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Internet Auctions with a Temporary Buyout Option[*]

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Abstract

We model an Internet auction with a *temporary* buyout option. Our main result shows that under certain parameter values, there exist two types of equilibria where offering a temporary buyout option with an appropriate reserve price enables the seller to increase expected revenue.

1 Introduction

This paper is motivated by Internet auctions, in which buyout options are present. The buyout option in the auctions allows a bidder to obtain the object immediately by exercising a pre-determined price. Mainly, there are two types of buyout options: the first is a permanent buyout option, which remains available throughout the whole auction and was offered on Yahoo. The second is a temporary buyout option and active as long as no bid has been put in the auction. This option is offered on eBay, called "Buy-It-Now". In this paper, we only focus on temporary buyout option.

eBay's quarterly reports for 2007-2008 show that fixed price trading, mainly consisting of "Buy-It-Now" purchases, accounts for more than 40% of gross merchandise volume. Moreover, accross different product categories, percentage of augmenting Internet auctions with a buyout option is between 20% and 60%. Our main interest is to investigate in which case it is attractive for the seller to combine an auction with a temporary buyout option.

Typically, Internet auction format is second price. However, Internet auctions also have some special characteristics. First, normally, in order to attract as many bidders as possible, an Internet auction lasts for a few days. During this period, each bidder sequentially arrives at the auction and endogenously decides whether or not to enter and put a bid. Second, before entry, each bidder has to spend some money and

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time identifying the seller's reliability and services, all of which can be seen as a participation cost for each bidder[1]. Bajari and Hortacsu (2003) have shown the significant empirical evidence of participation cost on eBay auctions, and also suggested that this cost is one of the main determinant factors of entry.

In this paper, we construct a simple two-valuation model with two risk neutral bidders to illustrate how a temporary buyout price is profitable for a seller in a second price auction, when participation is costly and bidders sequentially decide whether or not to enter. First, we characterize the maximum expected revenue of the seller under reserve price alone. We show that under certain parameter conditions, the presence of participation cost discourages future entry and thus decreases the expected revenue of the seller. With a temporary buyout option, since the first bidder with high valuation is supposed to exercise this option, when this option is not exercised, the future bidder knows that the initial bidder is of low valuation. This encourages entry of the second bidder in the auction and enables the seller to increase his expected revenue.

Till date, the main explanation for the use of buyout options is based on risk aversion of agents. In Budish and Takeyama (2001), it is assumed that bidders are risk averse while Mathews and Katzman (2006) study a risk averse seller sells an object via a second price auction with a temporary buyout option and show that "this (buyout) option may result in a Pareto improvement compared to a sealed bid second price auction"; Hidvégi, Wang, and Whinston (2006) investigate an English auction with a permanent buyout price, and show that "(a) buy price increases expected social welfare and the expected utility of each agent when either buyers or seller are risk-averse"; Reynolds and Wooders (2009) consider both temporary and permanent buyout options and claim that in an auction with a buyout option, the seller can extract more profit from risk averse bidders.

In the literature, some theoretical analysis on participation cost in auctions has been discussed by Tan and Yilankaya (2006); McAfee and McMillan (1987); Levin and Smith (1994); Samuelson (1985). Bulow and Klemperer (2009) discuss jump bidding in the case of costly and sequential entry in auctions.

2   The model

Consider a seller S selling an indivisible object to two *risk neutral* bidders, 1 and 2 by employing a second price auction with a reserve price $R \in [0, V_H]$. The valuation V_i for bidder i is private information and we assume that it is either V_L or V_H, where $V_L < V_H$ with $\alpha \in (0,1)$ being the probability that $V_i = V_L$. We assume that the auction lasts for two periods and bidders arrive sequentially and make a decision to either enter the auction or quit. We assume that the bidders are aware of their own private valuations and thus entry decisions can be conditioned on this variable. Moreover, we assume that bidder 2 also gets to observe whether entry has taken place in period 1 or not and thus bidder 2's entry decision in period 2 can also be conditioned on this information. We also assume that to enter the auction and submit a bid, each bidder has to incur a participation cost C, i.e., money and time associated with bidding, where $C \in (0, \Delta)$, $\Delta = V_H - V_L$ and $V_L - C \geq 0$. This cost is the same across both bidders. At the end of the second period, a bidder with highest bid wins and pays either the second highest bid or the reserve price R. If both bidders put the same bid, the object is allocated to one of the bidders with probability $1/2$. If there is no entry, the

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[1]Even thought eBay has a feedback system to assess sellers' reputation, eBay still suggests potential bidders to contact with the seller and check his creditability before bidding, since this reputation system might be easily manipulated by sellers.

seller keeps the object whose valuation of the object is normalized to zero. Throughout, we will restrict our attention to equilibrium outcomes in which each entering bidder bids his true valuation[2].

2.1 Maximum expected revenue under reserve price alone

In this section, we first characterize the maximum expected revenue of the seller when he only sets a reserve price R.

If $R > V_L - C$, a bidder with valuation V_L will never enter the auction and thus the seller can only sell if one of the bidders has the valuation V_H. It is clear that the optimal choice of R in such a case will be given by $R = V_H - C$. The seller's expected payoff given $R = V_H - C$ is given by

$$Y_1 = (1 - \alpha^2)(V_H - C) \tag{1}$$

Now with $R \leq V_L - C$, if entry takes place in period 1, then bidder 2 of valuation V_L will not enter the auction. However, the decision of bidder 2 of high valuation will depend on whether $\alpha(V_H - V_L)$ is greater or less than C and this will affect the optimal reserve price (as well as the expected payoff to the seller), we consider each in turn.

Case 1. $\alpha(V_H - V_L) < C$

In this case, if entry takes place in period 1, bidder 2 of either valuation makes a negative profit by entering the auction. Anticipating this, bidder 1 of both valuation must enter in the first period. The seller's payoff then is R. Thus, when optimal value of $R \leq V_L - C$ and $\alpha(V_H - V_L) < C$, the maximum expected revenue to the seller is given by

$$Y_2 = V_L - C \tag{2}$$

Case 2. $\alpha(V_H - V_L) \geq C$

In this case, even if entry takes place in period 1, bidder 2 of high valuation must enter the auction. This implies that bidder 1 with low valuation gets an expected payoff of $\alpha(V_L - R) - C$. Now if optimal choice of R is such that $R \leq V_L - \frac{C}{\alpha}$, there exists a pure strategy equilibrium in which bidder 1 of both valuations will enter with probability one, in period 2, bidder 2 enters if and only if he is of high valuation. This gives the seller an expected payoff of $\alpha R + (1 - \alpha)[\alpha V_L + (1 - \alpha)V_H]$. Clearly, this payoff is maximized at $R = V_L - \frac{C}{\alpha}$ which gives the seller an expected payoff of

$$Y_3 = (1 - \alpha)^2 V_H + \alpha(2 - \alpha)V_L - C \tag{3}$$

Now when R is strictly greater than $V_L - \frac{C}{\alpha}$ (but less than $V_L - C$), a pure strategy equilibrium will not exist. In this case, however, there exists a unique mixed strategy equilibrium in which bidder 1 of high

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[2]Other types of outcomes may arise in equilibrium, for instance, if $R \leq V_L - C$, an outcome in which bidder 1 enters and bids V_H (not truthful bidding), and bidder 2 staying out can be supported as an equilibrium outcome. In the equilibrium, bidder 1 wins the object by paying R. A large literature has been discussed about selection of "reasonable" equilibria in a sequential game. Riley (2001) gives a survey in this literature.

valuation enters with probability one, bidder 2 of low valuation does not enter the auction. Bidder 1 of low valuation enters the auction with probability q_L while bidder 2 of high valuation enters the auction with probability q_H. Let $Y(R)$ denote the expected payoff to the seller in this mixed strategy equilibrium. It is however, possible to show that $Y(R)$ is strictly decreasing in R in the interval $(V_L - \frac{C}{\alpha}, V_L - C)$. This allows us to conclude that the seller's expected payoff takes the maximum value when $R = V_L - \frac{C}{\alpha}$ and $q_L = q_H = 1$. Thus, when the optimal choice of R is no more than $V_L - C$ and $\alpha(V_H - V_L) \geq C$, the best choice of R is given by $R = V_L - \frac{C}{\alpha}$ and the maximum expected payoff to the seller is given by Y_3 in Eq (3.)[3].

We can summarize the above discussions in the following Proposition

Proposition 1. *If the seller uses only reserve price, his optimal choice of R is given by*

(1.) Suppose $\alpha(V_H - V_L) < C$:

if $\frac{1}{\alpha^2}[V_L - (1 - \alpha^2)V_H] \leq C$, the seller chooses $R = V_H - C$ and the expected payoff is Y_1;

if $\frac{1}{\alpha^2}[V_L - (1 - \alpha^2)V_H] > C$; the seller chooses $R = V_L - C$ and the expected payoff is Y_2.

(2.) Suppose $\alpha(V_H - V_L) \geq C$:

if $\frac{1}{\alpha}[(2 - \alpha)V_L - 2(1 - \alpha)V_H] \leq C$, the seller chooses $R = V_H - C$ and the expected payoff is Y_1;

if $\frac{1}{\alpha}[(2 - \alpha)V_L - 2(1 - \alpha)V_H] > C$; the seller chooses $R = V_L - \frac{C}{\alpha}$ and the expected payoff is Y_3.

Proof. To prove this Proposition, we first observe that $Y_3 > Y_2$. Now when $C > \alpha(V_H - V_L)$, once one bidder enters the auction, the second bidder (of either valuation) can not enter the auction. Therefore, the seller can get at most R and his payoff is either Y_1 or Y_2. It is easy to check that $Y_1 \geq Y_2$ if and only if $C \geq \frac{1}{\alpha^2}[V_L - (1 - \alpha^2)V_H]$. Now when $C \leq \alpha(V_H - V_L)$, bidder 2 (of valuation V_H) will enter the auction and thus the maximum payoff to the seller is given by $max\{Y_1, Y_3\}$. It is easy to check that $Y_1 \geq Y_3$ if and only if $C \geq \frac{1}{\alpha}[(2 - \alpha)V_L - 2(1 - \alpha)V_H]$. □

2.2 Temporary buyout option

In this section, we consider the same scenario but the seller can offer a temporary buyout option (price), denoted as B. This option is temporary as it will disappear as soon as the first bid is placed in the auction. The bidder who exercises this buyout price pays this price and obtains the object. The participation cost C still needs to be paid whether the object is acquired through bidding his private value or exercising the buyout price B. The seller chooses two variables to maximize his expected revenue: (B, R), where B is the buyout price, R is the reserve price, and $B > R$[4].

The use of a temporary buyout option in our model is profitable for the seller as it encourages entry of the second bidder. The seller can choose an appropriate buyout price which will be exercised if and only if the initial bidder is of high valuation. The first bidder of low valuation either does not enter the auction

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[3]A proof is available upon request.

[4]When $B = R$, in fact, the auction becomes a fixed price selling mechanism.

(as in the equilibrium play of Proposition 2) or else he enters but does not exercise the buyout option (as in the equilibrium play in Proposition 3). In either case, however, when the buyout option is not exercised, the second bidder correctly anticipates that the first bidder is of low valuation. This encourages future entry and raises the seller's expected revenue in the auction.

Proposition 2. *Suppose that* $(1-2\alpha)C > 2\alpha(V_H - V_L) - \alpha^2(2V_H - V_L)$, *then there exists a choice of* (B, R) *and an associated equilibrium in which the seller's expected revenue is strictly greater than his expected revenue in an auction which uses only reserve price.*

Proof. Let (B^*, R^*) be given by $R^* = V_L - C$ and B^* satisfying

$$V_H - B^* = \alpha(V_H - R^*) \tag{4}$$

and consider the following strategy profile for the bidders: bidder 1 with valuation V_H enters the auction and exercises the buyout option while bidder 1 of low valuation does not enter the auction. The entry strategy of bidder 2 of high valuation is to always enter the auction given that the buyout option is not exercised while bidder 2 with valuation V_L enters the auction if and only if there has been no entry in the first period.

Given the strategies of the second bidder, if the first bidder of high valuation exercises the buyout option, his payoff is $V_H - B^*$. On the other hand, if he enters and does not exercise the option, bidder 2 of high valuation will enter, which gives bidder 1 an expected payoff of $\alpha(V_H - R^*)$. Clearly, the strategy of high valuation bidder 1 is optimal. It is also clear that if bidder 1 of low valuation were to enter, his expected payoff will be negative given that the second bidder of high valuation would enter the auction.

Given bidder 1's strategies, either the auction is over with the buyout option being exercised in the first period, or there is no entry in the first period. Given no entry and the fact that $R^* = V_L - C$, it is optimal for bidder 2 of both valuations to enter and get the object at R^*. Thus, bidder 2' entry strategy is optimal as well. It is important to note that bidder 2 (valuation V_H) always enters the auction. Now if there is an entry in the first period and the buyout option is not exercised (this happens 'off the equilibrium path'), bidder 2's belief is that the first bidder is of low valuation and this justifies his entry decision.

The expected payoff to the seller from this equilibrium is given by $Y_B = \alpha R^* + (1-\alpha)B^*$. It is easy to check that at $R^* = V_L - C$ and $B^* = V_H(1-\alpha) + \alpha R^*$ that $Y_B = (1-\alpha)^2 V_H + \alpha(2-\alpha)(V_L - C)$ which is strictly greater than Y_3. Moreover, given the assumption that $(1-2\alpha)C > 2\alpha(V_H - V_L) - \alpha^2(2V_H - V_L)$, it is also possible to check that $Y_B > Y_1$. Since from Proposition 1, the maximum expected payoff to the seller is no more than $max\{Y_1, Y_3\}$, the Proposition follows. $\square$

While this Proposition tells us that a temporary buyout option can dominate an auction mechanism that uses only reserve price, the strategies used by the bidders in this equilibrium is somewhat problematic. Observe that in this equilibrium, the strategy of low valuation bidder 1 is not to enter the auction while bidder 2 of the same valuation enters the auction with probability one (along the equilibrium path) if the buyout option is not exercised. In the context of Internet auctions (which runs for a few days) where the buyers have an option to wait, the entry sequence of the bidders is not predetermined and may not be common knowledge. Consequently it might be difficult (if not impossible) to implement the above outcome as the bidders may not be sure whether he is the first or the second bidder to arrive at the auction. Thus, it

is of interest to know whether a temporary buyout option can still dominate a standard auction mechanism using strategies in which the bidders' strategies *can* depend only on (*a.*) bidders' private valuations and (*b.*) the entry history, i.e, whether some other bidder has already entered an auction but not on any other information. Given this restriction, a bidder with a given valuation and who faces the same entry history must use the same strategies independent of when he arrives at the auction. We refer to these strategies as the *time invariant* strategies. In what follows, we only consider pure strategy equilibrium.

Proposition 3. *Assume that the bidder uses time invariant strategies in which his choices could depend only on his valuation and the entry history but not on the time he arrives at the auction, then, there exists a choice of (B, R) and an associated equilibrium in which the seller's expected payoff is exactly $Y_3 = (1 - \alpha)^2 V_H + \alpha(2 - \alpha)V_L - C$. Moreover, Y_3 is also the maximum payoff that the seller can get when bidders are restricted to use time invariant strategies.*

Proof. Let $R^{**} = V_L - \frac{C}{\alpha}$ and B^{**} that satisfies $V_H - B^{**} = \alpha(V_H - R^{**})$. Consider the following *time invariant* strategy profile for the bidders: bidder 1 (of both valuations enters) exercises B^{**} when $V_1 = V_H$ and bids his valuation when $V_1 = V_L$. Bidder 2 of high valuation chooses to enter given that B^{**} is not exercised while bidder 2 of valuation V_L always chooses not to enter. Note that given bidder 1's entry strategies, along the equilibrium path, in bidder 2's turn, if B^{**} is not exercised and a bid is already in the auction, bidder 2 believes that $V_1 = V_L$ and his entry decision is optimal.

Given bidder 2's strategies, bidder 1 of high valuation obtains the same expected payoff between exercising B^{**} and putting his valuation. Clearly, the strategy of bidder 1 with valuation V_H is optimal. It is also straightforward to see that when $V_1 = V_L$, bidder 1 weakly prefers to enter (the expected payoff is zero) given bidder 2 of high valuation would enter the auction. Given the strategies of the first bidder, either the auction is over with the buyout option being exercised in the first period, or there is one bid in the first period. Given a bid already in the auction, it is optimal for bidder 2 of high valuation to enter and obtain the object by paying V_L, but chooses not to enter if $V_2 = V_L$. Thus, bidder 2's entry strategy is optimal.

The expected revenue of the seller in the equilibrium is given by $(1 - \alpha)B^{**} + \alpha[\alpha R^{**} + (1 - \alpha)V_L]$. Substituting $R^{**} = V_L - \frac{C}{\alpha}$ and $B^{**} = (1 - \alpha)V_H + \alpha V_L - C$ into the equation, we have the expected revenue of the seller $(1 - \alpha)^2 V_H + \alpha(2 - \alpha)V_L - C$, which is exactly equal to Y_3.

Observe also that in any equilibrium where bidders use *time invariant* strategies, it must be the case that a bidder who arrives at the auction firstly must enter immediately. The high valuation bidder must exercise the buyout option while the low valuation bidder will put in a bid. Since the buyout price is higher than the reserve price, it must be that if the buyout option is not exercised, the second bidder (of high valuation) must enter the auction. This means that the expected payoff of the first bidder (low valuation) is at most $\alpha(V_L - R) - C$. Since this payoff has to be non negative, we have $R \leq V_L - \frac{C}{\alpha}$. Given any such R, the maximum buyout price B equals $V_H - \alpha(V_H - R)$. Clearly, the seller's payoff is increasing in R in the range $[0, V_L - \frac{C}{\alpha}]$ and thus the best value for R equals $R^{**} = V_L - \frac{C}{\alpha}$, which yields an expected payoff of Y_3 to the seller. $\square$

Corollary 1. Suppose that $Y_1 < Y_3$ and that $\alpha(V_H - V_L) < C$, then there exists a temporary buyout option and an associated equilibrium in which the seller's expected payoff is strictly greater than the expected payoff that he can get using an auction that uses only reserve price.

Proof. In Proposition 1, under the condition of $\alpha(V_H - V_L) < C$, the expected payoff of the seller is no more than $max\{Y_1, Y_2\}$ when he uses only reserve price. From Proposition 3, in the equilibrium a temporary buyout option yields Y_3 to the seller. Note that Y_3 is strictly greater than Y_2. Therefore, if $Y_1 < Y_3$, then the seller is better off from offering a temporary buyout option.

Summarizing the parameter conditions of $Y_1 < Y_3$ and $\alpha(V_H - V_L) < C$, we have that the temporary buyout option raises the seller's expected revenue when $\alpha(V_H - V_L) < C < \frac{1}{\alpha}[(2 - \alpha)V_L - 2(1 - \alpha)V_H]$. $\quad\square$

Remark 1. When the seller uses a temporary buyout option, there also exist other equilibria where the buyout option may reduce the seller's expected revenue [5]. In particular, when $\alpha(V_H - V_L) < C < \frac{1}{\alpha}[(2 - \alpha)V_L - 2(1 - \alpha)V_H]$ so that the seller is better off from a temporary buyout option in one equilibrium, there always exists another equilibrium where bidder 1 with both valuations chooses to enter and never exercises the buyout option and bidder 2 is weakly better off not to enter. Compared with using reserve price alone, in the second type of equilibrium the buyout option decreases the seller's expected payoff, as bidder 1 only pays $R = V_L - \frac{C}{\alpha}$ to obtain the object after entry.

3   Conclusion

Our main result shows that under certain parameter values, there exist two types of equilibria where offering a temporary buyout option with an appropriate reserve price enables the seller to increase expected revenue.

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[5]I am grateful to an anonymous referee who pointed this out.

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