A STOCHASTIC APPROACH TO PENSION SCHEME FUNDING

by

I D WRIGHT

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Department of Actuarial Science and Statistics
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BY I.D. WRIGHT, B.SC, PH.D.

ABSTRACT

Stochastic asset-liability modelling is already widely used as a means of helping to frame strategic investment policy. The methodology allows an objective measure of the level of risk inherent in any particular investment strategy and, thus, enables an optimal investment strategy to be determined which maximises the expected investment return subject to a given level of risk. For a defined benefit pension scheme, the level of risk can be measured by, for example, the volatility of the required contribution rate or the probability of insolvency on a particular valuation basis (commonly, a wind-up basis).

However, stochastic asset-liability modelling has many other potential applications in the field of actuarial science. For a defined benefit pension scheme, one of these applications is in helping to frame funding strategy.

The method proposed uses stochastic simulation techniques to determine the level of the required contribution rate payable such that the probability that the scheme is insolvent on a wind-up basis at the following valuation date is at most α%, where the value of α is chosen in advance and, hence, determines the pace of funding. As such, the need to incorporate specific margins on the grounds of prudence in the deterministic parameters used in the valuation basis is removed.

Stochastic simulation techniques can be very time-consuming as a result of the large number of simulations required, and as such may be considered to have limited practical application. However, the proposed method is extended further to enable a deterministic approximation of the required contribution rate to be made, thereby avoiding the need for a large number of simulations.

1. INTRODUCTION

Stochastic asset-liability modelling is well established as a means of framing strategic investment policy for a defined benefit pension scheme. For a given level of future contributions, the level of risk (and reward) for different long term investment strategies can be compared objectively. An optimal asset allocation, which maximises the expected rate of return on the assets for a chosen level of risk, can then be determined.

For a defined benefit pension scheme, an appropriate measure of the level of risk could be the likelihood that the assets built up are insufficient to meet the accrued liabilities as and when they fall due.

However, as defined benefit pension schemes are usually considered to be largely passive investors, it may be more appropriate to fix the long-term investment strategy and compare the effect on the level of risk of different levels of future contributions. Then, the higher the level of future contributions, the larger the size of the fund built up and, thus, the lower the level of the risk that the fund is unable to cover the accrued liabilities.

The traditional approach adopted in the on-going valuation of a final salary pension scheme is to incorporate specific margins on the grounds of prudence in each of the deterministic parameters used in the valuation basis. The effect of these margins is that the fund built up is larger than that which would be needed, in normal circumstances, to cover the accrued liabilities and a valuation surplus can be expected to arise in future. However, as stated by Loades (1988), "... it is not easy to judge whether the combined effect of these margins is inadequate, adequate or excessive".

If the margins incorporated are inadequate, then the size of the fund built up may, indeed, be insufficient to meet the accrued liabilities as and when they fall due. For an on-going scheme, this is unlikely to be a problem, as the current benefit outgo can largely be met by the contribution income received and any shortfall in the value of the assets held compared with the value of the accrued liabilities made up gradually over time. However, in the event of a pension scheme being wound up, the assets held may well have to be realised quickly to purchase immediate annuities in respect of existing current pensioners and to meet transfer value obligations in respect of existing active members and deferred pensioners. In this case, if the size of the fund built up is insufficient to cover the accrued liabilities and the sponsoring employer is unwilling or unable to make up the shortfall, then the level of benefits provided will be below that expected.

On the other hand, if the margins incorporated are excessive, money which could better be used elsewhere in the company will be tied up unnecessarily within the pension scheme resulting in an 'opportunity cost' to the sponsoring employer, and, in extreme circumstances, the tax-exempt status of the fund may be jeopardised (or, excessive surplus, on the statutory Inland Revenue basis laid down in the Finance Act 1986, may have to be refunded to the company - subject to tax of 40%).

Loades (1988) states that "... there has been little discussion on what level of security is desirable".

The method proposed below is based on the approach suggested by Loades (1988), and, for a model final salary pension scheme, attempts to estimate the contribution rate required to be paid in the period up to the next valuation date so that the probability of insolvency on wind-up at that date is, at most, some pre-determined value α%.

For any particular value of α, the approach used to calculate the required contribution rate requires a large number of independent, but equally likely, simulations of the future progress of the scheme in the period up to the next valuation date using a suitable stochastic asset model. However, in practice, this is unlikely to be possible because of the constraints of time and, thus, a very simple method for estimating the resulting contribution rate deterministically is also proposed.

2. AN OVERVIEW OF THE PROPOSED METHOD

2.1 *Introduction*

2.1.1 *A model pension scheme*

Consider a model final salary pension scheme were it is assumed that :

a) death in service benefits (both lump sum and widow's pension) are insured annually with a life office by means of an additional recurrent single premium;
b) on withdrawal before retirement, a transfer value is paid immediately in lieu of the deferred pension accrued; and
c) on retirement, an immediate annuity is purchased from a life office which exactly matches the liability of the scheme.

Thus, the model scheme will contain only active members.

2.1.2 *The stochastic asset model used*

The stochastic asset model used for projecting the model scheme is the Wilkie (1995) model. This model is the best-known and most widely-used stochastic asset model in the UK at present, however, it has attracted a great deal of criticism since originally published in 1986 (see, for example, Kitts, 1990; Geoghegan *et al*, 1992 and Huber, 1997).

Both the assets and the liabilities of the model scheme are assumed to behave stochastically, as investment and salary growth experience is assumed to follow the series generated using this model. The Wilkie stochastic asset model generates the following key series :

a) the force of price inflation in year t, $I(t)$
   - then, the retail prices index at time t, $Q(t) = Q(t-1).\exp[I(t)]$
b) the force of earnings inflation in year t, $J(t)$
   - then, the average earnings index at time t, $W(t) = W(t-1).\exp[J(t)]$
c) the share dividend yield at time t, $Y(t)$

d) the force of share dividend growth in year t, $K(t)$

- then, the share dividend index at time t, $D(t) = D(t-1).\exp[K(t)]$
- then, the share price index at time t, $P(t) = \dfrac{D(t)}{Y(t)}$
- then, the gross 'rolled-up' share index at time t (assuming the re-investment of dividend income), $PR(t) = PR(t-1).\dfrac{P(t)+D(t)}{P(t-1)}$

e) the yield on undated fixed-interest gilts at time t, $C(t)$

- then, the gross 'rolled-up' undated fixed-interest gilt index at time t (assuming the re-investment of coupon income),
  $CR(t) = CR(t-1).\left(\dfrac{1}{C(t)}+1\right).C(t-1)$

f) the yield on cash deposits at time t, $B(t)$

- then, the gross 'rolled-up' cash index at time t (assuming the re-investment of income), $BR(t) = BR(t-1).[1+B(t-1)]$
- thus, cash is, effectively, treated as a one-year bond

g) the yield on long-dated index-linked gilts at time t, $R(t)$

- then, the gross 'rolled-up' long-dated index-linked gilt index at time t (assuming the re-investment of coupon income),
  $RR(t) = RR(t-1).\left(\dfrac{1}{R(t)}+1\right).R(t-1).\dfrac{Q(t)}{Q(t-1)}$

h) the property rental yield at time t, $Z(t)$

i) the force of property rental growth in year t, $EK(t)$

- then, the property rental index at time t, $E(t) = E(t-1).\exp[EK(t)]$
- then, the property price index at time t, $A(t) = \dfrac{E(t)}{Z(t)}$
- then, the gross 'rolled-up' property index at time t (assuming the re-investment of rental income), $AR(t) = AR(t-1).\dfrac{A(t)+E(t)}{A(t-1)}$

Appendix A gives details of the Wilkie model.

Subsequent to the work detailed below, alternative stochastic asset models have been proposed by, amongst others, Dyson & Exley (1995), Smith (1996) and Wright (1997). Whether or not the proposed method is as successful when used in conjunction with these or other models remains to be explored.

2.1.3 *Valuing the initial scheme liabilities on discontinuance*

The discontinuance funding level at the start of the projection period, denoted by $DFL(0)$, is given by :

$$DFL(0) = \frac{MVA(0)}{AL_{\text{dis}}(0)} = \frac{\text{initial market value of scheme assets}}{\text{initial value of accrued liabilities on a discontinuance basis}}$$

On discontinuance, it is common for each active member to be granted a transfer value in lieu of any deferred pension accrued. The size of this transfer value should be calculated in accordance with the guidelines laid down in GN11.

Thus, the initial value of the accrued liabilities on a discontinuance basis, $AL_{\text{dis}}(0)$, is given by the sum of the current cash-equivalent transfer values for each of the active members.

The key requirement of GN11 is that the basis used to calculate these cash-equivalent transfer values should be market-related. Thus :

a) since the Social Security Act 1990 requires that, during the period of deferment, a deferred pension is revalued in line with RPI increases (subject to a maximum of 5% *pa*), the net rate of interest in deferment should be based on the *current* real yield available on long-dated index-linked gilts (but with an allowance for re-investment);
b) as the model scheme is assumed to provide LPI increases on pensions in payment (in accordance with the requirements of the Social Security Act 1990), so that the net rate of interest in possession is also based on the *current* real yield available on long-dated index-linked gilts (but with an allowance for re-investment);
c) the mortality of deferred pensioners prior to retirement should be consistent with the assumptions used in the ongoing valuation basis - in this case, assume A67/70(ult) standard mortality;
d) the mortality of deferred pensioners (and spouses) after retirement should be consistent with the assumptions used in the ongoing valuation basis - in this case, assume PA(90) standard mortality, with spouses rated down 2 years;
e) the marital statistics should be consistent with the assumptions used in the ongoing valuation basis - in this case, assume, spouses are, on average, 3 years younger; and 90% of members are married at death.

Then, for a typical active member of age x at time 0, with current pensionable salary $S(x,0)$ and past pensionable service $n(x,0)$, the size of the cash-equivalent transfer value, $TV(x,0)$, is :

$$TV(x,0) = g \times n(x,0) \times S(x,0) \times v^{\text{NRA}-x}(0) \times \frac{l_{\text{NRA}}}{l_x} \times a_{\text{NRA}}(0)$$

where

a) g is the annual rate of pension accrual;
b) NRA is the scheme normal retirement age;
c) the discount factor in deferment at time 0, $v^{\text{NRA}-x}(0)$, is calculated using the current real yield on long-dated index-linked gilts;
   - the real yield on long-dated index-linked gilts at time 0 is given by $R(0)$
   - using the 'neutral' starting values for the Wilkie model, $R(0) = 4.0\%$
   - Appendix A.10 gives the 'neutral' starting values for the Wilkie model
d) $\dfrac{l_{\text{NRA}}}{l_x}$ is calculated using A67/70(ult) standard mortality;
e) the annuity factor in possession at time 0, $a_{\text{NRA}}(0)$, includes a contingent spouse's annuity and is calculated using the current real yield on long-dated index-linked gilts, given by $R(0)$, and PA(90) standard mortality.

Suppose that the initial market value of the assets, $MVA(0)$, is such that the initial discontinuance funding level, $DFL(0)$, is equal to $(100+f)\%$.

2.1.4 *The inter-valuation period*

Suppose that valuations of the model scheme are carried out triennially.

Then, for each realisation of the future investment experience generated using the Wilkie model, the first step is to calculate the contribution rate required to be paid during the 3 year inter-valuation period to give a discontinuance funding level at time 3, $DFL(3)$, of exactly 100%. This required contribution rate is denoted by CR.

Membership movements during the inter-valuation period (i.e. deaths, withdrawals and retirements) are not stochastic, but rather are assumed to occur at the end of each year during the inter-valuation period and to be in accordance with some chosen service table.

In addition, new entrants to the scheme during the inter-valuation period are assumed to be such that the scheme membership profile remains stable with respect to age, pensionable salary (in real terms) and past pensionable service.

As at time 0, the value of the accrued liabilities on a discontinuance basis at time 3, $AL_{\text{dis}}(3)$, is given by the sum of the current cash-equivalent transfer values for each of the active members.

Now, for a typical active member of age x at time 3, with current pensionable salary $S(x,3)$ and past pensionable service $n(x,3)$, the size of the cash-equivalent transfer value, $TV(x,3)$, is :

$$TV(x,3) = g \times n(x,3) \times S(x,3) \times v^{\text{NRA}-x}(3) \times \frac{l_{\text{NRA}}}{l_x} \times a_{\text{NRA}}(3)$$

where

a) the discount factor in deferment at time 3, $v^{\text{NRA}-x}(3)$, is calculated using the current real yield on long-dated index-linked gilts;
   - the real yield on long-dated index-linked gilts at time 3 is given by $R(3)$

b) the annuity factor in possession at time 3, $a_{\text{NRA}}(3)$, includes a contingent spouse's annuity and is calculated using the current real yield on long-dated index-linked gilts, given by $R(3)$.

As the model scheme remains stable with respect to age, pensionable salary and past pensionable service it can be shown that :

$$\begin{aligned} TV(x,3) &= g \times n(x,3) \times S(x,3) \times v^{\text{NRA}-x}(3) \times \frac{l_{\text{NRA}}}{l_x} \times a_{\text{NRA}}(3) \\ &\approx g \times n(x,0) \times \left[S(x,0) \times \frac{W(3)}{W(0)} \right] \\ &\quad \times \left[v^{\text{NRA}-x}(0) \times \left(1 + \left[R(0) - R(3)\right]\right)^{\text{NRA}-x} \right] \times \frac{l_{\text{NRA}}}{l_x} \times a_{\text{NRA}}(3) \end{aligned}$$

Also, a commonly-used 'rule of thumb' by pensions practitioners is that a decrease of 1% *pa* in the net rate of interest in possession leads to an approximate increase of 8% in the value of liabilities (if all other things remain equal). Thus, we have :

$$a_{\text{NRA}}(3) \approx a_{\text{NRA}}(0) \times \left(1 + 8 \times \left[R(0) - R(3)\right]\right)$$

Thus, as the model scheme is assumed to remain stable, the change in the value of the accrued liabilities on a discontinuance basis between time 0 and time 3 depends only on :

a) the *actual* growth in average earnings between time 0 and time 3, given by $\frac{W(3)}{W(0)}$;

   and

b) the change in the *actual* real yield on long-dated index-linked gilts between time 0 and time 3.

Suppose that the assets of the scheme are invested in the following proportions :

a) z_1 in UK equities, and

b) z_2 in UK undated fixed-interest gilts

where $z_1 + z_2 = 1$.

If the portfolio of assets is re-balanced annually, then the value of the 'rolled-up' investment index at time t (assuming that investment income received for each asset class is re-invested in the same asset class), denoted by $MR(t)$, is given by :

$$MR(t) = MR(t-1) \times \left[z_1 . \frac{PR(t)}{PR(t-1)} + z_2 . \frac{CR(t)}{CR(t-1)} \right]$$

Then, the market value of the scheme assets at time 3, $MVA(3)$, is given by :

$$MVA(0) \times \frac{MR(3)}{MR(0)}$$

+ the accumulated value of the *actual* contribution income received during the inter-valuation period

− the accumulated value of the *actual* benefit outgo paid during the inter-valuation period

Assuming a contribution rate of $k\%$ *pa* payable annually in advance, the accumulated value of the *actual* contribution income received during the inter-valuation period is given by :

$$\sum_{t=0}^{2} \left[\frac{k}{100} \times TS(t) \right] \times \frac{MR(3)}{MR(t)}$$

where $TS(t)$ is the total pensionable salary roll at time t.

As the scheme membership profile is assumed to remain stable with respect to pensionable salary, the total pensionable salary roll at time t, $TS(t)$, is given by :

$$TS(t) = TS(0) \times \frac{W(t)}{W(0)}$$

where $TS(0)$ is the total initial pensionable salary roll.

As mentioned above, membership movements are assumed to occur at the end of each year during the inter-valuation period and to be in accordance with some chosen service table.

Death in service benefits are insured, so that there is no benefit outgo from the fund during the inter-valuation period in respect of members who die in service.

On withdrawal, it is assumed that a transfer value is paid immediately in lieu of the deferred pension accrued. The deferred pension is based on pensionable salary and past pensionable service at the date of exit.

Thus, for an individual member, the cash-equivalent transfer value paid on withdrawal at some future time t is calculated in exactly the same way as the valuation of the accrued liabilities on a discontinuance basis, but with the net rate of interest in both deferment and in possession based on the current net yield on long-dated index-linked gilts, given by $R(t)$.

On retirement at normal retirement age, it is assumed that an immediate annuity is purchased from a life office which exactly matches the liability of the scheme.

As with the cash-equivalent transfer value basis above, the basis used by the life office to cost an immediate annuity will be market-related.

Thus, for simplicity, it is assumed that the basis used by the life office, at some future time t, is, where appropriate, identical to that which would be used to calculate a current transfer value. However, to reflect the margin for expenses, profit and other contingencies, the current net rate of interest in possession, given by $R(t)$, is reduced by 0.5%.

Hence, the total benefit outgo at time t, denoted by $BEN(t)$, is given by the sum of the cash-equivalent transfer values for each active member who withdraws from the scheme at time t prior to reaching normal retirement age plus the sum of the cost of buying an immediate annuity for each active member who retires from the scheme at time t on attaining normal retirement age.

Then, as membership movements are assumed to occur only at the end of each year of the inter-valuation period, the accumulated value of the *actual* benefit outgo paid during the inter-valuation period is given by :

$$\sum_{t=1}^{3} BEN(t) \times \frac{MR(3)}{MR(t)}$$

Thus, the market value of the scheme assets at time 3, $MVA(3)$, is given by :

$$MVA(0) \times \frac{MR(3)}{MR(0)} + \sum_{t=0}^{2} \left\{ \left[\frac{k}{100} \times TS(0) \times \frac{W(t)}{W(0)} \right] \times \frac{MR(3)}{MR(t)} \right\} - \sum_{t=1}^{3} \left[BEN(t) \times \frac{MR(3)}{MR(t)} \right]$$

Then, the value of the required contribution rate, CR, for each realisation of future investment and salary growth experience is found by solving for k in the equation :

$$MVA(3) = AL_{\text{dis}}(3)$$

2.2 *An empirical estimate of the required contribution rate*

By repeating the process of calculating the required contribution rate, *CR*, for a large number of separate independent realisations of the future investment and salary growth experience (each of which is equally likely in practice), an empirical approximation of the distribution function of the underlying random variable is obtained.

Consider the following variants on the model scheme outlined above :

(1) an initial discontinuance funding level of 100% (i.e. $f = 0$), and assets invested 80% in equities and 20% in long-dated fixed-interest gilts;

(2) an initial discontinuance funding level of 150% (i.e. $f = 50$), and assets invested 80% in equities and 20% in long-dated fixed-interest gilts;

(3) an initial discontinuance funding level of 80% (i.e. $f = -20$), and assets invested 80% in equities and 20% in long-dated fixed-interest gilts;

(4) an initial discontinuance funding level of 100%, and assets invested 100% in equities; and

(5) an initial discontinuance funding level of 100%, and assets invested 50% in equities and 50% in long-dated fixed-interest gilts.

Table 1 shows the mean, standard deviation, *central forecast* and main percentile points for the empirical distribution function of random variable representing the required contribution rate, *CR*, for each of the above model scheme variants.

The empirical distribution function is obtained by means of 1,000 independent simulations of the future investment experience generated using the Wilkie model. The same set of 1,000 simulations is used for each of the model scheme variants.

The *central forecast* of *CR* is the value obtained assuming that all the random components in the Wilkie model are set equal to zero.

Table 1. Empirical distribution function of the required contribution rate, *CR*, for each of the five variants of the model scheme

	(1)	(2)	(3)	(4)	(5)
mean	12.0%	-33.9%	30.4%	11.5%	13.3%
standard deviation	20.7%	23.0%	20.0%	25.3%	14.1%
central forecast	12.7%	-33.3%	31.1%	11.2%	15.1%
1st percentile	-27.0%	-80.4%	-7.2%	-34.3%	-13.9%
5th percentile	-19.5%	-68.4%	0.5%	-26.2%	-9.2%
10th percentile	-13.6%	-62.4%	6.1%	-19.0%	-4.4%
25th percentile	-3.3%	-51.4%	15.6%	-6.5%	2.3%
50th percentile	10.3%	-35.0%	28.7%	9.0%	12.5%
75th percentile	25.2%	-18.1%	42.9%	28.2%	22.9%
90th percentile	40.3%	-2.6%	56.8%	46.9%	33.3%
95th percentile	49.5%	6.5%	66.5%	58.3%	37.5%
99th percentile	57.3%	16.5%	74.2%	69.6%	45.0%

Then, the empirical distribution function for *CR* can be used to estimate the contribution rate which must be paid during the inter-valuation to give a probability of insolvency on wind-up at the valuation at time 3 of at most α%.

For example, for model scheme variant (1), the 95th percentile of the empirical distribution of *CR* is 49.5%, which corresponds to the empirical estimate of the contribution rate which must be paid during the inter-valuation to give a probability of insolvency on wind-up at the valuation at time 3 of at most $(100-95)=5\%$. Similarly, if the desired probability of insolvency on wind-up at time 3 is at most 1%, then the empirical estimate of the contribution rate which must be paid during the inter-valuation period rises to 57.3%.

Also, for model scheme variant (2), if the desired probability of insolvency on wind-up at time 3 is at most 1%, then the empirical estimate of the contribution rate which must be paid during the inter-valuation period is 16.5%. The reduction in the required contribution rate, in this case, is due to the higher initial discontinuance funding level.

Also, for model scheme variant (4), if the desired probability of insolvency on wind-up at time 3 is at most 1%, then the empirical estimate of the contribution rate which must be paid during the inter-valuation period is 69.6%. The increase in the required contribution rate (compared with 57.3% above), in this case, is due to the higher-risk investment strategy adopted (which increases the volatility of the market value of the assets held).

2.3 *A deterministic approach to estimating the required contribution rate*

While the approach adopted above is perfectly feasible in theory, time constraints mean that it is unlikely that a large number of simulations (even with only a 3 year projection period) could carried out in practice for every scheme.

Thus, the next step is to attempt to estimate the required percentile points of the distribution of *CR* (for any initial discontinuance funding level and chosen investment strategy) without requiring an empirical estimate of the entire distribution by means of a large number of simulations.

The *central forecast* of *CR* requires only that the random components of the Wilkie stochastic asset model are set equal to zero, and so, can be calculated easily and quickly. This value can be used as an approximation to the median of the underlying distribution of *CR*.

However, in order to estimate other percentile points of the distribution of *CR*, an assumption as regards the shape of the distribution must also be made.

By general reasoning, consider the effect on the value obtained for *CR* of, in turn, changing each of the main random components of the Wilkie model :

a) From Appendix A.1, if $QZ(t) > 0 \ \forall t$, then the force of price inflation, $I(t)$, will be an increasing function.

   An increase in the force of price inflation will filter through, primarily, to give an increase in both the force of average earnings growth, $J(t)$ - and, thus, an increase in the value of the accrued liabilities on a discontinuance basis - and in the 'rolled-up' equity index, $PR(t)$ - and, thus, an increase in the market value of equity holdings.

   As most on-going final salary pension schemes are likely to be invested predominantly in equities, these two effects can be considered to broadly cancel out, so that an increase in the force of price inflation will, all other things being equal, have little overall effect on the value of *CR*.

b) From Appendix A.2, if $WZ(t) > 0 \ \forall t$, then the force of average earnings growth, $J(t)$, will be an increasing function.

   An increase in the force of average earnings growth will lead *directly* to an increase in the value of the accrued liabilities on a discontinuance basis.

   Hence, the effect of an increase in the force of average earnings growth will, all other things being equal, be an increase in the value of *CR*.

c) From Appendix A.3, if $YZ(t) > 0 \ \forall t$, then the share dividend yield, $Y(t)$, will be an increasing function.

   An increase in the share dividend yield will lead *directly* to a decrease in the force of share dividend growth (as a result of the negative DY parameter).

   As a result, an increase in the share dividend yield will change the balance of the return on equities between income and capital growth, but the overall equity return is likely to be largely unaffected.

A secondary effect of an increase in the share dividend yield will be an increase in the yield on undated fixed-interest gilts (as a result of the *CY* parameter), leading to a decrease in the 'rolled-up' undated fixed-interest gilts index, *CR*(t) - and, thus, a decrease in the market value of undated fixed-interest gilt holdings.

Hence, an increase in the share dividend yield will, all other things being equal, have little overall effect on the value of *CR*.

d) From Appendix A.4, if $DZ(t) > 0 \ \forall t$, then the force of share dividend growth, *K*(t), will be an increasing function.

An increase in the force of share dividend growth will lead *directly* to an increase in the 'rolled-up' equity index, *PR*(t) - and, thus, an increase in the market value of equity holdings.

Hence, the effect of an increase in the force of share dividend growth will, all other things being equal, be a decrease in the value of *CR*.

e) From Appendix A.5, if $CZ(t) > 0 \ \forall t$, then the yield on long-dated fixed-interest gilts, *C*(t), will be an increasing function.

An increase in the yield on undated fixed-interest gilts will lead *directly* to a decrease in the 'rolled-up' undated fixed-interest gilts index, *CR*(t) - and, thus, an decrease in the market value of undated fixed-interest gilt holdings.

However, as most on-going final salary pension schemes are likely to hold a small proportion of their assets in the form of undated fixed-interest gilts, an increase in the yield on undated fixed-interest gilts will, all other things being equal, have little overall effect on the value of *CR*.

f) From Appendix A.7, if $RZ(t) > 0 \ \forall t$, then the yield on long-dated index-linked gilts, *R*(t), will be an increasing function.

An increase in the yield on long-dated index-linked gilts will lead *directly* to a decrease in the 'rolled-up' long-dated index-linked gilts index, *RR*(t) - and, thus, an decrease in the market value of long-dated index-linked holdings. However, most on-going final salary pension scheme are likely to hold a small proportion of their assets in the form of long-dated index-linked gilts.

More important, however, the yield on long-dated index-linked gilts is used as the basis for calculating the transfer value granted in lieu of a deferred pension on withdrawal. As a result, an increase in the yield on long-dated index-linked gilts will lead *directly* to a decrease in the value of the accrued liabilities on a discontinuance basis.

Hence, the effect of an increase in the yield on long-dated index-linked gilts will, all other things being equal, be an decrease in the value of *CR*.

g) Most on-going final salary pension schemes are likely to hold a very small proportion of their assets in both cash and property. As a result, the effect of any change in the random components *BZ*(t), *ZZ*(t) and *EZ*(t) is likely to have a negligible effect on the value of *CR*.

Thus, only changes in the random components *WZ*(t), *DZ*(t) and *RZ*(t) are likely to have a significant effect on the value of *CR*.

The random components in Wilkie stochastic asset model are independent $N(0,1)$ random variables. From statistical tables, the 51st percentile of the standard $N(0,1)$ distribution is 0.025, and, by symmetry, the 49th percentile is -0.025.

Repeating the process used to calculate the central forecast of *CR*, but setting $WZ(t) = 0.025\ \forall t$ and $DZ(t) = RZ(t) = -0.025\ \forall t$ with the other random components remaining set equal to zero, gives a first approximation of the 51st percentile of the underlying distribution of *CR*. However, the cumulative effect of the error terms means that this approximation can be expected to exceed the 'true' value.

Instead, a simple weighted average approach is used. An equal weighting could be given to each of the three random components (e.g. we can approximate the $50\frac{1}{3}$rd percentile of a standard $N(0,1)$ distribution by $\frac{0.025}{3}$, then repeating the process used to calculate *CR*, but setting $WZ(t) = \frac{0.025}{3}\ \forall t$ and $DZ(t) = RZ(t) = -\frac{0.025}{3}\ \forall t$, can be expected to give a better approximation of the 51st percentile of the underlying distribution of *CR*).

However, intuitively, the random component which will have the most significant effect on the value of *CR* is *RZ*(*t*). This is because a relatively small decrease (or increase) in the yield on long-dated index-linked gilts, will lead to a significant increase (or decrease) in the value of the accrued liabilities on a discontinuance basis. Indeed, if we assumed a discounted mean term to retirement of, say, 15 years, a decrease in the yield on long-dated index-linked gilts of 0.5% *pa* will lead to an increase in the value of the accrued liabilities on a discontinuance basis of around 12%. Thus, it is proposed that, for simplicity, 100% of the weighting is given to this random component.

Then, repeating the process used to calculate the central forecast of *CR*, but setting :

a) $RZ(t) = -0.025\ \forall t$, and

b) $QZ(t) = WZ(t) = YZ(t) = DZ(t) = CZ(t) = BZ(t) = ZZ(t) = EZ(t) = 0\ \forall t$

gives a reasonable approximation of the 51st percentile of the 'true' underlying distribution of *CR*.

Then, by making the simple assumption that the distribution of *CR* is Normal, the estimates of the median, denoted by $p^{\bullet}_{CR}(50)$, and the 51st percentile, denoted by $p^{\bullet}_{CR}(51)$, can be used to estimate the standard deviation of the distribution of *CR*, denoted by $\sigma^{\bullet}_{CR}$, as follows :

$$\sigma^{\bullet}_{CR} = \frac{p^{\bullet}_{CR}(51) - p^{\bullet}_{CR}(50)}{0.025}$$

For a Normal distribution, the mean is equal to the median, and given estimates of the mean and the standard deviation, estimates of percentiles of the distribution can be easily obtained.

Consider model scheme variant (1).

From Table 1, the central forecast of CR, $p^*_{CR}(50)$, is 12.73%.

Setting $RZ(t) = -0.025$ and repeating the process used to calculate CR, gives an estimate of the 51st percentile of the distribution of CR, $p^*_{CR}(51)$, of 13.22%.

Then, the estimated standard deviation of the distribution of CR, σ^*_{CR}, is :

$$\sigma^*_{CR} = \frac{p^*_{CR}(51) - p^*_{CR}(50)}{0.025} = \frac{0.1322 - 0.1273}{0.025} = 0.196 \text{ or } 19.6\%$$

The estimate of the standard deviation of the distribution of CR of $\sigma^*_{CR} = 19.6\%$ compares favourably with the 'true' value of $\sigma_{CR} = 20.7\%$, as shown in Table 1.

The 95th percentile of a standard $N(0,1)$ distribution is 1.645.

Then, the estimated 95th percentile of the distribution of CR, $p^*_{CR}(95)$, is :

$$p^*_{CR}(95) = p^*_{CR}(50) + 1.645 \times \sigma^*_{CR} = 0.1273 + 1.645 \times 0.196 = 0.450 \text{ or } 45.0\%$$

The estimate of the 95th percentile of the distribution of CR of $p^*_{CR}(95) = 45.0\%$ compares favourably with the 'true' value of $p_{CR}(95) = 49.5\%$, as shown in Table 1.

Table 2 shows the estimated 90th, 95th and 99th percentiles obtained using the above method for each of the five model scheme variants.

Table 2. Deterministic approximation of the percentiles of the distribution of the required contribution rate, CR, for each of the five variants of the model scheme

	(1)	(2)	(3)	(4)	(5)
central forecast, $p^*_{CR}(50)$	12.7%	-33.3%	31.1%	11.2%	15.1%
estimated 51st percentile, $p^*_{CR}(51)$	13.2%	-32.8%	31.6%	11.8%	15.5%
estimated standard deviation, σ^*_{CR}	19.6%	20.8%	20.8%	22.8%	15.2%
'true' standard deviation, σ_{CR}	20.7%	23.0%	20.0%	25.3%	14.1%
estimated 90th percentile, $p^*_{CR}(90)$	37.9%	-6.6%	57.8%	40.4%	34.6%
'true' 90th percentile, $p_{CR}(90)$	40.3%	-2.6%	56.8%	46.9%	33.3%
estimated 95th percentile, $p^*_{CR}(95)$	45.0%	0.9%	65.3%	48.7%	40.1%
'true' 95th percentile, $p_{CR}(95)$	49.3%	6.5%	66.5%	58.3%	37.5%
estimated 99th percentile, $p^*_{CR}(99)$	58.3%	15.1%	79.5%	64.2%	50.5%
'true' 99th percentile, $p_{CR}(99)$	57.3%	16.5%	74.2%	69.6%	45.0%

Consider model scheme variant (1).

The 'true' 95th percentile of the distribution of *CR* is 49.3%, which corresponds to the contribution rate required to paid during the 3-year inter-valuation period to give a probability of insolvency at the next valuation date of at most 5%.

The estimated 95th percentile of the distribution of *CR* is 45.0%. By interpolation from Table 1, if a contribution rate of 45.0% is paid during the 3-year inter-valuation period, then the *actual* probability of insolvency at the next valuation date can be estimated as follows :

$$1-\left[0.90+(0.95-0.90)\times\left(\frac{0.450-0.403}{0.495-0.403}\right)\right]=0.074 \text{ or } 7.4\%$$

Table 3 shows the actual probability of insolvency at the next valuation date if the contribution rates estimated in Table 2 for a desired probability of insolvency at the next valuation date of 1%, 5% and 10% are paid.

Table 3. Actual probability of insolvency for each of the five variants of the model scheme using the contribution rates obtained by the pseudo-stochastic valuation approach

	actual probability of insolvency				
	(1)	(2)	(3)	(4)	(5)
desired probability of insolvency = 10%	12.4%	13.9%	9.5%	15.2%	8.5%
desired probability of insolvency = 5%	7.4%	8.1%	5.6%	9.2%	3.6%
desired probability of insolvency = 1%	0.5%	1.6%	0.3%	2.9%	0.3%

From Table 2, it can be seen that, for each of the five variants of the model scheme and for each of the three chosen levels of risk, the contribution rate calculated using the pseudo-stochastic approach is not markedly different from the empirical estimate of the actual contribution rate required (calculated using 1,000 simulations).

Table 3 confirms that, for each of the five variants of the model scheme and for each of the three chosen levels of risk, if the contribution rate calculated using the pseudo-stochastic approach is paid (rather than the empirical estimate of the actual contribution rate required), the probability of insolvency achieved is very similar to that desired.

Also, from Table 3, there does not appear to be any discernible pattern to the difference between the required contribution rate as calculated using the pseudo-stochastic approach and the empirical estimate of the actual contribution rate required. For model scheme variant (4), the pseudo-stochastic approach consistently under-estimates the required contribution rate, albeit not to any great extent. The opposite is true, however, for model scheme variant (5), where the pseudo-stochastic approach consistently over-estimates the required contribution rate. Also, for model scheme variants (1) and (2), the pseudo-stochastic approach under-estimates the required contribution rate for a desired probability of insolvency of 5% and 10%, but, for a desired probability of insolvency of 1%, the pseudo-stochastic approach again under-estimates the required contribution rate for model scheme variant (2) *but* over-estimates the required contribution rate for model scheme variant (1).

3. CONCLUSIONS

While stochastic asset-liability modelling techniques as a means of helping to frame the long-term strategic asset allocation for pension funds are reasonably well-established, the development of other potential uses for this powerful new actuarial tool has been largely neglected.

The method proposed in this paper considers the use of stochastic asset-liability modelling as a means of framing funding strategy. In this case, for a chosen level of risk, stochastic asset-liability modelling enables the level of future contribution rate to the fund to be determined. However, as shown above, this requires a large number of simulations. Indeed, although the work detailed in this paper is based on 1,000 simulations, as we are essentially interested in the more extreme values of the distribution function representing the future contribution rate (for example, the 90^{th}, 95^{th}, or even 99^{th} percentiles) it may be more appropriate to use a larger number of simulations (possibly 5,000, or even 10,000).

The proposed method eliminates the need for incorporating specific margins on the grounds of prudence into the valuation basis, the combined effect of which can be difficult to determine. The level of security provided is explicit in the choice of the probability of insolvency.

One of the main drawbacks limiting the growth of stochastic asset-liability modelling as a practical tool in actuarial science is the time required to generate the large number of simulations necessary to obtain meaningful results (although the strength of this argument is constantly diminishing as computers become ever more powerful).

As a result, a deterministic approach to estimating the required future contribution rate for a given level of risk, which is based on very simple general reasoning and requires only two simulations, is shown to reasonably successful for a variety of different investment strategies and initial funding levels.

The key assumptions underlying this deterministic approximation are :

a) the random component of the index-linked gilt yield model, $RZ(t)$, has the most significant effect on the value of the required future contribution rate, CR
   - given the liability model used, this is likely to be case for most alternative stochastic asset models (e.g. Dyson & Exley, 1995; Smith, 1996; and Wright, 1997)
   - alternatively, the proposed method can also be shown to be reasonably successful when equal weight is given to each of the random error components of the chosen stochastic asset model (as suggested initially in Section 2.3)

b) the distribution function of the random variable representing the required future contribution rate, CR, is Normal
   - this assumption is likely to be fairly robust to a change in the stochastic asset model used

As a result, it is hoped that the deterministic approximation method proposed can be used equally well (albeit, perhaps, with slight modifications where appropriate) in conjunction with other stochastic asset models. However, as mentioned in Section 2.1.2, this remains to be investigated fully.

The principles underlying the work outlined above are not restricted to the field of pension funding, and it is hoped that this paper will lead to the development of further applications for stochastic asset-liability modelling techniques within the wider field of actuarial science.

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APPENDIX A. THE WILKIE (1995) STOCHASTIC ASSET MODEL

A.1 *Force of price inflation*

The model proposed by Wilkie (1995) for the force of price inflation is :

$$I(t) = QMU + QA.\left[I(t-1) - QMU\right] + QE(t)$$

where

a) $I(t)$ is the force of price inflation at time t;

b) $QE(t) = QSD.QZ(t)$; and

c) $QZ(t)$ is a $N(0,1)$ white noise series.

The parameter values suggested by Wilkie (1995) are :

$$\begin{aligned} QMU &= 0.047 \\ QA &= 0.58 \\ QSD &= 0.0425 \end{aligned}$$

A.2 *Force of earnings inflation*

The model proposed by Wilkie (1995) for the force of earnings inflation is :

$$J(t) = WW1.I(t) + \left(1 - WW1\right).I(t-1) + WN(t)$$

where

a) $J(t)$ is the force of earnings inflation at time t;

b) $WN(t) = WMU + WA.\left[WN(t-1) - WMU\right] + WE(t)$;

c) $WE(t) = WSD.WZ(t)$; and

d) $WZ(t)$ is a $N(0,1)$ white noise series.

The parameter values suggested by Wilkie (1995) are :

$$\begin{aligned} WW1 &= 0.69 \\ WMU &= 0.016 \\ WA &= 0.0 \\ WSD &= 0.0244 \end{aligned}$$

A.3 *Share dividend yield*

The model proposed by Wilkie (1995) for the share dividend yield is :

$$\ln Y(t) = YW.I(t) + YN(t)$$

where

a) $Y(t)$ is the share dividend yield at time t;

b) $YN(t) = \ln YMU + YA.[YN(t-1) - \ln YMU] + YE(t)$;

c) $YE(t) = YSD.YZ(t)$; and

d) $YZ(t)$ is a $N(0,1)$ white noise series.

The parameter values suggested by Wilkie (1995) are :

$$\begin{aligned} YW &= 1.8 \\ YMU &= 0.0375 \\ YA &= 0.55 \\ YSD &= 0.155 \end{aligned}$$

A.4 *Force of share dividend growth*

The model proposed by Wilkie (1995) for the force of share dividend growth is :

$$\begin{aligned} K(t) \;=\; & DW.DM(t) + (1-DW).I(t) + DMU \\ & + DY.YE(t-1) + DB.DE(t-1) + DE(t) \end{aligned}$$

where

a) $K(t)$ is the force of share dividend growth at time t;

b) $DM(t) = DD.I(t) + (1-DD).DM(t-1)$;

c) $DE(t) = DSD.DZ(t)$; and

d) $DZ(t)$ is a $N(0,1)$ white noise series.

The parameter values suggested by Wilkie (1995) are :

$$\begin{aligned} DW &= 0.58 \\ DD &= 0.13 \\ DMU &= 0.016 \\ DY &= -0.175 \\ DB &= 0.57 \\ DSD &= 0.07 \end{aligned}$$

A.5 *Yield on undated fixed-interest gilts*

The model proposed by Wilkie (1995) for the yield on undated fixed-interest gilts is :

$$C(t) = CW.CM(t) + CMU.\exp[CN(t)]$$

where

a) $C(t)$ is the yield on undated fixed-interest gilts at time t;

b) $CM(t) = CD.I(t) + (1 - CD).CM(t-1)$;

c) $CN(t) = CA1.CN(t-1) + CY.YE(t) + CE(t)$;

d) $CE(t) = CSD.CZ(t)$; and

e) $CZ(t)$ is a $N(0,1)$ white noise series.

The parameter values suggested by Wilkie (1995) are :

$$\begin{aligned} CW &= 1.0 \\ CD &= 0.045 \\ CMU &= 0.0305 \\ CA1 &= 0.9 \\ CY &= 0.34 \\ CSD &= 0.185 \end{aligned}$$

A.6 *Yield on cash deposits*

The model proposed by Wilkie (1995) for the yield on cash deposits is :

$$B(t) = C(t).\exp[-BD(t)]$$

where

a) $B(t)$ is the yield on cash deposits at time t;

b) $C(t)$ is the yield on undated fixed-interest gilts at time t;

c) $BD(t) = BMU + BA.[BD(t-1) - BMU] + BE(t)$;

d) $BE(t) = BSD.BZ(t)$; and

e) $BZ(t)$ is a $N(0,1)$ white noise series.

The parameter values suggested by Wilkie (1995) are :

$$\begin{aligned} BMU &= 0.23 \\ BA &= 0.74 \\ BSD &= 0.18 \end{aligned}$$

A.7 *Yield on long-dated index-linked gilts*

The model proposed by Wilkie (1995) for the yield on long-dated index-linked gilts is :

$$\ln R(t) = \ln RMU + RA.\left[\ln R(t-1) - \ln RMU\right] + RBC.CE(t) + RE(t)$$

where

a) $R(t)$ is the yield on long-dated index-linked gilts at time t;

b) $RE(t) = RSD.RZ(t)$; and

c) $RZ(t)$ is a $N(0,1)$ white noise series.

The parameter values suggested by Wilkie (1995) are :

$$\begin{aligned} RMU &= 0.040 \\ RA &= 0.55 \\ RBC &= 0.22 \\ RSD &= 0.05 \end{aligned}$$

A.8 *Property rental yield*

The model proposed by Wilkie (1995) for the property rental yield is :

$$\ln Z(t) = \ln ZMU + ZA.\left[\ln Z(t-1) + \ln ZMU\right] + ZE(t)$$

where

a) $Z(t)$ is the property rental yield at time t;

b) $ZE(t) = ZSD.ZZ(t)$; and

c) $ZZ(t)$ is a $N(0,1)$ white noise series.

The parameter values suggested by Wilkie (1995) are :

$$\begin{aligned} ZMU &= 0.074 \\ ZA &= 0.91 \\ ZSD &= 0.12 \end{aligned}$$

A.9 *Force of property rental growth*

The model proposed by Wilkie (1995) for the force of property rental growth is :

$$EK(t) = EW.EM(t) + (1 - EW).I(t) + EMU + EBZ.ZE(t) + EE(t)$$

where

a) $EK(t)$ is the force of share dividend growth at time t;

b) $EM(t) = ED.I(t) + (1 - ED).EM(t-1)$;

c) $EE(t) = ESD.EZ(t)$; and

d) $EZ(t)$ is a $N(0,1)$ white noise series.

The parameter values suggested by Wilkie (1995) are :

$$\begin{aligned} EW &= 1.0 \\ ED &= 0.11 \\ EMU &= 0.003 \\ EBZ &= 0.24 \\ ESD &= 0.06 \end{aligned}$$

A.10 *Neutral starting conditions*

The neutral starting conditions suggested by Wilkie (1995) are :

$$\begin{array}{lcllcl} I(0) & = & QMU & & = & 0.047 \\ J(0) & = & QMU + JMU & & = & 0.063 \\ Y(0) & = & \exp(YW.QMU).YMU & & = & 0.0408 \\ DM(0) & = & QMU & & = & 0.047 \\ CM(0) & = & QMU & & = & 0.047 \\ C(0) & = & QMU + CMU & & = & 0.0775 \\ B(0) & = & \exp(-BMU).C(0) & & = & 0.0616 \\ R(0) & = & RMU & & = & 0.04 \\ Z(0) & = & ZMU & & = & 0.074 \\ EM(0) & = & QMU & & = & 0.047 \end{array}$$

$$\begin{array}{lcl} YE(0) & = & 0 \\ DE(0) & = & 0 \end{array}$$

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