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Numerical study on screw machines with large helix angles

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Abstract. Modelling and performance calculation of screw machines with large helix angles such as a single and multiphase screw pumps by use of Computational Fluid Dynamics is challenging. The numerical procedures explained in literature are based on the 3D numerical meshes generated by series of 2D transverse cross sections which allows mesh to either follow the helix or be perpendicular to the rotor axis. This allows generating a conformal mesh. However, if the rotor helix angle is large, the cell skewness becomes prohibitively large which introduces errors in numerical simulation. The paper firstly attempts to generalize the generation method of rotor profiles with emphasis on producing a normal rack and rotors profiles. Then it introduces the method which uses series of 2D numerical meshes in the planes normal to each of the rotors and rack in order to decompose the working domain in two sub-domains. By this means it is possible to achieve 3D numerical mesh with extended capability of mesh refinement in clearances and alignment of the mesh to the main and leakage flows. However, special treatment is required to provide conformal interface between two moving meshes and with the casing. It is expected that it will greatly benefit accuracy and ease of performance calculation using a number of CFD solvers. In addition it is expected that it will allow generation of various different screw machine configurations like single screw machines or machines with conical rotors.

1. Introduction
Single and twin screw machines are positive displacement rotary machines comprising a meshing pair of rotors contained in a casing, which together form a working chamber whose volume depends on the angle of rotation. The performance and efficiency of screw machines are mostly related to the rotor profiles and machining precision. Generally, there are three different methods to generate rotor profiles namely, generation from a given curve of a main rotor, generation from a meshing line and generation from a rack on a transverse or on a normal plane. The envelope method is the basis for the design of rotor profiles of screw rotors. The envelope gearing method of the screw rotors is similar to the method used for helical gears. Litvin[1] described the process of coordinate transformation in matrix representation and implemented it in the generation of gear and rack cutter. Stosic[2] used the envelope gearing method to derive a general meshing condition for crossed helical gears which is then used to generate the profile of a hobbing or cutting tool. Stosic and Hanjalic[3]presented a rack based procedure to generate various lobe combination rotors. Refer to the rack-generated profile method of Stosic, Yu-ren[4] explicitly redefined the rack which including two normal-equidistant trochoids using at least one control parameter to each compound curve, which contributes to the flexible and adjustable rack. Then

a method using a rack defined in the normal plane to design the twin screw rotor profiles was proposed by him[5]. The advantage of this method is that the normal circular pitch is same for main and gate rotor as the helix angle varies.

In order to enable performance calculation by CFD, the working domain between the casing and the rotors has to be mapped with a numerical grid. A 3D flow domain between the casing and two rotors is discretized by a number of 2D cross sections. In each of these cross sections, the flow domain is divided in two subdomains for easier grid generation. Practically, three methods can be used to decompose a 2D cross section of the flow domain, namely, differential division line, transverse rack and a normal rack. Kovacevic[6] was the first to use the analytical rack in the transverse cross section for decomposition of the 2D cross section of the flow domain as shown in Figure 1. Recently, as shown by Sham[7, 8], algebraic grid generation is used for a variable profile rotors with parallel axis using transverse rack. In this procedure, the split line is the rack segment stretching between the bottom and top cusp point. In his case, each 2D cross section has a different rotor profile while the position of transverse planes can also vary depending on the lead of the rotors. The two O blocks together form a composite grid. Voorde et al [9]constructed a block structured mesh using similar decomposition but has applied gradient lines and the potential lines in the transverse planes to obtain numerical solution. This often leads to a large gradient of volumes around the CUSP.

In the case of machines with non-parallel axes such as single screw compressors or twin screw compressors with large helix angle, cells formed from the consecutive 2D transverse planes may become too skewed for conservative solution. The large helix angle can cause a twist of the mesh too large for reliable grid generation. Despite a significant increase in the number of papers published recently in the area of grid generation for screw machines, very few journal papers have been published on grid generation of screw machines with large helix angles. In this paper, the proposed normal rack grid generation method will be proposed to decompose the fluid domain using normal planes. Grid Generation in this 2D normal cross section is expected to be much easier and would be directly applicable for screw machines with large helix angles. This paper proposes use of a normal rack in order to separates flow domains of the male and female rotors.
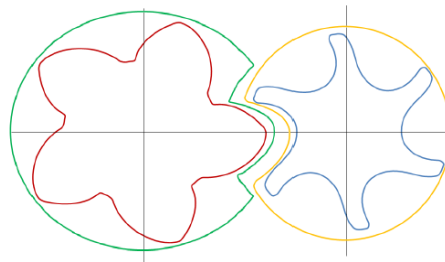
Reproduced from Kovacevic 1998
Figure 1. Transverse rack as a splitting curve.

2. Rotor generation

The performance of a screw compressor mainly depends on the design of rotor profiles and machining precision for their manufacturing. Nowadays, screw rotors are manufactured mainly by milling, grinding and hobbing tools for the rough and finish machining. Form tools for rotor manufacture can be regarded as crossed helical gears on non-parallel and non-intersecting axes system with a uniform lead such as in the case of form hobbing, or with no lead as in formed milling or grinding. The rotor profiles of twin screw compressors are usually with parallel and non-intersecting axes. Generally, there are three different mathematical methods for rotor profiles generation, namely, generation from a given main or gate rotor profile curve, generation from a meshing line and generation from a specifying rack on the transverse or normal plane. The envelop method is the basics for generation of conjugate rotor profiles.

It states that two rotors transforming in prescribed motion can generate or envelope each other under necessary conditions.

2.1. Applied coordinate systems

To start the procedure of rotor profiling, the coordinate system must be defined firstly. Single screw rotors and their cutting tools are normally defined in a non-parallel axes coordinate system while the twin screw rotors have parallel axes.

2.1.1. Non-parallel axes coordinate system

The coordinate system of non-parallel and nonintersecting axes is given in Figure 2. The shortest distance between the two axes is C and the angle between them is Σ. Consider coordinate system $S_1(X_1, Y_1, Z_1)$ and $S_2(X_2, Y_2, Z_2)$ that are rigidly connected to main rotor and gate rotor respectively and coordinate system $S_1(X_1, Y_1, Z_1)$ is the global coordinate system. And the coordinate system $R(X_{01}, Y_{01})$ rotates around Z_1 with main rotor and the coordinate system $R(X_{02}, Y_{02})$ rotates around Z_2 with gate rotor, which are local coordinate system. This global and local coordinate systems can be used to generate the single screw rotors and form cutter profiles from the rotor profiles.
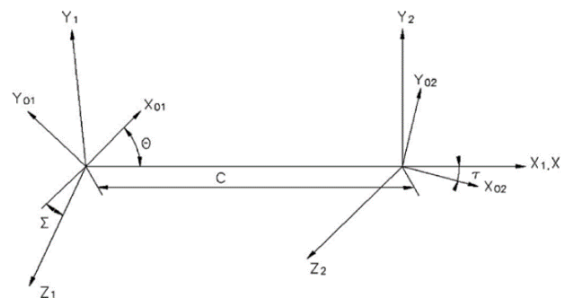

Figure 2. Coordinate systems of non-parallel and non-intersecting axes.

2.1.2. Parallel axis coordinate system.

When the angle between two axes is $\Sigma = 0$, the parallel and intersecting axes coordinate system can be obtained and the profiles of main and gate twin screw compressor rotors can be designed and generated in this coordinate system as shown in Figure 3. r_{1w} and r_{2w} are the radii of the pitch circle of main rotor profile and gate profile respectively. So the axes distance is $C = r_{1w} + r_{2w}$.
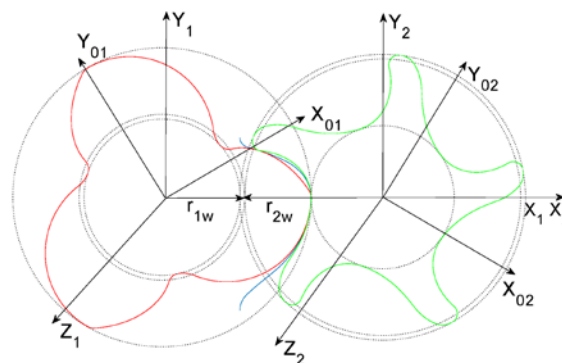

Figure 3. Coordinate systems of parallel and intersecting axes.

2.2. Coordinate transformation

With the coordinate system, the matrix of coordinate transformation provide the mathematical methods for generating profiles of the rack from the rotor profiles and conversely, generating the rotor profiles from the rack. With the coordinate systems showed in Figure 2, the coordinate transformation in transition from the main rotor profile coordinate S_1 to the gate rotor profile coordinate S_2 is based on the matrix equation:

$$\mathbf{r}_2 = M_{21}\mathbf{r}_1 = M_{2p}M_{pf}M_{f1}\mathbf{r}_1 \tag{1}$$

$$\mathbf{r}_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix},\ \mathbf{r}_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix},\ \mathbf{M}_{pf} = \begin{bmatrix} 1 & 0 & 0 & -C \\ 0 & \cos(\Sigma) & -\sin(\Sigma) & 0 \\ 0 & \sin(\Sigma) & \cos(\Sigma) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{M}_{2p} = \begin{bmatrix} \cos(\tau) & -\sin(\tau) & 0 & 0 \\ \sin(\tau) & \cos(\tau) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ \mathbf{M}_{f1} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where $r_1(t,\theta)$ and $r_2(t,\theta,\tau)$ are the generating and generated surfaces respectively and t is a profile parameter while θ and τ are motion parameters of main rotor profiles and gate rotor profile respectively. M_{2p} and M_{f1} are rotational matrices corresponding to rotation with the Z_1 and Z_2 axes respectively and M_{pf} is a translational matrix corresponding to the transformation from coordinate system $S_1(X_1, Y_1, Z_1)$ to $S_2(X_2, Y_2, Z_2)$. Here these matrices are used to generate the rack from the given rotor profiles.

3. Rack generation

The rack line is the curve representing a rotor with infinite radius. Namely, 3D rotors and rack surface do not change when the cutting plane changes. Only if they are cut with the particular plane, they form either transverse or normal 2D projections of rotors and rack. The rack generation procedure for screw rotor profiles was firstly proposed by Rinder[10]. He used the rack as a generating curve and both the main and the gate rotor profiles can be generated from the same rack in the same coordinate system. The rack line can mesh with the main rotor profile and gate rotor profile simultaneously. In addition to that convenience, a rack generation procedure offers the possibility to design various lobe combinations rotors using the same rack. As shown in the Figure 4, the main rotor profile and gate rotor profiles are both generated from the rack between the main and gate rotor profiles and the main rotor with three lobes shown in the left side is able to mesh with the gate rotors with five lobes or seven lobes shown in the right side. The red line between two rotor profiles is the rack line. The rack can be defined in the transverse plane system which is called transverse rack and in the normal plane system which is called normal rack. Yu-ren[5] proposed normal-rack generation method (NRGM) to design all the tangent continuous profile segments explicitly in the normal section to generate the conjugated rotors which can be utilised for various lobe combinations, centre distances even the different helix angles of the main and gate rotors.
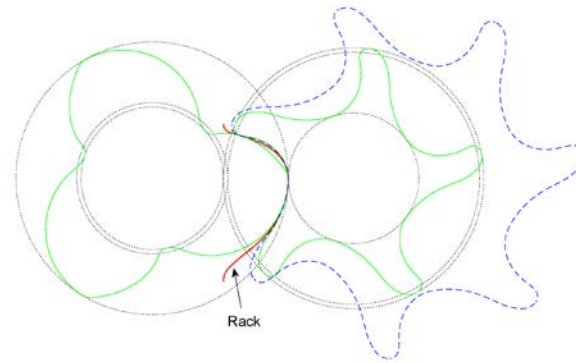
Figure 4 Rotor profiles of 3/5 and 3/7 with the same rack profile.

3.1. The transverse plane and the normal plane

The transverse plane is perpendicular to the rotor shaft. Usually the rotor profiles are generated in the transverse plane. The normal plane is perpendicular to the vector of the helix in the pitch circle. As showed in Figure 5, the transverse plane is indicated by the blue line and the normal plane is indicated by the red line. The angle between the transverse plane and normal plane is the helix angle β. For grid generation, if the transverse plane is used for the grid generation, then the transverse plane is unique for both rotors and rack. However, if the normal plane is used for grid generation, in that case each rotor will have a series of its own normal planes and the rack will have its own normal plane.
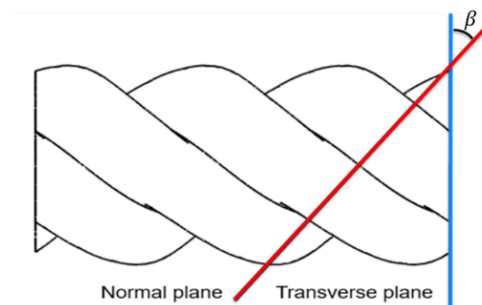

Figure 5. Transverse plane and normal plane.

3.2. Transverse rack

The transverse rack can be generated from the main rotor profile. The main rotor profile coordinate in system $R(X_{01}, Y_{01})$ is $r_1(x_{01}, y_{01})$ and the gate rotor profile coordinate in system $R(X_{02}, Y_{02})$ is $r_2(x_{02}, y_{02})$. For the parallel axes, since the angle between two axes is $\Sigma = 0$, gear ratio $i = \theta/\tau$ and $k = 1 + 1/i$. If the generated rotor profile is the main rotor, the meshing gate profile equations in the transverse plane can be obtained according to the Equation 1 as:

$$\begin{bmatrix} x_{02} \\ y_{02} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\tau) & -\sin(\tau) & 0 & 0 \\ \sin(\tau) & \cos(\tau) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -C \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{01} \\ y_{01} \\ 0 \\ 1 \end{bmatrix}$$

Equation 2 can be obtained by simplification of the above equation as:

$$\begin{cases} x_{02} = x_{01}\cos(k\theta) + y_{01}\sin(k\theta) - C\cos(\dfrac{\theta}{i}) \\\\ y_{02} = -x_{01}\sin(k\theta) + y_{01}\cos(k\theta) + C\cos(\dfrac{\theta}{i}) \end{cases} \tag{2}$$

Rack coordinates can be calculated from Equation 2 if the gear ratio i tends to infinity:

$$\begin{cases} x_{0r} = x_{01}\cos(\theta) + y_{01}\sin(\theta) - r_{1w}\theta \\ y_{0r} = -x_{01}\sin(\theta) + y_{01}\cos(k\theta) \end{cases} \tag{3}$$

The rack can be regarded as the rotor with infinite radius which means the gear ration i tends to infinity. The main rotor profile and gate rotor profile can be generated from the rack profile $r_{0r}(x_{0r}, y_{0r})$ respectively as:

$$\begin{cases} x_{01} = x_{0r}\cos(\theta) - (y_{0r} - r_{1w}\theta)\sin(\theta) \\ y_{01} = x_{0r}\sin(\theta) + (y_{0r} - r_{1w}\theta)\cos(\theta) \end{cases} \tag{4}$$

$$\begin{cases} x_{02} = -x_{0r}\cos(\theta) - (y_{0r} - r_{2w}\theta)\sin(\theta) \\ y_{02} = x_{0r}\sin(\theta) + (y_{0r} - r_{2w}\theta)\cos(\theta) \end{cases} \tag{5}$$

By solving the meshing equation and the rack profile equations, the transverse rack profile can be obtained. The rack surface between two rotors can be obtained by projecting the transverse rack to the rack normal plane as shown in Figure 6. The rack surface always lies between the main rotor and gate rotor.
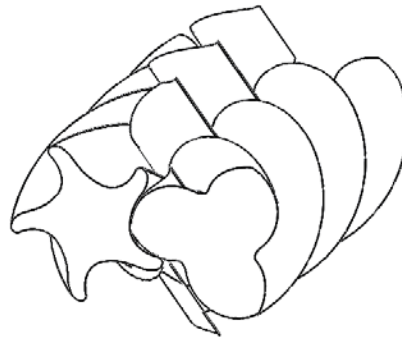

Figure 6. Rack surface.

3.3. Normal rack
The normal projection of the rack surface called the normal rack $r_{nr}(\theta, \beta)$ can be generated analytically from projecting the rack surface onto the normal plane. The normal rack profile equations are as follows:

$$\begin{cases} x_{nr} = x_{0r}\cos(\beta) \\ y_{nr} = y_{0r} \\ z_{nr} = x_{0r}\sin(\beta) \end{cases} \tag{6}$$

Where r_{nr} is the normal rack profile and β is the helix angle at the pitch circle. The normal rack for the main rotor and gate rotor in the normal plane is as shown separately in Figure 7.
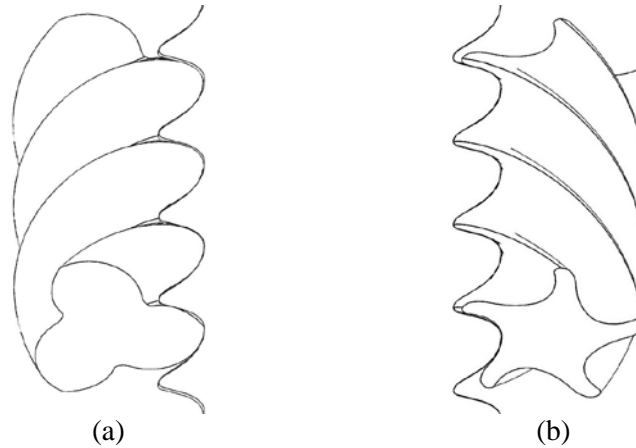

(a)   (b)
Figure 7. Normal rack of (a) main rotor and (b) gate rotor.

4. Domain decomposition

The grid generation process of screw machine is performed in the Cartesian coordinate system introduced at the beginning of the paper and starts from the process of replacing the working chamber with a set of grid points. The decomposition of the flow region between the casing and two rotors consists of dividing the region into a series of 2D cross sections. In practise three methods can be utilised to decompose a cross section of the working domain, namely, transverse differential division curve, transverse rack and the normal rack. So far the grids of the fluid domain are generated in transverse planes. A different method using the normal rack to split the fluid domain in the normal cross section is proposed in this paper. It is more convenient if the main rotor and gate fluid domains are cutting by their normal planes respectively, so the grids can be generated in the normal planes. This process will make the grid lines between the consecutive planes nearly normal to the planes which will completely render or at least reduce the skewness of the grids. The 2D cross sections are then combined together to construct the full 3D fluid domain representing the main and gate fluid domains.

4.1. The fluid domain

The fluid domain of a screw rotor is helical type volume generated by the simultaneous revolving of the interlobe space around the rotor axis and translation along the axis. And the interlobe space can be separated to two domains according to the normal rack splitting surface. And then the rack line appears in the normal cross section. The outer boundary is the composition of the normal rack and the casing circle and the inner boundary is the rotor profile. To visualize the changing of rotor domain, two sub-domains are established as shown in Figure 8. The sub-domain around one lobe of the main rotor denoted by the green colour and the sub-domain in one interlobe of the gate rotor denoted by the red colour are meshing with each other and the unique rack surface is between the two rotors.
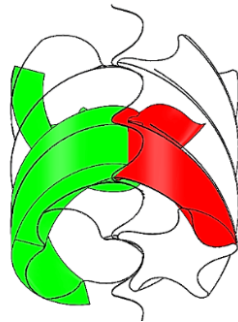
Figure 8. Two sub-domain between the rotors.

4.2. Decomposition method

The fluid domain between the rotors and casing is represented by a number of cross sections using the normal planes of the main rotor and gate rotor respectively. Then the normal rack line and the casing will form the outer boundary of the fluid domain while the rotor profile acts as the inner boundary. Figure 9 shows the sub-domain of the gate rotor cut by 20 consecutive normal planes to show how is the fluid domain changing. The orientation of the profile is aligned with the orientation of normal planes. The fist cross section is cut out at the end of the domain. Following that, the six normal cross sections are identical because the gate and the main sub-domains are apart. Once the two sub-domains interface, the normal rack replaces the outer circle as the outer boundary gradually cutting in and then out after which the domain returns to the shape formed by the outer circle and the rotor. If all of these 2D normal planes are meshed with a numerical grid, then a three dimensional fluid domain can be obtained by combining meshes in two dimensional cross sections. There are many advantages of the domain decomposition in normal planes. Firstly, the meshes in the clearance between rotors and casing can be refined by adjusting the density of nodes in required sections and the alignment of the mesh to the main and leakage flows can be achieved. Secondly, orthogonality of the mesh can be easily obtained. All of this advantages can contribute to a robust and efficient simulation of twin screw machines using a number of CFD solvers.
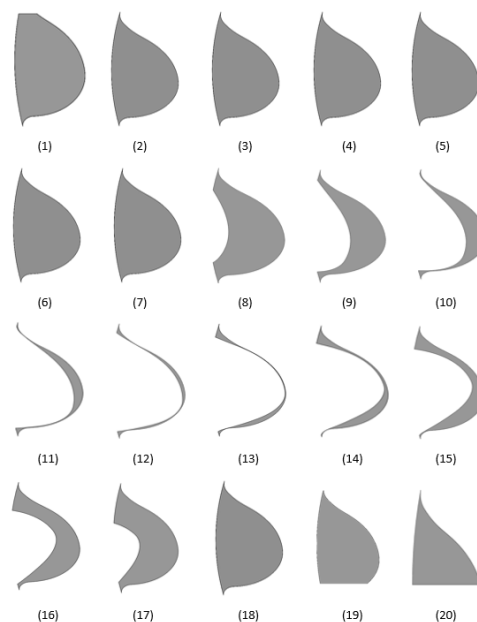

Figure 9. The cross section of the female sub-domain.

5. Conclusions

In this paper, the analytical racks of screw rotor in transverse plane and normal plane are firstly introduced. Then it introduces a method which uses normal rack to decompose the fluid domain between the twin screw rotors and the casing into two sub-domains in which numerical meshes can be generated in the planes normal to each of the rotors. By this means it is possible to achieve 3D numerical mesh with extended capability of mesh refinement in clearance and alignment of the mesh to the main and leakage flows. This method can also be used in the single screw and non-parallel axis compressors like internal conical rotary screw compressor. Application of the method will be demonstrated in the case of screw machines with large helix angles as used in single and multiphase screw pumps from the simulated and experimental results.

Acknowledgments

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