Simulation of Investment Returns for a Money Purchase Fund

by

M Z KHORASANEE

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Department of Actuarial Science and Statistics
City University
London

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Abstract

This paper examines the problem of investment risk in money purchase pension schemes.

The disadvantages of modelling equity returns as an independent, identically-distributed, random variable are considered, and a modified stochastic model is proposed. The modified stochastic model is used to estimate the variability in a scheme member's retirement fund, and to compare various alternatives to investing 100% of the assets in ordinary shares. Varying conclusions are drawn about the likely success of these alternative investment strategies in reducing investment risk.

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1. Introduction

Defined benefit and money purchase schemes

In defined benefit pension schemes, the pension is calculated from a pre-set formula. The most common approach is for the pension to equal to a fixed fraction of the member's salary close to retirement multiplied by the number of years of service with the employer. Such arrangements are usually described as final salary schemes.

From the employees' perspective, final salary schemes have the advantage of providing pensions linked to their retirement income needs. New entrants can predict what fraction of their earnings will be replaced by the scheme, should they stay in service until retirement. Moreover, provided that an employee's salary increases at a rate not lower than price inflation, the real value of the pension (in terms of its future purchasing power) has a lower bound.

The money purchase approach is fundamentally different: a retirement fund is accumulated from contributions paid into each member's account, the value of which depends on investment returns over the same member's period of service. A comparison given by Bodie (1989), based on historic UK investment and earnings data, for a money purchase scheme in which contributions of 10% of earnings are invested in ordinary shares, has shown that the pension of an employee with 20 years' past service, retiring in one of the years from 1970 to 1987, would have varied between 13% and 41% of final salary.

Nevertheless, money purchase schemes are becoming increasingly prevalent for a variety of reasons. The objective of this paper is to examine the problem of investment risk in such schemes, and assess the validity of various strategies which may be employed to limit this risk.

Outline of the paper

We first develop a stochastic investment model for equity returns net of wage inflation. The reasons for focusing on returns net of wage inflation are twofold. First, contributions to money purchase schemes are usually a fixed percentage of the employee's salary. Second, it is desirable for an employee's retirement fund to be measured relative to the projected salary at retirement. Thus, it is natural to use currency units adjusted for future wage inflation, in which case returns must also be measured relative to wage inflation.

In Section 2, we derive formulae for the expected value and variance of a money purchase fund, assuming annual investment returns net of earnings growth are independent and log-normally distributed. Parameters for the model are derived from past UK equity returns and earnings data. However, the model is rejected on the grounds that it overstates the variability in the retirement fund.

In Section 3, we develop a modified stochastic investment model, employing aspects of Wilkie's model, which is then used in the rest of the paper.

In Section 4, we use the model to simulate investment returns for a fund invested 100% in UK equities. These simulations are used to illustrate: (i) the variability in the retirement fund of an new entrant 40 years from retirement, and (ii) to what extent this variability reduces as the employee gets closer to retirement.

In Section 5, we investigate the consequences of switching the fund (and future contributions) to low risk assets at some point before retirement. The two criteria examined are: (i) the reduction in the expected value of the fund after switching, and (ii) the reduction in downside risk after switching.

In Section 6, we compare investing 100% of the fund in equities with balanced investment strategies, in which a fixed percentage of the fund is allocated to low risk assets throughout the period of service. The disadvantage of a lower expected fund is compared with the reduction in its variability.

In Section 7, we examine the use of derivative instruments as a means of reducing investment risk, by simulating investment returns from a Guaranteed Equity Product (GEP), similar in design those currently being marketed by UK life assurance companies.

Section 8 summarizes the main conclusions of this paper.

2. Variability in fund assuming independent log-normal returns

Let us assume that a contribution of 1 unit is paid annually in advance into a member's fund. All amounts and returns are expressed in terms of constant earnings.

Let:

F_t = ***projected fund at time t***

i_t = ***average force of interest in year t***

It follows that:

$$F_{t+1} = (F_t + 1)e^{i_t} \qquad (2.1)$$

Let:

x_0 = ***youngest permitted entry age to a money purchase scheme***

x = ***the age of a specific scheme member in mid-career*** $(x > x_0)$.

We now derive expressions for the expected value and variance of the fund at retirement for a member at any age, assuming that:

(i) the annual investment return net wage inflation is an independent, identically-distributed, log-normal random variable;

(ii) a member aged x has already accumulated a fund equal to its expected value at that age on entering the scheme at age x_0.

The second assumption is also intended to cover the case of members who enter the scheme in mid-career, bringing with them transfer values.

Expected value and variance of the fund at retirement

Let μ_t and σ_t^2 = mean and variance of normal distribution for i_t

From equation (2.1) we can deduce that:

$$E(F_{t+1}) = [E(F_t) + 1]e^{\mu_t + \frac{1}{2}\sigma_t^2} \tag{2.2}$$

Assuming μ_t and σ_t^2 are independent of t we can write:

$$r = e^{\mu + \frac{1}{2}\sigma^2}$$

Equation (2.2) then has the following solution.

$$E(F_t) = F_0 r^t + \frac{r(r^t - 1)}{r-1} \tag{2.3}$$

We have assumed a member aged x has accumulated a fund equal to its expected value at that age on entering the scheme. Hence for such a member:

$$F_0 = \frac{r(r^{(x-x_0)} - 1)}{r-1} \tag{2.4}$$

Let n = retirement age - x

From equations (2.3) and (2.4) we can deduce that:

$$E(F_n) = \frac{r(r^{n+x-x_0} - 1)}{(r-1)} \tag{2.5}$$

We now derive the variance of the retirement fund of the member aged $\boldsymbol{x}$.

From the recurrence formula for F_t , equation (2.1), we can deduce that:

$$F_{t+1}^2 = (F_t^2 + 2F_t + 1)e^{2I_t} \qquad (2.6)$$

$$E(F_{t+1}^2) = [E(F_t^2) + 2E(F_t) + 1]e^{2\mu + 2\sigma^2} \qquad (2.7)$$

Now let $s = e^{2\mu + 2\sigma^2}$

The recurrence formula for $E(F_t^2)$, equation (2.7), yields the following expression:

$$E(F_n^2) = F_0^2 s^n + \frac{s(s^n - 1)}{s - 1} + 2\sum_{t=0}^{n-1} E(F_t)s^{n-t} \qquad (2.8)$$

This expression can be further simplified by substituting for $E(F_t)$, but is already in a suitable form for evaluation in a spreadsheet software package.

The variance of the fund at retirement is then given by:

$$Var(F_n) = E(F_n^2) - [E(F_n)]^2 \qquad (2.9)$$

As one would expect, this is greatest at $x = x_0$, and reduces with increasing $\boldsymbol{x}$.

Parameters estimated from past equity returns

Estimators for the mean and standard deviation of the force of interest were obtained from UK equity index returns and average earnings data over 1950-1993. The equity returns were taken from the BZW equity index and the earnings data from Government statistics. The following estimators were obtained:

Mean {i}	=	0.052
Standard deviation {i}	=	0.2556

The standard deviation of i is perhaps larger than might have been expected, particularly if one believes that equity returns are correlated, to some extent, with wage and price inflation. However the data suggests that there is very little correlation when returns are measured over annual intervals. In addition, the period covered includes the crash/recovery scenario of 1974 and 1975, which has a significant effect on the measured standard deviation.

Setting $n = 20$ and $F_0 = 0$ in equations 2.7 and 2.8, gives:

$E(F_{20})$	=	54.5
$SD(F_{20})$	=	55.8

Although it is quite possible, given the skew nature of the distribution, for the standard deviation of the fund to exceed its expected value, the figure obtained is nevertheless implausibly high, and is not consistent with empirical studies of the kind to which Bodie refers.

It may be incorrect to assume that annual equity returns net of wage inflation are independent, for the purpose of estimating the variability in funds accumulated over long periods. In making such an assumption, we ignore the fact that the average the dividend yield on ordinary shares tends to fluctuate around a central value that may well be comparatively stable. This effect will tend to reduce the variability in returns over long periods, without necessarily effecting the measured variability in annual returns.

An explicit dividend yield model is a central feature of Wilkie's stochastic model for the simulation of equity returns. In the next Section, this aspect of Wilkie's approach is adopted to simulate equity returns net of wage inflation.

3. Modification of simple log-normal model

In 35 of the 44 years from 1950-1993, the end-year dividend yield on the BZW equity index lay in the range 4% to 6%. This has had a profound effect on long-term stability in equity returns, as there has been a tendency for the market to correct itself when overvalued or undervalued by historical standards.

UK actuaries have implicitly recognised this phenomenon by using a discounted cash flow (or "actuarial") value for equities in valuations of defined benefit schemes. Actuarial values differ from market values in that price changes arising from fluctuations in dividend yields (as opposed to a rise or fall in dividend income) are not recognised.

Following Thornton and Wilson (1992), the actuarial force of net interest, a_t, is defined by:

$$e^{a_t} = e^{I_t} . \frac{D_t}{D_{t-1}} \tag{3.1}$$

where D_t is the average equity index dividend yield at the end of year t.

We shall now model the actuarial return as an independent, identically-distributed, log-normal random variable. Looking at historical data over 1950-1993, gives the following estimators for the mean and standard deviation of this distribution.

Mean $\{a\}$	=	0.0428
Standard deviation $\{a\}$	=	0.0646

Compared with market value returns, the standard deviation is much reduced. What matters for a scheme member, however, is the market value of the fund at retirement, which is given by:

$$F_n = F_0.\frac{D_0}{D_n}.\exp[\sum_{t=1}^{n} a_t] + \sum_{t=0}^{n-1} \frac{D_t}{D_n}.\exp[\sum_{k=t+1}^{n} a_k] \tag{3.2}$$

In order to simulate market value returns, we therefore require a model for the way that dividend yields change over time.

Dividend yield model

Dividend yields must lie in the range zero to infinity, so it is reasonable is to assume that log($\boldsymbol{D_t}$) can be modelled as a normally distributed random variable.

Hence, we shall define:

$$d_t = log_e(D_t)$$

Wilkie (1986) observed that the average dividend yield on UK equities has tended to vary about a long-term average, and that yields in adjacent periods exhibit significant positive correlation. We shall estimate the autocorrelation of $\boldsymbol{d_t}$ from the year-end dividend yield on the BZW equity index over the period 1919-1993.

k	**correlation of d_t and d_{t+k}**
1	0.512
2	0.204
3	0.030
4	-0.008

The data confirms that an autoregressive model, as used by Wilkie, is appropriate. Wilkie used an autocorrelation parameter of 0.6 for $\boldsymbol{d_t}$ and $\boldsymbol{d_{t-1}}$, and also assumed that the rate of price inflation had a direct effect on $\boldsymbol{d_t}$. Since we require a model that operates in real values, we shall ignore the latter feature of Wilkie's model, and use an autocorrelation parameter of 0.5 for $\boldsymbol{d_t}$ and $\boldsymbol{d_{t-1}}$ in accordance with our own data.

This leads to the following first order autoregressive formula for $\boldsymbol{d_t}$:

$$d_t = 0.5d_{t-1} + 0.5\mu_d + \frac{\sqrt{3}}{2}\sigma_d.N_t \tag{3.3}$$

where N_t is an independent, normal random variable with mean zero and unit variance, and the coefficients have been selected so that:

$Mean\{d_t\} = \mu_d$

$Var\{d_t\} = \sigma_d^2$

The following estimators for the mean and standard deviation were obtained from the historic data:

$\mu_d = -3.008$

$\sigma_d = 0.240$

Initial fund of member in mid-career

As before, we assume that a member aged x has accumulated a fund equal to its expected value at this age on entering the scheme. Assuming that the change in the equity dividend yield over any period is independent of the actuarial return over the same period, the expected value of this fund is given by:

$$F_0 = \sum_{t=1}^{x-x_0} r_a^{\,t}.E\left(\frac{D_{-t}}{D_0}\right) \qquad (3.4)$$

where $r_a = E(e^a)$

Given the nature of our dividend yield model, one might suppose that the expected value of D_{-t}/D_0 must be unity for all t. This is not quite correct, but for the parameters used in our model it can be shown that such an assumption is a sufficiently good approximation for our purposes. Thus, we shall use the following formula for the initial fund of a member aged x:

$$F_0 = \frac{r_a(r_a^{\,x-x_0}-1)}{r_a-1} \qquad (3.5)$$

4. Use of simulation to obtain percentiles

As Bodie has noted, the standard deviation is not a particularly useful parameter for the skew distribution of the fund at retirement. What is really required are values of the fund at various percentiles, so that we can estimate the probability of a member's benefits lying within a particular range. The relevant probability density function is difficult to obtain, so these values have to be estimated through simulation.

For $n=40$ and $x\text{-}x_0$ = 0, 20, 30, and 35 respectively, 1000 simulations were carried out using the modified stochastic model described in Section 3. The values of the retirement fund at various percentiles, as a multiple of its mean value over each run of 1000, are shown in Table 1, below.

TABLE 1: VALUES OF F_{40} AT VARIOUS PERCENTILES

	5th	25th	50th	75th	95th
$x\text{-}x_0$ = 0	0.49	0.71	0.93	1.18	1.82
$x\text{-}x_0$ = 20	0.54	0.74	0.94	1.21	1.63
$x\text{-}x_0$ = 30	0.58	0.77	0.95	1.18	1.56
$x\text{-}x_0$ = 35	0.60	0.80	0.97	1.16	1.52

There are two main conclusions to be drawn from Table 1.

1) Even though the stochastic model allows for long-term stability in dividend yields, the variability in the projected fund of a new entrant 40 years from retirement is still high: the ratio of the 75th percentile to the 25th percentile is 1.66. Or in other words, an employee whose working career coincided with a period of moderately favourable equity returns would end up with a fund 66% greater than that of a similar employee whose working career coincided with a period of moderately unfavourable equity returns.

2) The variability in the projected retirement fund reduces only slowly as the as the employee gets closer to retirement. Even at only 5 years from retirement, the ratio of the 75th percentile to the 25th percentile is as high as 1.45. There is still a 1-in-4 chance that the fund will turn out to be less than 80% of its expected value, and a 1-in-20 chance that it will turn out to be less than 60% of its expected value.

The results obtained over a 40-year period of service are broadly consistent with those of Knox (1993), based on the experience of an Australian managed fund. However, this appears to be a co-incidence, as the stochastic model used by Knox assumed independent, identically-distributed returns, combined with a rather low standard deviation. Hence for periods of service of less than 40 years, Knox's model would imply significantly less variability.

<u>Practical problems created by investment risk</u>

Some practical implications of the results shown in Table 1 are discussed below.

(i) **Uncertainty in future benefit levels**

An employee in a money purchase scheme may have little idea of what the real value of his or her future pension will be, which makes planning for retirement more difficult. The projected future pension arising from a given rate of contribution can be estimated, but these estimates would need to be updated frequently, and may turn out to be wide of the mark. Even if the contribution rate is varied following regular benefit projections, the scheme member may find that either:

(a) the retirement fund is too small to purchase the required pension; or

(b) the retirement fund is larger than required, and the surplus savings it contains must now be used to purchase an annuity.

The problem mentioned in (b) is a consequence of UK legislation, which limits the amount of a member's fund that can be taken as a lump sum.

(ii) **Inequity between employees**

It could be argued that a money purchase scheme is the most equitable form of pension provision, as the same contribution rate can be paid for each employee, who would always receive his or her asset share by definition. I believe this definition of equity is valid only for individual pension contracts, where the member effectively hires an insurance company to manage his or her personal savings, and retains control over the choice of insurer and type of fund.

In an employer-sponsored scheme, the member usually has less control over the money invested on his or her behalf. Furthermore, the option to receive salary in lieu of pension contributions is not normally available. It follows that the benefit being provided by the employer is not the contributions, but the pension derived from these contributions. In a money purchase scheme, this pension will depend on whether the employee's period of service happens to coincide with a period of favourable or unfavourable investment experience. Thus, different generations of employees, with identical salary and service histories, may end up with very different pensions.

If a government requires its citizens to invest social security contributions in money purchase arrangements, the economic consequences of inequity between the generations could be severe, as an entire generation of newly-retired pensioners could end up with inadequate pensions, and may require additional financial support from the working population.

5. Switching to low risk assets

The results obtained in Section 4 were for a money purchase fund invested fully in ordinary shares. The first variant from this investment strategy that we shall examine is one very frequently employed - switching the existing fund and future contributions to low risk assets at some time fairly close to retirement.

Before investigating the optimal time for such a switch, we should consider what low-risk assets it would be appropriate to switch into. In the UK, most insurance companies writing unit-linked business have funds invested in cash and/or government bonds, specifically to meet the needs of risk-averse policyholders. Individuals with unit-linked pension policies can switch their assets into these funds at any time, sometimes subject to a small administration fee. However, cash and fixed interest bonds give no guaranteed protection against inflation, so switching into a fund investing in index-linked government bonds may be more appropriate.

The real yield (net of price inflation) on UK index-linked bonds has usually been in the range 3-4%, which is approximately 1% above the annual growth in UK average earnings over the post-war period. For modelling purposes, we shall assume that a scheme member can always switch into assets which guarantee a fixed return 1% above the increase in UK average earnings.

Let F_n^{sw} = fund at retirement after switching at age x

$$\text{Then } F_n^{sw} = F_0.1.01^n + \ddot{s}_{\overline{n}|}^{1\%} \tag{5.1}$$

Switching to index-linked assets partly solves the problem of having an unpredictable pension at retirement - at least the real value of the fund is now fairly predictable, although uncertain future annuity rates have still to be contended with. The earlier the switch is made, the easier it is to plan for retirement, and to afford any extra contributions which may be required to obtain the desired pension. If the switch is made too early, however, the projected fund at retirement will be far below the fund expected from continued investment in equities.

Under the stochastic model used in this paper, the equity dividend yield at the time of switching would have an important bearing on the decision. The argument for switching would be strengthened if the dividend yield were below its long-term average, because of the greater risk of a fall in the equity market. The reverse would apply if the dividend yield were above its long-term average.

Simulations were carried out to compare the fund obtained after switching into index-linked bonds at age x with that obtained by remaining in equities, assuming that the equity dividend yield the time of switching was either equal to, 1% below, or 1% above its long-term average.

Table 2, overleaf, shows the value of the fund obtained after switching into index-linked bonds as a fraction of the mean fund from continued investment in equities, for switches made at different durations from retirement and at different equity dividend yields. For comparison, the 25th and 50th percentiles of the fund obtained from continued equity investment (from Table 1) are also shown.

TABLE 2: FUND OBTAINED BY SWITCHING TO LOW RISK ASSETS

	SWITCH TO LOW RISK ASSETS			STAY IN EQUITIES	
	D_0 = 4.08%	D_0 = 5.08%	D_0 = 6.08%	25th %ile	50th %ile
$x\text{-}x_0 = 0$	0.40	0.40	0.40	0.71	0.93
$x\text{-}x_0 = 20$	0.61	0.52	0.47	0.74	0.94
$x\text{-}x_0 = 30$	0.82	0.67	0.58	0.77	0.95
$x\text{-}x_0 = 35$	0.98	0.79	0.67	0.80	0.97

As one might expect, the ratio of the switched retirement fund to the mean fund from continued investment in equities is always less than one. The amount by which this ratio falls below unity is effectively the "insurance premium" paid in order to obtain a guaranteed fund at retirement.

By comparing these ratios with the percentiles from continued investment in equities, we can assess the degree of risk protection obtained by switching. If the fund remains in equities, the probability of ending up with a retirement fund below, say, the 25th percentile is 0.25 - clearly a significant risk. If by switching to low risk assets we can guarantee a fund equal to or higher than this, the case for switching might be reasonably strong.

According to Young (1994), the most commonly recommended time for a switch to low risk assets is approximately 5 years before retirement, which corresponds to the case $x\text{-}x_0=35$. Table 2 confirms that, at this duration, the risk of a lower retirement fund by remaining in equities is significant, but the magnitude of this risk depends greatly on the prevailing equity dividend yield. (At $x\text{-}x_0 = 0$, however, the initial dividend yield is irrelevant as there is no fund to switch.)

Ideally, the following conditions would hold before switching into low risk assets.

(a) The projected fund after switching will meet the member's requirements.

(b) The equity market is overvalued by historic standards.

(c) There is less than 10 years to go before retirement.

If (a) is true, one would expect the member to be very risk-averse, as he or she can virtually guarantee the required fund without having to pay extra contributions. Thus, even a small probability of not achieving the necessary fund might be unacceptable.

If (b) is true, (a) is more likely to be true (as the market value of the accumulated fund will be greater), and the risk of ending up with a lower retirement fund by remaining in equities would be greater.

If (c) is true, the risk of ending up with a lower retirement fund by remaining in equities would be significant under most conditions. However, if (c) is true and (a) is not true, there is less time to obtain the required fund by paying extra contributions. A member might therefore prefer to risk continued equity investment in the hope of obtaining the target fund through superior investment performance - i.e. by taking a calculated gamble.

In summary, we can conclude that switching to low risk assets at some point within, say, 10 years of retirement is likely to be a suitable strategy for most members of money purchase schemes. However, the precise timing of this switch should flexible, depending on the member's projected fund after switching and the level of the equity market at the time of the switch.

6. Balanced investment strategies

In this Section, we shall examine the results of following a balanced investment strategy throughout an employee's period of service, and compare them with the results obtained for 100% investment in equities.

The following balanced investment strategies were considered:

(i) 75% equities, 25% index-linked bonds, realigned annually by market values;

(ii) 50% equities, 50% index-linked bonds, realigned annually by market values.

1000 simulations were carried out simultaneously for each investment strategy, so that each set of simulations was based on the same sequence of equity returns. This enabled the number of times that a particular investment strategy led to a higher retirement fund than an alternative strategy to be calculated.

The simulations were carried out for the case $n=40$, $x=x_0$, i.e. for a new entrant at the youngest permitted age of entry, with no accumulated fund. The values of the retirement fund at various percentiles, as a multiple of the mean fund from investing fully in equities, are shown in the Table 3, overleaf.

TABLE 3: COMPARISON OF DIFFERENT INVESTMENT STRATEGIES

	5th percentile	25th percentile	50th percentile	75th percentile	95th percentile	Mean
A: 100% equities	0.49	0.72	0.93	1.20	1.75	1.00
B: 75% equities	0.53	0.71	0.86	1.05	1.37	0.90
C: 50% equities	0.53	0.64	0.73	0.84	1.00	0.75

As one would expect, a lower allocation to equities reduces the mean value of the retirement fund, but also reduces its variability. In order to determine whether a balanced investment strategy has anything to offer the individual scheme member, the following probabilities were estimated from the simulations:

Investment strategy A
Probability of obtaining a fund of less than one-half the mean = 0.056.

Investment strategy B
Probability of obtaining a fund of less than one-half A's mean = 0.031.

Investment strategy C
Probability of obtaining a fund of less than one-half A's mean = 0.024.

Is there a case for a balanced investment strategy?

We can summarise the results obtained, by saying that a more balanced investment strategy would result in a lower retirement fund for the majority of members, but would also reduce the already small proportion of members who obtain a severely sub-standard fund. So, we could "sell" investment strategy B to a member by explaining that although his or her expected fund would be 10% lower, the risk of ending up with only half the expected fund is reduced from 5.6% to 3.1%. My feeling is that most members would not feel this was a good deal, and the case for strategy C would be even weaker.

The main advantage of investing in low risk assets is that inequity between different members is significantly reduced. The ratio of the retirement fund at the 75th percentile to that at the 25th percentile is 1.67 for strategy A, 1.48 for strategy B and 1.31 for strategy C. However, this has been achieved entirely by "levelling down": the fund at the 25th percentile is highest for strategy A.

The results obtained therefore suggest that the case for investing a significant proportion of the fund in low risk assets, as a long-term strategy, is weak. This does not necessarily argue against short-term tactical switches away from the equity market, based on the judgement of the fund manager.

7. Guaranteed Equity Products

The final investment strategy to be considered, as an alternative to 100% investment in equities, is one involving the use of Guaranteed Equity Products (GEPs).

GEPs have been marketed by UK insurance companies, as a means of allowing policyholders to participate in the underlying growth of an equity portfolio, while also benefitting from a guaranteed minimum fund, either at termination of the contract or at intermediate durations. These guarantees are designed to give protection against adverse movements in the equity market.

A typical contract might provide a return on the investor's capital equal to the increase in an ordinary share price index, while guaranteeing that the investor will be re-paid the initial capital should the index fall over the term of the contract. In such a contract, the absence of re-invested dividends would "pay" for the guarantee. Dodhia and Sheldon (1994) have described how the creative use of financial options has enabled the design of a wide variety of contracts, each offering a different type of guarantee.

Consider a contract which provides a rolling guarantee at one-year intervals, coinciding with the annual investment of contributions to the pension fund. We shall assume that the contract guarantees a fraction of the capital invested at the start of the year, plus the actual equity return (if positive) applied to the minimum guaranteed capital. For modelling purpose we shall further assume that:

(i) the guaranteed capital increases in line with UK average earnings over the year;

(ii) the equity return is based on the equity price index with dividends re-invested, as opposed to the more usual practice of using the price index alone.

Dodhia and Sheldon have commented on the feasibility and propriety of (i) for pension fund contracts.

The GEP investment return net of wage inflation in year t is thus given by:

$$R_t = f.\max(e^{I_t}, 1) - 1 \qquad (7.1)$$

where f is some fraction below one.

It is immediately apparent that the expected value of the retirement fund will be very sensitive to the value of *f* chosen, as this factor will compound over the years to retirement. We shall choose values for *f* which produce approximately the same expected fund as from investing in the equity portfolio alone, which after some trial simulations were found to be in the range 0.92 to 0.93.

Using the modified stochastic model, 1000 simulations were carried out simultaneously for contracts with f equal to 0.92, 0.925 and 0.93 respectively, and for investment in the underlying equities alone. As before, these were done for a new entrant at the youngest age, with 40 years to go until retirement. The values of the retirement fund at various percentiles, expressed as a multiple of the mean fund from investing in equities alone, are shown in Table 4, below.

TABLE 4: EFFECT OF INVESTING IN GUARANTEED EQUITY PRODUCTS

	5th percentile	25th percentile	50th percentile	75th percentile	95th percentile	Mean
GEP f=0.920	0.44	0.62	0.80	1.05	1.58	0.89
GEP f=0.925	0.50	0.71	0.91	1.21	1.82	1.02
GEP f=0.930	0.57	0.82	1.05	1.40	2.11	1.18
EQUITIES	0.50	0.70	0.93	1.20	1.78	1.00

Table 4 indicates that the expected fund from investing in a rolling, one year GEP contract is very sensitive to the level of guarantee offered. More importantly, there appears to be no reduction in the variability of the fund at retirement compared with a strategy of investing in the underlying shares alone.

Why is the GEP contact ineffective in reducing long-term investment risk?

Guaranteed Equity Products clearly reduce variability in investment returns over short periods, so it is perhaps surprising that a rolling, one-year contract fails to reduce the same variability over longer periods.

An intuitive explanation follows from the fact that the return from a rolling GEP contract depends on how variable the underlying equity returns are. The greater the variability in equity returns, the greater the return from the GEP, as the investor benefits from large positive equity returns while being protected against large negative ones.

However, over long periods of fixed length, the variability in equity returns might also be very variable - maybe there will be several crash/recovery scenarios as in 1974 and 1975, maybe there won't be any at all. It follows that the long-term return from a GEP might be just as variable as the long term return from the underlying shares.

8. Conclusions

The main findings of this paper are summarized below.

- [] Modelling equity returns as an independent, identically-distributed, log-normal random variable appears to seriously overestimate the variability in funds accumulated from the investment of annual contributions over relatively long periods.

- [] In the UK, stochastic models which allow for the tendency of the equity dividend yield to move towards a central value produce results which are more consistent with empirical studies. However, even when such models are used, the variability in the retirement fund of a new entrant to a money purchase scheme is found to be very large, and this variability reduces only slowly as the member approaches retirement.

- [] A strong case exists for the individual scheme member to switch his or her fund to low risk assets in the period close to retirement. Although the case for switching becomes stronger as the member approaches retirement, the optimal time to do so depends also on the member's target fund and the prevailing equity dividend yield.

- [] A balanced investment strategy, in which a significant proportion of the member's fund is invested in low-risk assets throughout his or her period of service, reduces both the expected value of the fund at retirement and its variability. However, most of the reduction in variability occurs from "levelling down" - the reduction in the member's downside risk is not very significant.

- [] Over a 40-year period, a rolling, one-year guaranteed equity contract of simple design, results in no significant reduction in the variability of the retirement fund, compared with investing purely in equities.

Implications for pension scheme design

The arguments for investing long-term savings in ordinary shares are very strong, both from the viewpoint of maximising returns and hedging against wage and price inflation. Equities are a highly appropriate asset class for pension schemes other than those which consist mainly of retired employees.

In money purchase pension schemes, however, investment in equities results in pension benefits which depend excessively on whether the employee's period of service happens to coincide with a period of favourable or unfavourable investment experience. This makes it difficult for individual members to plan for retirement, and results in inequity between different generations of employees.

Three strategies for reducing the investment risk associated with equities were examined in this paper:

(1) switching to low risk assets close to retirement;

(2) balanced investment strategies;

(3) the use of derivative-based investment products.

Of these three, only the first was found to offer significant advantages to the individual member. Moreover, a switching strategy does not deal with the fundamental problem - by the time a member gets close to retirement the damage may have already have been done!

A great advantage of defined benefit schemes is the implicit smoothing of variable investment returns for different generations of employees, brought about by the use of a fixed benefit formula. A good example of such a formula is found in the UK State Earnings-Related Pension Scheme, where a pension equal to a fixed fraction of career-averaged, revalued earnings is granted. The rate of revaluation applied to each year's earnings figure is the increase in national average earnings between the year concerned and the year prior to retirement. I mention this example because of its *similarity* to a money purchase scheme in which a fixed percentage of salary is invested for each employee. The only difference is that a *guaranteed* rate of interest, equal to the increase in the earnings index, is applied to each member's contributions.

However, defined benefit schemes are becoming less popular. Aside from the costs of complying with increasingly complex legislation, employers have been less willing to accept the open-ended liability of such schemes, which may require them to increase their contribution rate to cover a shortfall created by unfavourable experience.

In my view, a way must be found to apply the defined benefit principle to defined contribution schemes. In some ways, this would be similar to a with-profits insurance fund, and a small number of UK pension schemes are indeed run on this basis. However, unlike a with-profits fund, I believe that there should be explicit formulae for calculating the benefits paid-out, ideally based on career-average revalued earnings as used in UK State Scheme. In addition, there would have to be rules for varying the rate of benefit accrual, should the experience of the scheme deviate too far from the assumptions made by the actuary.

It is possible that a defined contribution scheme with a defined benefit scale that could be adjusted, from time to time, would represent a more equitable and secure form of pension provision than arrangements based purely on the money purchase principle.

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Appendix

UK Equity Dividend Yields & Index Returns

Year	(1)	(2)	Year	(1)	(2)
1919	4.8%		1957	6.3%	-6.5%
1920	9.5%		1958	4.8%	44.9%
1921	8.9%		1959	3.6%	46.6%
1922	6.0%		1960	4.5%	-5.2%
1923	6.4%		1961	4.8%	-3.6%
1924	5.4%		1962	5.0%	-2.9%
1925	4.8%		1963	4.1%	13.2%
1926	5.4%		1964	5.2%	-9.1%
1927	4.9%		1965	5.2%	3.6%
1928	4.6%		1966	5.9%	-9.3%
1929	6.1%		1967	4.2%	30.5%
1930	6.7%		1968	3.4%	29.8%
1931	6.8%		1969	4.0%	-18.7%
1932	4.7%		1970	4.6%	-13.7%
1933	3.9%		1971	3.4%	32.9%
1934	3.8%		1972	3.1%	5.3%
1935	3.7%		1973	4.4%	-39.9%
1936	3.4%		1974	11.8%	-60.8%
1937	4.6%		1975	5.7%	109.1%
1938	5.5%		1976	7.4%	-11.4%
1939	5.4%		1977	5.4%	43.7%
1940	6.3%		1978	5.6%	-1.1%
1941	5.2%		1979	6.8%	-8.4%
1942	4.4%		1980	5.8%	12.5%
1943	4.1%		1981	5.9%	1.8%
1944	3.8%		1982	5.2%	19.4%
1945	3.8%		1983	4.7%	18.9%
1946	3.5%		1984	4.5%	22.0%
1947	4.3%		1985	4.2%	11.6%
1948	4.3%		1986	4.1%	17.8%
1949	5.0%		1987	4.4%	-0.4%
1950	5.0%	5.1%	1988	4.7%	2.1%
1951	5.4%	-1.6%	1989	4.2%	24.5%
1952	6.1%	-7.1%	1990	5.4%	-15.3%
1953	5.4%	17.2%	1991	5.0%	13.4%
1954	4.4%	37.5%	1992	4.4%	14.8%
1955	4.8%	1.6%	1993	3.4%	24.7%
1956	5.7%	-14.8%			

(1) Equity index dividend yield at year-end.
(2) Return on equity index net of increase in average earnings.

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