Department of Economics

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Department of Economics
Discussion Paper Series
No. 18/02

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Exchange-rate exposure in a “Rule of Three” Model

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May 2018

Abstract.

We examine exchange-rate exposure in an international Bertrand model of differentiated goods using a “Rule of Three” (*RoT*) market structure that allows both *within* and *between* countries competition. We construct two versions of our model, a static and a dynamic one. In the latter, we explore how the intertemporal effects of exchange rates on the optimal prices of a firm’s domestic and international rivals will affect a firm’s long-run exposure in relation to its short-run exposure. We find that in the static version, the addition of a domestic competitor increases the firm’s exposure, while the effect on its foreign competitor is ambiguous. In the dynamic case, we find that the gap in exposure between the *RoT* model and the international duopoly case is larger in the long run than in the short run for the company facing a domestic rival, while the exposure for that firm can be either smaller or larger in the long run relative to the short run. Finally, the firm that remains a monopolist in its domestic market has a smaller exposure in the long run as compared to the short run.

Keywords Oligopoly Market Structure; Bertrand Model; Foreign Exchange; Long-Run Exposure; Short-Run Exposure

JEL Classification Numbers: L13, D21, F31

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1. Introduction

Both the marketing and economics literature study markets where three competitors are present - a market structure known as a triopoly in economics and a "*Rule of Three*" (*RoT*) in marketing.

The marketing strand argues that the majority of manufacturing industries have a few large companies that lead the market followed by a fringe of firms (Axtell, 2001; Buzzell, 1981; Gans and Quiggin, 2003; Ijiri and Simon 1977; Quandt 1966). Sheth and Sisodia (2002) argue that an industry structure consisting of three large generalists and numerous smaller specialists generates a competitive environment that is "optimal" for firm stability and profitability. Uslay et.al. (2010) show that industries with three firms financially outperform every other market structure, both in terms of operating returns to assets and in terms of annual cumulative abnormal returns. The *RoT* theory also appears in a variety of practitioner-oriented publications like Sheth and Sisodia (1998), Gordon (2001), Landry (2002), Saffersfone (2002) and Henricks (2002).

All these studies differ from our work in two main aspects. First, they study *RoT* market structures without dealing with the impact of exchange rates on profitability. They focus on the convergence to a *RoT* market structure, while we look at markets that already have converged to this market structure and study the impact of exchange rate on prices and profits (exposure) in an international Bertrand triopoly model.

The economics literature also looks at markets with three competitors (Shugan ,1989; Puu and Marin, 2006; Bouis et al., 2009; Ji, 2009; Elabbasy, et al., 2009; Elsadany et al., 2013; Matouk et. al., 2017; Shibata, 2016; Andaluz and Jarne, 2015; Andaluz, et. al. 2017). All these studies differ from our work in two main aspects. First, they study a triopoly (or quadropoly) but in a closed economy, while we focus on an international triopoly modelling the competition *between* domestic and foreign firms, and the combined effects of the *between* and *within* competition. Second, the focus of our paper is not on stability, but on the impact of exchange rate on profits (exposure) in a price-setting model. This is important from a financial point of view, as it explains the implications on the performance on markets as they evolve into *RoT* ones. To the best of our knowledge, no other study examines the impact of exchange rates on prices (passthrough) and profits (exposure) on a *RoT* market structure. We study two versions of the *RoT* model — a static and a dynamic one — to explore short-run and long-run exposure, which corresponds to consumable and durable goods respectively.

For the latter model, we employ a switching cost model constructing a dynamic framework following the rich literature on switching costs with seminal articles, such as Beggs and Klemperer (1992), Cabral and Villas-Boas (2005) and, more recently, Rhodes (2014). We produce a theoretical framework that connects economics with strategic management, focusing on a price-setting model.

As mentioned above, our paper is connected to the exchange-rate exposure literature. Bodnar et al.,(2002) develop and estimate a model for eight Japanese export industries during 1986-1995. They examine if the relation between exchange-rate exposure and passthrough as derived in their model is consistent with actual market behavior. Flode´n et al. (2008) study how changes on the supply side across industries affects the relationship between passthrough and exposure. Since pricing affects profitability, they argue that nonlinearities in the cost function when studying the relationship between exchange-rate passthrough and exposure across industries are important. However, they study only the cases of monopolistic competition and a Bertrand and a Cournot duopoly, in which one of the two firms only operates in its domestic country and the other operates in both domestic and exports abroad, while we look at a *RoT* market structure.

Bartram et al., (2008) expand the theoretical model of Bodnar, et al., to examine the exchange-rate exposures of a global firm that can compete and produce in both a foreign and local market. In the Bodnar, et al. model, the exporting firm cannot sell in its own market and the local firm cannot produce abroad. Bartram et al. derive optimal pass-through decisions and the resulting foreign exchange exposures of global firms in globally competitive industries. The Bartram et al. model shows exposure to be a function of market share, product substitutability, and passthrough in foreign currency. Moreover, the exposure of the Bartram et al. model is smaller than the exposure in the Bodnar, et al., model under most conditions. Interestingly, there are cases where the Bartram et al. model produces negative exchange-rate exposure.

Bodnar et al. do not study the impact of industry structure on exposure. Marston (2011) emphasizes the importance of the competitive structure of the industry in which a firm operates, on its economic exposure. However, he studies only the following cases: monopoly; a Cournot duopoly in which one of the two firms only operates in its domestic country and the other operates in both domestic and exports abroad; and the case of Stackelberg leadership by the exporting or the local firm. By contrast, we look at a *RoT* market structure.

The dynamic part of our *RoT* model also relates to the Froot and Klemperer (FK) (1989) and Gross and Schmitt (2000) (GS), (2000) switching cost models. FK are the first who construct an oligopoly model with dynamic demand-side effects by allowing the future demands of firms to depend on current market shares. Their model is a two-period dynamic game in which expected exchange rates affect the value of current market shares. However, their model only includes two producers — one foreign, one domestic — and the goods offered are homogenous.

GS use FK's switching cost model. They maximize the value of the firm, namely the present value of its profits, showing that an intertemporal link exists in pricing decisions in a durable goods two period model. The dynamic formulation allows the study of the exchange-rate passthrough both in the long run and in the short run; the result is that price interdependence matters and that exchange-rate fluctuations have significant feedback effects on prices, resulting in lowering passthrough in the long run rather than in the short run.

Both GS and Bénassy-Quéré et al. (2011) note that there is an intertemporal trade-off as far as switching costs are concerned. Both Rhodes and Cabral (2017) study how old customers lock into the products of a firm. Consequently, they are less sensitive to price changes by that firm and its competitors. On the other hand, new customers will be offered lower prices as a "firm's incentive to lock people in will outweigh the customer's incentive to avoid being locked in" (Rhodes, p. 172). How the firm responds to an exchange-rate change will depend on how it values its future profits as measured by the size of the firm's discount factor.

We note that our analysis differs from the GS paper, as they study two foreign producers of a homogenous good serving a market with no home production under Bertrand competition, looking only at exchange-rate passthrough and not foreign exchange exposure. Our model looks at foreign exchange exposure in a *RoT* setting where there are two home firms and one foreign firm competing in a Bertrand differentiated goods framework. We study how the intertemporal effects of exchange rates on the optimal prices of a firm's domestic and international rivals will, in turn, change the magnitude of long-run as compared to short-run exposure.

2. General Framework

We start with a simple static model, as it is appropriate to study the impact of the exchange rate on consumable goods where the effect is a single-period one.

We study the impact of the exchange rate, *S,* on prices and profits in the Bertrand model in a setting of differentiated goods and linear demands. We first look at a simple model with one home, *h*, and one foreign firm, *f*, setting prices P_h and P_f respectively. The demand functions in the home and foreign markets are respectively $q_h(P_h, P_f; S) = \theta_o + \theta_h P_h + \theta_f S P_f$ and $q_f(P_h, P_f; S) = \lambda_o + \lambda_f P_f + \lambda_h \frac{1}{S} P_h$, where $\theta_f, \lambda_h > 0$, $\theta_h, \lambda_f < 0$, and constant marginal costs c_h and c_f respectively, such that $0 < c_h < \theta_o$ and $0 < c_f < \lambda_o$, while the firms choose prices simultaneously. For the second order conditions (S.O.C.s) to be satisfied, $4\lambda_f\theta_h - \theta_f\lambda_h > 0$. An increase in *S*, namely a foreign exchange rate, appreciation increases (decreases) the optimal price of home (foreign) goods as $\frac{\partial P_h^{*B}}{\partial S} = \frac{\theta_f(\lambda_o - \lambda_f c_f)}{(4\lambda_f\theta_h - \theta_f\lambda_h)} > 0$ and $\frac{\partial P_f^{*B}}{\partial S} = \frac{\lambda_h(\theta_h c_h - \theta_o)}{(4\lambda_f\theta_h - \theta_f\lambda_h)S^2} < 0$. The positive (negative) derivative of the home (foreign) equilibrium price with respect to (*wrt*) *S* is the result of the fact that as this is a 1x1 case there is only *between* (countries) competition.

It is straightforward to show that an increase in *S* makes the *h* firm better (worse) off and the *f* firm worse (better) off if the equilibrium prices of their rivals are inelastic (elastic) in *S*. This is because $\frac{\partial \Pi_h^{*B}}{\partial S} = \theta_f(m_h^*)P_h^*P_f^*[1 + \varepsilon_{P_f^*,S}]$, where $\varepsilon_{P_f^*,S} = \frac{\partial P_f^*}{\partial S}\frac{S}{P_f^*}$<0, is the partial elasticity of the equilibrium foreign price *wrt S* and $m_h^* = \frac{(P_h^* - c_h)}{P_h^*}$ is the equilibrium price-cost-margin of *h*. Hence, the sign of $\frac{\partial \Pi_h^{*B}}{\partial S}$ depends on $\varepsilon_{P_f^*,S}$; if the latter is elastic (inelastic) *wrt S*, the equilibrium profits of *h* decrease (increase) in *S*. Clearly, as *S* increases, unless *f* reduces its price so that it more than offsets the increase in *S* to restore its competitiveness, *h* will enjoy an increase in its optimal profits. The size of the change in the equilibrium profits of *h* depends on the cross-price substitution parameter, θ_f, the price sensitivity of its rival to *S*, and its own price-cost margin.

Similarly, $\frac{\partial \Pi_f^{*B}}{\partial S} = \frac{1}{S^2}\lambda_h\left(m_f^*\right)P_h^*P_f^*[\varepsilon_{P_h^*,S} - 1]$, where $\varepsilon_{P_h^*,S} = \frac{\partial P_h^*}{\partial S}\frac{S}{P_h^*}$>0 is the partial elasticity of the equilibrium home price *wrt S* and $m_f^* = \frac{P_f^* - c_f}{P_f^*}$ is the equilibrium price-cost margin of *f*. Again, the sign of $\frac{\partial \Pi_f^{*B}}{\partial S}$ depends on $\varepsilon_{P_h^*,S}$. If the equilibrium price of *h* is elastic (inelastic) *wrt S*, the profits of *f* increase (decrease) in *S*. Hence, the profits of *f*, whose currency appreciates, will decrease unless *h* more than counteracts its gain in competitiveness through a more than offsetting increase in its equilibrium price. So, the change in the equilibrium profits of *f* will depend on its price-cost margin and the degree of substitutability between the two goods, λ_h, $\varepsilon_{P_h^*,S}$, and will inversely depend on the square of *S*.

To conclude, in the 1x1 Bertrand model there is a link between the ability of each firm to pass on the exchange-rate change in the price it charges and the direction and degree of the impact on the profits of the other firm as a result of *S*.

The above results cover the cases where one of the two firms is a monopoly in its home market and competes in the foreign market with the domestic firm there. Interestingly, exchange-rate fluctuations do not affect just exporting firms, but even firms which focus on their home country (Marston; Aggarwal and Harper; 2010).

3. A Rule of Three-Industry Structure

Now we study the impact of foreign exchange rates on profits and prices in a *RoT* market structure, with differentiated products and linear demands, when firms compete in prices.

3.1. Consumable Goods

3.1.1. Between- and Within-home Countries Bertrand Competition

There are three firms — we assume two home firms and one foreign. They offer differentiated consumable goods competing in a Bertrand model. Each *h* firm competes with the other *h* firm – *within* competition and with the *f* firm in the other country – *between* competition. The *f* firm faces only *between* competition. The two *h* firms and the *f* firm choose prices $P_{1,h}, P_{2,h},$ and P_f respectively, and the three demand functions in matrix form are as follows:

$$\begin{bmatrix} q_{1,h}(P_{1,h}, P_{2,h}, P_f; S) \\ q_{2,h}(P_{1,h}, P_{2,h}, P_f; S) \\ q_f(P_{1,h}, P_{2,h}, P_f; S) \end{bmatrix} = \begin{bmatrix} \theta_{1,o} \\ \theta_{2,o} \\ \lambda_o \end{bmatrix} + \begin{bmatrix} \theta_{11,h} & \theta_{12,h} & \theta_{1,f}S \\ \theta_{21,h} & \theta_{22,h} & \theta_{2,f}S \\ \lambda_{1,h}\frac{1}{S} & \lambda_{2,h}\frac{1}{S} & \lambda_f \end{bmatrix} \begin{bmatrix} P_{1,h} \\ P_{2,h} \\ P_f \end{bmatrix} \tag{1}$$

The direct terms parameters are all negative, i.e. $\theta_{11,h}, \theta_{22,h}, \lambda_f < 0$. The parameters pair, $\theta_{12,h}, \theta_{21,h}$ are the *within* countries cross-price effects, while the pairs $\theta_{1,f}, \theta_{2,f}$ and $\lambda_{1,h}, \lambda_{2,h}$ are the *between* countries cross-price effects involving the two *h* firms and the *f* firm. The firms produce substitutes, so all the cross-price effects are positive. We assume that each firm's own price has a greater absolute effect on its demand than that of the prices of the firms it competes with, both domestically and abroad. Hence, $|\theta_{11,h}| > \theta_{12,h}, \theta_{1,f}$, $|\theta_{22,h}| > \theta_{21,h}, \theta_{2,\mathrm{f}}$, $|\lambda_f| > \lambda_{1,h}, \lambda_{2,h}$.There are no fixed production costs, and marginal costs are constant, while $c_{1h} < \theta_{1,o}, c_{2h} < \theta_{2,o}$, $c_f < \lambda_o$. The optimization problems for the two *h* firms and the *f* firm are as follows:

Home Country:

$$\max_{p_{1,h}} \Pi_{1,h}(P_{1,h}, P_{2,h}, P_f; S) = \max_{P_{1,h}} [\theta_{1,o} + \theta_{11,h} P_{1,h} + \theta_{12,h} P_{2,h} + \theta_{1,f} S P_f][P_{1,h} - c_{1,h}]$$

$$\max_{p_{2,h}} \Pi_{2,h}(P_{1,h}, P_{2,h}, P_f; S) = \max_{P_{2,h}} [\theta_{2,o} + \theta_{21,h} P_{1,h} + \theta_{22,h} P_{2,h} + \theta_{2,f} S P_f][P_{2,h} - c_{2,h}]$$

Foreign Country:

$$\max_{P_{1,f}} \Pi_f(P_{1,h}, P_{2,h}, P_f; S) = \max_{P_{1,f}} \left[\lambda_o + \lambda_f P_f + \lambda_{1,h} \frac{1}{S} P_{1,h} + \lambda_{2,h} \frac{1}{S} P_{2,h}\right] [P_f - c_f]$$

Solving this gives the three equilibrium prices for the Bertrand model, which we use to study the effect of *S* on the three optimal prices. The vector $\frac{\partial P^{*B}}{\partial S} = [\frac{\partial P^{*B}_{1,h}}{\partial S}, \ \frac{\partial P^{*B}_{2,h}}{\partial S}, \ \frac{\partial P^{*B}_{f}}{\partial S}]'$ gives the impact of a change in *S* on the prices of *h* and *f* firms. In the 1x1 Bertrand model, an increase in *S* decreases (increases) the price of the firm in the appreciating (depreciating) country.

Proposition 1

An increase in *S* has a positive impact on the optimal prices of the home goods and a negative impact on the optimal price of the foreign goods.

Proof

Please refer to the MA.

In the 1x1 case, the existence of *between* competition only results in a positive (negative) derivative of the equilibrium price of the *h* (*f*) firm *wrt S*. The addition of a second home firm, which introduces *within* competition in the home market alone, does not affect the sign of the results. We next study $\frac{\partial \Pi^*_{1,h}}{\partial S}, \frac{\partial \Pi^*_{2,h}}{\partial S}, \frac{\partial \Pi^*_{f}}{\partial S}$ by using the first order conditions (F.O.Cs) and replacing the outputs with the demand functions.

Proposition 2

Profit exposure is dependent on the exchange-rate price elasticities of two rivals, *h* and *f* in the case of each *h* firm, and *wrt* elasticities of the two *h* firms in the case of the *f* firm.

Proof

For each *h* firm the impact of *S* on the profit of a firm depends on the sign (and if negative, on the size as well) of the elasticity of the *f* rival *wrt S*. It also depends on the sign and size of the elasticity of the price of the home rival *wrt S*:

$$\frac{\partial \Pi^*_{i,h}}{\partial S} = P^*_{i,h}\, m^*_{i,h} \left[\theta_{ij,h} P^*_{j,h} \frac{\varepsilon_{P^*_{j,h},S}}{S} + \theta_{i,f} P^*_f \left(1 + \varepsilon_{P^*_f,S}\right)\right]$$

for *i*=1,2, *j*=1,2, $j \neq i$, where $\varepsilon_{P^*_{i,h},S} = \frac{\partial P^*_{i,h}}{\partial S}\frac{S}{P^*_{i,h}} > 0, \varepsilon_{P^*_{j,h},S} = \frac{\partial P^*_{j,h}}{\partial S}\frac{S}{P^*_{j,h}} > 0$, are the partial elasticities of each home firm's optimal price *wrt S*. In the above relation $\varepsilon_{P^*_{j,h},S} = \frac{\partial P^*_{j,h}}{\partial S}\frac{S}{P^*_{j,h}} > 0$ is the *within*-market exchange-rate passthrough of firm *i*'s domestic competitor *j* and $\varepsilon_{P^*_f,S} = \frac{\partial P^*_f}{\partial S}\frac{S}{P^*_f} < 0$ is the exchange-rate passthrough of the foreign competitor. Finally, the term $m^*_{i,h} = \frac{(P^*_{i,h} - c_{i,h})}{P^*_{i,h}}$ is the equilibrium price-cost margin of the home firm *i*.

Hence, the impact of *S* on the profits of this firm depends on the firm's own price elasticity of demand (as reflected through its inverse relation to the price-cost margin, $m^*_{i,h}$) and on both of its two rivals' (home and foreign) price sensitivity to the exchange rate (i.e. passthrough). The addition of a home competitor increases the firm's exchange-rate exposure relative to the international duopoly (1x1) case. In the case of an increase in *S*, it will affect the profits positively because of the home currency's depreciation. We note that if $\varepsilon_{P^*_f,S}$ is in absolute value sufficiently larger than one, then the profits of the *i h* firm may decrease rather than increase because of an increase in *S*.

The impact of *S* on the profit of the *f* firm depends on the elasticities of the two *h* firms:

$$\frac{\partial \Pi^*_f}{\partial S} = \frac{1}{S^2} P^*_f\, m^*_f \left[\lambda_{i,h} P^*_{i,h}\left(\varepsilon_{P^*_{i,h},S} - 1\right) + \lambda_{j,h} P^*_{j,h}\left(\varepsilon_{P^*_{j,h},S} - 1\right)\right]$$

As noted above, $\varepsilon_{P^*_{i,h},S}$ and $\varepsilon_{P^*_{j,h},S}$ are both positive. Similarly, the term $m^*_f = \frac{\left(P^*_f - c_f\right)}{P^*_f}$ is the equilibrium price-cost margin of the *f* firm. Hence, the impact of *S* on profits depends on the *f*'s own price elasticity of demand (as reflected through its inverse relation to the price-cost margin m^*_f) and on the overseas rival's price sensitivity to the exchange rate (passthrough).

Consequently, as in the 1x1 case, the impact of *S* on the profits of the *f* firm may end up being either positive or negative depending on the size of the two *h* firms price elasticities *wrt S*. Given the ambiguity in the signs, it is impossible to determine whether the addition of a domestic competitor in one country increases or decreases exposure for the firm in the other country relative to the international duopoly case.

3.2. Durable Goods

3.2.1. Between- and Within-home Countries Bertrand Competition

In this section, we set a dynamic model that describes the behavior of the three firms in a durable goods context. The firms choose prices $P_{t,1,h}, P_{t,2,h}$ and $\mathrm{P}_{t,f}$ in periods $t = 1{,}2$ respectively. We look for a subgame perfect equilibrium using backward induction. The value of each firm V_t at time *t* is the present value of its future profits Π_{t+i} at the one-period discount factor $\delta_{1,h}$, $\delta_{2,h}, \delta_f$ for each of the three firms respectively. The value functions for the *h* and *f* firms are the present values of their profits and their optimization problems in a simple two-period game are as follows:

Home Country:

Firm 1

$$\max_{\{p_{1,1,h,}p_{2,1,h}\}} \mathrm{V}_{1,h} = \max_{\{p_{1,1,h,}p_{2,1,h}\}} \big[\Pi_{1,1,h}\big(P_{1,h}, P_{12,h}, P_{1,f}; S_1\big) \\ + \delta_{1,h} \Pi_{2,1,h}\{\big(P_{2,1,h}, P_{2,2,h}, P_{2,f}; S_2\big), q_{1,1,h}\big(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_1\big)\}\big]$$

Firm 2

$$\max_{\{p_{1,2,h,}p_{2,2,h}\}} \mathrm{V}_{2,h} = \max_{\{p_{1,2,h,}p_{2,2,h}\}} \big[\Pi_{1,2,h}\big(P_{1,h}, P_{12,h}, P_{1,f}; S_1\big) \\ + \delta_{2,h} \Pi_{2,2,h}\{\big(P_{2,1,h}, P_{2,2,h}, P_{2,f}; S_2\big), q_{1,2,h}\big(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_1\big)\}\big]$$

Foreign Country:

$$\max_{\{p_{1,f,}p_{2,f}\}} \mathrm{V}_f = \max_{\{p_{1,1,h,}p_{2,1,h}\}} \big[\Pi_{1,f}\big(P_{1,h}, P_{12,h}, P_{1,f}; S_1\big) \\ + \delta_f \Pi^*_{2,f}\{\big(P_{2,h}, P_{2,f}; S_2\big), q_{1,f}\big(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_1\big)\}\big]$$

where $q_{1,1,h}, q_{1,2,h}$ and $q_{1,f}$ are the outputs of the two *h* firms and the *f* firm respectively in the first period, and S_1 and S_2 the corresponding exchange rates in the first and second periods respectively.

We use backward induction starting from period $t = 2$. As there are switching costs, a higher consumer base in the first period would imply that consumers will be "locked-in" and buy from the same firm in the second period. Hence, we first maximize the profits of each firm in the second period~~,~~ bearing in mind that the prices set in the first period determine the position of the demand in the second period. This means that all the second-period demand intercepts $\theta_{2,1,o}$,$\theta_{2,2,o}$ and $\lambda_{2,o}$ are no longer exogenously determined and fixed, but are instead a function of the prices as set by the firms in the first period, *i.e.* for firm 1, $\theta_{2,1,o}(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_1)$, where $\frac{\partial\theta_{2,1,o}}{\partial P_{1,1,h}} < 0, \frac{\partial\theta_{2,1,o}}{\partial P_{1,2,h}} > 0, \frac{\partial\theta_{2,1,o}}{\partial P_{1,f}} > 0$. Similar results hold for the other two intercepts with a negative own-price impact and positive cross-price impact. This follows the approach of GS, where "prices set in the first period determine the position of the demand in the second period" (p.92).

We argue that this shift will only affect the intercept terms. This is a realistic assumption, as unless the relative prices of the goods change substantially between the two subsequent periods, we do not expect that the own- and cross-price effects parameters will change. The consumers will alter the degree that they substitute among different goods only when relative prices change substantially.[1] Hence, without loss of generality, and in order to retain tractability in our results, we assume that the own- and cross-substitution parameters are fixed during these two periods. We first focus on period $t = 2$, where each firm needs to find the prices in the second period that maximize the expression below:

Home Country:

Firm 1

[1] An episode of deep price cuts by The Times in 1993 (from 45p to 30p), severely affected the market share of Daily Express (typically classified as belonging in the mid-market range) rather than the upper range "quality" newspapers (The Times, Guardian, Telegraph and the Independent), thus altering the definition of the market (Behringer and Filistrucchi 2015).

$$\max_{p_{2,1,h}} \Pi_{2,1,h}(P_{2,1,h}, P_{2,2,h}, P_{2,f}; S_2)$$

$$= \max_{P_{1,h}}[\theta_{2,1,o}(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_1) + \theta_{11,h}P_{2,1,h} + \theta_{12,h}P_{2,2,h} + \theta_{1,f}S_2P_{2,f}][P_{2,1,h} - c_{2,1,h}]$$

Firm 2

$$\max_{p_{2,2,h}} \Pi_{2,2,h}(P_{2,1,h}, P_{2,2,h}, P_{2,f}; S_2)$$

$$= \max_{P_{2,h}}[\theta_{2,2,o}(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_1) + \theta_{21,h}P_{2,1,h} + \theta_{22,h}P_{2,2,h} + \theta_{2,f}S_2P_{2,f}][P_{2,2,h} - c_{2,2,h}]$$

Foreign Country:

$$\max_{P_{2,f}} \Pi_{2,f}(P_{2,1,h}, P_{2,2,h}, P_{2,f}; S_2)$$

$$= \max_{P_{1,f}}\left[\lambda_{2,o}(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_1) + \lambda_f P_{2,f} + \lambda_{1,h}\frac{1}{S_2}P_{2,1,h} + \lambda_{2,h}\frac{1}{S_2}P_{2,2,h}\right][P_{2,f} - c_{2,f}]$$

We set~~:~~ $[\frac{\partial \Pi_{2,1,h}}{\partial P_{2,1,h}}, \quad \frac{\partial \Pi_{2,2,h}}{\partial P_{2,2,h}}, \quad \frac{\partial \Pi_{2,f}}{\partial P_{2,f}},]' = [0, \quad 0, \quad 0]'$ ~~,~~ to obtain the optimal prices that satisfy the F.O.Cs for the second period. We derive prices $P_{21,h}^M, P_{22,h}^M, P_{2,f}^M$, i.e. "intermediate" optimal prices of the second period that are functions of prices in the first period.

Hence, the optimal prices M in the second period are functions of the prices and exchange rates in the first period through the impact of the latter on the second-period demand intercept parameters. We correspondingly define:

$$\pi_{2,1,h}^M(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_2),\ \pi_{2,2,h}^M(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_2),\ \pi_{2,f}^M(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_2)$$

as the intermediate profits for the second period, once the intermediate optimal prices for the second period have been inserted, thus making these profits only functions of the prices in the first period.

Consequently, to find the equilibrium prices in the first period we now feed the above relations into the value functions and write them as functions of the prices and exchange rates in period 1. The F.O.Cs for the two home and the foreign firm are:

$$\frac{\partial V_{1,h}}{\partial P_{1,1,h}} = \frac{\partial[\Pi_{1,1,h}(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_1)]}{\partial P_{1,1,h}} + \delta_{1,h}\frac{\partial[\pi_{2,1,h}^M(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_2)]}{\partial P_{1,1,h}} = 0 \qquad (2)$$

$$\frac{\partial V_{2,h}}{\partial P_{1,2,h}} = \frac{\partial[\Pi_{1,2,h}(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_1)]}{\partial P_{1,2,h}} + \delta_{2,h} \frac{\partial[\pi^{M}_{2,1,h}(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_2)]}{\partial P_{1,2,h}} = 0 \quad (3)$$

$$\frac{\partial V_{f}}{\partial P_{1,f}} = \frac{\partial[\Pi_{1,f}(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_1)]}{\partial P_{1,f}} + \delta_{f} \frac{\partial[\pi^{M}_{2,f}(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_2)]}{\partial P_{1,f}} = 0 \quad (4)$$

The dynamic optimal prices for the three firms, $P^{*D}_{1,1,h}$, $P^{*D}_{1,2,h}$, $P^{*D}_{1,f}$, are derived from solving the above equations. They will only depend on the exogenous variables of the model, *i.e.* the exchange rate as well as the cost of each firm.

In the case of durable goods, the specification retains the features that we encounter in the GS model. All other things being equal, a higher output today resulting from a lower price charged for the good translates to a higher demand for the same good tomorrow and thus a higher profit. In other words, the presence of a dynamic effect implies that pricing decisions today will have an impact on prices (and consequently profits) tomorrow by securing a larger customer base in subsequent periods.

Similarly, an increase in the price of a good in period 1 will decrease the demand for that same good, not only in the first period, but also in the second period. In other words, since $\frac{\partial P^{M}_{21,h}}{\partial P_{1,1,h}}, \frac{\partial P^{M}_{22,h}}{\partial P_{1,2,h}}, \frac{\partial P^{M}_{2,f}}{\partial P_{1,f}}$ are negative, this means that $\frac{\partial(\pi^{M}_{2,1,h}(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_2))}{\partial P_{1,1,h}}, \frac{\partial(\pi^{M}_{2,2,h}(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_2))}{\partial P_{1,2,h}}, \frac{\partial(\pi^{M}_{2,f}(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_2))}{\partial P_{1,f}}$ are negative too.

Consequently, the L.H.S. in Eqs. (2), (3) and (4) will become equal to zero at a point where the corresponding first-period derivatives, $\frac{\partial(\Pi_{1,1,h}(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_1))}{\partial P_{1,1,h}}$, $\frac{\partial(\Pi_{1,2,h}(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_1))}{\partial P_{1,2,h}}$, $\frac{\partial(\Pi_{1,f}(P_{1,1,h}, P_{1,2,h}, P_{1,f}; S_1))}{\partial P_{1,f}}$, will be positive (instead of zero as in the static one-period model). This means that the optimal prices in the first period are less than they would have been in the static model, i.e. the intertemporal effect tends to lower optimal prices.

We note that the impact of the first-period price on the demand function of its rivals implies a dynamic interdependence among the three firms, where all the terms $\frac{\partial P^{M}_{2i,h}}{\partial P_{1j,h}}, \frac{\partial P^{M}_{2i,h}}{\partial P_{1,f}}, \frac{\partial P^{M}_{2,f}}{\partial P_{1i,h}}$ are positive for $i = 1{,}2$ and i$\neq j$. We examine this interdependence in Proposition 3 below.

Proposition 3

The direction of the exchange-rate exposure in the first period on the value function of each firm from the currency depreciating country *i*:

$$\frac{\partial V_{i,h}^{*}}{\partial S_1} = m_{1i,h}^{*D} P_{1i,h}^{*D} [\theta_{ij,h} P_{12,h}^{*D} \frac{\varepsilon_{P_{1i,h,S_1}^{*D}}}{S_1} + \theta_{1,f} P_{1f}^{*D} (1 + \varepsilon_{P_f^{*D},S_1})] + \delta_{i,h} \left(\frac{\partial \pi_{2i,h}^{M}}{\partial P_{1i,h}^{*D}} \frac{\partial P_{1i,h}^{*D}}{\partial S_1} + \frac{\partial \pi_{2i,h}^{M}}{\partial P_{1f}^{*D}} \frac{\partial P_{1f}^{*D}}{\partial S_1} \right),$$

depends on three factors.

a) The first-period profit exposure: $\frac{\partial \Pi_{1i,h}^{*D}}{\partial S_1} = m_{1i,h}^{*D} P_{1i,h}^{*D} [\theta_{ij,h} P_{12,h}^{*D} \frac{\varepsilon_{P_{1j,h,S_1}^{*D}}}{S_1} + \theta_{1,f} P_{1f}^{*D} (1 + \varepsilon_{P_f^{*D},S_1})]$.
b) The positive impact on the second-period profit function of firm *i* via the intertemporal effect of S_1 on the first-period price of the *h* rival firm *j*: $\frac{\partial \pi_{2i,h}^{M}}{\partial P_{1i,h}^{*D}} \frac{\partial P_{1j,h}^{*D}}{\partial S_1}$, reinforcing the gains from a depreciation in the home currency for firm *i*.
c) The negative impact on the second-period profit function of firm *i* via the intertemporal effect of S_1 on the first-period price of the firm from the currency appreciating country: $\frac{\partial \pi_{2i,h}^{M}}{\partial P_{1f}^{*D}} \frac{\partial P_{1f}^{*D}}{\partial S_1}$. As the impact of an increase in S_1 on the optimal price chosen by *f* is negative, its optimal price in the first period decreases. This leads to a negative impact on the profits of the *h* firm, thus offsetting the gains from the depreciation in the home currency for firm *i.*

We draw two conclusions from the above result. First, we note that in the international duopoly (1x1) case the intertemporal effect of term (b) would not exist. This means that the difference in the exposure between the *RoT* model and the (1x1) model is larger in the long run than in the short run. Second, in the *RoT* case, the existence of two intertemporal effects with opposing signs means that we do not know whether the firm's long-run exposure is larger or smaller relative to its short-run exposure. This contrasts with the 1x1 case exposure, where the long-run exposure is unambiguously smaller than the short-run given the absence of an intertemporal effect from a domestic competitor.

The direction of the exchange-rate exposure in the first period on the value of firm *f* (the firm from the currency appreciating country):

$$\frac{\partial V_f^*}{\partial S_1} = \frac{1}{(S_1)^2} m_{1f}^{*D} P_{1f}^{*D} [\lambda_{1,h} P_{11,,h}^{*D} (\frac{\varepsilon_{P_{11,h,S_1}^{*D}}}{S_1} - 1) + \lambda_{2,h} P_{12,,h}^{*D} (\frac{\varepsilon_{P_{12,h,S_1}^{*D}}}{S_1} - 1)] + \delta_f (\frac{\partial(\pi_{2,f}^{M})}{\partial P_{1i,h}^{*D}} \frac{\partial P_{1i,h}^{*D}}{\partial S_1} + \frac{\partial(\pi_{2,f}^{M})}{\partial P_{1j,h}^{*D}} \frac{\partial P_{1j,h}^{*D}}{\partial S_1}),$$

depends on two factors.

a) The first-period profit exposure: $\frac{\partial \Pi_f^{*D}}{\partial S_1} = \frac{1}{(S_1)^2} m_{1f}^{*D} P_{1f}^{*D} \left\{ \lambda_{1,h} P_{11,,h}^{*D} \left(\frac{\varepsilon_{P_{11,h,S_1}^{*D}}}{S_1} - 1 \right) + \lambda_{2,h} P_{12,,h}^{*D} \left(\frac{\varepsilon_{P_{12,h,S_1}^{*D}}}{S_1} - 1 \right) \right\}$.

b) The sum of the positive impacts on the second-period profit function of the foreign firm via the intertemporal effects of S_1 on the first-period price of the two home rivals (firms from the currency depreciating country) *i* and *j*: $\frac{\partial \pi_{2,f}^{M}}{\partial P_{1i,h}^{*D}} \frac{\partial P_{1i,h}^{*D}}{\partial S_1} + \frac{\partial \pi_{2,f}^{M}}{\partial P_{1j,h}^{*D}} \frac{\partial P_{1j,h}^{*D}}{\partial S_1}$, thus ameliorating the losses experienced by firm *f* as a result of the depreciation in the currency of country *h*.

The above result implies that the exchange-rate exposure for the f firm in the long run will be smaller than in the short run. In other words, the stock price for the firm is less sensitive to exchange rates than the profits.

Proof

Please refer to the MA.

We note that the first term in the equation for each *h* firm includes the two terms found in the static relation in Proposition 2. The new terms in the second bracket are the positive (and correspondingly negative) impact on the value function of firm 1 via the intertemporal effect of S_1 on the optimal first-period price of the *h* rival firm (and the optimal first-period price of the *f* firm) feeding into the second-period profit of *h* firm 1. In other words, the rival *h* firm 2 and the *f* firm affect the profit of *h* firm 1 both directly and indirectly via the dynamic term. Written differently:

$$\frac{\partial V_{1.h}^*}{\partial S_1} = \frac{\partial \Pi_{1,h}^{*D}}{\partial S_1} + \delta_{1,h} \left(\frac{\partial \pi_{21,h}^{M}}{\partial P_{12,h}^{*D}} \frac{\partial P_{12,h}^{*D}}{\partial S_1} + \frac{\partial \pi_{21,h}^{M}}{\partial P_{1f}^{*D}} \frac{\partial P_{1f}^{*D}}{\partial S_1} \right).$$

This equation reads as long-run exposure = short-run exposure + intertemporal effect of S_1 on the first-period price of the h rival firm + intertemporal effect of S_1 on the first period of the *f* rival firm. Similarly for *h* firm 2. The equation for the *f* firm reads analogously as long-run exposure = short-run exposure + sum of intertemporal effects of S_1 on the optimal first-period price of *f* firm's two overseas rivals.

We note that in the case of an international duopoly, which we have studied above, there is only a *between* dynamic interaction effect, which is negative, and the long-run exposure is lower than the short-run. In other words, the stock price is less sensitive than the profits to changes in S_1.

4. Conclusions

This paper constructs a mathematical model of exchange-rate exposure of firms competing in an international *RoT* market structure. This framework extends previous approaches by allowing *within* and *between* countries competition. It constructs two versions of the *RoT* model, a static and a dynamic one, to explore consumable and durable goods producing firms respectively. The coexistence of *between* and *within* competition enhances our knowledge of competition interactions among firms in an international oligopoly. We establish a link between the ability of each firm to pass on the exchange-rate change in the price it charges its customers and the direction and degree of the impact on the profits of the other firms.

In particular, we find that in the static case the addition of a domestic competitor in one of the countries increases the firm's exposure in that country, while the effect on its foreign competitor is ambiguous as it depends on the sizes of the exchange-rate elasticities of its two overseas competitors. In the dynamic case, we find that the gap in exposure between the *RoT* model and the (1x1) case is larger in the long run than in the short run for the company that now faces a domestic rival. On the other hand, the two intertemporal effects (domestic and foreign) have opposing signs. This means that the foreign-exchange exposure can be either smaller or larger in the long run relative to the short run. Finally, the firm that remains a monopolist in its domestic market finds that its exposure in the long run is smaller than in the short run.

References

Aggarwal, R, and T. J. Harper. 2010. Foreign exchange exposure of “domestic” corporations. *Journal of International Money and Finance*, 29, 1619–1636.

Andaluz, J., A. A. Elsadany and G. Jarne. 2017. Nonlinear Cournot and Bertrand-type dynamic triopoly with differentiated products and heterogeneous expectations. *Mathematics and Computers in Simulation*, 132, 86–99.

Andaluz, J. and G. Jarne. 2015. On the dynamics of economic games based on product differentiation, *Mathematics and Computers in Simulation*, 113, 16–27.

Axtell, R., L. 2001. Distributions of U.S. Firm Sizes. *Science* 7, 1818–1820.

Bartram, S., M., G. W., Brown, and B.A., Minton. 2010. Resolving the exposure puzzle: The many facets of exchange rate exposure. *Journal of Financial Economics*, 95,148 –173.

Beggs, A. and P. Klemperer. 1992. Multi-period competition with switching costs. *Econometrica*, 60, 651–666.

Behringer, S. and L. Filistrucchi. 2015. Areeda-Turner in two-sided Markets. *Review of Industrial Organization* 46, 287–306.

Bénassy-Quéré A. and L. R. H. Fontagné. 2011. Exchange-rate misalignments in duopoly: the case of Airbus and Boeing. *World Economy*, *34*, 623–641.

Bodnar, G, M., B., Dumas, and R., C. Marston. 2002. Pass-through and Exposure. *Journal of Finance* 57, 199–231.

Bouis, R., K. J. M. Huisman and P. M. Kort. 2009. Investment in oligopoly under uncertainty: The accordion effect. *International Journal of Industrial Organization*, 27, 320–331.

Buzzell, R., D. 1981. Are There ‘Natural’ Market Structures? *Journal of Marketing* 45, 42–51.

Cabral, L. 2017. Introduction to Industrial Organization, second Ed. The MIT Press.

Cabral, L. and M. Villas-Boas. 2005. Bertrand Supertraps. *Management Science*, 51, 599–613.

Elabbasy, E. M., H. N. Agiza and A. A. Elsadany. 2009. Analysis of nonlinear triopoly game with heterogeneous players. *Computers and Mathematics with Applications*, 57, 488–499.

Elsadany, A. A., H. N. Agiza and E. M. Elabbasy. 2013. Complex dynamics and chaos control of heterogeneous quadropoly game. *Applied Mathematics and Computation*, 219, 11.110–11.118.

Flode´n, M, W., Simbanegavi and W. Wilander. 2008. When is a lower exchange rate pass-through associated with greater exchange rate exposure? *Journal of International Money and Finance*, 27,124–139.

Froot, K. A. and P. Klemperer. 1989. Exchange rate pass-through when market share matters. *American Economic Review,* 79, 637–654.

Gans, J. S., and J. Quiggin. 2003. A Technological and organizational explanation for the size distribution of firms. *Small Business Economics* 21, 243–256.

Gordon, R. M. 2001. The Rule of Three: Surviving and Thriving in Competitive Markets. *Marketing Management*, 10 (November– December), 53–54.

Gross, D. M. and N. Schmitt. 2000. Exchange rate pass-through and dynamic oligopoly: An empirical investigation. *Journal of International Economics,* 52, 89–112.

Henricks, M. 2002. Also-Rans. *Entrepreneur*, 30, 25.

Ijiri, Y. and H. A. Simon. 1977. Skew Distributions and the Sizes of Business Firms, New York: Elsevier-North-Holland.

Ji, W. 2009. Chaos and control of game model based on heterogeneous expectations in electric power triopoly. *Discrete Dynamics in Nature and Society*, Article ID 469564, 8.

Landry, J. T. 2002. The Rule of Three: Surviving and Thriving in Competitive Markets. *Harvard Business Review*, 80, 23.

Marston, R. C. 2001. The effects of industry structure on economic exposure. *Journal of International Money and Finance*, 20,149–164.

Matouk, E., A. A. Elsadany and X. Baogui. 2017. Neimark–Sacker bifurcation analysis and complex nonlinear dynamics in a heterogeneous quadropoly game with an isoelastic demand function. *Nonlinear Dynamics*, 89, 2533–2552.

Puu, T., M. R. Marin. 2006. The dynamics of a triopoly Cournot game when the competitors operate under capacity constraints. *Chaos, Solitons & Fractals*, 28, 403–413.

Rhodes, A. 2014. Re-examining the effects of switching costs. *Economic Theory,* 57, 161–194.

Quandt, R. E. 1966. On the Size Distribution of Firms. *American Economic Review,* 56, 416–432.

Saffersfone, M. J. 2002. The Rule of Three: Surviving and Thriving in Competitive Markets. *Academy of Management Executive*, 16, 161–162.

Sheth, J. N. and R. S. Sisodia. 1998. Only the Big Three Will Survive. *The Wall Street Journal*, Eastern ed., (November 5), A22.

Sheth, J. N. and R. S. Sisodia. 2002. The Rule of Three: Surviving and Thriving in Competitive Markets. New York: *The Free Press*.

Shibata, T. 2016. Strategic entry in a triopoly market of firms with asymmetric cost structures. *European Journal of Operational Research*, 249, 728–739.

Shugan, S. M. 1989. Product Assortment in a Triopoly. *Management Science*, 35, 304–320.

Uslay, C., A. Z. Altintig and R. D. Winsor. 2010. An Empirical Examination of the “Rule of Three”: Strategy Implications for Top Management, Marketers, and Investors. *Journal of Marketing* 74, 20–39.

Mathematical Appendix

PROOF OF PROPOSITION 1

We differentiate the optimal prices *wrt S*:

$$\frac{\partial P^*_{1,h}}{\partial S}=\frac{\partial\begin{vmatrix}\theta_{1,o}-\theta_{11,h}c_{1,h} & \theta_{12,h} & \theta_{1,f}S\\ \theta_{2,o}-\theta_{22,h}c_{2,h} & 2\theta_{22,h} & \theta_{2,f}S\\ \lambda_o-\lambda_f c_f & \lambda_{2,h}\frac{1}{S} & 2\lambda_f\end{vmatrix}}{\partial S}=\left[\left(\lambda_o-\lambda_f c_f\right)(\theta_{12,h}\theta_{2,f}-2\theta_{22,h}\theta_{1,f}\right]>0,$$

and,

$$\Delta\frac{\partial P^*_{2,h}}{\partial S}=\frac{\partial\begin{vmatrix}2\theta_{11,h} & \theta_{1,o}-\theta_{11,h}c_{1,h} & \theta_{1,f}S\\ \theta_{21,h} & \theta_{2,o}-\theta_{22,h}c_{2,h} & \theta_{2,f}S\\ \lambda_{1,h}\frac{1}{S} & \lambda_o-\lambda_f c_f & 2\lambda_f\end{vmatrix}}{\partial S}=-\left(\lambda_o-\lambda_f c_f\right)(2\theta_{11,h}\theta_{2,f}-\theta_{21,h}\theta_{1,f})>0,$$

where $\Delta=\begin{vmatrix}2\theta_{11,h} & \theta_{12,h} & \theta_{1,f}S\\ \theta_{21,h} & 2\theta_{22,h} & \theta_{2,f}S\\ \lambda_{1,h}\frac{1}{S} & \lambda_{2,h}\frac{1}{S} & 2\lambda_f\end{vmatrix}$.

Hence, the optimal price for two home firms increases in *S*.

$$\Delta\frac{\partial P^*_{f}}{\partial S}=\frac{\partial\begin{vmatrix}2\theta_{11,h} & \theta_{12,h} & \theta_{1,o}-\theta_{11,h}c_{1,h}\\ \theta_{21,h} & 2\theta_{22,h} & \theta_{2,o}-\theta_{22,h}c_{2,h}\\ \lambda_{1,h}\frac{1}{S} & \lambda_{2,h}\frac{1}{S} & \lambda_{1,o}-\lambda_f c_f\end{vmatrix}}{\partial S}=$$

$$-\frac{1}{S^2}\left(\theta_{1,o}-\theta_{11,h}c_{1,h}\right)(\theta_{21,h}\lambda_{2,h}-2\theta_{22,h}\lambda_{1,h})+\frac{1}{S^2}\left(\theta_{2,o}-\theta_{22,h}c_{2,h}\right)(2\theta_{11,h}\lambda_{2,h}-2\theta_{12,h}\lambda_{1,h})<0.$$

Therefore, the optimal price for the foreign firm decreases in *S*.

Q.E.D.

PROOF OF PROPOSITION 3

By solving Eqs. (2), (3), and (4), we deduce the optimal dynamic prices in the first period, i.e. $P^{*D}_{11,h}, P^{*D}_{12,h},$ and $P^{*D}_{1,f}$. Replacing them into the *V* (value) functions of each company, we examine the impact of exchange rates in period S_1 on the value functions:

For firm 1:

$$\frac{\partial V^*_{1.h}}{\partial S_1}=\frac{\partial(\left[\theta_{11,o}+\theta_{11,h}P^{*D}_{11,h}+\theta_{12,h}P^{*D}_{12,h}+\theta_{1,f}S_1P^{*D}_{1f}\right][P^{*D}_{11,h}-c_{1,h}])}{\partial S_1}+\delta_{1,h}\frac{\partial \pi^M_{21,h}}{\partial S_1}=$$

$$\left\{\frac{\partial\left(\left[\theta_{11,o}+\theta_{11,h}P_{11,h}^{*D}+\theta_{12,h}P_{12,h}^{*D}+\theta_{1,f}S_1P_{1f}^{*D}\right]\left[P_{11,h}^{*D}-c_{1,h}\right]\right)}{\partial P_{11,h}^{*D}}+\delta_{1,h}\frac{\partial \pi_{21,h}^{M}}{\partial P_{11,h}^{*D}}\right\}\frac{\partial P_{11,h}^{*D}}{\partial S_1}+$$

$$\left\{\frac{\partial\left(\left[\theta_{11,o}+\theta_{11,h}P_{11,h}^{*D}+\theta_{12,h}P_{12,h}^{*D}+\theta_{1,f}S_1P_{1f}^{*D}\right]\left[P_{11,h}^{*D}-c_{1,h}\right]\right)}{\partial P_{12,h}^{*D}}+\delta_{1,h}\frac{\partial \pi_{21,h}^{M}}{\partial P_{12,h}^{*D}}\right\}\frac{\partial P_{12,h}^{*D}}{\partial S_1}+$$

$$\left\{\frac{\partial\left(\left[\theta_{11,o}+\theta_{11,h}P_{11,h}^{*D}+\theta_{12,h}P_{12,h}^{*D}+\theta_{1,f}S_1P_{1f}^{*D}\right]\left[P_{11,h}^{*D}-c_{1,h}\right]\right)}{\partial P_{1f}^{*D}}+\delta_{1,h}\frac{\partial \pi_{21,h}^{M}}{\partial P_{1f}^{*D}}\right\}\frac{\partial P_{1f}^{*D}}{\partial S_1}+\theta_{1,f}P_{1f}^{*D}\left[P_{11,h}^{*D}-c_{1,h}\right]$$

Using the envelope theorem, the first term is equal to zero, and the above relation is reduced to the following:

$$\frac{\partial V_{1.h}^{*}}{\partial S_1}=$$

$$=\underset{(+)}{\frac{\partial P_{12,h}^{*D}}{\partial S_1}\theta_{12,h}\left(P_{11,h}^{*D}-c_{1,h}\right)}+\underset{(?)}{\theta_{1,f}P_{1f}^{*D}\left(1+\frac{\partial P_{1f}^{*D}}{\partial S_1}\frac{S_1}{P_{1f}^{*D}}\right)\left(P_{11,h}^{*D}-c_{1,h}\right)}+\underset{(+)}{\delta_{1,h}\frac{\partial \pi_{21,h}^{M}}{\partial P_{12,h}^{*D}}\frac{\partial P_{12,h}^{*D}}{\partial S_1}}+\underset{(-)}{\delta_{1,h}\frac{\partial \pi_{21,h}^{M}}{\partial P_{1f}^{*D}}\frac{\partial P_{1f}^{*D}}{\partial S_1}}$$

As $\frac{\partial P_{12,h}^{*D}}{\partial S_1}>0, \frac{\partial P_{1f}^{*D}}{\partial S_1}<0$, $\frac{\partial \pi_{21,h}^{M}}{\partial P_{12,h}^{*D}}>0$ $\left(\text{as } \frac{\partial \theta_{2,1,o}}{\partial P_{12,h}^{*D}}>0\right)$ and $\frac{\partial \pi_{21,h}^{M}}{\partial P_{1f}^{*D}}>0$ $\left(\text{as } \frac{\partial \theta_{2,1,o}}{\partial P_{1f}^{*D}}>0\right)$, this means that the first term is positive, the second can be either negative or positive, depending on the price elasticity of the foreign firm *wrt* S_1 and whether the latter is elastic (in which case the term $1+\frac{\partial P_{1f}^{*D}}{\partial S_1}\frac{S_1}{P_{1f}^{*D}}$ is negative) or inelastic (in which case $1+\frac{\partial P_{1f}^{*D}}{\partial S_1}\frac{S_1}{P_{1f}^{*D}}$ is positive), the third term is positive and the fourth negative. Hence, both the rival home firm and the foreign firm affect the profits of home firm 1, both directly and indirectly, via the dynamic term in opposite directions. We can re-write the above in terms of elasticities and price-cost margins:

$$\frac{\partial V_{1.h}^{*}}{\partial S_1}=m_{11,h}^{*D}P_{11,h}^{*D}\left[\theta_{12,h}P_{12,h}^{*D}\frac{\varepsilon_{P_{12,h,S_1}^{*D}}}{S_1}+\theta_{1,f}P_{1f}^{*D}\left(1+\varepsilon_{P_f^{*D},S_1}\right)\right]+\delta_{1,h}\left(\frac{\partial \pi_{21,h}^{M}}{\partial P_{12,h}^{*D}}\frac{\partial P_{12,h}^{*D}}{\partial S_1}+\frac{\partial \pi_{21,h}^{M}}{\partial P_{1f}^{*D}}\frac{\partial P_{1f}^{*D}}{\partial S_1}\right),$$

where $m_{11,h}^{*D}=\frac{P_{11,h}^{*D}-c_{1,h}}{P_{11,h}^{*D}}$ is the price-cost margin in the first period of firm 1.

The first term in the above relation includes the two terms also found in the static relation in Proposition 2. The two additional terms in the second bracket are the positive (and correspondingly negative) impact on the value function of the first firm via the intertemporal effect of S_1 on the optimal first-period price of the home rival firm (and the optimal first-period price of the foreign firm) feeding into the second-period profit of home firm 1. In other words, the rival home firm and the foreign firm affect the profit of home firm 1, both directly and indirectly, via the dynamic term. Written differently,

$$\frac{\partial V_{1.h}^{*}}{\partial S_1} = \frac{\partial \Pi_{1,h}^{*D}}{\partial S_1} + \delta_{1,h}\left(\frac{\partial \pi_{21,h}^{M}}{\partial P_{12,h}^{*D}}\frac{\partial P_{12,h}^{*D}}{\partial S_1} + \frac{\partial \pi_{21,h}^{M}}{\partial P_{1f}^{*D}}\frac{\partial P_{1f}^{*D}}{\partial S_1}\right)$$

This equation reads as long-run exposure = short-run exposure + intertemporal effect of S_1 on the first-period price of the home rival firm + intertemporal effect of S_1 on the first period of the foreign rival firm. Similarly, for the second *h* firm,

$$\frac{\partial V_{2.h}^{*}}{\partial S_1} = m_{12,h}^{*D} P_{12,h}^{*D}[\theta_{21,h} P_{11,h}^{*D}\frac{\varepsilon_{P_{11,h,S_1}^{*D}}}{S_1} + \theta_{2,f} P_{1,f}^{*D}(1+\varepsilon_{P_f^{*D},S_1})] + \delta_{2,h}\left(\frac{\partial \pi_{22,h}^{M}}{\partial P_{11,h}^{*D}}\frac{\partial P_{11,h}^{*D}}{\partial S_1} + \frac{\partial \pi_{22,h}^{M}}{\partial P_{1f}^{*D}}\frac{\partial P_{1f}^{*D}}{\partial S_1}\right),$$

where $m_{12,h}^{*D} = \frac{P_{12,h}^{*D} - c_{2,h}}{P_{12,h}^{*D}}$, with $\frac{\partial P_{11,h}^{*D}}{\partial S_1} > 0, \frac{\partial P_{1f}^{*D}}{\partial S_1} < 0$, $\frac{\partial \pi_{22,h}^{M}}{\partial P_{11,h}^{*D}} > 0$ $\left(\text{as } \frac{\partial \theta_{2,2,o}}{\partial P_{12,h}^{*D}} > 0\right)$ and $\frac{\partial \pi_{21,h}^{M}}{\partial P_{1f}^{*D}} > 0$ $\left(\text{as } \frac{\partial \theta_{2,2,o}}{\partial P_{1f}^{*D}} > 0\right)$.

Finally, for the *f* firm,

$$\frac{\partial V_f^{*}}{\partial S_1} = \frac{1}{(S_1)^2} m_{1f}^{*D} P_{1f}^{*D}[\lambda_{1,h} P_{11,,h}^{*D}(\frac{\varepsilon_{P_{11,h,S_1}^{*D}}}{S_1} - 1) + \lambda_{2,h} P_{12,,h}^{*D}(\frac{\varepsilon_{P_{12,h,S_1}^{*D}}}{S_1} - 1)] + \delta_f(\frac{\partial(\pi_{2,f}^{M})}{\partial P_{1i,h}^{*D}}\frac{\partial P_{11,h}^{*D}}{\partial S_1} + \frac{\partial(\pi_{2,f}^{M})}{\partial P_{12,h}^{*D}}\frac{\partial P_{12,h}^{*D}}{\partial S_1}),$$

where, $m_{1f}^{*D} = \frac{P_{1.f}^{*D} - c_{2,h}}{P_{1,f}^{*D}}$, with $\frac{\partial P_{11,h}^{*D}}{\partial S_1} > 0, \frac{\partial P_{12,h}^{*D}}{\partial S_1} > 0$, $\frac{\partial \pi_{2,f}^{M}}{\partial P_{11,h}^{*D}} > 0$ $\left(\text{as } \frac{\partial \lambda_{2,o}}{\partial P_{12,h}^{*D}} > 0\right)$ and $\frac{\partial \pi_{2,f}^{M}}{\partial P_{12,h}^{*D}} > 0$ $\left(\text{as } \frac{\partial \lambda_{2,o}}{\partial P_{12,h}^{*D}} > 0\right)$.

Written differently,

$$\frac{\partial V_f^{*}}{\partial S_1} = \frac{\partial \Pi_f^{*D}}{\partial S_1} + \delta_f(\frac{\partial(\pi_{2,f}^{M})}{\partial P_{11,,h}^{*D}}\frac{\partial P_{11,h}^{*D}}{\partial S_1} + \frac{\partial(\pi_{2,f}^{M})}{\partial P_{12,,h}^{*D}}\frac{\partial P_{12,h}^{*D}}{\partial S_1}).$$

This equation reads as long-run exposure = short-run exposure + sum of intertemporal effects of S_1 on the optimal first-period price of *f* firm's two overseas rivals.

Q.E.D.