Banking Privatization and Market Structure in Brazil: A Dynamic Structural Analysis *†

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Abstract

This paper examines the effects of bank privatization on the number of bank branches operating in small isolated markets in Brazil. We estimate a dynamic game played between Brazilian public and private banks. We find private banks compete with each other as expected. We also find public banks generate positive spillovers for private banks. The latter can at least partly be explained by complementarities between credit products offered by different types of banks in Brazil. Our counterfactual study shows that privatization substantially reduces the number of banks. More than half of the markets in our sample would end up without any bank branch if banks were privatized. The government can mitigate the effects of privatization by providing subsidies to private banks. Our model predicts subsidy policies that reduce operating costs are always more cost-effective than entry costs for isolated markets in Brazil.

JEL Classification Numbers: L11, L13, C14

Keywords: Entry, Exit, Market Structure, Competition, Markovian Games.

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1 Introduction

The rise of the privatization of public, state-owned, entities has been observed since the 1980s in many parts of the world – see Megginson and Netter (2001) for a survey on privatization programs in several countries. The conventional argument for privatization is that private companies are more efficient and able to deliver better products and services for final consumers. Privatization also raises funding that can be used to finance other public areas of need. Privatization is usually a contentious topic that generates political and social tensions. Depending on what is being privatized, there are several different, often subjective, aspects for evaluating the impact of privatization.

This paper studies the potential effects of privatizing public banks on small isolated markets in Brazil. We quantify the potential impact of privatization by studying changes in the number of bank branches[1]. The focus on the number of bank branches operating in small markets is of economic relevance. For instance, Burgess and Pande (2005), Pascali (2012) and Bruhn and Love (2014) have shown causal effects of branch expansion into unbanked rural areas on the reduction of poverty and local development. Additionally, anecdotal evidence suggests that the lack of bank branches in isolated markets has non negligible economic effects in Brazil[2].

To do this we estimate a dynamic entry game between Brazilian public and private banks. We use data from isolated markets in Brazil for 1995-2010 to estimate the primitives of the game. We then use the model to make predictions about the effects of privatization on the number of bank branches. Our model predicts that the privatization of public banks would cause a significant reduction in the total number of bank branches operating in these markets. One way that the government can mitigate this problem is to provide subsidies, to incentivize banks to operate in isolated markets. Our study demonstrates that subsidies to operating costs are more cost-effective than subsidies to entry costs.

The driving forces behind the market structure of the banking sector are complex; see La Porta, Lopez-de-Silanes and Shleifer (2002), Levy-Yeyati, Micco and Panizza (2004, 2007). On the one hand, a strand of the banking literature argues that public banks complement private banks. Public banks focus their activities on segments of the market with high social (but low private) returns, fostering financial development and eventually providing conditions for the growth of private banks. On the other hand, critics of public banks argue that they crowd-out more efficient, more competitive private banks, thus slowing the development of the financial system.

[1]There are other important aspects to privatization in the banking sector. For instance, many studies focus on the operating efficiency and profitability of banks following privatizations. See Clarke et al. (2005) and Megginson (2005) for surveys. Other authors focus on the effects of privatization on economic growth. For example, see Berkowitz et al. (2014) and the references therein.

[2]There have been several recent highly publicized bank robberies in isolated municipalities in Brazil, where the robbers also destroyed the bank during the act of crime. Details of disruptions and effects on economic activities in these cities have been reported, for examples, in: (i) http://diariodonordeste.verdesmares.com.br/cadernos/regional/ataque-a-bancos-em-choro-causa-prejuizos-ao-comercio-1.1398310; and, (ii) http://g1.globo.com/sao-paulo/itapetininga-regiao/noticia/2016/03/comerciantes-reclamam-de-prejuizos-com-explosoes-em-bancos.html

Building on this literature, we first perform a descriptive analysis to look for evidence whether public banks complement or crowd-out private banks in Brazil. We regress observed activity decisions of private banks in each market/year on the number of private and public competitors operating in the same market in the previous period. We find that the number of private competitors in a given market reduces the probability of a private bank being active in the same market but the number of public banks increases this probability. This suggests that while private banks compete with private banks, public banks appear to complement private banks. Therefore we expect that privatization would reduce the overall number of bank branches.

We then set up and estimate a dynamic entry game between Brazilian banks to precisely quantify the effect from privatization. We assume private banks are profit maximizers. It is not clear if public banks are necessarily profit maximizers – see Sapienza (2004), Micco, Panizza and Yañes (2007), Cole (2009) and Carvalho (2009). We therefore take two different approaches to model public banks. In the first, we do not model the objective function of public banks. We take their entry patterns in the data as an exogenous process. In the second, public banks are modeled as profit maximizers analogously to private banks. We estimate the profit functions for all profit maximizing banks.

The model estimates of the competition/complementarity parameters for public and private banks are qualitatively consistent with the descriptive study. For the private banks, we find the entry and operating costs are large and account for a major share of their profit function. When we assume public banks are maximizing profit, their entry cost estimate is similar to the private banks' in size but their operating cost has the wrong sign. There are two ways to interpret this. One, such model is misspecified as public banks are not profit maximizing. Two, public banks are profit maximizers but they receive subsidies from the government. Both of these views are complementary to our institutional knowledge that public banks can act according to a development mandate to operate in areas that are unattractive to private banks.

With the estimates of the primitives in hand, we can solve and simulate our model. Our main counterfactual question is: "What would be the number of bank branches operating in small markets if public banks were privatized?" Our study treats public bank actions as an exogenous process as, in addition to the lack of clarity in the objective function of public banks, this model fits the data better than when we assume public banks are profit maximizing. We find that the average number of bank branches would drop from 1.64 to 0.43 per market after public banks are privatized. Our model highlights three key factors that contribute to this reduction. First, after privatization, private banks lose the positive spillovers from public banks. Second, privatized public banks start competing with existing private banks. Third, entry and operating costs are substantial and private banks may not find it profitable to operate in some small markets even without any competing banks.

The negative effect of privatization on the number of banks can be mitigated if the government provides subsidies to reduce banking costs. We find that it is more cost-effective to subsidize operating costs (OC) than entry costs (EC), in the sense that the former can increase the number of active bank branches at a lower cost than the latter. Our model reveals OC and EC subsidies operate in different ways. A reduction in OC reduces the exit rates but has little effect on entry rates. The policy's main effect is to keep existing banks in the market. In contrast a reduction in EC increases both entry and exit rates. New entrants increase competition, which later on leads to exits.

We end our empirical analysis with a possible explanation for the positive spillover (or complementarity) effect private banks receive from public bank activities. We focus on the role of credit supply. We regress the credit supply of each private bank to the credit supply by its competing private banks and public banks. The qualitative results mirror the activity decisions of banks, namely private banks compete with private banks and public banks complement private banks. By disaggregating credits into subcategories, we find that the main contributors to the complementarity effect come from the public bank supply of credit for mortgages, infrastructures, other durable goods and investment, but not from personal credit and invoice discounting. Our finding is consistent with the institutional design of the Brazilian banking market, where public banks receive large subsidies and are forced by mandate to provide credit for mortgage/infrastructure and durable goods/investment, where they are responsible for approximately 94% of these credits in the market. On the other hand, private banks play a more prominent role in providing credit lines for other categories, such as personal credits and invoice discounting. This may explain why the spillover operates through mortgage/infrastructure and investment and not through personal credit and invoice discounting.

A more difficult question to answer is how increases in the public credit to mortgages, infrastructures and investment lead to the private bank credit supply. One possible explanation for this finding may stem from the demand side complementarities between the credit products provided by public banks and those from private banks. Firms carrying out an expansion plan or individuals investing in a new house may demand different credit lines at the same time. In this case, when public banks increase the credit supply for mortgages, infrastructures and investment, some individuals and firms will respond by simultaneously increasing their demand for credits in market segments where the presence of private banks is (relatively) more important. Unfortunately, to test this conjecture, we would need credit data at the individual/firm level but such dataset is not publicly available.

We estimate our model using the Asymptotic Least Squares (ALS) estimator proposed by Sanches, Silva Junior and Srisuma (2016). This is a two-step estimation procedure that is obtained by minimizing differences in observed and model implied expected payoffs instead of the conditional choice probabilities (CCPs) as proposed in the well-known paper Pesendorfer and Schmidt-Dengler (2008). The most attractive feature of the ALS estimator in Sanches et al. (2016) is that it has a closed-form expression. This estimator is therefore easy to compute and is known a priori to be the global solution to the optimization problem.

Throughout this paper we pay close attention to how unobserved heterogeneity may affect our results. For both our descriptive analysis of bank activities and the choice probabilities, we incorporate market/player unobserved heterogeneity directly through a two-step fixed effects logit approach.

In the first step we estimate a linear probability model using a fixed effects estimator. We then use these fixed effects as a control variable in the logit function in the second step. This parametric approach has been used in Collard-Wexler (2013), Dunne, Klimek, Roberts and Xu (2013), Lin (2015) and Minamihashi (2012) amongst others. A nonparametric alternative of this would be to model unobserved heterogeneity using finite mixtures, e.g. as done in Igami and Yang (2016). However, its practical implementation can be a delicate task, especially when it comes to modeling the number of types that each market must belong to – see Section 4.2 in Igami and Yang (2016) for further details.

The remainder of the paper is organized as follows. Section 2 describes our dataset and the institutional background of the Brazilian banking industry. Section 3 presents descriptive regressions of the banks' activity decisions. Section 4 introduces the model. Section 5 discusses identification and estimation of the model. Section 6 contains results from the counterfactual analysis. Section 7 analyzes possible explanations for the spillovers from public to private banks. Then we conclude the paper. The Appendix contains supplementary materials to support our empirical analysis and modeling choices.

2 Data and Institutional Background

■ **Data.** Our datasets come from the Brazilian Central Bank and from the Brazilian Ministry of Labor. The Brazilian Central Bank database has followed the activities of all Brazilian banks since 1900. These data contain the opening and closing dates and the name of the chain that operates each branch for all branches opened since 1900 in all Brazilian municipalities. The Brazilian Ministry of Labor provides the total payroll data for the formal sector in all Brazilian cities since 1985. We use the payroll data to construct a measure of market size, which we then deflate using the official inflation index, IPCA-IBGE. In what follows a municipality will serve as a market. We will therefore use the terms municipality and market interchangeably.

Following Bresnahan and Reiss (1991), our analysis examines small isolated markets. We select municipalities that are (i) at least 20 km away from the nearest municipality and (ii) that are at least 100 km away from state capitals. We exclude all state capitals and metropolitan areas. We also exclude municipalities that have had more than ten bank branches since 1900 as well as any municipalities with missing bank branch entry/exit date information. We use the data from 1995 to 2010.

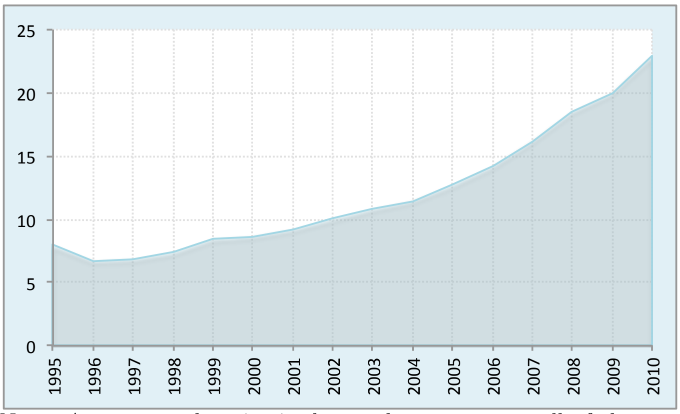
Our sample consists of 1,002 isolated markets. This corresponds to approximately 18% of the total number of municipalities in Brazil. The majority of municipalities in our sample contain either one or no branch per (bank) chain. In the municipalities with more than one branch operated by the same chain, which correspond to less than 4% of the total number of municipalities and around 0.2% of our sample, we aggregate the branch level information for each bank with more than one branch. We use the formal worker payroll to measure market size. All monetary values in this paper are reported in real terms using 2011 as the base year. Table 1 reports some basic statistics of our dataset.

Table 1: Basic Sample Statistics

	Average	5th Percentile	95th Percentile
Active Branches	1.50	0	4
Market Size	11.99	0.38	43.00
Entry	0.045	0	0
Exit	0.065	0	1
Municipalities		1002	
Municipalities × Periods		16032	

The sample has 1,503 branches per year on average (1.5 active branches per market times 1,002 markets). The average entries and exits are respectively 45 and 65 per year. The average market size stands at R$11.99 million (2011). This value is relatively low as we leave out the richest cities by excluding state capitals and metropolitan regions. The second and third columns in the table show that there are large variations in terms of market size and the number of active branches in our sample. We also note that during this period our measure of market size grew by 7.2% per year. Figure 1 illustrates its evolution between 1995 and 2010.

Figure 1: Market Size Evolution 1995-2010



Note: Average market size is the yearly average payroll of the municipalities and measured in R$ millions of Jan/2011. Sample period: 1995-2010.

Table 2 reports (i) the number of markets with a given market configuration (averaged across years between 1995 and 2010); and, (ii) the average market size (yearly average payroll of the municipality) corresponding to each market structure. This illustrates that:

1. Public banks are more frequently located in smaller markets (as measured by the municipality average payroll) than private banks; and,

2. Public banks are frequently the only providers of financial services in these isolated markets (on average 302 markets are public monopolies each year).

The reasons behind these features can be explained by the fact that public and private banks have some intrinsic differences. We shall elaborate on these next.

Table 2: Average Payroll and Number of Public/Private Banks

Nº of Public	Nº of Private	Average Market Size	Average Nº of Markets
0	0	3.45	228.56
0	1	7.68	81.06
0	2	18.18	5.13
1	0	7.39	302.50
1	1	18.19	94.31
1	2	32.78	29.44
2	0	10.20	93.94
2	1	25.23	49.63
2	2	32.30	17.19

Note: Average number of markets is the average number of markets with the corresponding market configuration. The table contains only the most frequent market structures reflecting approximately 90% of the total number of markets.

■ **Institutional background.** In 2010, our sample contains 17 different chains with at least one active branch. The biggest chain that year was the Bank of Brazil (BB), with 670 active branches; then came Bradesco, with 300 active branches; the third largest was Itau with 154 active branches and with the fourth largest being Caixa Economica Federal (CEF), with 122 active branches. BB and CEF are public and controlled by the Federal government. The other two, Bradesco and Itau, are private banks.

Public and private banks compete to provide the same financial services for many segments of the population. However, there are also some fundamental differences. Private banks are essentially profit maximizers. Public banks, by legal mandate, are responsible for providing services to market segments that may not be profitable for private banks. For example, BB plays an important role as the provider of government funding to the Brazilian agriculture sector. It has expanded its operations into smaller and poorer areas of the country based on central government policies aiming at "popularizing" banking services among poor workers and small businesses. In turn, CEF has a monopoly over a number of different government funds with their resources allocated to two main areas: housing and sanitation. CEF is also responsible for the distribution of the Bolsa Família benefits program that provides poor families with a monthly income. This was launched to reduce poverty in the most under-developed areas of the country. These motivations may explain why public banks are generally located in smaller markets and frequently the only financial service providers in these small isolated markets – as indicated by Table 2 (see Coelho, Melo and Rezende (2013) for further discussion of public bank operations determined by legal mandate).

Supplying credit is a traditional source of profits for all banks. To better understand the differences between public and private banks we look into their lending activities. Using data from the Brazilian Central Bank, Table 3 shows the total volume of public/private bank loans (Total Credit) averaged across the years. The credit total is also broken down into different credit lines: personal credit/invoice discounting for individuals/firms; credit for the purchase of durable goods and investment (except agriculture, livestock, mortgages and infrastructures); credit for agricultural production and investment, credit for livestock production and investment; credit for mortgage and infrastructure investment; and other credit lines. Personal credit/invoice discounting generally constitutes short-term loans used for individuals/firms facing cash flow problems. Interest rates for these lines are much higher than credits for investment purposes[3]. The other lines (goods/investment, agriculture, livestock and mortgage/infrastructure) are in general collateralized and used for the purchase of durable goods, production and investment.

Table 3 shows the supply of public and private bank loans overlap in the case of most credit lines. However, the composition of their lending activities differs substantially. In particular: (i) public banks are responsible for 90.6% of the credit in our sample – approximately R$195 billion per year against R$20 billion per year from private banks; (ii) 72% of public bank loans is directed towards the purchase of durable goods, production and investment and only 22% targets short-term operations such as personal credit and invoice discounting. On the other hand, (i) 53.3% of private bank loans are for short-term operations (personal credit and invoice discounting) and (ii) only 46% are for durable goods, investment and production. Note also that all the credits for mortgage and infrastructure lending came from public banks in our sample. Most of this credit is in fact subsidized and supplied by CEF for the reasons explained above. Indeed, interest rates on mortgage and credits for agriculture are, in general, much lower in public banks due to subsidies. For other products, however, private banks are more competitive.

Table 3: Average Annual Lending of Public/Private Banks from 1995-2010

	Variable	Mean	% Total
Public	Total Credit	195,750.00	100%
	Personal Credit/Invoice Discounting	43,400.00	22.17%
	Credit Goods/Investment	11,966.11	6.11%
	Credit Agriculture	79,600.00	40.66%
	Credit Livestock	38,368.75	19.60%
	Mortgage/Infrastructure Credit	11,346.64	5.80%
	Other Credits	11,109.35	5.68%
Private	Total Credit	20,278.14	100%
	Personal Credit/Invoice Discounting	10,838.25	53.45%
	Credit Goods/Investment	1,163.03	5.74%
	Credit Agriculture	5,538.64	27.31%
	Credit Livestock	2,621.53	12.93%
	Mortgage/Infrastructure Credit	-	-
	Other Credits	134.39	0.663%

[3]In 2010, average interest rates for personal loans were around 3.5% per month, 2.8% per month for invoice discounting compared to 1.5% per month for investment. Average rates were calculated using data from the 6 largest commercial banks in Brazil.

In summary, although public and private banks provide many of the same types of services in these markets, they are different in various aspects. These differences mainly stem from the institutional design of the Brazilian market. By mandate, public banks have to operate in smaller markets and supply subsidized credits for infrastructure, mortgage and investment. Private banks are frequently located in larger markets, providing basically short-term credits to individuals and firms. These observations motivate the second step in our analysis, in which we study how these differences between public and private banks affect the ways they interact in the market.

3 Descriptive Analysis

Our goal in this section is to uncover the first order effects of interactions between public and private banks. From the point of view of the banking literature – see La Porta et al. (2002), Levy-Yeyati et al. (2004, 2007) – it remains unclear whether public banks complement or crowd-out private banks. Our analysis in Section 2 also suggests that competition between public and private banks is not necessarily expected a priori. Motivated by this discussion, we would like to know whether public banks complement or crowd-out private banks. The answer to this question can offer a preliminary insight into whether privatization can have a negative impact in terms of reducing the number of bank branches.

Our analysis is based on studying how private bank activities are affected by their past activities and the number of different types of other banks in the market[4]. In addition to controlling for observed covariates, we pay close attention to how unobserved heterogeneity may affect our results. The most direct and transparent way to account for unobserved heterogeneity is to use time varying and time invariant unobserved fixed effects. However, our dependent variable here is binary. It is well-known that nonlinear binary choice models, such as logit and probit, have difficulties in accommodating all these effects due to the incidental parameter problem; e.g. see Heckman (1981). As an alternative, we employ a two-step fixed effects logit estimator, which uses fixed effects estimates from a first step linear probability model (LPM) to control for unobserved effects in the second step logit model. This approach can be seen as a compromise between estimating the linear probability model (LPM) and the ideal logit with fixed effects. A similar strategy was used in Minamihashi (2012), Collard-Wexler (2013) and Lin (2015).

The specification for the first stage LPM is:

$$a_{im}^{t} = \rho_0 + \rho_1 a_{im}^{t-1} + \rho_2 n_m^{pub,t-1} + \rho_3 n_m^{pri,t-1} + \rho_4 \mathbf{x_m^t} + \mu^t + \mu_{im} + \mu_m^t + \zeta_{im}^t. \quad (1)$$

For each municipality m, period t, and private bank i, the dependent variable, a_{im}^t, takes value 1 when the bank was active and zero otherwise; a_{im}^{t-1} is its action in the previous period; $n_m^{pub,t-1}$ and $n_m^{pri,t-1}$ respectively are the bank's number of public and private competitors in the previous

[4]Results for public banks can be found in Appendix A.

period; $\mathbf{x_m^t}$ is a vector of the municipality's characteristics; μ^t, μ_{im} and μ_m^t respectively are time, market/bank and market/time specific effects; ζ_{im}^t is an idiosyncratic term varying across banks, markets and time periods. The parameters to be esimated are $(\rho_0,\rho_1,\rho_2,\rho_3,\boldsymbol{\rho_4})$. The data include all municipalities where bank i was active for at least one period since 1900.

The vector $\mathbf{x_m^t}$ contains the municipality payroll, federal and state government transfers to the municipality, municipal government expenditure and municipality agricultural production. The municipality payroll provides a measure of market size. The inclusion of transfers and municipal expenditure controls for the fact that the entry of public banks can be correlated with an increase of federal/state investment in the municipality, which can also affect entry of private banks. We include agricultural production as a large fraction of the income in our isolated municipalities comes from agricultural activities. This variable represents a different indicator to market size. A fixed effect in this model is μ_{im}. We model the terms μ^t as year dummies while the terms μ_m^t are modeled as an interaction between a time trend and market dummies.

In the second stage we compute the logit estimates obtained by inserting the linear index of observable terms on the right hand side of equation (1) along with an estimate of μ_{im} obtained from the first stage regression inside the logit function. The estimate of μ_{im} controls for other unobserved market/bank effect in each market; our fixed effects are fully flexible in the sense that each market/bank constitutes an unobserved type.

Table 4 reports the estimates for private banks. Only the coefficients associated with $n_m^{pub,t-1}$ and $n_m^{pri,t-1}$ are shown. For the linear model, specification (I) to (IV) increasingly controls for fixed effects, time effects and other observed covariates. For the logit specifications: (I) is a standard logit regression; (II) in addition includes a time dummy in the logit regression; (III) is the two-step fixed effect logit that includes a time dummy; (IV) is the two-step fixed effect logit that furthermore includes other observed covariates. Our two-step fixed effect logit uses the fixed effect from specification (IV) of the linear model.

We see in Table 4 that the coefficients associated with $n_m^{pub,t-1}$ are always *positive* and significant. The coefficients attached to $n_m^{pri,t-1}$ become *negative* and strongly significant after accounting for the market/bank fixed effects. Similar observations that ignoring the unobserved heterogeneity attenuates competition estimates upwards (along the positive real line) have also been reported in the literature. The LPM and logit type estimates are in agreement with the general direction of the estimates. We also note that the coefficients attached to $n_m^{pub,t-1}$ become more pronounced when we control for higher levels of unobserved effects.

Table 4: Activity Probabilities of Private Banks as a Function of the Number of Public Banks and Private Competitors

Linear Probability Models				
	(I)	(II)	(III)	(IV)
Nº Public	0.007***	0.011***	0.019***	0.019***
	[0.00]	[0.00]	[0.00]	[0.00]
Nº Private	-0.001	0.002	-0.027***	-0.028***
	[0.00]	[0.00]	[0.01]	[0.01]
Bank/Market Fixed Effects	No	No	Yes	Yes
Year Dummies	No	Yes	Yes	Yes
Trend*Market Dummies	No	No	Yes	Yes
Transfers, Expenditure, Agric. Prod.	No	No	No	Yes
Observations	22,528	22,528	22,528	22,441
Two-Step Fixed Effects Logit				
	(I)	(II)	(III)	(IV)
Nº Public	0.193***	0.323***	0.461***	0.443***
	[0.04]	[0.04]	[0.05]	[0.04]
Nº Private	-0.010	0.087*	-0.231***	-0.227***
	[0.05]	[0.05]	[0.07]	[0.07]
Bank/Market Fixed Effects	No	No	Yes	Yes
Year Dummies	No	Yes	Yes	Yes
Transfers, Expenditure, Agric. Prod.	No	No	No	Yes
Observations	22,528	22,528	22,528	22,441

Note: (***) Significant at 1%; (**) significant at 5%; (*) significant at 10%. Standard-errors of the two-step models calculated by bootstrap.

Results in Table 4 suggest that private banks compete with other private banks but public banks *complement* them. Competition between private banks is certainly expected, however, complementarity may not be obvious a priori. As seen with the competition estimates, including more controls for unobserved heterogeneity emphasizes the spillover effect. We would expect that if these results were caused by unobserved heterogeneity, the inclusion of time varying and time invariant fixed effects in our models would push the spillover estimates towards the negative side of the real line[5]. It is important to note, however, that descriptive analysis based on regressions with bank activities as dependent variables does not explain the potential source of the spillovers from public to private banks. In Section 7, we will return to this topic and illustrate how the spillovers from public to private banks may operate through the banks' lending activities.

In summary, our descriptive analysis indicates that the privatization of public banks will reduce

[5]We also investigate whether these results – particularly the positive effect on $n_m^{pub,t-1}$– are robust against the potential bias that may arise from the estimation of dynamic models with fixed effects – Nickell (1981) shows that the fixed effects estimator produces biased estimates in dynamic linear panels with large "N" and small "T". To do this, we estimate specification (IV) in Table 11 using the GMM estimator developed in Blundell and Bond (1998). In the first GMM specification we only have lagged activity, a_{im}^{t-1}, in the set of endogenous variables; in the second we have a_{im}^{t-1}, $n_m^{pub,t-1}$ and $n_m^{pri,t-1}$ in the set of endogenous variables. In both models the coefficients associated with $n_m^{pub,t-1}$ are positive and significant at 1%.

the number of branches operating in isolated markets. Existing private banks not only have to compete with the now privatized public banks but they also lose the positive spillovers from public banks. To precisely quantify the effect of privatization we need to develop a dynamic structural model that explains the entry and exit decisions of public and private banks.

4 The Model

This section develops a model to explain the entry and exit decision of Brazilian banks in small isolated markets. We model the activity decisions of public and private banks using a dynamic game of incomplete information, where decisions are made simultaneously[6]. The basic premise for a profit maximizing bank is to optimally decide whether or not to be active in each time period based on its expected discounted profits.

We assume throughout that private banks are profit maximizing. We take two approaches to modeling public banks. First, we take the public bank entry patterns in the data as given and do not model their decision rules. This does not necessarily mean that public banks are not strategic. We simply model their entry decisions as an exogenous stochastic process within our game. In the second, public banks are assumed to be profit maximizers analogous to private banks. We will later compare the estimates and fit to the data in order to choose between the two models.

We characterize the equilibrium of this model and estimate the primitives of the game. We later use the model to quantify the equilibrium number of branches when public banks are privatized. The primitives in our model include entry and operating costs. Their estimates enable us to understand and compare how different cost subsidies can be used to mitigate the potential effects of the privatization. We will now briefly explain why we choose to employ a model of *incomplete information* as opposed to a model of *complete information*, such as Bresnahan and Reiss (1991), to study the market structure of Brazilian banks following privatizations.

At the end of Section 3, we highlight two distinct effects that privatization can have on the market structure. Firstly, private banks will lose the spillovers from public banks. Then, newly privatized banks will exert competitive pressures on the market. Our *incomplete information* model accounts for both effects explicitly because we distinguish between private and public banks; thus we are able to make clear the role of incumbents and potential entrants for both types of banks. This stands in contrast to a two-period model of *complete information*, which uses the zero profit condition to determine the number of firms in equilibrium without having to model the identity of incumbents or potential entrants. More specifically, these models do not capture well the competitive pressures that arise from when potential entrants change status from public to private banks. To illustrate this point, consider a market with no active public banks before privatization. Suppose now that all

[6]From 1990 - 2010, public banks were first entrants in 53% of the markets in our sample. If either public or private banks were first entrants at a much higher percentage than the other, then a sequential game would be a more suitable model.

public banks are privatized. Then the number of public banks in that market is zero both *ex-ante* and *ex-post*, meaning there is no change in the equilibrium structure of this market through the lens of a two-period complete information model. In a model of incomplete information, like ours, the whole probability distribution of the number of firms in equilibrium will change with the change in the status of potential entrants. This is because, after privatization, the incumbent banks will update their beliefs in response to a change in the status of the potential entrants, leading to changes in their entry/exit decisions. Furthermore, a two-period model, which uses a reduced-form profit function to model the long-run profit, cannot be used to perform our second set of counterfactual study that compares the effects of different cost subsidies. More specifically, these models aggregate both entry costs and operating costs into the same *fixed costs* parameter. Thus they are not able to distinguish between policies that target entry costs from operating costs and vice versa.

4.1 Elements of the Game

The elements of the game are as follows. Time is discrete, $t = 1, 2, ..., \infty$. There are $m \in \mathbf{M} = \left\{1, 2, 3, .., \overline{M}\right\}$ markets. In each market, there is a total of $N_{pub} + N_{pri}$ banks. We denote the total number of public and private banks by N_{pub} and N_{pri} respectively. The set of public and private banks is indexed by i_{pub} and i_{pri}. A bank's action in market m, period t is denoted by $a^t_{im} \in \{0, 1\}$, where 0 means that the bank is inactive; and 1 means the bank is active. The $1 \times (N_{pri} + N_{pub})$ vector $\mathbf{a^t_m}$ denotes the action profile in market m, period t. We sometimes use $\mathbf{a^t_{-im}}$ to denote the actions of all banks other than bank i. We use $\mathbf{s^t_m}$ to denote a vector of the state space in market m, which includes past actions as well as other variables such as demand shifters, $\mathbf{x^t_m}$. When necessary we use N_s to express the number of different possible states in market m. Note that there are only a finite number of states in our model. The vector $\mathbf{s^t_m}$ evolves according to the transition matrix $p_m(\mathbf{s^{t+1}_m}|\mathbf{s^t_m}, \mathbf{a^t_m})$, described by the next period distribution of possible values for the vector $\mathbf{s^t_m}$ conditional on each possible current state and actions in municipality m. We sometimes use $\mathbf{p_m}$ to denote the vector of transitions, $p_m(\mathbf{s^{t+1}_m}|\mathbf{s^t_m}, \mathbf{a^t_m})$, for every possible future state $\mathbf{s^{t+1}_m}$ given any possible combinations of $(\mathbf{s^t_m}, \mathbf{a^t_m})$. In each period, each bank draws a profitability shock ε^t_{im}. The shock is privately observed while its distribution is publicly known.

A private bank's period payoff is:

$$\Pi_{im}(\mathbf{a^t_m}, \mathbf{s^t_m}, \boldsymbol{\varepsilon^t_{im}}; \boldsymbol{\Theta_{im}}) = \pi_{im}(\mathbf{a^t_m}, \mathbf{x^t_m}) + a^t_{im}\varepsilon^t_{im} + a^t_{im}\left(1 - a^{t-1}_{im}\right)F$$

Here, $\pi_{im}(\mathbf{a^t_m}, \mathbf{x^t_m})$ denotes bank i's deterministic profits in market m and F denotes entry costs. $\boldsymbol{\Theta_{im}}$ denotes the parameters in the model. An incumbent bank deciding to stay in the market receives period profits of $\pi_{im}(a^t_{im} = 1, \mathbf{a^t_{-im}}, \mathbf{x^t_m}) + \varepsilon^t_{im}$. A new bank entering (or re-entering) the market in addition has to pay a sunk entry cost F. Since operating costs, entry costs, and scrap values cannot be jointly identified (e.g. see Aguirregabiria and Suzuki (2014) and Komarova, Sanches, Silva Junior and Srisuma (2017)), we assume that banks leaving the market get a scrap value of zero. We specify $\pi_{im}(\mathbf{a^t_m}, \mathbf{x^t_m})$ as follows:

$$\pi_{im}(\mathbf{a^t_m}, \mathbf{x^t_m}) = a^t_{im}\left(\theta_0 + \theta_1^{pub}\left(\sum_{j\in i_{pub}} a^t_{jm}\right) + \theta_1^{pri}\left(\sum_{j\neq i, j\in i_{pri}} a^t_{jm}\right) + \theta_2 x^t_m\right). \tag{2}$$

Here, $\theta^k \in \mathbb{R}^k$ are parameters and x^t_m is a demand shifter. In our application, the demand shifter is the municipality payroll. We interpret the constant, θ_0, as the operating costs associated to action $a^t_{im} = 1$. The parameters θ_1^{pub} and θ_1^{pri} capture respectively the effects of a new public bank and of a new private bank on the payoffs of bank i. This specification allows for the different "competition" effects of public and private banks. The profitability shock ε^t_{im} has two components:

$$\varepsilon^t_{im} = \mu_{im} + \xi^t_{im}, \tag{3}$$

where, μ_{im} is a term that varies only across markets and banks but not over time and ξ^t_{im} is an iid extreme value variable randomly drawn across banks, time and markets. ξ^t_{im} is the only source of asymmetric information in the model. The term μ_{im} is known to the banks. This captures the correlation of the profitability shocks for the same bank in the same market across time. With this formulation for the profitability shock, operating costs of bank i can be written as $(\theta_0 + \mu_{im})$, i.e., we allow the operating costs to vary across players and markets. The payoff parameters for a private bank are $\mathbf{\Theta_{im}} = \left(F, \theta_0, \theta_1^{pub}, \theta_1^{pri}, \theta_2, \mu_{im}\right)$. The period payoff is discounted by the factor $\beta \in [0, 1)$ after each time period. When public banks are profit maximizers, we model their profit function analogously to private banks; we allow private and public banks to possibly have different payoff parameters.

The game then proceeds as follows:

1. States are observed.
2. Each competing bank draws a private profitability shock ε^t_{im}.
3. The actions of all banks are simultaneously chosen.
4. After these actions are chosen, the law of motion for $\mathbf{s^t_m}$ determines the distribution of states in the next period; the problem then restarts.

4.2 Equilibrium

We restrict our attention only to *stationary pure Markovian strategies.* This means that bank decisions at time t only depends on $(\mathbf{s^t_m}, \varepsilon^t_{im})$, past information does not matter, and their optimal actions will be the same in any other time periods whenever they have the same draw of the state variables as $(\mathbf{s_m^t}, \varepsilon_{im}^t)$. Therefore bank i's best response solves the following Bellman equation:

$$V_i(\mathbf{s_m^t}, \varepsilon_{\mathbf{im}}^{\mathbf{t}}; \sigma_{\mathbf{im}}, \mathbf{p_m}, \mathbf{\Theta_{im}}) = \underset{a_i^t=k\in\{0,1\}}{Max} \left\{ \begin{array}{c} \sum_{\mathbf{a_{-im}^t}} \sigma_{im}(\mathbf{a_{-im}^t}|\mathbf{s_m^t})\Pi(a_{im}^t{=}k, \mathbf{a_{-im}^t}, \varepsilon_{\mathbf{im}}^{\mathbf{t}}, \mathbf{s_m^t}; \mathbf{\Theta_{im}}) + \\ \beta \mathbf{z_k}\left(\mathbf{s_m^{t+1}}|\mathbf{s_m^t}; \sigma_{\mathbf{im}}, \mathbf{p_m}\right) \mathbf{E}_\xi \mathbf{V_{im}}\left(\sigma_{\mathbf{im}}, \mathbf{p_m}\right) \end{array} \right\}. \quad (4)$$

Here $\Pi(\cdot)$ is the bank's period payoff; the function $\sigma_{im}(\mathbf{a_{-im}^t}|\mathbf{s_m^t})$ accounts for i's beliefs on other private and public bank actions given current states; $\sigma_{\mathbf{im}}$ is a vector that contains the beliefs for all possible combinations of actions given any possible state in market m; $\mathbf{z_k}\left(\mathbf{s_m^{t+1}}|\mathbf{s_m^t}; \sigma_{\mathbf{im}}, \mathbf{p_m}\right)$ is a $1 \times N_s$ vector containing the transitions $\sigma_{im}(\mathbf{a_{-im}^t}|\mathbf{s_m^t})p_m(\mathbf{s_m^{t+1}}|a_{im}^t = k, \mathbf{a_{-im}^t}, \mathbf{s_m^t})$ and $\mathbf{E}_\xi \mathbf{V_{im}}\left(\sigma_{\mathbf{im}}, \mathbf{p_m}\right)$ is a $N_s \times 1$ vector with the expected continuation value for private bank i in market m, $E_\xi V_i(\mathbf{s_m^{t+1}}; \sigma_{\mathbf{im}}, \mathbf{p_m}, \mathbf{\Theta_{im}})$, for all $\mathbf{s_m^{t+1}}$.

When we do not model public bank explicitly as profit maximizers, the equilibrium of the game can be found by solving (4) for private banks. We take the beliefs of private banks on the behavior of public banks as given from the data. When we model public banks as profit maximizers, problem (4) is also solved for public banks. In the latter case, the equilibrium beliefs of both public and private banks must be consistent with the solution of problem (4) for all banks.

Formally, the solution to this dynamic problem is a collection of vectors of all strategic bank's optimal actions when this bank faces each possible configuration for the state vector $\mathbf{s_m^t}$ and has consistent beliefs about other banks actions in the same states of the world. A proof of the existence of such vector follows from the equilibrium existence result in Aguirregabiria and Mira (2007) and Pesendorfer and Schmidt-Dengler (2008). Equilibrium uniqueness, however, is not guaranteed. This is a common feature of this class of dynamic entry games.

5 Identification and Estimation

This section describes the identification and estimation of the model parameters. We begin by discussing the identification of our models in Subsection 5.1. Our identification strategy is constructive and guides estimation. We describe the estimation of the parameters focusing on the Conditional Choice Probabilities (CCPs) in Subsection 5.2. We report and discuss the structural estimates in Subsection 5.3.

5.1 Identification

Our identification strategy follows Pesendorfer and Schmidt-Dengler (2008). We proceed in two steps. We first identify the CCPs, i.e. the vector of activity probabilities for public and private banks, and the transition probabilities directly from the data. The identification of period payoff parameters can then be verified using matrix algebra as described in Section 5 of Pesendorfer and Schmidt-Dengler (2008).

We identify the CCPs and the transitional probabilities by pooling data across different markets. We assume the same equilibrium is played in each market. In order to ensure the single equilibrium assumption is not a priori violated, we do not pool markets with different numbers of potential players, which is defined as the sum of N_{pri} and N_{pub} that respectively denote the potential number of private and public banks. We only use data from markets where $N_{pri} = 2$ and $N_{pub} = 2$. This market capacity serves to answer the primary interest of our paper, which is to understand how the interactions between private-private and private-public banks affect the banking market structure.

The potential number of banks in each market is not observed and have to be estimated. We estimate them by taking the maximum of the number of banks observed across all time periods in the sample.[7] In our sample, we have 46 markets where the potential number of private and public banks are both two. There are practical benefits despite us having to reduce the sample size. First, as the cardinality of our state space is proportional to the number of markets, reducing the number of markets implies a reduction in the state space of our models and makes our counterfactual exercises feasible. Second, this subset is necessarily more homogenous than the entire sample. This latter point helps to alleviate problems related to unobserved heterogeneity.

5.2 Estimation

Our estimation procedure takes two steps. First, we estimate the choice and transition probabilities. Once these are available we follow Sanches et al. (2016), who show that, when the payoff function is linear in the parameters as in (2), the parameters of interest can be estimated in closed-form using an OLS formula. We refer the reader to Sanches et al. (2016) for other implementation and statistical details of our estimator. We will only discuss the estimation of the choice and transitoin probabilities.

For the CCPs, we use the same two-step fixed effects logit approach to account for unobserved heterogeneity as done in Section 3. The CCP specification is:

$$P\left(a_{im}^{t} = 1 | a_{im}^{t-1}, n_{m}^{pub,t-1}, n_{m}^{pri,t-1}, x_{m}^{t}, \mu_{im}; \rho\right) = \Lambda\left(\rho_0 + \rho_1 a_{im}^{t-1} + \rho_2 n_{m}^{pub,t-1} + \rho_3 n_{m}^{pri,t-1} + \rho_{\mathbf{4}} x_{m}^{t} + \mu_{im}\right), \tag{5}$$

where $\left(a_{im}^{t}, a_{im}^{t-1}, n_{m}^{pub,t-1}, n_{m}^{pri,t-1}\right)$ are defined as in equation (1), and x_{m}^{t} is the municipality payroll; μ_{im} captures the market/bank fixed effects. $\Lambda(\cdot)$ is the logistic distribution and $\rho = (\rho_0, \rho_1, \rho_2, \rho_3, \rho_{\mathbf{4}})$ denote the parameters to be estimated. The market fixed effects are not observed. We first estimate μ_{im} using a linear probability model, where we use the full set of covariates in $\mathbf{x_m^t}$ along with fixed and time effects. I.e. we used the same specification as column (IV) in Table 4 (and also Table 11 in Appendix A) for the 46 markets with $N_{pri} = 2$ and $N_{pub} = 2$. We then use these fixed effects as a control variable in the logit function. This variable controls for 92 unobserved market/private bank

[7] This is a super-consistent estimator in the sense that it converges at a rate faster than $\sqrt{T}$. It has been used in Pesendorfer and Schmidt-Dengler (2003) and Dunne et al. (2013).

Table 5: Logits/two-step Fixed Effects Logit CCPs

	Public Banks		Private Banks	
	(I)	(II)	(I)	(II)
Lagged Activity	6.798***	6.240***	6.525***	5.812***
	[0.28]	[0.32]	[0.39]	[0.27]
Nº Public	-0.818***	-1.675***	0.452*	0.453*
	[0.27]	[0.39]	[0.26]	[0.24]
Nº Private	-0.018	-0.141	-0.326	-1.350**
	[0.26]	[0.19]	[0.43]	[0.64]
Market Payroll	0.009	0.002	0.016*	0.002
	[0.01]	[0.01]	[0.01]	[0.01]
Market/Bank Fixed Effect		5.600***		10.367***
		[0.91]		[2.28]
Constant	-3.459***	-2.502***	-4.260***	-3.743***
	[0.41]	[0.50]	[0.44]	[0.45]
Observations	1,472	1,470	1,472	1,470

Note: (***) Significant at 1%; (**) significant at 5%; (*) significant at 10%.

types and 92 unobserved market/public bank types – two public and two private banks in each of the 46 markets.

The CCPs are shown in Table 5. For each bank type, we report the estimates for models with and without our control for market/bank fixed effects. The lagged activities and the control for market/bank fixed effects are clearly important factors in determining the entry/activity probabilities of banks. The former can partly be attributed to fixed costs, such as sunk and operating costs, which we shall estimate in the second step. Ignoring the fixed effects would lead to a substantial understatement of the competition effects that banks have on each other. For both types of banks, the coefficients attached to $n_m^{pub,t-1}$ and $n_m^{pri,t-1}$ are qualitatively the same as those shown in Section 3 (for private banks) and in Appendix A (for public banks); our comments and interpretations given in those sections also apply here. Subsequently, in what follows,we use only those CCPs that control for market/bank heterogeneity.[8]

The vector of state variables $\mathbf{s}_{im}^t$ for any bank consists of $\left(a_{im}^{t-1}, \mathbf{a_{-im}^{t-1}}, x_m^t, u_{im}\right)$ where: u_{im} is the bank/market fixed effect that we can estimate according to the procedure described above; a_{im}^{t-1} is bank i's action in market m in $t-1$, $\mathbf{a_{-im}^{t-1}}$ are the actions of bank i's public and private competitors in the same market in period $t-1$ and x_m^t is the market payroll.

The law of motion for x_m^t is calculated using an auto-regressive ordered logit structure. The variable x_m^t can take 10 possible values. Its support corresponds to the observed municipality payroll in the last ten years for each municipality.

Given the CCP and the transition probabilities estimates described above, we can compute the expected discounted payoffs for each element of the state space. The structural parameters estimates

[8]We provide further robustness checks by including different controls for observed market level heterogeneity and different formulations for the market/bank fixed effect variable in Appendix B.

are then obtained using the OLS expression as given in equation (7) in Sanches et al. (2016).

5.3 Structural Estimates

We now present the payoff parameters estimates. To compute our estimates we set the discount factor to be 0.9. All standard-errors in this section are calculated by block bootstrapping CCPs and state transitions 50 times. Our estimates do not have a level interpretation due to the scale of the extreme value distribution of the profit shock. Hence, only the relative magnitudes matter. To facilitate the interpretation of the coefficients, in the second and fourth columns of the tables below, we show the coefficient divided by the absolute value of the entry costs estimate. All the models estimated in this section include a set of 91 market/player fixed effects[9] (denoted by μ_{im} in equations (3) and (7)). For brevity, they are not shown. Table 6 below reports the estimates for both private and public banks.

■ **Private banks.** The model predicts that a new private competitor entering the market reduces private bank profits by 11% of the entry costs. In turn, the entry of a new public bank increases the profits of a private incumbent by 4.3% of the entry cost. The signs of the coefficients measuring the effects of entry of public and private banks on the profits of a private incumbent are consistent with the descriptive evidence reported in Section 3. The constant term, which measures operating costs, is negative and relatively large. Entry costs are also negative and large as expected. Market experts estimate that the costs of opening a new bank branch vary somewhere around R\$1.5 million[10]. This value is relatively high compared to the market size of an average municipality in our sample (around R\$11 million per year; see Section 2). The size of these cost estimates may help rationalize the fact that the presence of private banks in small isolated markets is low.

■ **Public banks.** When we assume that public banks are profit maximizers, the model predicts that both new public and private competitors reduce the profits of public incumbents by comparable amount, respectively by 11.8% and 9.2% of the entry costs. Public banks' entry costs are negative and slightly larger than the entry costs of private players. An important difference between the estimates for public and private banks is that the constant, which measures operating costs, is positive and relatively large for public banks. There are two possible explanations for this unusual finding. First, the objective function/behavior of public banks is misspecified. Public banks are in fact not profit maximizers. Second, if public banks are profit maximizers, non-negative operating costs can be rationalized by the existence of subsidies for public bank to operate in small isolated markets. In order to decide whether we should perform our counterfactual study with public banks acting as a profit maximizer, we will solve the dynamic models (i) taking the behavior of public banks as given (as an exogenous stochastic process) and (ii) assuming that public banks are profit maximizers, and

[9]We have 46 different markets where $N_{pri} = 2$ and $N_{pub} = 2$. For each market we have two different private/public banks. We exclude one of the market/bank fixed effect to avoid perfect collinearity.

[10]See: http://exame.abril.com.br/negocios/bradesco-planeja-abrir-mais-1-500-correspondentes-bancarios/.

Table 6: Structural Parameters

	Private Banks		Public Banks	
	Coefficients	Coef/Entry Cost	Coefficients	Coef/Entry Cost
Nº Public	0.190	4.3%	-0.565	-11.8%
	[0.10]		[0.10]	
Nº Private	-0.487	-11.0%	-0.437	-9.2%
	[0.31]		[0.16]	
Market Payroll	0.001	0.0%	0.004	0.01%
	[0.00]		[0.00]	
Constant	-0.732	-16.5%	0.319	6.7%
	[0.18]		[0.18]	
Entry Costs	-4.442	-100.0%	-4.769	-100.0%
	[0.21]		[0.30]	
Market/Bank Fixed Effect	Yes		Yes	

Note: The column labeled Coeff/Entry Costs reports the coefficients as a % of the absolute value of entry costs.

compare the moments generated by both models to the data.

6 Counterfactual Studies

We now use the model to analyze how the privatization of public banks would affect the number of bank branches operating in small isolated markets. We also study how costs subsidies to entry and operating costs of bank branches can mitigate the impact of privatization. We then perform a sensitivity analysis comparing the predictions of our model to models built alternative assumptions.

To perform the counterfactual exercises we solve the model with the relevant configuration of its structural parameters. We then use the bank optimal decision rules to forward simulate the game. Our analysis will be based on the implied average number of private and public banks in the market, as well as their entry and exit rates.

6.1 Model Fit

We begin by comparing two models that differ in how we model public banks. For the first model, we take the behavior of public banks as given in the data and solve the model for the activity probabilities of private banks. For each market the solution to the model is a vector of $N_{pri} \cdot N_s$ entry probabilities that solves the system of best responses derived from problem (4). For the second model, public banks are assumed to be profit maximizers. The solution of this model is a vector of $(N_{pri} + N_{pub}) \cdot N_s$ entry probabilities for private and public players that solves an analogous system of best responses.

For each market in our sample, we simulate the average number of private and public banks, private and public bank entries, and private and public bank exits from 1995 to 2010. Starting from the state vector of each market in 1995 we simulate 100 paths for these variables for 16 time periods.

Table 7: Model Fit

		Behavior of Public Banks Fixed		Profit Maximizer Public Banks	
	Real Data (1995-2010)	Simulated Data	Difference	Simulated Data	Difference
Private Branches	0.568	0.525	-0.042	0.595	0.027
Public Branches	1.163	1.109	-0.054	1.409	0.246
Total Branches	1.731	1.635	-0.096	2.004	0.273
Entry Rate Private	0.042	0.030	-0.012	0.035	-0.007
Entry Rate Public	0.016	0.019	0.003	0.012	-0.004
Entry Rate Total	0.058	0.049	-0.009	0.047	-0.012
Exit Rate Private	0.030	0.033	0.003	0.030	0.000
Exit Rate Public	0.061	0.062	0.001	0.029	-0.033
Exit Rate Total	0.091	0.095	0.004	0.059	-0.032

Note: Data are simulated taking the state vector in each market in 1995 as the initial conditions.

We compute the average value across paths, years and markets to generate the relevant moments.

The equilibria we compute are locally stable. We check for stability by re-solving our models in the following way: first, we solve the model for the entry probabilities using the logit probabilities as the initial guess; second, we perturb the logit probabilities; third, we compute again the solution for the model using the "perturbed" vector of logit probabilities as the initial guess; fourth, we compare the "perturbed" solution with the original solution. In all such experiments, the resulting equilibrium did not change.

Table 7 compares the model moments and the data moments. The first column contains the relevant averages from the observed data. The second and third columns give the same statistics from the model we treat the behavior of public banks as given and their differences between the observed data, respectively. The fourth and fifth columns report the analogous statistics when we solve the game with public banks acting as profit maximizers.

The simulated moments implied by the model are generally close to those observed in the data when we treat the behavior of public banks as given. When public banks are assumed to be profit maximizers, we observe a similar prediction for the moments of private banks. However, the simulated moments for public banks perform substantially worse as some are off by more than an order of magnitude when compared to the model where the behavior of public banks is treated as given. The aggregate differences are in favor of the model where we do not model public banks as profit maximizers in both absolute and relative terms. In what follows, we use only the model where the behavior of public banks is taken as given.

6.2 Counterfactuals

Our privatization counterfactual study assumes that payoff parameters are invariant to the policy change. The newly privatized public banks use the same policy functions as the existing private banks. We find the optimal decisions for all banks by solving a system of best responses analogous to equation (4). We use the same procedures to simulate moments and to verify that our equilibria are locally stable as described in the study of model fit above.

■ **Privatization.** The first column of 8 contains the simulated moments prior to privatization, which is identical to those in Table 7. The second column of Table 8 reports the moments from the privatization experiment.

Our model predicts that privatization would reduce the average number of bank branches from 1.64 to 0.43. What causes this reduction? At the end of our descriptive analysis section we alluded to two different channels that can contribute to the reduction of bank branches. First, when public banks are privatized, the spillover from public to private banks disappears. In what follows, we refer to this effect as the "spillover effect". Second, the privatized public banks begin operating as private banks and thereby increasing the competitive pressures on the market. We refer to this effect as the "competitive effect".

Table 8: Counterfactual Experiments: Privatization

	Simulated	Privatization	No Spillover
Private Branches	0.525	0.430	0.301
Public Branches	1.109		1.120
Total Branches	1.635	0.430	1.421
Entry Rate Private	0.030	0.037	0.019
Entry Rate Public	0.019		0.019
Entry Rate Total	0.049	0.037	0.039
Exit Rate Private	0.033	0.157	0.039
Exit Rate Public	0.062		0.061
Exit Rate Total	0.095	0.157	0.101

Note: Data are simulated taking the state vector in each market in 1995 as the initial conditions.

Next, we attempt to quantify the impact from the spillover effect. We do this by solving and simulating the game while θ_1^{pub} is constrained to be zero. The third column of Table 8 reports the relevant moments of this exercise. We see that, without the spillover effect, the average number of bank branches operating in the market falls slightly from 1.6 to 1.4. This suggests the competition effect cannot on its own account for the prediction that there will only be 0.4 private banks left per market either. Therefore our model implies public banks must be willing to operate as the lone bank in some markets where private banks are not. A reason for this could be due to high entry and/or operating costs. Our institutional knowledge of the Brazilian market would support this view as public banks are known to receive subsidies to operate in some isolated markets. We next analyze two cost subsidies the government be can used to mitigate the effect of privatization.

■ **Subsidies.** The first policy is a subsidy for operating costs (OC). The second is a subsidy for sunk entry costs (EC). We are mainly interested in understanding (i) the effects of these policies on the entry and exit decisions of private banks following privatization, and (ii) the cost-effectiveness of each policy – i.e. the cost of each policy vis-a-vis their impacts on the equilibrium number of branches.

Specifically, what we do is to reduce the entry and operating costs by different factors and compute the privatization counterfactuals according to different entry and operating cost configurations. The results of this analysis are shown in Table 9.

Table 9: Counterfactual Experiments: Subsidies

OC Factor	EC Factor	Branches	Entry Rate	Exit Rate	Subsidy/Branches
1.00	1.00	0.430	0.037	0.157	0.000
1.00	0.90	0.448	0.054	0.169	0.569
1.00	0.80	0.478	0.078	0.191	0.612
1.00	0.70	0.521	0.109	0.219	0.682
0.90	1.00	0.519	0.041	0.154	0.229
0.80	1.00	0.663	0.043	0.147	0.215
0.70	1.00	0.886	0.042	0.131	0.211

Note: Data are simulated taking as initial conditions the state vector in each market in 1995.

The first two columns have the factors we are using to multiply the operating costs (OC Factor) and the sunk entry costs (EC factor). Columns 3-5 contain, respectively, the average number of branches (across markets and periods), the average entry rates and the average exit rates. The last column has our measure of the cost-effectiveness of each policy. This is defined as the total policy cost (in present values) divided by the increase in the total number of bank branches induced by the subsidies. The numbers in the first row of Table 9 (with OC and EC costs equal to one) are equal to the numbers in the second column of Table 8. The second, third and fourth rows present results for counterfactual policies where, after privatization, banks receive a subsidy equal to 10%, 20% and 30% of their original entry costs, respectively. The last three rows present analogous results for the scenario where banks receive subsidies for their operating costs.

From Table 9, we see that any reduction in OCs increases the entry rate by only a small amount relative to lowering the exit rate. Therefore the policy's main effect is keeping existing banks in the market. On the other hand, a reduction in ECs increases both entry and exit rates on a comparable scale. New private bank entrants increase competition that then leads to exits. Taking the absolute values of EC and OC into account, the overall implication of this exercise is summarized by the last column in Table 9. It shows that subsidies to OC are more cost-effective than subsidies to EC. An increase in bank branches through reduction in OC is around 3 times more cost-effective than EC subsidies. Furthermore, we also see that EC subsidies appear to have decreasing returns, while OC subsidies have increasing returns – i.e. the cost-effectiveness of EC (OC) subsidies decreases (increases) in accordance with the total value of the subsidies.

6.3 Robustness Checks

Our counterfactual studies thus far are based on a dynamic game under a stationary environment. We also compare the performance of our model with two other models, one static and one non-stationary. We provide a summary of them below. A complete description of the two models, along with estimation results, can be found in Appendix C and D, respectively. The remaining part of this Section will focus on counterfactual predictions using different parameter estimates related to the non-stationary model as a robustness check.

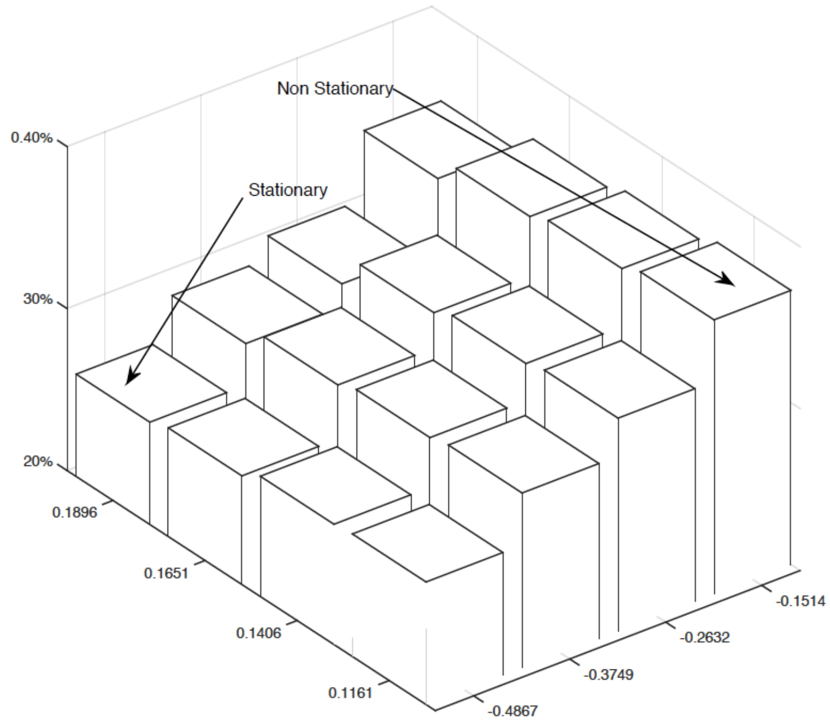
In order to highlight the importance of dynamics in our model Appendix C compares the results above with a static model. The static model is a version of our model where the discount factor is set to zero – see Seim (2006). There, we show that the estimates of the two models are similar for most parameters, but the static model estimates the operating cost to be very close to zero. This magnitude of the latter cost is hard to justify as high operating costs is a well-known reason for the lack of bank branches in small markets in Brazil; e.g. see Andrade (2007), Gonçalves and Sawaya (2007), and Gouvea (2007). Numerically, the reason for this can be traced to the fact that continuation values for banks are found to be positive in the dynamic model, and the static model compensates for their absence by reducing the operating costs. Furthermore, the static model generates moments that fit the data worse than the dynamic model. Even with a lower estimate of operating costs, the static model underestimates entry and overestimates exit of private banks compared to the dynamic model. The latter observation may be explained by the fact that the parameter capturing competition between private banks is much larger in the static model than the estimate in the dynamic model.

We consider a non-stationary model in Appendix D. There, we propose and estimate a finite-horizon non-stationary game where we let the market payroll (x_m^t) grow deterministically and the shock (ε_{im}^t) is allowed to have a time trend component. Specific to our dataset, we have at least two reasons to believe some of our variables may be non-stationary. First, Section 2 shows that our demand shifter, in the form of the payroll, grows over time. Our analysis in Section 3 also shows that the estimates of the descriptive regressions explaining the activity decisions of private banks are sensitive to the inclusion of different controls for time varying unobserved heterogeneity. Moreover, in some specifications of these regressions where we included a time trend instead of year dummies[11], we find that the coefficient attached to the time trend is positive and significant at 1%. This may be explained by the fact that bank technologies improve over time (the advent of ATMs, credit cards etc.). The implementation of modern banking activities can reduce operating costs and increase bank profits in all markets over time.

The estimates of the non-stationary model are qualitatively the same as the estimates of the stationary model. In particular, the non-stationary estimates of competition/spillovers are consistent with the results presented in Table 6. The main difference is that the non-stationary estimates of competition/spillovers are lower (in absolute values) than the stationary ones. However, we do not perform our counterfactual study directly with the non-stationary model due to the complications described in Igami (2014). Instead, as a sensitivity analysis, we rerun the privatization counterfactuals

[11]For brevity, we do not show these specifications in Section 3.

Figure 2: Sensitivity Analysis



Note: Each bar shows the ratio between the total number of branches after and before privatization for the different configurations of the competition/spillover effect. The left axis displays the different values for the spillover effect. The right axis has different values for the competition effect.

using a range of parameter values guided by the non-stationary estimates.

Specifically, we redo the privatization counterfactuals using different configurations for the competition/spillover effects. In the stationary model, the estimate of the business stealing effect of private competitors on the profits of a private bank is -0.4867; in the non-stationary model this estimate is -0.1514. The estimates for the spillover effects are 0.1896 in the stationary model and 0.1161 in the non-stationary model respectively. We interpolate two values -0.3749 and -0.2632 between the two estimates of competition effects and the two values 0.1651 and 0.1406 between the two estimates of the spillovers. This gives us 16 combinations of competition/spillover effects – i.e. $\left(\theta_1^{pri}, \theta_1^{pub}\right) \in \{-0.4867, -0.3749, -0.2632, -0.1514\} \times \{0.1896, 0.1651, 0.1406, 0.1161\}$. We solve and simulate the model for each configuration of $\left(\theta_1^{pri}, \theta_1^{pub}\right)$ before and after privatization. We compute averages of the total number of branches in each scenario following the procedures described in Subsections 6.1 and 6.2. Figure 2 shows the average number of branches after privatization divided by the average number of branches before privatization. The arrows "Stationary" and "Non-stationary" indicate the results of the exercises where the competition/spillover estimates of the stationary and non-stationary models, respectively, are used.

When using the configuration of competition/spillover implied by our stationary model, the ratio between the number of branches after and before privatization is 26.32% – i.e. 0.43 (after) divided by 1.64 (before) as shown in Table 8. Instead, when the configuration of competition/spillover parameters implied by the non-stationary model is used this ratio increases to 37.67% – 0.61 (after) divided by 1.77 (before). More generally, Figure 2 suggests that the privatization would cause a significant fall in the number of branches operating in small Brazilian markets independently of the configuration of the competition/spillover estimates. This fall ranges from approximately 73%, when we use the stationary competition/spillover estimates to compute the privatization counterfactuals, to 63%, when using the non-stationary competition/spillover estimates. Even though the counterfactual predictions of the stationary and non stationary models are not directly comparable the sensitivity analysis above suggests that our counterfactual results are robust to a wide set of competition/spillover effects, including those that are obtained from the non-stationary model.

7 A Mechanism Behind the Spillovers from Public to Private Banks

We have highlighted the positive spillovers effects from public to private banks as observed in both the descriptive and structural studies. Given that credit supply is a traditional source of profits for banks, we look closer at the bank lending activities for a potential explanation for the spillovers. The question we ask in this section is: "How is the credit supply of private banks affected by the credit supply of public banks?" This exercise will help us to understand whether the spillovers operate through the credit markets.

We use the same dataset that generates Table 3 to estimate a series of credit supply models for private banks. We consider a specification that runs in parallel with equation (1):

$$credit^{t}_{im} = \rho_0 + \rho_1 credit^{t-1}_{im} + \rho_2 credit^{pub,t-1}_{im} + \rho_3 credit^{pri,t-1}_{im} + \rho_{\mathbf{4}} \mathbf{x^t_m} + \mu^t + \mu_{im} + \mu^t_m + \zeta^t_{im}. \quad (6)$$

For each municipality m, period t: the dependent variable, $credit^{t}_{im}$, is the credit supply of bank i; $credit^{t-1}_{im}$ is the credit supply of the same bank in the previous period; $credit^{pub,t-1}_{im}$ and $credit^{pri,t-1}_{im}$ respectively are the credit supply of the other competing public and private banks in the previous period; the vector $\mathbf{x^t_m}$ and the components μ_{im}, μ^t and μ^t_m are the same and have the same interpretation as those in equation (1).

The results of this regression are shown in the first two columns of Table 10. The coefficients exhibit the same pattern as in Table 4. An increase in the credit supply of private competitors leads to a decrease in the credit supply of private banks. An increase in the credit supply of public banks leads to an increase in the credit supply of private banks. These results are more pronounced and become statistically significant in the second specification, where we control for different forms of observed and unobserved heterogeneity between players, markets and time. According to specification (II), the implied elasticities of $credit^{t}_{im}$ with respect to $credit^{pri,t-1}_{im}$ and $credit^{pub,t-1}_{im}$ – evaluated at the average values of $credit^{pub,t-1}_{im}$ and $credit^{pri,t-1}_{im}$ across players, markets and time – are -0.0314 and

Table 10: Credit Supply of Private Banks as a Function of the Credit Supply of Public Banks and Private Competitors

	(I)	(II)	(III)	(IV)
Private Competitors: Total Credit	-0.002	-0.038**	-0.007	-0.040**
	[0.01]	[0.02]	[0.01]	[0.02]
Public: Total Credit	0.002	0.004*		
	[0.00]	[0.00]		
Public: Personal Credit/Invoice Discounting			0.010	-0.011
			[0.01]	[0.01]
Public: Goods/Investment			0.004**	0.009*
			[0.00]	[0.00]
Public: Mortgage/Infrastructure			-0.002***	0.002***
			[0.00]	[0.00]
Public: Other			0.003	0.001
			[0.00]	[0.00]
Bank/Market Fixed Effects	No	Yes	No	Yes
Year Dummies	Yes	Yes	Yes	Yes
Trend*Market Dummies	No	Yes	No	Yes
Transfers, Expenditure, Agric. Prod.	No	Yes	No	Yes
Observations	6,644	6,621	6,644	6,621

Note: (***) Significant at 1%; (**) significant at 5%; (*) significant at 10%.

0.0413, respectively – i.e. credit supply of a private bank is, in absolute terms, more elastic to $credit_{im}^{pub,t-1}$ than to $credit_{im}^{pri,t-1}$.

We have shown in Section 2 that public banks operate in different credit markets; see Table 3. In order to see which types of credit offered by public banks are most relevant for explaining credit spillovers, we break down $credit_{m}^{pub,t-1}$ into four different components: personal credit and invoice discounting for individuals/firms; credit for the purchase of durable goods and investment excluding mortgage and infrastructure; credit for mortgage and infrastructure investment; and other credit lines. We then run a regression equation similar to equation (6) using these credit variables in place of total public bank credits. The regression results are presented in the third and fourth columns of Table 10. The estimated coefficients show that an increase in credit from public competitors for durable goods and investment and mortgage and infrastructure positively affects the credit supply of private banks. The total credit of private competitors negatively affects the credit supply of private banks.

The results in Table 10 show that the positive spillovers from the public to the private banks we have observed previously also appear in bank lending activities. More specifically, column (IV) in Table 10 indicates that credit spillovers may be coming from the lending operations of public banks in the mortgage/infrastructure and durable goods/investment segments. We have already seen from Table 3 in Section 2 that public banks are mostly responsible for the lending for mortgage/infrastructure and durable goods/investment with little competition from private banks. Private banks' lending activities focus on personal credit and invoice discounting. We may therefore expect more competition between public and private banks in the personal credit and invoice discounting segments than in the other credit types. This would support the finding that the positive spillovers operates through mortgage/infrastructure and investment and not through personal credit and invoice discounting.

8 Conclusions

We build a dynamic structural model to study the entry and exit decisions of public and private banks in Brazil. We estimate it using using data of Brazilian small isolated markets from 1995-2010. Our model predicts that privatization of public banks would cause a substantial reduction in the supply of bank branches in small markets. After privatization, more than half of these markets would end up without bank branches. We suggest the government can incentivize private banks to operate in small markets by offering subsidies. Our model shows that subsidies that reduce bank operating costs are more cost-effective than analogous subsidies that reduce entry costs.

The fall in the number of branches after the privatization can partly be explained by the nature of the strategic interactions between public and private banks. Specifically, our structural estimates suggest that private banks compete with private banks but public banks generate profit spillovers to private banks. Therefore, when public banks are privatized, these spillovers disappear. At the same time, the privatized branches will start to exert competitive pressures on the market. We also show the extent of the fall in the number of bank branches goes beyond the combination of these two effects as we find evidence some private banks will not operate in some small isolated markets alone without public banks. These banks would need some subsidies in order to operate in such markets.

Throughout the paper we have highlighted the empirical finding that there is a positive spillover from public to private banks. Firstly, our regression analysis showed that the activity probabilities of private banks are higher in markets where the number of public banks is higher. Secondly, we showed that the credit supply of private banks is also higher in markets where there is a higher level of public bank credit supply. These effects also get more pronounced as we control for various forms of unobserved heterogeneity. The latter finding suggests that profit spillovers from public to private banks may be explained by demand side complementarities between those credit lines offered by private banks and those available from public banks. In order to test this we ideally would need the demand for credit at the individual level. However, as we do not have such data, we leave this important question for future research.

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Appendix

This Appendix has four parts: A (regressions on activities of public banks), B (CCP robustness checks), C (static model), and D (non-stationary model).

Appendix A: Regressions on Activities of Public Banks

Appendix A concerns the estimates of equation (1) for public banks. Table 11 reports the results. We see that the coefficients associated with $n_m^{pub,t-1}$ are always negative and significant. The coefficients attached to $n_m^{pri,t-1}$ become negative and significant only after the market/bank fixed effects are accounted for. The LPM and the logit type estimates are in agreement with the general direction of the estimates. The inclusion of bank/market fixed effects, the interaction between the time trend and market dummies and additional controls for market level (observed) heterogeneity increases the absolute value of the estimates.

Table 11: Activity Probabilities of Public Banks as a Function of the Number of Private Banks and Public Competitors

Linear Probability Models				
	(I)	(II)	(III)	(IV)
Nº Public	-0.008***	-0.006***	-0.105***	-0.105***
	[0.00]	[0.00]	[0.01]	[0.01]
Nº Private	0.000	-0.002	-0.042***	-0.042***
	[0.00]	[0.00]	[0.01]	[0.01]
Bank/Market Fixed Effects	No	No	Yes	Yes
Year Dummies	No	Yes	Yes	Yes
Trend*Market Dummies	No	No	Yes	Yes
Transfers, Expenditure, Agric. Prod.	No	No	No	Yes
Observations	28,704	28,704	28,704	28,622
Logit/Two Step Fixed Effects Logit				
	(I)	(II)	(III)	(IV)
Nº Public	-0.239***	-0.173***	-0.794***	-0.822***
	[0.03]	[0.03]	[0.06]	[0.07]
Nº Private	0.009	-0.046	-0.469***	-0.514***
	[0.05]	[0.05]	[0.05]	[0.06]
Bank/Market Fixed Effects	No	No	Yes	Yes
Year Dummies	No	Yes	Yes	Yes
Transfers, Expenditure, Agric. Prod.	No	No	No	Yes
Observations	28,704	28,704	28,622	28,622

Note: (***) Significant at 1%; (**) significant at 5%; (*) significant at 10%. Standard-errors of the two-step models calculated by bootstrap.

It is important to note, however, that the interpretation of negative estimates for $n_m^{pub,t-1}$ and $n_m^{pri,t-1}$ in Table 11 depends on how we perceive public banks to behave. If public banks are assumed to be profit maximizers the negative estimates suggest the usual competition effects as all banks are rivals. On the other hand, if public banks act only based on a development (and/or political) mandate the negative estimates may indicate that public banks focus their operations in unbanked areas (i.e. without or with few public and private branches) in order to increase the coverage of the banking system in underdeveloped areas. Our institutional analysis in Section 2 supports the latter interpretation. In reality we expect both views to be relevant as public banks has to optimize in some respect (for example, with regard to some cost control) while also being tasked to operate in less attractive areas.

Appendix B: CCPs Robustness Checks

Appendix B studies the sensitivity of our CCP logits with respect to different controls for market level heterogeneity and alternative formulations for market/bank unobserved heterogeneity.

■ **Inclusion of different controls.** The relevant results are in Table 12. We analyzed how the inclusion of different controls for (observed) market level heterogeneity affects our CCP estimates. Columns in Table 12 are equivalent to columns labeled (II) in Table 5. The only difference is that in Table 12 we included transfers of the Federal and State governments to the municipality, municipal government expenditure and agricultural production of the municipality. These variables control for (observed) market level heterogeneity that may affect activity decisions of public and private banks – see also Section 3. In general, these variables are not significant in all the specifications (only expenditure is significant for public banks in the model without time trend). Additionally, the inclusion of these variables barely affects the coefficients associated with $n_m^{pub,t-1}$ and $n_m^{pri,t-1}$.

■ **Alternative formulations for bank/market unobserved heterogeneity.** Table 13 provides sensitivity analyses of our CCP estimates with respect to changes in the way we are formulating the variable market/bank fixed effect – which controls for market/bank unobserved heterogeneity. Columns (I) and (II) in Table 13 are equivalent to columns labeled (II) in Table 5. The differences are:

1. In Table 13, columns labeled (I), the variable market/bank fixed effect is obtained as the estimated fixed effects from a linear probability model with a specification similar to that shown in column (II) in tables 11 and 4 – i.e. we run a linear probability model for public and private banks in the sample where $N_{pri} = 2$ and $N_{pub} = 2$ using a linear specification similar to (II) in the upper part of tables 11 and 4, recover the fixed effects and use them as a control variable in the logit function; and,

2. In Table 13, columns labeled (II), the variable market/bank fixed effects is obtained as the estimated fixed effects from a linear specification similar to that shown in column (III) in the upper part of tables 11 and 4 – i.e. we run a linear probability model for public and private banks in the sample where $N_{pri} = 2$ and $N_{pub} = 2$ using a specification similar to (III) in Table 11 and 4, recover the fixed effects and use it as a control in the logits.

Table 12: Logits for Activity Decisions of Public and Private Banks (Alternative Control Variables)

	Public Banks	Private Banks
Lagged Activity	6.236***	5.885***
	[0.44]	[0.37]
N Public	-1.820***	0.573*
	[0.68]	[0.34]
N Private	-0.146	-1.428**
	[0.27]	[0.68]
Market Payroll	-0.011	0.001
	[0.01]	[0.02]
Market/Bank Fixed Effect	5.914***	11.012***
	[1.82]	[2.82]
Expenditure	0.0630*	-0.046
	[0.03]	[0.04]
Transfers	-0.030	0.046
	[0.05]	[0.08]
Agricultural Prod	-2.583	3.654
	[5.18]	[9.23]
Constant	-2.722***	-3.721***
	[0.92]	[0.45]
Observations	1,470	1,470

Note: (***) Significant at 1%; (**) significant at 5%; (*) significant at 10%. Standard-errors of the two-step models calculated by bootstrap.

Table 13: Logits for Activity Decisions of Public and Private Banks (Alternative Formulations for Unobserved Heterogeneity)

	Public Banks		Private Banks	
	(I)	(II)	(I)	(II)
Lagged Activity	5.932***	6.193***	6.575***	5.820***
	[0.19]	[0.46]	[0.39]	[0.33]
N Public	-1.565***	-1.817***	0.503***	0.505
	[0.48]	[0.64]	[0.13]	[0.31]
N Private	-0.494**	-0.118	0.020	-1.457*
	[0.21]	[0.29]	[0.33]	[0.84]
Market Payroll	-0.042***	-0.009	0.011	0.008
	[0.01]	[0.01]	[0.01]	[0.01]
Market/Bank Fixed Effect	23.390***	6.217***	21.798***	10.628***
	[2.48]	[1.87]	[1.66]	[2.95]
Constant	-1.098***	-2.274**	-4.988***	-3.879***
	[0.33]	[0.99]	[0.27]	[0.57]
Observations	1,472	1,472	1,472	1,472

Note: (***) Significant at 1%; (**) significant at 5%; (*) significant at 10%. Standard-errors of the two-step models calculated by bootstrap.

Qualitatively, for public banks, our results appear to be similar independently of the formulation we used to construct the variable market/bank fixed effect. For private banks, the coefficients attached to $n_m^{pri,t-1}$ are sensitive to the way the variable market/bank fixed effect is constructed. While in Table 5 estimates for the effects of $n_m^{pri,t-1}$ are negative and significant, point estimates of the coefficients associated with $n_m^{pri,t-1}$ are positive (but not statistically significant) in Table 13, column (I), equation for private banks. Positive estimates for $n_m^{pri,t-1}$ may suggest that our control for market/bank unobserved heterogeneity is not effective. The second formulation – column (II) for private banks in Table 13 – shows results that are close to those shown in Table 5. For these reasons we prefer the specification of the control for unobserved heterogeneity that was used in Table 5.

Appendix C: Static Model of Entry and Exit

Appendix C considers a static version of the model described in Section 4 except for the discount factor, β, that is set to be zero. A similar model is studied in Seim (2006). We estimate the parameters of private banks' payoff function using the same method to estimate the dynamic model. The same CCPs are used in both models. We then solve this model for private banks (assuming that the behavior of public banks is fixed) and compare its fitting with the fitting of the dynamic model (see Table 7).

Table 14: Structural Parameters for Private Banks, Dynamic and Static Models

	Dynamic		Static	
	Coefficients	Coefficients/Entry Cost	Coefficients	Coefficients/Entry Costs
Nº Public	0.190	4.3%	0.496	11.3%
	[0.10]		[0.27]	
Nº Private	-0.487	-11.0%	-1.285	-29.3%
	[0.31]		[0.69]	
Market Payroll	0.001	0.0%	0.002	0.0%
	[0.00]		[0.01]	
Constant	-0.732	-16.5%	-0.189	-4.3%
	[0.18]		[0.47]	
Entry Costs	-4.442	-100.0%	-4.387	-100.0%
	[0.21]		[0.21]	
Market/Bank Fixed Effects	Yes		Yes	

Note: The column labeled Coeffs/Entry Costs reports the coefficients as a % of the absolute value of entry costs.

Table 14 puts together the estimates of the dynamic model (taken from Table 6) and the estimates of the static model. We observe that the two models produce qualitatively similar estimates. In particular, the static model predicts that private banks compete and that public banks generate profit spillovers for private banks. The magnitude of entry costs is also very close across the two models. The main differences appear to be in the magnitude of the coefficients measuring competition, spillovers and operating costs. The static model tends to overestimate competition and spillovers and to underestimate operating costs. In fact, operating costs in the static model are not statistically different from zero[12]. From the institutional perspective this result is hard to rationalize. Operating costs for banks are expected to be relatively large in these small markets (see Subsection 5.3). These discrepancies may reflect on the ability of the static model to explain the observed data.

Table 15 compares the fitting of the static model with the fitting of the dynamic model. The first column gives the moments from the data (taken from the first column in Table 7); the second and third columns give the simulated moments from the dynamic model and its difference with the observed data respectively (taken from the second and third columns in Table 7 respectively); the fourth and fifth columns give the analogous simulated moments from the static model and its difference with the observed data respectively. Both models produce similar moments for most statistics apart from the first row of Table 15. The static model substantially underestimates the number of private banks operating in the market. This happens because the static framework underestimates entry rates and overestimates exit rates compared to the dynamic model.

Table 15: Model Fit – Static and Dynamic Models of Incomplete Information

	Real Data	Dynamic	Difference Dynamic	Static	Difference Static
Private Branches	0.568	0.525	-0.042	0.400	-0.168
Public Branches	1.163	1.109	-0.054	1.116	-0.047
Total Branches	1.731	1.635	-0.096	1.516	-0.215
Entry Rate Private	0.042	0.030	-0.012	0.024	-0.018
Entry Rate Public	0.016	0.019	0.003	0.019	0.003
Entry Rate Total	0.058	0.049	-0.009	0.043	-0.015
Exit Rate Private	0.030	0.033	0.003	0.035	0.005
Exit Rate Public	0.061	0.062	0.001	0.061	0.000
Exit Rate Total	0.091	0.095	0.004	0.097	0.005

In addition to the numerical support we find for the dynamic model, there are also theoretical concerns for using a static version of our model. For example, Einav (2010) and Ellickson and Misra (2011) argue that static models with asymmetric information are vulnerable to the so-called ex-post regret. In models of incomplete information firms make their decisions based on expectations about rivals' actions and not on rivals' actual actions. In certain situations, firms would like to change their actions ex-post, after observing actual rivals' actions. By construction, the one-shot nature of the static model rules out this possibility. For example, firms regretting the decision of staying out of the market are unable to change their actions – and there is no apparent justification for this irreversibility (Einav (2010)). In the dynamic model, on the other hand, ex-post regret will affect future decisions – see Berry and Reiss (2007) – and regrets can be reversed – see Sweeting (2013) and Ellickson and Misra (2011).

In summary, these results indicate that the structural estimates of competition/spillovers in the

[12]The average continuation value for incumbents and entrants is positive in the dynamic models. In the static model, by construction, this object is equal to zero. Therefore, to rationalize the activity probabilities observed in the data, the estimates of operating costs from the static models must compensate the exclusion of the positive continuation values.

static model are consistent with the analogous estimates obtained from the dynamic model. On the other hand, the former seems to overestimate competition/spillovers compared to the latter. Importantly, the static estimate of operating costs is statistically undistinguishable from zero. This result seems counterintuitive. As mentioned before, we would expect significant operating costs in these markets. Lastly, the fitting of the static model to the data is relatively poor when compared to the fitting of the dynamic model. Consistent with the anecdotal evidence presented in the paper, the latter finding suggests that the dynamic framework is more appropriate to model private banks' entry and exit decisions.

Appendix D: A Non-stationary Model

Our stationary model assumes that all variables in the model are stationary, as most empirical dynamic games in the literature do. Recent works by Igami (2014) and Igami and Uetake (2017) emphasize the importance of non-stationary components in a dynamic structural model. Below we describe a non stationary version of our model, the procedures we use to estimate it and the results of the structural estimation.

■ **Model.** We consider a non-stationary game similar to the model used in Igami (2014). The basic elements of the non-stationary game are the same as in the stationary model. The only modifications we make to accommodate non-stationary features are the following:

1. Time is finite, $t = 1, 2, ..., T$, for some $T < \infty$. From period $T + 1$ onwards, the discounted payoffs are assumed to be zero.

2. The demand shifter in the payoffs, x_m^t, evolves following a deterministic rule $x_m^{t+1} = (1 + \gamma_{xm}) \cdot x_m^t$, where γ_{xm} is the long-run growth rate of x_m^t.

3. The profitability shock ε_{im}^t is modeled as:

$$\varepsilon_{im}^t = \mu_{im} + \eta^t + \xi_{im}^t. \tag{7}$$

We assume the time horizon time is finite in order for payoffs to be bounded. The term η^t in equation (7) captures correlations in bank actions in different markets at the same period of time. Bank payoffs are the same as in the stationary model but the adjustments above make the model non-stationary. In particular, this specification for the profitability shock implies that bank operating costs are given by $(\theta_0 + \mu_{im} + \eta^t)$, i.e. this formulation allows operating costs to vary across banks, markets and time – see the payoff function (2). The assumption on the demand shifter is reasonable because our data proxies for the demand shifters appear to be growing steadily over time – see Figure 1. The sequence of events faced by banks remains the same as the stationary model apart from the game ending at time T. Our finite time horizon game can be solved backwards regardless of whether or not public banks are assumed to be strategic.

■ **Estimation.** We can estimate the non-stationary game in two steps similarly to the stationary one. We first estimate the CCP estimates for the non-stationary model. Table 16 shows the CCP estimates for the non-stationary model. The only difference between the estimation of the CCPs in this case compared to the stationary model is that we include year dummies to the set of explanatory variables.

Table 16: Logits/two-step Fixed Effects Logit CCPs – Non-stationary Model

	Public Banks		Private Banks	
	(I)	(II)	(I)	(II)
Lagged Activity	7.028***	5.748***	9.397*	6.089***
	[0.74]	[0.39]	[5.17]	[0.31]
Nº Public	-2.149**	-2.273***	1.317***	1.113***
	[0.89]	[0.47]	[0.46]	[0.31]
Nº Private	-0.513	-0.282	-1.704*	-1.265**
	[0.38]	[0.26]	[0.98]	[0.64]
Market Payroll	0.024	0.021	-0.040	-0.029*
	[0.02]	[0.02]	[0.03]	[0.02]
Market/Bank Fixed Effect	7.148**	6.549***	16.866***	11.194***
	[3.28]	[1.20]	[6.07]	[3.23]
Time Trend		-0.1400***		0.193***
		[0.05]		[0.03]
Constant	-3.024**	-0.968	-5.463**	-5.770***
	[1.21]	[0.72]	[2.73]	[0.60]
Year Dummies	Yes	No	Yes	No
Observations	1,470	1,470	1,470	1,470

Note: (***) Significant at 1%; (**) significant at 5%; (*) significant at 10%.

The columns labeled (I) in Table 16 show the results for the model with year dummies. These estimates are qualitatively similar to Table 5. In particular, they suggest that the effects of $n_m^{pub,t-1}$ and $n_m^{pri,t-1}$ on bank activity decisions tend to be more pronounced when time dummies are included.

In practice, using year dummies as a control for time varying heterogeneity is problematic as, to simulate the model forward, we have to attribute a value for these dummies in the years not covered by our sample (from 2011 onwards). Clearly, this would not be possible without making strong and ad hoc assumptions. Therefore, instead of modeling η^t as year dummies we model this component as a constant times a time trend – i.e. we assume that the shock process in equation (7) is given by $\varepsilon_{im}^t = \mu_{im} + \eta \cdot t + \xi_{im}^t$, where η is a constant and t is a time trend. The columns labeled (II) in Table 16 show the results for the CCPs with this time trend instead of the year dummies. For both bank types, the results are qualitatively similar across the columns. We use only the CCPs with time trend to estimate the non-stationary model.

The state variables in the non-stationary model are the time trend and the municipality payroll. We model both variables to grow deterministically. As previously described in this Appendix, the demand shifter evolves according to the rule: $x_m^{t+1} = (1 + \gamma_{xm}) \cdot x_m^t$, where γ_{xm} is the average growth rate of the payroll in municipality m during the period 1995-2010. The time trend assumes value 0 in 1995, 1 in 1996, 2 in 1997 and so on.

We use the CCPs and deterministic transitions to forward simulate the value functions. We set $T = 100$. For each market we simulate one value function for each bank – starting from each year available in our sample. In total we simulate $2 \cdot 16 \cdot 46 = 1472$ value functions. The payoff parameters remain linear in the discounted expected payoffs structure. They can also be estimated in closed-form as in Sanches et al. (2016). Our estimator here is essentially the least squares counterpart to the moment-based (instrumental variable) estimator of Hotz, Miller, Sanders and Smith (1994) for dynamic games. See also Bajari, Benkard and Levin (2007) for another estimator for dynamic games that uses forward simulation.

The estimation results are reported in Table 17. The first and second columns in the table show the estimates for private banks. The fourth and fifth columns in the table present the results of the model when public banks are considered profit maximizers, labeled on the table as public banks.

Table 17: Structural Parameters for Private and Public Banks, Non-Stationary Model

	Private Banks		Public Banks	
	Coefficients	Coeff/Entry Cost	Coefficients	Coeff/Entry Cost
Nº Public	0.116	2.9%	-0.184	-4.4%
	[0.03]		[0.04]	
Nº Private	-0.151	-3.8%	-0.008	-0.2%
	[0.07]		[0.02]	
Market Payroll	-0.004	-0.1%	0.003	0.1%
	[0.00]		[0.00]	
Time Trend	0.022	0.6%	-0.021	-0.5%
	[0.01]		[0.01]	
Constant	-0.889	-22.3%	-0.010	-0.2%
	[0.08]		[0.07]	
Entry Costs	-3.987	-100.0%	-4.194	-100.0%
	[0.26]		[0.32]	
Market/Bank Fixed Effect	Yes		Yes	

Note: The column labeled Coeff/Entry Costs reports the coefficients as a % of the (absolute value of) entry costs.

For private banks, the results for the non-stationary model are similar to the results of the stationary model. Public competitors increase the profits of private incumbents. Private competitors reduce the profits of private incumbents. Entry and operating costs are also negative and relatively large. However, the time trend is positive and relatively large. A possible interpretation of this finding is that the introduction of technological innovations over time may have forced operating costs downwards, making these markets more profitable for private banks. Quantitatively, however, there are some notable differences between the estimates from the stationary and non-stationary models. Operating costs are higher in the non-stationary model. The entry costs and the complementarity effects are both lower in the non-stationary model. In absolute terms, the biggest difference between both sets of estimates is in the business stealing effect caused by the entry of a new competitor into the market. This effect appears to be much higher in the stationary model.

When we assume public banks are profit maximizers there are more notable differences. In both stationary and non-stationary models, a new public competitor reduces the profits of public incumbents. But while we see that a new private competitor significantly reduces the profits of a public incumbent in the stationary model, this effect for the non-stationary model is not statistically different from zero. Once again, the stationary model appears to overestimate competition effects. The constant term, which measures operating costs of public banks, is positive for both models. While this estimate is relatively large in the stationary model, it is statistically zero in the non-stationary model.

Another difference between the models for private and public banks in Table 17 is that the time trend for public banks is negative and significant. It is hard to interpret this finding from the point of view of sectoral technological changes. Public banks, in general, were also exposed to the same types of technological innovations adopted by private banks and they also adopted these technologies to some extent. Thus, it is possible that the time trend is capturing characteristics of public banks behavior that are not considered in the payoff function specified above.

The results presented here suggest that ignoring non-stationary aspects of the data may lead to biased estimates. To understand how our counterfactual results are affected by possible biases in the competition/spillover estimates we run a series of sensitivity checks in Subsection 6.3. Our results appear to be robust to a wide configuration of competition/spillover estimates. Second, it is important to note that the structural estimates for public banks must be interpreted with care. Some of the estimates – for instance, the positive (or zero) operating costs and the negative time trend – for public banks are hard to interpret.

■ **Counterfactuals.** Our non-stationary models have finite time horizon. Although they can be solved backwards, the number of equilibrium paths accumulates and multiplies at each time period, making the problem computationally complicated. We refer to the older working versions of Igami (2014) for further details.