On the Lifetime and One-Year Views of Reserve Risk, with Application to IFRS 17 and Solvency II Risk Margins

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Abstract

This paper brings together analytic and simulation-based approaches to reserve risk in general (P&C) insurance, applied to the traditional actuarial view of risk over the lifetime of the liabilities and to the one-year view of Solvency II. It also connects the lifetime and one-year views of risk. The framework of the model in Mack (1993) is used throughout, although the results have wider applicability.

The advantages of a simulation-based approach are highlighted, giving a full predictive distribution, which is used to estimate risk margins under Solvency II and risk adjustments under IFRS 17. We discuss methods for obtaining capital requirements in a cost-of-capital risk margin, and methods for estimating risk adjustments using risk measures applied to a simulated distribution of the outstanding liabilities over their lifetime.

Keywords: Stochastic reserving, IFRS 17 risk adjustment, Solvency II risk margin, Bootstrap, Cost-of-capital, Coherent risk measure.

1. Introduction

Within the Solvency II regulatory regime in Europe, a risk margin is required in addition to considering reserving risk within internal capital models or when applying the Standard Formula (see EU Commission (2009)). Furthermore, International Financial Reporting Standards 17 (IFRS 17), which sets requirements for financial reporting for insurance entities in countries where the International Accounting Standards Board has a mandate, requires a risk adjustment within the estimates of liabilities, effective from 1 January 2021 (see IASB (2017)). Whereas Solvency II considers risk over a one-year time horizon, IFRS 17 is based on the fulfilment cash-flows over their lifetime, which requires careful consideration of an appropriate time horizon for risk quantification.

When quantifying reserve risk, there are several concepts to consider: there are analytic formula-based approaches that provide a standard deviation, there are simulation-based approaches that give a full predictive distribution, there is the traditional view over the lifetime of the liabilities, and there is the one-year view of Solvency II. This paper explores the concepts, highlighting the connections between

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them, and illustrates the usefulness of simulation-based approaches with application to Solvency II risk margins and, in particular, IFRS 17 risk adjustments.

2. The Traditional Actuarial View of Risk Over the Lifetime of the Liabilities

Early work on reserve risk focused on statistical models for the incremental or cumulative claims in so-called run-off triangles, deriving formulae for the root mean square error of prediction (RMSEP) until the liabilities are extinguished for a given model, where the RMSEP is a standard deviation of the forecasts, taking account of parameter uncertainty. The RMSEP, also known simply as the *prediction error*, can be approximated using the formula:

$$RMSEP = Prediction\ Error\ \approx\ \sqrt{Process\ Variance + Estimation\ Variance} \qquad (1)$$

where the estimation variance is the uncertainty associated with the estimation of parameters, and the process variance is a measure of uncertainty associated with the underlying claims generating process. Note that model error is not quantified; the prediction error is calculated conditional on a proposed model.

Thus, the problem of estimating the standard deviation of the forecasts analytically for a given model reduces to deriving formulae for the two components. Examples of models that have been proposed include the lognormal models of Kremer (1982), Verrall (1991), Zehnwirth (1994), and Barnett and Zehnwirth (2000); the gamma model of Mack (1991), Renshaw (1994), and England and Verrall (2001); the so-called distribution free approach of Mack (1993); the over-dispersed Poisson model of Renshaw and Verrall (1998); and many others. An overview can be found in Taylor (2000), England and Verrall (2002), and Wüthrich and Merz (2008).

With the advent of powerful computers and simulation-based techniques such as bootstrapping and Markov-chain Monte Carlo (MCMC) methods, the emphasis changed from the standard deviation of the forecasts to a full predictive distribution. Although Ashe (1986) first proposed bootstrapping to assist quantifying reserve uncertainty, the popularity of the technique increased after the publication of England and Verrall (1999), where an appropriate residual definition was used in association with a special case of an over-dispersed Poisson generalised linear model (GLM) that gives the same forecasts as the traditional chain-ladder model (under certain constraints). The bootstrap procedure was explained, and the standard deviation of the bootstrap results was compared to analogous results obtained analytically for the same model, showing remarkable similarity and thereby validating the procedure. Bootstrapping was only used as a computationally simple way of obtaining an estimate of the prediction error; the usefulness of a full predictive distribution was not considered. That was rectified in England (2002) and England and Verrall (2002) where a second simulation step was included when forecasting to provide a full predictive distribution, not just a standard deviation. The appeal of simulation-based approaches is that they bypass the challenging mathematics and provide a full predictive distribution directly. When applied correctly, the standard deviation from such a distribution for a given model will match the RMSEP calculated analytically (subject to simulation error). Since a full distribution is available, any statistic of interest or risk measure can be estimated at no extra cost, for example, value-at-risk, tail value-at-risk, and so on.

Bootstrapping the over-dispersed Poisson model of England and Verrall (2002) quickly became a popular technique for quantifying reserve risk in simulation-based capital models for Solvency II, and it became known simply as “the ODP model” or “the bootstrap model”. These are unfortunate monikers since England and Verrall (2002) only considered a special case of an over-dispersed Poisson GLM, and bootstrapping is simply a statistical procedure that can, in fact, be applied to any well-specified statistical model. Attempting to rectify the misunderstanding, England and Verrall (2006) focused on predictive distributions for GLMs in general, including a GLM representation of the model of Mack (1993), called "Mack's model" hereafter. In addition, the paper described how to obtain predictive distributions using log-normal regression models.

In this paper, we focus solely on Mack's model applied analytically, and a bootstrap representation of Mack's model, since the model is straightforward, and analytic formulae for the main results are available. We recognise that the chain ladder model and the assumptions in Mack's model are not always the most appropriate to use in practice, but at the same time the chain ladder model is a good starting point, and we believe the insights gained in this paper are applicable quite generally.

2.1 A Distribution-Free Approach to Estimating the RMSEP of Reserve Estimates over their Lifetime

For ease of exposition, and without loss of generality, we assume that the data consist of a triangle of cumulative claims:

$$\begin{array}{llll} C_{1,0}, & C_{1,1}, & \dots, & C_{1,J} \\ C_{2,0}, & \dots, & C_{2,J-1} & \\ \vdots & & & \\ C_{n,0} & & & \end{array}$$

This can be written as $\mathcal{D}_n = \{C_{i,j}: i = 1, \dots, n; j = 0, \dots, n-i\}$, where $n = J + 1$ is the number of origin periods. Note that the development periods use a zero-based array for consistency with Merz and Wüthrich (2008, 2014). This simplifies a discussion of calendar period since it starts at one when calendar period is defined as origin period plus development period.

The aim of reserving is to populate the missing lower portion of the triangle and to extrapolate beyond the maximum development period where necessary. One traditional actuarial technique that does this is the well-known chain-ladder technique. Mack (1993) proposed a stochastic version of the chain-ladder technique and focused on the cumulative claims with mean and variance:

$$E\left[C_{i,j+1}|C_{i,0}, \dots, C_{i,j}\right] = \lambda_j C_{i,j} \text{ and } Var\left[C_{i,j+1}|C_{i,0}, \dots, C_{i,j}\right] = \sigma_j^2 C_{i,j} \quad \text{for } 0 \le j \le J-1. \qquad (2)$$

The expected value of the cumulative claims is proportional to the cumulative claims at the previous development period, and the variance is also proportional to the previous cumulative claims. Mack (1993) considered the model to be "distribution free" since only the first two moments of the cumulative claims are specified, not the full distribution. Mack (1993) derived estimators of the unknown parameters λ_j and σ_j^2 and, making further limited assumptions, provided formulae for the prediction errors of the forecast payments and reserve estimates.

The estimators for λ_j and σ_j^2 derived by Mack (1993) are given by:

$$\hat{\lambda}_j = \frac{\sum_{i=1}^{n-j-1} w_{i,j} f_{i,j}}{\sum_{i=1}^{n-j-1} w_{i,j}} \qquad (3)$$

where $w_{i,j} = C_{i,j}$ and $f_{i,j} = \frac{C_{i,j+1}}{C_{i,j}}$

and

$$\hat{\sigma}_j^2 = \frac{1}{n-j-2}\sum_{i=1}^{n-j-1} w_{i,j}\left(f_{i,j} - \hat{\lambda}_j\right)^2 \tag{4}$$

The estimator for the final unknown parameter σ_{J-1}^2 (in the case $n = J + 1$) is conventionally given by:

$$\hat{\sigma}_{J-1}^2 = \min\left(\hat{\sigma}_{J-3}^2, \hat{\sigma}_{J-2}^2, \hat{\sigma}_{J-2}^4 \big/ \hat{\sigma}_{J-3}^2\right)$$

The estimator for the development factors λ_j is the standard volume weighted chain-ladder estimator. Then, the variance component σ_j^2 is estimated as a residual sum of squares divided by degrees of freedom (within each development period). Mack (1993) derived formulae for the RMSEP of the chain-ladder forecasts under the model in the usual way by providing estimators for the process variance and the estimation variance, and combining them using equation (1).

From Mack (1993), the prediction variance (MSEP) of the reserves in origin period i at time n is given by:

$$MSEP\left[\hat{R}_i|\mathcal{D}_n\right] \approx \hat{C}_{i,J}^2 \sum_{j=n-i}^{J-1} \frac{\hat{\sigma}_j^2}{\hat{\lambda}_j^2}\left(\frac{1}{\hat{C}_{i,j}} + \frac{1}{\sum_{l=1}^{n-j-1} C_{l,j}}\right)$$

Where $\hat{R}_i = \hat{C}_{i,J} - C_{i,n-i}$ and both process variance and estimation variance are included.

For the overall reserve prediction variance, a covariance adjustment is needed for the estimation variance, giving:

$$MSEP\left[\hat{R}_+|\mathcal{D}_n\right] \approx \sum_{i=2}^{n}\left\{MSEP\left[\hat{R}_i|\mathcal{D}_n\right] + 2\hat{C}_{i,J}\left(\sum_{q=i+1}^{n} \hat{C}_{q,J}\right) \times \sum_{j=n-i}^{J-1} \frac{\hat{\sigma}_j^2}{\hat{\lambda}_j^2 \sum_{l=1}^{n-j-1} C_{l,j}}\right\}$$

where the overall reserve estimate is given by:

$$\hat{R}_+ = \sum_{i=2}^{n} \hat{R}_i$$

2.2 Bootstrapping Mack's Model

By expressing Mack's model as a GLM, England and Verrall (2006) showed how to bootstrap Mack's model, providing a predictive distribution consistent with the assumptions in Mack (1993). Non-parametric bootstrapping is a statistical procedure that involves creating many simulated sets of pseudo-data by sampling with replacement from the original data sample, then obtaining the parameters of interest for the given model from each set of pseudo-data, giving a simulated joint distribution of the parameters.

When bootstrapping, the data used for re-sampling must be independent and identically distributed. With regression-type models (such as GLMs), the "observations" are usually assumed to be independent, but are not identically distributed since the means (and possibly the variances) depend on covariates. Therefore, with regression-type models, it is common to bootstrap the residuals (suitably scaled such that the variance is approximately equal to one), rather than the data themselves, since the scaled residuals are approximately independent and identically distributed. The residual definition must be consistent with the model being fitted. A random sample of the scaled residuals is taken (using sampling with replacement), and new pseudo-data values are obtained by inverting the definition of the scaled residuals in terms of the variable of interest. The model is then re-fitted to the pseudo data, giving revised parameter estimates. When repeated many times, a joint distribution of the parameter estimates is obtained.

To obtain a predictive distribution, a second simulation is performed when forecasting into the future by simulating from the assumed process distribution, conditional on the parameters. This two-step simulation procedure (bootstrapping to obtain a distribution of the parameters followed by forecasting from the process distribution) automatically includes both parameter and process uncertainty.

The key to understanding how to bootstrap Mack's model is to re-express it as a model of the ratios f, rather than a model of the cumulative amounts C. Using equation (2) and taking $C_{i,j}$ to the left-hand side gives:

$$E\left[f_{i,j}|C_{i,0},\dots,C_{i,j}\right]=\lambda_j \text{ and } Var\left[f_{i,j}|C_{i,0},\dots,C_{i,j}\right]=\frac{\sigma_j^2}{C_{i,j}} \text{ for } j\geq 0.$$

It should be noted that Mack (1999) also re-expressed the model in terms of the ratios. The ratios $f_{i,j}$ then become the focus of attention, with the associated $C_{i,j}$ considered as fixed and known weights. It is then possible to define appropriate residuals for use in a bootstrap procedure for Mack's model. Given a triangle of scaled residuals, the bootstrap procedure involves creating new triangles of scaled residuals, by sampling with replacement from the original scaled residuals triangle. The pseudo data are then obtained by inverting the formula for the scaled residuals in terms of the data of interest, f. The original model used to obtain the residuals can be re-fitted to each triangle of pseudo data, to obtain new fitted development factors using equation (3).

Since Mack's model is a recursive model, forecasting proceeds one step at a time. Starting from the latest cumulative claims, the one-step-ahead forecasts can be obtained for each bootstrap iteration either non-parametrically by re-sampling from the residuals again, or by drawing a sample from an assumed parametric process distribution. The forecast claims can be used to provide predictive distributions of the outstanding liabilities by origin period, calendar period, or in total. A complete list of steps can be found in Appendix 1.

Tables 2 and 4 in Section 4 demonstrate that the standard deviation of the simulated distribution of the reserves using bootstrapping matches the analytic results of Mack (1993), connecting the analytic and simulation-based approaches for the lifetime view of risk.

3. The One-Year View of Risk and Beyond

The Solvency II regulatory regime in the European Union introduced the concept of the one-year view of risk. According to Article 101 of the Solvency II Directive:

> *"The Solvency Capital Requirement (SCR) … shall correspond to the value-at-risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period",*

where the "basic own funds" is a Solvency II definition of the net assets on an insurer's balance sheet. Although many details are missing, the definition of the SCR is complete from a theoretical perspective since it includes all four elements needed to estimate capital requirements:

1. A risk profile (distribution of the basic own funds)
2. A risk measure (value-at-risk)
3. A risk tolerance criterion (99.5%)
4. A time horizon (one year)

The selections made under Solvency II for the 4 items are not the only choices that could be made, but at least all 4 are included. Using simulation-based internal models, it seems straightforward (at least in theory) to estimate the SCR: just simulate the balance sheet one-year ahead (under the Solvency II rules), then apply value-at-risk at the 99.5th percentile level.

The liabilities side of the opening Solvency II balance sheet contains an estimate of the expected outstanding liabilities. Each simulated balance sheet one year ahead also contains an estimate of the expected outstanding liabilities at that time, conditional on the payments that have emerged in the year. This introduces the concept of the profit or loss on the reserves, which is known as the claims development result (CDR) or simply the run-off result.

For a given origin period i:

$$CDR_i^{(n+1)} = \hat{R}_i^{(n)} - I_i^{(n+1)} - \hat{R}_i^{(n+1)} \tag{5}$$

where superscripts represent calendar time $\tau = i + j;\ \tau \geq n$ and:

$\hat{R}_i^{(n)}$ represents the estimated (undiscounted) reserves at the start of calendar year $n+1$

$I_i^{(n+1)} = C_{i,n-i+1} - C_{i,n-i}$ represents the incremental payments made during calendar year $n+1$

$\hat{R}_i^{(n+1)}$ represents the estimated (undiscounted) reserves at the end of calendar year $n+1$

Adding and subtracting the cumulative claims up to calendar year n gives:

$$CDR_i^{(n+1)} = \left(C_{i,n-i} + \hat{R}_i^{(n)}\right) - \left(C_{i,n-i} + I_i^{(n+1)} + \hat{R}_i^{(n+1)}\right) = \hat{U}_i^{(n)} - \hat{U}_i^{(n+1)} \tag{6}$$

where:

$\hat{U}_i^{(n)}$ represents the estimated (undiscounted) ultimate cost of claims at the start of calendar year $n+1$

$\hat{U}_i^{(n+1)}$ represents the estimated (undiscounted) ultimate cost of claims at the end of calendar year $n+1$

If at the end of the year, the estimated ultimate cost of claims has gone up, there is a loss on the reserves, since $CDR_i^{(n+1)} < 0$, which must be made up from capital. Similarly, if the estimated ultimate cost of claims at the end of the year has gone down, there is a profit on the reserves, since $CDR_i^{(n+1)} > 0$.

Under Solvency II, it is the change in the ultimate cost of claims over a one-year time horizon (the profit or loss over one year) that is important, and the Solvency II definition of reserve risk is in that context. The analogy on the assets side of the balance sheet is the change in the value of assets over one year.

Clearly, the Solvency II definition of reserve risk is different from the traditional actuarial view of risk, which considers the outstanding payments over their lifetime. Several authors have discussed this, including Böhm and Glaab (2006), De Felice and Moriconi (2003, 2006), Merz and Wüthrich (2007, 2008), Ohlsson and Lauzeningks (2008a, 2008b, 2009), and Merz *et al*. (2009). In particular, as Ohlsson and Lauzeningks observe, a study from the mutual insurers' organisation AISAM-ACME (2007) notes that

"Only a few members were aware of the inconsistency between their assessment on the ultimate costs and the Solvency II framework which uses a one-year horizon".

3.1 The RMSEP of the Claims Development Result over One-Year under the Distribution Free Approach

Recognising the difference between the lifetime view of risk and the one-year view of Solvency II, Merz and Wüthrich (2008) derived formulae for the RMSEP (*ie* standard deviation) of the CDR over one year, using the same model structure and underlying assumptions as Mack (1993), thereby providing the one-year analogue to Mack's model. For each origin period i, Merz and Wüthrich (2008) showed that at time n:

$$MSEP\left[CDR_i^{(n+1)}|\mathcal{D}_n\right] \approx \hat{C}_{i,J}^2 \frac{\hat{\sigma}_{n-i}^2}{\hat{\lambda}_{n-i}^2}\left[\frac{1}{C_{i,n-i}} + \frac{1}{\sum_{l=1}^{i-1} C_{l,n-i}}\right] + \hat{C}_{i,J}^2 \sum_{j=n-i+1}^{J-1} \frac{\hat{\sigma}_j^2}{\hat{\lambda}_j^2}\left[\alpha_j^{(n)} \frac{1}{\sum_{l=1}^{n-j-1} C_{l,j}}\right]$$

where

$$\alpha_j^{(n)} = \frac{C_{n-j,j}}{\sum_{l=1}^{n-j} C_{l,j}} \quad \in \ (0,1]$$

For the total across all origin periods, Merz and Wüthrich (2008) showed that:

$$\begin{aligned} MSEP\left[CDR_+^{(n+1)}|\mathcal{D}_n\right] \\ &\approx \sum_{i=2}^{n} MSEP\left[CDR_i^{(n+1)}|\mathcal{D}_n\right] \\ &+ 2\sum_{2\le i<q\le n} \hat{C}_{i,J}\hat{C}_{q,J}\left[\frac{\hat{\sigma}_{n-i}^2}{\hat{\lambda}_{n-i}^2}\frac{1}{\sum_{l=1}^{i-1} C_{l,n-i}} + \sum_{j=n-i+1}^{J-1}\frac{\hat{\sigma}_j^2}{\hat{\lambda}_j^2}\left[\alpha_j^{(n)}\frac{1}{\sum_{l=1}^{n-j-1} C_{l,j}}\right]\right] \end{aligned}$$

where

$$CDR_+^{(n+1)} = \sum_{i=2}^{n} CDR_i^{(n+1)}$$

3.2 Simulating the Claims Development Result over One-Year

To obtain a predictive distribution consistent with Merz and Wüthrich (2008), it is necessary to simulate the CDR in a way that is consistent with Mack's assumptions. Modifying equations (5) and (6):

$$CDR_{i,s}^{(n+1)} = \hat{R}_i^{(n)} - I_{i,s}^{(n+1)} - \hat{R}_{i,s}^{(n+1)} = \hat{U}_i^{(n)} - \hat{U}_{i,s}^{(n+1)} \tag{7}$$

for each simulation s. The reserves at the start of the year are fixed, so it is only necessary to simulate the payments that emerge over the next calendar period, and also the estimated reserves at the end of the year, conditional on the payments that emerge.

The payments that emerge over the next calendar period can be obtained by bootstrapping Mack's model, as described in Section 2.2. All that remains is to estimate the outstanding liabilities at the end of the year conditional on what has emerged, for each simulation. For this, it is necessary to augment the original payments triangle by the simulated payments that emerge over the next calendar period for each origin period, which are all an actuary sees over a one-year period. Conditional on the payments that emerge (for each simulation), it is then necessary to estimate the reserves at the end of the period. At this point, an automated reserving methodology is required; an actuary in the computer is required, or an "actuary-in-the-box", as the procedure is known[4]. To remain consistent with the underlying methodology described in this paper, the standard chain-ladder method is adopted for this purpose. That is, for each new simulated triangle, the chain-ladder model is re-fitted conditional on the claims that emerge in the year, giving the reserves at the end of the year. This automatic re-fitting of the reserving methodology has led to the "actuary-in-the-box" procedure also being known as "re-reserving" (see, for example, Diers (2009)).

It should be noted that it is not necessary to simulate again, conditional on the claims that have emerged over the next calendar period for each simulation (nested simulation), since the expected liabilities given by such a process would match the chain-ladder estimates and it is only the expectation that is required to compute the CDR.

Given the payments that emerge in the year (using the bootstrap results) and the reserves at the end of the year (using the re-reserving approach), the CDR for each simulation can be evaluated using equation (7), giving a distribution of the CDR over one year. Once created, any risk measure can be applied to the distribution, not just the standard deviation. Although Ohlsson and Lauzeningks (2008a, 2008b) are usually credited with first describing the re-reserving approach, they note that *"this simulation algorithm is by no means new"*.

In Section 4, we demonstrate that the standard deviation of the simulated distribution of the CDR using the re-reserving approach matches the analytic approach of Merz and Wüthrich (2008), connecting the analytic and simulation-based approaches for the one-year view of risk. Connecting the one-year view of risk and the traditional lifetime view is all that remains.

3.3 The RMSEP of the Claims Development Result Beyond One-Year Ahead

Merz and Wüthrich (2014) extended the results of Merz and Wüthrich (2008) beyond one-year ahead and provided formulae for the MSEP of the CDRs for a sequence of one-year views over the lifetime of the liabilities. Similar results were obtained by Röhr (2016) and Diers *et al*. (2016).

Generalising equations (5) and (6), for each origin period i:

$$CDR_i^{(n+k+1)} = \hat{R}_i^{(n+k)} - I_i^{(n+k+1)} - \hat{R}_i^{(n+k+1)} = \hat{U}_i^{(n+k)} - \hat{U}_i^{(n+k+1)} \quad (8)$$

Thus, the CDR in calendar year $n + k + 1$ is the expected ultimate cost of claims at the start of the year less the expected ultimate cost of claims at the end of the year.

Using a Bayesian representation of the chain-ladder model, Merz and Wüthrich (2014) showed that for each origin period i:

[4] The term "actuary-in-the-box" was originally coined by Esbjörn Ohlsson.

$$MSEP\left[CDR_i^{(n+k+1)}|\mathcal{D}_n\right]$$
$$\approx \hat{C}_{i,J}^2 \frac{\hat{\sigma}_{n-i+k}^2}{\hat{\lambda}_{n-i+k}^2}\left[\frac{1}{\hat{C}_{i,n-i+k}} + \prod_{m=1}^{k}\left(1-\alpha_{n-i+m}^{(n)}\right)\frac{1}{\sum_{l=1}^{i-k-1} C_{l,n-i+k}}\right]$$
$$+ \hat{C}_{i,J}^2 \sum_{j=n-i+k+1}^{J-1} \frac{\hat{\sigma}_j^2}{\hat{\lambda}_j^2}\left[\alpha_{j-k}^{(n)} \prod_{m=0}^{k-1}\left(1-\alpha_{j-m}^{(n)}\right)\frac{1}{\sum_{l=1}^{n-j-1} C_{l,j}}\right]$$

For the total across all origin periods:

$$MSEP\left[CDR_+^{(n+k+1)}|\mathcal{D}_n\right]$$
$$\approx \sum_{i=2}^{n} MSEP\left[CDR_i^{(n+k+1)}|\mathcal{D}_n\right]$$
$$+ 2\sum_{2\le i<q\le n} \hat{C}_{i,J}\hat{C}_{q,J}\frac{\hat{\sigma}_{n-i+k}^2}{\hat{\lambda}_{n-i+k}^2}\prod_{m=1}^{k}\left(1-\alpha_{n-i+m}^{(n)}\right)\frac{1}{\sum_{l=1}^{i-k-1} C_{l,n-i+k}}$$
$$+ 2\sum_{2\le i<q\le n} \hat{C}_{i,J}\hat{C}_{q,J}\sum_{j=n-i+k+1}^{J-1}\frac{\hat{\sigma}_j^2}{\hat{\lambda}_j^2}\left[\alpha_{j-k}^{(n)}\prod_{m=0}^{k-1}\left(1-\alpha_{j-m}^{(n)}\right)\frac{1}{\sum_{l=1}^{n-j-1} C_{l,j}}\right]$$

Merz and Wüthrich (2014) also showed that the sum of the MSEPs of the CDRs for each future calendar year equals the MSEP from the model of Mack (1993). Thus, the MSEP from Mack's model can be partitioned into a sequence of one-year views, showing how the lifetime risk emerges over time, and linking the traditional lifetime view of risk with the one-year view of Solvency II (see Tables 2 and 3 in Section 4).

3.4 Recursively Simulating the Claims Development Result Beyond One Year Ahead

Again, it is desirable to have a full predictive distribution of the CDR at any future time period, from which any risk measure can be obtained. As Ohlsson and Lauzeningks (2008b, 2009) observe, *"it is straightforward to extend the* [re-reserving] *simulation method to two or more years"*. To achieve this, it is necessary to modify equation (8) such that:

$$CDR_{i,s}^{(n+k+1)} = \hat{R}_{i,s}^{(n+k)} - I_{i,s}^{(n+k+1)} - \hat{R}_{i,s}^{(n+k+1)} = \hat{U}_{i,s}^{(n+k)} - \hat{U}_{i,s}^{(n+k+1)} \quad (9)$$

for each simulation s.

For $k \ge 1$, the reserves at the start of year $n+k+1$, $\hat{R}_{i,s}^{(n+k)}$, and the ultimate cost of claims at the start of the year, $\hat{U}_{i,s}^{(n+k)}$, are now different for each simulation s. Note that when $k=0$, the re-reserving procedure of Section 3.2 provides $\hat{R}_{i,s}^{(n+1)}$ and $\hat{U}_{i,s}^{(n+1)}$, giving the starting values required for evaluating $CDR_{i,s}^{(n+1)}$, when $k=1$. Again, note that the payments that emerge over each calendar period, $I_{i,s}^{(n+2)}$, are available from bootstrapping Mack's model, so to complete the evaluation of $CDR_{i,s}^{(n+2)}$, it is only necessary to evaluate $\hat{R}_{i,s}^{(n+2)}$ or $\hat{U}_{i,s}^{(n+2)}$. For this, another iteration of the re-reserving procedure is performed. That is, the original claims triangle is augmented by a second payments diagonal (from bootstrapping Mack's model), and the reserves (or ultimate cost of claims) are evaluated using the chain-ladder model for each simulation, conditional on the claims that have emerged over the two-year period.

To evaluate $CDR_{i,s}^{(n+k+1)}$ for remaining values of k, the re-reserving procedure is repeated recursively, each time augmenting the original claims triangle by another diagonal (from bootstrapping Mack's model) and applying the chain-ladder model for each simulation, conditional on what has emerged. This recursive approach provides a distribution of the CDR for each future calendar period, from which any risk measure can be calculated. Diers *et al*. (2013) also use the multi-year re-reserving approach to evaluate the cumulative emergence of the CDR, not the incremental one-year movements (see Section 7).

Tables 3 and 6 in Section 4 demonstrate that the standard deviation of the simulated distributions of the incremental CDRs using the recursive re-reserving approach match the analytic results from the Merz and Wüthrich (2014) formulae, again connecting the analytic and simulation-based approaches, and connecting the one-year view of risk and the traditional lifetime view.

4. An Illustration of the Analytic and Simulation Based Approaches to Quantifying Reserve Risk

To illustrate the methodology described in Sections 2 and 3, consider the data shown in Table 1, taken from Taylor and Ashe (1983), and used in several other previously published papers. Also shown in Table 1 are the estimated chain-ladder factors, $\hat{\lambda}_j$, and the parameters $\hat{\sigma}_j$ from Mack's model. It is not implied that Mack's model is optimal for this dataset. In particular, the data have not completely run-off by the final development period and a model that allows extrapolation into the tail would be more predictive. The results shown here are purely illustrative of the models of Mack (1993) and Merz and Wüthrich (2008, 2014).

In Table 2, the chain-ladder reserves are shown, together with the standard deviations of the forecasts (RMSEPs) from Mack's model, giving a coefficient of variation of the total reserves under the lifetime view of risk of 13.1%. In addition, the RMSEPs of the CDRs over 1 year using the formulae from Merz and Wüthrich (2008) are shown in Table 2. The RMSEPs divided by the expected reserves at the start of the year are also shown, giving 9.52% for the total CDR. This one-year measure of risk is lower than the traditional lifetime view, and there has been some discussion about whether this is always the case.

Table 3 shows the RMSEPs (*ie* standard deviations) of the CDRs for each future calendar period (the "full picture") using the formulae from Merz and Wüthrich (2014). The result of squaring the values (to give variances), adding up across all columns within each row, and taking the square root is shown in the final column. A comparison with Table 2 shows that the square root of the sum of squares of the CDRs gives the same result as the RMSEP from Mack's model over the lifetime of the liabilities. This demonstrates how the lifetime view of risk under Mack's model can be partitioned into a sequence of one-year views. It also shows that the one-year view of risk must always be lower than the lifetime view since variances cannot be negative. This is an interesting result and links the lifetime view of risk with the one-year view of Solvency II using analytic approaches.

Acc. Year	Development Year 0	1	2	3	4	5	6	7	8	9
1	357,848	1,124,788	1,735,330	2,218,270	2,745,596	3,319,994	3,466,336	3,606,286	3,833,515	3,901,463
2	352,118	1,236,139	2,170,033	3,353,322	3,799,067	4,120,063	4,647,867	4,914,039	5,339,085	
3	290,507	1,292,306	2,218,525	3,235,179	3,985,995	4,132,918	4,628,910	4,909,315		
4	310,608	1,418,858	2,195,047	3,757,447	4,029,929	4,381,982	4,588,268			
5	443,160	1,136,350	2,128,333	2,897,821	3,402,672	3,873,311				
6	396,132	1,333,217	2,180,715	2,985,752	3,691,712					
7	440,832	1,288,463	2,419,861	3,483,130						
8	359,480	1,421,128	2,864,498							
9	376,686	1,363,294								
10	344,014									
$\hat{\lambda}_j$	3.491	1.747	1.457	1.174	1.104	1.086	1.054	1.077	1.018	
$\hat{\sigma}_j$	400.35	194.26	204.85	123.22	117.18	90.48	21.13	33.87	21.13	

Table 1: Cumulative amounts, estimated chain-ladder factors $\hat{\lambda}_j$, and parameters $\hat{\sigma}_j$.

Acc. Year	Reserves	Mack RMSEP	Mack RMSEP %	Merz-Wüthrich RMSEP	Merz-Wüthrich RMSEP %
1	0	0	0%	0	0%
2	94,634	75,535	79.82%	75,535	79.82%
3	469,511	121,699	25.92%	105,309	22.43%
4	709,638	133,549	18.82%	79,846	11.25%
5	984,889	261,406	26.54%	235,115	23.87%
6	1,419,459	411,010	28.96%	318,427	22.43%
7	2,177,641	558,317	25.64%	361,089	16.58%
8	3,920,301	875,328	22.33%	629,681	16.06%
9	4,278,972	971,258	22.70%	588,662	13.76%
10	4,625,811	1,363,155	29.47%	1,029,925	22.26%
Total	18,680,856	2,447,095	13.10%	1,778,968	9.52%

Table 2: Chain-ladder reserves, Mack RMSEP, and Merz-Wüthrich RMSEP.

Table 4 shows the expected reserves, standard deviation (prediction error), and coefficient of variation from bootstrapping Mack's model using 500,000 simulations. Also shown are the standard deviations of the one-year ahead CDRs using the re-reserving approach described in Section 3.2, and the standard deviations expressed as a proportion of the expected reserves at the start of the year. Comparison with Table 3 shows that the expected reserves are very close to the chain-ladder reserves, and the standard deviations of the simulated reserves from bootstrapping Mack's model are very close to the analytic results given by Mack's model. In addition, the standard deviations of the one-year ahead CDRs are very close to the analytic results given by the formulae from Merz and Wüthrich (2008).

Note that since the bootstrap approach provides distributions of all future cash-flows (not just the reserves), it is straightforward to obtain a distribution of the discounted reserves. For example, Table 5 shows a summary of the results of discounting the future cash-flows at 3% (assuming payments are made mid-way through the year).

Table 6 shows the standard deviations of the CDRs for each future calendar period using the recursive re-reserving approach. Again, the result of squaring the values (to give variances), adding up across all columns within each row, and taking the square root is shown in the final column. A comparison with Table 4 shows that the square root of the sum of squares of the CDRs are very close to the standard deviations from bootstrapping Mack's model over the lifetime of the liabilities, and again demonstrates how the lifetime view of risk under Mack's model can be partitioned into a sequence of one-year views.

The values in Tables 3 and 6 show remarkable similarity, validating the simulation approaches described in Sections 3.2 and 3.4, and connecting the lifetime and one-year views of risk for analytic and simulation-based approaches associated with Mack's model.

Again, an advantage of the simulation-based approach is that a full predictive distribution is available, from which any risk measure can be obtained. For example, Table 7 shows value-at-risk of the CDRs at 99.5% (where VaR at 99.5% is the negative of the 0.5th percentile of the distribution of the CDR).

Acc. Year	Future Calendar Year 1	2	3	4	5	6	7	8	9	Sqrt(Sum Squares)
1										
2	75,535									75,535
3	105,309	60,996								121,699
4	79,846	91,093	56,232							133,549
5	235,115	60,577	82,068	51,474						261,406
6	318,427	233,859	57,825	82,433	51,999					411,010
7	361,089	328,989	243,412	59,162	85,998	54,343				558,317
8	629,681	391,249	359,352	266,320	64,443	94,166	59,533			875,328
9	588,662	554,574	344,763	318,493	236,576	56,543	83,645	52,965		971,258
10	1,029,925	538,726	511,118	317,142	293,978	218,914	51,661	77,317	49,055	1,363,155
Total	1,778,968	1,177,727	885,178	607,736	428,681	267,503	128,557	96,764	49,055	2,447,095

Table 3: RMSEP of one-year ahead CDRs for all future years, together with the square root of the sum of squares.

Acc. Year	Expected Reserves	Bootstrap Std Dev	Bootstrap Std Dev %	One-Year CDR Std Dev	One-Year CDR Std Dev %
1	0	0	0%	0	0%
2	94,740	75,502	79.69%	75,502	79.69%
3	469,419	121,842	25.96%	105,505	22.48%
4	709,488	133,525	18.82%	79,900	11.26%
5	984,602	261,623	26.57%	235,182	23.89%
6	1,418,656	410,932	28.97%	318,385	22.44%
7	2,178,489	558,356	25.63%	360,974	16.57%
8	3,922,105	875,881	22.33%	629,558	16.05%
9	4,277,964	972,731	22.74%	588,355	13.75%
10	4,629,277	1,365,691	29.50%	1,030,505	22.26%
Total	18,684,738	2,448,700	13.11%	1,778,428	9.52%

Table 4: Bootstrap expected reserves, Bootstrap standard deviation, and One-Year CDR standard deviation.

Acc. Year	Expected Reserves	Bootstrap Std Dev	Bootstrap Std Dev %
1	0	0	0%
2	93,350	74,394	79.69%
3	459,844	118,704	25.81%
4	683,244	127,227	18.62%
5	937,363	252,853	26.97%
6	1,334,650	393,732	29.50%
7	2,033,376	527,020	25.92%
8	3,657,999	819,344	22.40%
9	3,954,984	894,036	22.61%
10	4,230,360	1,239,033	29.29%
Total	17,385,171	2,247,923	12.93%

Table 5: Bootstrap reserves, discounted at 3%.

Acc. Year	Future Calendar Year 1	2	3	4	5	6	7	8	9	Sqrt(Sum Squares)
1										
2	75,502									75,502
3	105,505	61,064								121,902
4	79,900	91,187	56,220							133,640
5	235,182	60,586	82,023	51,433						261,447
6	318,385	234,199	57,958	82,493	51,975					411,198
7	360,974	329,445	243,466	59,341	86,029	54,325				558,556
8	629,558	391,657	359,801	266,771	64,517	94,290	59,571			875,765
9	588,355	555,320	345,533	319,501	236,760	56,698	83,899	53,115		972,186
10	1,030,505	540,566	514,208	318,327	295,623	219,723	51,898	77,667	49,300	1,366,280
Total	1,778,428	1,180,046	887,767	609,443	430,087	267,978	128,952	97,132	49,300	2,449,520

Table 6: RMSEP of simulated one-year ahead CDRs for all future years, together with the square root of the sum of squares.

Acc. Year	Future Calendar Year 1	2	3	4	5	6	7	8	9
1									
2	184,986								
3	275,133	157,719							
4	207,422	235,915	145,566						
5	620,860	157,606	213,851	133,345					
6	850,057	620,832	150,694	214,796	135,117				
7	965,896	879,328	644,331	155,554	225,075	141,832			
8	1,710,846	1,045,126	964,395	707,002	168,242	247,251	155,974		
9	1,611,437	1,521,873	932,618	857,537	632,367	149,831	221,923	139,124	
10	2,989,241	1,508,467	1,442,823	881,416	811,586	602,299	139,917	208,894	132,911
Total	4,867,412	3,173,257	2,383,086	1,626,872	1,150,758	715,193	337,560	256,643	132,911

Table 7: Value-at-Risk @ 99.5% of simulated one-year ahead CDRs for all future years.

5. Solvency II Risk Margins using the Cost-of-Capital Approach

Solvency II stipulates that risk margins must be calculated using a cost-of-capital approach. The mechanics of the approach are straightforward. Given capital requirements for each future year as the reserves run-off, the risk margin is the sum of the discounted costs of capital, where the costs of capital are the capital requirements multiplied by the cost-of-capital rate. The cost-of-capital rate and discount rates that should be used in the calculations are prescribed and fixed. The main difficulty is obtaining the opening capital required, and the future capital requirements as the liabilities run-off. An example of the calculations for the Taylor and Ashe data is shown in Table 8.

Future Time	Projected Reserve	Projected Capital Requirement	Capital Profile	Projected Cost of Capital	Discounted Cost of Capital
0	17,381,602	4,867,412	100.0%	292,045	283,539
1	12,598,695	3,528,043	72.5%	211,683	199,531
2	8,735,034	2,446,093	50.3%	146,766	134,311
3	5,818,790	1,629,450	33.5%	97,767	86,865
4	3,834,408	1,073,759	22.1%	64,426	55,574
5	2,364,307	662,082	13.6%	39,725	33,269
6	1,239,956	347,228	7.1%	20,834	16,940
7	521,786	146,117	3.0%	8,767	6,921
8	85,285	23,883	0.5%	1,433	1,098
Total					818,047

Table 8: Cost-of-capital risk margin calculations using the “Best Estimate” basis.

The second column of Table 8 shows the expected reserves remaining in each future period, discounted to the start of that period at 3% (assuming payments occur half way through each year), and evaluated using the cash-flows from the chain ladder model applied deterministically. Discounting is used for consistency with Solvency II, where the “best estimate” is defined as the present value of future cash flows. The third column shows opening capital requirements, and the future capital requirements as the reserves run-off. The basis for these capital requirements is explained later in this section. The fourth column shows the capital requirements at each future period expressed as a percentage of the opening capital requirements. We call this the “capital profile”, and display it in Figure 1 (“best estimate” basis). The fifth column shows the costs-of-capital, calculated as the capital requirements multiplied by 6% (the Solvency II cost-of-capital rate). The final column shows the costs-of-capital discounted at 3% (chosen for simplicity), and the sum of the discounted costs-of-capital, giving a risk margin of 818,047.

For Solvency II, the opening capital requirements should come from an “internal model” (usually simulation-based), or from the Standard Formula, where the inputs are in respect of the legally obliged business only (*ie* no new business). Insurance risk, catastrophe risk (in respect of unearned but legally obliged business only), and reinsurance default risk should be considered, but asset risk is generally excluded. In the example used in Table 8, we have simply used value-at-risk at 99.5% of the total CDR over one year (shown in Table 7) as a suitable proxy; as such, we are only considering risk associated with running off the losses in the claims triangle.

There are several approaches that could be used for estimating the future capital requirements. The internal model or Standard Formula could be used repeatedly, each time adjusting the inputs to reflect the reducing reserve volumes over time. Alternatively, various approximations have been prescribed and can be used, where justified. A popular approximation is to calculate future capital requirements as a proportion of the opening capital requirement, where the proportions are calculated in accordance with the reduction in the best estimate of the reserves over time. That is, the opening capital requirement is multiplied by a capital profile, where the capital profile is obtained by dividing the best estimate of the reserves at each future year by the opening best estimate. With this "best estimate" basis, capital requirements as a proportion of outstanding reserves are constant at each time period. The best estimate basis was used to obtain the capital profile (and hence the future capital requirements) in Table 8, but it is not the only capital profile that could be used.

Although the best estimate basis is popular, it needs to be justified. For example, in a supervisory statement issued by the Prudential Regulation Authority (2014), the UK regulator stated that:

> *"Firms should not approximate the future Solvency Capital Requirements used to calculate the risk margin as proportional to the projected best estimate unless this has been shown not to lead to a material misstatement of technical provisions."*

Some insurance companies adjust the best estimate basis to increase capital requirements as a percentage of reserves as the reserves reduce, but there is no generally accepted approach for the adjustments. In this paper, we propose alternative methods to obtain both the opening capital requirement, and the capital profile for obtaining future capital requirements (which could then be used to justify, or otherwise, the best estimate basis).

Returning to the four items required for a theoretically sound basis for capital requirements (see Section 3), we need a risk profile, a risk measure, a risk tolerance criterion, and a time horizon. When obtaining capital requirements for risk margins, Merz and Wüthrich (2014) proposed using a distribution of the CDR at each future time period as the risk profile, and a multiple of the standard deviation or variance as risk measures, where a given multiplier controls the risk tolerance. Their formulae provided the standard deviations (RMSEPs) and variances (MSEPs) required. Merz and Wüthrich (2014) did not calculate the capital requirements nor risk margins explicitly, nor discuss the capital profile, but instead focused on how a sequence of risk margins calculated at each future period would compare to an opening risk margin, giving a "risk margin run-off pattern". By doing so, the risk tolerance multiplier adopted is irrelevant, since it cancels out. Merz and Wüthrich (2014) also used the (undiscounted) best estimate basis, and graphically compared the three risk margin run-off patterns.

In this paper, we focus on the capital profile for obtaining a risk margin (given an opening capital requirement), not the risk margin run-off pattern. Although graphs of the capital profile and risk margin run-off pattern look similar (starting at 100% and dropping to zero), the difference between them should be understood.

Like Merz and Wüthrich (2014), we consider the best estimate basis, and multiples of standard deviations of the distributions of a sequence of CDRs at each future time period (multiples of variance are omitted since using a scale based on units squared could be criticised). In addition, we consider VaR at 99.5%, since this is more consistent with Solvency II's choice of risk measure and risk tolerance criterion and is available from the simulation results. Implications for IFRS 17 are considered later.

Table 9 shows capital requirements under the three bases. For VaR at 99.5%, the values were taken from the final row of Table 7. The standard deviations for the standard deviation basis were taken from Table

6 and the respective multiplier was set such that the opening capital requirement is the same as for VaR at 99.5%. Similarly, for the best estimate basis, the opening capital requirement was set to be the same.

Future Time	"Best estimate"	St Dev	Var @ 99.5%
0	4,867,412	4,867,412	4,867,412
1	3,528,043	3,229,690	3,173,257
2	2,446,093	2,429,744	2,383,086
3	1,629,450	1,667,996	1,626,872
4	1,073,759	1,177,111	1,150,758
5	662,082	733,433	715,193
6	347,228	352,932	337,560
7	146,117	265,843	256,643
8	23,883	134,929	132,911
Risk Margin	818,047	822,321	810,816

Table 9: Future capital requirements under different bases.

Figure 1 shows the respective capital profiles, where for each basis, the capital requirements at each time period are expressed as a proportion of the opening capital requirement (the values can be calculated easily from Table 9). It should be noted that it was not necessary to select a multiplier for the standard deviation basis such that the opening capital requirements match in all cases, since the multiplier cancels out when calculating the capital profile. However, using the same opening capital requirement in each case provides a meaningful comparison of the respective risk margins, also shown in Table 9.

It is clear from Figure 1 that the capital requirements drop rapidly. The standard deviation and VaR at 99.5% bases are almost indistinguishable (in this example). It should be remembered that the total CDR is the sum of individual CDRs, and since their dependence is low, we expect the distribution of the total CDR to tend to normality, and therefore VaR at a fixed percentile to be a fixed multiple of the standard deviation. In this example, the best-estimate basis is close to the standard deviation and VaR bases, but that is not always the case with other data sets.

Given the similarity of the risk profiles, it is no surprise that the risk margins associated with all three bases in Table 9 are close to each other (using the same opening capital amounts).

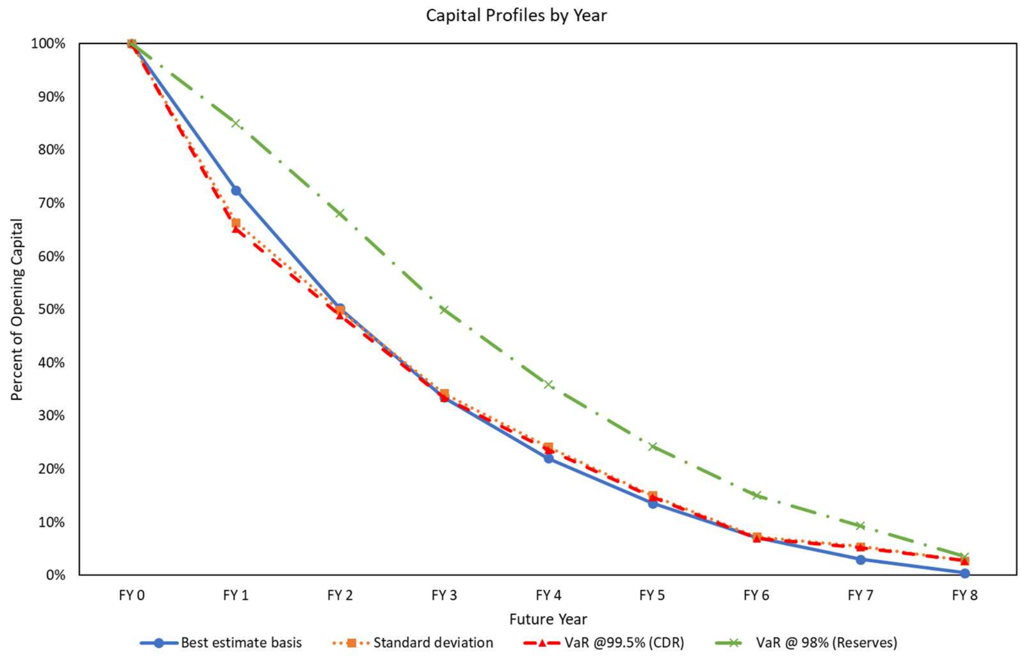

Figure 1: Capital profiles associated with Table 9, including VaR at 98% of the discounted outstanding reserves in each future year (see Section 6).

6. IFRS 17 Risk Adjustments

According to IFRS 17, an entity is required to estimate the discounted expected value (*ie* probability weighted mean) of the full range of possible outcomes of all future cash flows. In addition:

> *"An entity shall adjust the estimate of the present value of the future cash flows to reflect the compensation that the entity requires for bearing the uncertainty about the amount and timing of the cash flows that arises from non-financial risk."*

IFRS 17 is more principles based than Solvency II, and does not specify the techniques for calculating the "risk adjustment", which is just a risk margin by another name. Although IFRS 17 does not specify the techniques that should be used, it does state that:

> *"If the entity uses a technique other than the confidence level technique for determining the risk adjustment for non-financial risk, it shall disclose the technique used and the confidence level corresponding to the results of that technique."*

The "confidence level" is the percentile level of a value-at-risk measure, although the risk profile associated with the risk measure is not directly specified. However, since the basis of the measurement of insurance contracts under IFRS 17 is the discounted expected value of all future cash flows, we can infer that the most appropriate risk profile is the distribution of the discounted future cash-flows over their lifetime, which is consistent with the traditional actuarial view of reserve risk.

There are two main candidates for calculating the risk adjustment: a cost-of-capital approach, or a risk measure applied to the distribution of the discounted future cash-flows over their lifetime. A cost-of-capital approach is likely to be popular given its use for Solvency II, however it is not clear that the one-year view of risk under Solvency II is an appropriate basis for the underlying capital requirements under IFRS 17. Insurance entities will need to choose an appropriate basis for capital requirements in an IFRS

17 context if a cost-of-capital approach is used. We elaborate on this below. Furthermore, the cost-of-capital rate and discount rates will be entity specific and will likely be different from Solvency II.

The most straightforward approach to calculating a risk adjustment under IFRS 17 is to apply a risk measure to the distribution of the discounted future cash-flows. This implicitly takes the traditional lifetime view of reserve risk. In an Educational Monograph produced by the International Actuarial Association (2018), the following three risk measures were proposed:

1. VaR: Value-at-risk ("confidence level technique")
2. TVaR: Tail value-at-risk (conditional tail estimation)
3. PHT: Proportional hazards transform[5]

Clearly, there are other possibilities, including a multiple of the standard deviation. Given the choice of risk measure, the only other input is the associated risk tolerance level (that is, percentile level for VaR or TVaR, and proportional hazards parameter for PHT). The risk adjustment is then the risk measure evaluated at the selected risk tolerance level less the mean (sometimes called a *relative* risk measure).

Where a distribution of the future cash-flows has been obtained using simulation techniques, the three risk measures listed above can all be expressed as a weighted average of the simulations, sorted into ascending order. Given a percentile level, VaR has a single weight at a single simulation, and zero elsewhere. TVaR has equal weights above a given percentile level, and zero elsewhere, and PHT has a non-zero weight for each simulation, with increasing weights as the simulation values increase (see Appendix 2 and Figure 2).

Value-at-risk is easy to explain to a non-technical audience, and has the advantage of simplicity, but since it is based on a single simulation, could be prone to simulation error (although there are techniques to mitigate this). It has a range from the minimum simulated value to the maximum, as the percentile level changes. It has been criticised since it does not adequately recognise skewness nor extremes, nor is it a coherent risk measure (as defined by Artzner *et al*. (1999)) since it does not obey the sub-additivity property.

Tail value-at-risk is straightforward to calculate. It has a range from the mean to the maximum simulated value as the percentile level changes, and is better at recognising skewness and extremes since all values above a given percentile level are included in the calculation. It also has the advantage of being a coherent risk measure, and can be used for allocations of risk to sub-groups, where distributions have been combined before the risk measure is applied.

The proportional hazards transform, introduced by Wang (1995) in the context of insurance, also has a range from the mean to the maximum simulated value as the associated parameter increases from 1 to infinity. Although the PH parameter does not have a natural interpretation, the PHT itself can be interpreted as a risk-adjusted expected value (see Appendix 2). It could be argued that the PHT is even better at recognising skewness and extremes since the weights increase as the simulation values increase, unlike TVaR where the weights are constant above a given percentile level. It is also a coherent risk measure, and again can be used for allocations of risk to sub-groups. Wang's proportional hazards transform was first mentioned as a way of estimating a "prudential margin" for claims reserves in Wright (1997).

Table 10 shows risk adjustments obtained by applying VaR, TVaR and PHT risk measures to the distribution of discounted total outstanding claims from bootstrapping Mack's model. For VaR, the 75% "confidence level" (percentile) was chosen, to be consistent with the requirements of the Australian

[5] For a description of the proportional hazards transform, see Appendix 2

Prudential Regulation Authority (see APRA (2015)). For TVaR, the 40th percentile was used, to give a risk adjustment that is approximately the same as VaR at 75%. Likewise, a proportional hazards parameter of 1.85 was used to give a similar risk adjustment. The risk adjustments are higher than the cost-of-capital risk margins shown in Table 9. A graph of the TVaR and PHT weights are shown in Figure 2.

Whatever technique is used to calculate the risk adjustment for IFRS 17, it is straightforward to obtain the equivalent "confidence level" from a simulated distribution of future cash-flows, as required for disclosure purposes. For example, using the "best estimate basis" cost-of-capital risk margin from Table 9 as the risk adjustment, Table 11 shows that the equivalent "confidence level" is 65.3%. The analogous risk tolerance levels for the TVaR and PHT measures are 21.7% and 1.44 respectively.

	Value-at-Risk	Tail Value-at-Risk	Proportional Hazards Transform
Risk Tolerance Level	75%	40%	1.85
Risk Adjustment	1,468,622	1,431,645	1,455,235
Best Estimate (Disc)	17,385,171	17,385,171	17,385,171
Total	18,853,794	18,816,816	18,840,406
Risk Adjustment %	8.45%	8.23%	8.37%

Table 10: Risk adjustments using VaR, TVaR and PHT.

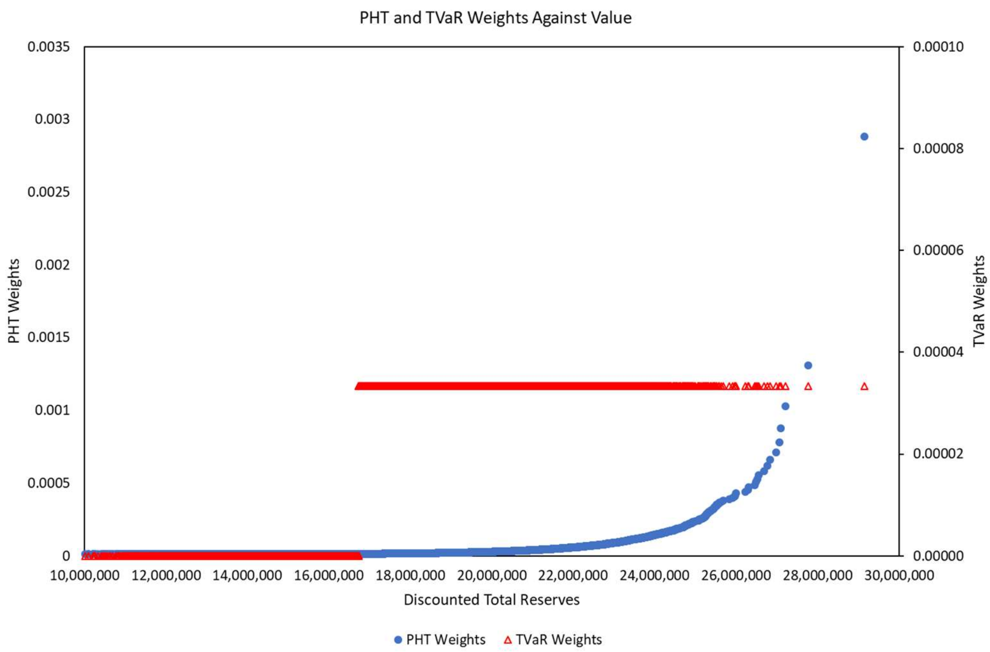

Figure 2: TVaR and PHT weights against simulated total discounted reserves.

	Cost-of-Capital	Value-at-Risk	Tail Value-at-Risk	Proportional Hazards Transform
Risk Tolerance Level		65.3%	21.7%	1.44
Risk Adjustment	818,047	818,062	818,048	818,047
Best Estimate (Disc)	17,381,602	17,385,171	17,385,171	17,385,171
Risk Measure	18,199,649	18,203,233	18,203,219	18,203,218
Risk Adjustment %	4.71%	4.71%	4.71%	4.71%

Table 11: Equivalent risk tolerance levels required to obtain the cost-of-capital risk margin.

For this example, the equivalent confidence level of the Solvency II cost-of-capital risk margin looks quite low. The inputs of the cost-of-capital risk margin can be scenario tested to investigate the difference. For example, using a cost-of-capital rate of 11% instead of 6% would bring the equivalent confidence level up to 75%. Although equality can be ensured by manipulating the control parameters, there is no fundamental significance to such an equality. Nevertheless, it is informative to investigate differences between the approaches.

Instead of viewing the Solvency II cost-of-capital risk margin as too low, an alternative explanation is that the distribution given by Mack's model is too wide, and a narrower distribution would give a higher equivalent confidence level. It would be important in practice to investigate this further, for example, identifying outliers or specific data points unduly influencing the width of the distribution. Furthermore, Mack's model may not be optimal; an alternative model might improve the precision of the forecasts.

Returning to the basis for capital requirements if a cost-of-capital approach is used, an alternative for IFRS 17 is to calculate capital requirements from distributions of the outstanding future cash-flows at each future time period, not distributions of the (incremental) CDRs. That is, insurance entities need to choose whether capital requirements should be based on the lifetime view of risk, or the one-year view of Solvency II. The remainder of this section considers cost-of-capital risk adjustments for IFRS 17 under the lifetime view of risk.

Estimating capital requirements under the lifetime view of risk requires distributions of outstanding payments at each time period *given data up to that time period*, which is not available at calendar period n. However, bootstrapping provides a simulated distribution of all future cash-flows *given data up to calendar period* n, which can be used to approximate the distributions required at each future time period.

From the perspective of calendar period n, the second column of Table 12 shows the expected value of the distribution of discounted reserves remaining at each future year (discounted at 3%, and assuming payments occur mid-way through each year), which can be compared with the deterministic values in the second column of Table 8. The third and fourth columns show the standard deviation of discounted and undiscounted reserves respectively. The fifth column shows the VaR at 98% of the discounted reserves, and the sixth column shows the VaR at 99.5%. The risk tolerance level of 98% was selected such that the value in the first year is close to the opening capital requirement in Table 9, allowing corresponding risk adjustments to be compared.

Capital profiles associated with each of the columns in Table 12 are not shown but are easily calculated from the values in the table. Then using VaR at 98% of the initial distribution of discounted reserves as the opening capital requirement in all cases except the final column, the corresponding cost-of-capital risk adjustments are shown in the final row of Table 12.

Future Yr	Expected (Disc. Reserve)	St Dev (Disc. Reserves)	St Dev (Undisc. Reserves)	VaR @ 98.0% (Disc. Reserves)	VaR @ 99.5% (Disc. Reserves)
0	17,385,171	2,247,923	2,448,700	4,867,402	6,241,677
1	12,601,574	1,900,311	2,044,912	4,142,042	5,329,406
2	8,737,791	1,508,462	1,607,022	3,313,603	4,289,988
3	5,819,876	1,101,628	1,165,327	2,432,676	3,155,930
4	3,835,089	786,505	825,688	1,747,725	2,277,027
5	2,364,929	528,488	550,999	1,181,030	1,538,848
6	1,240,343	327,295	338,944	732,636	952,069
7	522,078	198,068	202,648	454,451	594,302
8	85,381	74,825	75,939	170,089	221,198
Risk Adjustment	818,069	1,030,736	1,011,176	1,044,314	1,348,743

Table 12: Expected value, standard deviation and value-at-risk of the discounted reserves at each future period. Also standard deviation of the undiscounted reserves. Cost-of-capital risk adjustments are shown for each basis.

Using VaR at 98% for the capital requirements in all years gives a risk adjustment of 1,044,314 (using a cost-of-capital rate of 6%), with an equivalent "confidence level" of 68.9%. Although this is higher than the value shown in Table 11, it is still considerably below the risk adjustments shown in Table 10. Given the same opening capital requirement, the risk adjustment associated with the standard deviation basis (discounted) is similar to the VaR basis at 98%, indicating that, again, the capital profiles associated with these two bases are almost indistinguishable. The capital profile using the VaR basis at 98% applied to the distribution of the discounted reserves is included in Figure 1, and appears to the right of the other profiles (based on the CDR), as expected given the higher risk adjustment.

Using VaR at 99.5% for the capital requirements in all years instead gives a risk adjustment of 1,348,743, which is closer to the risk adjustments shown in Table 10, but still below them. The capital profile associated with this method is very close to the equivalent profile using VaR at 98%; the higher risk adjustment is essentially a result of the higher opening capital requirement.

In Section 8.1 of Merz and Wüthrich (2014), the standard deviations of the undiscounted outstanding payments *given data up to each time period* were approximated using the square root of the reverse sum of future CDR MSEPs at each period. The results of that approximation are shown in the second column of Table 13 (using the final row of Table 3). The third column shows the analogous results using simulation, taking the standard deviation of the reverse sum of simulated CDRs. The fourth and fifth columns show the VaR of the reverse sum of simulated CDRs at the 97.1% and 99.5% levels. The risk tolerance level of 97.1% was selected such that the value in the first year is close to the opening capital requirement in Table 9, again allowing corresponding risk adjustments to be compared.

Future Yr	----------------------- Reverse Sum of CDRs -----------------------			
	Merz-Wüthrich Analytic RMSEP	St Dev (Simulated)	VaR @ 97.1%	VaR @ 99.5%
0	2,447,095	2,448,700	4,867,430	6,822,322
1	1,680,341	1,684,741	3,297,114	4,601,011
2	1,198,543	1,202,430	2,342,761	3,269,502
3	808,063	810,293	1,565,416	2,171,462
4	532,562	534,128	1,028,854	1,430,349
5	315,998	317,089	609,497	844,694
6	168,216	168,878	322,039	443,098
7	108,489	108,943	207,041	288,425
8	49,055	49,300	94,456	132,911
Risk Adjustment	806,591	807,802	795,836	1,110,778

Table 13: The square root of the reverse sum of the CDR MSEPs, together with the standard deviation and VaR of the reverse sum of simulated CDRs. Cost-of-capital risk margins are shown for each basis.

Using VaR at 97.1% of the reverse sum of CDRs for the capital requirements in all years gives a risk adjustment of 795,836 (using a cost-of-capital rate of 6%), which is close to the values shown in Table 9 for VaR at 99.5% applied to the distribution of incremental one-year CDRs (indicating that the respective capital profiles must be close in this example). Using the same opening capital requirement, but capital profiles obtained from the standard deviation of the reverse sum of CDRs (or the analytic equivalent) again gives similar risk adjustments. Using VaR at 99.5% of the reverse sum of CDRs for the capital requirements in all years gives a higher risk adjustment, as expected.

It should be noted that the distribution of the discounted outstanding future cash-flows *given data up to calendar period* n is the appropriate risk profile for the opening capital requirement in a cost-of-capital risk adjustment calculation for IFRS 17 (under the lifetime view of risk); it is only the remaining capital requirements that would ideally be based on data up to each time period ($> n$), not on the data available up to calendar period n only. It should also be noted that it is not clear what risk tolerance level is appropriate under a cost-of-capital risk adjustment for IFRS 17; the choice is entity specific and is not prescribed.

Appendix 3 shows that the square root of the reverse sum of CDR MSEPs is a prudent approximation to the expected standard deviation of the (undiscounted) outstanding payments at each time period, *given data up to each time period*, and is more appropriate under IFRS 17 than the standard deviation of the (undiscounted) outstanding payments at each time period, *viewed from the perspective of time period* n (except at time period n, when the two will be equivalent). Appendix 3 also shows that we expect the square root of the reverse sum of future CDR MSEPs at each period to be less than (or equal to) the standard deviation of the undiscounted outstanding payments *given data up to calendar period* n, and comparison between the second (or third) column of Table 13 and the fourth column of Table 12 demonstrates that this is the case.

Given the same opening capital requirement, we can infer from Appendix 3 that a cost-of-capital risk adjustment associated with the (undiscounted) outstanding payments at each time period, *viewed from the perspective of time period* n will be prudent compared to a cost-of-capital risk adjustment associated with the reverse sum of CDRs (although as shown in Tables 9, 10 and 11, both may be lower than a risk adjustment calculated simply from a risk measure applied to the distribution of outstanding payments given data up to calendar period n).

It should be noted that the recursive re-reserving approach is computationally expensive. Therefore, although it may be better to use a capital profile obtained from a risk measure applied to a distribution of the reverse sum of CDRs to estimate future capital requirements for IFRS 17 (under the lifetime view of risk), using the distribution of the discounted outstanding future cash-flows *given data up to calendar period* n may be expedient (with the risk tolerance level being used to control the level of prudence).

Obtaining a risk adjustment for IFRS 17 using a cost-of-capital approach seems more problematic than simply using a risk measure applied to a distribution of the discounted future cash-flows in any event.

7. Discussion

In this paper, various concepts associated with the quantification of reserve risk have been connected. The analytic formula-based approaches of Mack (1993) for the lifetime view of reserve risk, and Merz and Wüthrich (2008) for the one-year view of Solvency II, have been compared to simulation-based results obtained by bootstrapping Mack's model, supplemented with the re-reserving approach. Furthermore, the lifetime and one-year views were brought together by considering a sequence of one-year views until the liabilities are extinguished. Again, this was considered analytically, using Merz and Wüthrich (2014), and using a simulation-based approach by applying re-reserving recursively.

The assumptions of Mack (1993) were used throughout this paper for simplicity, but it is not the only model that could be used. The MSEPs of the CDRs over multiple years for a range of models are considered in Wüthrich and Merz (2015). Analytic formulae for the MSEPs of the CDRs over multiple years for the additive model of Mack (2002) have been derived by Diers and Linde (2013): it is possible to apply a recursive simulation-based re-reserving approach as well since it is straightforward to bootstrap the additive model of Mack (2002). Simulation studies (by the first author of this paper) for the over-dispersed Poisson model of Renshaw and Verrall (1998) also demonstrate that the lifetime view of risk can be partitioned into a sequence of uncorrelated one-year views when bootstrapped using England (2002) and applying the recursive re-reserving approach. We welcome similar investigations with other models.

As Ohlsson and Lauzeningks (2008b, 2009) observe, the simulation approach is useful since it can be generalised. For example, the principles behind the re-reserving approach can be used when curves are fitted to the chain-ladder factors to allow extrapolation into the tail, or when non-constant scale parameters and curves are used with the over-dispersed Poisson model. Generalisations of the chain ladder model are also mentioned in Diers *et al*. (2016), where simulation methods are recommended for quantifying the various measures of uncertainty, due to difficulties with the analytic approach. With the simulation approach, it is important that the original model is bootstrapped appropriately when obtaining the forecasts, and that the same model is fitted to each simulation within the re-reserving procedure. There is no guarantee, however, that the expected CDR will be zero for each future year (this should be checked). When simulating, there is also no guarantee that the lifetime view of risk can be partitioned into a sequence of independent one-year views. Where a dependence is observed, it is more informative to consider the CDR cumulatively instead of incrementally. That is, consider the variance of $\widehat{U}_i^{(n)} - \widehat{U}_{i,s}^{(n+k)}$ as k increases, which will naturally converge on the lifetime view of risk. The cumulative emergence of the CDR was considered by Diers *et al*. (2013), Röhr (2016), and Section 6.1 of Merz and Wüthrich (2014).

In the context of Solvency II, the results of Sections 2 and 3 were used to derive different bases for a capital profile that can be applied to an opening capital requirement to obtain future capital requirements for a cost-of-capital risk margin calculation. These can be compared to the "best estimate" basis commonly used as an approximation. It was shown that the simulation approach can be used to obtain a VaR measure, and that this gives a very similar capital profile to a standard deviation measure. A problem with the simulation-based approach is that very many simulations are required to reduce simulation error in a value-at-risk measure at extreme percentiles. A stable measure of standard deviation requires far fewer simulations. We can therefore summarise our findings in the following recommendations:

1. For Solvency II at least, VaR at 99.5% applied to a sequence of distributions of the 1 year-ahead CDRs is an appropriate risk measure for risk margin capital requirements.
2. A recursive re-reserving approach can be used to obtain the distributions.
3. VaR at 99.5% is an extreme percentile, and requires very many simulations for stability, but a stable measure of standard deviation requires far fewer simulations.
4. Since capital profiles given by standard deviation and VaR measures are almost indistinguishable, a standard deviation measure could be used as a proxy, instead of VaR at 99.5%.
5. When using the model of Mack (1993), the analytic formulae of Merz and Wüthrich (2014) giving the standard deviations of the CDRs may be sufficient without requiring simulation at all.
6. Any opening capital amount (that can be justified) can be "plugged-in", for example using an internal capital model or the Solvency II Standard Formula.
7. Then a capital profile obtained using risk measures applied to a sequence of distributions of the CDR can be used to estimate future capital requirements for the risk margin calculation.
8. This will be more justifiable than a capital profile obtained using "best estimates", or it could justify using a "best estimate" profile as a proxy.

A note of caution should be given concerning extreme percentiles. In this paper, results have been shown for various quantities at a variety of extreme percentiles, conditional on a given model. A model is merely a representation of reality with a small number of parameters, and the extreme percentiles should be viewed with an appropriate level of scepticism. This is true for any model, including economic scenario generators, underwriting loss models, and natural catastrophe models, commonly used to assist estimating Solvency II capital requirements.

The estimation techniques used to determine the risk adjustment under IFRS 17 are not specified. In addition to a cost-of-capital approach, a risk measure applied to a distribution of the discounted future cash-flows is clearly a suitable candidate. Under the disclosure requirements, such a distribution is required anyway since whatever technique is used, the equivalent "confidence level" must be disclosed. In this paper, three risk measures were considered, which are easily applied to a simulated distribution.

It should be noted that IFRS 17 requires a risk adjustment in respect of all fulfilment cash-flows, including expenses, and other outward costs, and reinstatement premiums and other recoveries. In the example in this paper, we have only considered a payments triangle, and a risk adjustment calculated solely from that would either need modifying to allow for other cash-flows involved in fulfilling the insurance contracts, or the triangle would need to include all relevant cash-flows. We have also assumed that provisions for unexpired risks will be calculated using the Premium Allocation Approach (PAA): if the Building Block Approach (BBA) is used instead, then a distribution of the discounted future cash-flows in respect of those risks will need to be included.

IFRS 17 risk adjustments are also required on a gross and reinsurance basis (taking account of reinsurance credit risk). Clearly, it is the net position that is most relevant for the interpretation of an insurance entity's financial position, so it seems appropriate to estimate risk adjustments from distributions of gross and net discounted future cash-flows, then taking the difference as the reinsurance risk adjustment. Reinsurance modelling to obtain an accurate distribution of the net discounted future cash-flows (together with an assessment of credit risk) could be complex. In particular, the current actuarial practice of applying an approximate net-to-gross ratio looks increasingly inadequate (where non-proportional reinsurance treaties exist), and triangle methods for attritional claims may need to be supplemented by individual claims modelling for large claims, with accurate reinsurance modelling.

Furthermore, risk adjustments are required for groups of contracts, not just at the aggregate entity level (or holding company level for a multinational group), which raises questions about allocation of risk and diversification. Again, a simulation framework can be used (using copulae to apply dependencies when aggregating), but the issues are complex and beyond the scope of this paper.

If the cost-of-capital technique is used for IFRS 17 risk adjustments, insurance entities will need to choose between the one-year and lifetime views of risk when estimating capital requirements. Solvency II considers the one-year view of risk, whereas the lifetime view of risk could be used under IFRS 17. A distribution of the remaining total cash-flows at each future time period is appropriate as a basis for estimating capital requirements under the lifetime view (although as discussed in Section 6 and Appendix 3, the time perspective becomes important). Furthermore, cost-of-capital and discount rates are entity specific under IFRS 17, but prescribed under Solvency II. The cost-of-capital technique is considerably more complex than simply applying a risk measure to a distribution of discounted future cash-flows, and requires more parameters to select and justify: it requires an opening capital requirement, future capital requirements, a cost-of-capital rate and a yield curve for discounting. Since the equivalent "confidence level" is required anyway under IFRS 17, it questions why the cost-of-capital method would be used at all. A distribution of discounted future cash-flows is required for the equivalent confidence level (which implicitly takes the lifetime view of risk), so it seems more straightforward to calculate IFRS 17 risk adjustments simply from a risk measure applied to that distribution. Given the distribution and choice of risk measure, the only input to select and justify is the entity specific risk tolerance level.

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Appendix 1 – Bootstrapping Mack's Model

Mack's model can be bootstrapped by completing the following steps, which can be performed without difficulty in a spreadsheet:

1. Obtain the ratios $f_{i,j}$ and standard chain-ladder development factors $\hat{\lambda}_j$ from cumulative data using equation (3). Note that the weights $w_{i,j}$ are the denominator of the ratios $f_{i,j}$.

2. Calculate the unscaled Pearson residuals using:

$$r_P(f_{i,j}, \hat{\lambda}_j, w_{i,j}) = \sqrt{w_{i,j}}(f_{i,j} - \hat{\lambda}_j)$$

3. Adjust the Pearson residuals using

$$r_P^{adj}(f_{i,j}, \hat{\lambda}_j, w_{i,j}) = \sqrt{\frac{n_j}{n_j - 1}}\sqrt{w_{i,j}}(f_{i,j} - \hat{\lambda}_j)$$

   where $n_j = n - j - 1$, being the number of residuals at development period j, $0 \leq j \leq J - 2$. This is required to replicate the bias correction in Mack (1993) using the analytic approach (see the Appendix of England and Verrall (2006) for further details).

4. Calculate the parameters σ^2 from Mack (1993), where

$$\hat{\sigma}_j^2 = \frac{1}{n_j - 1}\sum_{i=1}^{n_j} w_{i,j}\left(f_{i,j} - \hat{\lambda}_j\right)^2 = \frac{1}{n_j}\sum_{i=1}^{n_j} \frac{n_j}{n_j - 1} w_{i,j}\left(f_{i,j} - \hat{\lambda}_j\right)^2$$

   That is, the parameters σ^2 can be calculated as the sum of the unscaled Pearson residuals squared divided by the number of residuals minus 1, or simply the average of the bias-adjusted unscaled residuals squared. Calculate the final unknown parameter σ_{J-1}^2 as described in Section 2.2.

5. Calculate scaled residuals using:

$$r_{PS}(f_{i,j}, \hat{\lambda}_j, w_{i,j}, \hat{\sigma}_j) = \frac{\sqrt{w_{i,j}}(f_{i,j} - \hat{\lambda}_j)}{\hat{\sigma}_j} \qquad (A1.1)$$

6. Calculate scaled bias-adjusted residuals using:

$$r_{PS}^{adj}(f_{i,j}, \hat{\lambda}_j, w_{i,j}, \hat{\sigma}_j) = \frac{\sqrt{\frac{n_j}{n_j - 1}}\sqrt{w_{i,j}}(f_{i,j} - \hat{\lambda}_j)}{\hat{\sigma}_j}$$

7. Begin iterative loop, to be repeated *N* times (*N*=100,000, say):

   a. Resample the scaled bias adjusted residuals with replacement, creating a new triangle of residuals. It is important that the scaled bias-adjusted residuals are used at this stage to replicate the bias correction from Mack (1993) used in the analytic formulae.

   b. For each cell in the past triangle, solve equation (A1.1) for f, giving a set of pseudo-ratios. That is:

   $$f_{i,j}^B = r_{i,j}^B \frac{\hat{\sigma}_j}{\sqrt{w_{i,j}}} + \hat{\lambda}_j$$

   where $r_{i,j}^B$ are the resampled scaled bias-adjusted residuals from the previous step.

   c. Obtain new chain-ladder development factors using equation (3) applied to the set of pseudo-ratios. That is:

   $$\tilde{\lambda}_j = \frac{\sum_{i=1}^{n-j+1} w_{i,j} f_{i,j}^B}{\sum_{i=1}^{n-j+1} w_{i,j}}$$

   It should be noted that the same values of $w_{i,j}$ are used for each bootstrap iteration, being the weights defined at step (1).

   d. Starting from the latest cumulative claims, forecast one-step-ahead either by re-sampling from the residuals again, or by drawing a sample from an assumed parametric process distribution. Using the non-parametric approach, for $i = 2,3, \dots, n$:

   $$\tilde{C}_{i,n-i+1} | C_{i,n-i} = \tilde{\lambda}_{n-i} C_{i,n-i} + r_{i,n-i+1}^F \hat{\sigma}_{n-i} \sqrt{C_{i,n-i}}$$

   where r^F is resampled with replacement from the scaled bias-adjusted residuals.

   Using the parametric approach instead, it is necessary to select a parametric process distribution. Since it is desirable for the cumulative amounts to be positive, a Gamma or lognormal distribution would be reasonable choices. Using a Gamma distribution with parameters (a, b):

   $$\tilde{C}_{i,n-i+1} | C_{i,n-i} \sim Gamma(a_{i,n-i+1}, b_{i,n-i+1})$$

   where a and b are selected such that:

   $$E[\tilde{C}_{i,n-i+1} | C_{i,n-i}] = \tilde{\lambda}_{n-i} C_{i,n-i}$$

   $$Var[\tilde{C}_{i,n-i+1} | C_{i,n-i}] = \hat{\sigma}_{n-i}^2 C_{i,n-i}$$

   Note that a and b are different for each simulation.

   e. Continue forecasting recursively from the cumulative amounts simulated at the previous step. That is, with a non-parametric approach, the two-steps-ahead forecasts, and beyond, can be obtained using:

$$\tilde{C}_{i,j+1}|\tilde{C}_{i,j} = \tilde{\lambda}_j\tilde{C}_{i,j} + r^F_{i,j+1}\hat{\sigma}_j\sqrt{\tilde{C}_{i,j}}$$

for $i = 3,4, \dots, n$ and $j = n - i + 2, n - i + 3, \dots, J - 1$.

Alternatively, with a parametric approach and a Gamma distribution, the two-steps-ahead forecasts, and beyond, can be obtained using:

$$\tilde{C}_{i,j+1}|\tilde{C}_{i,j} \sim Gamma\left(a_{i,j+1}, b_{i,j+1}\right)$$

where a and b are selected such that:

$$E\left[\tilde{C}_{i,j+1}|\tilde{C}_{i,j}\right] = \tilde{\lambda}_j\tilde{C}_{i,j}$$
$$Var\left[\tilde{C}_{i,j+1}|\tilde{C}_{i,j}\right] = \hat{\sigma}_j^2\tilde{C}_{i,j}$$

Note that a and b are different for each simulation.

f. Calculate the incremental amounts by differencing in the usual way to give the simulated cash-flows (which can be discounted if required).

g. Sum the simulated incremental amounts in the future triangle by origin year and overall to give the origin year and total reserve estimates.

h. Store the results, and return to the start of the iterative loop until *N* bootstrap replications have been created and stored.

The set of stored results forms the predictive distribution. The mean of the stored results should be compared to the standard chain-ladder reserve estimates to check for errors. The standard deviation of the stored results gives an estimate of the prediction error (RMSEP) analogous to Mack's model.

Note that when using a Gamma or lognormal process distribution, the incremental amounts can be negative since the cumulative claims at development period $j + 1$ can be less than the cumulative claims at development period j while still being positive. This is a characteristic of bootstrapping Mack's model that can be useful when claims triangles include negative incremental amounts.

Note also that using a Gamma or lognormal distribution for the cumulative amounts does not imply that the reserves are Gamma or lognormally distributed, or even skewed. The reserves are the sum of incremental payments, and even if the distributions of incremental payments are themselves markedly skewed, the distribution of the sum of incremental payments will tend towards normality unless the dependencies are very high. With Mack's model (and many other stochastic reserving models), dependencies appear through the estimation uncertainty component only and are generally not high enough to result in a distribution of total reserves that is markedly skewed.

Appendix 2 – The Proportional Hazards Transform

According to Wang (1995):

Given a non-negative loss random variable *X*, with survival function $S_X(u)$ such that

$S_X(u) = Pr\{X > u\} = 1 - Pr\{X \leq u\}$

Then $E[X] = \int_0^\infty S_X(u)du$

The *PH-mean* with parameter ρ is given by $H_\rho(X)$ where

$$H_\rho(X) = \int_0^\infty [S_X(u)]^{1/\rho} du \qquad \text{(for } \rho \geq 1\text{)}$$

where the *PH-mean* refers to the expected value under the transformed distribution.

To apply Wang's proportional hazards transform to a set of N non-negative simulated values, first order the simulations from the minimum, X_1, to the maximum, X_N.

Then $\hat{S}(X_s) = 1 - s/N$ for ordered simulation s, and the transformed empirical survival function $t\left(\hat{S}(X_s)\right) = \left(1 - s/N\right)^{1/\rho}$.

Calculate *PH-weights*:

$$\omega_1 = 1 - t\left(\hat{S}(X_1)\right)$$
$$\omega_s = t\left(\hat{S}(X_s)\right) - t\left(\hat{S}(X_{s-1})\right) \qquad s \geq 2$$

Then the estimated *PH-mean* is given by:

$$\widehat{H}_\rho(X) = \frac{\sum_{s=1}^N \omega_s X_s}{\sum_{s=1}^N \omega_s} = \sum_{s=1}^N \omega_s X_s$$

since $\sum_{s=1}^N \omega_s = 1$. Note that the weights are independent of the magnitude of the values X, depending only on their order.

Note also that the proportional hazards transform $Z \mapsto t(z) = z^{1/\rho}$ is concave for $\rho > 1$, and provides a convex game (see Delbaen, F (2000)).

Appendix 3 – IFRS 17 and the Standard Deviation of Outstanding Payments at Each Future Time Period

We denote incremental payments in origin period i and development period j by $I_{i,j}$ and cumulative payments by $C_{i,j}$ for $i = 1, \dots, n; j = 0, \dots, J$. The available data at time $\tau \geq n$ is denoted by $\mathcal{D}_\tau$, and $n = J + 1$ (for simplicity).

The future cash-flows at time $\tau \geq n$ over the remaining lifetime of the liabilities are given by:

$$\sum_{i+j>\tau} I_{i,j} = \sum_{i=\tau-J+1}^{n} \sum_{j=\tau-i+1}^{J} I_{i,j} = \sum_{i=\tau-J+1}^{n} C_{i,J} - C_{i,\tau-i}$$

The total uncertainty at time $\tau \geq n$, measured by the variance, is given by:

$$Var\left(\sum_{i+j>\tau} I_{i,j} | \mathcal{D}_\tau\right) = Var\left(\sum_{i=\tau-J+1}^{n} \left(C_{i,J} - C_{i,\tau-i}\right) | \mathcal{D}_\tau\right) = Var\left(\sum_{i=\tau-J+1}^{n} C_{i,J} | \mathcal{D}_\tau\right)$$

Since the value-at-risk measure (in this context) is approximately a fixed multiple of standard deviation (see Section 5), ideally, we would like to determine the risk measure:

$$\varrho_\tau = \left(Var\left(\sum_{i+j>\tau} I_{i,j} | \mathcal{D}_\tau\right)\right)^{1/2} = \left(Var\left(\sum_{i=\tau-J+1}^{n} C_{i,J} | \mathcal{D}_\tau\right)\right)^{1/2}$$

for the future cash-flows at time $\tau \geq n$ over their remaining lifetime. Viewed from time n, this provides expected risk measure:

$$E(\varrho_\tau | \mathcal{D}_n) = E\left[\left(Var\left(\sum_{i+j>\tau} I_{i,j} | \mathcal{D}_\tau\right)\right)^{1/2} | \mathcal{D}_n\right]$$

Adopting Jensen's inequality:

$$E(\varrho_\tau | \mathcal{D}_n) \leq \left(E\left[Var\left(\sum_{i+j>\tau} I_{i,j} | \mathcal{D}_\tau\right) | \mathcal{D}_n\right]\right)^{1/2}$$

and using the tower property of conditional expectations we obtain the following lemma:

Lemma A3.1: Assuming second moments exist, we have:

$$E(\varrho_\tau | \mathcal{D}_n) \leq \left(E\left[Var\left(\sum_{i+j>\tau} I_{i,j} | \mathcal{D}_\tau\right) | \mathcal{D}_n\right]\right)^{1/2} \leq \left(Var\left(\sum_{i+j>\tau} I_{i,j} | \mathcal{D}_n\right)\right)^{1/2}$$

From Section 6 of Merz and Wüthrich (2014), choosing a fixed origin period $i \in \{\tau - J + 1, \dots, n\}$, we have:

$$E\left[Var\left(\sum_{j=\tau-i+1}^{J} I_{i,j} | \mathcal{D}_\tau\right) | \mathcal{D}_n\right] = E\left[Var\left(C_{i,J} | \mathcal{D}_\tau\right) | \mathcal{D}_n\right]$$

$$= E\left[Var\left(\sum_{k=\tau+1}^{i+J} CDR_i^{(k)} | \mathcal{D}_\tau\right) | \mathcal{D}_n\right]$$

Since the claims development results are uncorrelated (not necessarily independent),

$$E\left[Var\left(\sum_{j=\tau-i+1}^{J} I_{i,j} | \mathcal{D}_\tau\right) | \mathcal{D}_n\right] = \sum_{k=\tau+1}^{i+J} E\left[Var\left(CDR_i^{(k)} | \mathcal{D}_\tau\right) | \mathcal{D}_n\right]$$

Then since the expected value of the CDRs is zero, and using the tower property of conditional expectations, the following corollary is obtained (see Merz and Wüthrich (2014) for further details):

Corollary A3.2: Assuming second moments exist, for the standard deviation risk measure $\varrho_{i,\tau}$ restricted to origin period $i \in \{\tau - J + 1, \dots, n\}$, we have:

$$E\left(\varrho_{i,\tau}|\mathcal{D}_n\right) \leq \left(E\left[Var\left(\sum_{j=\tau-i+1}^{J} I_{i,j}\,|\mathcal{D}_\tau\right)|\mathcal{D}_n\right]\right)^{1/2} = \left(\sum_{k=\tau+1}^{i+J} Var\left(CDR_i^{(k)}|\mathcal{D}_n\right)\right)^{1/2}$$

$$\leq \left(Var\left(\sum_{j=\tau-i+1}^{J} I_{i,j}\,|\mathcal{D}_n\right)\right)^{1/2}$$

where

$$\varrho_{i,\tau} = \left(Var\left(\sum_{j=\tau-i+1}^{J} I_{i,j}\,|\mathcal{D}_\tau\right)\right)^{1/2}$$

In the Bayesian representation of the chain ladder model of Merz and Wuthrich (2014), for $k \geq 0$ and $i \geq n+k-J$, $Var\left(CDR_i^{(n+k+1)}|\mathcal{D}_n\right)$ is estimated by $MSEP\left[CDR_i^{(n+k+1)}|\mathcal{D}_n\right]$, shown in Section 3.3.