Use of a Parametric Risk Measure in Assessing Risk Based Capital and Insolvency Constraints for With Profits Life Insurance

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USE OF A PARAMETRIC RISK MEASURE IN ASSESSING RISK BASED CAPITAL AND INSOLVENCY CONSTRAINTS FOR WITH PROFITS LIFE INSURANCE

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ABSTRACT

This paper defines a risk measure derived from that proposed by Clarkson (1989), which assesses risk as a function of the intensity of expected unfavourable outcomes. The risk measure is used to assess Risk-Based Capital (RBC) for a hypothetical model office consisting of a single tranche of Unitised With-Profits life insurance business, and makes comparisons with RBC assessed in relation to a fixed probability of ruin. It is concluded that the proposed (parametric) measure leads to significantly different RBC for most factors, and that decisions made in order to satisfy a given probability of ruin can be inconsistent with exercising adequate control of the office's parametric risk. The application of the methodology in practical and theoretical investigations is briefly discussed.

KEYWORDS

Capital; Life Insurance; Risk; Risk-Based Capital; Solvency; Unitised With-Profits; Utility Theory

1. INTRODUCTION

1.1 *Risk-Based Capital*

The amount of capital a financial institution must hold in order to meet its solvency criteria is referred to as *Risk-Based Capital* (RBC). RBC concepts first emerged in Europe in the banking industry in the 1960s (Hooker *et al.*, 1995), and its principles were adopted in the USA for insurance regulation in 1993. Members of the European Union are currently committed to reviewing the minimum solvency margin formulae for both life and non-life insurance business, and Risk-Based Capital is one of the likely candidates for the new approach.

Estimating the amount of RBC for an insurance company is a difficult task, and is harder to carry out for certain sources of risk than for others. The nature of risk for a general insurer is well described by Hooker *et al.* (1995), but many of the concepts are equally applicable to life insurance (see Booth *et al.*, 1996.) Hooker *et al.* (1995) loosely define risk as "*the possibility that events will develop worse than planned*". Booth *et al.* (1996) assume the premium basis assumptions to represent the 'planned outcomes' in the life insurance context, and that less favourable outcomes therefore contribute to the insurer's risk.

Risk is essentially the consequence of *uncertainty* (not to be confused with *variability*), and this uncertainty can take a number of forms. As described by Hooker *et al.* (1995), many processes in the insurance business can be described by probabilistic models, such as the well known Binomial or Poisson models for the random number of death claims (see Institute and Faculty of Actuaries, 1995), or the more complex models for simulating asset returns such as those of Wilkie (1995). Any probabilistic model is subject to three types of risk (see Hooker *et al*, 1995; and similarly Cairns, 1995) namely:

(i) *process risk*: the uncertainty due to the random nature of the process;

(ii) *parameter uncertainty*: the risk that the wrong parameter values are assumed in the model; and

(iii) *specification error*: the risk that the wrong model structure has been used to describe the process.

The contributions of each type of risk to the overall risk will vary according to the process under consideration. For example, it would be generally accepted that for a life insurer the greatest contribution to the mortality risk would arise from incorrect parameterisation (ie from parameter uncertainty), while the investment risk would be much more likely to be significantly affected by the specification error (see Huber, 1995; Kitts, 1990; Kemp, 1996.)

There are further significant contributions to an insurer's risk which cannot be represented by any probabilistic model. Examples in life insurance would include the risk of expense overruns arising from unfavourable changes in future business volume (see Chadburn, 1993); future losses due to unpredictable changes in taxation; etc. The method of assessing RBC for such risks would probably be necessarily judgemental and subjective.

The task of assessing RBC for probabilistic processes is, however, more tractable, at least for the first two types of risk listed above. Two alternative criteria for assessing RBC are considered in this paper, and as an example of their application a life office model is used in which only the process risk is considered, although there is no theoretical reason why these criteria cannot be applied in RBC assessments involving other risk types. The fact that these other risk types have been ignored in the examples quoted in this paper should, therefore, be borne in mind before laying too great a stress on the absolute values of the results presented.

1.2 *Non-Parametric RBC*

Needleman & Roff (1995) show example RBC calculations for single tranche models of with-profits offices, where the risk criterion is that 95% of the simulated outcomes should predict positive assets to be held by the model office at the moment of run-off of the in-force business. Higher or lower RBC would obviously be necessary were a greater or lesser frequency of non-negative outcomes required. In this paper it is found convenient to refer to the 95% level of RBC of Needleman and Roff as the '5% level': ie the criterion is simply stated in terms of the frequency of simulations in which *negative* assets (or *losses*) are predicted. This could synonymously be referred to as the RBC level which corresponds to a 5% *probability of ruin*. This criterion will be referred to here as a *non-parametric* RBC criterion, as it takes no account of the *values* of the predicted losses from the model. It could, of course, be at some level other than 5%: it will, however, be assumed to be 5% for the purpose of illustration here. This 5% non-parametric RBC criterion will be referred to as the NPC, for convenience.

1.3 *Parametric RBC*

As intimated above, a possible criticism of the NPC must be that it takes no account of the magnitude, or intensity, of the losses which are predicted to arise in the 5% of simulations which are insolvent. Hence, according to the NPC, the same level of RBC would be required for 50 losses of size X as for 50 losses of size $100X$ (if these were the only losses predicted out of 1000 equally likely simulations.) However the size of the losses cannot be considered immaterial to the risk. Should the office be wound up in such an event, then the losses would have to be borne by the outgoing policyholders through a reduction in their benefits (at least down to the level which would be supported under the Policyholders' Protection Act, 1975.) Furthermore, while a formula-based stochastic model office may predict a loss, the office would in reality be expected to undertake some crisis management actions which would probably reduce the size of the expected loss, and quite possibly avoid it altogether. However the office would be much more likely to be able to avoid a loss of size X than a loss of size $100X$ through its crisis management responses. Hence the NPC is theoretically flawed; however if the distributions of expected losses (in relative terms) are essentially the same for any model office which satisfies the NPC, then the NPC would still lead to consistent assessments of the RBC. Whether this is likely to occur in reality will be investigated in this paper.

A risk measure which takes into account both the incidence and intensity of the expected unfavourable outcomes was described by Clarkson (1989), with respect to the measurement of investment risk, as:

$$R = \int_{-\infty}^{h} W(h - x).dF(x) \tag{1.1}$$

where h is some target outcome, x the (random) outcome, and $F(x)$ the cumulative probability density of x. Hence we could take:

$$W(s) = s^r \quad (r > 1);$$

and $$x = \frac{A}{B} \tag{1.2}$$

where A = assets of the office projected to exist at the run-off of the existing business; and

B = a measure which is proportionate to the value of the benefit payments made under the in-force business, as at the date of run-off.

The value of $-x$ is therefore the amount of loss predicted by a given insolvent simulation, measured as a proportion of the overall benefit level paid. It is necessary to measure the loss in relative terms as the significance of any absolute loss will vary depending on the monetary volume of the business to which the loss is attributed, which could vary significantly between different simulations of the same model.

1.4 *Utility Theory*

R, as defined here, is essentially the expected value of the following risk function:

$$\begin{aligned} R_r(x) &= (-x)^r \quad &\text{for } x < 0 \\ &= 0 \quad &\text{for } x \geq 0 \end{aligned} \tag{1.3}$$

However, any risk function can also be expressed as a utility function (see Booth, 1996). For the particular risk function defined above, the equivalent implied utility function is:

$$\begin{aligned} U_r(x) &= -(-x)^r \quad &\text{for } x < 0 \\ &= 0 \quad &\text{for } x \geq 0 \end{aligned} \tag{1.4}$$

According to utility theory, each outcome has a *relative* utility to the life office's investors which is proportionate to $U_r(x)$, which must satisfy the conditions:

(i) $U_r'(x) > 0$; and

(ii) $U_r''(x) < 0$.

The above conditions are satisfied by $U_r(x)$ for all ($r = 2,3,\ldots$) as:

$$U_r'(x) = r(-x)^{r-1} > 0; \quad \text{and} \tag{1.5}$$

$$U_r''(x) = -r(r-1)(-x)^{r-2} < 0. \tag{1.6}$$

The level of absolute risk aversion implied by a utility function is defined as:

$$\alpha(x) = \frac{-U_r''(x)}{U_r'(x)} = \frac{r-1}{-x} \tag{1.7}$$

while the level of relative risk aversion is:

$$\rho(x) = -x.\alpha(x) = r-1 \tag{1.8}$$

Hence $U_r(x)$ displays increasing absolute risk aversion and constant relative risk aversion so that, for example, 10% of any investor's wealth will have the same utility regardless of its absolute size. The higher the value of r, the greater the degree of risk aversion implied. The proposed risk function is therefore fully consistent with the principles of utility theory.

Hence we could define a parametric RBC criterion (a 'PC') to be some chosen level K such that:

$$E[R_r(x)] = K$$

The value of $E[R_r(x)]$ for any given model can easily be estimated from S simulations as:

$$\frac{1}{S}\sum_{i=1}^{S} R_r(x_i)$$

where x_i is the projected realised value of x in the ith simulation. For convenience the division by the constant S can be omitted (provided the same number of simulations is always performed) so the risk measure can finally be written as:

$$P(r) = \sum_{i=1}^{S} R_r(x_i) \tag{1.9}$$

and the RBC would then be chosen which will satisfy the condition:

$$P(r) = S.K = L. \tag{1.10}$$

1.5 *Aims*

The aims of the work reported in this paper are as follows:

(i) to assess the extent to which non-parametric criteria are likely to produce RBC assessments which are consistent with parametric risk criteria;

(ii) to identify any consistent relationships which may exist between RBC assessed according to non-parametric or parametric criteria;

(iii) to compare the relative sensitivities of RBC assessed under non-parametric and parametric criteria to variations in the conditions (assumptions) of the model;

(iv) to assess the implications to a life office of basing RBC on non-parametric or parametric criteria; and

(v) to investigate the possible consequences of using parametric or non-parametric criteria in decision-making.

2. Model Assumptions

2.1 *Contract Details*

The assessments will be based upon a single tranche model of unitised with-profits (UWP) business (see Booth *et al.*, 1996.) The model is an earlier version of that used by the Institute of Actuaries working party on UWP for its reserving investigations. The initial assumptions (which constitute the 'standard' model) are now described.

The assumed contract is a fifteen year term policy, with annual premium £1000 and contractual claim at the end of the term or on earlier death. The contractual claim benefit is equal to the face value of accumulated units at the date of claim plus any terminal bonus. On surrender the policy pays the full claim benefit, provided this is less than 105% of the policy asset share at that time; otherwise the surrender value is equal to the asset share. (This process represents the application of a market value adjuster (MVA) to the claim benefit on surrender when benefit levels exceed asset shares.) All profits or losses from surrender are shared among the remaining policyholders at the end of each year.

The tranche of existing business was assumed to have been in force for five years at the valuation date; policyholders were aged 30 at entry.

2.2 *Calculating Benefit Levels*

Let $roa(t)$ = the return on assets between times $[t\text{-}1,t]$, where $(t=0)$ is the valuation date;

$i(t)$ = the return attributed to policyholders over $[t\text{-}1,t]$

$$= roa(t) - CC \tag{2.1}$$

where CC is an annual charge for capital (see Needleman & Roff, 1995);

$ir(t)$ = notional reduced return over $[t\text{-}1,t]$

$$= F.i(t) \qquad \text{for } i(t) > 0$$

$$= i(t)/F \qquad \text{for } i(t) < 0; \text{ where } F \leq 1; \tag{2.2}$$

and $is(t)$ = the geometrically smoothed policyholders' return over times $[t\text{-}1,t]$, calculated as the geometric mean of $i(t)$ over the last yma years, ie over times $[t\text{-}yma,t]$.

Then $PAS(s)$ = policy asset share at time s (calculated by accumulating cash flows over times $[0,s]$ at the rates $i(t)$, $t = 1,2,\ldots s$);

$RPAS(s)$ = reduced policy asset share at time s (calculated by accumulating cash flows at the rates $ir(t)$, $t = 1,2,\ldots s$);

$SPAS(s)$ = smoothed policy asset share at time s (calculated by accumulating cash flows over times $[0,s]$ at the rates $is(t)$, $t = 1,2,\ldots s$);

$UF(s)$ = face value of policy units at time s (calculated by accumulating cash flows over times $[0,s]$ at the rates $[1+g][1+r(t)]-1$, $t = 1,2,\ldots s$);

where g = guaranteed annual rate of unit growth; and

$r(t)$ = declared reversionary bonus over $[t-1,t]$, declared annually in advance.

The total benefit level at time s is calculated as:

$$CBL(s) = \max[SPAS(s),\ UF(s)] \tag{2.3}$$

so that the terminal bonus is:

$$TB(s) = CBL(s) - UF(s).$$

2.3 *Determining the Reversionary Bonus*

The reversionary bonus rate for the year $[t,t+1]$ is determined at time t as:

$$r(t+1) = r(t) + DRT(t) \tag{2.4}$$

where $DRT(t) = DRB(t) + DRU(t)$, (rounded down to the next .0025, with a maximum value of .015.)

Now $DRB(t)$ and $DRU(t)$ are contributions to the *change* in bonus rate deduced by reference to the yield on consols and to the difference between the reduced policy asset share and the unit fund respectively. Specifically:

$$DRB(t) = 0.5\times F1\times\left\{\max\left(\frac{ic(t)-g}{1+g},\ 0\right) - r(t)\right\} \tag{2.5}$$

and $$DRU(t) = 0.05\times F2\times\left(\frac{RPAS(t) - UF(t)}{RPAS(t)}\right) \qquad \text{for } RPAS(t) < UF(t)$$

$$= \quad 0.025 \times F2 \times \left(\frac{RPAS(t) - UF(t)}{RPAS(t)} \right) \quad \text{for } RPAS(t) \geq UF(t)$$

(2.6)

where $ic(t)$ = yield on consols at time t;

and $F1$ and $F2$ are appropriate weighting factors.

The declared reversionary bonus therefore responds partly to changes in the consols yield and partly to the difference between the reduced policy asset share and the unit fund. The reduced policy asset share (*RPAS*) therefore acts as a target level for the unit fund, with the difference between *RPAS* and the full policy asset share representing the target terminal bonus.

The values for the standard model parameters were chosen by visual inspection of a range of individual simulations of the model, so that the standard model displayed the following properties:

(i) *SPAS* showed significant but not excessive smoothing compared to the *PAS*; and
(ii) changes to reversionary bonus rates were reasonably cautious responses to events, especially when increasing, while displaying a sufficient degree of smoothing.

The Institute of Actuaries working party has subsequently revised its assumptions for fixing reversionary bonus rates, as the present version was found to exhibit some year-on-year inconsistencies and to show possibly too much oscillation from year to year. These changes are, however, likely to be of little significance for the present analysis, although the absolute values of the RBC may be altered.

The standard model parameter values are:

CC = .005;
F = .75;
yma = 3;
g = .03;
$F1$ = 1; and
$F2$ = 1.

2.4 *Experience Assumptions*

The stochastic investment model specified by Wilkie (1995) was used to generate 1000 pseudo-random scenarios of investment returns for each of the four asset types: equities, consols, index-linked gilts and cash. While this model is not free from criticism (see for example Kitts, 1990; Huber, 1995; Kemp, 1996) relative results tend to be more reliable than absolute values so that a comparison of the two risk criteria should be reliable enough using this model. Note that the variance reduction for the equity model used by the Institute of Actuaries working party and by some other workers was not incorporated here. The standard model assumes an intial asset mix of 75% equities and 25% consols by value. In each projection year the total net cash flow is reinvested as far as possible to restore this asset mix as at the end of the year.

Deterministic mortality was assumed according to the A1967/70 ultimate table; surrenders were assumed to occur at a constant independent rate of 5% per annum. Per-policy expenses were assumed to be £300 initial, plus £24 per annum varying with the simulated retail prices index from the Wilkie (1995) model. The existing funds at the valuation date (ie at the end of the fifth policy year) were calculated by assuming mean rates of return had operated deterministically over the past five years.

RESULTS

3.1 *Calculation of RBC based upon the Non-Parametric Criterion*

Example RBC values were calculated for a range of assumptions for the model parameters, according to the 5% NPC. The distributions of the relative asset values (x) were plotted in each case for the 50 loss-producing simulations. (Note that the sum of all benefits paid at maturity was used for the value of B in expression 1.2.) Example distributions, along with their means (M) and standard deviations (SD), are shown in Figure 1.

Figure 1a demonstrates that the NPC does *not* necessarily lead to similar levels of parametric risk. The 100% consols and 100% equities investment strategies demonstrate the most extreme contrasts in terms of the distributions of losses out of all the variations in assumptions investigated. Hence it would seem clear that the actual levels of risk implied by the RBC according to a constant non-parametric criterion can differ very markedly in different situations.

Several of the factors show a consistent pattern of change in the distribution of losses with respect to changes in the factors concerned. Increases in the value of F (Figure 1b), for example, are seen to produce losses with increasing mean and standard deviation. Similar monotonic relationships are observed for variations in the parameters CC, $F1$ and $F2$ (not shown.)

Variations in the unexpired term of the contract produce a less clear pattern in terms of changes to the mean and standard deviation (Figure 1c.) However, examination of the distributions reveals a clear tendency for increasing skewness as the unexpired term increases. Hence term 5 has a higher mean loss but a lower standard deviation than term 20. In this case it becomes unclear as to which scenario has the higher risk, as the choice between them depends upon the investors' relative preference for expected loss as opposed to variability: ie upon their level of *risk aversion* (see Markowitz 1952, 1991.) If we now compare the $P(2)$ values (also shown on the Figure) it can be seen that term 5 has higher risk than term 20, whereas for investors with higher risk aversion (eg with $r = 3$) the 20-year term produces the greater risk, as shown by the difference in $P(3)$ values. The usefulness, at least in theory, of the parametric risk measure $P(r)$ in measuring the risk is clear from this example; however it may not be easy to assess with confidence the actual level of risk aversion to assume in any practical situation, and hence the risk function itself may be very difficult to define.

Variation in the overall degree of smoothing, represented by the smoothing period *yma*, shows a similar effect (Figure 1d): a *yma* of 5 has a higher mean and a lower variance than where *yma* is 3 or 4. Of these, the less risk averse investors ($r = 2,3$) would treat *yma* = 4 as the least risky, whereas for $r > 3$ the scenario where *yma* = 5 produces the lowest risk.

Figures 2, 3 and 4 show the values of the parametric risk $P(2)$ for a variety of risk factors, along with the corresponding non-parametric RBC levels, expressed as a proportion of the policy asset share at the valuation date. Figure 2 shows factors which appear to show a positive correlation with RBC; Figure 3 shows cases of negative correlations; and cases

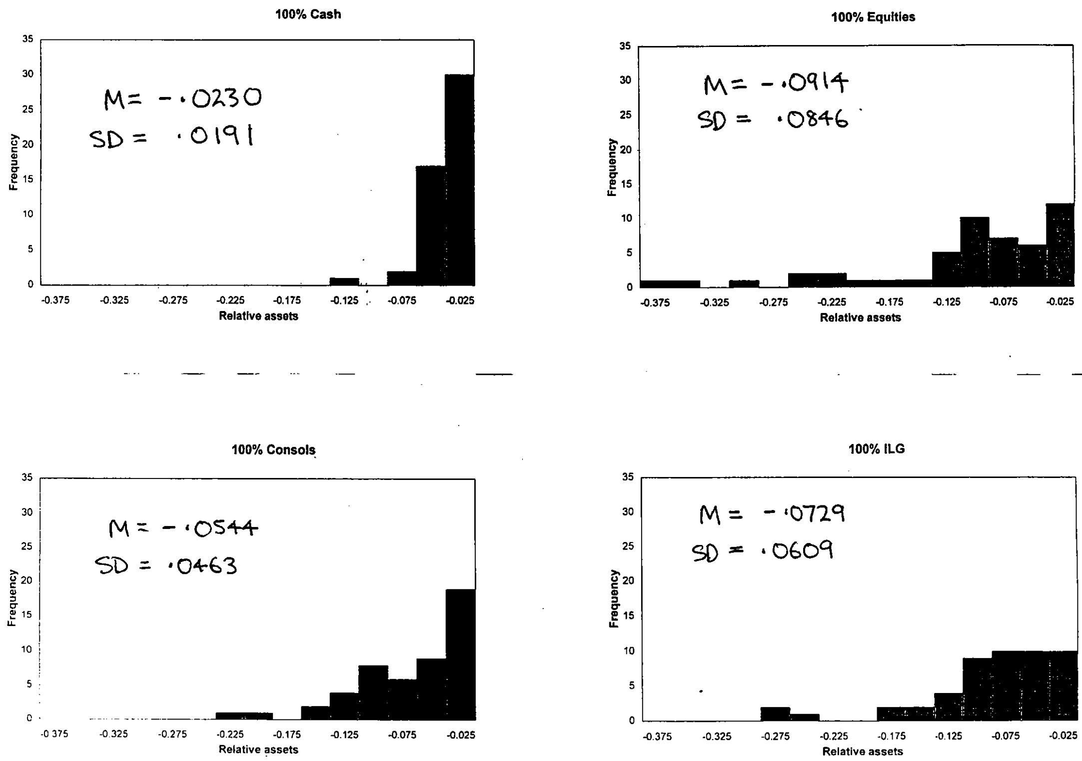


FIGURE 1a : DIFFERENT ASSET CLASSES

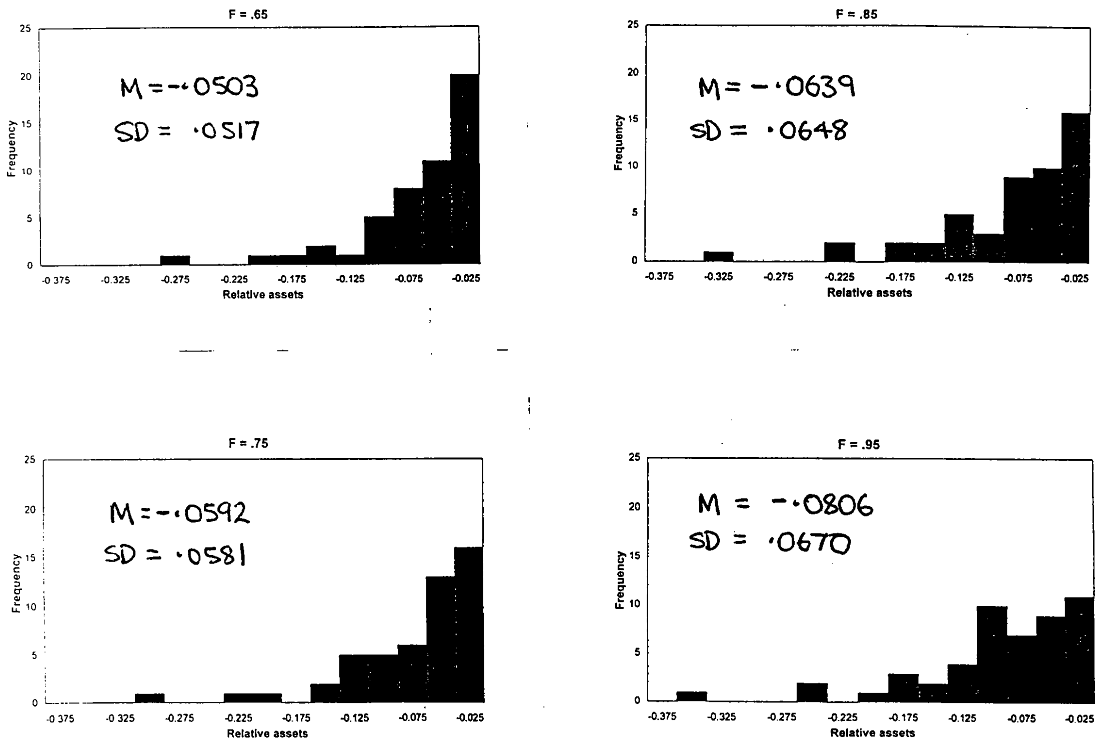


FIGURE 1b: F FACTOR

FIGURE 1 (CONTINUED)

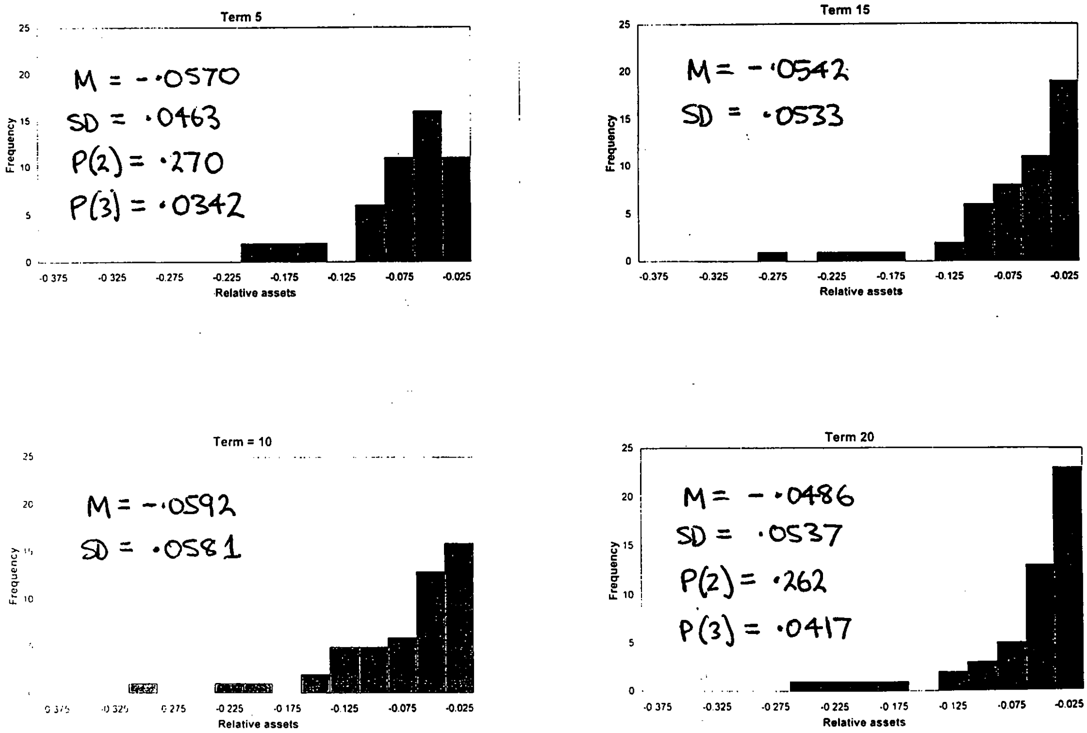
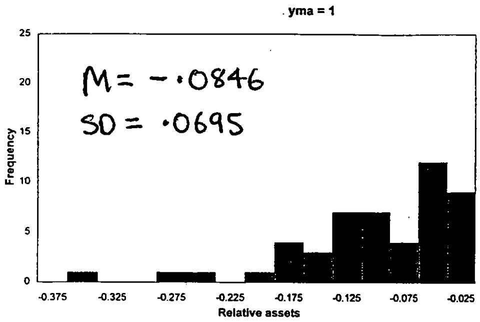
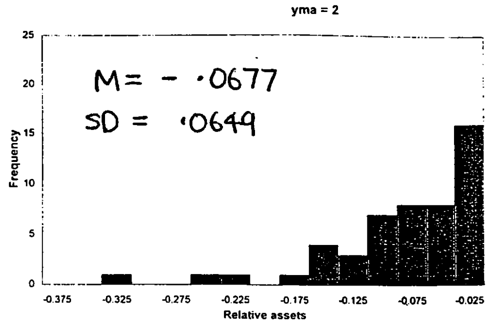
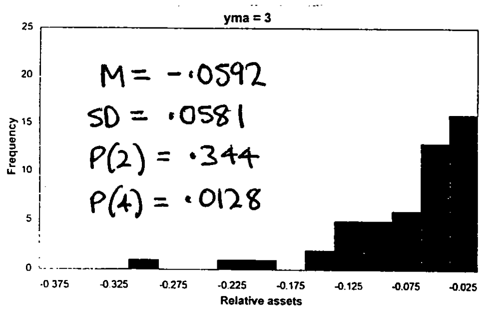


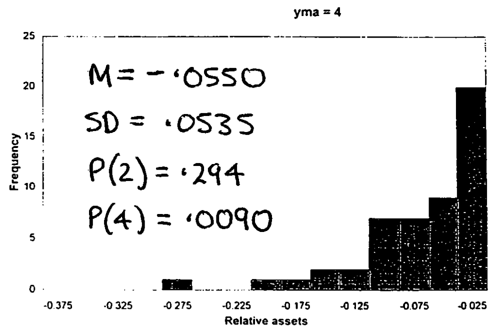
FIGURE 1c: UNEXPIRED TERM.

FIGURE 1 (END).









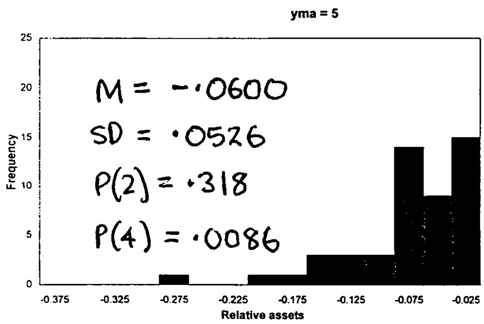
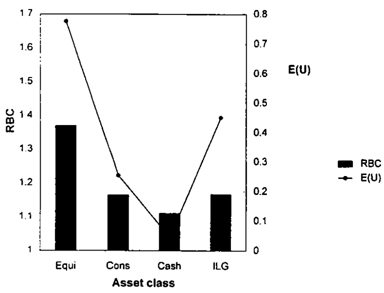


FIGURE 1d: yma FACTOR

WITH POSITIVE ASSOCIATION

2a Asset class



2b cons/equi

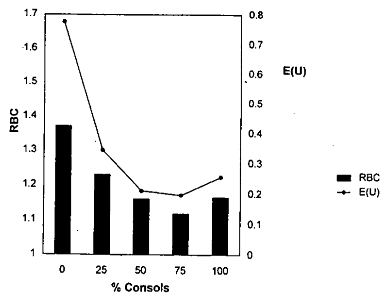
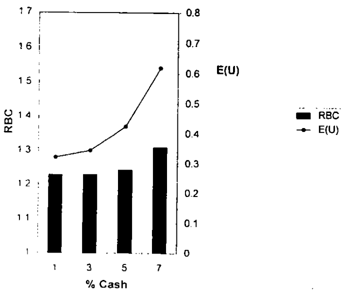
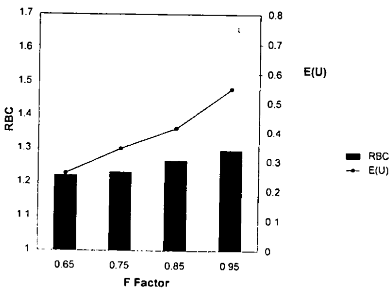


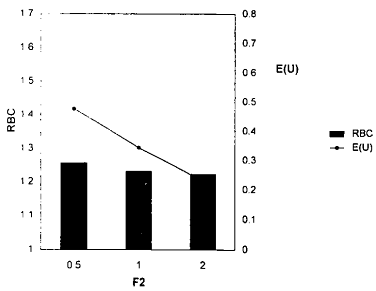
Figure 2c: g



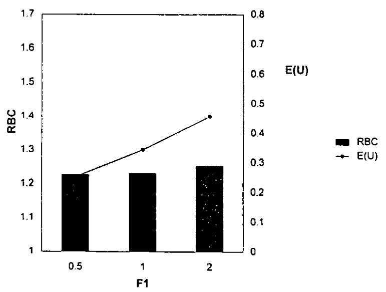
2d F Factor



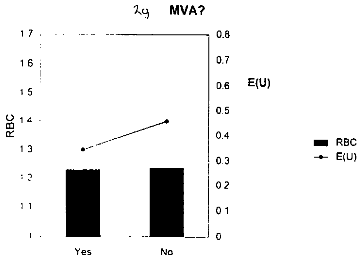
2e F2



2f F1



2g MVA?



2h expired term

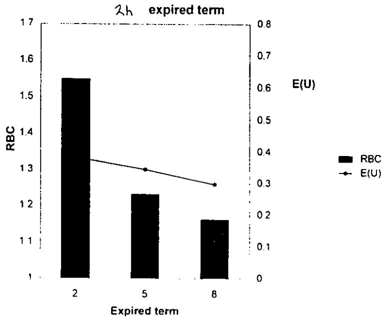


FIGURE 3: NON-PARAMETRIC RBC AND PARAMETRIC RISK (E(U) = P(2)) WITH NEGATIVE ASSOCIATION

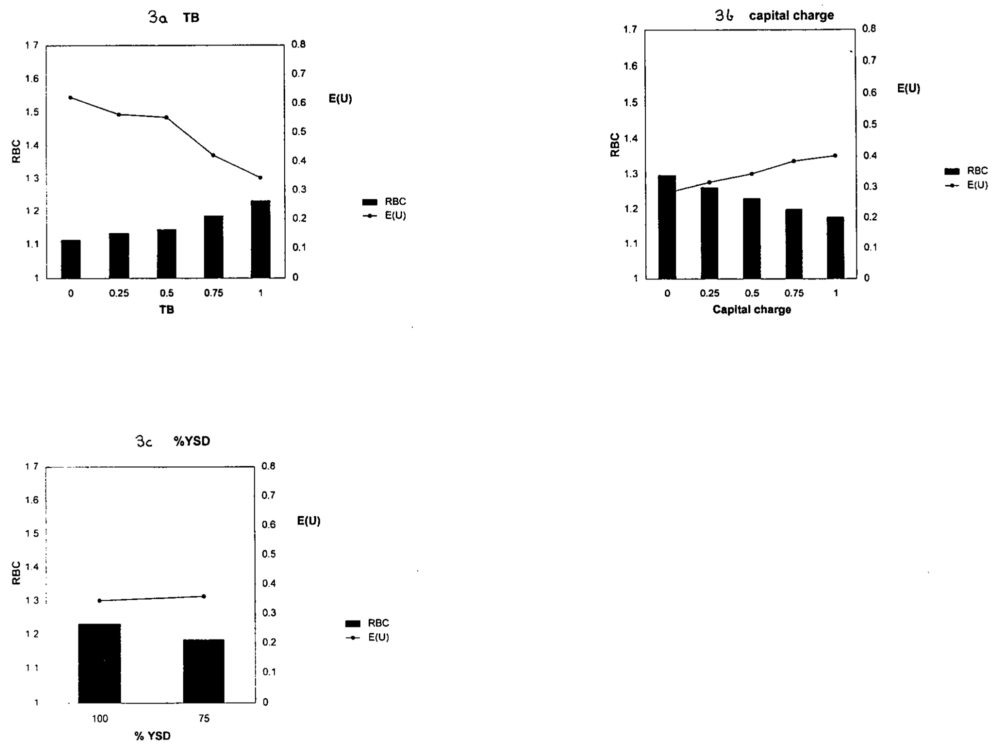
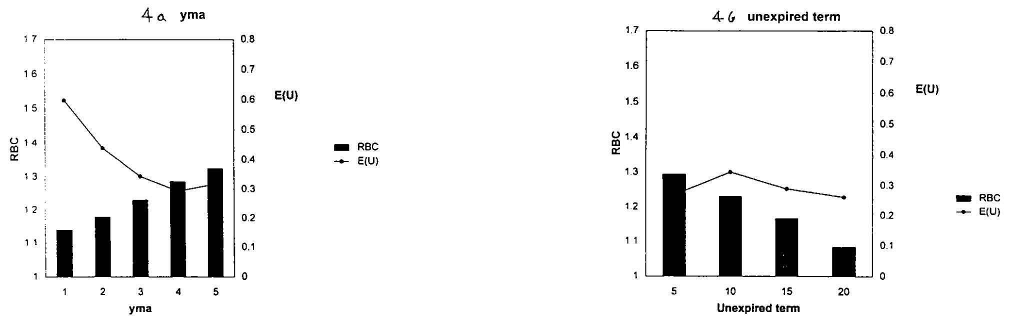


FIGURE 4: NON-PARAMETRIC RBC AND PARAMETRIC RISK WITH MIXED ASSOCIATION (E(U) = P(2)).



where the relationship varies or is uncertain are shown in Figure 4. In each Figure the factors are ranked according to the size of the range of $P(2)$ values observed, with Figure *a* having the highest value in each case. Table 1 shows the full list of factors tested, ranked in the same way.

Table 1. Variables ranked by size of range of $P(2)$ observed, for RBC fixed by the 5% NPC

Rank	Factor/variable	Range of $P(2)$	Nature of association (+/-)
1.5	Asset class	.731	+
1.5	Cash/equities	.731	+
3	Consols/equities	.580	+
4	ILG/equities	.558	+
5	Cash/ILG	.406	+ (-)
6	Consols/ILG	.321	+
7	*yma*	.305	- (+)
8	*g*	.298	+
9	*F*	.289	+
10	*TB* proportion	.279	-
11	*F2*	.247	+
12	Consols/cash	.220	+
13	*F1*	.205	+
14	*CC*	.118	-
15	MVA (yes or no)	.113	+
16	Expired term	.086	+
17	Unexpired term	.082	+ (-)
18	%*YSD* (100% or 75%)	.015	-

Key to table (where not otherwise described in the text):

ILG: Index-linked gilts.

TB proportion: The proportion of the projected terminal bonus which is assumed to be paid in the current benefit level (*CBL*), with 100% representing the standard basis; 0% represents a valuation of future benefits which ignores the expected cost of terminal bonus.

MVA: The 'yes' assumptioin is the standard model, where an MVA is applied on surrender. The 'no' assumption is where full current benefit levels are assumed to be payable on surrender, regardless of whether these are higher or lower than policy asset shares at the time.

%*YSD*: The proportion of the value of the parameter *YSD* assumed in the equity investment model (see Wilkie, 1995.) *YSD* affects the variance of the predicted equity returns, with 75% of *YSD* producing a standard deviation in returns which is approximately 83% of that of the standard equity model.

The largest ranges of $P(2)$ are associated with variations in asset mix, and these are generally (but not invariably) positively correlated with the non-parametric RBC (Figures 2a and 2b.) Significant variation in $P(2)$ is also observed for the factors which affect the benefit structure, its smoothing properties and guarantee levels: namely *yma*, *g*, *F*, *F*1 and *F*2. It is also clear from Figures 2, 3 and 4 that there is no universal consistent relationship between $P(2)$ and the non-parametric RBC, as correlations may be positive, negative, or both, although out of the factors tested (18), the majority (12) showed a positive association, as seen in Table 1.

In several cases, changes in the various factors appear to produce little appreciable effect on the non-parametric RBC, but nevertheless do vary the risk significantly according to $P(2)$. A key example is the MVA case (Figure 2g.) There is surprisingly little effect on the non-parametric RBC level of removing the MVA; however there is a considerable increase in $P(2)$ indicating that, despite appearances, the intensity of the risk has certainly increased. Factors *g*, *F*1 and *F*2 also show examples of the same phenomenon. There is therefore considerable danger that the non-parametric risk criterion will produce responses to the RBC which will tend to understate real *incresaes* in the risk: indeed, this will be the case *wherever a positive correlation between the non-parametric RBC and the parametric risk exists.*

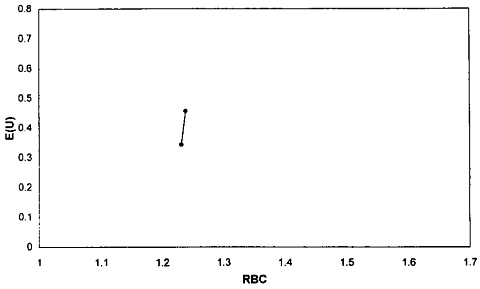
Where a negative correlation exists, then the non-parametric RBC will tend to exaggerate changes to the parametric risk. The overall smoothing factor (*yma*) is the most marked example of this, at least over the range [1,4] (see Figure 4a.) This situation can have equal dangers to the case of a positive correlation: for example, while a reduction in *yma* (ie less smoothing) implies a reduction in the non-parametric risk, the parametric risk will reduce by less or, possibly, even increase. Figures 3a and 3b show other, possibly important, examples of this phenomenon. In all these cases, therefore, *reductions* in the RBC can be overstated where these are based on non-parametric criteria.

It is interesting to consider the case for the unexpired term (Figure 4b.) The relationship between RBC and $P(2)$ seems to be negative between terms 5 and 10, and positive thereafter. However, as the absolute size of the variation in $P(2)$ is small relative to most other factors, then the non-parametric RBC would probably be reasonably appropriate in this case. The other cases in which the non-parametric RBC would appear consistent with the parametric risk are the expired term (Figure 2h) and the *%YSD* (Figure 3c.)

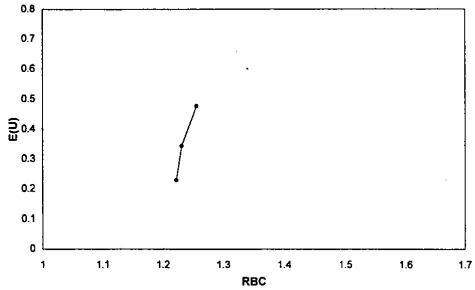
It is apparent from the above discussion that of paramount importance to the appropriateness of the non-parametric RBC is the *extent* to which $P(2)$ varies with the RBC: ie the gradient of the regression of $P(2)$ on RBC. The $P(2)$:RBC relationships are plotted in Figure 5, in rank order, with Figure 5a having the highest gradient. There are a number of significant changes to the ordering of the various factors in Figure 5 compared with Table 1. In particular the factors which affect the reversionary bonus benefits (*F*, *F*1 and *F*2) have been consistently elevated from middle to high rank, while variations in asset mix now occupy middle ranking status. The largest single change has been to the MVA factor, moving from 15th position to 1st. The higher the rank a factor is placed in Figure 5, the greater is the inability of the non-parametric risk criterion to reflect the parametric risk appropriately when deriving the RBC. This Figure is therefore of considerable significance.

IN RANK ORDER OF GRADIENT.

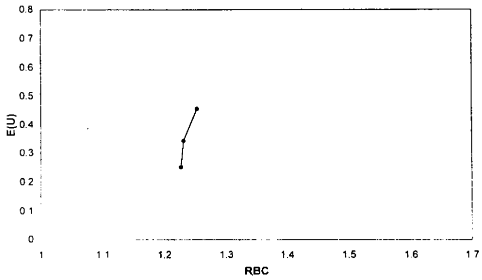
5a MVA?



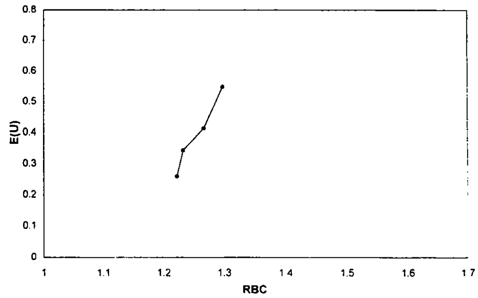
5b F2



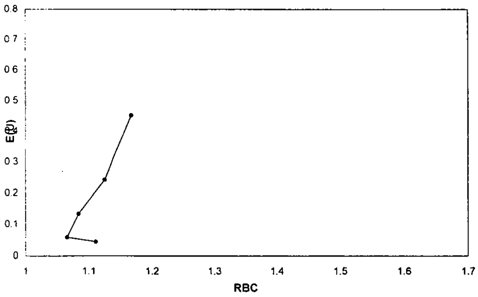
5c F1



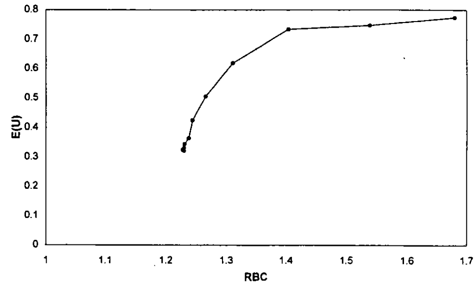
5d F Factor



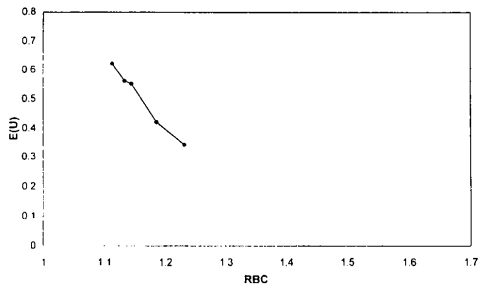
5e cash/ILG



5f g



5g TB



5h cash/equi

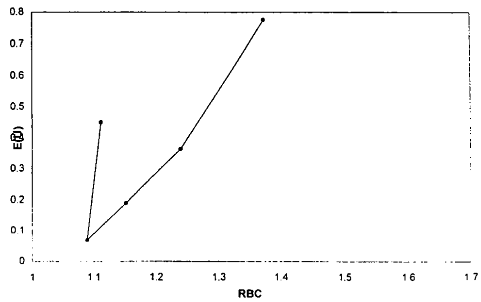
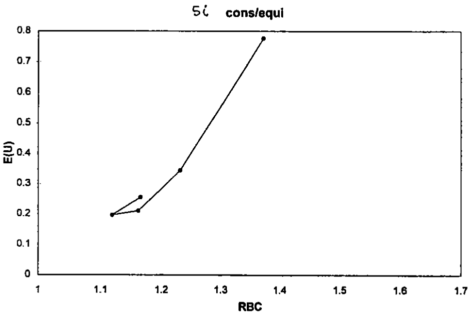
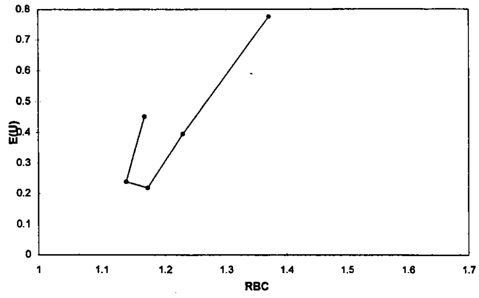


FIGURE 5 (CONTINUED)

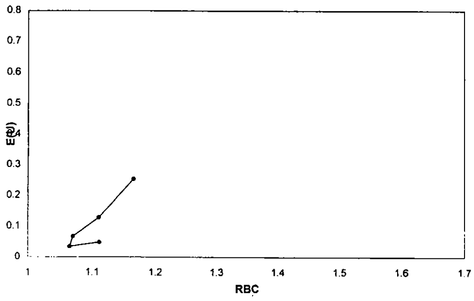
5i cons/equi



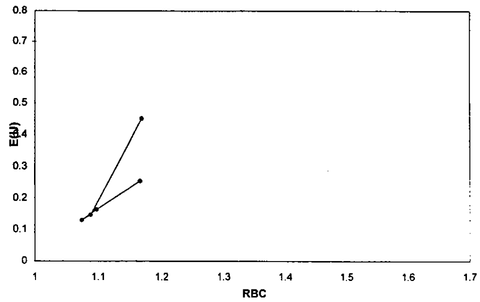
5j ILG/equi



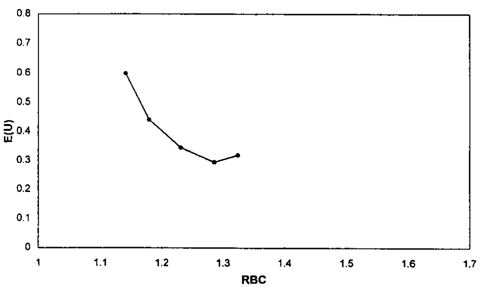
5k cons/cash



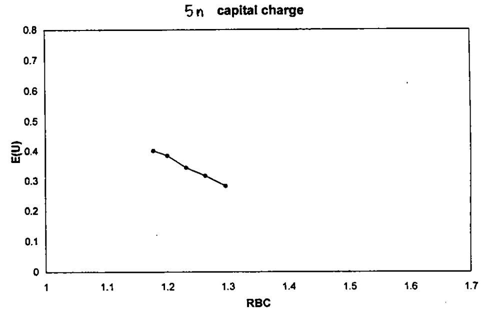
5l cons/ILG



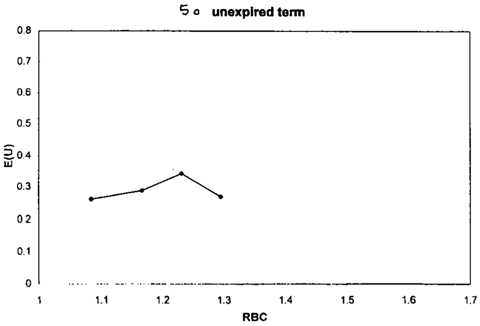
5m yma



5n capital charge



5o unexpired term



5p expired term

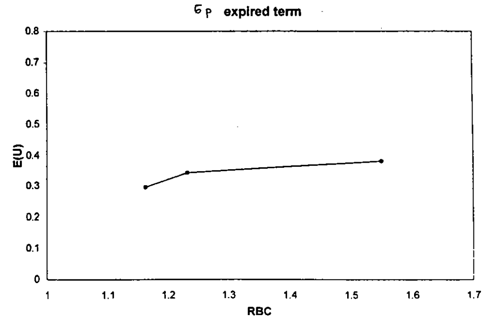
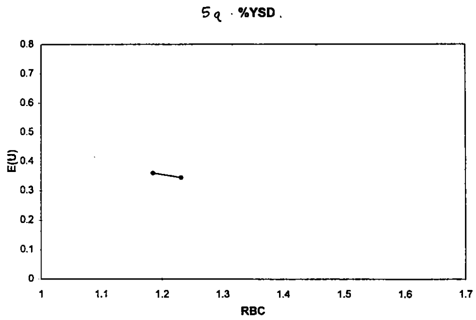


FIGURE 3



3.2 *Calculation of RBC based on the parametric criterion.*

Revised RBC calculations were made on the basis of a fixed parametric criterion: $P(2) = .344$. This was chosen, arbitrarily, as the $P(2)$ value obtained for the standard model according to the non-parametric criterion at a 5% level. Hence for the standard model, RBC calculated according to the two criteria are equal.

Figure 6 shows a comparison between non-parametric and parametric RBC for various factors, in the same rank order as Figure 5. Where positive relationships exist between the non-parametric RBC and $P(2)$, then *the parametric RBC displays more sensitivity to changes in the assumptions than the non-parametric RBC* (Figures 6: a-f,h,i). Cases having negative association show the opposite feature (Figure 6: g,j). Despite the higher *relative* sensitivity of the reversionary bonus factors, the largest absolute differences are observed for variations in the asset mix, consistent with Table 1.

Figure 6g is an excellent example of the problem with negative associations, as the parametric RBC is essentially unchanged with reducing terminal bonus proportions, while the non-parametric RBC reduces considerably. The same effect occurs at low levels of *yma*.

The choice of risk criterion will clearly have an effect upon the office's overall management policy, according to these results. Should the office be concerned to stabilise its RBC, which is a reasonable management aim, and it uses non-parametric techniques to measure its risk, then it would concentrate on maintaining a stable terminal bonus smoothing policy (*yma*), as it is to this factor that the RBC would be most sensitive. On the other hand, should its risk measurement be parametric, then it would concentrate on maintaining a stable reversionary bonus distribution policy (ie F, $F1$ and $F2$.) Which policy is appropriate, of course, depends upon which criterion is felt to be the most realistic approach to measuring the risk, but clearly there are dangers in considering only one type of risk measure when formulating decisions.

Figure 7 shows the parametric RBC for each factor along with their corresponding non-parametric risk values, ie the frequency of ruin. The factors are ranked according to the range of ruin probabilities shown; the full ordered table of factors is shown as Table 2, in which the maximum ruin probabilities are also shown.

By far the largest probabilities of ruin are associated with variations in the asset mix. In Figure 7b, for example, it can be seen that in the case of cash the office would accept a large frequency of ruin in the knowledge that the expected extent of the loss would only be small; whereas for equities a much lower ruin frequency is required as the intensity of the losses would be expected to be that much greater. While this clearly reflects the logic of the parametric approach (see section 1.3), it may be open to doubt as to whether an office would consider a ruin frequency of nearly 30% as 'acceptable' (as in the case of the 25:75 consols/ cash mix shown in Figure 7a.) It should not be forgotten, however, that the chosen parametric risk criterion ($P(2) = .344$) was arbitrarily determined by reference to the standard model, which is a 75:25 equities/consols mix. A criterion based upon a more conservative standard (for example, with less equities in the portfolio) would certainly produce much lower ruin frequencies for most alternative scenarios than the ones shown here.

It can also be seen from Table 2 that the bonus smoothing parameters are of middle rank in terms of the variability in ruin frequency. Of these the reversionary bonus factors appear again to be more significant than the *yma*.

FIGURE 6 : COMPARISON BETWEEN NON-PARAMETRIC (STANDARD) AND PARAMETRIC RBC

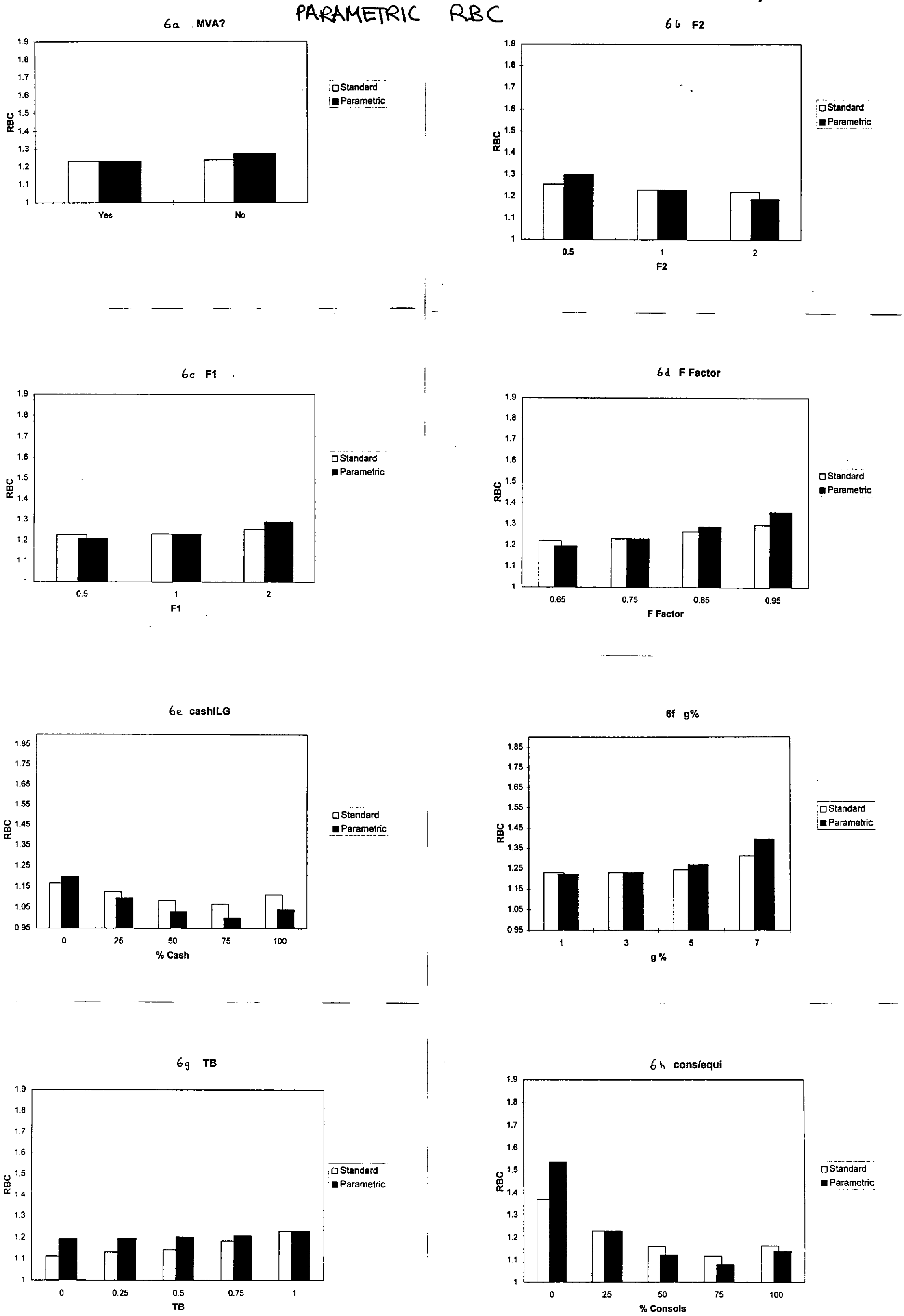
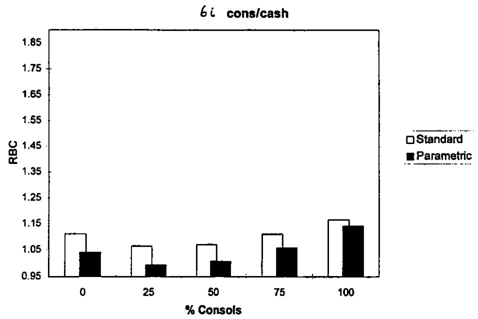
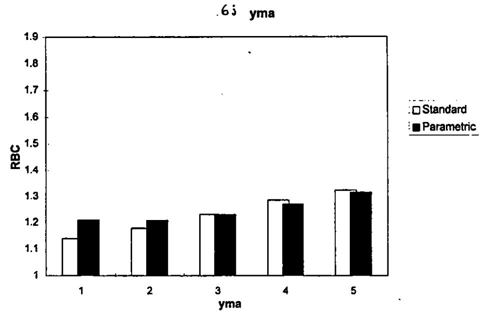


FIGURE 6 (END)

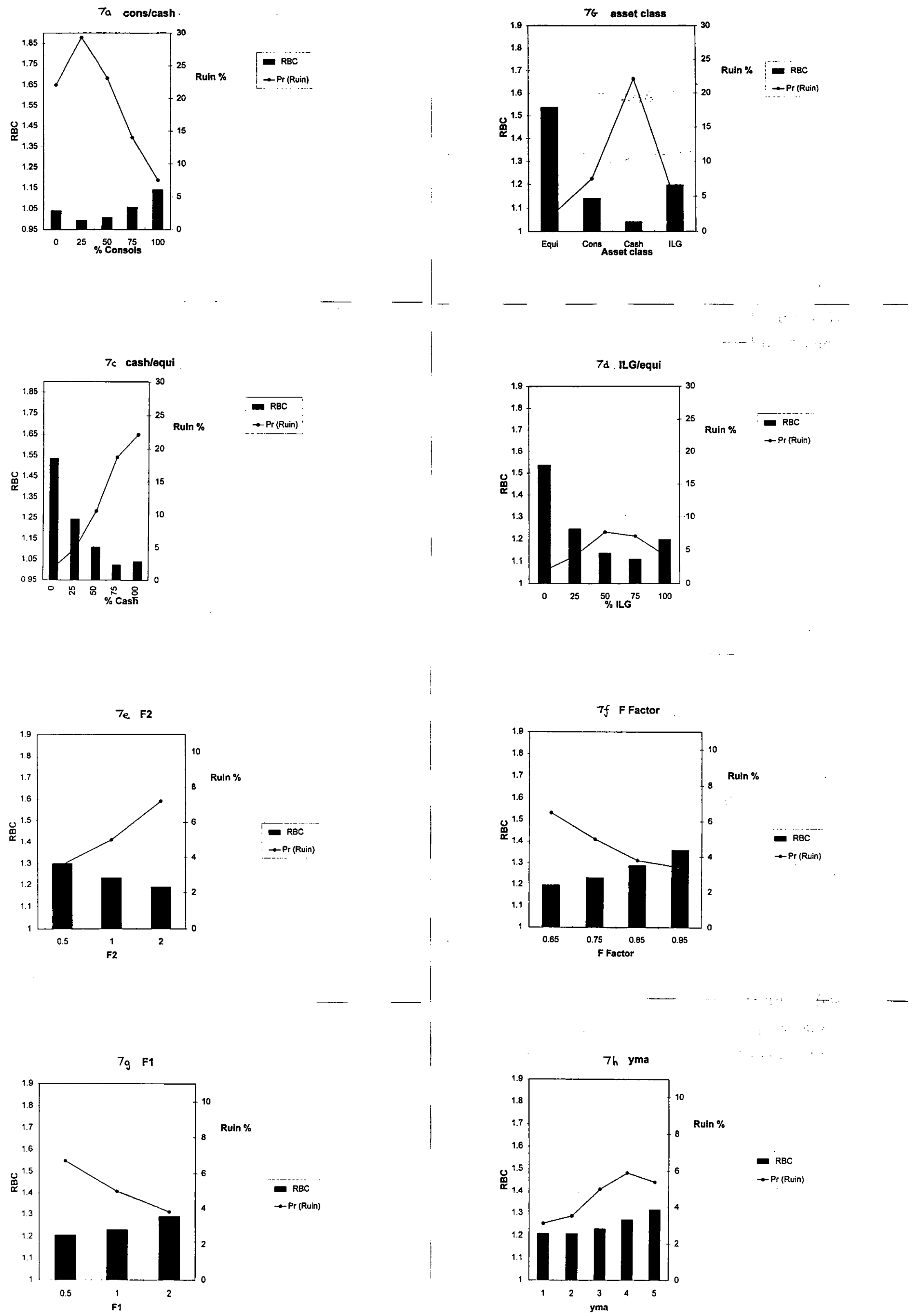


6i cons/cash
RBC
% Consols
Standard
Parametric



6j yma
RBC
yma
Standard
Parametric

FIGURE 7: PARAMETRIC RBC AND Pr(RUIN).



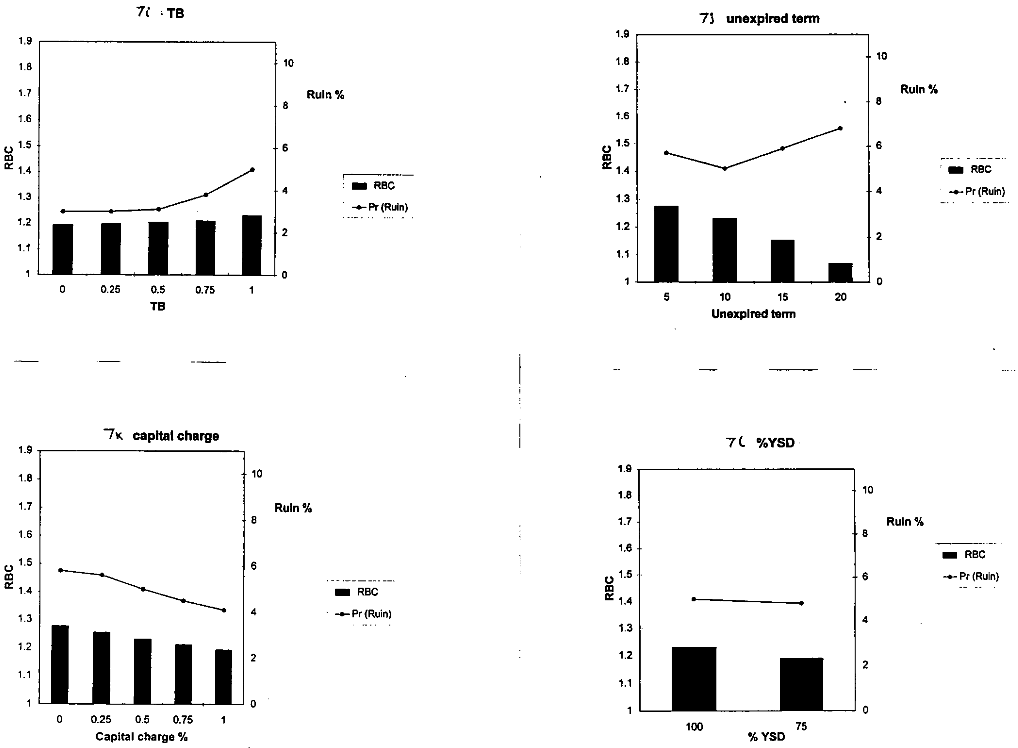

TB
RBC
Ruin %
RBC
Pr (Ruin)
TB
unexpired term
RBC
Ruin %
RBC
Pr (Ruin)
Unexpired term
capital charge
RBC
Ruin %
RBC
Pr (Ruin)
Capital charge %
%YSD
RBC
Ruin %
RBC
Pr (Ruin)
% YSD

Table 2. Variables ranked according to the size of range of Pr(ruin) observed, for RBC fixed by $P(2) = .344$.

Rank	Factor/variable	Range of Pr(ruin) %	Maximum Pr(ruin) %
1	Consols/cash	21.8	29.3
2.5	Asset classes	20.1	22.1
2.5	Cash/equities	20.1	22.1
4	Cash/ILG	19.6	23.8
5	Consols/ILG	11.2	15.4
6	Consols/Equities	8.3	10.3
7	Equities/ILG	5.7	7.7
8	*F2*	3.6	7.2
9	*F*	3.1	6.5
10	*F1*	2.9	6.7
11	*yma*	2.8	5.9
12	*g*	2.3	5.6
13	*TB* proportion	2.0	5.0
14	Unexpired term	1.8	6.8
15	*CC*	1.7	5.8
16	Expired term	1.6	6.0
17	MVA (yes or no)	1.5	5.0
18	*%YSD* (100% or 75%)	0.2	5.0

Examination of Figure 7 show that opportunities may exist for reducing both the parametric RBC and the frequency of ruin. In Figure 7c a 75:25 cash/equity mix requires lower parametric RBC and produces a lower ruin frequency than 100% cash. Similarly a 75:25 index-linked gilt/equity mix would be preferred to a 50:50 mix. The office would also clearly prefer less terminal bonus smoothing ($yma << 4$), and higher charges for capital (Figures 7h and 7k.) A policy with a ten year unexpired term also requires less capital and produces a lower ruin frequency than a five year unexpired term.

3.3 *The Effect of using Non-Parametric or Parametric Criteria as Constraints in Decision-Making*

An example of the way in which management decisions might vary according to the risk measurement criterion used has already been described (see section 3.2.) Furthermore it was suggested at the end of that section that certain decisions might be preferred in which both the capital requirements and the probability of ruin can be minimised. However management decisions are not made purely on the criteria of reducing risk and/or capital. The returns to the office's investors, be they policyholders or shareholders, must also be considered.

Booth, Chadburn & Ong (1996) described a methodology for decision-making which, as its objective, seeks to maximise the utility of the investors' returns subject to maintaining an acceptable probability of ruin: ie a non-parametric risk criterion is used as a constraint in the decision-making process. The question then arises as to how such a decision might be affected by framing the constraint in terms of parametric rather than non-parametric criteria.

It was decided to use the following quantity as proportionate to the utility of the policy proceeds to the policyholders:

$$PU \quad = \quad \frac{1}{S}\sum_{i=1}^{S} \ln\left(CBL(m) + \min\left(0, \frac{A(m)}{N(m)}\right)\right) \tag{3.1}$$

where m = the maturity date of the tranche of business;

$A(m)$ = total projected assets of the office remaining at the run-off of business at time m; and

$N(m)$ = the number of maturing policies at time m.

This formula therefore allows for any losses on termination to be shared among its outgoing policyholders. The logarithmic utility function is used because of its characteristic of constant relative risk aversion, and can be applied in this case as all outcomes are almost certain to be positive (see Booth, Chadburn and Ong, 1996.) While in theory benefits paid on death and surrender before maturity should also be included as part of the policyholders' utility, these amounts are relatively small and would introduce considerable additional complexity. As the present exercise is to provide only an illustration of the possible effect of applying different risk constraints upon decision-making, then the chosen approach was felt to be adequate.

PU can be used as a relative measure of the expected utility to policyholders of any given scenario provided that any other payments to or from the policyholders are constant (ignoring death and surrender claims, as above.) Maturing policyholders in each simulation will always have paid the same total premium; however it is also necessary to assume a constant amount of assets (ie capital) at the valuation date, as this capital can be considered to have been contributed from the past premium payments. (Alternatively the capital which is in excess of policy asset shares might be considered as being contributed by the shareholders in a proprietary company: however for simplicity the presence of shareholders will be ignored for the purpose of this illustration.)

The decision concerned in this example is the choice of asset mix between consols and equities, assumed to stay as constant as possible through the projection period (see section 2.4.) Hence for a given amount of intial assets (RBC) the office is assumed to choose the asset mix which maximises the expected utility (*PU*) for the policyholders, subject to the chosen risk constraint being satisfied. Again, for illustration, the non-parametric risk constraint used is that the probability of ruin should not exceed 5%, while the parametric constraint is that $P(2) \leq .344$.

The results of this exercise, for different initial levels of assets, are shown in Table 3 and in Figures 8 and 9. In the event it was found that the utility maximising decision for all the cases considered always equated to the maximum allowable risk under the given constraint. Hence the cases under the different initial conditions all exhibit equal risk

Figure 8
Utility under Pr(ruin) = 5%, P(2) = .344

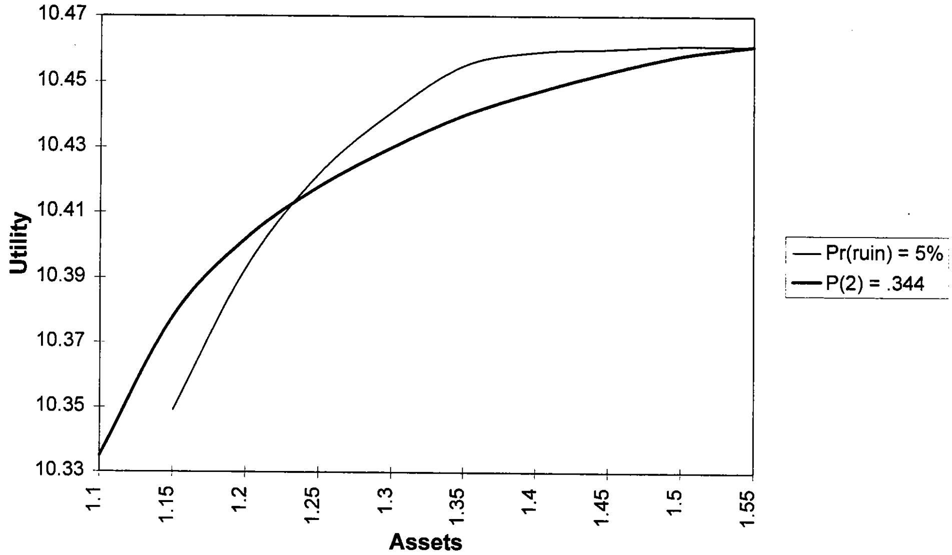
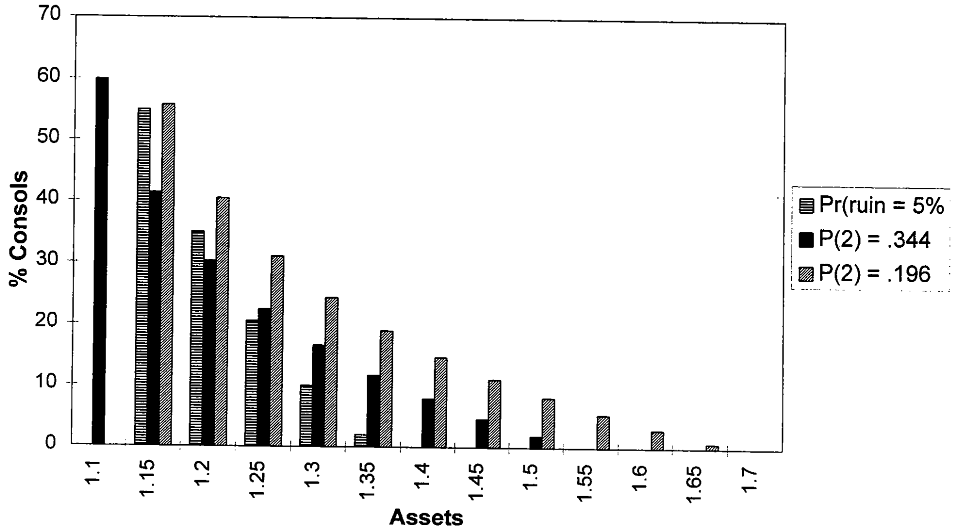


Figure 9
Optimal consols/equities decisions under different risk constraints



according to the criterion used.

Table 3. Optimal consols/equities decisions under different risk constraints

Table 3a. Optimal decisions according to non-parametric constraint

Initial assets	% consols	Expected utility	Mean	Standard deviation
1.15	55	10.349	31702	5679
1.20	35	10.393	33507	7986
1.25	20.5	10.422	34856	9938
1.30	10	10.441	35879	11548
1.35	2	10.455	36666	12905
1.40	0	10.459	36887	13246
1.45	0	10.460	36914	13233
1.50	0	10.461	36933	13223
1.55	0	10.461	36946	13217

Table 3b. Optimal decisions according to parametric constraint

Initial assets	% consols	Expected utility	Mean	Standard deviation
1.10	59.9	10.335	31203	5200
1.15	41.41	10.378	32886	7219
1.20	30.3	10.402	33923	8595
1.25	22.45	10.418	34677	9655
1.30	16.6	10.430	35255	10506
1.35	11.72	10.440	35748	11256
1.40	7.93	10.447	36133	11863
1.45	4.65	10.453	36469	12414
1.50	1.9	10.458	36749	12888
1.55	0	10.461	36946	13217

Key to Table 3:

Initial assets:	total initial assets as a proportion of policy asset shares;
Expected utility:	expected utility of policyholders' returns;
Mean:	mean of policyholders' returns; and
Standard deviation:	standard deviation of policyholders' returns.

Figure 8 shows the variation in policyholders' expected utility associated with the optimal decisions under each risk constraint. Maximum utility under both criteria are equal at an asset level of 1.231 of policy asset share, which equates to the standard basis of 25% consols and 75% equities. At other asset levels the optimal decisions diverge (see Figure 9) with corresponding divergence between maximum expected utilities, as shown in Figure 8. For example, with initial assets of 1.35, the non-parametric constraint will allow the office to invest in as much as 98% equities, with expected utility of 10.455. Adherence to the parametric constraint would, however, restrict the investment policy to about 88% equities, with expected utility of 10.440. (While the difference in expected utility appears slight, this translates into appreciable differences in expected return and in the variability of return to policyholders, as seen in Table 3.) At asset levels below 1.231, the effect is reversed, with the parametric constraint allowing the greater proportion in equities to be invested. Indeed, at asset levels of 1.10, there is no equity/consols mix which will meet the non-parametric constraint. According to these examples, an absolute difference in optimal assets mix of as much as almost 14% can be obtained (see Table 3, initial assets = 1.15.)

Given the assumptions made in this example, and if it is assumed that the parametric criterion is the more appropriate of the two, then it can be seen that the non-parametric criterion can lead to overly conservative decisions at low asset levels (ie below the cross-over point), and to overly risky decisions where solvency is higher (ie above the cross-over point.) The former case can therefore be considered as imposing an unfair penalty on policyholders' returns, while the latter would constitute an unreasonable risk to solvency.

Of critical importance to these considerations is the cross-over point between the curves such as shown in Figure 8. It was suggested earlier that this point was probably on the high side, as it bases the parametric risk constraint on a standard asset mix of 75% equities. A more conservative standard would cause the cross-over point to be moved significantly to the south west of the curves in Figure 8; for example, if the 25:75 equity/consols mix were used as a standard, the constraint would be $P(2) = .196$ instead of $P(2) = .344$, and the curves would cross at initial assets of 1.118 instead of 1.231. This situation is shown in Figure 10 (with optimal decisions also shown in Figure 9), and shows that here decisions made according to the non-prametric constraint would be *too risky at almost every solvency level*, as almost all viable asset levels are to the right of the cross-over point. The safety of using a non-parametric constraint in life office decision-making must therefore be seriously questioned from these results.

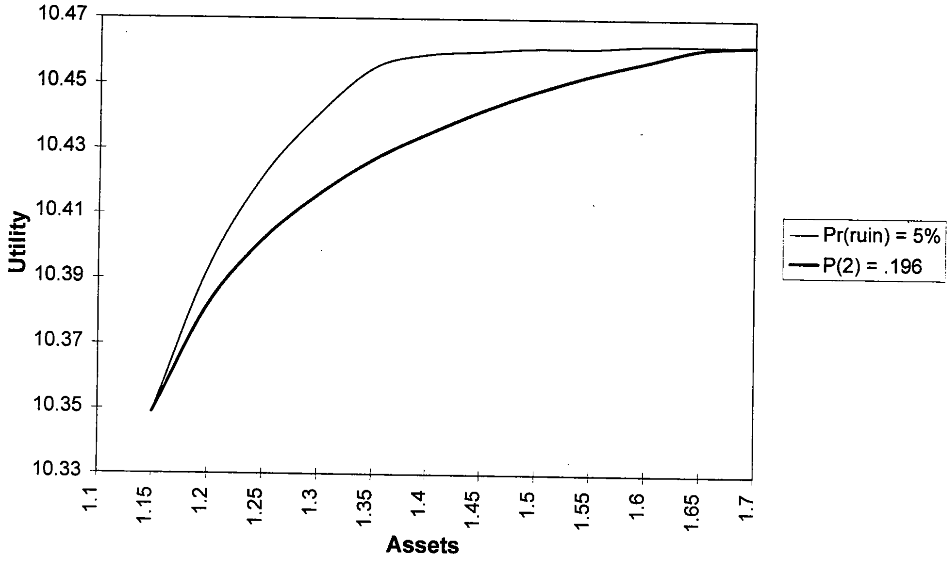
4. SUMMARY

4.1 A non-parametric risk criterion is not a satisfactory proxy for parametric risk in many cases.

4.2 A positive correlation between non-prametric RBC and $P(r)$ means that increases in RBC are understated when using a non-parametric risk criterion. A negative correlation means that decreases in RBC are overstated when using a non-parametric criterion. Two-thirds of the eighteen factors investigated in this paper showed a positive correlation.

4.3 The higher the absolute value of the gradient of the regression of $P(r)$ on the non-parametric RBC for any factor, the lower the ability of the non-parametric risk criterion to reflect the parametric risk adequately. The higher this gradient the more the RBC calculated according to the two criteria will diverge in responses to changes in any factor. The factors which exhibited greatest sensitivity in this respect were those associated with

Figure 10
Utility under Pr(ruin) = 5%, P(2) = .196



determining guaranteed benefits and reversionary bonus levels; factors affecting terminal bonuses and investment mix showed less sensitivity; the term of the contract and the implied volatility of the investment model showed relatively little sensitivity, although differences in the distribution of losses for a given non-parametric risk level could still occur (for example, as with the unexpired term.)

4.4 Factors with a positive gradient (as in 4.3) will tend to lead to more stable RBC according to non-parametric rather than parametric criteria; factors with a negative gradient produce the opposite effect. This can affect management decisions which may be concerned with minimising the variability of the office's capital requirements.

4.5 RBC calculated according to parametric criteria can produce large variations in the probability of ruin, particularly for variations in the asset mix; this behaviour was shown to be a reasonable effect of the parametric approach. Possibilities may sometimes exist to make decisions which can both reduce the capital required and reduce the probability of insolvency, although such decisions should not be made in isolation of the other interests of the life office, such as in maximising the utility of returns to its policyholders or shareholders.

4.6 The use of non-parametric risk constraints in decision-making can lead to overly conservative or risky decisions, depending upon the office's solvency level and also upon what is considered to be an acceptable level of parametric risk. It is argued that a reasonably conservative assessment of this level could lead, for example, to optimal investment decisions which were too risky at nearly all solvency levels, if these decisions were based solely on meeting a non-parametric risk constraint.

5. CONCLUSION

A parametric risk criterion (such as $P(r)$ as defined in this paper) would seem to have considerable theoretical advantage over a non-parametric criterion (such as the probability of ruin) when assessing life office risk. The various dangers involved in the use of a non-parametric risk measure for assessing Risk-Based Capital and other aspects of decision-making are well illustrated by the examples presented here. However there remains two fundamental difficulties with the method in practice:

(i) how should an appropriate risk function (such as $R_r(x)$) be obtained; and

(ii) given that an appropriate *measure* of parametric risk can be obtained (such as $P(r)$) then how should the appropriate *level* for this measure be ascertained for the purpose of defining a risk constraint?

A perceived problem with a parametric risk measure such as $P(r)$ is that it is much more difficult to visualise than a 'probability of ruin'. While this may lead practitioners to be suspicious of the parametric approach, an aversion purely on these grounds would be illogical. However, the parametric method requires more assumptions to be made regarding the risk preferences of the life office's investors, and can therefore be viewed as more subjective. Clarkson (1989) illustrates ways in which the risk preferences of investors could be ascertained in practice, leading to a construction of the risk function based upon relatively objective information. Application of these procedures should therefore lead to a more objective and credible parametric basis of risk assessment, which should lead to more appropriate decision-making by the life office on behalf of its investors.

Fixing the level of the parametric risk constraint could be done by the method outlined in this paper: ie to choose a reasonably conservative standard set of assumptions and to use the level of parametric risk which equates to a given non-parametric risk level under these circumstances.

It would appear that the parametric risk measure $P(r)$ defined here is a flexible and rational tool for theoretical work. However the fact that it forms part of the investigator's model assumptions, both in terms of the structure of the risk function and of its parameterisation, must never be overlooked when drawing inferences from the results of such investigations. This warning must therefore be applied to the results presented in this paper.

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