OPTIMISATION OF SCREW COMPRESSORS

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SYNOPSIS

Increasing demands for more efficient screw compressors require that compressor designs are tailored upon their duty, capacity and manufacturing capability. A suitable procedure for optimisation of the screw compressor shape, size, dimension and operating parameters is described here, which results in the most appropriate design for a given compressor application and fluid. It is based on a rack generation algorithm for rotor profile combined with a numerical model of the compressor fluid flow and thermodynamic processes. Some optimisation issues of the rotor profile and compressor parts are discussed, using 5/6 screw compressor rotors to present the results. It is shown that the optimum rotor profile, compressor speed, oil flow rate and temperature may significantly differ when compressing different gases or vapours or if working at the oil-free or oil flooded mode of operation. Compressors thus designed achieve higher delivery rates and better efficiencies than those using traditional approaches, which is illustrated in an example of the 3/5 screw rotors designed for a family of dry air compressors, produced and marketed by a renown British compressor manufacturer.

Key Words: Screw compressor design, rotor lobe optimal profiling, numerical modelling

1. INTRODUCTION

The screw compressor is a positive displacement rotary machine. It consists essentially of a pair of meshing helical lobed rotors, which rotate within a fixed casing that totally encloses them. Screw compressor rotors of various profiles can be today conveniently manufactured with small clearances at an economic cost. Internal leakages have been reduced to a small fraction of their values in earlier designs. Screw compressors are therefore efficient, compact, simple and reliable. Consequently, they have largely replaced reciprocating machines in industrial applications and in refrigeration systems.

The oil injected compressors rely on relatively large masses of oil injected with the compressed gas in order to lubricate the rotor motion, seal the gaps and reduce the temperature rise during compression. It requires no internal seals, it is simple in mechanical design, cheap to manufacture and highly efficient. Consequently it is widely used in both the compressed air and refrigeration industries.

In the oil free machine, there is no mixing of the working fluid with oil and contact between the rotors is prevented by timing gears which mesh outside the working chamber and are lubricated externally. In addition, to prevent lubricant entering the working chamber, internal seals are required on each shaft between the working chamber and the bearings. In the case of process gas compressors, double mechanical seals are used. Even with elaborate and costly systems such as these, successful internal sealing is still regarded as a problem by established process gas compressor manufacturers. It follows that such machines are considerably more expensive to manufacture than those which are oil injected.

Screw compressors can be either single or multistage machines. Multistage are used for the compressor working with higher pressure ratios, while the single stages are used either for low pressure oil free machines or moderate pressure oil flooded compressors. A special challenge is imposed upon the multistage compressor optimisation, because not only the compressor geometry parameters and operational conditions, but also the interstage pressures are optimised.

As other design processes, the design of screw compressors is an interactive feedback process where the performance of the compressor is compared with those specified in advance. Usually this is a manual process where the designer makes a prototype system which is tested and modified until it is satisfactory. With the help of a simulation model the prototyping can be reduced to a minimum. If the desired behaviour can be expressed as a figure of merit, as an object function, optimisation as a tool can be introduced to help the designer to reach an optimal solution.

Recent advances in mathematical modelling and computer simulation can be used to form a powerful tool for the screw compressor process analysis and design optimisation. Such models have evolved greatly during the past ten years and, as they are better validated, their value as a design tool has increased. Their use has led to a steady evolution in screw rotor profiles and compressor shapes which should continue in future to lead to further improvements in machine performance. Evidence of this may be seen in the publications by *Tang and Fleming, 1992 and 1994, Sauls, 1994, Fleming et al, 1995 and Fujiwara and Osada, 1995.* In order to make such computer models more readily accessible to designers and engineers, as well as specialists, the authors have developed a suite of subroutines for the purpose of screw machine design, *Hanjalic and Stosic, 1997*, which duly can be used in screw compressor optimisation.

A problem in optimisation is a number of calculations which must be performed to identify and reach an optimum. Another problem is how to be certain that the optimum calculated is the global optimum. Among the optimisation methods frequently used in engineering are steepest descent, Newton's method, Davidon-Fletcher-Powell's method, random search, grid search method, search along coordinate axes, Powell's method, Hooke-Jeeves's method. A widely used method for optimisation of functions with several optima is the genetic algorithm. It requires only a value of the target function and it can conveniently handle discontinuities, however this method is slow in converging to a solution. Alternatively a constrained simplex method, known as Box complex method can be conveniently used. It also requires the function value only and not its gradient. The disadvantage is that it is less suitable for discrete parameters, for example, if a choice between discrete component sizes is required.

Box complex method was therefore used here to find the local minima, which were input to an expanding compressor database. This finally served to estimate a global minimum. That database may be used later in conjunction with other results to accelerate the minimization.

The constrained simplex method emerged form the evolutionary operation method which was introduced already in the 1950s by *Box*, 1957 and *Box and Draper*, 1969. The basic idea is to replace the static operation of a process by a continuous and systematic scheme of slight perturbations in the control variables. The effect of these perturbations is evaluated and the process is shifted in the direction of improvement. The basic simplex method was originally developed for evolutionary operation, but it was also suitable for the constrained simplex method. Its main advantage is that only a few starting trials are needed, and the simplex immediately moves away from unsuitable trial conditions. The simplex method is especially appropriate when more than three control variables are to be perturbed and the process requires a fresh optimisation with each new set of input data.

There are several criteria for screw profile optimisation which are valid irrespective of the machine type and duty. Thus, an efficient screw machine must admit the highest possible fluid flow rates for a given machine rotor size and speed. This implies that the fluid flow cross-sectional area must be as large as possible. In addition, the maximum delivery per unit size or weight of the machine must be accompanied by minimum power utilization for a compressor and maximum power output for an expander. This implies that the efficiency of the energy interchange between the fluid and the machine is a maximum. Accordingly unavoidable losses such as fluid leakage and energy losses must be kept to a minimum. Therefore, increased leakage may be more than compensated by greater bulk fluid flow rates. However, specification of the required compressor delivery rate requires simultaneous optimisation of the rotor size and speed to minimise the compressor weight while maximising its efficiency. Finally, for oil-flooded compressors, the oil injection flow rate, inlet temperature and position needs to be optimised. It follows that a multivariable minimisation procedure is needed for screw compressor design with the optimum function criterion comprising a weighted balance between compressor size and efficiency or specific power.

2. MINIMISATION METHOD USED IN SCREW COMPRESSOR OPTIMISATION

The power and capacity of contemporary computers is only just sufficient to enable a full multivariable optimisation of both the rotor profile and the whole compressor design to be performed simultaneously in one pass.

The optimisation of a screw compressor design is generically described as a multivariable constrained optimisation problem. The task is to maximise a target function $f(x_1, x_2, ..., x_n)$, subjected simultaneously to the effects of the explicit and implicit constraints and limits, $g_i \leq x_i \leq h_i,\ i = 1, n$ and $g_i \leq y_i \leq h_i,\ i = n+1, m$ respectively, where the implicit variables $y_{n+1},\ldots,y_m$ are dependent functions of x_i. The constraints g_i and h_i are either constants or functions of the variables x_i.

When attempting to optimise a compressor design a criterion for a favourable result must be decided, for instance the minimum power consumption, or operation cost. However, the power consumption is coupled to other requirements which should be satisfied, for example a low compressor price, or investment cost. The problem becomes obvious if the requirement for low power consumption conflicts with the requirement of low compressor price. For a designer, the balance is often completed with sound judgement. For an optimisation program the balance must be expressed in numerical values. This is normally done with weights on the different parts of the target function.

As an example of the usage of weights is a target function $F = w_1 L + w_2 C$, where L is calculated power loss and C is measure of the compressor price. The choice of weights may substantially change the target function, and some choices can lead to a target function which is difficult or impossible to optimise. Moreover, it is likely that many combinations of weights w_1 and w_2 will result in a target function with several equally good optima. It is obvious that with a large number of conflicting performance criteria, the tasks of the optimisation program and its user will be more difficult. When using multi-target optimisation, the separate parts of the objective function are evaluated which would eliminate some of the difficulties in the defining of the target function.

Another important issue for real-world optimisation problems is constraints. In the general case, there are two types of constraints, explicit or implicit. The explicit ones are limitations in the range of optimised parameters, for example available component sizes. These two different constraints can, in theory, be handled more or less in the same way. In practice, however, they are handled differently.

The implicit constraints are often more difficult to manage than explicit constraints. The most convenient and most common way is to use penalty functions and thus incorporate the constraints in the objective function. Another way is to tell the optimisation algorithm when the evaluated point is invalid and generate a new point according to some predetermined rule. Generally, it can be said that constraints, especially implicit constraints, make the optimisation problem harder to solve, since it reduces the solution space.

In the early 1960s, a method called the simplex method emerged as an empirical method for optimisation, this should not be confused with the simplex method for linear programming. The simplex method was later extended by *Box*, 1965 to handle constrained problems. This constrained simplex method was appropriately called the complex method, from constrained simplex. Since then, several versions have been used. Here, the basic working idea is outlined for the complex method used. If the nonlinear problem is to be solved, it is necessary to use k points in a simplex, where k=2n. These starting points are randomly generated so that both the implicit and explicit conditions in are satisfied. Let the points x^h and x^g be defined by

$$f(x^h) = \max\ f(x^1), f(x^2), ..., f(x^k)$$
$$f(x^g) = \min\ f(x^1), f(x^2), ..., f(x^k)$$

calculate the centroid $\bar{x}$ of these points except x^l by

$$\bar{x} = \frac{1}{k-1}\sum_{i=1}^{k} x_j^i, \quad x^i \neq x^l$$

The main idea of the algorithm is to replace the worst point x^l by a new and better point. The new point x^r is calculated as a reflection of the worst point through the centroid. This is done as

$$x^r = \bar{x} + \alpha(\bar{x} - x^l)$$

where the reflection coefficient α is chosen according to Box as α=1.3.

The point x^r is examined with regard to explicit and implicit constraints and if it is feasible x^l is replaced with x^r unless $f\left(x^r\right) \le f\left(x^l\right)$. In that case, it is moved halfway towards the centroid of the remaining points. This is repeated until it stops repeating as the lowest value. However, this cannot handle the situation where there is a local minimum located at the centroid. The method used here is to gradually move the point towards the maximum value if it continues to be the lowest value. This will, however, mean that two points can come very close to each other compared to other points, with a risk of collapsing the complex. Therefore, a random value is also added to the new point. In this way, the algorithm will take some extra effort to search for a point with a better value, but in the neighbourhood of the point of the maximum value. It is consequently guaranteed that a point better than the worst of the remaining points will be found. Expressed as an equation

$$x^{r(new)} = 0.5\left[x^{r(old)} + c\bar{x} + (1-c)x^h\right] + (\bar{x} - x^h)(1-c)(2R-1)$$

where

$$c = \left(\frac{n_r}{n_r + k_r - 1}\right)^{\frac{n_r + k_r - 1}{n_r}}$$

and k_r is the number of times the point has repeated itself as lowest value and n_r is a constant. Here n_r=4 has been used. R is a random number in the interval [0,1].

If a point violates the implicit constraints, it is moved halfway towards the centroid. In order to handle the case of the centroid violating the implicit constraint, the point is gradually moved towards the maximum value. If the maximum value is located very close to the implicit constraint, this will take many iterations and the new point will be located very close to the maximum value and will not really represent any new information. Therefore a random value is added also in this case. Now

$$c = \left(\frac{n_r}{n_r + k_c - 1} \right)^{\frac{n_r + k_c - 1}{n_r}}$$

where k_c is now the number of times the point has violated the constraint.

These modifications of the complex method have led to a robust method which has already been used in many engineering applications.

3. CALCULATION OF ROTOR PROFILES IN SCREW COMPRESSOR OPTIMISATION

Screw machine rotors have parallel axes and a uniform lead and they are a form of helical gears. The rotors make line contact and the meshing criterion in the transverse plane perpendicular to their axes is the same as that of spur gears. A procedure to get the required meshing condition as described in *Stosic, 1998*. More detailed information on the envelope method applied to gears can be found in *Litvin, 1994*.

To start the procedure of rotor profiling, the profile point coordinates in the transverse plane of one rotor, and their first derivatives, must be known. This profile can be specified on either the male or female rotors or in sequence on both. Also the primary profile may also be defined as a rack as shown in Fig 1.

A helicoid surface and its derivatives for the given rotor profile can be found from the transverse plane rotor coordinates. The envelope meshing condition for screw machine rotors defines the meshing condition obtainable either numerically, if the generating curves are given on the compressor rotors, or directly, if the curves are given on the rotor rack. This enables a variety of primary arc curves to be used and basically offers a general procedure. Moreover, numerical derivation of the primary arcs permits such an approach even when only the coordinates of the primary curves are known, without their derivatives.

The following are the elements of the rack-generated 'N' profile. The primary curves are specified on the rack: D-C is a circle with radius r_3 on the rack, C-B is a straight line, B-A is a parabola constrained by radius r_1, A-H-G are trochoids on the rack generated by the small circles of radii r_2 and r_4 from the male and female rotors respectively, G-E is a straight line and E-F and E-D are circles on the rack. A full description of the rack generation procedure and rotor geometry is given in *Stosic* and *Hanjalic, 1997*. These three rotor radii, r_1, male rotor lobe radius, r_2, male rotor tip radius and r_3, rack root radius and the female rotor addendum r_0, as presented in Fig. 1, are used as variables for the rotor optimisation.

Fig 1 Distribution of generating profile curves on the rack for 'N' rotors

Full rotor and compressor geometry, like the rotor throughput cross section, rotor displacement, sealing lines and leakage flow cross section, as well as suction and discharge port coordinates are calculated from the rotor transverse plane coordinates and rotor length and lead. They are later used as input parameters for calculation of the screw compressor thermodynamic process. Four rotor parameters were used as independent variables for optimisation of the rotor profile in the calculations presented here, i.e. radii r_0, r_1, r_2, and r_3 were rotor profile parameters. For any variation of input parameters r_0 to r_3, the primary arcs must be recalculated and a full transformation performed to obtain the current rotor and compressor geometry. Additionally the compressor built-in volume ratio is also used as a geometrical optimisation variable.

Therefore, built–in volume ratio is another compressor geometry variable, while compressor speed is an operating variable and oil flow, temperature and injection position are oil optimisation parameters. Each of these rotor or compressor variables has its own influence upon the compressor process which is explained in the following qualitative diagrams, Figs 2 to 5.

Fig. 2 Qualitative dependence of the rotor displacement and compressor volumetric efficiency upon the rotor radius r_0

Radius r_0, which is a female rotor addendum simultaneously increases the compressor displacement and length of a sealing line between the rotors. These are two conflicting effects, therefore there exists an optimum value of r_0 for which the compressor performance is the best.

Fig. 3 Qualitative dependence of the blow hole and sealing line leakages upon the rotor radius r_1

A somewhat similar effect is caused by change of the male tip radius r_1. It simultaneously increases the blow-hole area and decreases sealing line length which are two opposing effects. Therefore, a value of r_1 exists for which the smallest compound leakage through the blow-hole and sealing line is obtained, which gives the best compressor performance.

Fig. 4 Mismatch in the built-in volume ratio: too large, overcompression, too small, undercompression

A mismatch of the built-in volume ration and actual compressor pressure ratio causes either overcompression or undercompression, both of which cause higher compressor indicated losses. Only a matched built-in volume and pressure ratio give the best compressor performance. This value of the built-in volume ratio is at the same time the optimal one.

Fig. 5 Influence of the compressor built-in volume ratio upon the compressor indicated efficiency

4. CALCULATION OF THERMODYNAMIC PROCESSES IN SCREW COMPRESSOR OPTIMISATION

The algorithm of the thermodynamic and flow processes used in optimisation calculations is based on a mathematical model comprising a set of equations which describe the physics of all the processes within the screw compressor. The mathematical model gives an instantaneous operating volume, which changes with rotation angle or time, together with the differential equations of conservation of mass and energy flow through it, and a number of algebraic equations defining phenomena associated with the flow. These are applied to each process that the fluid is subjected to within the machine; namely, suction, compression and discharge. The set of differential equations thus derived cannot be solved analytically in closed form. In the past, various simplifications have been made to the equations in order to expedite their numerical solution. The present model is more comprehensive and it is possible to observe the consequences of neglecting some of the terms in the equations and to determine the validity of such assumptions. This provision gives more generality to the model and makes it suitable for optimisation applications.

A feature of the model is the use the energy equation in the form which results in internal energy rather than enthalpy as the derived variable. This was found to be computationally more convenient, especially when evaluating the properties of real fluids because their temperature and pressure calculation is not explicit. However, since the internal energy can be expressed as a function of the temperature and specific volume only, pressure can be calculated subsequently directly. All the remaining thermodynamic and fluid properties within the machine cycle are derived from the internal energy and the volume and the computation is carried out through several cycles until the solution converges.

The working fluid can be any gas or liquid-gas-mixture, i.e. any ideal or real gas or liquid-gas mixture of known properties. The model accounts for heat transfer between the gas and compressor and for leakage through the clearances in any stage of the process. The model works independently of the specification of compressor geometry. Liquid can be injected during any of the compressor process stages. The model also takes in consideration the gas solubility in the injected fluid. The thermodynamic equations of state and change of state of the fluid and the constitutive relationships are included in the model.

The following forms of the conservation equations have been employed in the model. The conservation of internal energy is:

$$\omega\left(\frac{dU}{d\theta}\right)=\dot{m}_{in}h_{in}-\dot{m}_{out}h_{out}+Q-\omega p\frac{dV}{d\theta}$$

$$\dot{m}_{in}h_{in}=\dot{m}_{suc}h_{suc}+\dot{m}_{l,g}h_{l,g}+\dot{m}_{oil}h_{oil}$$

$$\dot{m}_{out}h_{out}=\dot{m}_{dis}h_{dis}+\dot{m}_{l,l}h_{l,l}$$

where θ and ω are angle and angular speed of rotation of the male rotor respectively, h=h(θ) is specific enthalpy, $\dot{m} = \dot{m}(\theta)$ is mass flow rate going in or out, p=p(θ) is fluid pressure in the working chamber control volume, $\dot{Q} = \dot{Q}(\theta)$, heat transfer between the fluid and the compressor surrounding, $\dot{V} = \dot{V}(\theta)$ local volume of the compressor working chamber. In the above equation the index in denotes inflow and the index out the fluid outflow. Oil and leakage are denoted by indices oil and l.

The mass continuity equation is:

$$\omega \frac{dm}{d\theta} = \dot{m}_{in} - \dot{m}_{out}$$

The instantaneous density ρ=ρ(θ) is obtained from the instantaneous mass m trapped in the control volume and the size of the corresponding instantaneous volume V as ρ=m/V.

The suction and discharge port flows are defined by velocity through them and their cross section area $\dot{m}_{in} = w_{in}\rho_{in}A_{in}$, $\dot{m}_{out} = w_{out}\rho_{out}A_{out}$. The cross-section area A is obtained from the compressor geometry and it was considered as a periodical function of the angle of rotation θ.

Leakage in a screw machine forms a substantial part of the total flow rate and plays an important role because it affects the delivered mass flow rate and compressor work and hence both the compressor volumetric and adiabatic efficiencies.

$$\dot{m}_l = \rho_l w_l A_l = \sqrt{\frac{p_2^2 - p_1^2}{a^2\left(\zeta + 2\ln\frac{p_2}{p_1}\right)}}$$

where a is the speed of sound, ζ is a compound resistance coefficient and indices l, 1 and 2 represent leakage, upstream and downstream conditions.

Injection of oil or other liquids for lubrication, cooling or sealing purposes, modifies the thermodynamic process in a screw compressor substantially. Special effects, such as gas or its condensate mixing and dissolving in or flashing out of the injected fluid must be accounted for separately if they are expected to affect the process. In addition to lubrication, the major purpose for injecting oil into a compressor is to seal the gaps and cool the gas.

Flow of the injected oil, oil inlet temperature and injection position are additional optimisation variables if the oil-flooded compressors are in question. Heat transfer between oil and gas is modelled as a first order dynamic system.

$$\frac{dT_{oil}}{d\theta}=\frac{h_{oil}A_{oil}\left(T-T_{oil}\right)}{\omega m_{oil}c_{oil}} \qquad T_{oil}=\frac{T-kT_{oil,p}}{1+k}$$

$$k=\frac{\omega m_{oil}c_{oil}}{h_{oil}A_{oil}\Delta\theta}=\frac{\omega d_{oil}c_{oil}}{6h_{oil}\Delta\theta}$$

K is, therefore, a time constant and h and A are the heat transfer coefficient between oil and gas and effective area surface based on the mean Sauter diameter d of the oil droplet. C is specific heat. $\Delta\theta$ is a time step and index p denotes previous.

The solution of the equation set in the form of internal energy U and mass m is performed numerically by means of the Runge-Kutta 4th order method, with appropriate initial and boundary conditions. As the initial conditions were arbitrary selected, the convergence of the solution is achieved after the difference between two consecutive compressor cycles becomes sufficiently small.

Once solved, internal energy U(θ) and mass in the compressor working chamber m(θ) serve to calculate the fluid pressure and temperature. Since $U(\theta)=(mu)+(mu)_{oil}$, specific internal energy is:

$$u=\frac{U-\left(mcT\right)_{oil}}{m}$$

As volume V(θ) is known, a specific volume is calculated as v=V/m. Therefore, temperature T and pressure p for ideal gas can be calculated as:

$$T=\left(\gamma-1\right)\frac{u}{R} \qquad p=\frac{RT}{v}$$

where R and γ are gas constant and isentropic exponent respectively. In the case of a real gas, $u=f_1(T,v)$ and $p=f_2(T,v)$ are known functions and should be solved to obtain the fluid temperature and pressure T and p. This task is simplified because internal energy u is not a function of pressure, therefore, f_1 and f_2 can be solved in a sequence. In the case of a wet vapour because of the fluid phase change either through evaporation or condensation, the saturation temperature and pressure determine each other between themselves and also the liquid and vapour internal energy and volume, u and v. Indices f and g denote liquid and gas phases. Therefore, vapour quality x can be calculated by successive approximations of u. Variables T or p and v can be obtained from:

$$u = (1-x)u_f + xu_g \qquad v = (1-x)v_f + xv_g$$

Numerical solution of the mathematical model of the physical process in the compressor provides a basis for a more exact computation of all desired integral characteristics with a satisfactory degree of accuracy. The most important of these properties are the compressor mass flow rate $\dot{m}$ [kg/s], the indicated work W_{ind} [kJ] and power P_{ind} [kW], specific indicated power P_s [kJ/kg], volumetric efficiency η_v, adiabatic efficiency η_i and isothermal efficiency η_t. Z_1 and n are the number of lobes in the main rotor and main rotor rotational speed. F_1 and F_2 and L are the main and gate rotor cross section and length. Index s means theoretical and indices t and a denote isothermal and adiabatic. Isothermal and W_t adiabatic work W_a and are given here for ideal gas.

$$m = m_{in} - m_{out} \qquad W_{ind} = \int_{cycle} V dp$$

$$\dot{m} = m z_1 n / 60 \qquad P_{ind} = \frac{W_{ind} z_1 n}{60}$$

$$\dot{m}_s = \frac{(F_{1n} + F_{2n}) L n z_1 \rho}{60}$$

$$W_t = R T_1 \ln \frac{p_2}{p_1} \qquad W_a = \frac{\gamma}{\gamma - 1} R (T_2 - T_1)$$

A full and detailed description of the presented model of the compressor thermodynamics is given in *Hanjalic and Stosic, 1997*.

$$\eta_v = \frac{\dot{m}}{\dot{m}_s} \qquad \eta_t = \frac{W_t}{W_{ind}} \qquad \eta_i = \frac{W_a}{W_{ind}}$$

$$P_s = \frac{P_{ind}}{\dot{V}}$$

Compressor speed is used as the compressor operating variable and oil flow, temperature and oil injection position are oil optimisation parameters. Each of these rotor variables has its own influence upon the compressor process which is qualitatively explained in the following qualitative diagrams, Figs 6 and 7.

Compressor shaft speed increases dynamic losses and decreases relative leakages. These two opposing effects cause that therefore, an optimum value of the shaft speed exists which gives the best compressor performance.

Fig. 6 Influence of the compressor shaft speed upon the compressor volumetric and dynamic losses

In oil flooded compressors oil is used to lubricate the rotors, seal the leakage gaps and cool the gas compressed. Therefore its influence upon the compressor process is complex. More oil improves the compressor volumetric efficiency and also improves cooling, however, it increase the friction drag between the rotors themselves and between the rotors and housing. Obviously an oil flow rate exists which will produce the best compressor performance.

Fig. 7 Influence of the injected oil flow upon leakage and friction drag losses

Each of the described geometry and operating parameters influences the compressor process on its own way and only a simultaneous minimization, which takes into consideration all the influences together will produce the best overall compressor performance. Therefore only a multivariable optimisation finds its full sense in the evaluation of the best compressor performance.

5. EXAMPLES OF OPTIMISATION OF THE ROTOR PROFILE, COMPRESSOR DESIGN AND OPERATING CONDITIONS

Nine optimisation variables were used in the calculation of a single stage compressor presented here, radii r_0, r_1, r_2, and r_3 were four rotor profile parameters, while built–in volume ratio was another compressor geometry variable, compressor speed is an operation variable and oil flow, temperature and injection position are oil optimisation parameters. Each of these rotor variables has its own influence upon the compressor process which is explained in the qualitative diagrams given in Figs. 3 to 7. In the case of a two-stage compressor, a number of variables used were 19, two sets of the compressor stage variables plus the interstage pressure.

Box constrained simplex method was used here to find the local minima. The Box method stochastically selects a simplex, which is a matrix of independent variables and calculates the optimisation target. This is later compared with those of previous calculations and then their minimization is performed. One or more optimisation variables may be limited explicitly or implicitly by the calculation results in the constrained Box method. This gives additional flexibility and maneuverability to the compressor optimisation.

5.1 Optimisation of a Single Stage Compressor for Oil-Free and Oil-Flooded Air and Refrigeration Applications

A 5/6 rotor configuration of 128 mm outer diameter of the male rotor was optimised to obtain the best compressor performance if used either in the dry air compressor, or oil-flooded air compressor or oil-flooded refrigeration compressor. The optimisation results, after being input to an expandable compressor database, finally served to estimate a global minimum. The database may be used later in conjunction with other results to accelerate the minimization.

The suction and discharge pressures were 1 - 3 bar for the dry air compressor and 1 - 8 bar for the oil flooded compressor, while the evaporation and condensation temperatures were 5 and 40 ^{o}C for R-134A. The centre distance and male rotor outer diameters were kept constant for all compressors, 90 and 128 mm respectively.

The optimisation criterion was the lowest compressor specific power. As a result, three distinctively different rotor profiles were calculated, one for oil-free compression and the other two for oil-flooded air and refrigeration compression. They are presented in Fig. 8.

Fig. 8 Rotor profiles for the oil-free and oil-flooded air and refrigeration compressor duty, detail in bold, oil-flooded air, light, oil-free and refrigeration compressor duty

As it can be noticed from the Fig. 8, although the profiles somewhat look alike, there is a substantial difference between their geometry which is given in the following table as well as further results of the compressor optimisation. These profiles are compared in Table 1.

Table 1: Results of optimisation calculations for dry and oil flooded air compressors and oil flooded refrigeration compressor

	DryAir	Oil-Flooded Air	Refrigeration
r_0 [mm]	2.62	0.74	0.83
r_1 [mm]	19.9	17.8	19.3
r_2 [mm]	6.9	5.3	4.5
r_3 [mm]	11.2	5.5	5.2
Built-in volume ratio	1.83	4.1	3.7
Rotor speed [rpm]	7560	3690	3570
Oil flow [lit/min]	-	12	8
Injection position [^{o}]	-	65	61
Oil temperature [^{o}]	-	33	32

As in the case of any result of multivariable optimisation, the calculated screw compressor profile and compressor design parameters must be considered with the extreme caution. This is because multivariable optimisation usually finds only local minima, which may not necessarily be globally the best optimisation result. Therefore, extensive calculations should be carried out before a final decision on the compressor design is made.

As an additional example, if the female rotor addendum is analysed in detail, it can be concluded that, the size of the rotor blow-hole area is proportional to the addendum. Therefore r_0 should be made as small as possible in order to minimise the blow-hole. It would therefore appear that ideally, r_0 should be equal to zero or even be 'negative'. However, reduction in r_0 also leads to a decrease the fluid flow cross-sectional area and hence a reduction in the flow rate and the volumetric efficiency. It follows that there is a lower limit to the value of r_0 to obtain the best result. More details of that single variable optimisation of screw compressor rotors can be found in *Hanjalic and Stosic, 1994.*

Fig. 10 Variation of the compressor specific power as function of the compressor rotor parameters

The dry air compressor was chosen for that analysis. This is because the compression process within it is close to that of an ideal gas compressed adiabatically in which γ, the isentropic exponent, has the relatively large value of approximately 1.4. As an example of how the optimisation variables influence the compressor specific power, the radii r_0-r_3 are considered. The influence of the female rotor tip addendum r_0, and the female rotor radius r_3 are presented in Fig. 9, as well as the male rotor radii r_1 and r_2. In Fig. 10, the influence of the compressor built-in volume ratio, as well as compressor speed is presented. Some details of a similar optimisation can be found in *Stosic et al 2001.*

Fig. 11 Specific power as function of the compressor built-in volume and speed

5.2 Optimisation of a Two Stage Oil-Flooded Air Compressor

Only a brief description of the complex optimisation of the two-stage screw compressor is given here. A task was to find the best design of a family of the two-stage compressors which covers the shaft powers between 22 and 312 kW at 8 – 15 bars discharge pressure.

Altogether 19 variables were used for this multivariable optimisation, 9 for each compressor stage plus the interstage pressure. The target function was minimum specific power without any other compromises. Therefore the compressor speed inevitably remained very low, rotor clearances are the lowest possible, rotor size is somewhat large and a variable frequency drive was chosen for each compressor stage to minimize the power required in the compressor part load.

The first calculations were used to determine a number of compressor frames to form the compressor family. It was calculated that with the appropriate use of the variable frequency drives only three compressor frames will cover the whole range.

A more detailed calculation was then used to find out the compressor stage sizes which will the best cover the range within the three compressor frames. A complex multivariable optimisation was used to determine the stage speed, built-in volume ratio and oil parameters together with the rotor profile details.

The rotor configuration of 5 lobes in the male rotor and 6 lobes in the female rotor emerged as the best mutual configuration for both stages. Therefore, only the profile details are varied between the compressor stages. Once the profiles were determined, the rotor centre distances for both of the compressor stages were additionally constrained to accommodate the best possible choice of the compressor bearings. Both rotors are presented in Fig. 11, where it can bee seen that a distinctive difference between them exists. The rotor profile of the first stage is somewhat slender to achieve the maximum possible displacement, while the second stage rotors are stronger to survive the high pressure loads of the second compressor stage. A final optimisation calculation was performed to tune up all of the optimisation variables to the best compressor performance.

Fig. 11 Rotor profiles optimised for a two stage oil-flooded air compressor duty

Since the optimisation calculation of the two-stage screw compressor presented in this paper is the first of such character reported in the open literature, every calculation triggered more questions than offered available answers and additional analyses had to be performed to explain all the phenomena encountered.

For example, since the oil of relatively low temperature and high pressure was injected into both compressor stages and due to the postponed air-oil mixing between the stages, it appeared that the air discharge temperature after the second stage was lower than the suction temperature to the same compressor stage. Since the first stage was extremely sensitive to the oil drag effect, the oil injection point to the first stage was chosen to be very late, contrary of the second stage, where the oil was injected very early. This even indicated a possibility to inject oil only to the discharge of the first stage.

5.3 Optimisation of a Family of Oil-Free Air Compressors Based on 3/5 Rotors

A successful example of optimisation of a family of two oil-free compressors is presented here. XK12 and XK18 compressors, based on 3/5 rotor profiles, presented in Fig 12, have been developed by a renown British screw compressor manufacturer. Together, the two machines cover the discharge range of 350-1000 m^3/h.

Fig. 12 3/5 Rotor profile optimised for an oil-free compressor

Prototype tests showed that both the volumetric and adiabatic efficiencies of these machines were higher than the published values of any equivalent compressors currently manufactured and marketed. This confirmed the advantages of both the rotor profile and the design optimisation procedure.

In Fig. 13 the performance of these compressors at discharge pressure of 3 bars was compared with the reference compressor R2, D-9000 of the same manufacturer, R1, C-80 of GHH based on SRM ‘A’ profile rotors, which despite its age outperformed other reference compressors, for example the compressors R3, Typhoon by Mouvex which is based on modern screw compressor profiles and R4, GHH CS1000, which is again based on SRM ‘A’ profile.

Fig. 13 Performance of the optimised compressors XK12 and XK18 compared with their market competition: R1, C-80, R2, D-9000, R3, Typhoon and R4, CS-1000

As it can be seen, the flow of the optimised compressors is at least 10% higher than of any other competitor for the same compressor power, which is actually greater than the expected and predicted value. The measured performance values were found to compare very favourably with published information for equivalent machines at present commercially available. More information about this optimisation can be found in *McCreath et al, 2001*.

6. CONCLUSIONS

A full multivariable optimisation of screw compressor geometry and operating conditions has been performed to establish the most efficient compressor design for any given duty. This has been achieved with a computer package for modelling compressor processes, developed by the authors, which provides the general specification of the lobe segments in terms of several key parameters and which can generate various lobe shapes and simultaneously calculates compressor thermodynamics. Computation of the instantaneous cross-sectional area and working volume could thereby be calculated repetitively in terms of the rotation angle. A mathematical model of the thermodynamic and fluid flow process is contained in the package, as well as models of associated processes encountered in real machines, such as variable fluid leakages, oil flooding or other fluid injection, heat losses to the surroundings, friction losses and other effects. All these are expressed in differential form in terms of an increment of the rotation angle. Numerical solution of these equations enables the screw compressor flow, power and specific power and compressor efficiencies to be calculated.

Rack generated profiles in 5/6 configuration rotors were used in the paper as examples to show how optimisation may permit both better delivery and higher efficiency for the same tip speed. Several rotor geometrical parameters, namely the male and female tip radii, as well as the compressor built-in ratio and compressor speed and oil flow and temperature and injection position were used as optimisation variables and applied to the multivariable optimisation of the machine geometry and its working parameters for a defined optimisation target. In the case of the example given, this was minimum compressor specific power. It has thereby been shown that for each application, a different rotor design is required to achieve optimum performance.

Finally, the rotors in 3/5 configuration optimised for an oil-free compressor duty were presented here to illustrate superiority of the optimised screw compressors over other compressors of similar size designed by use of classical design methods.

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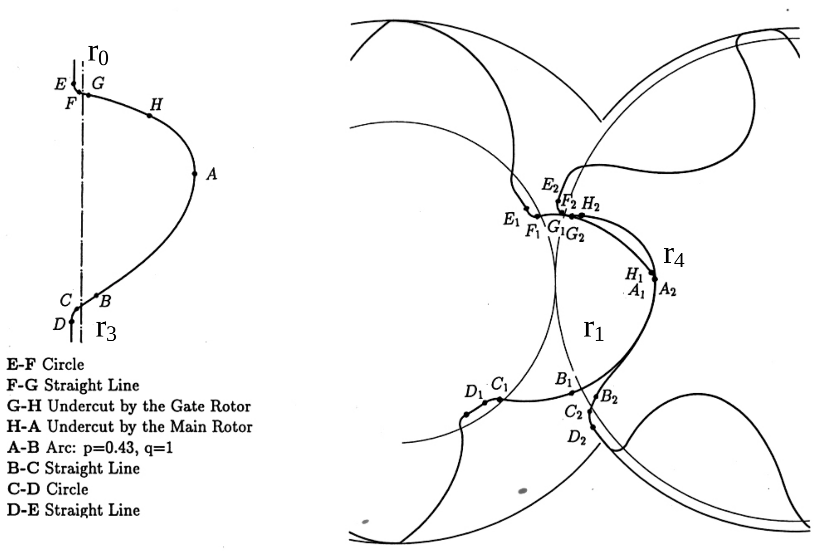

Fig. 1 N rotor variables used in optimisation calculations:

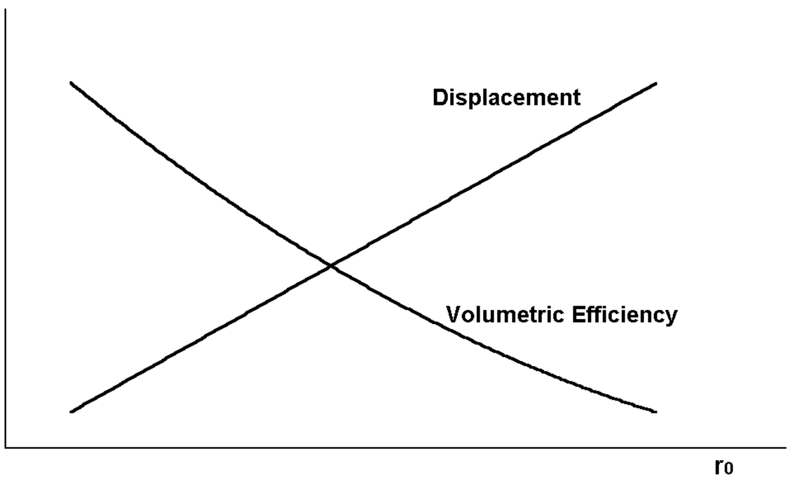

Fig. 2 Qualitative dependence of the rotor displacement and compressor volumetric efficiency upon the rotor radius r_0

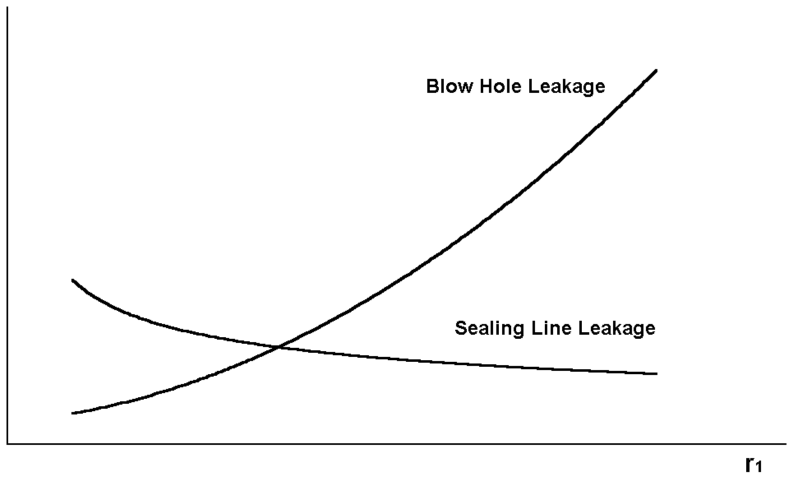

Fig. 3 Qualitative dependence of the blow hole and sealing line leakages upon the rotor radius r_1

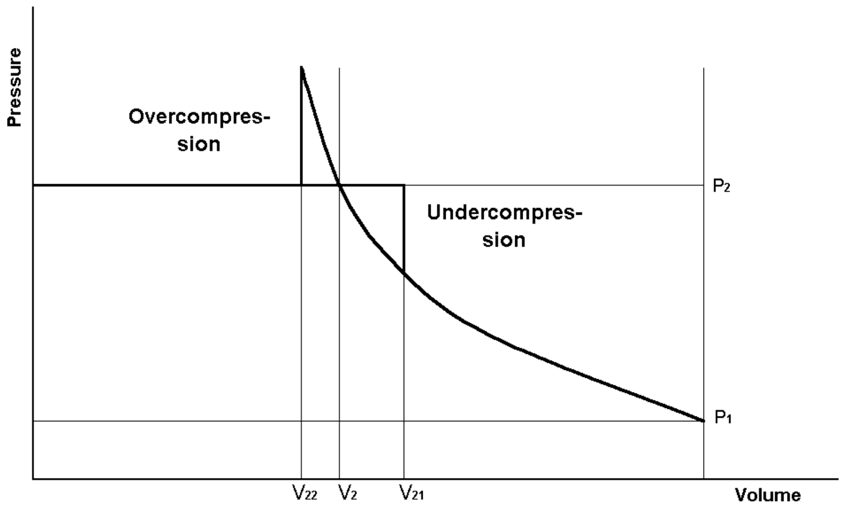

Fig. 4 Mismatch in the built-in volume ratio: too large, overcompression, too small, undercompression

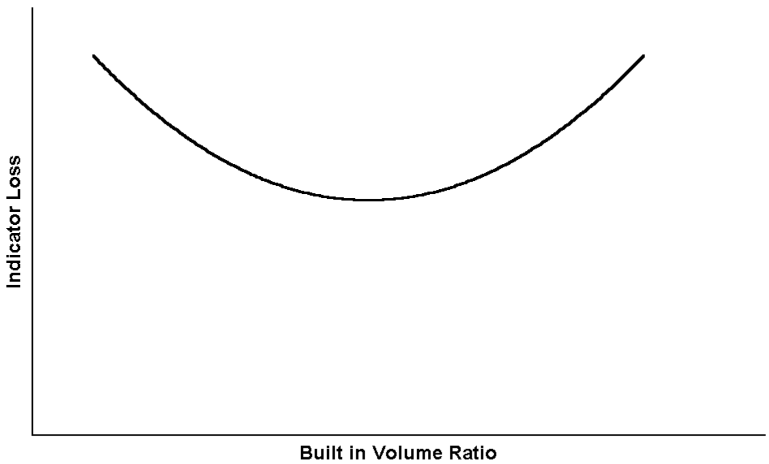

Fig. 5 Influence of the compressor built-in volume ratio upon the compressor indicated efficiency

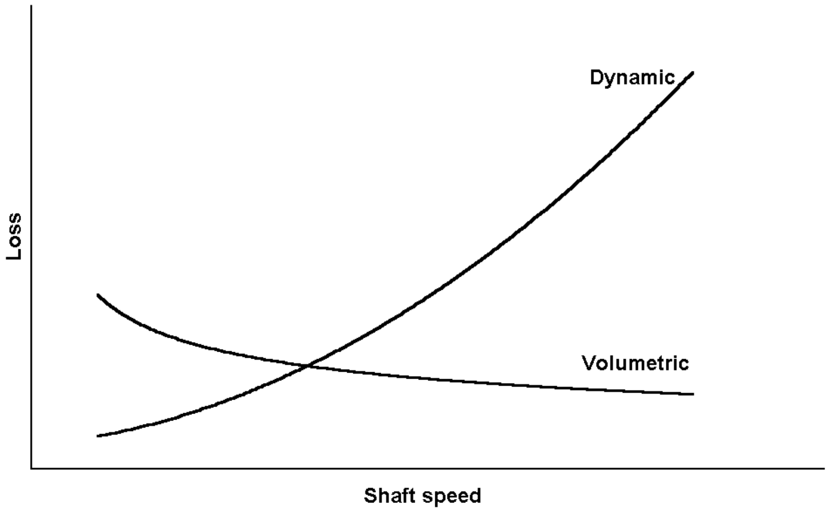

Fig. 6 Influence of the compressor shaft speed upon the compressor volumetric and dynamic losses

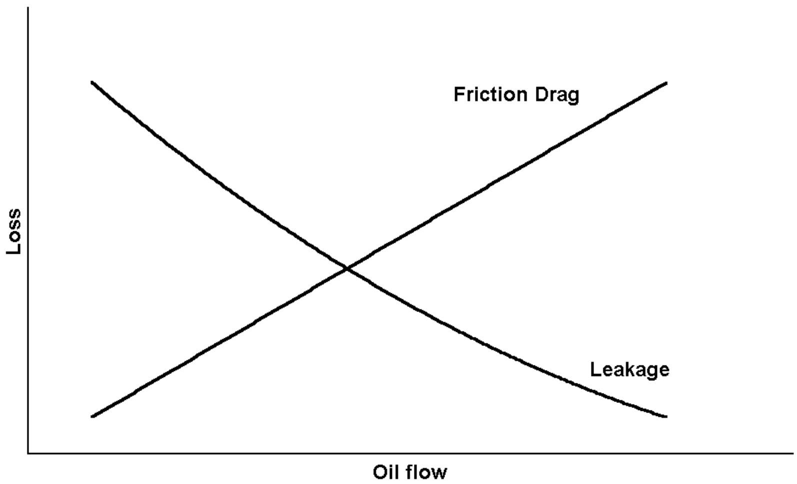

Fig. 7 Influence of the injected oil flow upon leakage and friction drag losses

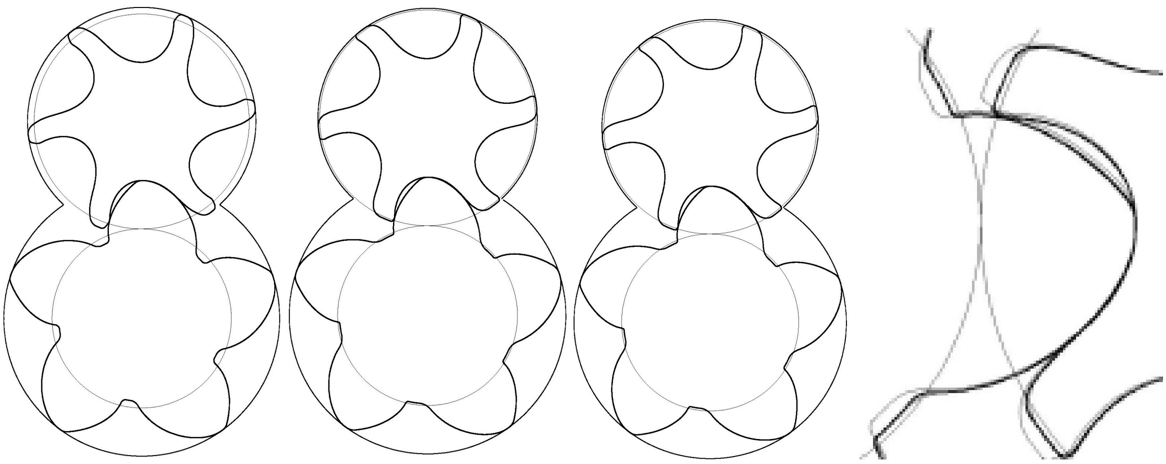

Fig. 8 Rotor profiles optimised for oil-free and oil-flooded air and refrigeration compressor duty

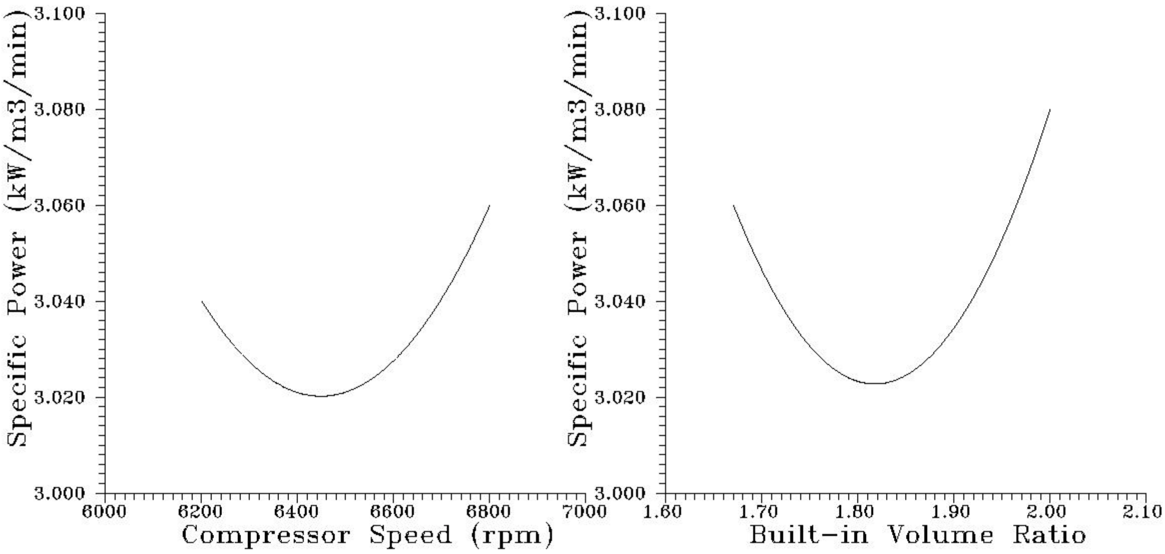

Fig. 9 Specific power as function of the compressor built-in volume and speed

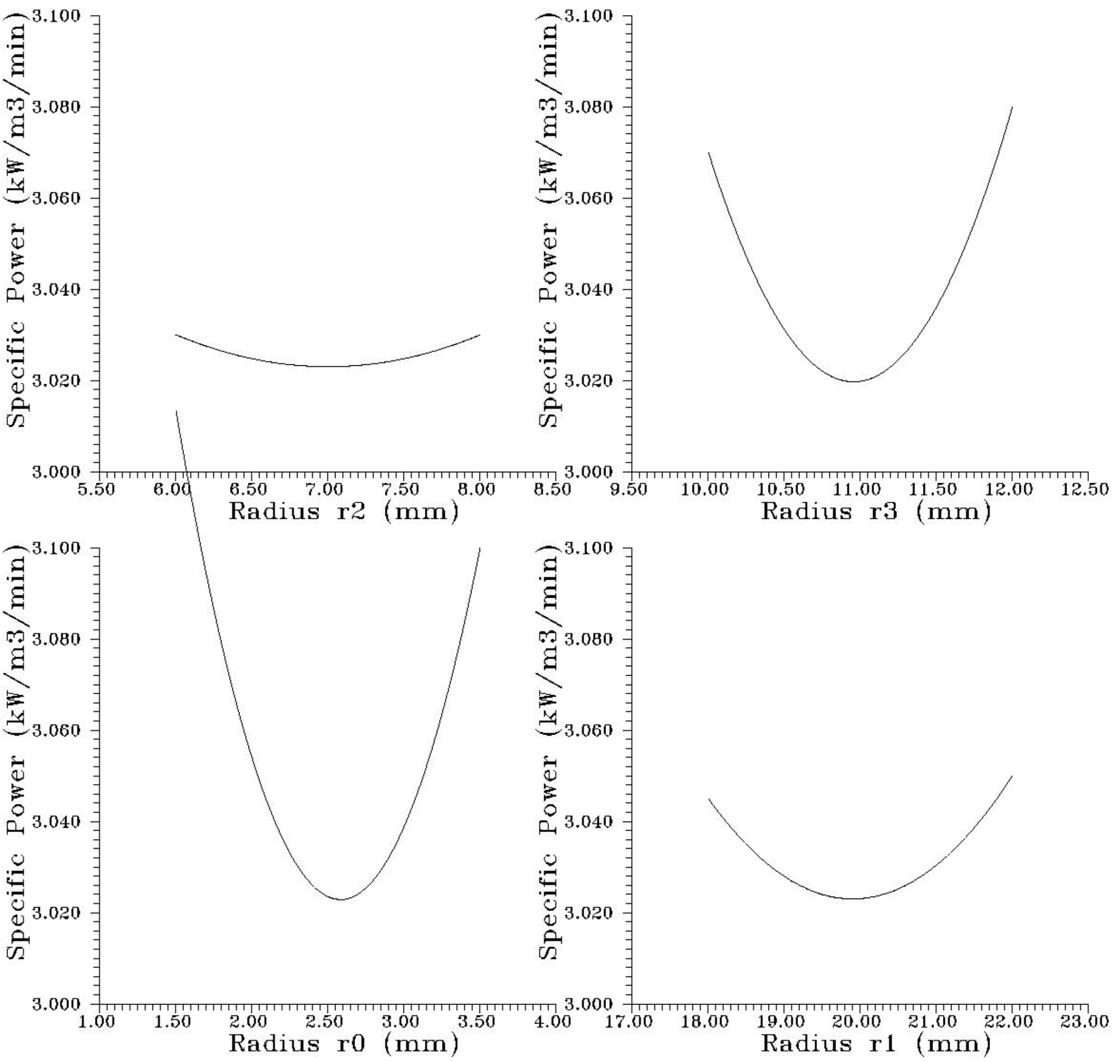

Fig. 10 Variation of the compressor specific power as function of the compressor rotor parameters

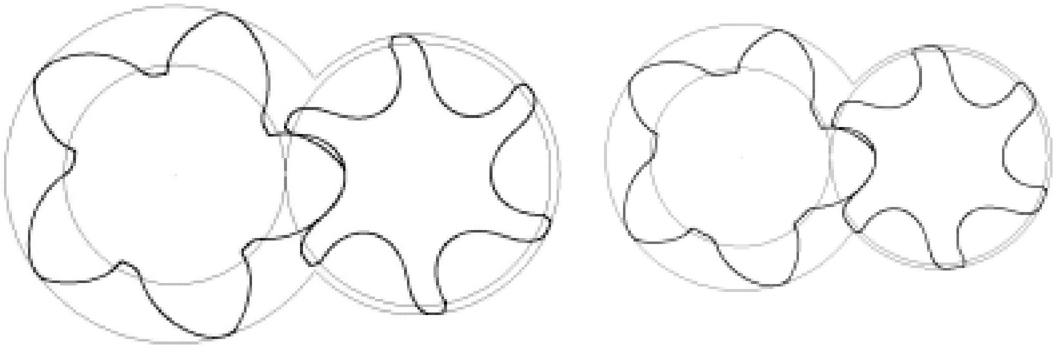

Fig. 11 Rotor profiles optimised for a two stage oil-flooded air compressor

Fig. 12 3/5 Rotor profile optimised for an oil-free compressor

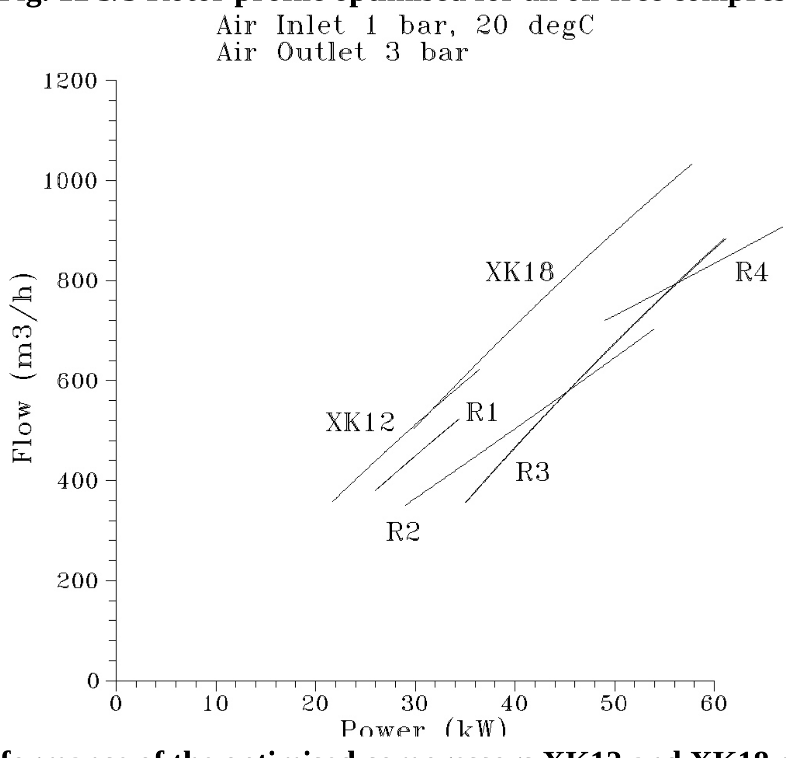

Fig. 13 Performance of the optimised compressors XK12 and XK18 compared with their market competition: R1, C-80, R2, D-9000, R3, Typhoon and R4, CS-1000

Figure and Table Captions

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Table 1: Results of optimisation calculations for dry and oil flooded air compressors and oil flooded refrigeration compressor