text{hours} \qquad (9.3)$$

where n is the speed of rotation in rev/min. The equation is more useful in the form

$$C = \frac{P}{f_\phi}\left(\frac{60nL_{hR}}{10^6}\right)^{1/\varepsilon} \tag{9.4}$$

Since the service life of a bearing is unfavourably affected by shocks and high temperature during operation, we also introduce the load-application factor $f_p$ and the temperature factor $f_t$ to increase the required value of basic dynamic capacity before selecting a bearing. The values of $f_p$ and $f_t$ are given in Table 9.3 and Table 9.4 [45]. Thus, Eq. (9.4) becomes

$$C = \frac{f_p P}{f_t f_\phi}\left(\frac{60nL_{hR}}{10^6}\right)^{1/\varepsilon} \tag{9.5}$$

Therefore if a certain application requires a bearing to last for $L_{hrR}$ hours at a speed of n rpm with a reliability of R, then the

Table 9.3

Load-Application Factor $f_p$

| Type of application | No impact or light impact | Moderate impact | Heavy impact |
|---|---|---|---|
| Load factor | 1.0 - 1.2 | 1.2 - 1.8 | 1.8 - 3.0 |

Table 9.4

Temperature Factor $f_t$

| Brg Working Temperature($^oC$) | <=120 | 125 | 150 | 175 | 200 | 225 | 250 | 300 | 350 |
|---|---|---|---|---|---|---|---|---|---|
| Temperature Factor $f_t$ | 1 | .95 | .90 | .85 | .80 | .75 | .70 | .60 | .50 |

required dynamic capacity $C_r$ of the bearing selected for this application should be

$$C_r = \frac{f_p P}{f_t f_\phi} \left( \frac{60nL_{hrR}}{10^6} \right)^{1/\varepsilon} \qquad (9.6)$$

### 9.2.3.3 Calculation of Equivalent Load

In the above discussion it has been assumed that the load acting on a bearing is a purely radial one for a radial bearing (or a purely thrust one for a thrust bearing). The more general situation might be that of combined radial and axial loading. In this case, it is necessary to determine the equivalent load **P** which has the same effect on bearing life as the combined radial and axial loading. The equivalent load may be calculated from the following equation [45]:

$$P = X\,R + Y\,A \qquad (9.7)$$

where R = actual radial bearing load

A = actual axial bearing load

X = radial factor

Y = thrust factor.

The factors **X** and **Y** depend upon the type of bearing and the ratio of (**A**/**R**). Values for these factors are listed in the manufacturers' catalogues [44, 76] and some of them are reproduced in Table 9.5 [45]. In the special case where a purely radial load acts on a bearing, i.e. **A** = 0, the value for **P** is simply equal to **R**, reference to the table indicating that **X** = 1 for this

Table 9.5

Radial Factor X and Thrust Factor Y

| Bearing Type | Code No. of Bearing Type | $\frac{A}{C_o}$ | e | $\frac{A}{R} <= e$ | | $\frac{A}{R} > e$ | |
|---|---|---|---|---|---|---|---|
| | | | | X | Y | X | Y |
| Single Row Radial Ball Bearing | 0000 | 0.025<br>0.04<br>0.07<br>0.13<br>0.25<br>0.50 | 0.22<br>0.24<br>0.27<br>0.31<br>0.37<br>0.44 | 1 | 0 | 0.56 | 2.0<br>1.8<br>1.6<br>1.4<br>1.2<br>1.0 |
| Single Row Angular Contact Ball Bearing | 36000 | 0.025<br>0.04<br>0.07<br>0.13<br>0.25<br>0.50 | 0.34<br>0.36<br>0.39<br>0.43<br>0.49<br>0.55 | 1 | 0 | 0.45 | 1.61<br>1.53<br>1.40<br>1.26<br>1.12<br>1.00 |
| | 46000 | | 0.70 | 1 | 0 | 0.41 | 0.85 |
| | 66000 | | 1.00 | 1 | 0 | 0.36 | 0.64 |

Notes: (1) $C_o$ is the basic static capacity

(2) Values of $(A/C_o)$, X, Y and e other than shown in Table 9.4 may be obtained by linear interpolation.

instance. Further, the effect of an axial load does not significantly influence the equivalent load P until its magnitude is such that the ratio (A/R) exceeds a specified value; this value is designated e in Table 9.4. Note that for a single row radial ball bearing and a certain type of single row angular contact ball bearing the value for e is a function of the ratio $(A/C_o)$, where $C_o$ is the basic static capacity which will be described in the following section. When the ratio of $(A/C_o)$ exceeds the limiting value of e corresponding to the particular bearing under consideration, the appropriate values for X and Y can be obtained from the table, and the equivalent load P can be calculated from Eq. 9.7. However, when selecting a bearing, since we do not know which bearing will be used, we have to assume a value of **Y** and check it later using the proper value instead of an assumed one. If the results are not satisfactory, we have to test another size of bearing.

In the case of angular contact ball bearings and taper roller bearings, an imposed radial load will give rise to an induced axial load, in addition to any external axial load, and this must frequently be included in the calculations. The induced axial load produced by a pure radial load can be determined from the equations given in Table 9.6.

Table 9.6

Formulae for Determining the Induced Axial Load S

| Taper Roller Bearing | Angular Contact Ball Bearings | | |
|---|---|---|---|
| 7000 | 36000 | 46000 | 66000 |
| S = 0.5R/Y | S = 0.4R | S = 0.7R | S = R |

These types of bearings are usually arranged so that they can be adjusted against each other, as indicated in Figure 9.1, where bearing 1 is subjected to a radial load $R_1$ and an external axial load **P**, bearing 2 to a radial load $R_2$. Using the notation

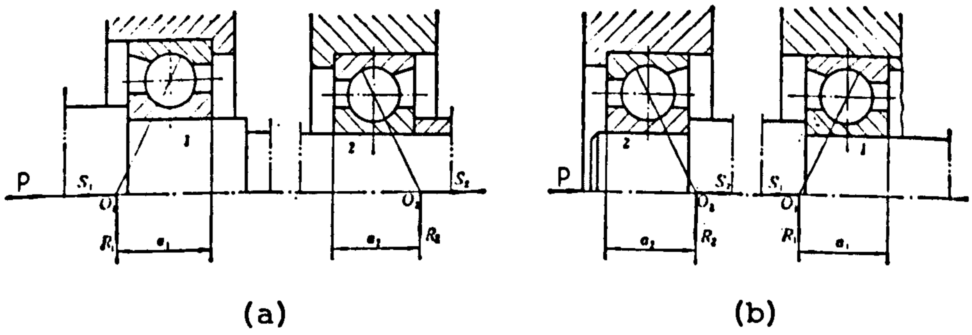


Figure 9.1 Bearing loads acting on radial thrust bearings

indicated in this figure, the axial bearing loads $A_1$ and $A_2$ may be obtained through the use of the formulae listed in Table 9.7 [45]. If the external axial load P acts in the opposite direction to that indicated in Figure 9.1, the formulae listed in Table 9.7 are still valid provided that the bearing position numbers are reversed, i.e. bearing 1 becomes bearing 2, and vice versa. After the axial bearing loads are found, the equivalent radial loads on these types of bearings may still be computed using Eq. (9.7).

Table 9.7

Formulae for Determining the Axial Bearing Load A

| Axial bearing load acting on bearing 1 ($A_1$) | Axial bearing load acting on bearing 2 ($A_2$) |
|---|---|
| $A_1 = \text{Max}\{ S_2 + P, S_1 \}$ | $A_2 = \text{Max}\{ S_2, S_1 - P \}$ |

#### 9.2.3.4 Calculation of Static Capacity

With the high peak stresses in rolling bearings, it does not take a large load to produce a permanent deformation on the surface. If the bearing rotates, the deformations are distributed over all the race and ball surfaces; if it is not rotating but subject to changing load or vibration, the deformations are concentrated, and surface depressions are formed which ruin the bearing. It is suggested that to ensure the proper operation of the bearing the permanent deformation in the ring and rolling element combined be less than 0.0001 times the diameter of the rolling element. The basic static capacity of a rolling bearing is defined by I.S.O. as that load which will give a maximum permanent deformation of about 0.0001 of the rolling element diameter at the most heavily stressed contact. This load is designated as $C_o$ and may be found in the manufacturers' catalogs as well.

The formula of selecting a bearing based on the basic static capacity is

$$C_{or} = S_o P_o <= C_o \tag{9.8}$$

where $C_{or}$ = required static capacity

$S_o$ = safety factor for static load (see Table 9.8) [46]

$P_o$ = equivalent static load.

Table 9.8

Safety Factor $S_o$ on Static Loads

| Type of Application and Load Characteristic | $S_o$ |
|---|---|
| Applications with high accuracy of rotation and smoothness or heavy impact | 1.2 - 2.5 |
| General Applications | 0.8 - 1.2 |
| Applications with low accuracy of rotation and smoothness and no impact | 0.5 - 0.8 |

The equivalent static load $P_o$ is computed by

$$P_o = X_o R + Y_o A \tag{9.9}$$

where $X_o$ and $Y_o$ are the radial and thrust factors for static load respectively. These factors depend upon the type of bearing and their values are listed in the manufacturers' catalogues. If the actual radial load R on a bearing should happen to be larger than the value of $P_o$, then the actual radial load R is used instead of $P_o$ for that bearing.

## 9.3 Description of Bearing Selection Program

The bearing selection program described here takes into account the most frequently encountered mounting situation where each end of a shaft is supported by one bearing [70], and these two bearings are capable of providing sufficient radial support to limit shaft bending and deflection to acceptable values. This is highly desirable and simplifies manufacturing [41].

The entire information on various types of standardised ball and roller bearings available from the Handbook of Machine Elements Design [44] has been stored as data bases in the bearing selection program. It contains over 1500 bearings, whose bores range from 3mm to 1020mm. The bearing data covers the bore, outside diameter, width, fillet radius, shoulder diameters, shoulder fillet radius, basic dynamic and static capacities, radial and thrust factors, weight and designation for each bearing.

Within the bearing selection program developed, bearings can be selected on basic dynamic capacity and basic static capacity conditions. Figure 9.2 shows the bearing selection flow chart. The program requests information from the designer on the loads acting on the bearings, operating speed of the shaft, safety factor on static loads, and designation of the bearing to be checked or type and bore of the bearing selected. If the bearing does not rotate, it will be checked or selected in accordance with basic static capacity. Otherwise it will be checked or selected according to the basic dynamic capacity and the designer must also specify the relevant information such as the expected bearing life and reliability, and the load factor.

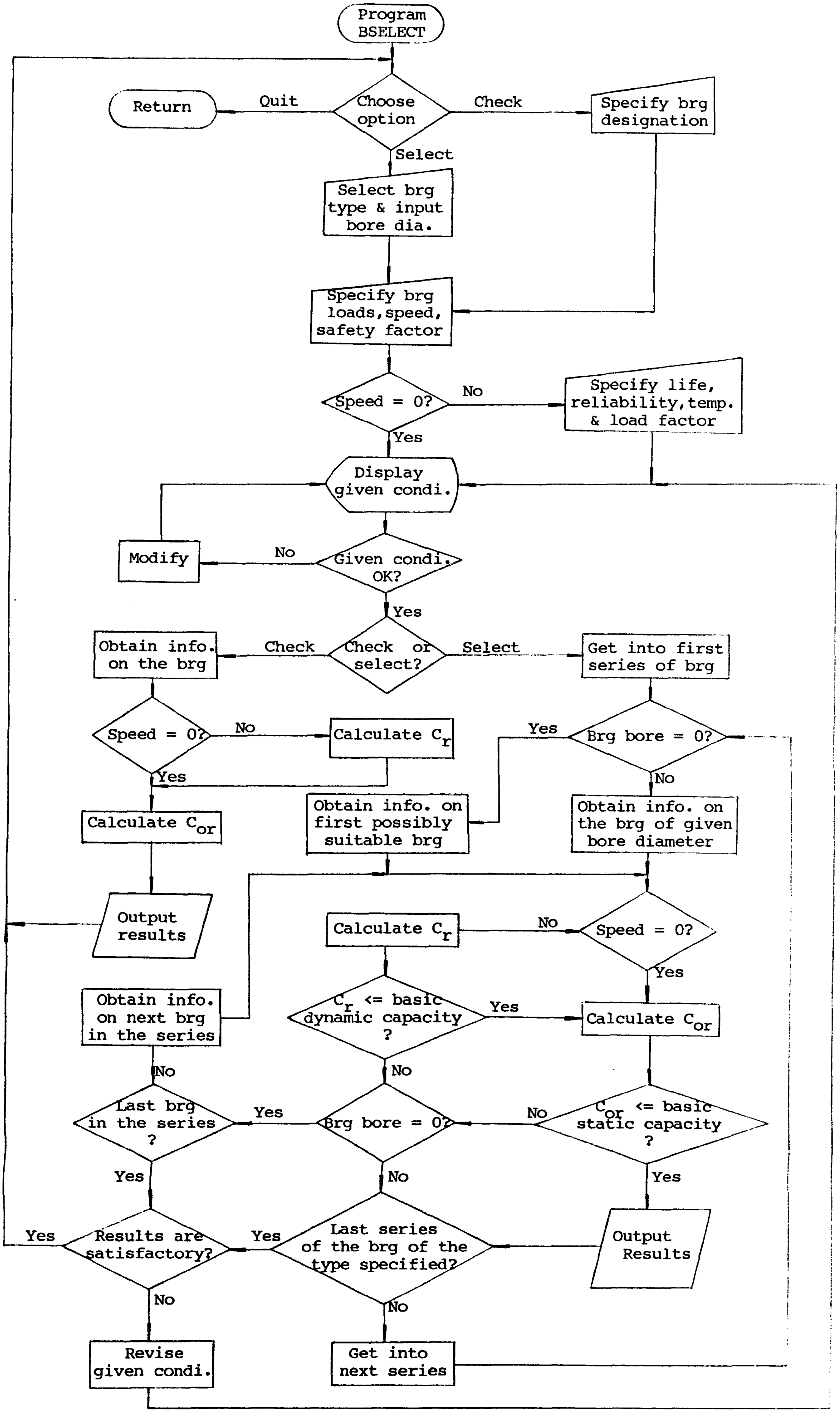


Figure 9.2 Bearing selection flowchart

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After that, the program calculates the equivalent load P (or $P_o$) and the required dynamic (or static) capacity, and immediately checks the specified bearing or searches for a suitable bearing which meets the designer's specification. In the case of selecting bearings, since the basic dynamic and static capacities of the different series of the same kind of bearing overlap, usually in the specified type there will be several series of bearings which satisfy the operating conditions. If this is the case, the information on all suitable bearings will be presented to the designer. From these data, the designer may choose the most suitable bearing he supposes according to the space requirements or other considerations, or reject them if he doesn't satisfy all the bearings presented. If the latter is the case or there is no suitable bearing in the type specified which can match the requirements, a message will be displayed and the designer should change the bearing type, the life expectancy, or the expected reliability. Then the program will search for any other suitable bearings again. In general, before a final decision is made, designs should be made or considered for all appropriate types of bearings which can carry the load.

If no bearing of the type chosen by the designer satisfies the load and running conditions and the designer is unwilling to change these conditions except the bearing bore diameter by any reason, he may return to editing menu to take the bearing bore as zero. This will cause the program to test every bearing starting from the first possibly suitable bearing in every series of bearing of the type specified until an acceptable bearing in each series is found. The bearings thus selected may, of course, have different bore diameters. The first possibly suitable bearing mentioned above is the bearing whose basic dynamic (or static) capacity is greater than or equal to the required value obtained by using the actual radial load R and nearer to it than any other bearing's in the series. The flow chart of finding the first possibly suitable bearing is shown in Figure 9.3.

Note that a bearing selected on the basic dynamic capacity will also be checked for static strength in this program. If the bearing fails to meet the requirements of static strength, which is likely to happen to a bearing rotating at very low speed, we may set the bearing speed to zero so that it will be selected on the basic static capacity.

The shaft design proceeding prior to the bearing selection is tentative because of the lack of exact bearing dimensions such as width and fillet. After the bearing selection has been finished, the shaft design may need to be modified. The modification of the shaft geometry is carried out based on the corresponding bearing data displayed on the screen. Figure 9.4 shows a sample output of information on bearing 313. The given shaft shoulder diameter $D_1$ should be used whenever possible to secure adequate support for the bearing and to resist the maximum axial loads. The shoulder fillet radius $r_g$ listed is the maximum radius at the shoulder on the shaft which is cleared by the corner radius on the bearing. If necessary, the shaft strength and stiffness should be checked again.

Appendix D shows how the program BSELECT to be used to select a bearing for a given specification.

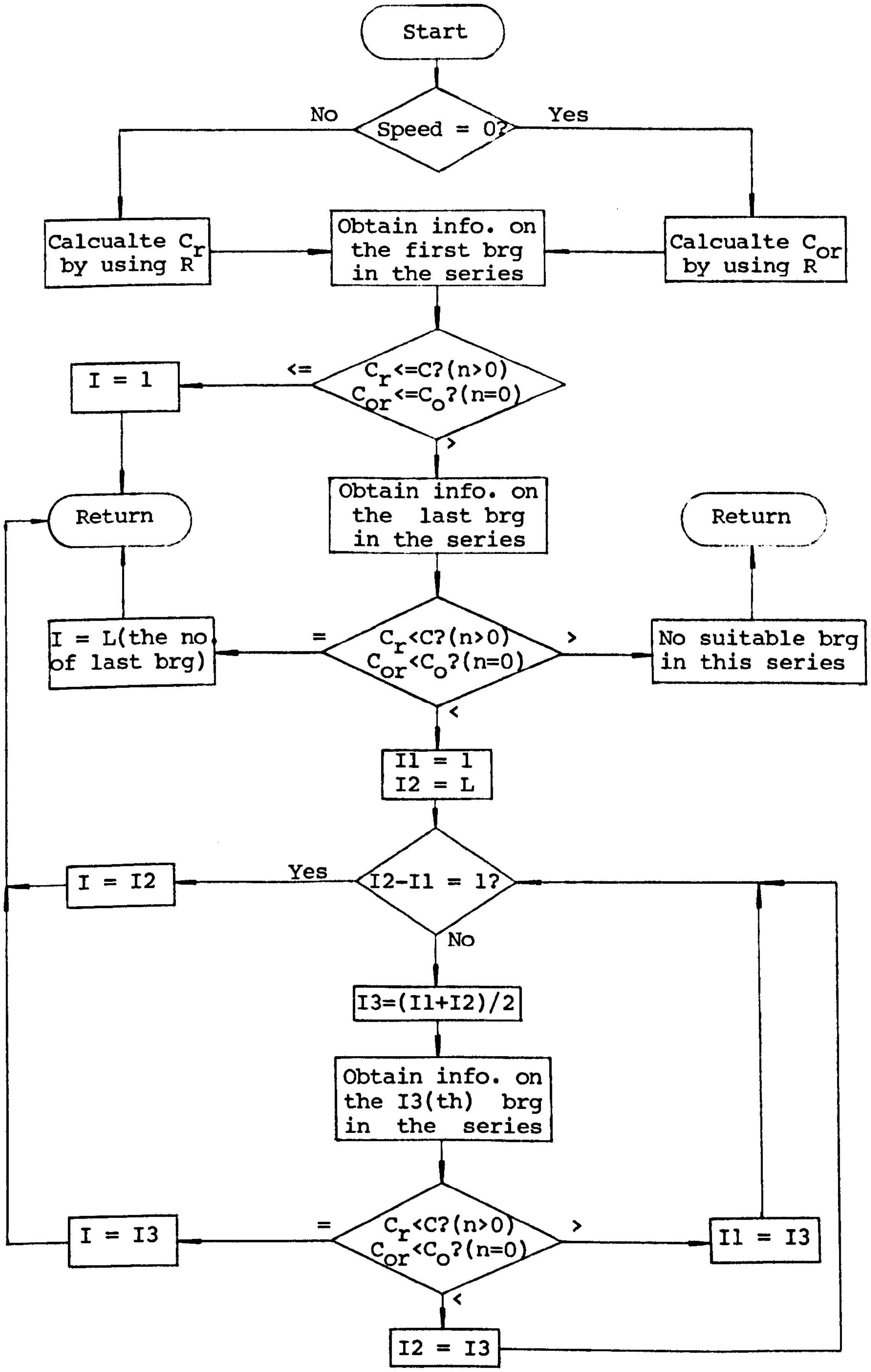
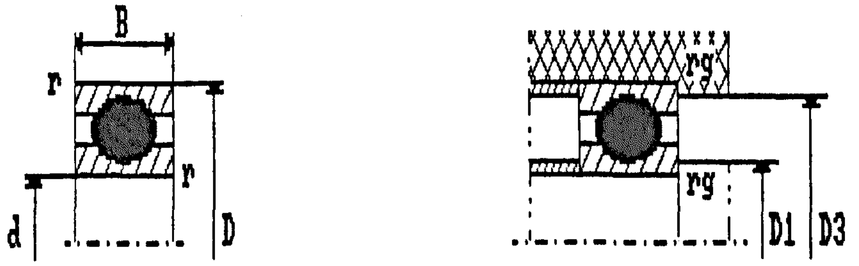



Figure 9.3 Flowchart of finding the first possibly suitable bearing in the given series




| Brg Designation | Brg Dimensions (mm) | | | | Shoulder Dimensions (mm) | | |
|---|---|---|---|---|---|---|---|
| # | d | D | B | r | D1 | D3 | rg |
| 313 | 65 | 140.00 | 33.00 | 3.50 | 77.00 | 128.00 | 2.00 |

| Basic Dyn. Cap. | Basic Sta. Cap. | Limit Speeds for Diff.Lubri. Methods | | Weight |
|---|---|---|---|---|
| C (kg) | C0 (kg) | n (rpm) (Grease) | n (rpm) (Oil) | W (kg) |
| 7260.0 | 5670.0 | 4500.0 | 5600.0 | 2.09 |

Figure 9.4 Sample output of information on bearing 313

# Chapter 10

# COMPUTER AIDED DRAUGHTING

## 10.1 Introduction

An important part of engineering is the formation of graphical data. Drawings have always been the main method of communication in the engineering world, although in some specialised areas where numerically controlled machines are employed, this method of communication may change [9].

Designers have traditionally developed their final designs via rough preliminary sketches. It is in turning the rough sketches into working drawings that the computer can play a useful role, because this is a tedious and time-consuming process. Furthermore, a large part of draughting is concerned with the use and modification of existing designs. If it is considered where time is spent in the design/manufacture cycle, many industries spend 80% and more of this time meticulously creating or referencing drawings [77].

The use of computers by engineers to produce design drawings began about 30 years ago. Computer-produced drawings are of a high standard with information often presented with greater clarity than with conventional drawing practice [8]. Once graphical data has been stored in the computer it can be re-used, modified, and edited, and parts can be added to produce a new drawing. Over the past decade, automated draughting applications, particularly for computer-aided design; have been justified because they can save money and time and can improve the quality of the drawings. The computer-aided draughting techniques are playing a significant role in this area.

As mentioned in Chapter 2, the whole computer-aided design and draughting package is composed of design programs and draughting programs. This chapter describes the development and features of the draughting programs. Some sample drawings are also given. As the detailed working drawings should be superior in quality, the draughting programs are written in FORTRAN 77 and GINO-F [33] so that the Imlac PDS4 refresh display, Tektronix 4010 graphical terminal, and Calcomp 1051 (4 colour) drum plotter available in the City University of London's computer centre may be used for graphical interaction and to produce the hard copies for design drawings.

## 10.2 Basic Geometric Construction Techniques

At present there is no standard graphics package specially designed for computer-aided mechanical engineering drawing. The author chose GINO-F, the most prominent general-purpose graphics package developed by the Computer-Aided Design Centre at Cambridge, as the graphics software for the project applications because it is already installed at lots of computer centres.

GINO-F is an acronym for Graphical INput/Output-Fortran version. It is a systematically designed graphics package that takes the form of a library of drawing and administrative subroutines. Most of the routines are written in standard ANSI FORTRAN making GINO-F virtually machine independent. GINO-F is also device independent — a change to one line of a user program being all that is required to convert the program to produce output on a different device. The routines in GINO-F that produce this output are code-generators, there being one for each device available on each machine. GINO-F provides for all commonly needed graphical activities including two-dimensional drawing, three-dimensional drawing and interaction for both refresh and non-refresh displays.

When trying to develop a graphics program to produce engineering drawings the first difficulty encountered is the variety of drawing types and styles in existence. Even a cursory glance at a few drawings picked up at random will show that they have very little in common. However, the kind of drawings used in mechanical engineering are composed largely of simple geometrical constructions, mostly curves and straight lines. Almost invariably sharp corners of an engineering component are rounded off with a fillet, and drawing fillets and lines tangential to circles or arcs is basic to mechanical engineering drawing. Table 10.1 [27] gives some statistics on some engineering drawings chosen at random, indicating the number of dimension symbols, circles, fillets, straight lines, and alphanumeric characters included as comments on the drawing.

Table 10.1

Statistics on Some Typical Engineering Drawings

| Drawing No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Dimension Symbols | 44 | 14 | 31 | 139 | 41 | 42 | 81 | 12 |
| Circles | 10 | 9 | 6 | 0 | 20 | 15 | 11 | 0 |
| Fillets | 20 | 13 | 0 | 48 | 4 | 16 | 6 | 11 |
| Straight Lines | 141 | 16 | 17 | 62 | 57 | 62 | 324 | 250 |
| Alphanumerics | 385 | 58 | 77 | 80 | 36 | 110 | 103 | 450 |

Although GINO-F provides facilities for producing graphical output ranging from two-dimensional graphs to complex three-dimensional interactive systems, the plotting requirements for machine components in a gearbox can not be satisfied by using the existing computer subroutines. Thus the author resorted to writing his own special-purpose draughting subroutines on the basis of the GINO-F package to enable the designer to produce working drawings much more easily and quickly. The purposes of some basic special-purpose geometric construction subroutines are listed below:

| | |
|---|---|
| ARODIM | draws annotated dimension lines terminated with an arrow |
| AROHD | draws arrowheads |
| CASTG1 | draws one type of enclosed geometric shape formed by lines and arcs for gears |
| CASTG2 | draws the other type of enclosed geometric shape formed by lines and arcs for gears |
| CIRCLE | draws a circle |
| CIRKEY | draws a view of a chamfered gear bore with keyway or a view of a chamfered shaft end with keyway |
| CYLARC | draws arcs and lines connected each other for shafts |
| DIMEN | draws annotated dimension lines |
| FILDIM | draws annotated dimension lines for chamfers |
| FILLET | draws a chamfer |
| GBSPE | provides gearbox specification annotation |
| GEARSP | provides gear specification annotation |
| KEYWAY | draws a keyway |
| LINCIR | draws lines connected with arcs |
| PHI | plots a symbol indicating a diameter |
| RECSHAD | draws section lines for rectangular-shaped areas |

RECT2 draws a rectangle

SKSHADE draws a sectional view with keyway of shafts

TITLE draws one type of title block

TITLE1 draws the other type of title block.

Each of the geometric construction subroutines described above has a list of calling parameters(arguments). They are as follows:

```
AROD IM(X,Y,R,THETA,ITYPE,SCALE)
AROHD(X,Y,R,THETA,SCALE,XNEW,YNEW)
CASTG1(X1,X2,R1,R2,ALPHA,R3)
CASTG2(X,R1,R2,R3,ALPHA)
CIRCLE(X,Y,D)
CIRKEY(X,Y,D1,D2,WIDKEY,HIGKAS,DBKAC1,DBKAC2)
CYLARC(X,Y,D,W,R,THETA)
DIMEN(X,Y,HL,HR,DIMENS,IPDECI,ITYDIM,IUPCEN,THETA,SCALE)
FILDIM(X,Y,W,FILSIZ,ALPHA,THETA,ITYPE,SCALE)
FILLET(X,Y,D,FILSIZ,ALPHA,THETA,IOUTIN,IOPCLO)
GBSPE(T1,GN1,U,A)
GEARSP(XP,YP,ITYPE,Z,GMN,X,BETNOM,IDERBT,WHODEP,IQUCL)
KEYWAY(X,Y,W,D,THETA)
LINCIR(X,Y,W,R,HLEFT,HRIGHT,THETA)
PHI(X,Y,THETA,SCALE)
RECSHAD(X,Y,WD,HT,FILSIZ,SCALE)
RECT2(X,Y,W,H,THETA,IUPDOW)
SKSHADE(X,Y,D,WIDKEY,DMTKEY,SCALE)
TITLE(IDATE,IMONTH,IYEAR,PLOSCA)
TITLE1(IDATE,IMONTH,IYEAR,PLOSCA)
```

Figure 10.1 illustrates the geometric shapes of the twenty basic geometric construction subroutines. These routines listed here together with some others will be used to create the draughting

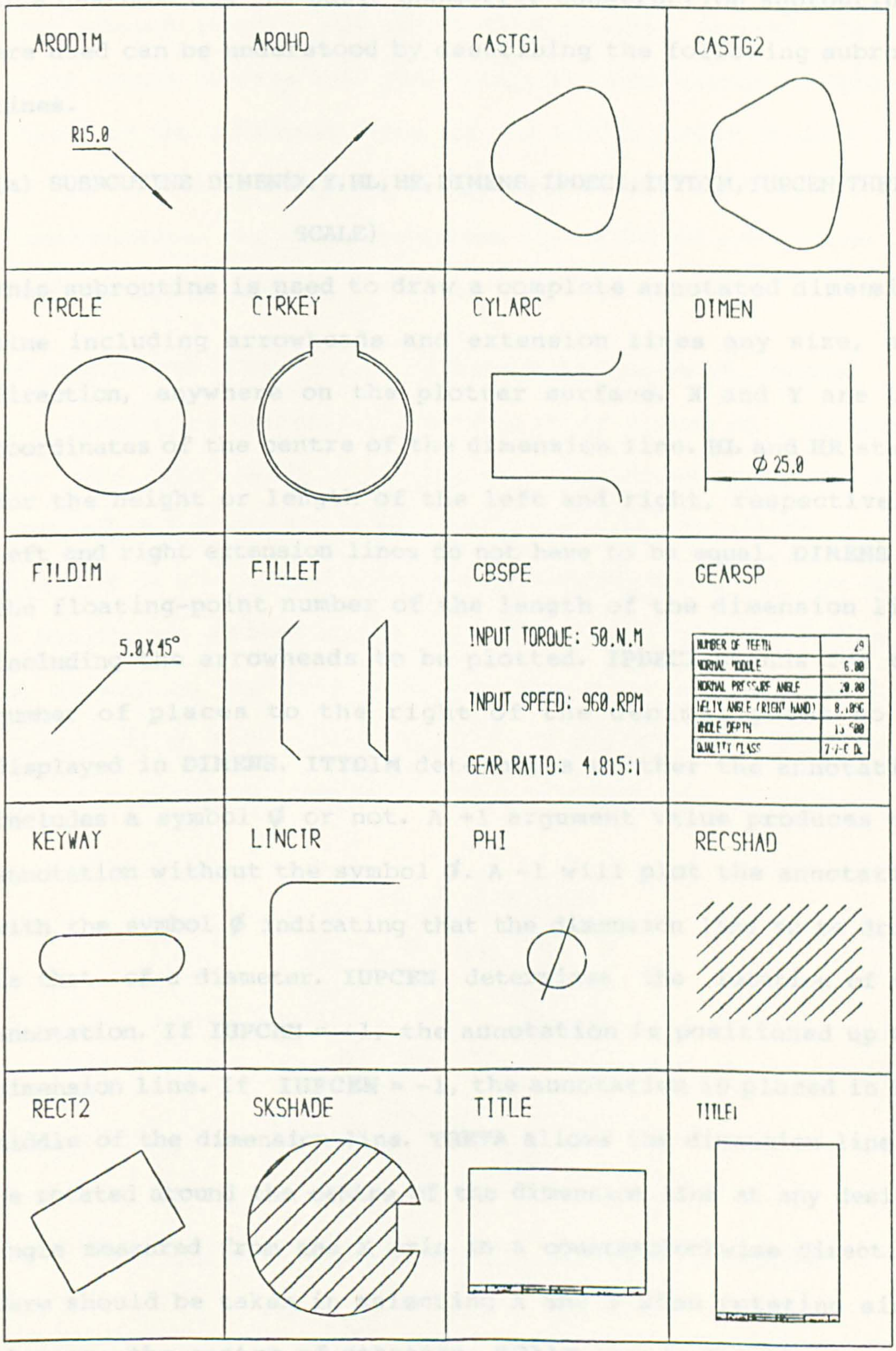



**Figure 10.1 Graphic outputs from the twenty geometric construction subroutines used on a Calcomp1051 drum plotter**

programs for generating working drawings of the main components in a gearbox. How the basic geometric construction subroutines are used can be understood by describing the following subroutines.

(a) SUBROUTINE DIMEN(X,Y,HL,HR,DIMENS,IPDECI,ITYDIM,IUPCEN,THETA, SCALE)

This subroutine is used to draw a complete annotated dimension line including arrowheads and extension lines any size, any direction, anywhere on the plotter surface. **X** and **Y** are the coordinates of the centre of the dimension line. **HL** and **HR** stand for the height or length of the left and right, respectively. Left and right extension lines do not have to be equal. **DIMENS** is the floating-point number of the length of the dimension line including the arrowheads to be plotted. **IPDECI** stands for the number of places to the right of the decimal point to be displayed in **DIMENS**. **ITYDIM** determines whether the annotation includes a symbol ∅ or not. A +1 argument value produces the annotation without the symbol ∅. A −1 will plot the annotation with the symbol ∅ indicating that the dimension line to be drawn is that of a diameter. **IUPCEN** determines the location of the annotation. If **IUPCEN** = +1, the annotation is positioned up the dimension line. If **IUPCEN** = −1, the annotation is placed in the middle of the dimension line. **THETA** allows the dimension line to be rotated around the centre of the dimension line at any desired angle measured from the **X** axis in a counterclockwise direction. Care should be taken in selecting **X** and **Y** when rotating since they are the centre of rotation. **SCALE** stands for the drawing scale chosen. The reason why this argument should be introduced is to ensure that the sizes of the annotation and arrowheads are not affected by the scaling factor.

The actual length of the dimension line is the product of **DIMENS** (the labeled length) and **SCALE** (the scaling factor). If the actual length is less than 12mm long, the arrowheads are placed outside of the extension lines. If the actual length is less than the space required for plotting the annotation, the annotation is placed outside. The examples of the use of DIMEN subroutine are shown in Figure 10.2.

(b) SUBROUTINE CIRKEY(X,Y,D1,D2,WIDKEY,HIGKAS,DBKAC1,DBKAC2)
This geometric construction technique draws a view of a chamfered gear bore with keyway or a view of a chamfered shaft end with keyway. **X** and **Y** are the centre of the circle. **D1** and **D2** are the diameters of the circles to be drawn. **WIDKEY** stands for the width of the keyway. **HIGKAS** stands for the distance from the bottom of the keyway to the bottom of the gear bore or the shaft, as illustrated in Figure 10.3. **DBKAC1** and **DBKAC2** are the returned arguments, whose meanings are shown in Figure 10.3 as well.

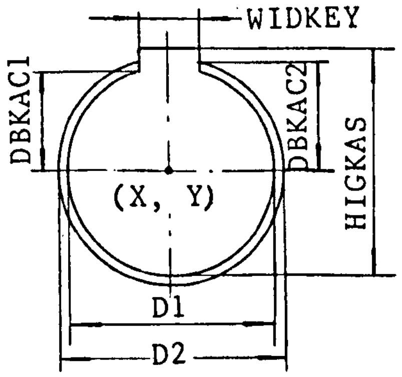



Figure 10.3 Schematic diagram of CIRKEY arguments

If **D2 > D1**, this subroutine draws a view of a chamfered gear bore with keyway; if **D1 > D2**, this subroutine draws a view of a chamfered shaft end with keyway. Figure 10.4 is the samples of CALL CIRKEY.

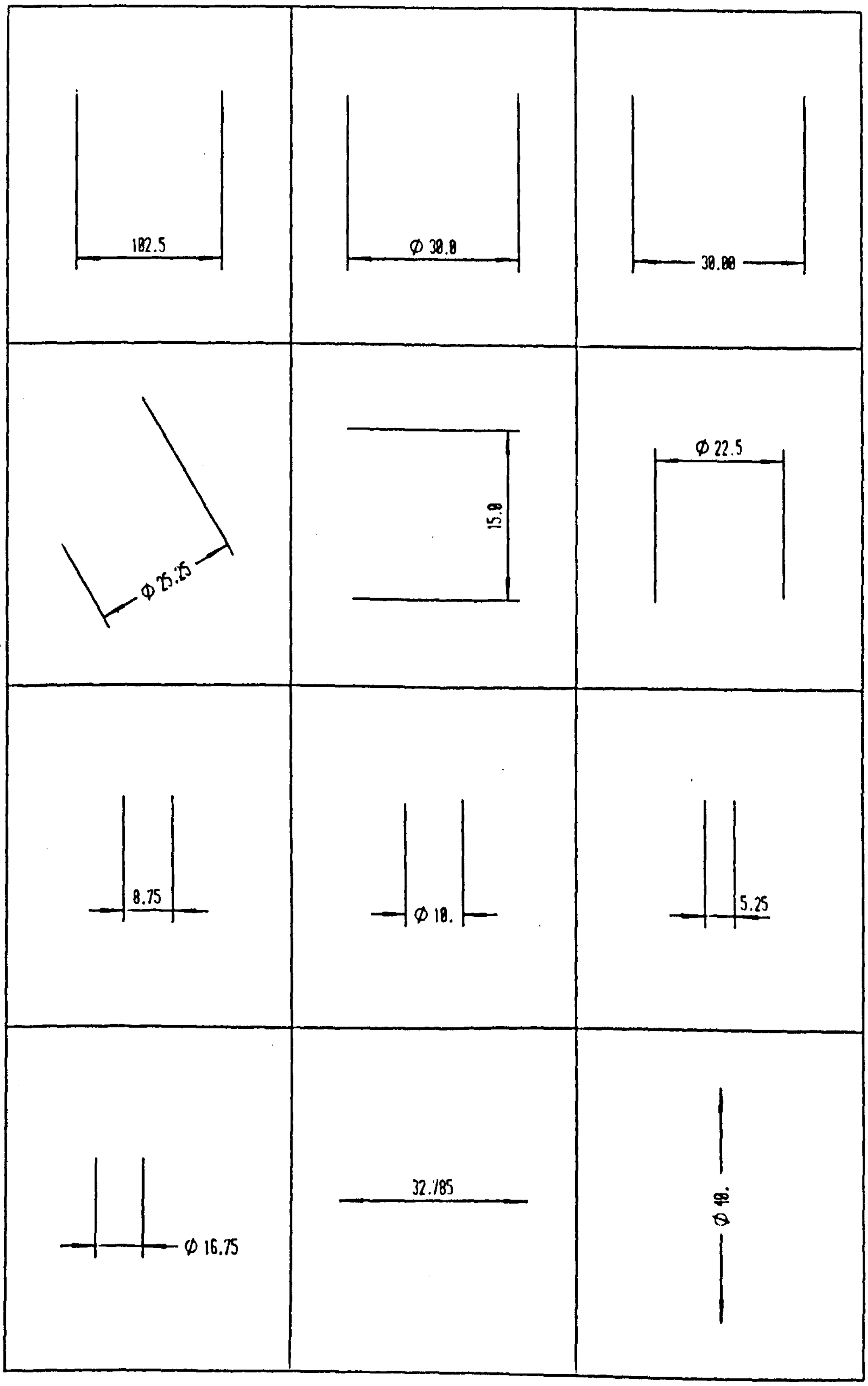



Figure 10.2  Examples of the use of subroutine DIMEN

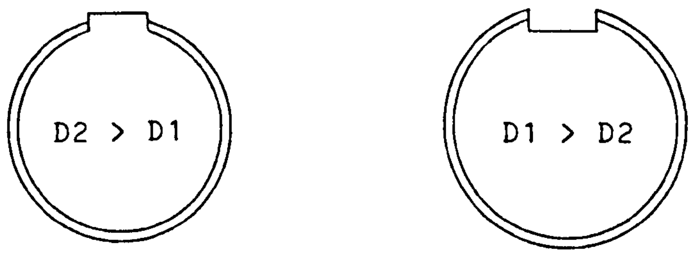



Figure 10.4 Samples of CALL CIRKEY

(c) SUBROUTINE SKSHADE(X,Y,D,WIDKEY,DMTKEY,SCALE)

This subroutine is used to draw a sectional view with keyway for shafts. **X** and **Y** are the coordinates of the centre of the circle. **D** is the diameter of the circle to be plotted. **WIDKEY** stands for the width of the keyway. **DMTKEY** stands for the distance from the bottom of the keyway to the bottom of the arc, as indicated in Figure 10.5. In order to remain the spacing between section lines unchanged under all circumstances, the scaling factor **SCALE** is introduced. The examples of the use of this subroutine are shown in Figure 10.6.

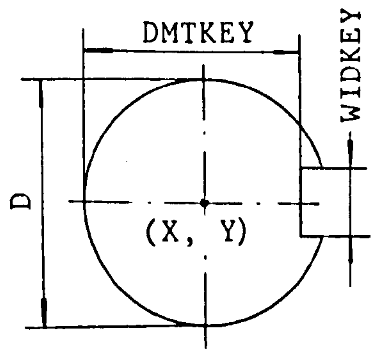



Figure 10.5 Schematic diagram of SKSHADE arguments

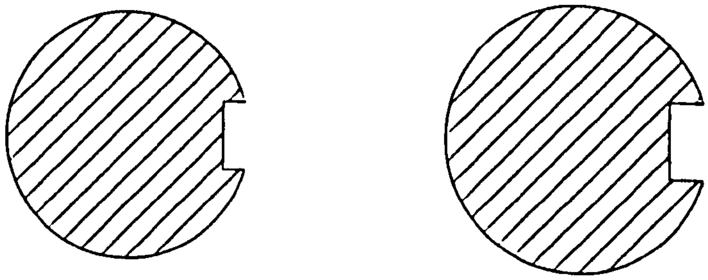


Figure 10.6 Examples of the use of subroutine SKSHADE

## 10.3 Description of Draughting Programs

Graphical output forms an important and integral part of the whole gearbox design package. Draughting programs are developed on the basis of the basic geometric construction subroutines. The design programs described previously are capable of producing a suitably formatted data file for input to the draughting programs. This allows display on the Imlac PDS4 and Tektronix 4010 graphic terminals, or the Calcomp drum plotter if a hard copy is necessary. The maximum drawing dimensions preset for a hard copy are 250 by 175 millimeters, but they may be changed in the light of specific requirements. Moreover, the maximum drawing dimensions also can be readily changed to match the screen sizes on the graphic terminal to be used. Whether for a hard copy or for screen display, the draughting programs will select a suitable scale based on the given drawing area to produce the graphical output.

The draughting software mainly consists of three programs, which automatically make a gearbox schematic drawing, gear working drawing and shaft working drawing without the intervention of the designer when it runs.

### 10.3.1 Gearbox Schematic Drawing Program

The gearbox schematic drawing program provides the designer with the principal characteristics of the gearbox to be designed. The characteristics contains general layout of the gearbox, its major dimensions and specifications such as input and output torques, input and output speeds, and total gear ratio. Besides, by the gearbox schematic drawing produced the designer can easily judge whether interference occurs or not between the first-reduction gear and the output shaft (see Figure 10.7), and whether it is possible to apply the splash lubrication to both the first-reduction gear train and the second-reduction gear train when a double reduction gearbox is adopted. If interference happens, a suggestion is immediately presented on the screen, and the designer should redistribute the gear ratio and redesign the gearbox. If not, the gear working drawings and shaft working drawings will be produced. When the splash lubrication is not applicable to the second-reduction gear train, a warning is output. In this case, the gear (not immersed in oil) should be lubricated by special devices or by pinions, rings, etc. (see Figure 10.8) [20].

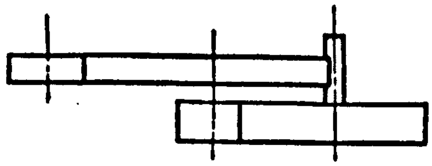


Figure 10.7 Interference between gear and shaft

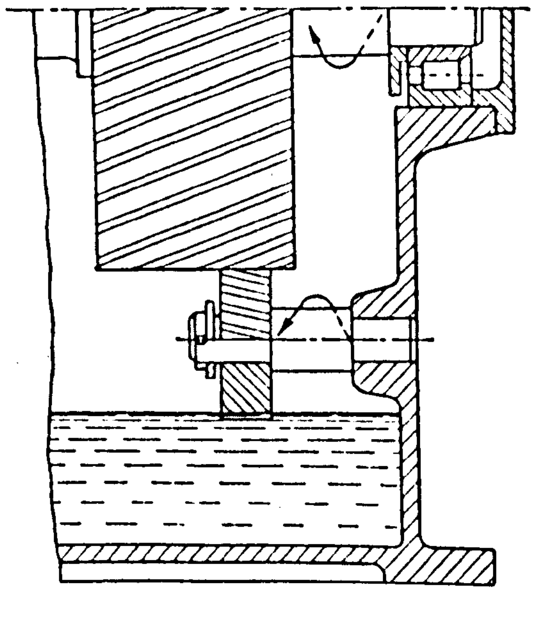


Figure 10.8 A gear oiled by a pinion

### 10.3.2 Gear Working Drawing Program

There are many types of physical forms for gears. However, only four of them have been taken into account in the gear working drawing program (GPLOT). Empirical formulae for determining proportions of gear elements of these types are derived from [45, 75]. Which type of physical form will be adopted for a specific gear is decided by the computer, depending upon the addendum circle diameter ($d_a$) of the gear. Cast gears with crossed spokes (spoked gears, see Figure 10.9) are used when the gears are $d_a >$ 400mm in diameter. When 160mm $< d_a <=$ 400mm, forged gears cored with round holes (webbed gears, see Figure 10.10) are selected. If the diameter of the pinion dedendum circle is large enough for the pinion to be secured directly to the shaft by any method and $d_a <=$ 160mm, the pinion is made solid and separate from the shaft (solid gear, see Figure 10.11). This is often the case with medium and high power transmissions at a low velocity ratio. If the diameter of the pinion dedendum circle differs but little from the shaft diameter or $e < 2m_t$ ($m_t$ is the transverse module, see Figure 10.12), the pinion is made integral with the shaft (integral pinion shaft, see Figure 10.13). This design is more advantageous because it reduces the amount of machining, does away with key or other joints, increases rigidity and accuracy of contact. Note that no graphical output on an integral pinion shaft is made until the shaft working drawing program is reached because of the lack of the data related to the shaft at the moment.

Gear working drawing produced by the program includes not only dimensioning and shading but also gear specifications, which are as follows:

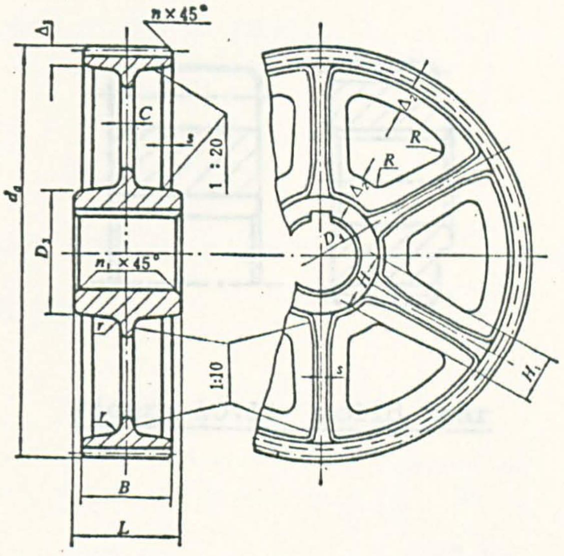



Figure 10.9 Spoked gear

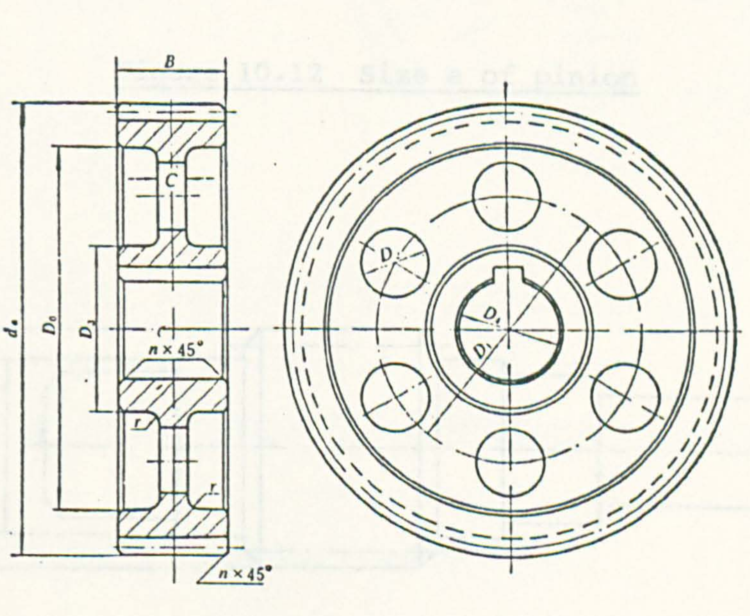



Figure 10.10 Webbed gear

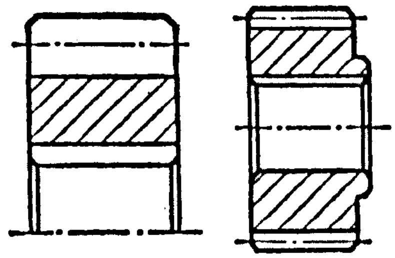


Figure 10.11 Solid gear

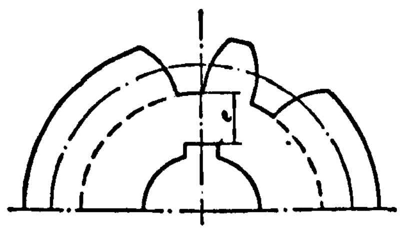



Figure 10.12 Size e of pinion

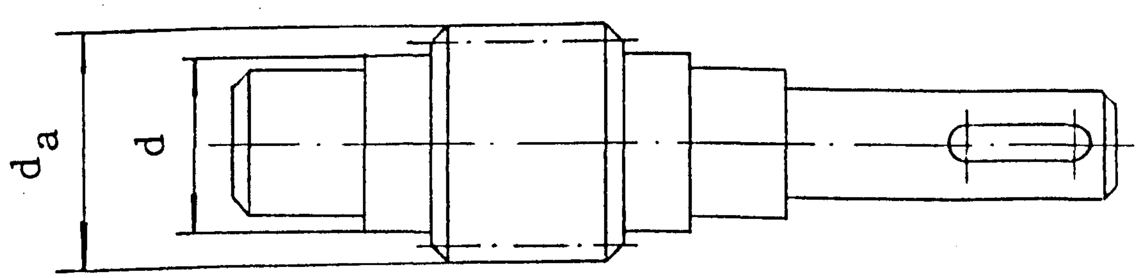



Figure 10.13 Integral pinion shaft

Number of Teeth

Normal Module

Normal Pressure Angle

Helix Angle (Left or Right Hand)

Whole Depth

Accuracy Grade.

Figure 10.14 shows an example of a gear working drawing drawn from the program GPLOT.

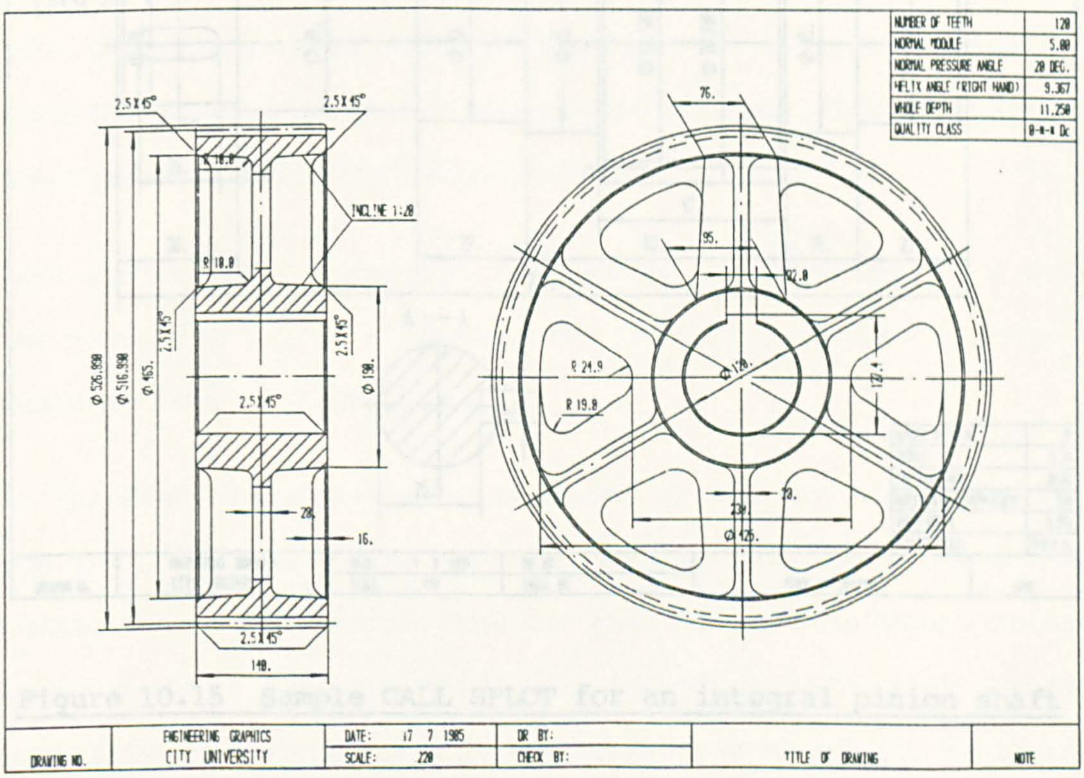



Figure 10.14 Sample CALL GPLOT for a spoked gear

### 10.3.3 Shaft Working Drawing Program

When the shaft working drawing program SPLOT is called, it will make relevant input shaft, intermediate shaft (for a double reduction gearbox) and output shaft working drawings in accordance with the data provided by the gearbox design programs. In addition, sectional views, dimensioning and shading are accomplished by the computer as well. If a shaft is made integral with a pinion, the pinion specifications are indicated in the shaft working drawing. A plotter output for this program is shown in Figure 10.15.

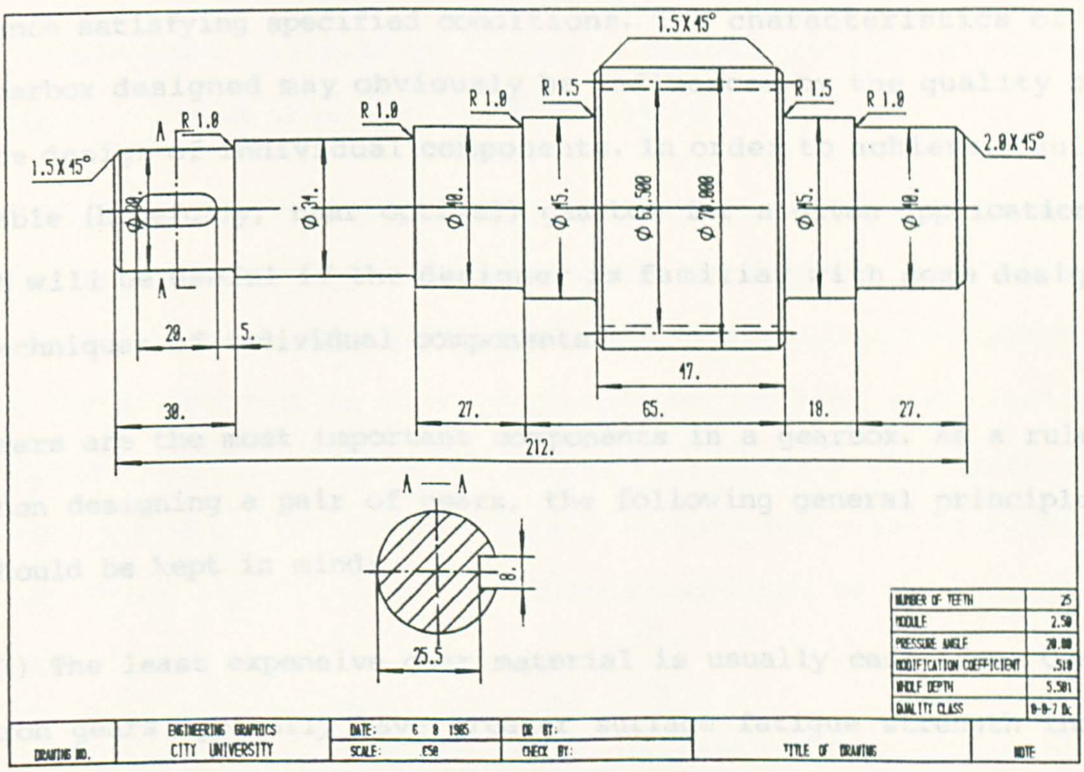



Figure 10.15 Sample CALL SPLOT for an integral pinion shaft

It is worth mentioning that in general, the hard copies for the graphical output of the gearbox to be designed should not be produced until the designer has examined the graphical output on the graphic terminal and has been satisfied with the results. If the results are not satisfactory, the designer should make appropriate modifications to the original design.

## Chapter 11

## DESIGN TECHNIQUES

The design of a complete gear transmission is a creative process, involving the use of scientific principles, technical information and imagination to produce the manufacturing instructions for the gear transmission, which will have a minimum cost and a performance satisfying specified conditions. The characteristics of a gearbox designed may obviously be influenced by the quality of the design of individual components. In order to achieve a suitable (hopefully, near optimal) gearbox for a given application, it will be useful if the designer is familiar with some design techniques of individual components.

Gears are the most important components in a gearbox. As a rule, when designing a pair of gears, the following general principles should be kept in mind:

(1) The least expensive gear material is usually cast iron. Cast iron gears typically have greater surface fatigue strength than bending fatigue strength. Nodular cast iron gears have substantially greater bending strength, together with good surface durability. A good combination is often a steel pinion mated to a cast iron gear under certain circumstances.

(2) Unheat-treated steel gears are relatively inexpensive, but have low surface endurance capacity. Heat-treated steel gears must be designed to resist warpage; hence alloy steels and oil quenching are usually preferred. Surface or case-hardened gears are usually processed by flame hardening, induction hardening, carburizing, or nitriding.

(3) Increasing the surface hardness of gears pays off handsomely in terms of surface fatigue strength (allowable Hertz stress); doubling the allowable Hertz stress quadruples the load capacity.

(4) Increases in surface hardness also increase bending fatigue strength, but the increase is far less. For example, the hardened case may effectively increase surface fatigue strength, yet be too shallow to contribute much to bending fatigue strength.

(5) Increasing tooth size (using a coaser pitch) increases bending strength more than surface strength.

(6) In general, the harder the gears the more costly they are to manufacture. On the other hand, harder gears can be smaller and still do the same job. And if the gears are smaller, the housing and other associated parts may also be smaller and lighter. Furthermore, if the gears are smaller, pitch-line velocities are lower, and this reduces the dynamic loading and rubbing velocities. Thus, overall cost can often be reduced by using harder gears.

(7) The size of the gears is usually a major factor controlling the size of the transmission. If minimum size gears are desired (for any given gear materials and application), it is best in general to start by choosing the minimum acceptable number of teeth for the pinion and then solving for the module required.

(8) With the same diameter of the gears found from the strength design of the teeth surfaces, a larger number of teeth improves the operating characteristics of the gear: losses due to friction are reduced (since they are inversely proportional to the number of teeth) and the contact ratio increases.

(9) Helical gears cost somewhat more than spur gears of the same accuracy. Therefore, it is pointless to take an excessively small inclination since greater production and control cost will not be compensated for by the advantages of their application — at a small helix angle the smoothness of contact does not noticeably increase. On the other hand, an excessive helix angle will cause a high axial force and will require larger proportions of the gear, larger bearings or additional thrust bearings. In practice, therefore, the helix angle is taken from $8^o$ to $20^o$.

(10) For a double reduction gearbox, the intermediate shaft will usually carry two gears, one a driver and the other a follower. If both are helical gears, the resultant axial thrust on the shaft is reduced if the gears are of the same hand, and increased if they are of opposite hand. If the teeth of the two gears have the same lead of helix, and the same hand, the resultant axial thrust is zero [51].

In brief, if it is found that a gear tooth will not be strong enough, the design may be altered as follows:

(a) Increase the module (this increases the tooth size)

(b) Increase the tooth width factor (this usually increases load-carrying capacity)

(c) Modify the selection of materials (or hardnesses).

By the same token, if an overdesigning happens, any or all of the previously mentioned factors may be changed.

Shafts are essential mechanical components and should be carefully designed. A few general principles that should be borne in mind for the overall shaft design are:

(1) Keep shafts as short as possible, with bearings close to the applied loads. This reduces the bending moment, and hence the deflection and bending stress.

(2) Place necessary stress raisers away from highly stressed shaft regions if possible. Otherwise, use generous radii and good surface finishes.

(3) Use inexpensive steels for deflection-critical shafts, as all the steels have essentially the same elastic modulus.

The design of a gearbox, or even a member of a gearbox, is not an exacting science. Usually for any gear train design there is no unique solution. Several designers, each given the same problem, would arrive at different solutions using alternative schemes. This is the reason there are various designs of a gear unit to perform the same function. The gearbox design package developed encourages the designer to seek alternative solutions on the basis of the principles described above. Using this technique, the design of a gearbox can be extensively studied and the best design can be selected from a wide range of designs very quickly. The thoroughness of the design and the large reduction in design time are the most important reasons for using this package.

An example of a single reduction gearbox design shown below is used to demonstrate the operation, design sequence and various conversational aspects of the package. The specification for the gearbox to be design is as follows:

| | |
|---|---|
| Input torque | = 295 N.m |
| Input speed | = 750 r/min |
| Nominal gear ratio | = 3.54 |

| | |
|---|---|
| Gear ratio tolerance | = 0.005 |
| Nominal centre distance | = 150 mm |
| Centre distance tolerance | = 0.001 mm |
| Working life | = 8 years |
| Working days | = 300 days/year |
| Duty | = 16 hours/day |
| Load characteristic on power source | = uniform |
| Load characteristic on driven machine | = uniform |
| Minimum demanded safety factor | |
| for contact stress | = 1.0 |
| for bending stress | = 1.25 |

The output from the interactive session of this design problem is completely presented in Appendix E. Figure E.1 shows the printed output for gear pair design, input and output shaft design, and bearing selection. The graphical output for the gearbox schematic, integral pinion shaft working drawing, gear working drawing, and output shaft working drawing is shown in Figure E.2, E.3, E.4 and E.5 respectively.

# Chapter 12

## CONCLUSIONS

The main point of this thesis is that computer-aided design and computer-aided draughting techniques can greatly contribute to the gearbox design process. The advantages of the interactive computer package developed for gearbox design have been demonstrated. It has been found that the user requires little computing knowledge and the simple program structure is easy to grasp. This enables the designer to make an effective use of computers. The interactive features of the package enable a designer to iterate to a near optimum design solution much faster. The direct flow of data within the design and draughting processes enables the designer to produce large reductions in times and increases the reliability of information transfer. Therefore the objectives of better designs have been achieved.

### 12.1 Future Development

This computer-aided design and draughting package is mainly used to design industrial gearboxes for transmitting power between shafts having parallel axes. To increase the flexibility of this package, considerable program developments remain to be done. They may be summarized as follows:

(1) The shaft design in the package is based on the strength requirements. However, the deflection of a machine shaft is, in many cases, more important than its strength. In most cases a shaft that is rigid enough is also strong enough, therefore a machine shaft should be designed for stiffness. This situation

should be taken into account for the further development of the package.

(2) Help facilities need to be added such as guidance on the type of gearing required, tooth numbers to choose or avoid, accuracy grades, materials, etc. that are particularly useful to the designer with little experience and knowledge on gearbox design.

(3) The draughting programs of the package should be able to operate in an interactive mode and their functions must be extended so that the computer-aided draughting techniques may be well applied to the gearing design.

(4) The possibility of computer produced geometric tolerances, specifications for inspection and material on working drawings is attractive. The problems associated with providing an acceptable user aid in performing this task has been investigated. It has been concluded that considerably more effort would be required to produce such an effective system.

(5) The facility to design the gearbox case and other commonly used components such as shaft seal and end cap would be essential when attempting to fulfil the overall gearbox design.

Gearbox design is but one aspect of mechanical engineering design that can be assisted and speeded up by computer techniques. It cannot be questioned but that computer techniques will play an important part in the future of mechanical engineering design.

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# Appendices

## Appendix A

## AN EXAMPLE OF THE USE OF THE PROGRAM GCHECK

The following pair of single helical gears are to be rated against the ISO gear standard to find the actual safety factors.

### Operating conditions

| | | | |
|---|---|---|---|
| Power rating: | 43kW | Pinion speed: | 1500rpm |
| Working life: | 15years | Working days: | 300days/year |
| Duty: | 16hours/day | Load characteristic: | uniform |
| Bearing span: | 214mm | Pinion offset distance: | 49.5mm |

Lubricant viscosity at $50^{\circ}C$: 100cSt

Minimum demanded safety factor for contact stress: 1.0

Minimum demanded safety factor for bending stress: 1.25

### Gear details

| | **Pinion** | **Gear** |
|---|---|---|
| Normal module: | 2.75mm | 2.75mm |
| Number of teeth: | 19 | 99 |
| Nominal pressure angle: | $20^{\circ}$ | $20^{\circ}$ |
| Helix angle: | $14^{\circ}$ | $14^{\circ}$ |
| Addendum modification coefficient: | 0.425 | 0.2471 |
| Facewidth: | 68mm | 68mm |
| Construction: | solid | solid |
| Material: | 40CrMnMo | 40CrMnMo |
| Heat treatment: | both case-hardened | |
| Surface hardness: | 572HB | 572HB |
| Manufacturing accuracy: | 6 | 6 |
| Surface roughness grade: | 7 | 7 |
| Fillet roughness grade: | 5 | 5 |

**Basic rack and hob details**

Addendum coefficient: 1.0 Clearance coefficient: 0.25

Fillet radius coefficient: 0.38

Hob protuberance coefficient: 0.05 Angle of protuberance: $6^{\circ}$

From the given conditions above the following checking results are obtained by using the program GCHECK.

| | PINION | WHEEL |
|---|---|---|
| NORMAL MODULE(mm).................... | 2.7500 | 2.7500 |
| TRANSVERSE MODULE(mm)................ | 2.8342 | 2.8342 |
| NUMBER OF TEETH...................... | 19.0000 | 99.0000 |
| VIRTUAL NUMBER OF TEETH.............. | 20.6488 | 107.5910 |
| NORMAL PRESSURE ANGLE(deg.).......... | 20.0000 | 20.0000 |
| TRANSVERSE PRESSURE ANGLE(deg.)...... | 20.5617 | 20.5617 |
| HELIX ANGLE(deg.).................... | 14.0000 | 14.0000 |
| BASE HELIX ANGLE(deg.)............... | 13.1401 | 13.1401 |
| ADDENDUM MODIFICATION COEFFICIENT.... | .4250 | .2471 |
| REFERENCE DIAMETER(mm)............... | 53.8496 | 280.5846 |
| PITCH DIAMETER(mm)................... | 54.4237 | 283.5764 |
| TIP DIAMETER(mm)..................... | 61.5565 | 287.3131 |
| ROOT DIAMETER(mm).................... | 49.3121 | 275.0686 |
| BASE DIAMETER(mm).................... | 50.4190 | 262.7098 |
| ADDENDUM(mm)......................... | 3.8535 | 3.3643 |
| TOOTH DEPTH(mm)...................... | 6.1222 | 6.1222 |
| FACEWIDTH(mm)........................ | 68.0000 | 68.0000 |
| TRANSVERSE P.A. AT THE HIGHEST POINT OF SINGLE TOOTH PAIR CONTACT(deg.) | 28.6984 | 22.4624 |
| SPAN MEASUREMENT: | | |
| TOOTH NUMBER OF SPAN MEASUREMENT.. | 4.0000 | 13.0000 |
| BASE TANGENT LENGTH(mm)........... | 30.0113 | 106.0999 |
| CONSTANT CHORD MEASUREMENT: | | |
| CONSTANT CHORDAL THICKNESS(mm).... | 4.5656 | 4.2512 |
| CONSTANT CHORDAL HEIGHT(mm)....... | 3.0226 | 2.5906 |
| PIN MEASUREMENT: | | |
| PIN DIAMETER(mm).................. | 4.6200 | 4.6200 |
| DIAMETER OVER PINS(mm)............ | 61.3632 | 287.7332 |

| | |
|---|---|
| WORKING TRANSVERSE CONTACT ANGLE(deg.)........... | 22.1171 |
| CENTRE DISTANCE(mm).............................. | 169.0001 |
| TRANSVERSE CONTACT RATIO......................... | 1.4626 |
| OVERLAP RATIO.................................... | 1.9042 |
| TOTAL CONTACT RATIO.............................. | 3.3667 |

Figure A.1 Typical application of the program GCHECK

| | PINION | WHEEL |
|---|---|---|
| BASIC ENDURANCE LIMIT($N/mm^2$): | | |
| FOR CONTACT STRESS............. | 1300.0000 | 1300.0000 |
| FOR BENDING STRESS............. | 310.0000 | 310.0000 |
| ACTUAL SAFETY FACTOR: | | |
| FOR CONTACT STRESS............. | 1.1264 | 1.1264 |
| FOR BENDING STRESS............. | 1.6842 | 1.5412 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Ft = | 10166.770N | Fr = | 3813.684N | Fa = | 2534.860N | | |
| l = | 214.00mm | s = | 49.50mm | V = | 4.2293m/s | | |
| V50 = | 100.0cSt | Fβ = | 12.00μm | | | | |
| Ff1 = | 8.00μm | Ff2 = | 9.00μm | Fpb1= | 9.00μm | Fpb2= | 10.00μm |
| SRG1= | 7 | SRG2= | 7 | FRG1= | 5 | FRG2= | 5 |
| Rz1 = | 6.30μm | Rz2 = | 6.30μm | Rzf1= | 20.00μm | Rzf2= | 20.00μm |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| KA = | 1.0000 | KV = | 1.0373 | KHβ = | 2.1500 | KFβ = | 2.0078 |
| KHα = | 1.2062 | KFα = | 1.2062 | ZH = | 2.3381 | ZE = | 189.8684 |
| Zε = | .8269 | Zβ = | .9850 | Yε = | .7628 | Yβ = | .8833 |
| ZX = | 1.0000 | YST = | 2.0000 | | | | |
| ZN1 = | 1.0000 | ZN2 = | 1.0000 | ZL1 = | 1.0000 | ZL2 = | 1.0000 |
| ZV1 = | .9784 | ZV2 = | .9784 | ZR1 = | .9554 | ZR2 = | .9554 |
| ZW1 = | 1.0000 | ZW2 = | 1.0000 | | | | |
| YF1 = | 1.3833 | YF2 = | 1.5267 | YS1 = | 2.1073 | YS2 = | 2.0912 |
| YFα1= | 2.4047 | YFα2= | 2.2833 | YSα1= | 1.7300 | YSα2= | 1.8258 |
| YNT1= | 1.0000 | YNT2= | 1.0000 | Yδr1= | .9985 | Yδr2= | 1.0008 |
| YRr1= | .9567 | YRr2= | .9567 | YX1 = | 1.0000 | YX2 = | 1.0000 |

Figure A.1, continued

Having studied the above information we find that the values of the face load distribution factors $K_{H\beta}$ and $K_{F\beta}$ under the given conditions are greater than 2, which means that only a part of the facewidth takes the entire load [39]. Therefore, the original design is not an ideal one. To improve the design quality, we should reduce the values of the face load distribution factors. This, in general, may be achieved by revising the design parameters, changing the arrangement configuration, increasing the manufacturing accuracy, choosing suitable amounts of tooth reliefs and corrections, etc. Here we shall modify the facewidth of the gear pair for demonstration purposes.

The original value of the facewidth **b** is greater than the upper limit of the range ($3\pi m_n <= b <= 5\pi m_n$) recommended by Shigley [70]. If we decrease the facewidth from 68mm to 36mm (= $4.17\pi m_n$), the following results may be achieved.

| | PINION | WHEEL |
|---|---|---|
| NORMAL MODULE(mm)...................... | 2.7500 | 2.7500 |
| TRANSVERSE MODULE(mm).................. | 2.8342 | 2.8342 |
| NUMBER OF TEETH........................ | 19.0000 | 99.0000 |
| VIRTUAL NUMBER OF TEETH................ | 20.6488 | 107.5910 |
| NORMAL PRESSURE ANGLE(deg.)............ | 20.0000 | 20.0000 |
| TRANSVERSE PRESSURE ANGLE(deg.)........ | 20.5617 | 20.5617 |
| HELIX ANGLE(deg.)...................... | 14.0000 | 14.0000 |
| BASE HELIX ANGLE(deg.)................. | 13.1401 | 13.1401 |
| ADDENDUM MODIFICATION COEFFICIENT.... | .4250 | .2471 |
| REFERENCE DIAMETER(mm)................. | 53.8496 | 280.5846 |
| PITCH DIAMETER(mm)..................... | 54.4237 | 283.5764 |
| TIP DIAMETER(mm)....................... | 61.5565 | 287.3131 |
| ROOT DIAMETER(mm)...................... | 49.3121 | 275.0686 |
| BASE DIAMETER(mm)...................... | 50.4190 | 262.7098 |
| ADDENDUM(mm)........................... | 3.8535 | 3.3643 |
| TOOTH DEPTH(mm)........................ | 6.1222 | 6.1222 |
| FACEWIDTH(mm).......................... | 36.0000 | 36.0000 |
| TRANSVERSE P.A. AT THE HIGHEST POINT OF SINGLE TOOTH PAIR CONTACT(deg.) | 28.6984 | 22.4624 |
| SPAN MEASUREMENT: | | |
| TOOTH NUMBER OF SPAN MEASUREMENT.. | 4.0000 | 13.0000 |
| BASE TANGENT LENGTH(mm)........... | 30.0113 | 106.0999 |
| CONSTANT CHORD MEASUREMENT: | | |
| CONSTANT CHORDAL THICKNESS(mm).... | 4.5656 | 4.2512 |
| CONSTANT CHORDAL HEIGHT(mm)....... | 3.0226 | 2.5906 |
| PIN MEASUREMENT: | | |
| PIN DIAMETER(mm).................. | 4.6200 | 4.6200 |
| DIAMETER OVER PINS(mm)............ | 61.3632 | 287.7332 |

| | |
|---|---|
| WORKING TRANSVERSE CONTACT ANGLE(deg.)........... | 22.1171 |
| CENTRE DISTANCE(mm).............................. | 169.0001 |
| TRANSVERSE CONTACT RATIO......................... | 1.4626 |
| OVERLAP RATIO.................................... | 1.0081 |
| TOTAL CONTACT RATIO.............................. | 2.4707 |

Figure A.1, continued

| | PINION | WHEEL |
|---|---|---|
| BASIC ENDURANCE LIMIT(N/mm²): | | |
| FOR CONTACT STRESS............. | 1300.0000 | 1300.0000 |
| FOR BENDING STRESS............. | 310.0000 | 310.0000 |
| ACTUAL SAFETY FACTOR: | | |
| FOR CONTACT STRESS............. | 1.0227 | 1.0227 |
| FOR BENDING STRESS............. | 1.3881 | 1.2703 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Ft = | 10166.770N | Fr = | 3813.684N | Fa = | 2534.860N | | |
| l = | 214.00mm | s = | 49.50mm | V = | 4.2293m/s | | |
| V50 = | 100.0cSt | Fβ = | 9.00μm | | | | |
| Ff1 = | 8.00μm | Ff2 = | 9.00μm | Fpb1= | 9.00μm | Fpb2= | 10.00μm |
| SRG1= | 7 | SRG2= | 7 | FRG1= | 5 | FRG2= | 5 |
| Rz1 = | 6.30μm | Rz2 = | 6.30μm | Rzf1= | 20.00μm | Rzf2= | 20.00μm |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| KA = | 1.0000 | KV = | 1.0273 | KHβ = | 1.5087 | KFβ = | 1.4092 |
| KHα = | 1.1146 | KFα = | 1.1146 | ZH = | 2.3381 | ZE = | 189.8684 |
| Zε = | .8269 | Zβ = | .9850 | Yε = | .7628 | Yβ = | .8833 |
| ZX = | 1.0000 | YST = | 2.0000 | | | | |
| ZN1 = | 1.0000 | ZN2 = | 1.0000 | ZL1 = | 1.0000 | ZL2 = | 1.0000 |
| ZV1 = | .9784 | ZV2 = | .9784 | ZR1 = | .9554 | ZR2 = | .9554 |
| ZW1 = | 1.0000 | ZW2 = | 1.0000 | | | | |
| YF1 = | 1.3833 | YF2 = | 1.5267 | YS1 = | 2.1073 | YS2 = | 2.0912 |
| YFα1= | 2.4047 | YFα2= | 2.2833 | YSα1= | 1.7300 | YSα2= | 1.8258 |
| YNT1= | 1.0000 | YNT2= | 1.0000 | Yδr1= | .9985 | Yδr2= | 1.0008 |
| YRr1= | .9567 | YRr2= | .9567 | YX1 = | 1.0000 | YX2 = | 1.0000 |

Figure A.1, continued

Now not only the face load distribution factors are kept less than 2 but the actual safety factors are still satisfactory. So the revised design is much better than the original one. This example also shows that increasing the power capacity of the gear pair by increasing the facewidth blindly will not work.

## Appendix B

## A TYPICAL DATA FILE FOR LUSAS

```
PROB TITLE BENDING STRESS ANALYSIS OF GEAR TOOTH
            UNIT N MM
            OPTION 30
            QPM8 ELEM TOPO
            FIRST 1 3 2 1 18 35 36 37 20
            INC 1 2 2 2 2 2 2 2 2 8
            INC 8 34 34 34 34 34 34 34 34 20
            NODE COOR
            1 -1.7038 49.971
            17 1.7038 49.971
            35 -2.5165 48.4677
            51 2.5165 48.4677
            69 -3.1966 46.9645
            85 3.1966 46.9645
            103 -3.7385 45.4613
            119 3.7385 45.4613
            137 -4.0461 44.3317
            153 4.0461 44.3317
            171 -4.253 43.2022
            187 4.253 43.2022
            205 -4.3135 42.0727
            221 4.3135 42.0727
            239 -4.2925 41.2628
            255 4.2925 41.2628
            273 -4.3522 40.6418
            289 4.3522 40.6418
            307 -4.4605 40.1529
            323 4.4605 40.1529
            341 -4.6009 39.7558
            357 4.6009 39.7558
            375 -4.7657 39.4245
            391 4.7657 39.4245
            409 -4.9329 39.1682
            425 4.9329 39.1682
            443 -5.1154 38.9462
            459 5.1154 38.9462
            477 -5.3119 38.7547
            493 5.3119 38.7547
            511 -5.5211 38.5914
            527 5.5211 38.5914
            545 -5.7418 38.4554
            561 5.7418 38.4554
            579 -5.9722 38.3462
            595 5.9722 38.3462
            613 -6.2108 38.2638
            629 6.2108 38.2638
            647 -6.4558 38.2084
            663 6.4558 38.2084
```

Figure B.1 Typical data file for gear tooth stress analysis using LUSAS

```
681 -6.7289 38.1613
697 6.7289 38.1613
689 0.0 31.25
SPACING
FIRST 1 35 17 2*1
INC 34 34 (20)
SPACING
FIRST 17 51 17 2*1
INC 34 34 (20)
SPACING
FIRST 1 17 1 16*1
INC 17 17 (17)
SPACING
281 689 17 24*1
SPACING
FIRST 290 298 1 8*1
INC 17 17 (24)
SPACING
FIRST 298 306 1 8*1
INC 17 17 (24)
QPM8 GEOM PROP
1 160 1 1 1 1 1 1 1 1 1
MATE PROP
1 160 1 2.06E5 0.3
SUPP NODE
681 689 1 R R
689 697 1 R R
LOAD CASE 1
CONC LOAD
119 0 0 -382 -119
ELEM OUTPUT ASCENDING
8 160 8 1
PLOT FILE
END
```

<u>Figure B.1, continued</u>

# Appendix C

## AN EXAMPLE OF THE USE OF THE PROGRAM SDESIGN

The stepped shaft carrying a worm gear as shown in Figure C.1 is to be checked for strength and stiffness. The forces acting upon the tooth of the worm gear with a pitch diameter of 410mm are $F_{t2}$ = 2928N, $F_{r2}$ = 1066N, and $F_{a2}$ = 841N. The shaft is made from steel 45 with the tensile ultimate strength of 600MPa and the tensile yield strength of 300MPa.

Figure C.2 shows the printed output of the checking calculation process using the program SDESIGN.

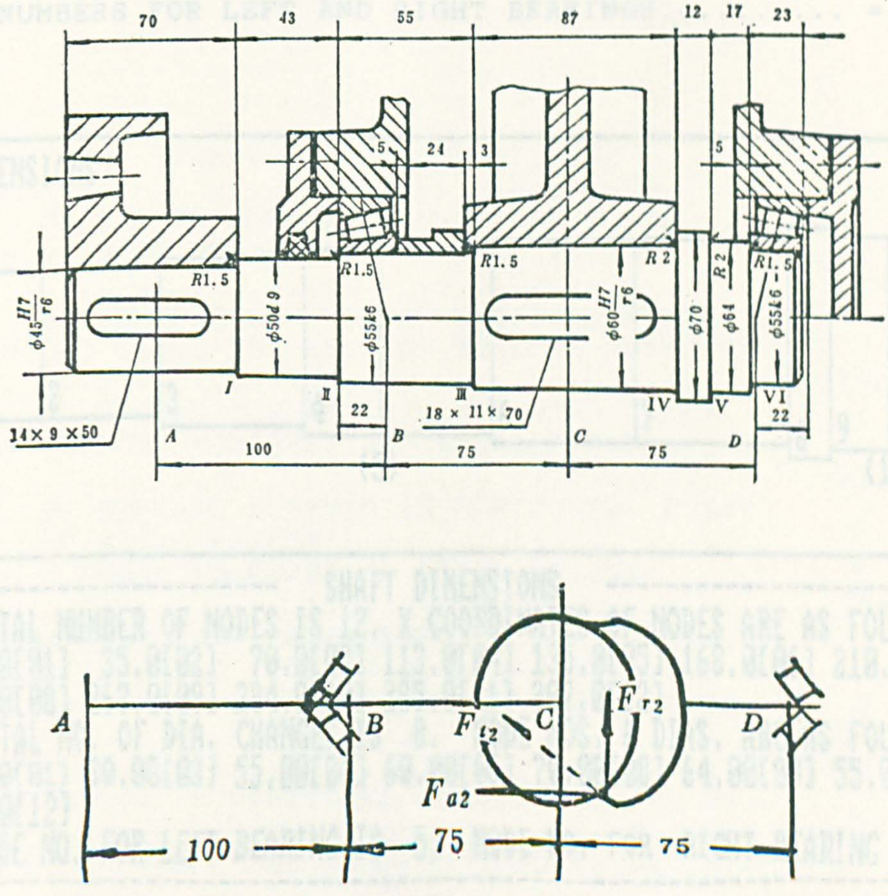
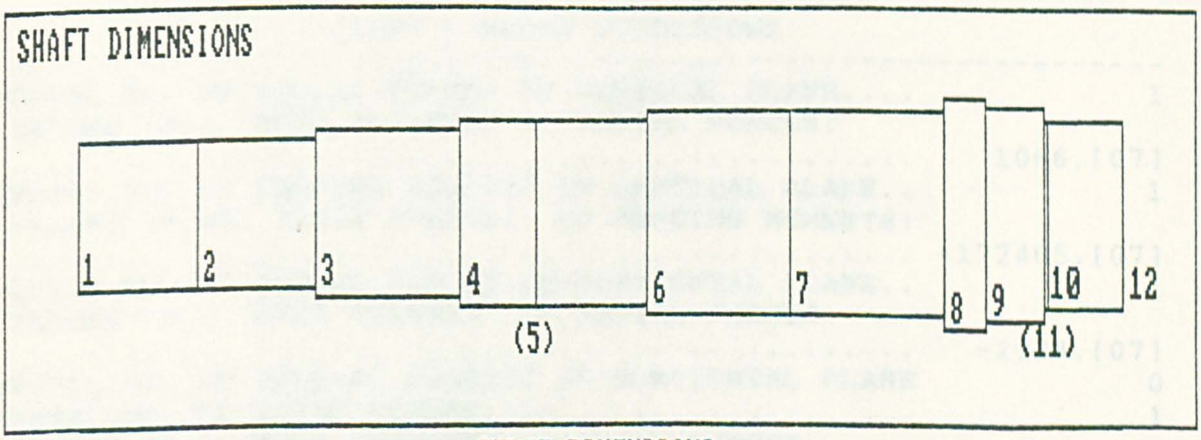



Figure C.1 Schematic of sample shaft checking

```
************  INPUT  SHAFT  DIMENSIONS  ************

TOTAL NUMBER OF NODES.................................... = 12
X COORDINATE (mm) FOR NODE NUMBER  2............... = 35
X COORDINATE (mm) FOR NODE NUMBER  3............... = 70
X COORDINATE (mm) FOR NODE NUMBER  4............... = 113
X COORDINATE (mm) FOR NODE NUMBER  5............... = 135
X COORDINATE (mm) FOR NODE NUMBER  6............... = 168
X COORDINATE (mm) FOR NODE NUMBER  7............... = 210
X COORDINATE (mm) FOR NODE NUMBER  8............... = 255
X COORDINATE (mm) FOR NODE NUMBER  9............... = 267
X COORDINATE (mm) FOR NODE NUMBER 10............... = 284
X COORDINATE (mm) FOR NODE NUMBER 11............... = 285
X COORDINATE (mm) FOR NODE NUMBER 12............... = 307

TOTAL NUMBER OF DIAMETER CHANGES....................... = 8
NODE NO. & DIAMETER(mm) FOR DIAMETER CHANGE NO.  1 = 1,45
NODE NO. & DIAMETER(mm) FOR DIAMETER CHANGE NO.  2 = 3,50
NODE NO. & DIAMETER(mm) FOR DIAMETER CHANGE NO.  3 = 4,55
NODE NO. & DIAMETER(mm) FOR DIAMETER CHANGE NO.  4 = 6,60
NODE NO. & DIAMETER(mm) FOR DIAMETER CHANGE NO.  5 = 8,70
NODE NO. & DIAMETER(mm) FOR DIAMETER CHANGE NO.  6 = 9,64
NODE NO. & DIAMETER(mm) FOR DIAMETER CHANGE NO.  7 = 10,55

NODE NUMBERS FOR LEFT AND RIGHT BEARINGS.......... = 5,11
```




```
------------------------  SHAFT DIMENSIONS  ------------------------
(1)TOTAL NUMBER OF NODES IS 12. X COORDINATES OF NODES ARE AS FOLLOWS:
 .0[01]  35.0[02]  70.0[03] 113.0[04] 135.0[05] 168.0[06] 210.0[07]
 255.0[08] 267.0[09] 284.0[10] 285.0[11] 307.0[12]
(2)TOTAL NO. OF DIA. CHANGES IS  8.  NODE NOS. & DIAS. ARE AS FOLLOWS:
 45.00[01] 50.00[03] 55.00[04] 60.00[06] 70.00[08] 64.00[09] 55.00[10]
 .00[12]
(3)NODE NO. FOR LEFT BEARING IS  5,  NODE NO. FOR  RIGHT BEARING IS 11
----------------------------------------------------------------------
          DO YOU WANT TO CHANGE THE DIMENSIONS? (Y/N)   = N
```

Figure C.2  Typical application of the program SDESIGN

```
*********  INPUT SHAFT LOADING CONDITIONS  *********

TOTAL NUMBER OF RADIAL FORCES IN VERTICAL PLANE... = 1
NODE NUMBER AND VALUE OF RADIAL FORCE (N.) NO. 1.. = 7,1066

TOTAL NUMBER OF BENDING MOMENTS IN VERTICAL PLANE. = 1
NODE NUMBER AND VALUE OF BENDING MOMENT(N.mm) NO.1 = 7,-172405

TOTAL NUMBER OF RADIAL FORCES IN HORIZONTAL PLANE. = 1
NODE NUMBER AND VALUE OF RADIAL FORCE (N.) NO. 1.. = 7,-2928

TOTAL NUMBER OF BENDING MOMENTS IN HORIZONTAL PLANE=

TOTAL NUMBER OF AXIAL FORCES...................... = 1
NODE NUMBER AND VALUE OF AXIAL FORCE (N.)  NO. 1.. = 7,-841
NODE NUMBER OF TAKING AXIAL FORCE................. = 5

TOTAL NUMBER OF APPLIED TORQUES................... = 2
NODE NUMBER AND VALUE OF APPLIED TORQUE(N.mm) NO.1 = 2,-600240
NODE NUMBER AND VALUE OF APPLIED TORQUE(N.mm) NO.2 = 7,600240
THE NATURE OF CHANGE IN TORSIONAL STRESS:
   CONSTANT (1), PULSATING (2), SYMMETRIC (3)..... = 2
```

```
                    SHAFT LOADING CONDITIONS
-----------------------------------------------------------------
(1)TOTAL NO. OF RADIAL FORCES IN VERTICAL PLANE....             1
   VALUES (N.) [NODE NUMBERS] OF RADIAL FORCES:
   ...........................................         1066.[07]
(2)TOTAL NO. OF BENDING MOMENTS IN VERTICAL PLANE..             1
   VALUES (N.mm) [NODE NUMBERS] OF BENDING MOMENTS:
   ...........................................    -172405.[07]
(3)TOTAL NO. OF RADIAL FORCES IN HORIZONTAL PLANE..             1
   VALUES (N.) [NODE NUMBERS] OF RADIAL FORCES:
   ...........................................        -2928.[07]
(4)TOTAL NO. OF BENDING MOMENTS IN HORIZONTAL PLANE             0
(5)TOTAL NO. OF AXIAL FORCES.......................             1
   VALUES (N.) [NODE NUMBERS] OF AXIAL FORCES:
   ...........................................         -841.[07]
   NODE NUMBER OF TAKING AXIAL FORCE...............             5
(6)TOTAL NO. OF APPLIED TORQUES....................             2
   VALUES (N.mm) [NODE NUMBERS] OF APPLIED TORQUES:
   ...........................................    -600240.[02]
   ...........................................     600240.[07]
   NATURE OF CHANGE IN TORSIONAL STRESS: (C, P, S).             2
-----------------------------------------------------------------
 DO YOU WANT TO CHANGE THE LOADING CONDITIONS?(Y/N) = N
```

Figure C.2, continued

```
******  INPUT INFORMATION ON SHAFT MATERIAL  *******

TYPE OF MATERIAL: CARBON STEEL(1), ALLOY STEEL(2). = 1

TENSILE ULTIMATE STRENGTH (MPa) (=0.335HBMPa)..... = 600

TENSILE YIELD STRENGTH (MPa) (DEFAULT=0.585σb).... = 300

TORSIONAL YIELD STRENGTH (MPa) (DEFAULT=0.585σs).. =

BENDING ENDURANCE LIMIT(MPa) (DEFAULT=0.27(σb+σs)) =

TORSIONAL ENDURANCE LIMIT(MPa)(DEFAULT=0.156(σb+σs)=

YOUNG'S MODULUS OF ELASTICITY(MPa)(DEFAULT=206000) =

SHEARING MODULUS OF ELASTICITY(MPa)(DEFAULT=81000) =


              INFORMATION ON SHAFT MATERIAL
-------------------------------------------------------------
(1)TYPE OF MATERIAL: CARBON (1), ALLOY STEEL (2)...         1
(2)TENSILE ULTIMATE STRENGTH (MPa)..................    600.0
(3)TENSILE YIELD STRENGTH (MPa).....................    300.0
(4)TORSIONAL YIELD STRENGTH (MPa)...................    175.5
(5)BENDING ENDURANCE LIMIT (MPa)....................    243.0
(6)TORSIONAL ENDURANCE LIMIT (MPa)..................    140.4
(7)YOUNG'S MODULUS OF ELASTICITY (MPa).............. 206000.0
(8)SHEARING MODULUS OF ELASTICITY (MPa).............  81000.0
-------------------------------------------------------------
 DO YOU WANT TO CHANGE THESE INFORMATION? (Y/N)   = N
```

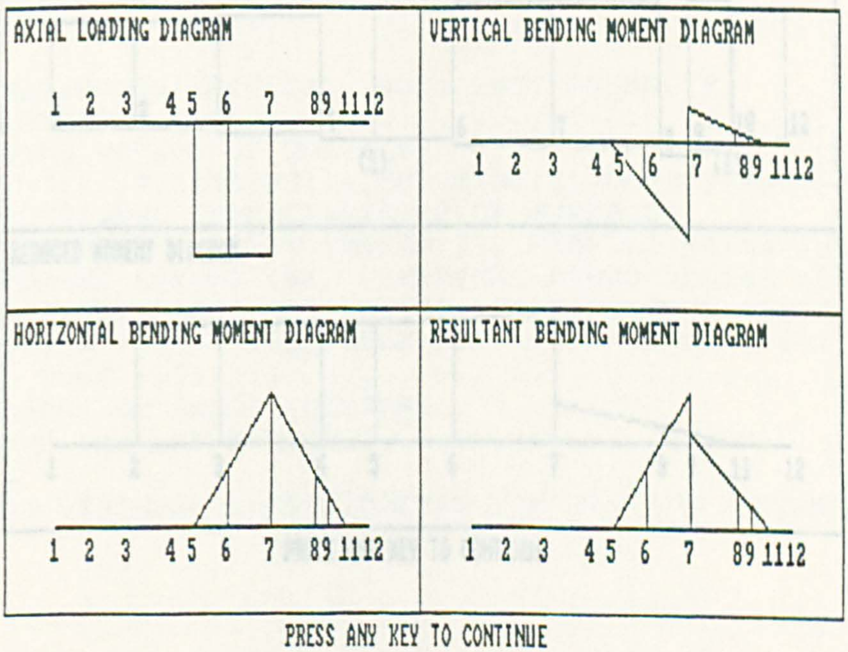
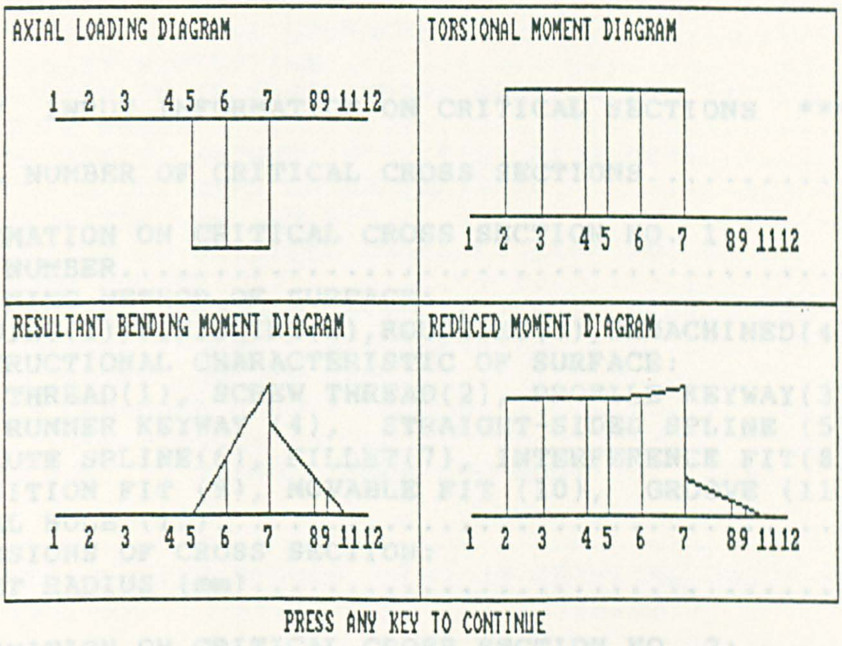



Figure C.2, continued



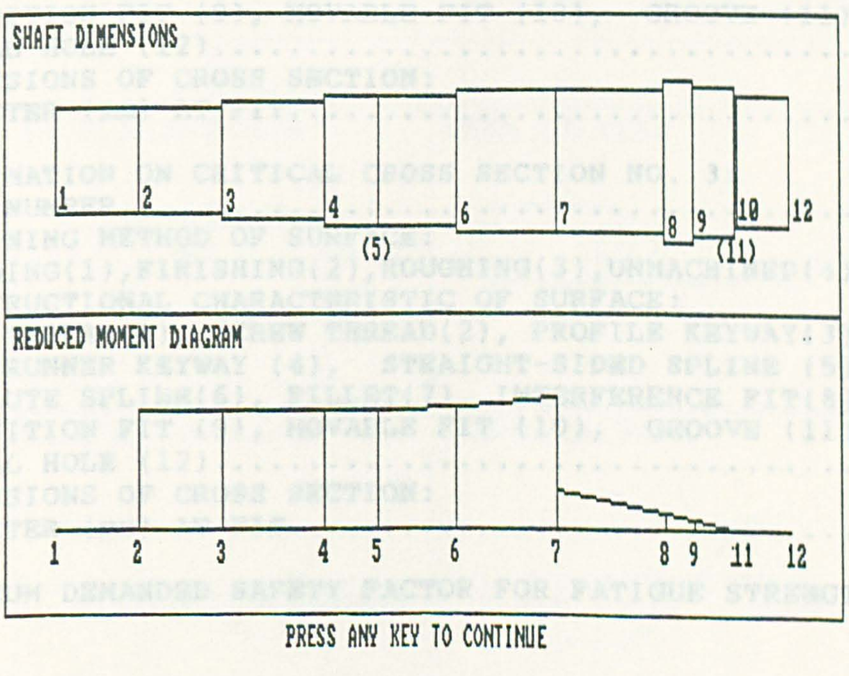




Figure C.2, continued

```
*****  INPUT INFORMATION ON CRITICAL SECTIONS  *****

TOTAL NUMBER OF CRITICAL CROSS SECTIONS........... = 3

INFORMATION ON CRITICAL CROSS SECTION NO. 1:
NODE NUMBER....................................... = 6
MACHINING METHOD OF SURFACE:
GRINDING(1),FINISHING(2),ROUGHING(3),UNMACHINED(4) = 1
CONSTRUCTIONAL CHARACTERISTIC OF SURFACE:
WORM THREAD(1), SCREW THREAD(2), PROFILE KEYWAY(3)
SLED RUNNER KEYWAY (4),  STRAIGHT-SIDED SPLINE (5)
INVOLUTE SPLINE(6), FILLET(7), INTERFERENCE FIT(8)
TRANSITION FIT (9), MOVABLE FIT (10),  GROOVE (11)
RADIAL HOLE (12).................................. = 7
DIMENSIONS OF CROSS SECTION:
FILLET RADIUS (mm)................................ = 1.5

INFORMATION ON CRITICAL CROSS SECTION NO. 2:
NODE NUMBER....................................... = 6
MACHINING METHOD OF SURFACE:
GRINDING(1),FINISHING(2),ROUGHING(3),UNMACHINED(4) = 1
CONSTRUCTIONAL CHARACTERISTIC OF SURFACE:
WORM THREAD(1), SCREW THREAD(2), PROFILE KEYWAY(3)
SLED RUNNER KEYWAY (4),  STRAIGHT-SIDED SPLINE (5)
INVOLUTE SPLINE(6), FILLET(7), INTERFERENCE FIT(8)
TRANSITION FIT (9), MOVABLE FIT (10),  GROOVE (11)
RADIAL HOLE (12).................................. = 8
DIMENSIONS OF CROSS SECTION:
DIAMETER (mm) AT FIT.............................. = 60

INFORMATION ON CRITICAL CROSS SECTION NO. 3:
NODE NUMBER....................................... = 7
MACHINING METHOD OF SURFACE:
GRINDING(1),FINISHING(2),ROUGHING(3),UNMACHINED(4) = 1
CONSTRUCTIONAL CHARACTERISTIC OF SURFACE:
WORM THREAD(1), SCREW THREAD(2), PROFILE KEYWAY(3)
SLED RUNNER KEYWAY (4),  STRAIGHT-SIDED SPLINE (5)
INVOLUTE SPLINE(6), FILLET(7), INTERFERENCE FIT(8)
TRANSITION FIT (9), MOVABLE FIT (10),  GROOVE (11)
RADIAL HOLE (12).................................. = 8
DIMENSIONS OF CROSS SECTION:
DIAMETER (mm) AT FIT.............................. = 60

MINIMUM DEMANDED SAFETY FACTOR FOR FATIGUE STRENGTH= 1.5
```

Figure C.2, continued

```
                   INFORMATION ON CRITICAL SECTIONS
-----------------------------------------------------------------
(1)TOTAL NUMBER OF CRITICAL CROSS SECTIONS..........          3
   INFORMATION ON CRITICAL CROSS SECTION NO. 1:
     [1]NODE NUMBER.................................          6
     [2]MACHINING METHOD: G(1),F(2),R(3),U(4).......          1
     [3]CONSTRUCTIONAL CHARACTERISTIC...............          7
     [4]FILLET RADIUS (mm)..........................      1.500
   INFORMATION ON CRITICAL CROSS SECTION NO. 2:
     [1]NODE NUMBER.................................          6
     [2]MACHINING METHOD: G(1),F(2),R(3),U(4).......          1
     [3]CONSTRUCTIONAL CHARACTERISTIC...............          8
     [4]DIAMETER (mm) AT FIT........................     60.000
   INFORMATION ON CRITICAL CROSS SECTION NO. 3:
     [1]NODE NUMBER.................................          7
     [2]MACHINING METHOD: G(1),F(2),R(3),U(4).......          1
     [3]CONSTRUCTIONAL CHARACTERISTIC...............          8
     [4]DIAMETER (mm) AT FIT........................     60.000
(2)MINIMUM DEMANDED SAFETY FACTOR...................      1.500
-----------------------------------------------------------------
   DO YOU WANT TO CHANGE THESE INFORMATION? (Y/N)   = N
```

```
 CALCULATED BENDING STRESS (MPa) AT CRITICAL CROSS SECTIONS
     22.145[06]      17.057[06]      18.469[07]

    CALCULATED SAFETY FACTORS AT CRITICAL CROSS SECTIONS
      7.029[06]       7.562[06]       6.234[07]

        FORCES ACTING ON LEFT BEARING & ITS BORE DIAMETER
R =     2230.169N       A =     -841.000N       d =    55.000mm

        FORCES ACTING ON RIGHT BEARING & ITS BORE DIAMETER
R =     1588.460N       A =         .000N       d =    55.000mm
```

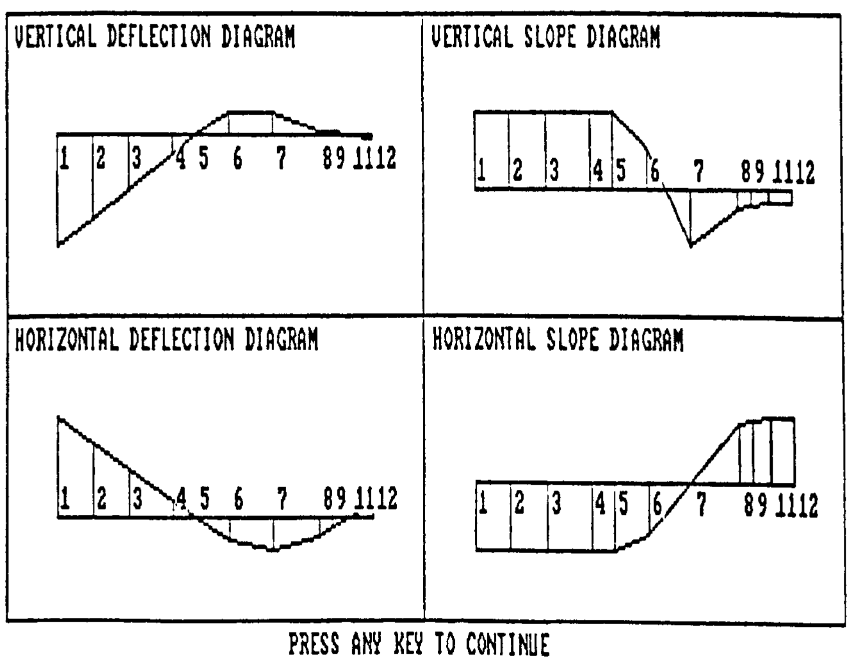
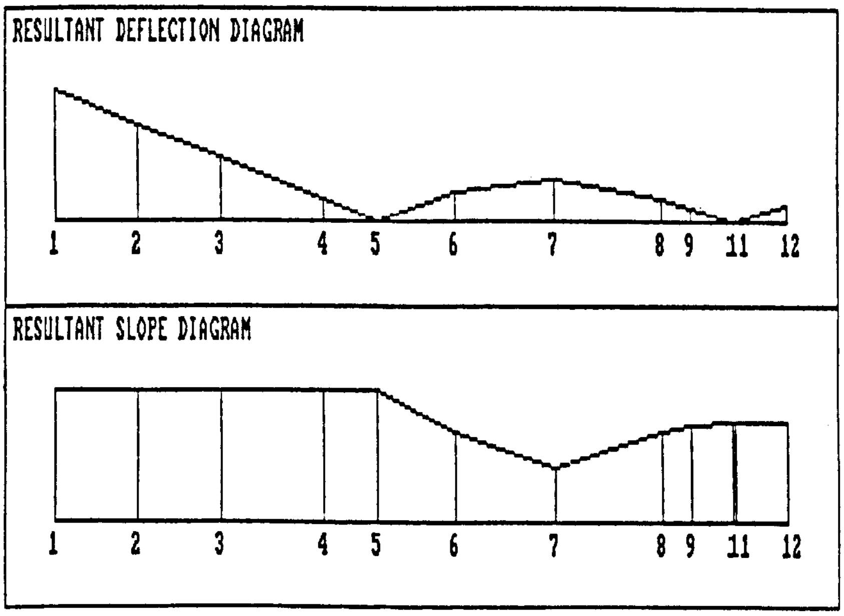



Figure C.2, continued




| | | | | |
|---|---|---|---|---|
| DEFLECTIONS (mm) AT ALL NODES ALONG THE SHAFT | | | | |
| .5408E-02[01] | .4006E-02[02] | .2604E-02[03] | .8813E-03[04] | .2328E-09[05] |
| .1183E-02[06] | .1693E-02[07] | .8824E-03[08] | .5409E-03[09] | .3048E-04[10] |
| .4657E-09[11] | .6707E-03[12] | | | |
| SLOPES (rad) AT ALL NODES ALONG THE SHAFT | | | | |
| .4006E-04[01] | .4006E-04[02] | .4006E-04[03] | .4006E-04[04] | .4006E-04[05] |
| .2761E-04[06] | .1677E-04[07] | .2762E-04[08] | .2919E-04[09] | .3048E-04[10] |
| .3048E-04[11] | .3048E-04[12] | | | |
| ANGLE OF TWIST PER UNIT LENGTH (deg/m) | | | | |
| .0000E+00[01] | .0000E+00[02] | .1055E+01[03] | .6920E+00[04] | .4726E+00[05] |
| .4726E+00[06] | .3337E+00[07] | .0000E+00[08] | .0000E+00[09] | .0000E+00[10] |
| .0000E+00[11] | .0000E+00[12] | | | |
| ANGLE OF TWIST (deg) | | | | |
| .0000E+00[01] | .0000E+00[02] | .3691E-01[03] | .2975E-01[04] | .1040E-01[05] |
| .1560E-01[06] | .1402E-01[07] | .0000E+00[08] | .0000E+00[09] | .0000E+00[10] |
| .0000E+00[11] | .0000E+00[12] | | | |

DO YOU WANT TO CHANGE THE GIVEN CONDITIONS? (Y/N)N

Figure C.2, continued

## Appendix D

## AN EXAMPLE OF THE USE OF THE PROGRAM BSELECT

A single row radial ball bearing is to be selected for the following working conditions:

radial bearing load = 5500 N    axial bearing load = 2700 N

load characteristic = light shock    bearing speed = 1250 r/min

working temperature < 120$^{o}$C    life expectancy = 5000 h

demanded reliability = 0.9

demanded safety factor for static strength = 1.0

The printed record of the bearing selection process is shown in Figure D.1.

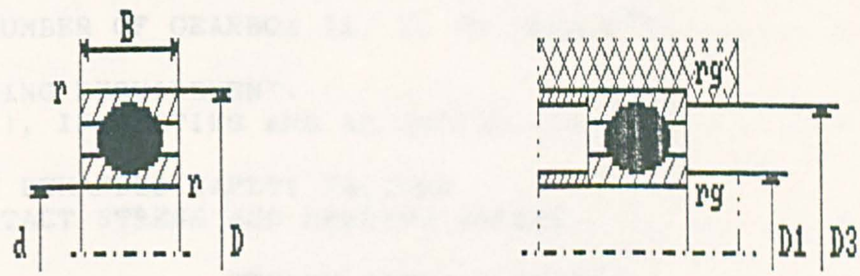
```
       ******  INPUT  BEARING  INFORMATION  ******

FOR  BEARING  TYPE:

BEARING TYPE                                              CODE NO.
SINGLE ROW RADIAL BALL BEARING.............................0
DOUBLE ROW SELF-ALIGNING BALL BEARING......................1
SINGLE OR DOUBLE ROW RADIAL SHORT CYLINDRICAL BEARING.....2
DOUBLE ROW RADIAL SELF-ALIGNING ROLLER BEARING............3
NEEDLE BEARING.............................................4
HELICAL ROLLER BEARING.....................................5
SINGLE ROW ANGULAR CONTACT BALL BEARING...................6
SINGLE ROW TAPER ROLLER BEARING...........................7
THRUST BALL BEARING........................................8
THRUST SPHERICAL ROLLER BEARING............................9

PLEASE INPUT THE CODE NO. OF THE SELECTED BEARING TYPE  = 0

REQUIRED BEARING BORE DIAMETER (mm). . . . . . . . . =

BEARING SPEED (r/min). . . . . . . . . . . . . . . . = 1250

LOADING CONDITIONS:
RADIAL FORCE (N.). . . . . . . . . . . . . . . . . . = 5500
AXIAL FORCE (N.) . . . . . . . . . . . . . . . . . . = 2700

DEMANDED SAFETY FACTOR FOR STATIC STRENGTH . . . . . = 1
```

Figure D.1  Typical application of the program BSELECT

```
LOAD FACTOR:   UNIFORM LOAD OR LIGHT SHOCK (1.0-1.2)
MODERATE SHOCK (1.2-1.8), HEAVY SHOCK (1.8-3.0). YOU
MAY EITHER CHOOSE THE VALUE FROM THE FOLLOWING TABLE
OR INPUT OTHER VALUE BY PRESSING THE RELEVANT NUMBER
-----------------------------------------------------
NO.  1    2    3    4    5    6    7    8    9    0
IS  1.0  1.1  1.2  1.5  1.8  2.1  2.4  2.7  3.0  OWN
-----------------------------------------------------
THE NUMBER YOU CHOOSE. . . . . . . . . . . . . . . . . = 3

WORKING TEMPERATURE (°c) (DEFAULT FOR <=120°c) . . =

LIFE EXPECTANCY (hours). . . . . . . . . . . . . . . . = 5000

DEMANDED RELIABILITY (DEFAULT FOR 0.9) . . . . . . . =

                  BEARING  INFORMATION
------------------------------------------------------------
(1)CODE NUMBER OF SELECTED BEARING TYPE..........          0
   REQUIRED BEARING BORE DIAMETER (mm)...........          0
(2)BEARING SPEED (r/min).........................   1250.000
(3)LOAD CONDITIONS:
   RADIAL FORCE (N)..............................   5500.000
   AXIAL FORCE (N)...............................   2700.000
(4)DEMANDED SAFETY FACTOR........................      1.000
(5)LOAD FACTOR: U(1.0-1.2),M(1.2-1.8),H(1.8-3.0).      1.200
(6)WORKING TEMPERATURE (°c)......................    120.000
(7)LIFE EXPECTANCY (hours).......................   5000.000
(8)DEMANDED RELIABILITY..........................       .900
------------------------------------------------------------
   DO YOU WANT TO CHANGE THE INFORMATION?  (Y/N)    = N


(01)
BEARING   CODE  NUMBER:          #          126
DYNAMIC RATING LOAD(N):        Cr =.69531E+05     C =.78064E+05
BEARING   LIFE  (hour):        Lr =.50000E+04     L =.70758E+04
STATIC RATING LOAD (N):       C0r =.55000E+04    C0 =.73062E+05
MAIN   DIAMETERS  (MM):         d =    130.000    D =    200.000
WIDTH(MM) & WEIGHT(Kg):         B =     33.000    W =      3.700

DO YOU WANT TO VIEW THE ENTIRE INFORMATION ON THIS BEARING? (Y/N)N

(02)
BEARING   CODE  NUMBER:          #          218
DYNAMIC RATING LOAD(N):        Cr =.67583E+05     C =.73847E+05
BEARING   LIFE  (hour):        Lr =.50000E+04     L =.65231E+04
STATIC RATING LOAD (N):       C0r =.55000E+04    C0 =.60509E+05
MAIN   DIAMETERS  (MM):         d =     90.000    D =    160.000
WIDTH(MM) & WEIGHT(Kg):         B =     30.000    W =      2.200

DO YOU WANT TO VIEW THE ENTIRE INFORMATION ON THIS BEARING? (Y/N)N
```

Figure D.1, continued

```
(03)
BEARING    CODE   NUMBER:          #         313
DYNAMIC RATING LOAD(N):        Cr =.66741E+05      C =.71199E+05
BEARING    LIFE   (hour):      Lr =.50000E+04      L =.60703E+04
STATIC RATING LOAD (N):       C0r =.55000E+04     C0 =.55606E+05
MAIN    DIAMETERS   (MM):       d =    65.000      D =    140.000
WIDTH(MM) & WEIGHT(Kg):         B =    33.000      W =      2.090

DO YOU WANT TO VIEW THE ENTIRE INFORMATION ON THIS BEARING? (Y/N)N

(04)
BEARING    CODE   NUMBER:          #         410
DYNAMIC RATING LOAD(N):        Cr =.66689E+05      C =.70414E+05
BEARING    LIFE   (hour):      Lr =.50000E+04      L =.58855E+04
STATIC RATING LOAD (N):       C0r =.55000E+04     C0 =.55311E+05
MAIN    DIAMETERS   (MM):       d =    50.000      D =    130.000
WIDTH(MM) & WEIGHT(Kg):         B =    31.000      W =      1.910

DO YOU WANT TO VIEW THE ENTIRE INFORMATION ON THIS BEARING? (Y/N)Y
```




| Brg Designation | Brg Dimensions (mm) | | | | Shouder Dimensions (mm) | | |
|---|---|---|---|---|---|---|---|
| # | d | D | B | r | D1 | D3 | rg |
| 410 | 50 | 130.00 | 31.00 | 3.50 | 66.00 | 116.00 | 2.00 |

| Basic Dyn. Cap. | Basic Sta. Cap. | Limit Speeds for Diff.Lubri. Methods | | Weight |
|---|---|---|---|---|
| C (kg) | C0 (kg) | n (rpm) (Grease) | n (rpm) (Oil) | W (kg) |
| 7180.0 | 5640.0 | 5300.0 | 7600.0 | 1.91 |

PRESS ANY KEY TO CONTINUE

Figure D.1, continued

## Appendix E

## COMPLETE RECORD OF A SINGLE-REDUCTION GEARING DESIGN PROCESS

```
      *****  INPUT  DESIGN  SPECIFICATIONS  *****

GEARBOX INPUT TORQUE(N.m)......................... = 295

GEARBOX INPUT SPEED (r/min)....................... = 750

NOMINAL TRANSMISSION RATIO........................ = 3.54

TRANSMISSION RATIO TOLERANCE...................... = 0.005

WORKING LIFE(yr), WORKING DAYS(dy/yr), DUTY(hr/dy) = 8,300,16

ROTATING DIRECTION OF INPUT SHAFT:
CLOCKWISE (1), COUNT-CLOCKWISE (2), REVERSIBLE (3) = 1

LOAD CHARACTERISTIC OF POWER SOURCE:
UNIFORM (1), LIGHT SHOCK (2), MEDIUM SHOCK (3).... = 1

LOAD CHARACTERISTIC OF DRIVEN MACHINE:
UNIFORM (1), MODERATE SHOCK (2), HEAVY SHOCK (3).. = 1

STAGE NUMBER OF GEARBOX (1, 2, OR DEFAULT)........ = 1

ASSEMBLING REQUIREMENT:
NONE (1), INSPECTING AND ADJUSTING (2)............ = 2

MINIMUM DEMANDED SAFETY FACTORS
FOR CONTACT STRESS AND BENDING STRESS............. = 1,1.25

                  DESIGN SPECIFICATIONS
-----------------------------------------------------------
(1)GEARBOX INPUT TORQUE(N.m)....................  295.0000
(2)GEARBOX INPUT SPEED(r/min)...................  750.0000
(3)NOMINAL TRANSMISSION RATIO...................    3.5400
   TOTAL TOLERANCE ON TRANSMISSION RATIO........     .0050
(4)WORKING LIFE(years)..........................    8.0000
   WORKING DAYS(days/year)......................  300.0000
   DUTY(hours/day)..............................   16.0000
(5)ROTATING DIRECTION OF INPUT SHAFT(CW,CCW,REV).        1
(6)LOAD CHARACTERISTIC OF POWER SOURCE(U,L,M)....        1
(7)LOAD CHARACTERISTIC OF DRIVEN MACHINE(U,M,H)..        1
(8)STAGE NUMBER OF GEARBOX.......................        1
(9)ASSEMBLING REQUIREMENT(NONE,INS & ADJ)........        2
(0)MINIMUM DEMANDED SAFETY FACTOR
   FOR CONTACT STRESS............................   1.0000
   FOR BENDING STRESS............................   1.2500
-----------------------------------------------------------
   DO YOU WANT TO CHANGE THE DESIGN SP.?  (Y/N)    = N
```

Figure E.1 Complete record of a single-reduction gearing design process

***** INPUT GIVEN CONDITIONS(1) *****

GEAR MATERIAL (AND HARDENING PROCESS)

```
CAST STEEL.............................................1
STRUCTURAL STEEL(NORMALIZING)..........................2
GREY CAST IRON.........................................3
BLACK HEART MALLEABLE CAST IRON........................4
SPHEROIDAL GRAPHITE CAST IRON..........................5
CARBON CAST STEEL......................................6
CARBON STEEL(HARDENING AND TEMPERING)..................7
ALLOY CAST STEEL.......................................8
ALLOY STEEL(HARDENING AND TEMPERING)...................9
QUENCHED AND TEMPERED STEEL(FLAME,INDUCTION HARDENING)...10
ALLOY STEEL(CASE HARDENING)............................11
QUENCHED AND TEMPERED STEEL(LIQUID-NITRIDING)..........12
QUENCHED AND TEMPERED STEEL(AIR-NITRIDING).............13
NITRIDED STEEL(AIR-NITRIDING)..........................14

TYPE THE CODE NO. OF PINION AND WHEEL MATERIAL.... = 10

MATERIAL QUALITY LEVEL: LOW(1), MEDIUM(2), HIGH(3) = 2

SURFACE HARDNESS (HB OR HV) OF PINION AND WHEEL... = 463

PERMISSIBLE PITTING: NONE(1), SOME(2)............. = 2

MANUFACTURING ACCURACY:
KINEMATIC, SMOOTHNESS, AND CONTACT ACCURACY....... = 7

CODE NO. OF GEAR LAYOUT (1 TO 5).................. = 1

TOOTH MODIFICATION: NONE(1),
HELIX CORRECTION(2), CROWNING(3), END RELIEF(4)... = 1
```

GIVEN CONDITIONS(1)

| | PINION | WHEEL |
|---|---|---|
| (1)GEAR MATERIAL..................... | 10 | 10 |
| MATERIAL QUALITY LEVEL(L,M,H)..... | 2 | 2 |
| SURFACE HARDNESS(HB OR HV)........ | 463.0000 | 463.0000 |
| (2)PERMISSIBLE PITTING(NONE,SOME).... | 2 | 2 |
| (3)MANUFACTURING ACCURACY............ | 7- 7- 7 | 7- 7- 7 |
| (4)CODE NO. OF GEAR LAYOUT..................... | | 1 |
| (5)TOOTH MODIFICATION(NONE,HELIX,CROWN,END)...... | | 1 |

DO YOU WANT TO CHANGE THE GIVEN COND.(1)? (Y/N) = N

Figure E.1, continued

```
          *****  INPUT  GIVEN  CONDITIONS(2)  *****

GEARING TYPE: EXTERNAL (1), INTERNAL (2).......... = 1

GEAR TYPE: SPUR(1), HELICAL(2), DOUBLE HELICAL(3). = 2

PARAMETERS  OF BASIC RACK TOOTH PROFILE:  <RETURN>
FOR  STANDARD  TOOTH  PROFILE,  ELSE  TYPE  NORMAL
PRESSURE  ANGLE  (deg.),   ADDENDUM   COEFFICIENT,
CLEARANCE  COEFFICIENT,  FILLET RADIUS COEFFICIENT =

PARAMETERS  OF  TOOL  PROTUBERANCE:  <RETURN>  FOR
WITHOUT  PROTUBERANCE,    ELSE  TYPE  PROTUBERANCE
COEFFICIENT   AND   ANGLE  OF  PROTUBERANCE (deg.) =

NUMBER OF TEETH ON PINION......................... = 26

HELIX ANGLE (deg.): DEFAULT <RETURN>.............. = 10

ADDENDUM MODIFICATION COEF. OF PINION AND WHEEL... =

FACEWIDTH FACTOR (FACEWIDTH/CD): DEFAULT <RETURN>. = .32

SPECIFIED CENTRE DISTANCE(mm): DEFAULT <RETURN>... = 150

                    GIVEN CONDITIONS(2)
-----------------------------------------------------------
(1)GEARING TYPE(EXTERNAL,INTERNAL)...............         1
(2)GEAR TYPE(SPUR,HELICAL,DOUBLE HELICAL)........         2
(3)PARAMETERS OF BASIC RACK TOOTH PROFILE:
   NORMAL PRESSURE ANGLE(deg.)...................   20.0000
   ADDENDUM COEFFICIENT..........................    1.0000
   CLEARANCE COEFFICIENT.........................     .2500
   FILLET RADIUS COEFFICIENT.....................     .3800
(4)PARAMETERS OF TOOL PROTUBERANCE:
   PROTUBERANCE COEFFICIENT......................     .0000
   ANGLE OF PROTUBERANCE(deg.)...................   20.0000
   ========================================================
                                    PINION        WHEEL
   --------------------------------------------------------
(5)NUMBER OF TEETH..................   26.0000
(6)HELIX ANGLE(deg.)................   10.0000      10.0000
(7)ADDENDUM MODIFICATION COEFFICIENT.     .0000        .0000
   --------------------------------------------------------
(8)FACEWIDTH FACTOR.................     .3200
   SPECIFIED CENTRE DISTANCE(mm).....  150.0000
-----------------------------------------------------------
   DO YOU WANT TO CHANGE THE GIVEN COND.(2)? (Y/N) = N
```

Figure E.1, continued

| | PINION | WHEEL |
|---|---|---|
| NORMAL MODULE(mm)...................... | 2.5000 | 2.5000 |
| TRANSVERSE MODULE(mm)................. | 2.5424 | 2.5424 |
| NUMBER OF TEETH....................... | 26.0000 | 92.0000 |
| VIRTUAL NUMBER OF TEETH............... | 27.2357 | 96.3723 |
| NORMAL PRESSURE ANGLE(deg.)........... | 20.0000 | 20.0000 |
| TRANSVERSE PRESSURE ANGLE(deg.)....... | 20.3115 | 20.3115 |
| HELIX ANGLE(deg.)..................... | 10.4753 | 10.4753 |
| BASE HELIX ANGLE(deg.)................ | 9.8371 | 9.8371 |
| ADDENDUM MODIFICATION COEFFICIENT.... | .0000 | .0000 |
| REFERENCE DIAMETER(mm)................ | 66.1017 | 233.8983 |
| PITCH DIAMETER(mm).................... | 66.1017 | 233.8983 |
| TIP DIAMETER(mm)...................... | 71.1017 | 238.8983 |
| ROOT DIAMETER(mm)..................... | 59.8517 | 227.6483 |
| BASE DIAMETER(mm)..................... | 61.9915 | 219.3544 |
| ADDENDUM(mm).......................... | 2.5000 | 2.5000 |
| TOOTH DEPTH(mm)....................... | 5.6250 | 5.6250 |
| FACEWIDTH(mm)......................... | 54.0000 | 48.0000 |

| | | |
|---|---|---|
| TRANSVERSE P.A. AT THE HIGHEST POINT OF SINGLE TOOTH PAIR CONTACT(deg.) | 21.5500 | 21.0216 |
| SPAN MEASUREMENT: | | |
| TOOTH NUMBER OF SPAN MEASUREMENT.. | 4.0000 | 11.0000 |
| BASE TANGENT LENGTH(mm)........... | 26.7863 | 80.8730 |
| CONSTANT CHORD MEASUREMENT: | | |
| CONSTANT CHORDAL THICKNESS(mm).... | 3.4676 | 3.4676 |
| CONSTANT CHORDAL HEIGHT(mm)....... | 1.8689 | 1.8689 |
| PIN MEASUREMENT: | | |
| PIN DIAMETER(mm).................. | 4.2000 | 4.2000 |
| DIAMETER OVER PINS(mm)............ | 71.5330 | 239.3832 |

| | |
|---|---|
| WORKING TRANSVERSE CONTACT ANGLE(deg.)........... | 20.3115 |
| CENTRE DISTANCE(mm).............................. | 150.0000 |
| TRANSVERSE CONTACT RATIO......................... | 1.6902 |
| OVERLAP RATIO.................................... | 1.1112 |
| TOTAL CONTACT RATIO.............................. | 2.8013 |

| | | |
|---|---|---|
| BASIC ENDURANCE LIMIT(N/mm²): | | |
| FOR CONTACT STRESS............. | 1109.9710 | 1109.9710 |
| FOR BENDING STRESS............. | 288.0294 | 288.0294 |
| ACTUAL SAFETY FACTOR: | | |
| FOR CONTACT STRESS............. | 1.0023 | 1.0501 |
| FOR BENDING STRESS............. | 1.4162 | 1.4218 |

| | | | |
|---|---|---|---|
| Ft = 8925.641N | Fr = 3303.730N | Fa = 1650.292N | |
| l = 130.00mm | s = .00mm | V = 2.5958m/s | |
| V50 =177.0cSt | Fß = 16.00µm | | |
| Ff1 = 11.00µm | Ff2 = 13.00µm | Fpb1= 13.00µm | Fpb2= 14.00µm |
| SRG1= 7 | SRG2= 7 | FRG1= 6 | FRG2= 6 |
| Rz1 = 6.30µm | Rz2 = 6.30µm | Rzf1= 10.00µm | Rzf2= 10.00µm |

Figure E.1, continued

```
-------------------------------------------------------------
KA   =  1.0000  KV   =  1.0324   KHß  =  1.7688  KFß  =  1.6558
KHα  =  1.3249  KFα  =  1.3249   ZH   =  2.4604  ZE   =189.8684
Zε   =   .7692  Zß   =   .9916   Yε   =   .6937  Yß   =   .9127
ZX   =  1.0000  YST  =  2.0000

ZN1  =  1.0000  ZN2  =  1.0417   ZL1  =  1.0515  ZL2  =  1.0515
ZV1  =   .9594  ZV2  =   .9594   ZR1  =   .9423  ZR2  =   .9477
ZW1  =  1.0000  ZW2  =  1.0000

YF1  =  1.3525  YF2  =  1.1985   YS1  =  1.9463  YS2  =  2.2004
YFα1=  2.5742  YFα2=  2.1940    YSα1=  1.6081  YSα2=  1.7917
YNT1=  1.0000  YNT2=  1.0000    Yδr1=   .9934  Yδr2=   .9991
YRr1=  1.0017  YRr2=  1.0017    YX1  =  1.0000  YX2  =  1.0000
-------------------------------------------------------------

                    DO YOU WANT TO CHANGE

THE DESIGN SPECIFICATIONS OR THE GIVEN CONDITIONS? (Y/N) = N




************  INPUT  SHAFT  DIMENSIONS  ************

TOTAL NUMBER OF NODES................................ = 12
X COORDINATE (mm) FOR NODE NUMBER  2.............. = 30
X COORDINATE (mm) FOR NODE NUMBER  3.............. = 60
X COORDINATE (mm) FOR NODE NUMBER  4.............. = 105
X COORDINATE (mm) FOR NODE NUMBER  5.............. = 119.5
X COORDINATE (mm) FOR NODE NUMBER  6.............. = 134
X COORDINATE (mm) FOR NODE NUMBER  7.............. = 157.5
X COORDINATE (mm) FOR NODE NUMBER  8.............. = 184.5
X COORDINATE (mm) FOR NODE NUMBER  9.............. = 211.5
X COORDINATE (mm) FOR NODE NUMBER 10.............. = 235
X COORDINATE (mm) FOR NODE NUMBER 11.............. = 249.5
X COORDINATE (mm) FOR NODE NUMBER 12.............. = 264

TOTAL NUMBER OF DIAMETER CHANGES.................. = 8
NODE NUMBER FOR DIAMETER CHANGE NO.  1............ = 1
NODE NUMBER FOR DIAMETER CHANGE NO.  2............ = 3
NODE NUMBER FOR DIAMETER CHANGE NO.  3............ = 4
NODE NUMBER FOR DIAMETER CHANGE NO.  4............ = 6
NODE NUMBER FOR DIAMETER CHANGE NO.  5............ = 7
NODE NUMBER FOR DIAMETER CHANGE NO.  6............ = 9
NODE NUMBER FOR DIAMETER CHANGE NO.  7............ = 10

NODE NUMBERS FOR LEFT AND RIGHT BEARINGS.......... = 5,11
```

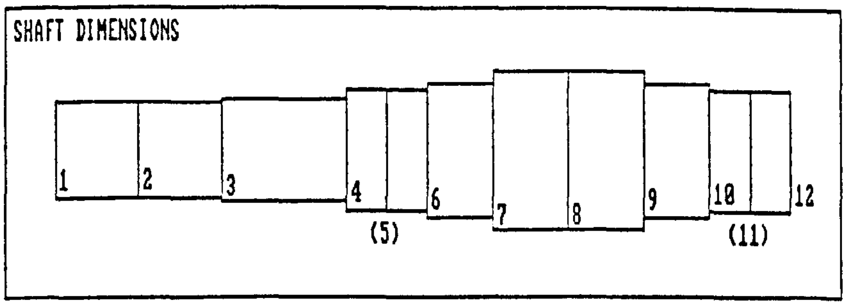
Figure E.1, continued

```
------------------------  SHAFT DIMENSIONS  ------------------------
(1)TOTAL NUMBER OF NODES IS 12. X COORDINATES OF NODES ARE AS FOLLOWS:
   .0[01]  30.0[02]  60.0[03] 105.0[04] 119.5[05] 134.0[06] 157.5[07]
 184.5[08] 211.5[09] 235.0[10] 249.5[11] 264.0[12]
(2)TOTAL NO. OF DIA. CHANGES IS  8.  NODE NOS. & DIAS. ARE AS FOLLOWS:
   .00[01]   .00[03]   .00[04]   .00[06]   .00[07]   .00[09]   .00[10]
   .00[12]
(3)NODE NO. FOR LEFT BEARING IS  5,  NODE NO. FOR  RIGHT BEARING IS 11
--------------------------------------------------------------------
       DO YOU WANT TO CHANGE THE DIMENSIONS? (Y/N)   = N

     *********  INPUT SHAFT LOADING CONDITIONS  *********

     TOTAL NUMBER OF RADIAL FORCES IN VERTICAL PLANE... = 1
     NODE NUMBER AND VALUE OF RADIAL FORCE (N.) NO. 1.. = 8,-3304

     TOTAL NUMBER OF BENDING MOMENTS IN VERTICAL PLANE. = 1
     NODE NUMBER AND VALUE OF BENDING MOMENT(N.mm) NO.1 = 8,-54544

     TOTAL NUMBER OF RADIAL FORCES IN HORIZONTAL PLANE. = 1
     NODE NUMBER AND VALUE OF RADIAL FORCE (N.) NO. 1.. = 8,-8926

     TOTAL NUMBER OF BENDING MOMENTS IN HORIZONTAL PLANE=

     TOTAL NUMBER OF AXIAL FORCES...................... = 1
     NODE NUMBER AND VALUE OF AXIAL FORCE (N.)  NO. 1.. = 8,1650
     NODE NUMBER OF TAKING AXIAL FORCE................. = 11

     TOTAL NUMBER OF APPLIED TORQUES................... = 2
     NODE NUMBER AND VALUE OF APPLIED TORQUE(N.mm) NO.1 = 2,295000
     NODE NUMBER AND VALUE OF APPLIED TORQUE(N.mm) NO.2 = 8,-295000
     THE NATURE OF CHANGE IN TORSIONAL STRESS:
        CONSTANT (1), PULSATING (2), SYMMETRIC (3)..... = 2

                    SHAFT LOADING CONDITIONS
   -----------------------------------------------------------------
   (1)TOTAL NO. OF RADIAL FORCES IN VERTICAL PLANE....             1
      VALUES (N.) [NODE NUMBERS] OF RADIAL FORCES:
      ...............................................    -3304.[08]
   (2)TOTAL NO. OF BENDING MOMENTS IN VERTICAL PLANE..             1
      VALUES (N.mm) [NODE NUMBERS] OF BENDING MOMENTS:
      ...............................................   -54544.[08]
   (3)TOTAL NO. OF RADIAL FORCES IN HORIZONTAL PLANE..             1
      VALUES (N.) [NODE NUMBERS] OF RADIAL FORCES:
      ...............................................    -8926.[08]
   (4)TOTAL NO. OF BENDING MOMENTS IN HORIZONTAL PLANE             0
   (5)TOTAL NO. OF AXIAL FORCES......................              1
      VALUES (N.) [NODE NUMBERS] OF AXIAL FORCES:
      ...............................................     1650.[08]
      NODE NUMBER OF TAKING AXIAL FORCE..............             11
   (6)TOTAL NO. OF APPLIED TORQUES...................              2
      VALUES (N.mm) [NODE NUMBERS] OF APPLIED TORQUES:
      ...............................................   295000.[02]
      ...............................................  -295000.[08]
      NATURE OF CHANGE IN TORSIONAL STRESS: (C, P, S).             2
   -----------------------------------------------------------------
     DO YOU WANT TO CHANGE THE LOADING CONDITIONS?(Y/N) = N
```

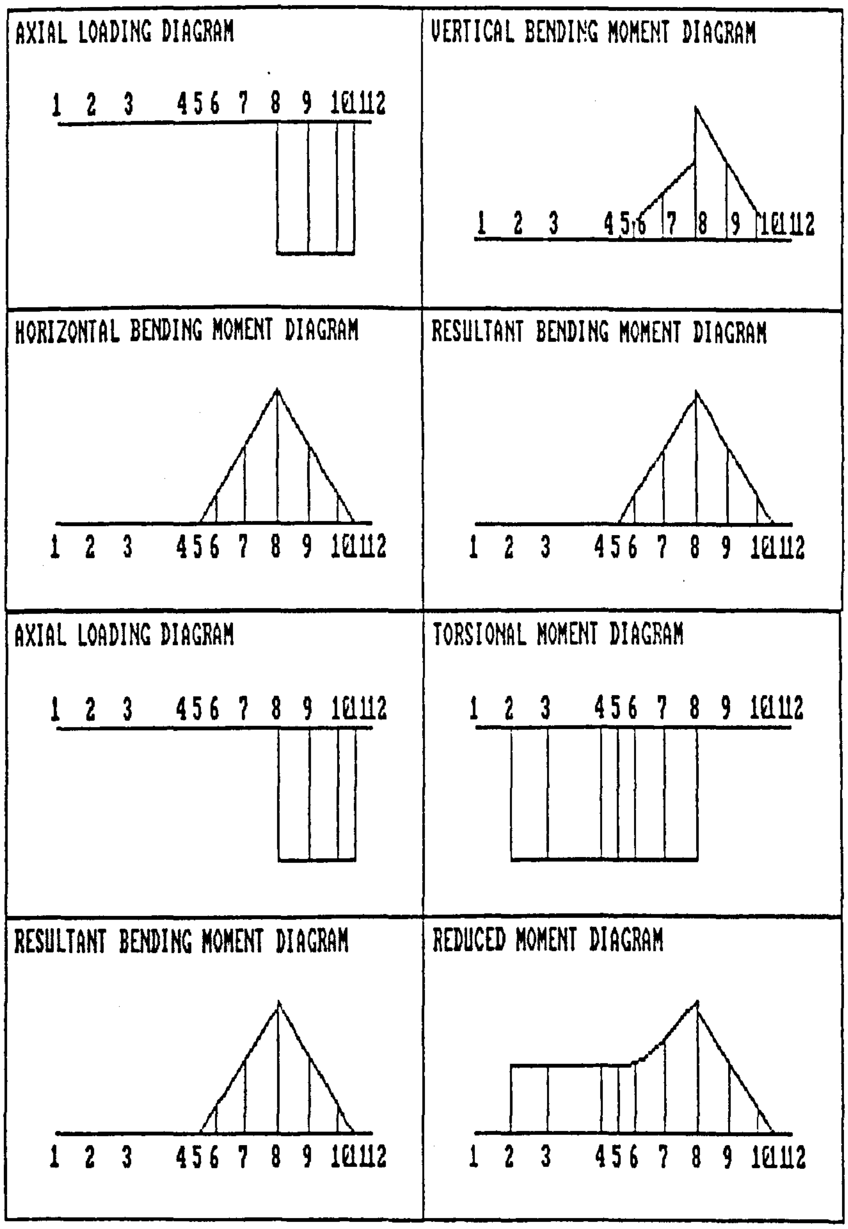
Figure E.1, continued

```
******  INPUT INFORMATION ON SHAFT MATERIAL  *******

TYPE OF MATERIAL: CARBON STEEL(1), ALLOY STEEL(2). = 1

TENSILE ULTIMATE STRENGTH (MPa) (=0.335HBMPa)..... = 600

TENSILE YIELD STRENGTH (MPa) (DEFAULT=0.585σb).... = 300

TORSIONAL YIELD STRENGTH (MPa) (DEFAULT=0.585σs).. =

BENDING ENDURANCE LIMIT(MPa) (DEFAULT=0.27(σb+σs)) =

TORSIONAL ENDURANCE LIMIT(MPa)(DEFAULT=0.156(σb+σs)=

YOUNG'S MODULUS OF ELASTICITY(MPa)(DEFAULT=206000) =

SHEARING MODULUS OF ELASTICITY(MPa)(DEFAULT=81000) =

               INFORMATION ON SHAFT MATERIAL
------------------------------------------------------------
(1)TYPE OF MATERIAL: CARBON (1), ALLOY STEEL (2)...        1
(2)TENSILE ULTIMATE STRENGTH (MPa)..................   600.0
(3)TENSILE YIELD STRENGTH (MPa)......................  300.0
(4)TORSIONAL YIELD STRENGTH (MPa)....................  175.5
(5)BENDING ENDURANCE LIMIT (MPa).....................  243.0
(6)TORSIONAL ENDURANCE LIMIT (MPa)...................  140.4
(7)YOUNG'S MODULUS OF ELASTICITY (MPa)............. 206000.0
(8)SHEARING MODULUS OF ELASTICITY (MPa)............  81000.0
------------------------------------------------------------
   DO YOU WANT TO CHANGE THESE INFORMATION? (Y/N)  = N


---  MINIMUM DEMANDED DIAMETERS (mm) [NODE NUMBERS] AT DIAMETER CHANGES  ---
    31.825[01]  31.825[03]  32.569[04]  35.787[06]  40.080[07]  32.594[09]
    23.641[10]     .000[12]

PLEASE INPUT NODE NUMBERS AND DIAMETERS AT DIAMETER CHANGES  ON THE BASIS OF
THE MINIMUM DEMANDED DIAMETERS AND THE STRUCTURE REQUIREMENTS OF THE SHAFT.


********************  INPUT SHAFT DIAMETERS   ********************

NODE NUMBER      MINIMUM DEMANDED DIAMETER(mm)      SELECTED VALUE(mm)
     1                    31.825                          35
     3                    31.825                          39
     4                    32.569                          45
     6                    35.787                          50
     7                    40.080                          60
     9                    32.594                          50
    10                    23.641                          45
```

Figure E.1, continued

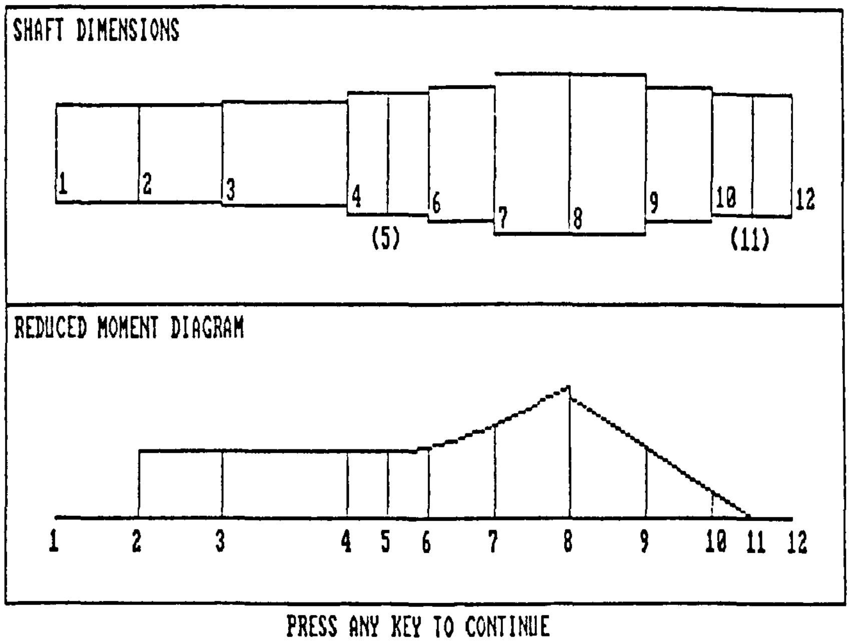



---------- DIAMETERS (mm) [NODE NUMBERS] ARE AS FOLLOWS: -----------
35.0[01] 39.0[03] 45.0[04] 50.0[06] 60.0[07] 50.0[09] 45.0[10]
.0[12]
--------------------------------------------------------------------
DO YOU WANT TO CHANGE THE DIAMETERS? (Y/N) = N




PRESS ANY KEY TO CONTINUE

Figure E.1, continued

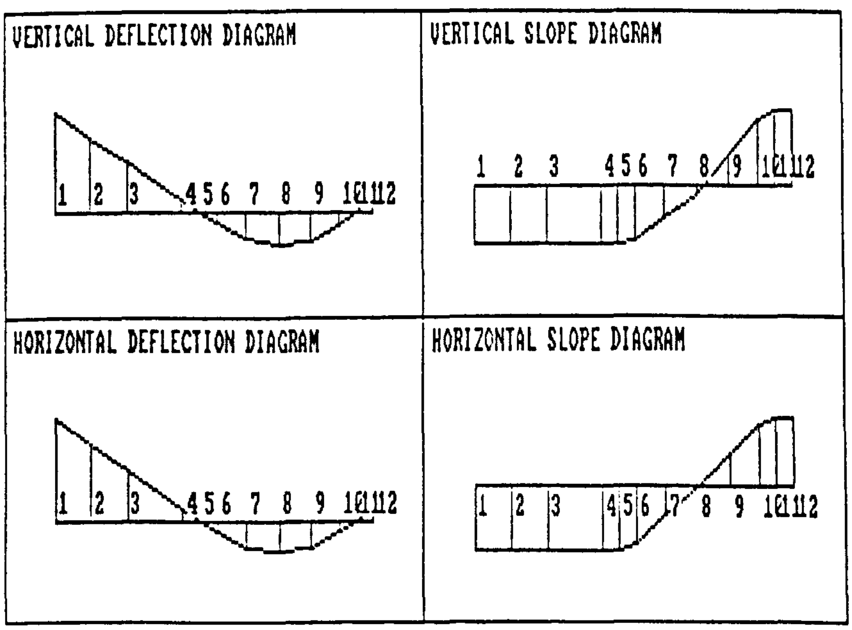



```
*****  INPUT INFORMATION ON CRITICAL SECTIONS  *****

TOTAL NUMBER OF CRITICAL CROSS SECTIONS........... = 2

INFORMATION ON CRITICAL CROSS SECTION NO. 1:
NODE NUMBER....................................... = 7
MACHINING METHOD OF SURFACE:
GRINDING(1),FINISHING(2),ROUGHING(3),UNMACHINED(4) = 1
CONSTRUCTIONAL CHARACTERISTIC OF SURFACE:
WORM THREAD(1), SCREW THREAD(2), PROFILE KEYWAY(3)
SLED RUNNER KEYWAY (4),  STRAIGHT-SIDED SPLINE (5)
INVOLUTE SPLINE(6), FILLET(7), INTERFERENCE FIT(8)
TRANSITION FIT (9), MOVABLE FIT (10),  GROOVE (11)
RADIAL HOLE (12).................................. = 7
DIMENSIONS OF CROSS SECTION:
FILLET RADIUS (mm)................................ = 1.5

INFORMATION ON CRITICAL CROSS SECTION NO. 2:
NODE NUMBER....................................... = 8
MACHINING METHOD OF SURFACE:
GRINDING(1),FINISHING(2),ROUGHING(3),UNMACHINED(4) = 1
CONSTRUCTIONAL CHARACTERISTIC OF SURFACE:
WORM THREAD(1), SCREW THREAD(2), PROFILE KEYWAY(3)
SLED RUNNER KEYWAY (4),  STRAIGHT-SIDED SPLINE (5)
INVOLUTE SPLINE(6), FILLET(7), INTERFERENCE FIT(8)
TRANSITION FIT (9), MOVABLE FIT (10),  GROOVE (11)
RADIAL HOLE (12).................................. = 8
DIMENSIONS OF CROSS SECTION:
DIAMETER (mm) AT FIT.............................. = 60

MINIMUM DEMANDED SAFETY FACTOR FOR FATIGUE STRENGTH= 1.5
```

Figure E.1, continued

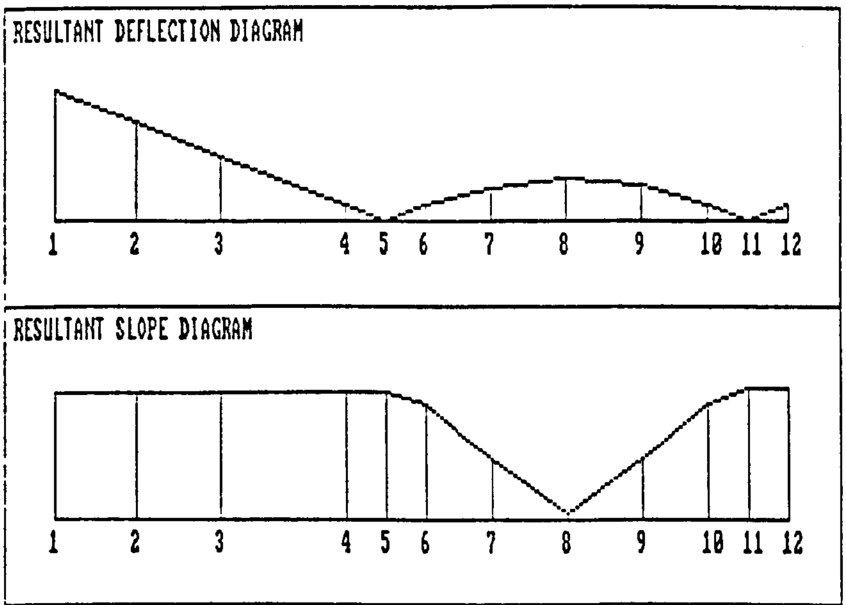
```
                INFORMATION ON CRITICAL SECTIONS
-------------------------------------------------------------
(1)TOTAL NUMBER OF CRITICAL CROSS SECTIONS..........        2
   INFORMATION ON CRITICAL CROSS SECTION NO. 1:
     [1]NODE NUMBER..................................        7
     [2]MACHINING METHOD: G(1),F(2),R(3),U(4).......         1
     [3]CONSTRUCTIONAL CHARACTERISTIC...............         7
     [4]FILLET RADIUS (mm)..........................     1.500
   INFORMATION ON CRITICAL CROSS SECTION NO. 2:
     [1]NODE NUMBER..................................        8
     [2]MACHINING METHOD: G(1),F(2),R(3),U(4).......         1
     [3]CONSTRUCTIONAL CHARACTERISTIC...............         8
     [4]DIAMETER (mm) AT FIT........................    60.000
(2)MINIMUM DEMANDED SAFETY FACTOR...................     1.500
-------------------------------------------------------------
   DO YOU WANT TO CHANGE THESE INFORMATION? (Y/N)  = N
```

```
  CALCULATED BENDING STRESS (MPa) AT CRITICAL CROSS SECTIONS
      20.167[07]                                 17.171[08]

     CALCULATED SAFETY FACTORS AT CRITICAL CROSS SECTIONS
       5.693[07]                                  4.929[08]

      FORCES ACTING ON LEFT BEARING & ITS BORE DIAMETER
R =     4630.038N       A =         .000N       d =   45.000mm

      FORCES ACTING ON RIGHT BEARING & ITS BORE DIAMETER
R =     4920.342N       A =     1650.000N       d =   45.000mm
```




PRESS ANY KEY TO CONTINUE

Figure E.1, continued




DEFLECTIONS (mm) AT ALL NODES ALONG THE SHAFT

| | | | | |
|---|---|---|---|---|
| .1287E-01[01] | .9636E-02[02] | .6406E-02[03] | .1561E-02[04] | .9313E-09[05] |
| .1504E-02[06] | .3308E-02[07] | .4077E-02[08] | .3366E-02[09] | .1542E-02[10] |
| .1041E-08[11] | .1602E-02[12] | | | |

SLOPES (rad) AT ALL NODES ALONG THE SHAFT

| | | | | |
|---|---|---|---|---|
| .1077E-03[01] | .1077E-03[02] | .1077E-03[03] | .1077E-03[04] | .1077E-03[05] |
| .9594E-04[06] | .5088E-04[07] | .5530E-05[08] | .5013E-04[09] | .9803E-04[10] |
| .1105E-03[11] | .1105E-03[12] | | | |

ANGLE OF TWIST PER UNIT LENGTH (deg/m)

| | | | | |
|---|---|---|---|---|
| .0000E+00[01] | .0000E+00[02] | -.1416E+01[03] | -.9188E+00[04] | -.5183E+00[05] |
| -.5183E+00[06] | -.3401E+00[07] | -.1640E+00[08] | .0000E+00[09] | .0000E+00[10] |
| .0000E+00[11] | .0000E+00[12] | | | |

ANGLE OF TWIST (deg)

| | | | | |
|---|---|---|---|---|
| .0000E+00[01] | .0000E+00[02] | -.4249E-01[03] | -.4134E-01[04] | -.7516E-02[05] |
| -.7516E-02[06] | -.7992E-02[07] | -.4428E-02[08] | .0000E+00[09] | .0000E+00[10] |
| .0000E+00[11] | .0000E+00[12] | | | |

Figure E.1, continued

****** INPUT BEARING INFORMATION ******

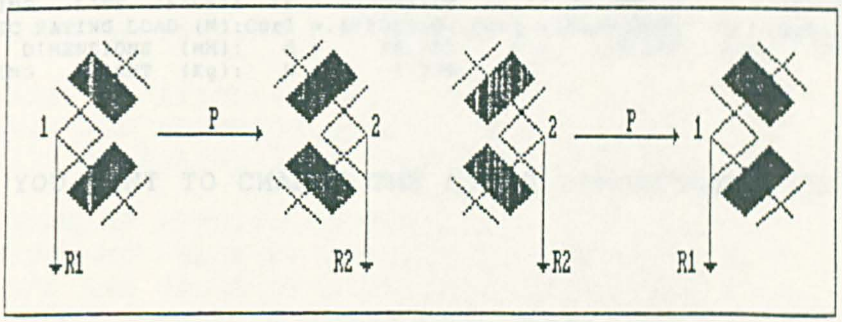
FOR BEARING TYPE:

BEARING TYPE CODE NO.
SINGLE ROW RADIAL BALL BEARING.................................0
DOUBLE ROW SELF-ALIGNING BALL BEARING..........................1
SINGLE OR DOUBLE ROW RADIAL SHORT CYLINDRICAL BEARING.....2
DOUBLE ROW RADIAL SELF-ALIGNING ROLLER BEARING.............3
NEEDLE BEARING.....................................................4
HELICAL ROLLER BEARING...........................................5
SINGLE ROW ANGULAR CONTACT BALL BEARING...................6
SINGLE ROW TAPER ROLLER BEARING..............................7
THRUST BALL BEARING..............................................8
THRUST SPHERICAL ROLLER BEARING...............................9

PLEASE INPUT THE CODE NO. OF THE SELECTED BEARING TYPE = 6

REQUIRED BEARING BORE DIAMETER (mm). . . . . . . . . . = 45

BEARING SPEED (r/min). . . . . . . . . . . . . . . . . = 750




NOW YOU ARE READY TO INPUT RADIAL LOADS (R1 AND R2) AND AXIAL LOAD (P) ACTING ON THE BEARINGS. THE BEARING POSITION NUMBERS DEPEND ON THE ARRANGEMENT OF THE BEARINGS. WHEN ENTERING DATA, YOU SHOULD OBEY THE DESIGNATION AS INDICATED IN THE ABOVE FIGURE. IF THE AXIAL LOAD ACTS IN THE OPPOSITE DIRECTION TO THAT INDICATED, IT MUST BE SPECIFIED BY A NEGATIVE VALUE.

RADIAL FORCES (R1 AND R2) (N.) . . . . . . . . . . = 4920,4630
AXIAL FORCE (N.) . . . . . . . . . . . . . . . . . . =1650

DEMANDED SAFETY FACTOR FOR STATIC STRENGTH . . . . = 1

LOAD FACTOR: UNIFORM LOAD OR LIGHT SHOCK (1.0-1.2) MODERATE SHOCK (1.2-1.8), HEAVY SHOCK (1.8-3.0). YOU MAY EITHER CHOOSE THE VALUE FROM THE FOLLOWING TABLE OR INPUT OTHER VALUE BY PRESSING THE RELEVANT NUMBER

| NO. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 |
|---|---|---|---|---|---|---|---|---|---|---|
| IS | 1.0 | 1.1 | 1.2 | 1.5 | 1.8 | 2.1 | 2.4 | 2.7 | 3.0 | OWN |

THE NUMBER YOU CHOOSE. . . . . . . . . . . . . . . . . = 1

WORKING TEMPERATURE (°c) (DEFAULT FOR <=120°c) . . =

LIFE EXPECTANCY (hours). . . . . . . . . . . . . . . . = 20000

DEMANDED RELIABILITY (DEFAULT FOR 0.9) . . . . . . =

Figure E.1, continued

```
                         BEARING  INFORMATION
-----------------------------------------------------------
(1)CODE NUMBER OF SELECTED BEARING TYPE..........           6
   REQUIRED BEARING BORE DIAMETER (mm)...........          45
(2)BEARING SPEED (r/min)........................      750.000
(3)LOAD CONDITIONS:
   RADIAL FORCES (N)..................  4920.000     4630.000
   AXIAL FORCE (N)..............................     1650.000
(4)DEMANDED SAFETY FACTOR.......................        1.000
(5)LOAD FACTOR: U(1.0-1.2),M(1.2-1.8),H(1.8-3.0).       1.000
(6)WORKING TEMPERATURE (°c).....................      120.000
(7)LIFE EXPECTANCY (hours)...................... 20000.000
(8)DEMANDED RELIABILITY.........................         .900
-----------------------------------------------------------
   DO YOU WANT TO CHANGE THE INFORMATION?  (Y/N)    = N


(01)
BEARING   CODE  NUMBER:   #        46409
DYNAMIC RATING LOAD(N): Cr1 =.59615E+05  Cr2 =.44702E+05   C =.66393E+05
BEARING   LIFE  (hour):  Lr =.20000E+05   L1 =.27628E+05  L2 =.65527E+05
STATIC RATING LOAD (N):C0r1 =.49200E+04 C0r2 =.46300E+04  C0 =.52566E+05
MAIN  DIMENSIONS  (MM):   d =     45.000    D =    120.000   B =    29.000
BEARING   WEIGHT  (Kg):   W =      1.770


   DO YOU WANT TO CHANGE THE GIVEN CONDITIONS? (Y/N)N


************  INPUT  SHAFT  DIMENSIONS  ************

TOTAL NUMBER OF NODES.............................. = 12
X COORDINATE (mm) FOR NODE NUMBER  2.............. = 35
X COORDINATE (mm) FOR NODE NUMBER  3.............. = 70
X COORDINATE (mm) FOR NODE NUMBER  4.............. = 115
X COORDINATE (mm) FOR NODE NUMBER  5.............. = 126.5
X COORDINATE (mm) FOR NODE NUMBER  6.............. = 170.5
X COORDINATE (mm) FOR NODE NUMBER  7.............. = 191.5
X COORDINATE (mm) FOR NODE NUMBER  8.............. = 215.5
X COORDINATE (mm) FOR NODE NUMBER  9.............. = 230.5
X COORDINATE (mm) FOR NODE NUMBER 10.............. = 245
X COORDINATE (mm) FOR NODE NUMBER 11.............. = 256.5
X COORDINATE (mm) FOR NODE NUMBER 12.............. = 268

TOTAL NUMBER OF DIAMETER CHANGES.................. = 8
NODE NUMBER FOR DIAMETER CHANGE NO.  1............ = 1
NODE NUMBER FOR DIAMETER CHANGE NO.  2............ = 3
NODE NUMBER FOR DIAMETER CHANGE NO.  3............ = 4
NODE NUMBER FOR DIAMETER CHANGE NO.  4............ = 6
NODE NUMBER FOR DIAMETER CHANGE NO.  5............ = 8
NODE NUMBER FOR DIAMETER CHANGE NO.  6............ = 9
NODE NUMBER FOR DIAMETER CHANGE NO.  7............ = 10

NODE NUMBERS FOR LEFT AND RIGHT BEARINGS.......... = 5,11
```

Figure E.1, continued

```
------------------------  SHAFT DIMENSIONS  -----------------------
(1)TOTAL NUMBER OF NODES IS 12. X COORDINATES OF NODES ARE AS FOLLOWS:
    .0[01]  35.0[02]  70.0[03] 115.0[04] 126.5[05] 170.5[06] 191.5[07]
 215.5[08] 230.5[09] 245.0[10] 256.5[11] 268.0[12]
(2)TOTAL NO. OF DIA. CHANGES IS  8.  NODE NOS. & DIAS. ARE AS FOLLOWS:
   .00[01]   .00[03]   .00[04]   .00[06]   .00[08]   .00[09]   .00[10]
   .00[12]
(3)NODE NO. FOR LEFT BEARING IS  5,  NODE NO. FOR  RIGHT BEARING IS 11
----------------------------------------------------------------------
        DO YOU WANT TO CHANGE THE DIMENSIONS? (Y/N)   = N

     *********  INPUT SHAFT LOADING CONDITIONS  *********

     TOTAL NUMBER OF RADIAL FORCES IN VERTICAL PLANE... = 1
     NODE NUMBER AND VALUE OF RADIAL FORCE (N.) NO. 1.. = 7,-3304

     TOTAL NUMBER OF BENDING MOMENTS IN VERTICAL PLANE. = 1
     NODE NUMBER AND VALUE OF BENDING MOMENT(N.mm) NO.1 = 7,-193000

     TOTAL NUMBER OF RADIAL FORCES IN HORIZONTAL PLANE. = 1
     NODE NUMBER AND VALUE OF RADIAL FORCE (N.) NO. 1.. = 7,8926

     TOTAL NUMBER OF BENDING MOMENTS IN HORIZONTAL PLANE=

     TOTAL NUMBER OF AXIAL FORCES...................... = 1
     NODE NUMBER AND VALUE OF AXIAL FORCE (N.)  NO. 1.. = 7,1650
     NODE NUMBER OF TAKING AXIAL FORCE................. = 11

     TOTAL NUMBER OF APPLIED TORQUES................... = 2
     NODE NUMBER AND VALUE OF APPLIED TORQUE(N.mm) NO.1 = 2,-1043846
     NODE NUMBER AND VALUE OF APPLIED TORQUE(N.mm) NO.2 = 7,1043846
     THE NATURE OF CHANGE IN TORSIONAL STRESS:
        CONSTANT (1), PULSATING (2), SYMMETRIC (3)..... = 2

                      SHAFT LOADING CONDITIONS
   -----------------------------------------------------------------
   (1)TOTAL NO. OF RADIAL FORCES IN VERTICAL PLANE....             1
      VALUES (N.) [NODE NUMBERS] OF RADIAL FORCES:
      ............................................    -3304.[07]
   (2)TOTAL NO. OF BENDING MOMENTS IN VERTICAL PLANE..             1
      VALUES (N.mm) [NODE NUMBERS] OF BENDING MOMENTS:
      ............................................. -193000.[07]
   (3)TOTAL NO. OF RADIAL FORCES IN HORIZONTAL PLANE..             1
      VALUES (N.) [NODE NUMBERS] OF RADIAL FORCES:
      ............................................      8926.[07]
   (4)TOTAL NO. OF BENDING MOMENTS IN HORIZONTAL PLANE             0
   (5)TOTAL NO. OF AXIAL FORCES.......................             1
      VALUES (N.) [NODE NUMBERS] OF AXIAL FORCES:
      ............................................      1650.[07]
      NODE NUMBER OF TAKING AXIAL FORCE...............            11
   (6)TOTAL NO. OF APPLIED TORQUES....................             2
      VALUES (N.mm) [NODE NUMBERS] OF APPLIED TORQUES:
      .............................................-1043846.[02]
      ............................................. 1043846.[07]
      NATURE OF CHANGE IN TORSIONAL STRESS: (C, P, S).             2
   -----------------------------------------------------------------
     DO YOU WANT TO CHANGE THE LOADING CONDITIONS?(Y/N) = N
```

<u>Figure E.1, continued</u>

****** INPUT INFORMATION ON SHAFT MATERIAL *******

TYPE OF MATERIAL: CARBON STEEL(1), ALLOY STEEL(2). = 1

TENSILE ULTIMATE STRENGTH (MPa) (=0.335HBMPa)..... = 600

TENSILE YIELD STRENGTH (MPa) (DEFAULT=0.585σb).... = 300

TORSIONAL YIELD STRENGTH (MPa) (DEFAULT=0.585σs).. =

BENDING ENDURANCE LIMIT(MPa) (DEFAULT=0.27(σb+σs)) =

TORSIONAL ENDURANCE LIMIT(MPa)(DEFAULT=0.156(σb+σs)=

YOUNG'S MODULUS OF ELASTICITY(MPa)(DEFAULT=206000) =

SHEARING MODULUS OF ELASTICITY(MPa)(DEFAULT=81000) =

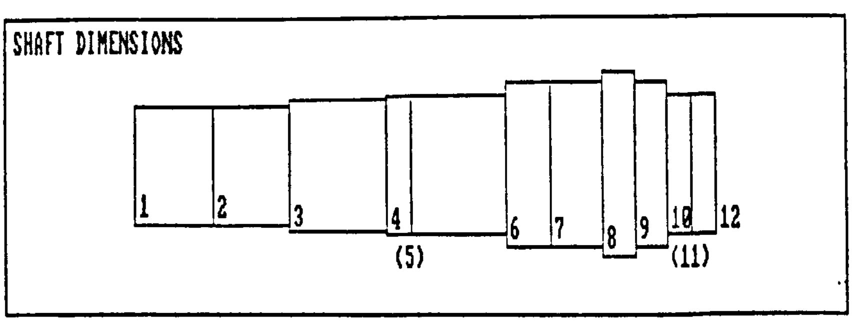
INFORMATION ON SHAFT MATERIAL

| | |
|---|---|
| (1)TYPE OF MATERIAL: CARBON (1), ALLOY STEEL (2)... | 1 |
| (2)TENSILE ULTIMATE STRENGTH (MPa)................. | 600.0 |
| (3)TENSILE YIELD STRENGTH (MPa).................... | 300.0 |
| (4)TORSIONAL YIELD STRENGTH (MPa).................. | 175.5 |
| (5)BENDING ENDURANCE LIMIT (MPa)................... | 243.0 |
| (6)TORSIONAL ENDURANCE LIMIT (MPa)................. | 140.4 |
| (7)YOUNG'S MODULUS OF ELASTICITY (MPa)............. | 206000.0 |
| (8)SHEARING MODULUS OF ELASTICITY (MPa)............ | 81000.0 |

DO YOU WANT TO CHANGE THESE INFORMATION? (Y/N) = N

******************** INPUT SHAFT DIAMETERS ********************

| NODE NUMBER | MINIMUM DEMANDED DIAMETER(mm) | SELECTED VALUE(mm) |
|---|---|---|
| 1 | 48.496 | 55 |
| 3 | 48.496 | 62 |
| 4 | 49.286 | 65 |
| 6 | 50.145 | 75 |
| 8 | 34.600 | 85 |
| 9 | 29.726 | 75 |
| 10 | 22.649 | 65 |




---------- DIAMETERS (mm) [NODE NUMBERS] ARE AS FOLLOWS: -----------
55.0[01] 62.0[03] 65.0[04] 75.0[06] 85.0[08] 75.0[09] 65.0[10]
.0[12]

DO YOU WANT TO CHANGE THE DIAMETERS? (Y/N) = N

Figure E.1, continued

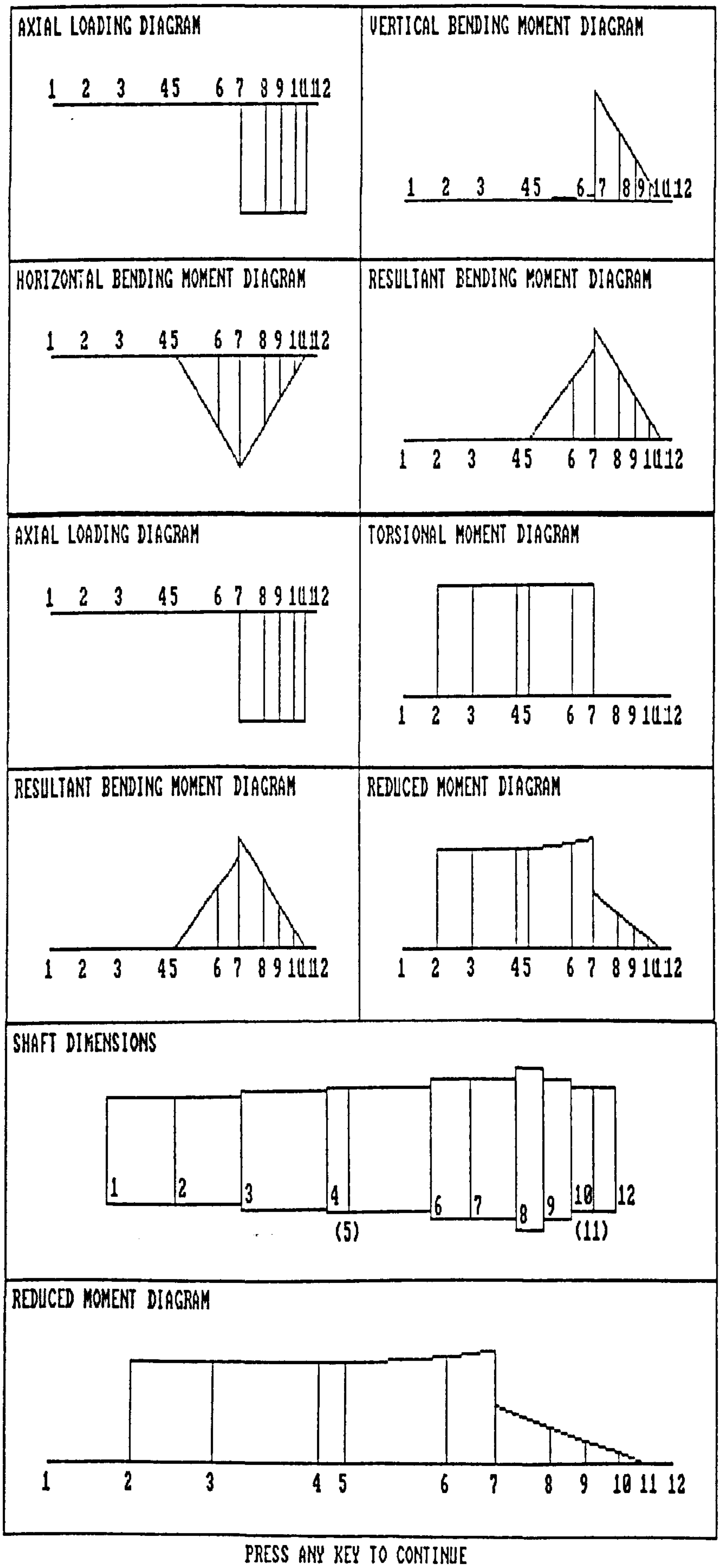



Figure E.1, continued

```
*****  INPUT INFORMATION ON CRITICAL SECTIONS  *****

TOTAL NUMBER OF CRITICAL CROSS SECTIONS........... = 2

INFORMATION ON CRITICAL CROSS SECTION NO. 1:
NODE NUMBER....................................... = 6
MACHINING METHOD OF SURFACE:
GRINDING(1),FINISHING(2),ROUGHING(3),UNMACHINED(4) = 1
CONSTRUCTIONAL CHARACTERISTIC OF SURFACE:
WORM THREAD(1), SCREW THREAD(2), PROFILE KEYWAY(3)
SLED RUNNER KEYWAY (4),  STRAIGHT-SIDED SPLINE (5)
INVOLUTE SPLINE(6), FILLET(7), INTERFERENCE FIT(8)
TRANSITION FIT (9), MOVABLE FIT (10),  GROOVE (11)
RADIAL HOLE (12).................................. = 7
DIMENSIONS OF CROSS SECTION:
FILLET RADIUS (mm)................................ = 2

INFORMATION ON CRITICAL CROSS SECTION NO. 2:
NODE NUMBER....................................... = 7
MACHINING METHOD OF SURFACE:
GRINDING(1),FINISHING(2),ROUGHING(3),UNMACHINED(4) = 1
CONSTRUCTIONAL CHARACTERISTIC OF SURFACE:
WORM THREAD(1), SCREW THREAD(2), PROFILE KEYWAY(3)
SLED RUNNER KEYWAY (4),  STRAIGHT-SIDED SPLINE (5)
INVOLUTE SPLINE(6), FILLET(7), INTERFERENCE FIT(8)
TRANSITION FIT (9), MOVABLE FIT (10),  GROOVE (11)
RADIAL HOLE (12).................................. = 8
DIMENSIONS OF CROSS SECTION:
DIAMETER (mm) AT FIT.............................. = 75

MINIMUM DEMANDED SAFETY FACTOR FOR FATIGUE STRENGTH= 1.5

              INFORMATION ON CRITICAL SECTIONS
-------------------------------------------------------------
(1)TOTAL NUMBER OF CRITICAL CROSS SECTIONS..........        2
   INFORMATION ON CRITICAL CROSS SECTION NO. 1:
     [1]NODE NUMBER.................................        6
     [2]MACHINING METHOD: G(1),F(2),R(3),U(4).......        1
     [3]CONSTRUCTIONAL CHARACTERISTIC...............        7
     [4]FILLET RADIUS (mm)..........................    2.000
   INFORMATION ON CRITICAL CROSS SECTION NO. 2:
     [1]NODE NUMBER.................................        7
     [2]MACHINING METHOD: G(1),F(2),R(3),U(4).......        1
     [3]CONSTRUCTIONAL CHARACTERISTIC...............        8
     [4]DIAMETER (mm) AT FIT........................   75.000
(2)MINIMUM DEMANDED SAFETY FACTOR...................    1.500
-------------------------------------------------------------
   DO YOU WANT TO CHANGE THESE INFORMATION? (Y/N)  = N
```

Figure E.1, continued

CALCULATED BENDING STRESS (MPa) AT CRITICAL CROSS SECTIONS

23.977[06] 17.158[07]

CALCULATED SAFETY FACTORS AT CRITICAL CROSS SECTIONS

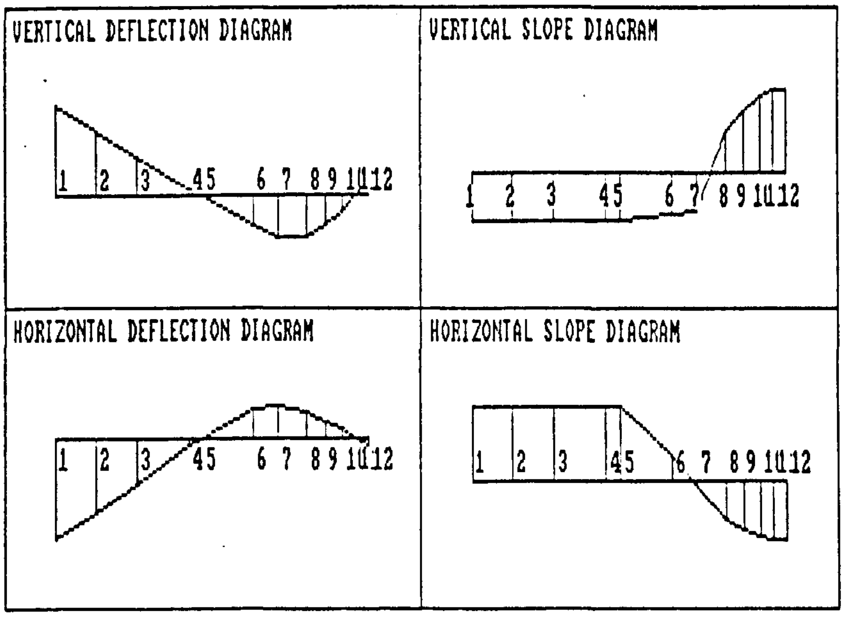
5.765[06] 6.047[07]

FORCES ACTING ON LEFT BEARING & ITS BORE DIAMETER

R = 4466.138N A = .000N d = 65.000mm

FORCES ACTING ON RIGHT BEARING & ITS BORE DIAMETER

R = 5454.972N A = 1650.000N d = 65.000mm




PRESS ANY KEY TO CONTINUE

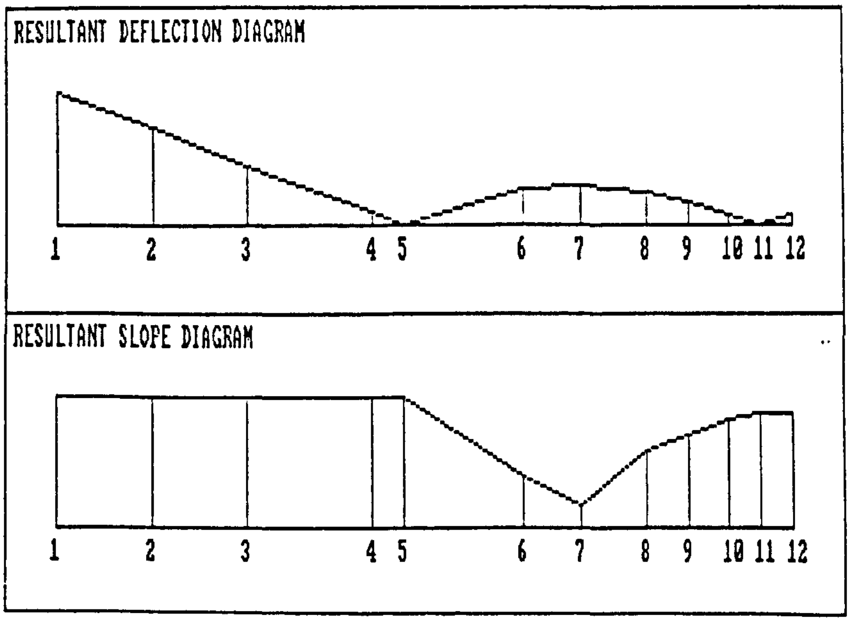



Figure E.1, continued

| | | | | |
|---|---|---|---|---|
| DEFLECTIONS (mm) AT ALL NODES ALONG THE SHAFT | | | | |
| .4758E-02[01] | .3441E-02[02] | .2125E-02[03] | .4325E-03[04] | .4693E-09[05] |
| .1309E-02[06] | .1456E-02[07] | .1166E-02[08] | .7996E-03[09] | .3747E-03[10] |
| .4657E-09[11] | .3822E-03[12] | | | |
| SLOPES (rad) AT ALL NODES ALONG THE SHAFT | | | | |
| .3761E-04[01] | .3761E-04[02] | .3761E-04[03] | .3761E-04[04] | .3761E-04[05] |
| .1450E-04[06] | .6695E-05[07] | .2177E-04[08] | .2677E-04[09] | .3128E-04[10] |
| .3323E-04[11] | .3323E-04[12] | | | |
| ANGLE OF TWIST PER UNIT LENGTH (deg/m) | | | | |
| .0000E+00[01] | .0000E+00[02] | .8219E+00[03] | .5090E+00[04] | .4213E+00[05] |
| .4213E+00[06] | .2377E+00[07] | .0000E+00[08] | .0000E+00[09] | .0000E+00[10] |
| .0000E+00[11] | .0000E+00[12] | | | |
| ANGLE OF TWIST (deg) | | | | |
| .0000E+00[01] | .0000E+00[02] | .2877E-01[03] | .2290E-01[04] | .4845E-02[05] |
| .1854E-01[06] | .4992E-02[07] | .0000E+00[08] | .0000E+00[09] | .0000E+00[10] |
| .0000E+00[11] | .0000E+00[12] | | | |

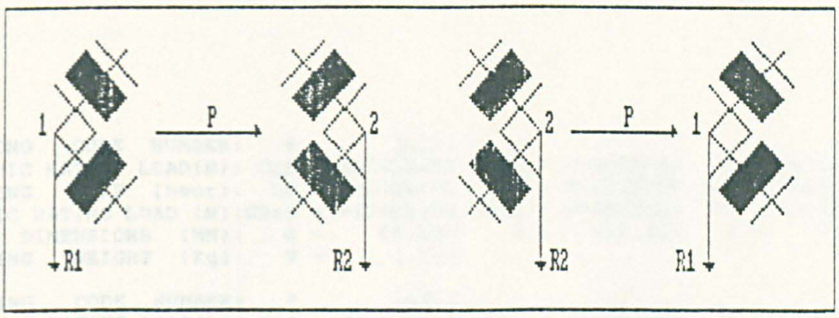
PRESS ANY KEY TO CONTINUE

****** INPUT BEARING INFORMATION ******

FOR BEARING TYPE:

```
BEARING TYPE                                               CODE NO.
SINGLE ROW RADIAL BALL BEARING...............................0
DOUBLE ROW SELF-ALIGNING BALL BEARING........................1
SINGLE OR DOUBLE ROW RADIAL SHORT CYLINDRICAL BEARING.....2
DOUBLE ROW RADIAL SELF-ALIGNING ROLLER BEARING............3
NEEDLE BEARING..............................................4
HELICAL ROLLER BEARING......................................5
SINGLE ROW ANGULAR CONTACT BALL BEARING....................6
SINGLE ROW TAPER ROLLER BEARING............................7
THRUST BALL BEARING.........................................8
THRUST SPHERICAL ROLLER BEARING............................9

PLEASE INPUT THE CODE NO. OF THE SELECTED BEARING TYPE  = 6

REQUIRED BEARING BORE DIAMETER (mm). . . . . . . . . = 65

BEARING SPEED (r/min). . . . . . . . . . . . . . . . = 212
```

Figure E.1, continued




NOW YOU ARE READY TO INPUT RADIAL LOADS (R1 AND R2) AND AXIAL LOAD (P) ACTING ON THE BEARINGS. THE BEARING POSITION NUMBERS DEPEND ON THE ARRANGEMENT OF THE BEARINGS. WHEN ENTERING DATA, YOU SHOULD OBEY THE DESIGNATION AS INDICATED IN THE ABOVE FIGURE. IF THE AXIAL LOAD ACTS IN THE OPPOSITE DIRECTION TO THAT INDICATED, IT MUST BE SPECIFIED BY A NEGATIVE VALUE.

RADIAL FORCES (R1 AND R2) (N.) . . . . . . . . . . . = 5455,4466
AXIAL FORCE (N.) . . . . . . . . . . . . . . . . . = 1650

DEMANDED SAFETY FACTOR FOR STATIC STRENGTH . . . . = 1

LOAD FACTOR: UNIFORM LOAD OR LIGHT SHOCK (1.0-1.2) MODERATE SHOCK (1.2-1.8), HEAVY SHOCK (1.8-3.0). YOU MAY EITHER CHOOSE THE VALUE FROM THE FOLLOWING TABLE OR INPUT OTHER VALUE BY PRESSING THE RELEVANT NUMBER

| NO. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 |
|---|---|---|---|---|---|---|---|---|---|---|
| IS | 1.0 | 1.1 | 1.2 | 1.5 | 1.8 | 2.1 | 2.4 | 2.7 | 3.0 | OWN |

THE NUMBER YOU CHOOSE. . . . . . . . . . . . . . . . . = 1

WORKING TEMPERATURE (°c) (DEFAULT FOR <=120°c) . . =

LIFE EXPECTANCY (hours). . . . . . . . . . . . . . . = 20000

DEMANDED RELIABILITY (DEFAULT FOR 0.9) . . . . . . =

BEARING INFORMATION

| | | |
|---|---|---|
| (1)CODE NUMBER OF SELECTED BEARING TYPE.......... | | 6 |
| REQUIRED BEARING BORE DIAMETER (mm)........... | | 65 |
| (2)BEARING SPEED (r/min)....................... | | 212.000 |
| (3)LOAD CONDITIONS: | | |
| RADIAL FORCES (N).................. | 5455.000 | 4466.000 |
| AXIAL FORCE (N)............................ | | 1650.000 |
| (4)DEMANDED SAFETY FACTOR...................... | | 1.000 |
| (5)LOAD FACTOR: U(1.0-1.2),M(1.2-1.8),H(1.8-3.0). | | 1.000 |
| (6)WORKING TEMPERATURE (°c)...................... | | 120.000 |
| (7)LIFE EXPECTANCY (hours)...................... | | 20000.000 |
| (8)DEMANDED RELIABILITY......................... | | .900 |

DO YOU WANT TO CHANGE THE INFORMATION? (Y/N) = N

```
(01)
BEARING    CODE  NUMBER:   #        36213
DYNAMIC RATING LOAD(N): Cr1 =.46142E+05  Cr2 =.30300E+05   C =.56783E+05
BEARING    LIFE  (hour):  Lr =.20000E+05   L1 =.37272E+05  L2 =.13163E+06
STATIC RATING LOAD (N):C0r1 =.54550E+04 C0r2 =.44660E+04  C0 =.50016E+05
MAIN  DIMENSIONS  (MM):    d =    65.000    D =   120.000   B =    23.000
BEARING    WEIGHT  (Kg):   W =     1.010
(02)
BEARING    CODE  NUMBER:   #        36313
DYNAMIC RATING LOAD(N): Cr1 =.48641E+05  Cr2 =.30958E+05   C =.92480E+05
BEARING    LIFE  (hour):  Lr =.20000E+05   L1 =.13746E+06  L2 =.53315E+06
STATIC RATING LOAD (N):C0r1 =.54550E+04 C0r2 =.44660E+04  C0 =.81594E+05
MAIN  DIMENSIONS  (MM):    d =    65.000    D =   140.000   B =    33.000
BEARING    WEIGHT  (Kg):   W =     2.230
(03)
BEARING    CODE  NUMBER:   #        46213
DYNAMIC RATING LOAD(N): Cr1 =.39896E+05  Cr2 =.28298E+05   C =.50898E+05
BEARING    LIFE  (hour):  Lr =.20000E+05   L1 =.41530E+05  L2 =.11638E+06
STATIC RATING LOAD (N):C0r1 =.54550E+04 C0r2 =.44660E+04  C0 =.42857E+05
MAIN  DIMENSIONS  (MM):    d =    65.000    D =   120.000   B =    23.000
BEARING    WEIGHT  (Kg):   W =     1.010

(04)
BEARING    CODE  NUMBER:   #        46313
DYNAMIC RATING LOAD(N): Cr1 =.39896E+05  Cr2 =.28298E+05   C =.87478E+05
BEARING    LIFE  (hour):  Lr =.20000E+05   L1 =.21084E+06  L2 =.59083E+06
STATIC RATING LOAD (N):C0r1 =.54550E+04 C0r2 =.44660E+04  C0 =.75024E+05
MAIN  DIMENSIONS  (MM):    d =    65.000    D =   140.000   B =    33.000
BEARING    WEIGHT  (Kg):   W =     2.570


                    PRESS ANY KEY TO CONTINUE
```

Figure E.1, continued

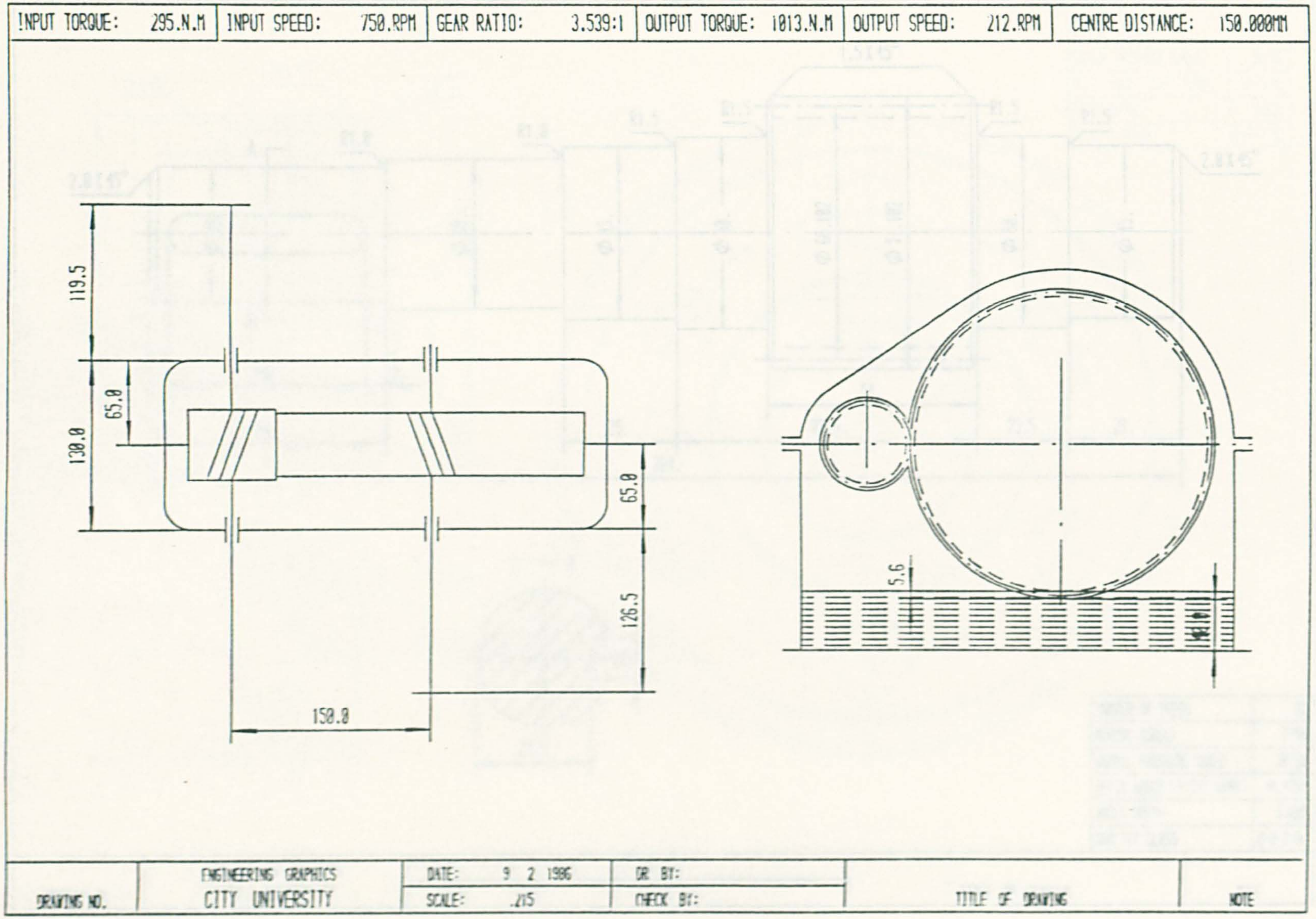



Figure E.2 Computer output of gearbox schematic

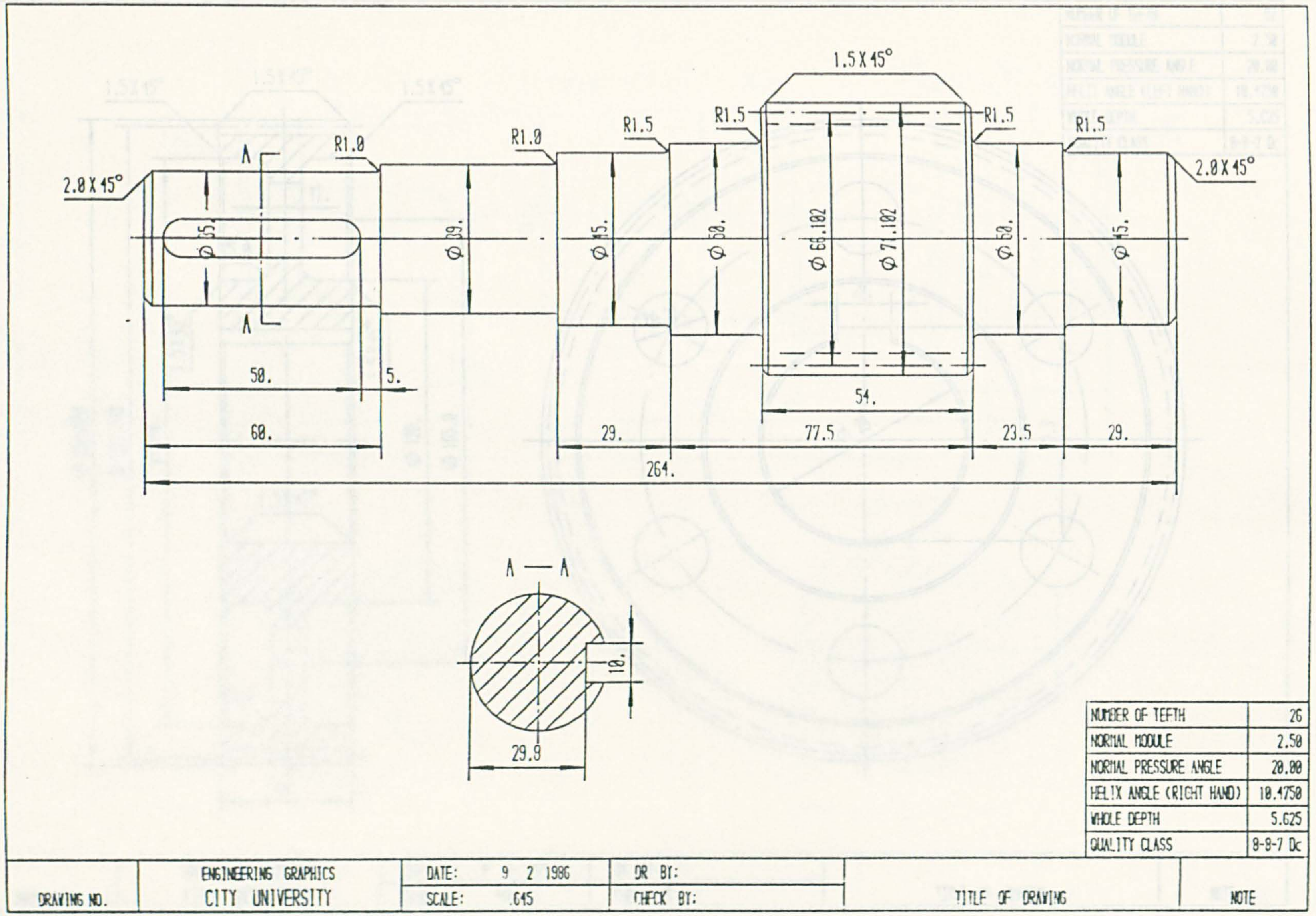



Figure E.3 Computer output of integral pinion shaft working drawing

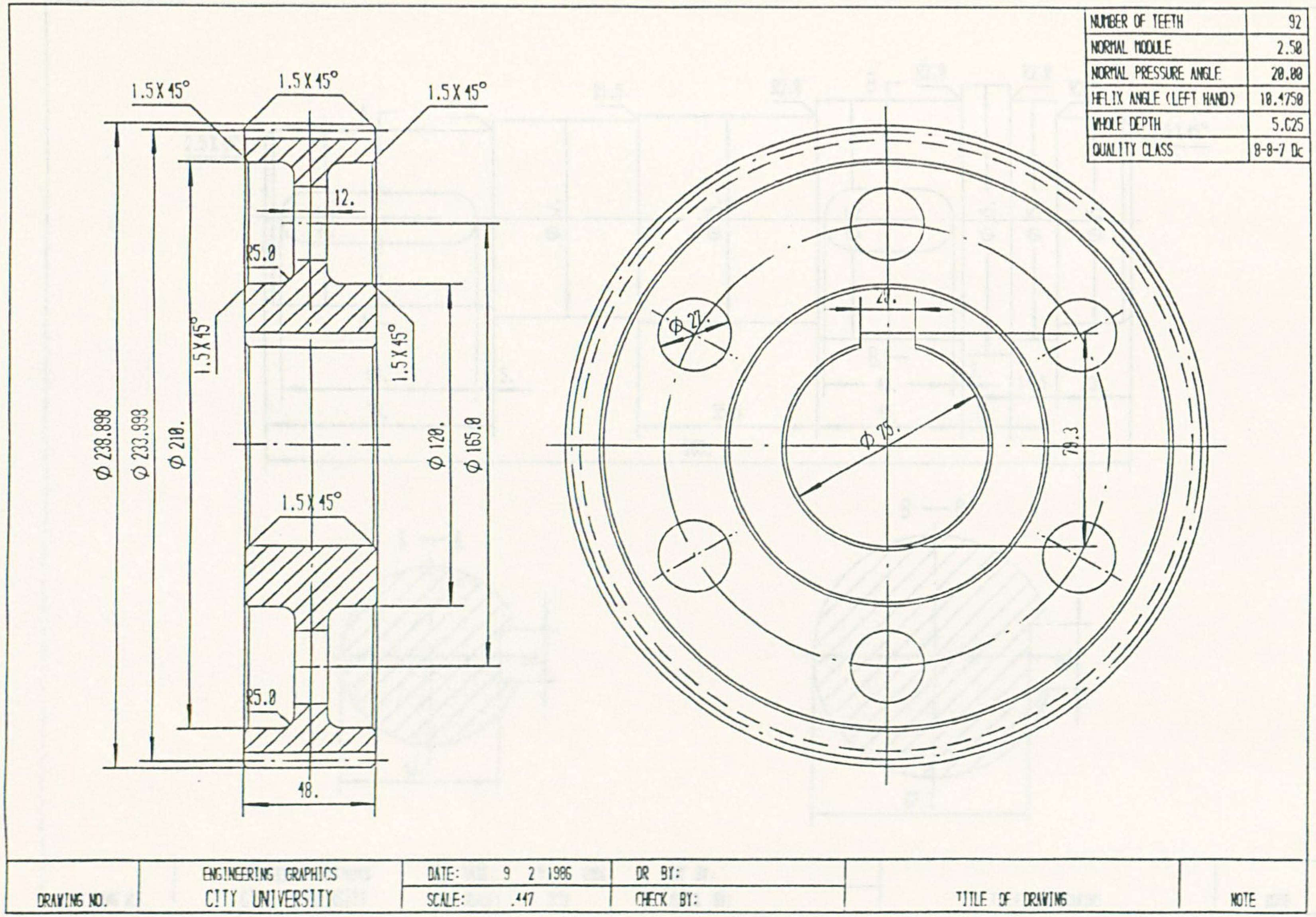



Figure E.4 Computer output of gear working drawing

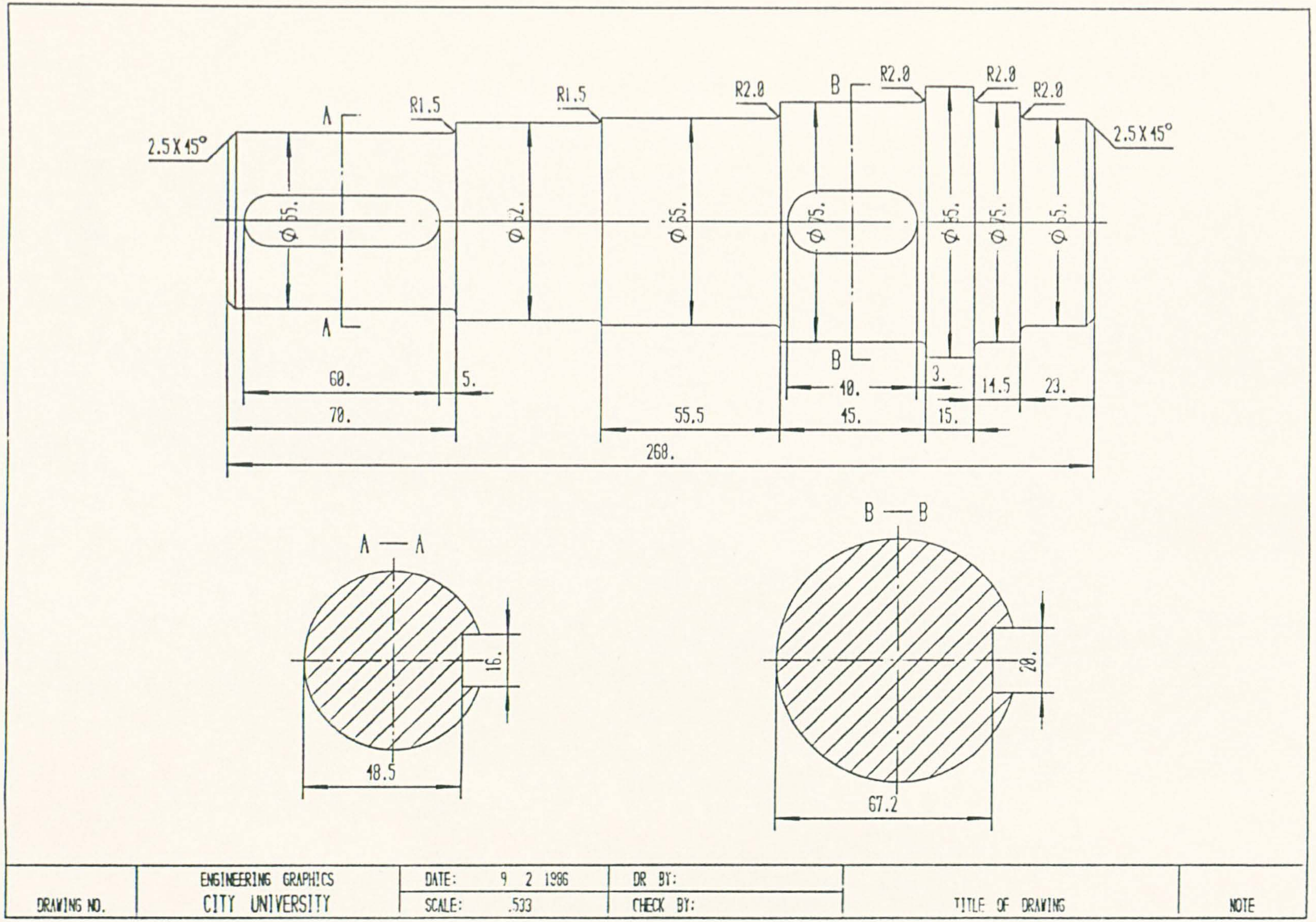



Figure E.5 computer output of output shaft working drawing