Market area analysis under orbital-radial routing with applications to the study of airport location

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The opinions expressed in this paper represent only those of the authors.

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Abstract
The study of market areas provides one indication of the economic and strategic value of a facility or attraction and is a commonly used tool in geographical and economic analysis. In this paper we study a class of models for use in stereotypical urban regions with an associated orbital-radial network. The aim of the paper is to provide the theoretical and analytical tools needed to understand the shapes, size and markets of an attraction as a function of both its location and the speeds of radial and orbital access to competing attractions. As a result we need to identify several hitherto unrecognised constructs such as eclipsing in which, due to proximity to a fast orbital road, one attraction can eliminate the market of another. We outline a facility location methodology in a case study, based on London, concerning access to airports serving the metropolis. Among other things we identify inner and outer eclipsing envelopes for a new airports, which substantially narrows the area of search for an optimum location.

Introduction

The economic and strategic value of a major facility depends on the geographical size of its market. Market area analysis is a commonly used geographical technique for identifying the set of locations that can be reached in quickest time or lowest cost. Market areas can be subdivided to reflect alternative route catchments, which in turn can be used to indicate for example whether the motorway or some alternative route is quicker to a given attraction. The properties of route catchments have been previously examined in Hyman and Mayhew (2000) and in an agricultural context in O'Kelly (1989). In this paper, we are concerned primarily with the properties of market areas although, as we shall see, route catchments are an important secondary consideration.

Earlier studies, in the tradition of regional science and urban economics, tend to adopt models that incorporate only weak differentiation in travel or transport costs. The resulting simplified models may hide emergent features of many developed urban regions. In contrast, the complex network models developed for transportation studies require large and expensive data sets. By virtue of their very complexity such models may also obscure fundamental morphological characteristics of urban regions.

Important classes of models meriting further study are based on continuous space (Angel and Hyman, 1976; Mayhew 1981; Hyman and Mayhew 1982) but incorporate certain limited network features (Smeed, 1963; Vaughan, 1987; Mayhew and Hyman, 2000; Hyman & Mayhew 2000). Models that permit, for example, only combinations of orbital-radial routing are examples of this that can be used to represent some of the complexity of a real-world urban network.

An orbital-radial routing pattern is a common in many towns and cities (Ministry of Transport, 1963; Tripp, 1942), and therefore our adoption of it here is potentially of wide relevance. In this paper we adopt the term 'radial routing' to mean travel restricted to the transport spokes emanating from the centre of a city. We use the term orbital route to mean travel that makes use of a ring road for some part of a journey, the other legs being accomplished radially.

Our analysis considers the market areas of urban facilities based on this kind of routing. The model that emerges predicts conditions under which one location ceases to be viable as it becomes 'eclipsed' by another - a characteristic finding that is not identified using traditional methods. We use the model to evaluate the shape, form and pattern of market areas in an orbital-radial transport system, and then apply the methods to a contemporary problem: the siting of airports serving cities.

The problem of airport location is an ideal illustration of the ideas and concepts put forward. Airports are strategic attractions that are typically located near the edge of an urban area. They attract passengers from an entire region and also provide a major source of employment. As aircraft traffic grows and urban areas expand, planning authorities face choices about whether to increase the capacity of existing airports, relocate them or construct new ones.

Our aim is not to produce a 'definitive planning study', but rather to apply the technique in a strategic setting so as to see if it gives insights into an ongoing planning problem and how that problem is influenced by the transport character of the urban area. As a simplification, we initially assume all airports have equal attractiveness, in other words each location provides equivalent services in terms of cost and quality. Clearly, this will not always be the case, for example where one airport concentrates on flights to selected destinations, or where there are restrictions in aircraft size. We return to the issue of 'unequal attractiveness' during the case study and associated discussion.

The plan of this paper is as follows. We briefly review a small selection of previous relevant work on market areas, provide a general framework for extending the analysis of market areas to incorporate the effect of alternative types of route, and give some simple illustrations. The concept of eclipsing is then defined and illustrated. Up to this point the illustrations are limited to two attractions (i.e. facilities), so the next step is to extend the notation to an arbitrary number of attractions. This extension leads to the notion of an eclipse envelope, which circumscribes the geographical limits at which new market entrants are viable. A fundamental geometrical property linking eclipse envelopes and market area boundaries is then derived, for a specific class of orbital-radial routing models, which greatly simplifies the construction of market areas. Details of the software developed for the case study are given in an annex.

Relevant Previous Work on Market Areas

A particularly valuable review of market area analysis, which describes the literature and techniques developed in a number of disciplines, is contained in Okabe, Boots and Sugihara (1992). More formally Okabe et al define the concept as follows:

"Given a set of distinct isolated points in a continuous space, we associate all locations in that space with the closest member in the point set. The result is a partitioning of the space into a set of regions" (Boots et al 1992, page 1).

Although we use the term market area throughout this paper, its definition is equivalent to terms such as Dirichlet or Voronoi regions, named after the discoverers. Thiessen (1911) is another who gave his name to the construction of polygons with the required properties for undifferentiated areas (see also Haggett, 1965, p247).

An important strand in market area analysis is focussed on spatial competition (Hotelling, 1929; Okabe et al, pp 385-387) in which firms adjust their locations to increase profits until an equilibrium pattern of locations is reached. For example, for two competing firms Eaton and Lipsey, 1975 obtain a solution where both firms are located at the centre of the region. We do not directly address competition issues in this paper, at least at the level of the firm, and as noted above, neither do we consider differences in attractivity. However, it is clear there are significant competitive implications arising out of our work based on new concepts introduced such as 'eclipsing' and market area sharing.

Studying the properties of ‘idealized’ urban routing patterns is not new. The identification of market areas for Manhattan routing goes back some years (for example, see Anjoumani,1981); orbital-radial routing models on the other hand are more recent. Klein (1988) considers basic properties of the so-called 'Karlsruhe metric' (Karlsruhe’s road network consists entirely of rings and spokes), and so his paper provides a starting point for the current work. However, as our previous paper points out, our approach is more general and also more realistic (Hyman and Mayhew, 2000). For example, we use time rather than distance, in order to reflect differences in speed. Also, instead of assuming unlimited orbital movement we restrict orbital movement to a limited number of strategic roads.

Market Areas Associated with a Routing Typology

Let U denote a continuous urban region with an associated travel time metric t. Consider a finite set L ={L0 … LN} of N+1 fixed attractions within the urban region. The market area for a given attraction is defined to be the set of all locations in U that have a smaller travel time to the given attraction than to any other attraction in L. Formally, this can be expressed as: Mn = {x(U | t(x, Ln) < t(x, Lm) $\forall L_m \in L$, $m \neq n$}.

The geometric properties of market areas is determined by the family of *break-even sets*, that is the locus of points that have equal travel time to each of two different attractions. Formally, for each pair of attractions L_n and L_m we can write $\mathbf{Y}_{nm} = \{x \in \mathbf{U} \mid t(x, L_n) = t(x, L_m)\}$. A break even set is a series of smooth arcs joined at cusps. As changes in routing produce such cusps, the geometry of the break-even set needs to reflect the *type of route* used to each of the attractions. So we let **P** denote a set of route types and let $p \in \mathbf{P}$ denote a specific type of route. Let $\mathrm{Rte}(x, y) \in \mathbf{P}$ denotes the type of route used between points x and y. The typology **P** gives rise to a partition **Z** of each market area into subsets associated with the type of route used to access the attraction: $\mathbf{Z}_{np} = \{x \in \mathbf{U} \mid t(x, L_n) < t(x, L_m)\ \forall L_m \in \mathbf{L}, m \neq n \wedge \mathrm{Rte}(x, L_n) = p\}$

The corresponding break-even sets become extended to include not only the break-even arcs and cusps between alternative attractions but also the break-even arcs and cusps between alternative types of route. The extended break-even set is formally defined by: $\mathbf{Y}_{nmpq} = \{x \in \mathbf{U} \mid t(x, L_n) = t(x, L_m) \wedge \mathrm{Rte}(x, L_n) = p \wedge \mathrm{Rte}(x, L_m) = q\}$.

Previously (Hyman and Mayhew, 2000), we adopted a fourfold *operational* routing typology:

1.	Radial	Travel through the centre, not using an orbital
2.	Inner Orbital	Through/strategic travel: inwards/orbital/outwards
3.	Outer Orbital	Local travel: outwards/orbital/inwards

4. Cross Orbital Arriving or departing travel, partly on an orbital.

All other types of routing are excluded as only types 1-4 correspond to minimum time routes. Routes that involve travel on a single radial leg are excluded as this arises only for a negligible proportion of trips.

For the purposes of the current analysis the three types of orbital routing will be condensed into a single explicit type, although they will still need to be given implicit consideration in the design of the algorithms.

The resulting typology P then reduces to a distinction between radial and orbital routing: the simplest practical extension of the break-even set. There are thus two types of market area: radial market areas, where the attraction is accessed via a radial route, and orbital market areas, where the attraction is accessed via an orbital route. This will be illustrated shortly, but first we need to specify a simple mathematical model of orbital-radial routing.

The KT1 Metric

In the KT1 metric (Hyman and Mayhew, 2000) the city is circular and locations are identified using polar co-ordinates, with the city centre as the origin. The angular co-ordinate varies between $-\pi$ and π, with zero pointing due north. Travel is either radial at a constant speed V_R or along a single orbital road, of radius R and constant speed V_O. Define the speed ratio $k=V_R/V_O$. Let (r,θ) be a varying point and let $(r_1, 0)$ be a fixed point. To obtain the minimum time route between these points we confine attention to:

a) *radial routes*: two radials, meeting at the city centre,
b) *orbital routes*: a radial leg to the ring road, followed by an orbital leg (along the ring road), followed by a radial leg away from the ring road.

In both cases (a) and (b), the radial legs are at angles 0 and θ. It can be sometimes be helpful to visualise these radial legs as two spokes of a wheel formed by the ring road.

The minimum travel time between the fixed and varying points is:

$$KT1 = \text{Min}\ [(r_1 + r)/V_R,\ R|\theta|/V_O + (|R - r_1| + |R - r|)/V_R]$$

where |x| denotes the absolute value of x. There is a critical *switching angle* at which the minimum travel time using an orbital route is equal to that on a radial route. At angular separations less than the switching angle, the orbital route takes less time, at greater angles, the radial route takes less time.

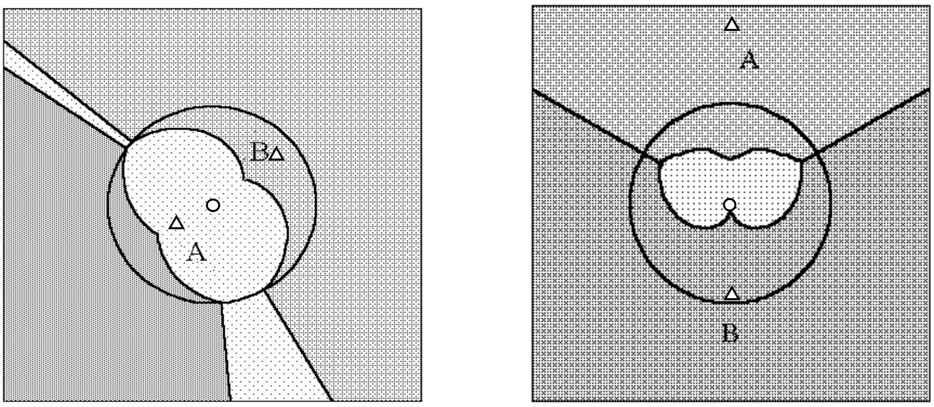
Table 1 gives the travel times on orbital routes and the corresponding switching angles. The results depend on the fixed radius r_1, the variable radius r, and their relationship between the radius R of the orbital. Under certain conditions, discussed later, switching angles may determine the orientation of radial legs in the break-even sets.

Domain	Travel Time	Switching Angle
$r_1 \le R$, $r \le R$	$(2R - r_1 - r + kR\|\theta\|) / V_R$	$2(r_1+r-R) / kR$
$r_1 \le R$, $r \ge R$	$(r - r_1 + kR\|\theta\|) / V_R$	$2r_1 / kR$
$r_1 \ge R$, $r \le R$	$(r_1 - r + kR\|\theta\|) / V_R$	$2r / kR$
$r_1 \ge R$, $r \ge R$	$(r_1 + r - 2R + kR\|\theta\|) / V_R$	$2 / k$

Table 1: Orbital Travel Times and Switching Angles for the KT1 metric

Initial Illustrations Based on Two Attractions

To help understand the properties of KT1 market areas we start with some simple illustrations based on two attractions and then develop the associated analysis. Note that only the attraction that is closest to the city centre has a radial market area. Such a location will generally have an orbital component as well, except when it is precisely at the city centre, in which case it is only accessed by radial routes.



(a): Fast radials (b): Slow radials

Figure 1: Market areas with two attractions, A & B. Light shading indicates the radial market area.

Figure 1 gives illustrations for the most common cases that arise. The large circle denotes the orbital road and a small circular symbol the city centre. The two attractions A and B are marked by triangles. The market areas are shaded, the lighter shading for the radial market area and the darker shading for orbital market areas. There is only one radial market area.

In a) attraction A is nearer the centre and captures the radial market area, plus it also captures an orbital area to south-west. The radial market area cuts across the ring road because the radial speed is assumed to be high relative to the orbital speed. So, in this case, the orbital market areas are completely separated by the radial market area.

The situation is quite different in diagram b), where a lower radial speed has been assumed. Now the radial market area no longer cuts across the ring road and, as the diagram shows, it is contained entirely within the ring road area. The orbital market areas of A and B now share common boundaries as they are no longer separated by a radial market area. In such cases the orbital market areas are said to 'collide'.

From the perspective of competition between attractions at different locations, there are some interesting limiting cases to consider. Firstly, assume that there are two attractions, both with the same radius, but at different angular coordinates. These attractions will share the radial component of their market areas, since a radial route to either of them takes the same time. Secondly, suppose that two distinct attractions are equidistant from the ring road, on the same radial leg. (This can only happen if the attractions lie on opposite sides of the ring road). In such cases the attractions will share their orbital market area. This is because the journey times from any point on the ring road to either attraction are equal.

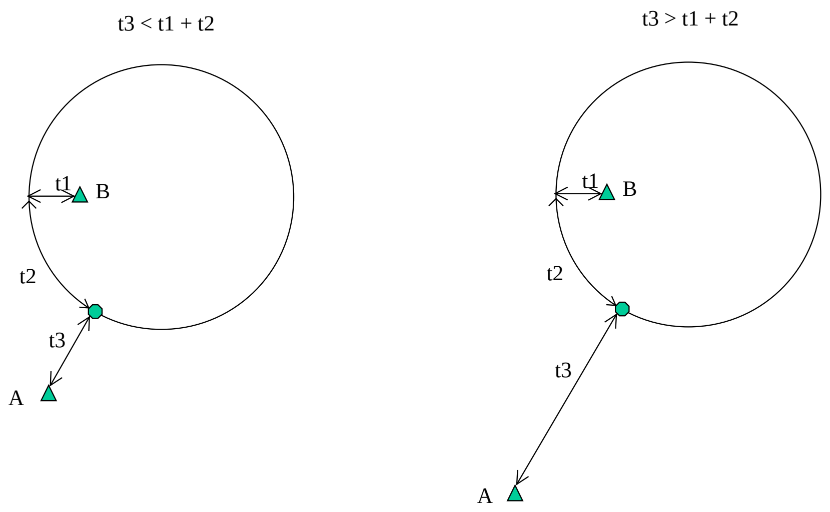
Eclipsing

Eclipsing is a major property of the KT metric that sets it apart from uniform metrics traditionally used for market area analysis. Suppose that there are two attractions A and B, which are on different radials and are at different distances from the ring road. Suppose also that the best route between them makes use of the ring road. Now imagine two travellers starting simultaneously, one going from A to B and the other from B to A. At some point on their trip they must pass each other. Is this point on the ring road or is it on a radial? If they pass on a radial then one of the attractions will be *eclipsed* and will have no orbital market area.

Now, more formally, we may note that the orbital route between an attraction A and an attraction B consists of three legs:

1) A radial journey leg between B and the ring road taking time t1
2) An orbital journey leg taking time t2
3) Another radial leg between A and the ring road, taking time t3

The attraction A is *eclipsed* by the attraction B whenever t3 > t1 + t2, as in figure 2b. Alternatively, if this condition is not met then A is not eclipsed, as in figure 2a.



(a) No Eclipsing *(b) Attraction A eclipsed*

Figure 2: Eclipsing of Attractions

If A is eclipsed then an orbital route from any third location to B will always take less time than an orbital route from the third location to A. It follows that attractions that are eclipsed by other attractions will have no orbital market area.

What are the implications for the location of a new attraction? Should it seek to be close to the orbital, close to the centre, close to an existing attraction or somewhere else? It is helpful to distinguish the following situations:

1. If a new attraction locates nearer the city centre than an existing attraction then it captures its radial market area.

2. If the new attraction locates farther from the ring road than an existing attraction, then depending on their angular separation and routing speeds, the new attraction may be eclipsed by the existing attraction and therefore have no orbital market area.

3. Conversely, if the new attraction locates closer to the ring road than an existing attraction, with an angular separation that is sufficiently small, then it may eclipse the existing attraction, capturing its entire orbital market area.

The Market Area Size Function

We now turn to the procedure for calculating the sizes of market areas. The size of a market area and the population contained within are key indicators of market share. The methodology will be extended to many attractions in due course but first we deal with the two-attraction case.

The size of the area that is bounded by a 'spiral' arc $r(\theta)=a+b|\theta|$ and the ring road and is within angular sector $(0, \phi)$ is given by the integral:

$$A(\phi,R,a,b) = \int_{0}^{\varphi} \int_{r(\theta)}^{R} r\, dr d\theta = \{(R^2 - a^2)|\phi| - ab|\phi|^2 - b^2|\phi|^3/3\}/2 \qquad (1)$$

Assume now that $r(\theta)$ is a break-even curve in the KT1 metric. The value of the spiral gradient $(dr/d\theta)$ is given by $b=kR/2$.

Consider just two attractions, numbered in order of increasing distance from the centre. The value of the spiral start radii 'a' are given by:

$$a_0 = \mathrm{Max}(0, R - r_0), \qquad (2)$$

$$\begin{aligned} a_1 &= (r_1 - r_0)/2 && \text{If } r_1 > R \\ &= \mathrm{Max}(0, R - (r_1 + r_0)/2)) && \text{If } r_1 \le R \qquad (3) \end{aligned}$$

The parameters a_0 and b are identical to those for the radial *route catchment area* for L_0, (see Hyman and Mayhew 2000). The radial market area can therefore be identified as the radial route catchment area associated with the innermost attraction L_0. It follows that a sufficient condition for the radial market area to be contained within the ring road is that $r(\pi) < R$, so that $k<2r_0/\pi R$. This precludes situations like Figure 1 a).

Example
Let V_R=66.5 kph, V_O=95 kph, R=28 km and r_0=25 km. Then k=0.558 and $2r_0/\pi R$=0.568, which exceeds k. Hence the radial market area is contained within the ring road.

In order to apply equation (1) it is necessary to determine appropriate values for the angular limits ϕ of the integral. This is illustrated in the following sections.

The Computation of KT1 Market Area Sizes

The determination of market area sizes involves the following steps:

- Testing for eclipsing.
- Testing for orbital collisions.
- Calculation of switching angles.

If an attraction is eclipsed it has no orbital market area. When collisions occur between any pair of attractions, there may be one or two collisions and both of them need to be identified. When collisions do not occur, the orbital market area is bounded by a switching angle.

Traverses and circuits

We can analyse eclipsing behaviour of attractions in several ways. While previous illustrations were suggestive of market area analysis in specific cases they do not illustrate when eclipsing arises, or the variation in the size of the market area. We therefore devised two sensitivity tests: traverses and circuits. In both tests attraction A is at a fixed location. In a traverse the attraction B is moved along a diameter that extends from one edge of the region to the other. In a circuit, B has a fixed radius but is rotated in a complete circle around the centre.

a) A Traverse

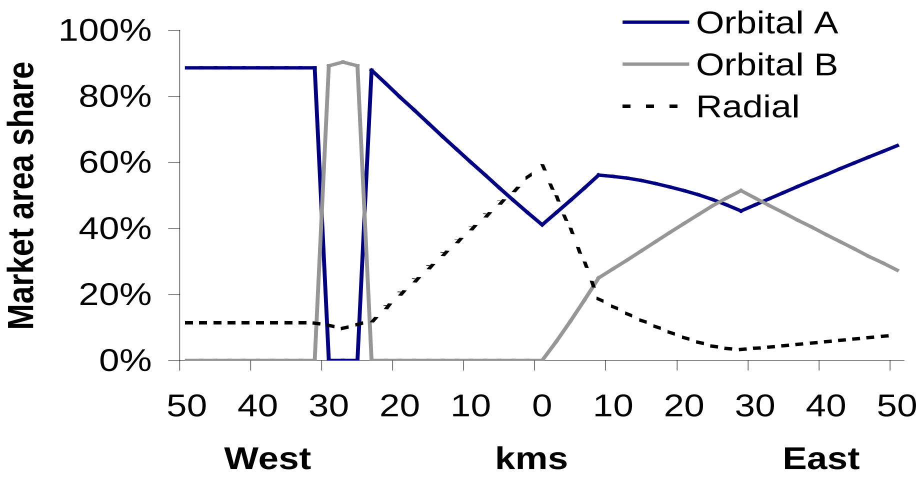


Figure 3: A Traverse

Figure 3 shows the percentage market area shares of two attractions split into orbital and radial segments. Attraction A is fixed at 25 km due west of the centre whereas B varies along a 100 km traverse, west to east. The ring road has a radius of 28 kms. The speed ratio k equals 0.55. The radial market area is shown separately form the orbital areas and is always captured by the attraction nearest the centre.

The following points can be noted: 1) On the western traverse there is a short segment where B is closer to the orbital than A and eclipses its orbital market area. Discontinuities in market area size occur at both ends of this segment; 2) Adjacent to this segment attraction A captures either all, or the majority, of the orbital market area; 3) On the eastern traverse, B increases its orbital market area continuously until it reaches the ring road and then declines; 4) The radial market area is a maximum when the variable attraction is at the city centre. It is clear that market areas are more equal and stable when the attractions are on opposite sides of the city and that competition for the orbital market area is strongest when both attractions are on the same side of the city.

b) A Circuit

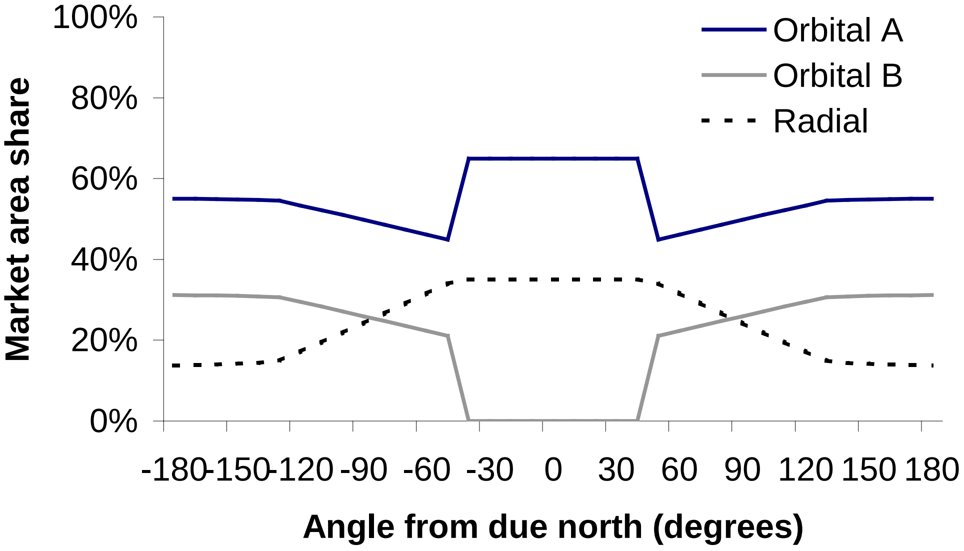


Figure 4: A Circuit.

In Figure 4, attraction A is fixed (in this case 25 kms due north) whilst B is rotated through 360 degrees at a radius of 12.5kms. The points arising in this example are that: 1) Attraction B, because it is nearest the centre, always captures the radial market area whose size varies over a limited angular range. 2) Attraction B's orbital market area is eclipsed whenever its radial market share is at its maximum value. 3) Attraction A captures the largest orbital market area as it is nearest the orbital and this area is greatest when attractions B is eclipsed. 4) The resulting patterns are symmetrical.

We conclude that for two attractions there is a rich range of market share behaviour and that market shares depends on both which attraction is nearest the centre and which is nearest the orbital. When the angular separation between attractions is small eclipsing behaviour takes place. In summary, the decision on where to locate must take account of both the distances from the city centre and the orbital, and also the angle of separation with a rival attraction.

Extension to Two Radial Speeds – The XKT1 Metric

So far we have assumed radial speeds are constant both inside and outside the orbital. We know however, that speeds inside an orbital are often significantly lower due to smaller road capacity and higher congestion. This in turn will affect market areas differentially. We adjust for this as follows. Let VI denote a constant radial speed within the ring road (of radius R), VX a constant radial speed outside the ring road and VO a constant orbital along the ring road. In subsequent sections this metric will be assumed. Let $t_R(r,s)$ and $t_O(r,\theta; s, \phi)$ denote radial and orbital travel times, respectively. The XKT1 metric is defined by:

$$t(r,\theta; s, \phi) = \mathrm{Min}(t_R(r; s), t_O(r,\theta; s, \phi)) \qquad (4)$$

where $t_R()$ and $t_O()$ are specified as follows. Define location-specific radial speeds by: $VR(r) = VI$ if $r \leq R$, $VR(r) = VX$ otherwise. Then the radial travel time from an arbitrary location of radius r to the city centre is given by $t_R(r,0) = (r - R) / VR(r) + R / VI$. The radial travel time between a general location of radius s and a location of radius r is:

$$t_R(r; s) = (r - R) / VR(r) + (s - R) / VR(s) + 2 R / VI \qquad (5)$$

The orbital travel time between (r,θ) and (s,ϕ) is given by:

$$t_O(r,\theta; s, \phi) = |r - R| / VR(r) + |s - R| / VR(s) + R|\theta - \phi| / VO \qquad (6)$$

Extending the Number of Attractions: Market Area Boundaries

Thus far we have examined examples with only two-attractions. We now extend the notation to deal with any number of attractions. Let $L=\{L_n; n=0..N\}$ denote a set of N+1 attractions where $L_n = (r_n, \theta_n)$. Number the attractions in a clockwise sequence, based on their angular coordinate, so that $\theta_{n+1}>\theta_n$. Define L_0 to be the attraction with the smallest radius. Attraction L_0 has a market area consisting of a radial and an orbital component, separated by a pair of spiral arcs. Every other non-eclipsed attraction has a market area with just an orbital component, which shares a spiral arc market boundary with the radial component of L_0. We define radial speeds in the vicinity of any attraction L_n by:

$$VR_n = VI \text{ if } r_n \leq R,\ VR_n = VX \text{ otherwise.} \qquad (7)$$

Equating radial and orbital times we obtain the following equation of such a spiral arc, for attraction L_n:

$$r = VI\left(\frac{R-r}{2VR_0} + \frac{|r_n - R|}{2VR_n}\right) + \frac{RVI}{2VO}|\theta - \theta_0| \quad n = 0..N,\ r \leq R \qquad (8)$$

Switching Angles

If a spiral arc market boundary intersects the ring road, the angle at which this occurs is referred to as a *switching angle*, given by:

$$\theta^*_0 = \theta_0 \pm \mathrm{Min}(\pi, 2VO\, r_0 / VI\, R), \qquad r_0 < R$$
$$= \theta_0 \pm \mathrm{Min}(\pi, 2VO / VI), \qquad r_0 \geq R \qquad (9)$$

$$\theta^*_n = \theta_n \pm \mathrm{Max}(0, \mathrm{Min}(\pi, (R-a_n)/b)), \qquad n>0 \qquad (10)$$

These angles form market area boundaries only when radial speeds are sufficiently fast and the attractions have a sufficiently wide angular spread so that they do no impinge on each other, as in Figure 1 a. Under such conditions these angles bound orbital market areas, separated by portions of the radial component of the market area for L_0.

Collision Angles

As already noted when the spiral arc market boundaries for different attractions intersect inside the ring road, the angle at which this occurs is referred to as a *collision angle* and are given by:

$$\theta^c_n = (\theta_{n+1} + \theta_n + (a_{n+1} - a_n) / b) / 2 \qquad 0 \leq n < N$$
$$\theta^c_N = (\theta_0 + \theta_N + (a_0 - a_N) / b) / 2 \qquad (11)$$

These angles only apply when the pairs of attractions involved have a sufficiently narrow angular spread so that they impinge on each other, as in fig 1 b). However their angular spread must not be so narrow that one attraction eclipses another. It may be verified that:

$$\theta^c_n = (\theta_{n+1} + \theta_n + (|R-r_{n+1}|/VR_{n+1} - |R-r_n|/VR_n)\, VO / R) / 2 \quad \forall n, \forall r_0 \quad (12)$$

Eclipsing in the XKT1 metric

In an earlier section we analysed eclipsing conditions for just two attractions using the KT1 metric. The analysis in this section deals with the conditions under which one attraction is eclipsed by another, under the XKT1 metric assuming many attractions. Attraction L_n is (orbitally) eclipsed by attraction L_m whenever:

$$|r_n - R| / VR_n > R|\theta_m - \theta_n| / VO + |r_m - R|./ VR_m \qquad (13)$$

or, equivalently,

$$|\theta_m - \theta_n| < K_{m,n} \text{ where } K_{m,n} = (|r_n - R|./ VR_n - |r_m - R|./ VR_m)\, VO / R \qquad (14)$$

The array $K_{m,n}$ is referred to as the *eclipse angle matrix*. Using (13) it can be verified that the eclipsing relation is *transitive*, i.e. if L_1 is eclipsed by L_2 but L_2 is itself eclipsed by L_3 then L_1 will be eclipsed by L_3.
The region eclipsed by attraction L_m consists of both an *external* component (outside the ring road) and an *internal* component (inside the ring road). The external eclipsed region is the set of all points (r,θ) such that $r > rX^e_m(\theta)$ where:

$$rX^e_m(\theta) = R + \frac{|r_m - R|VX}{VR_n} + \frac{RVX}{VO}|\theta - \theta_m| \qquad (15)$$

The internal eclipsed region is the set of all points (r,θ) such that
$r < rI^e_m(\theta)$ where:

$$rI^e_m(\theta) = Max(0,\ R - \frac{|r_m - R|VI}{VR_n} + \frac{RVI}{VO}|\theta - \theta_m|) \qquad (16)$$

The external and internal eclipsed regions are illustrated in Figure 5, for an attraction due west of the city centre. If the attraction were located outside the ring road at an equal distance the eclipse regions would be unchanged. On the radial containing the attraction the outer eclipse radius has an inward pointing cusp and the inner eclipse radial has an outward-pointing cusp. The maximum radius of the region that is not externally eclipsed is on the opposite side of the city to the attraction.

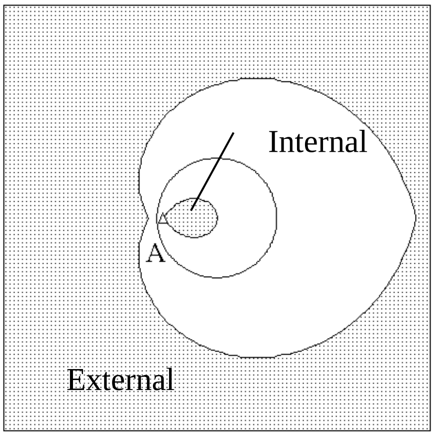


Figure 5: External and Internal Eclipsed Regions for one attraction A
The circle shows the ring road.

The Eclipsing of Candidate Sites for a New Attraction

As eclipsed locations have no orbital market area they would tend to be particularly poor sites for the location of a new attraction. In searching for good sites the elimination of eclipsed sites will result in a substantial reduction in the area of search. It is therefore of interest to construct the set of all locations (r, θ) that are eclipsed by one or more attractions from an existing set **L**={L_n, n = 0…N}. Define the *External eclipse envelope*:

$$RX^e(\theta) = \mathrm{Max}(R, \mathrm{Min}(rX^e_n(\theta), n=0\ldots N)) \tag{17}$$

and the *Internal eclipse envelope*:

$$RI^e(\theta) = \mathrm{Max}(0, \mathrm{Max}(rI^e_n(\theta), n=0,\ldots N)) \tag{18}$$

Then all locations (r, θ) such that $r > RX^e(\theta)$ or $r < RI^e(\theta)$ are orbitally eclipsed by at least one attraction in the set L. Usually the external eclipse envelope is of greater interest for several reasons. Firstly, the external envelope can be expected to eliminate a much wider area of search than the internal envelope. Secondly, the internal eclipse envelope may contain sites with good radial access and an appreciable local market, so such sites may be premature to eliminate. Thirdly, there are more likely to be physical, economic or environmental constraints on sites within the internal envelope, which would need to be considered in a more detailed analysis of options. A program for computing eclipse envelopes is described in Annex 1. Figure 6 shows eclipsing boundaries for a 3-attraction example.

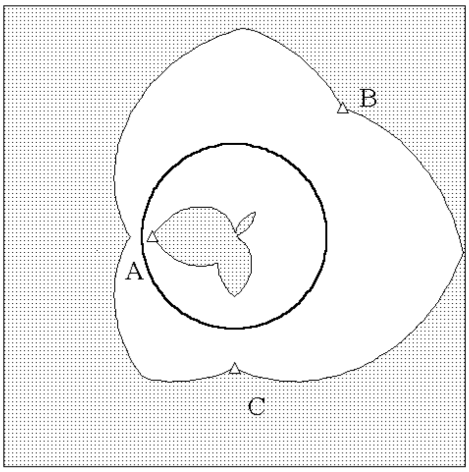


Figure 6. Map showing inner and outer eclipse envelopes. White areas are non-eclipsed and therefore potential sites.

Figure 6 illustrates eclipse envelopes for three attractions (A,B,C). The shaded areas are eclipsed – the inner eclipse envelope being the shaded area contained inside the ring road.

Extreme Locations for a New Attraction

The potential sites for an non-eclipsed new attraction with maximum feasible radius form a finite set of locations. These occur at the outer cusps of the external eclipse envelope. Similarly, the potential sites for an non-eclipsed new attraction with minimum feasible radius occur at the inner cusps of the internal eclipse envelope.

To identify sites ($RX^e{}_n$, $\theta X^e{}_n$) and ($RI^e{}_n$, $\theta I^e{}_n$) we equate the radial coordinates on the sections of each envelope, for adjacent existing attractions L_n and L_{n+1}. Solving for the angular coordinate we obtain the angular coordinate for next the pair of cusps (external and internal) clockwise from L_n:

$$\theta X^e{}_n = (\theta_{n+1} + \theta_n + (AX_{n+1} - AX_n) / BX) / 2 \qquad (19)$$

$$\theta I^e{}_n = (\theta_{n+1} + \theta_n + (AI_n - AI_{n+1}) / BI) / 2 \qquad (20)$$

The parameters AX, BX, AI and BI were given in section 3.2, from which we obtain the general expressions: $(AX_{n+1} – AX_n) / BX = (AI_n – AI_{n+1}) / BI = K_{n,\,n+1}$ where, from the definition of K in section 3.2:

$$K_{n,\,n+1} = (|r_{n+1} - R|./ VR_{n+1} - |r_n - R|./ VR_n) VO / R \qquad (21)$$

Hence:

$$\theta I^e{}_n = \theta X^e{}_n = (\theta_{n+1} + \theta_n + K_{n,\,n+1}) / 2 \qquad (22)$$

So the internal and external cusp angles are identical. Compare this expression with:

$$\theta^c{}_n = (\theta_{n+1} + \theta_n + (|R-r_{n+1}|/VR_{n+1} - |R-r_n|/VR_n) VO / R) / 2 \qquad (23)$$

as given at the end of section 2.5. This is equivalent to:

$$\theta^c{}_n = (\theta_{n+1} + \theta_n + K_{n,\,n+1}) / 2 \qquad (24)$$

It can be deduced that:

$$\theta^c_n = \theta I^e_n = \theta X^e_n. \qquad \forall n, \forall r_0 \qquad (25)$$

Equation (25) states that the collision angles are equal to the inner cusp angles of the internal eclipse envelope and to the outer cusp angles of the external eclipse envelope. This fundamental result both simplifies the geometrical constructions and provides checks on computations.

Figure 7 is a 3-attraction example showing the eclipse envelopes and radial-orbital market area boundaries and illustrates the results established in equation (25). The radials represent the orbital market area boundaries, and it is noted that these connect the outward cusps of the radial market area with the outward cusps of the outer eclipse envelope and the inward cusps of the inner eclipse envelope.

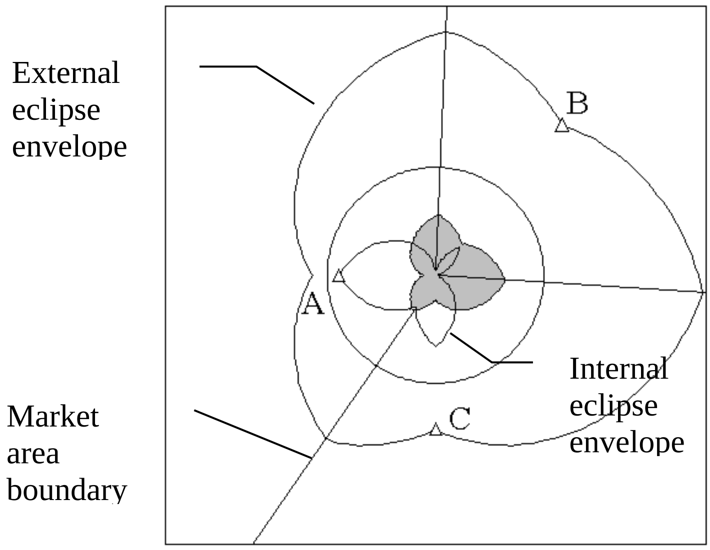


Figure 7: Alignment of envelope cusps along orbital market area boundaries. The shaded area shows the radial market area.

Figure 7 illustrates the alignment of cusps of the internal and external eclipse envelopes along the market area boundaries. Three attractions (A,B,C) are depicted. The circle represents the ring road. The unshaded spiral arcs are the inner and outer eclipse envelopes. The three radial lines show the position of the orbital market area boundaries. These go through cusps of both eclipse envelopes.

The Maximum Radius for a New Attraction

To derive the radii RX^e_n of the outer cusps of the external eclipse envelope, we can write:

$$RX^e_n = rX^e_n(\theta X^e_n) = AX_n + BX\ (\theta X^e_n - \theta_n) \qquad (26)$$

From above we have:

$$\theta X^e_n = (\theta_{n+1} + \theta_n + (AX_{n+1} - AX_n) / BX\ ) / 2 \qquad (27)$$

Substituting and simplifying we obtain:

$$RX^e_n = (BX\ (\theta_{n+1} - \theta_n) + AX_{n+1} + AX_n\ ) / 2 \qquad (28)$$

Giving a maximum radius of:

$$RX^e_{max} = \mathrm{Max}((BX\ (\theta_{n+1} - \theta_n) + AX_{n+1} + AX_n\ ) / 2,\ n=0..N) \qquad (29)$$

where attraction L_{N+1} is identified with L_0.

Example
Assume R = 20km, kX = 0.5 and 2 existing attractions: $L_0=(40, 0^o)$ and $L_1=(50, 180^o)$. Then BX = R*kX = 10km, AX_0 = 40, AX_1 = 50.
$\theta X^e_0 = (\theta_1 + \theta_0 + (AX_1 - AX_0) / BX\ ) / 2 = 119^0$
$RX^e_0 = (BX\ (\theta_1-\theta_0) + AX_1 + AX_0\ ) / 2 = 61$ km.
The location $(61, 119^0)$ is an outer cusp of the external eclipse envelope and there is a second cusp symmetrically to the west at $(61, -119^0)$, so 61km is the maximum radius for a new attraction.

Case Study

We now turn our attention to the case study of major civil airports in southeast England. As previously noted in our introduction, it is not our aim to produce a detailed planning study but rather to provide a test of the concepts described above in a real world setting. In particular we begin the study by limiting our attention to the question of evaluating the market areas of existing airports and examining the potential for new sites. Whilst the results are intuitively plausible, there are of course other social and operational factors that need to be considered. One or two of the more important these are briefly examined to show how the range of locational options may be further narrowed down from the initial set of feasible locations. A major assumption is that the airports have identical intrinsic attractiveness – that is they offer equivalent services so that the only thing differentiating them is their accessibility. It turns out that this is sufficient to produce meaningful results when the four major airports in Southeast England are considered.

Airport locations provide an interesting illustration for several reasons. For example, they tend to be publicly owned and are therefore often the subject of public scrutiny. The safety of passengers and local inhabitants, and environmental factors are particular concerns. These and the fact that airports need large areas of land means that they are normally located at the edge of cities rather than in them. On the other hand airports are commercially operated, with frequent and intense competition occurring between airline operators to attract as much business as possible. The combination of these factors puts a premium on the quality of surface access to airports. Simply put any existing airport that has poor access will tend lose its market to one that has better access. Similarly, a new airport in an in a location with poor access will fail to capture a significant number of passengers.

South-East and Eastern England contains several international airports, and supported 102 million passengers and 876,000 aircraft movements in 1998. With year on year growth in traffic there is a need to provide extra capacity at some existing sites and new sites are currently under consideration for a possible new airport**.** The region contains a substantial conurbation with about 7m people, which can take in excess of an hour to cross in typical traffic conditions. London itself has a road network that fits reasonably well with the orbital-radial model described here: the M25 ring road having a radius of about 25kms with up to 30 radial intersections. In addition there are fast rail connections linking three of the airports to the centre of London.

The region's civil passenger airports and their locations in polar co-ordinates are shown in Table 2. Note that two of the airports, Heathrow and City fall within the M25 London orbital and rest outside. Because it caters for a specialised business market and only accounts for a little over 1% of passenger traffic, City airport may be considered a special case. To begin with we exclude it from our analysis but then return to it later to look at the effect that City airport would have had on the model results.

Airport	*Distance from centre (kms)*	*Angle θ*
Gatwick	40	180
Heathrow	25	-90
City	15	95
Luton	40	-30
Stansted	51	40

Table 2: Locations of existing airports in polar co-ordinates.

Data

We used detailed 1998 data published by the Civil Aviation Authority on aircraft and passenger movements by airport. Information on residential densities was derived from the Post Office Address File for postal areas in and around the South East. Travel time data were derived from a commercial route finding package called AutoRoute 2000 which uses long run journey averages. The fact that it only includes car journey times is a weakness although, with the notable exceptions of fast rail routes to Heathrow and Gatwick, studies indicate travel times to be broadly comparable.

Radial speeds exterior to the M25 were estimated on average to be 66.5kph, radial speeds on the interior of the M25, 53 kph, and orbital speeds (using the M25), 95 kph. To check the accuracy of the speeds we made a comparison of actual and observed inter-airport speeds with the results shown in Figure 7.

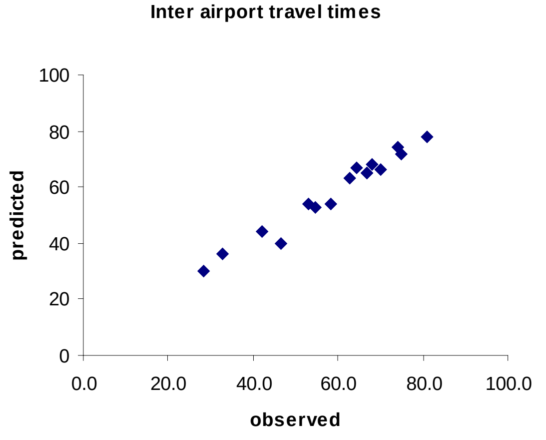


Figure 7: Scatterplot showing predicted and observed inter-airport travel times (minutes).

The best fit equation is (standard errors are shown in brackets):

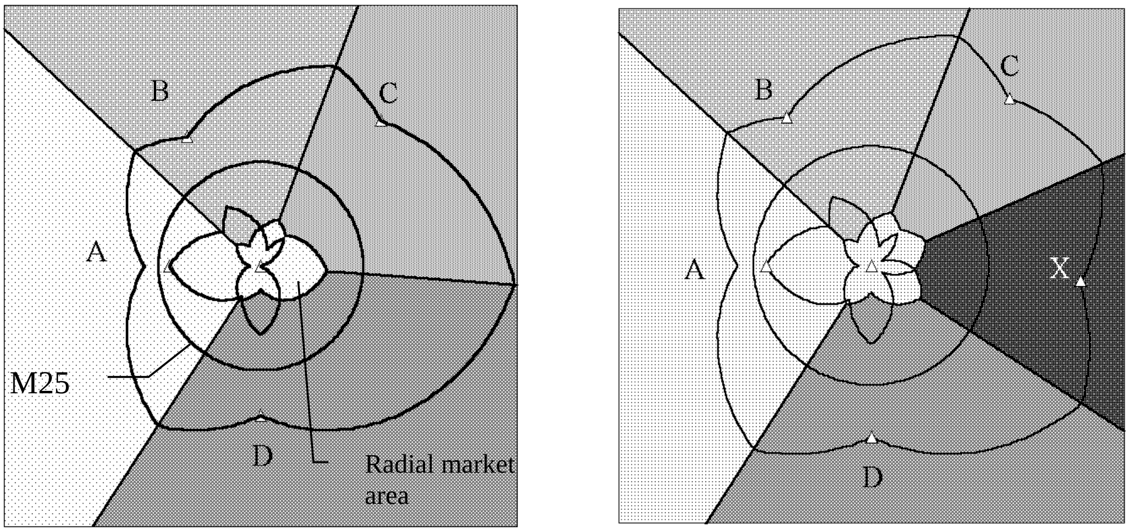
Observed time = 3.44 + 0.925 x predicted time
(2.71) (0.045)

$R^2 = 0.962$

In searching for possible locations for a new airport we adopted the assumption that the new airport should be of comparable capacity, should minimise the impact on other airports, and should maximise accessibility. This means that, in the language of the model, it should not be eclipsed by its neighbours and nor should it eclipse its neighbours. The range of feasible locations meeting these conditions, as defined by the inner and outer eclipsing boundaries, is limited, and will be illustrated below.

We now consider a scenario in which there is a new airport. We take, for illustrative purposes, a location X at 50 kms from London and at angle of 95degrees from due north, between Stansted C and Gatwick D. Figure 8, a and b, show the market areas predicted by the model before and after the new airport and the associated inner and outer eclipse envelopes. The 'petal-shaped' radial market area is shown to belong to Heathrow in either case, as it is the most accessible airport from the central area. As is seen it is fairly small and contained entirely within the M25 orbital, which means that for the majority of locations both inside and outside, the M25 is the favoured route.

We also compared market area boundaries from our model with those that would be directly obtained using travel times from AutoRoute 2000. We found that the general shape and orientation of the orbital market areas gave a satisfactory match. Comparisons between the modelled radial market boundary and Autoroute 2000 were less satisfactory particularly for routes between Heathrow and locations west of the city centre. This is principally because our model does not take into account London's inner but less effective orbital, the North & South Circular Roads**.** Note that these boundary validations are only intended to check on the reasonability of the routing predictions. It is not intended to be a check on where passengers actually come from. Such an analysis would require survey data on the origins of trips to each of the airports. We would expect the observed trip patterns to be much less distinct and to overlap to a substantial extent at the margins.



(a) Four existing airports (b) Adding a new airport

Figure 8: Market areas for four (a) and (b) five regional airports. The shaded sectors show their market areas.

In figure 8 the locations A, B C and D represent Heathrow. Luton, Stansted and Gatwick while X represents a potential site for a new airport. The circle represents the M25 The shaded sectors show the market areas for each airport location, the lightest shading showing the radial market area which is captured by attraction A. The inner and outer eclipse envelopes are also illustrated.

Numbers of residences, derived from the Post Office Address File, were converted to densities, giving:

	Central /Radial	*Inside M25*	*Outside M25*
	3100		
North		590	250
East		890	260
South		1200	154
West		1490	330

Table 3: Residential Densities (units no. per sq km)

These densities were used to predict market shares, by multiplying them by the predicted market area sizes.

Airport	*Name*	*Passenger Traffic (actual %)*	*Flights (actual %)*	*Passenger Market share (predicted %)*	*Predicted Share with new airport %*
A	Heathrow	60	53	51	49
B	Luton	4	6	13	13
C	Stansted	7	13	13	8
D	Gatwick	29	29	23	18
X	'North Kent'	n.a	n.a	n.a	12

Table 4: Actual and predicted market shares for existing airports and a possible new airport.

Table 4 compares the resulting market shares with the actual percentages of passengers and aircraft movements. Given the approximate nature of the modelling assumptions, an observed market share found to be within 25% of that predicted was taken to represent a broad correspondence. On this basis, the results indicate that:

1. Traffic at Heathrow is broadly in line with predictions. The new airport would cause only a small reduction in its market share.

2. Traffic levels at Luton suggest there is a substantial degree of under – utilisation, compared with the market size predicted by the model. Luton’s market would not be seriously affected by the new airport.

3. Passenger traffic levels at Stansted likewise indicate a slight degree of under utilisation, although the number of flights are close to model predictions. However, the results suggest its market share would fall significantly in the presence of a new airport, with the new airport capturing the greater market share. Stansted may lose its position as the third most important airport in the region.

4. The market share at Gatwick is broadly in line with expectations but like Stansted, Gatwick’s market share would be expected to fall significantly if a new airport were built. It is predicted that Gatwick would still remain the second most important airport in the region.

5. The model’s slight under-prediction of the market shares for Heathrow and Gatwick appears to be simply a direct consequence of its joint over-prediction of Stansted and Luton.

Sensitivity tests using traverse and circuit analysis

a) Traverse

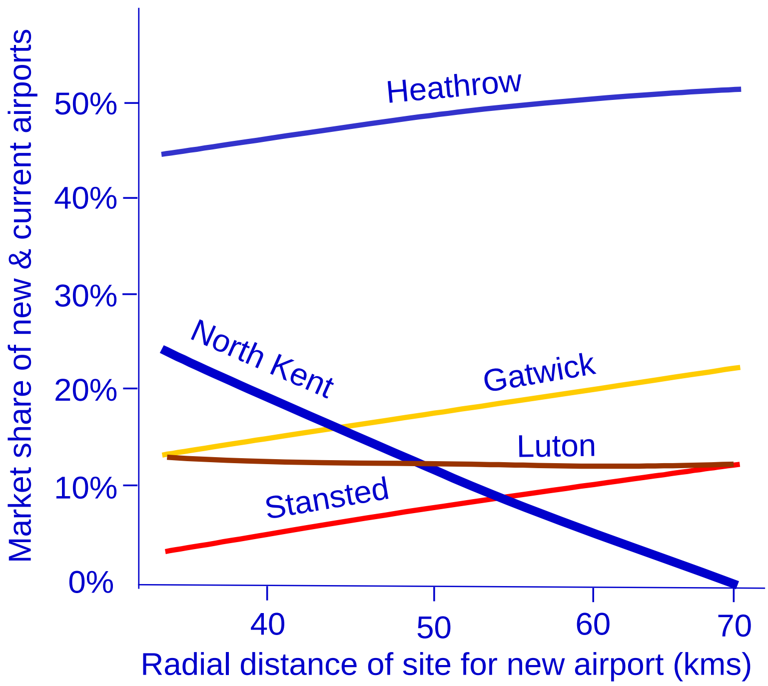


Figure 9: Market shares for alternative radii of the new airport.

The results presented are only indicative. It is of interest to assess its sensitivity to changes of possible location, either closer to the centre, further out or at a different orientation. We therefore used traverse and circuit analysis to test the sensitivity of our results, as outlined earlier in the paper. The traverse in Figure 9 shows how the radius of the new airport affects its market share and the market share of others. Not surprisingly North Kent's market share is maximised when it is sited near the M25 but then declines until, at around 70 kms, it falls to zero. The effect on neighbouring airports shows that Gatwick and Stansted stand to gain as the new airport's radius increases. Heathrow's market share varies as North Kent's market area boundary is adjacent to Heathrow' radial market area. Luton's market share is not affected as Luton's market area shares no common boundary with North Kent.

b) Circuit

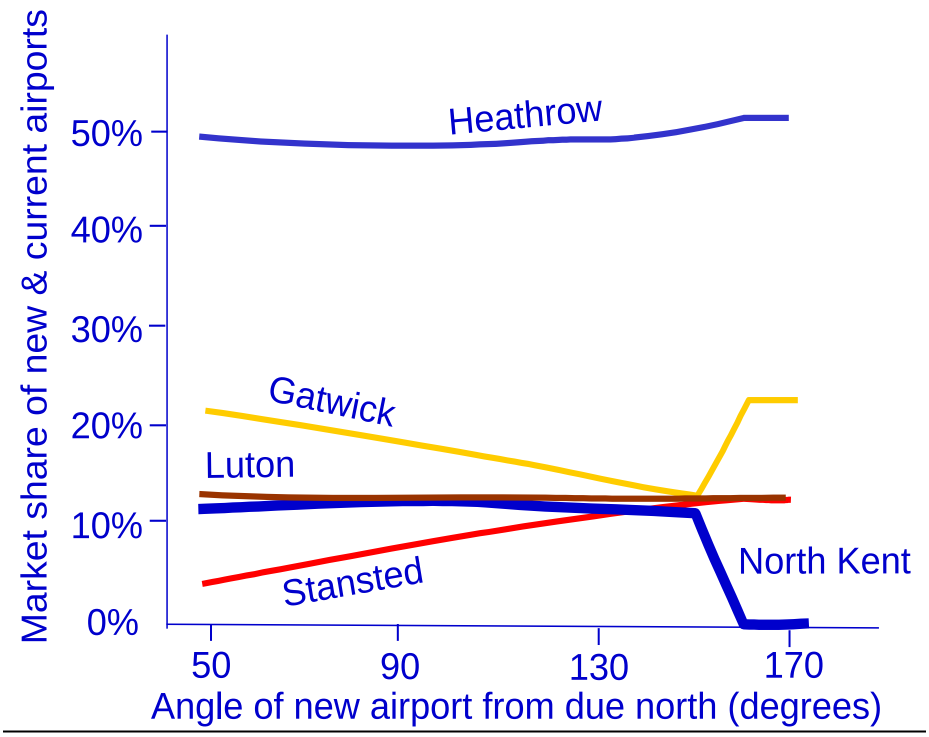


Figure 10: Market shares for varying angles of the new airport.

Figure 10 shows how the market share of each airport is affected as the location of the new airport is rotated at a 50kms radius between Stansted in the north-east and Gatwick in the south. As is seen Heathrow and Luton are barely affected as compared with Gatwick and Stansted. As the new airport rotates further towards Gatwick, Stansted's share goes up and Gatwick's down as is to be expected. Note however, that as the new airport rotates to within 20 degrees of Gatwick its market area is suddenly eclipsed and Gatwick gains as a result.

Travel time from the orbital

We have omitted to discuss why we chose to locate the new airport at 95 degrees. Figure 11 shows the travel time to the nearest airport from different points on the orbital circumference. In the 'before' situation travel time is a maximum (37 minutes) at a location 95 degrees from due north whereas it is a minimum at -90 degrees due north (corresponding to Heathrow). Following the introduction of the new airport the maximum journey time from the orbital falls to 29 minutes. This reduction is maximised by locating the new airport at 95 degrees. However this is only one of many factors that might bear on the actual location of any possible new airport, so our choice of this particular criterion is only given for illustrative purposes.

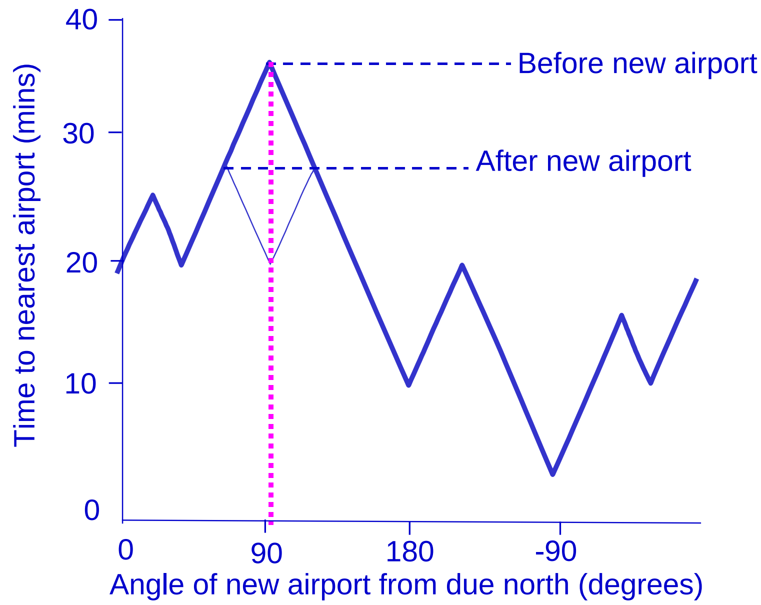


Figure 11: Travel time to nearest airport from a point on the orbital before and after introduction of new airport.

The Effect of City airport

Earlier we noted that City airport was a special case. Because it is closer to the city centre than Heathrow it is a potential threat to Heathrow's dominance. We now examine the effect of including City airport in the model, assuming that no new airport is built. For this exercise we can realistically assume that City airport enjoys faster radial surface access to the centre of London than Heathrow. But we also need to make a less realistic assumption: that it would have no limitation in passenger and aircraft handling capacity or any substantial environmental constraints on expansion . The model predicts that City would then capture the radial market area and gain an overall a market share in excess of 40%, whilst Heathrow's market share would halve.

Airspace Design Footprints

In our introduction to the case study we highlighted safety factors. These have the effect of introducing additional spatial constraints, so narrowing even further the range of feasible airport sites. We illustrate this briefly as follows simply to show how, in an actual application, the model would need to interface with other planning considerations. Figure 13 shows typical airspace design footprints for take-offs and landings (based on DETR, 2000 p21). The footprint for City has been reduced to avoid an overlap with Heathrow and may be justified in terms of the smaller aircraft that operate from City airport. However, when airport locations are so close that their footprints overlap, their takeoff and landing control systems would need to be closely integrated.

Of course this is just for illustrative purposes and accuracy is only intended to be approximate. However, it can be noted that new airport locations to the east of London that are appreciably closer than 48km from the centre may need to be ruled out as their airspace footprints would overlap that of City airport. The location marked in figure 13, which falls within the non-eclipsed area, just avoids this. It is in the Medway Towns area, roughly equidistant between Rochester and Gillingham.

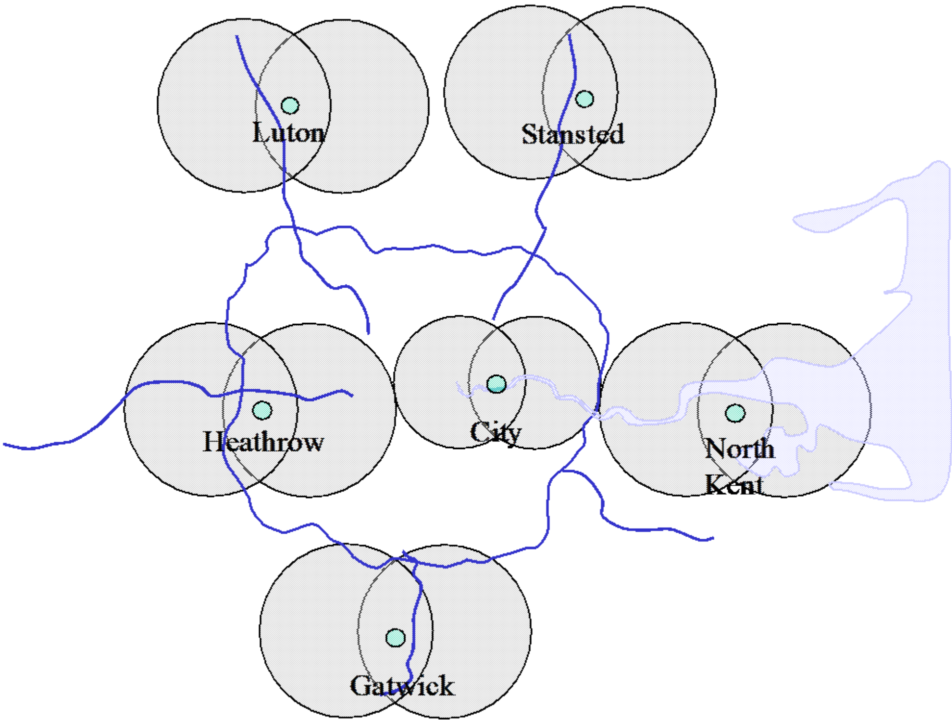


Figure 12: Airspace Design Footprints for five existing airports and a new airport in North Kent.

Summary and conclusions

In this paper we have developed an analytical framework for determining the characteristics of market areas under orbital-radial routing: a common feature of many cities. Orbital-radial routing characteristics depend on how many alternative ring roads are available. The framework provided is quite general but it has only been elaborated for models with a single ring road. This is a sufficiently good first approximation for the level of detail required for this case study. The study of airport location is of particular interest because airports are becoming increasingly strategic features of most cities and exercise their influence beyond city boundaries to the whole of a surrounding region, generating demands for both radial and orbital access.

Our conclusions therefore fall into three distinct categories. The first is the theory itself and the policy implications that arise in terms of planning and competition. Here we were able to demonstrate a number of hitherto unidentified factors, particularly the conditions under which one attraction may eclipse another and therefore capture all or a substantial part of its market. A deliberate omission was the incorporation of differential attractiveness although there appears to be no fundamental problems in extending the analysis to account for such factors.
This would be worthwhile extension to the theory if, for example, the model were to be applied to commercial facilities such as major retail and distribution centres.

The second set of conclusions concern the results of the case study. Here we are not suggesting that the ideal location for a new airport is the one identified in the case study. There are many other economic, financial and environmental factors to take into account, whether a new site is built at all or existing airports are expanded. However the factors incorporated in our analysis are expected to have a significant bearing on strategic airport location decisions, and may in themselves be sufficient to rule out a number of potential sites.

The third set of conclusions relates to the methods and software. This paper has described only basic software tools that can help to implement models with low data requirements. It would clearly be dangerous to rely on such tools as a substitute for a more complete analysis. However it is always of value to use simple methods to give a 'reasonability check' on the results of a more complex model and the tools developed here are expected to have further applications of this kind.

Annex 1: A Market Area Analysis Program

Metric: XKT1
Attractions: N+1
Outputs: Market Area Sizes, Plots of Eclipse Envelopes and Radial Market Area
Assumes: L_0 is inside the ring road. All orbital markets collide

```
Begin
   Read general fixed inputs: R, VI, VX, VO
   Calculate derived constants: kI, kX, BI, BX, b
   Loop over attractions n=0..N
      Read coordinates of each attraction: (r_n, θ_n)
      Calculate radial speeds at each attraction: VR_n
      Calculate eclipse envelope start radii AX_n , AI_n
      Calculate radial market area start radii a_n
      Calculate collision angles and angular limits of orbital markets
      Output size of market areas
   End loop
   Loop over all angles θ
      Loop over attractions n=0..N
         Calculate external eclipse projections rX^e_n(θ)
         Calculate internal eclipse projections rI^e_n(θ)
         Calculate radial market area projections rMR_n(θ)
      End loop over attractions
      Output external envelope RX^e(θ)
      Output internal envelope RI^e(θ)
      Output radial market area radius RMR(θ)
   End loop over angles
End
```

General Fixed Inputs

R	Radius of Ring Road	Km
VI	Internal Radial Speed	Kph
VX	External Radial Speed	Kph
VO	Orbital Speed	Kph

Derived Constants

kI	VI/VO	Internal Speed Ratio	Pure number
kX	VX/VO	External Speed Ratio	Pure number
BI	kI R	Internal Eclipse Gradient	Kms/Radian
BX	kX R	External Eclipse Gradient	Kms/Radian
b	BI / 2	Spiral Market Gradient	Kms/Radian

Inputs for each attraction

R_n	Radius of attraction n	Km
θ_n	Angle of attraction n	Degree

Derived Variables

VR_n	If(r_n < R, VI, VX)	Radial Speed at Attraction n	Kph
AX_n	R + $\lvert r_n$ - R$\rvert$ VX / VR_n	External envelope start radius n	Km
AI_n	Max(0, R - $\lvert r_n$ - R$\rvert$ VI / VR_n)	Internal envelope start radius n	Km
a_n	((R - r_0) / VR_0 + $\lvert r_n$ - R$\rvert$ / VR_n) VI / 2	Start radius of market area spiral n	Km
$\theta^c{}_n$	(θ_{n+1} + θ_n + Deg((a_{n+1} – a_n)) / b) / 2	Collision angle $0\underline{<}n<N$	Degree
$\theta^c{}_N$	(θ_0 + θ_N + Deg((a_0 – a_N) / b)) / 2	Collision angle N	Degree
$\phi_n{}^{Rt}$	Rad(Min($\lvert\theta^c{}_n$ - $\theta_n\rvert$, 360 - $\lvert\theta^c{}_n$ - $\theta_n\rvert$)))	Right angular limit of orbital market	Radian
$\phi_n{}^{Lt}$	Min($\lvert\theta^c{}_{n-1}$ - $\theta_n\rvert$, 360 - $\lvert\theta^c{}_{n-1}$ - $\theta_n\rvert$)	Left angular limit of orbital market n>0	Radian
$\phi_0{}^{Lt}$	Min($\lvert\theta^c{}_N$ - $\theta_0\rvert$, 360 - $\lvert\theta^c{}_N$ - $\theta_0\rvert$)	Left angular limit of orbital market n=0	Radian
$RX^e{}_n(\theta)$	AX_n + BX Rad(Min($\lvert\theta$ - $\theta_n\rvert$, 360 - $\lvert\theta$ - $\theta_n\rvert$)	External eclipse radius projected by attraction n at angle θ	Km
$rI^e{}_n(\theta)$	AI_n - BI Rad(Min($\lvert\theta$ - $\theta_n\rvert$, 360 - $\lvert\theta$ - $\theta_n\rvert$)	Internal eclipse radius projected by attraction n at angle θ	Km
$rMR_n(\theta)$	Min(R, a_n + b Rad(Min($\lvert\theta$ - $\theta_n\rvert$, 360 - $\lvert\theta$ - $\theta_n\rvert$))	Radial market area radius projected by attraction n at angle θ	Km

Outputs			
AI_n	$A(\phi_n^{Lt}, R, a_n, b) + A(\phi_n^{Rt}, R, a_n, b)$	Internal area of orbital market	Km^2
AR	$\pi R^2 - \sum_n AI_n$	Internal radial market area	Km^2
$RX^e(\theta)$	$Max(R, Min(rX^e_n(\theta), n=0..N)$	External eclipse radius at angle θ	Km
$RI^e(\theta)$	$Max(0, Max(rX^e_n(\theta), n=0..N)$	Internal eclipse radius at angle θ	Km
$RMR(\theta)$	$Min(rMR_n(\theta), n=0..N)$	Radial market area radius at angle θ	Km

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