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AN EMPIRICAL STUDY OF CLAIM AND SICKNESS INCEPTION TRANSITION INTENSITIES (ASPECTS OF THE UK PERMANENT HEALTH INSURANCE EXPERIENCE)

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ABSTRACT

We consider the relationship between the claim inception transition intensity and the sickness inception transition intensity. Using data from the CMI Bureau for Permanent Health Insurance (PHI) policies, we propose models for representing the underlying trends in the sickness inception intensities over an extended recent period, *viz* 1975-94. As part of this study, we note the dependence of this relationship on the underlying recovery intensities and we consider, in detail, the extent to which the recovery intensities for the policies with different deferred periods differ significantly (when matched by sickness duration and age).

KEYWORDS

Sickness inception intensities; Claim inception intensities

1. INTRODUCTION

We focus on the relationship between claim and sickness inception transition intensities from the healthy state, and explore its possible implications for modelling. This work may be seen as the natural extension to Renshaw & Haberman (1999) in which time trends, over the period 1975-94, are identified and modelled in respect of claim inception transition intensities (as well as for sickness recovery transition intensities and for the force of mortality when sick) for UK permanent health insurance (PHI) data. We also acknowledge aspects of the pioneering work of the CMI Bureau which, using the 1975-78 data base, established their PHI multiple state model (CMI Committee (1991)) and which are relevant to this work. Note, however, that we choose to operate wholly within a generalised linear modelling setting: as in Renshaw & Haberman (1995,1999).

In this study, it should be noted that we focus only on the male PHI UK experience, modelling the data either in 'dynamic' mode represented by individual calendar years, or in 'static' mode with the data grouped into one of five consecutive quadrennia defined sequentially within the twenty year period 1975-94 prior to analysis and with each quadrennium modelled separately. It should be noted that it is a straight forward matter to move from the first mode to the second mode by suppressing the relevant suffix in t (actual or implied) in any expressions and by pre-setting any time related parameters in the linear predictors to zero.

Given our stated interest in the 'dynamic' mode of analysis, it is a relatively simple matter to generalise the definition of the underpinning semi-Markovian process (CMI Committee (1991)), and to

develop the associated Kolmogorov forward equations, so as to accommodate this feature. Summary details are presented in Appendix I.

Issues concerning the implementation of the basic identity connecting claim and sickness transition intensities are discussed in Section 2. The chief of these, concerning the extent to which the recovery intensities, for different deferred periods based on a specific quadrennium, differ significantly, when matched by sickness duration and age is investigated in Section 3. The practical determination of the identity is described in Section 4. The sickness transition intensity is modelled as a function of age and calendar period effects in Section 5, while the implications of a viable alternative way of achieving this under the proposed modelling assumptions, are investigated in Section 6. The paper is summarised in Section 7.

2. A BASIC RELATIONSHIP

Let τ_{tx} and σ_{tx} denote the respective claim and sickness inception transition intensities, for a healthy male, aged x in calendar year t, at sickness inception. They are related by the formula

$$\tau_{tx} = \pi_{txd}\,\sigma_{tx}$$

in which π_{txd} denotes the probability that the time spent by an individual in the sick state exceeds the deferred period d weeks, before which benefits become payable under the terms of the sickness insurance policy. This relationship plays a role in the calculation of expected present values of disability annuities in Sweden: see Mattsson (1956), Dillner (1969), Haberman & Pitacco (1999). It is also implicit in the approach used to graduate sickness inception intensities for the 1975-78 UK experience based on a normal approximation to an overdispered Poisson distribution: see CMI Committee (1991). Typically, such UK experiences involve deferred periods $d = 1$, 4, 13, or 26 weeks, also referred to as DP1, DP4, DP13 and DP26 respectively.

The computation of π_{txd} is crucial to the targeting of σ_{tx}. Within the context of a suggested 'dynamic' version of the multiple state PHI model, it is defined by

$$\pi_{txd} = \exp - \int_0^{d/52} (\rho_{t+u,x+u,u} + \nu_{t+u,x+u,u})\,du \qquad (2.1)$$

where ρ_{txz} and ν_{txz} denote the respective recovery transition intensity when sick and the force of mortality when sick, for an individual with attained age x years, for calendar year t and current sickness duration z weeks. When we consider a specific quadrennium, we shall suppress the first of the suffices. Note also that the expression for π_{txd} partitions into two multiplicative components, with the recovery component dominant over the mortality component, given the parameter estimates in CMI Committee (1991), Renshaw & Haberman (1995,1999).

As recognised within the context of the data set for the 1975-78 quadrennium (CMI Committee 1991), the evaluation of the integral in (2.1) poses intrinsic practical difficulties because data associated with the deferred period concerned are not available for the estimation of the functional forms of ρ and σ within the range of the integral. That is, observations are only available for sickness durations in excess of the respective deferred periods. Consequently, a plausible way forward is to resort to the use of functional forms for ρ and ν based on the data associated with those deferred periods which are shorter than the one in question, where this is feasible. In so far as the contribution to π_{txd} from ν is concerned, the difficulty is resolved by the practical necessity of having to group the data over all deferred periods prior to the modelling of ν, matched by calendar period, age and sickness duration, because of the paucity of reported deaths.

With respect to the more dominant contribution from ρ, the CMI Committee (1991) study, based on the 1975-78 experience, has identifed two characteristics in the data which are subsequently exploited in the determination of the contribution to π_{txd} from ρ. The first characteristic is the presence of tapering convergent sickness duration 'run-in' periods in which the recovery rates for DP4, DP13 and DP26 policies, in the first four weeks when claims are allowed under the terms of these policies, lag significantly behind the corresponding recovery rates for DP1 policies, matched for sickness duration and age. The second characteristic is that, once beyond these tapering 'run-in' thresholds, the recovery rates for DP1, DP4, DP13 and DP26 policies do not differ significantly when matched by sickness duration and age. Hence, the data in these cells effectively converge and may be grouped, prior to the modelling of ρ as a function of x and z. It is of interest to ascertain the extent to which these two characteristics extend from the 1975-78 experience to subsequent quadrennia. This problem is considered in Section 3.

3. A Comparision of Matching Recovery Intensities

In this section, our objective is designed to ascertain the extent to which the recovery intensities, for different deferred periods based on a specific quadrennium (so that the suffix t is suppressed throughout this section), differ significantly, when matched by sickness duration and age. In this respect we are mindful of the pattern of significant differences established for the 1975-78 quadrennium *viz* the 'run-in' thresholds described above. The approach used is based on the analysis of paired differences in log crude recovery intensities. It is preferred as an alternative to the approach adopted by the CMI Committee (1991) since it acknowledges the interactive nature of the dependence of ρ on x and z prior to testing. By way of cross-reference, we also reinvestigate the 1975-78 experience in addition to the other four quadrennial experiences.

For the purpose of the analysis we focus on the following cross-classification of data cells

gender	male
quadrennia	75-78, 79-82, 83-86, 87-90, 91-94
deferred periods (dp)	1, 4, 13, 26 wks
ages (x)	18-, 25-, 30-, 35-, 40-, 45-, 50-, 55-, 60-64 yrs (coded 1,2, ..., 9)
sickness durations(z)	4,5,6,7,8,9,10,11-12; 13-14,15-17,18-21,22-25; 26-29,30-38,39-51 wks; 1yr,2-11yrs

This selection of categories agrees with that used by the CMI Committee (1996). In particular, we note that the four duration categories labelled 4, 5, 6 & 7 weeks constitute the so-called four week 'run-in' period for DP4, that the two duration categories labelled 13-14 & 15-17 effectively constitute the four week 'run-in' period for DP13, and that the single duration category labelled 26-29 constitutes the four week 'run-in' period for DP26.

For a specific quadrennium and for deferred period d ($d = 1, 4, 13, 26$), let

$$\begin{aligned} r^d_{xz} &= \text{number of recoveries in cell } (d, x, z) \\ e^d_{xz} &= \text{exposure (in years) to the possibility of recovery in cell } (d, x, z) \end{aligned}$$

and compute the differences in log crude recovery intensities

$$y^d_{xz} = \log\left\{\frac{r^1_{xz}}{e^1_{xz}}\right\} - \log\left\{\frac{r^d_{xz}}{e^d_{xz}}\right\}$$

where $d = 4$, 13 or 26, as appropriate. By this means, we can compare log crude recovery intensities, matched by age and sickness duration, for each deferred period DP4, DP13, DP13, relative to DP1.

Let

$$\mathrm{E}(Y^d_{xz}) = \eta^d_{xz}$$

denote the expected value of the difference in the log crude recovery intensities, Y^d_{xz}. Then, in the light of the CMI Committee (1991) findings described above, we formulate the null hypothesis:

$$\mathrm{H}_0\text{: } \eta^d_{xz} \begin{cases} \neq 0 \ \forall\ (z, x), \text{ within the 4 week 'run-in' threshold} \\ = 0 \ \forall\ (z, x), \text{ otherwise} \end{cases} \qquad \forall\ d = 4, 13, 26$$

Given the focus on *differences*, matched age for age as well as by sickness duration, we can test this hypothesis by fitting the one-way main effects parameterised structure

$$\mathrm{E}(Y^d_{xz}) = \eta^d_{xz} = \alpha_z \tag{3.1}$$

for $d = 4$, 13, 26 respectively. Under H_0, it is expected that the resulting predictor estimates $\hat{\eta}^d_{xz} = \hat{\alpha}_z$ will differ significantly from zero inside the sickness duration 'run-in' threshold *only*. Further, the

establishment of (statistically significant) decreasing predictor estimates with increasing duration within the 'run-in' period (observable for DP4 and DP13) would be consistent with the tapering of recovery rates within the 'run-in' period. Clearly, non-significant predictor estimates beyond the 'run-in' threshold would be consistent with the convergence of the respective matched recovery rates.

As an added precaution, we also test for main age effects using the two-way additive structure *viz*

$$\mathrm{E}(Y_{xz}^{d}) = \eta_{xz}^{d} = \alpha_z + \beta_x. \tag{3.2}$$

Given the emphasis here on matched differences in log recovery intensities, we do not anticipate this to be a statistically significant factor on the basis of the CMI Committee (1991) findings in relation to the 1975-78 experience.

The structures are fitted by the method of weighted least squares, with the y_{xz}^{d} statistics as the responses, and with weights

$$\frac{r_{xz}^{l} r_{xz}^{d}}{r_{xz}^{l} + r_{xz}^{d}}.$$

One aspect of this choice of weights is to ensure that in the event of a reported zero recovery cell count, for which the response variable goes undefined, such cells are weighted out of the analysis.

The parameter estimates $\hat{\alpha}_z$, coupled with their t-statistics, for the main sickness duration effects model structure (3.1) are presented systematically in Tables 3.1(a,b,c). The F-statistics associated with the addition of main age effects into the model structure are systematically reported in Table 3.2.

Scrutiny of the first column in each of Tables 3.1(a,b,c), representing the 1975-78 quadrennium, reveals that with one exception, all of the α_zs differ significantly from zero within the durational 'run-in' periods and that again with one exception, all of the α_zs do not differ significantly from zero outside the 'run-in' period. In addition, the values of the predictor estimates imply a pronounced tapering effect within the 'run-in' for the DP4 and DP13 experiences. We note, however, that there is no statistically significant support for a four week 'run-in' period for the DP26 experience from this analysis. Scrutiny of the first column in Table 3.2, again representing the 1975-78 quadrennium, provides strong evidence that, as anticipated, age effects are not statistically significant. Such results provide strong confirmation of the convergent 'run-in' periods for the DP4 and DP13 experiences identified by the CMI Committee (1991) for their PHI model and also lend strong support for the modelling of a common set of recovery intensities across deferred periods outside of the four week durational 'run-in' periods.

The situation is not quite so clear cut, however, in the more recent quadrennia under investigation, the results for which are recorded in the remaining columns of Tables 3.1(a,b,c) and Table 3.2. Tables 3.1(a,b,c) confirm the presence of statistically significant non-zero α_zs, with one

exception (DP13 for 1991-94), within the durational 'run-in' periods for each of the four quadrennia. These results indicate the persistence of the initial four week differential durational effect for recovery intensities when comparing DP4, DP13, and DP26 policies on the one hand and DP1 policies on the other hand, matched by sickness duration and by age. However, the distinctive tapering effect for the DP4 experiences across the four individual weeks concerned is not quite so pronounced for the 1983-86 and 1991-94 quadrennia (Table 3.1(a)). More importantly, there is a noteworthy occurrence of statistically significant non-zero α_zs outside the durational 'run-in' period coupled with the added complication of one significant main age effects F-statistic (and a further two marginally significant main age effects F-statistics: see Table 3.2). This runs somewhat counter to the rationale of grouping matching data cells across different deferred periods, outside the durational 'run-in' periods, prior to the modelling of recovery rates. We summarise these results by tabulating the tally of statistically significant α_zs outside the 'run-in' period thus:

	Quadrennium					*Total*
	75-78	*79-82*	*83-86*	*87-90*	*91-94*	*possible*
DP4	0	1*	0	4	3	13
DP13	0	2	0†	3†	4	7
DP26	1	3	1	2	0	4

(* denotes statistical significant age effects which should be taken into account)
(† denotes marginally significant age effects which are also investigated)

Tally of statistically significant non-zero α_zs outside the durational 'run-in' period

This summary table provides support for grouping in the case of the 1983-86 quadrennium (in addition to the 1975-78 quadrennium). The case for grouping is not well supported by these results when it comes to the other three quadrennia, although there are no obviously discernible, systematic patterns in the non-zero α_zs (outside of the four week 'run-in' periods) with one exception. This exception concerns DP4 policies in the 1987-90 quadrennium for which there is evidence of an extended 'run-in' of up to 12 weeks duration.

Next, we investigate the implications of the statistically significant F-statistic for DP4 policies in the 1979-82 quadrennium, established on fitting the two-way main effects structure, equation (3.2). On contrasting the resulting age effects, it is possible to reduce the number of effective age parameters $\beta_{x'}$, without a significant loss of information, to just two, by setting

$\beta_1 = 0$ representing jointly the youngest age group [18,30) and the combined oldest ages [50,65]

$\beta_2 \neq 0$ representing the single age group [30,35)

$\beta_3 \neq 0$ representing the combined age groups [35,50)

(Note that the first of these parameters is set equal to zero out of necessity in order to avoid over-parameterisation in a technical sense). The resulting estimated parameter values

$$\hat{\eta}^{4}_{x'z} = \hat{\alpha}_z + \hat{\beta}_{x'}$$

together with the t-statistics are reported, in the appropriate format, in Table 3.3. While these results are supportive of a tapering four week 'run-in' period, the results relating in particular to the 30-34 age group, and to a slightly lesser degree, the 35-49 age group, also establish significant differences in recovery intensities beyond the four week 'run-in' threshold.

Finally, for the record, it is also of interest to record the implications of the marginally significant F-statistic (p-value 5.5%) for DP13 policies in the 1983-86 quadrennium. This time, following an analysis of the contrasts in age effects, we are prompted to use five effective parameters $\beta_{x'}$, achieved by using the same parameter for the youngest and the oldest age groups, [18,30)∪[60,65], and by combining the adjacent age groups [45,50)∪[50,55) into a single grouping. The resulting estimated parameter values $\hat{\eta}^{13}_{x'z}$, together with the associated t-statistics are recorded in Table 3.4.

We have not pursued further the marginally significant F-statistic (p-value 8.3%) for DP13 policies in the 1987-90 quadrennium.

4. COMPUTATION OF π_{txd}

In this section, we need to consider the underlying transition intensities necessary for the calculation of π_{txd}. For the purpose of this study, we make use of equation (2.1) in combination with the formulae

$$\log \rho_{t+z,x+z,z} = \mu + \alpha\sqrt{z} + \beta z + \gamma_1 t + \gamma_2 t\sqrt{z} + \gamma_3 tz + \theta x + \theta_2 x^2 + \theta_3 x^3 + \phi x\sqrt{z} + \psi xz \qquad (4.1)$$

$$\log \nu_{t+z,x+z,z} = \mu + \gamma t + \theta x + \beta z + \sum_{j=1}^{n} \beta_j (z - z_j)_+ \qquad (4.2)$$

to compute values of π_{txd}. These formulae are established in Renshaw & Haberman (1999) in an attempt to model the respective trends in the recovery intensities for the male 1975-94 DP1 experience, (4.1), and the mortality from sickness intensities for the male 1975-94 experience for all DPs combined, (4.2). Given the somewhat inconclusive nature of the test results of Section 3 for sickness durations in excess of the 'run-in' thresholds, we have opted not to group the data over deferred period in any matching cells, prior to fitting the first of these formulae. In addition, with the 'static' mode of analysis in mind, the data have been grouped by quadrennia and modelled separately. It transpires that each of these cases is satisfactorily represented by a member of the same class of formulae, with obviously γ_1, γ_2, γ_3, γ pre-set to zero together with certain of the other parameters, as appropriate. The results are summarised in Table 4.1 and Table 4.2 in which details of the parameters estimates are presented, and from which details of the formulae structures for each case may be deduced. Details of

the integration of (2.1) are outlined in Appendix II. It is a relatively simple matter to program the integration under the 'static' mode as a special case of the integration under the 'dynamic' mode.

5. TARGETING σ_{tx}

The same data sets are used to target both the sickness inception intensity σ_{tx} and the claim inception intensity τ_{tx}. For a specific gender and specific deferred period, they comprise claim inception counts i_{tx} representing the number of sicknesses starting in cell (t, x) and which last beyond the deferred period of the class of policies concerned, together with matching exposures e_{tx} measured in years.

It should be noted that the exposures do not include any allowance for the exposure time spent as sick but not claiming, the effect of which is reportedly (CMI Committee 1996) "to overstate the exposure by about 0.5%". In addition, as with the targeting of τ_{tx} (Renshaw & Haberman 1999), we have not inflated the reported claim inception counts by a hypothetical figure for "the number of recoveries within four weeks after the end of the differed period which were not reported as claims or recoveries", as described for the 1975-78 quadrennial (DP4, DP13, DP26) experiences in Part C Section 3 CMI Committee (1991). In particular, it should be noted that this adjustment is not insubstantial, being of the order of at least 25% over the whole of the reported age range in the case of the DP4 experience (Table C1 p56, CMI Committee 1991), and consequently has a material effect on the resulting 'standard' sickness intensities based on the 1975-78 experience (Tables C7, C8, C14, CMI Committee 1991). We take as justification for our approach, the subsequent need to re-adjust any *expected* claim inception counts based on the 'standard' sickness inception intensities, in order to counteract the effects of the hypothetical unreported claims built into the 1975-78 'standard' tables, prior to comparison with the actual claim counts in quadrennia subsequent to 1975-78 (CMI Committee 1996).

The data have been cross-classified in the first instance by

gender	male
deferred periods (dp)	1, 4, 13, 26 wks
age at inception (x)	18-, 25-, 30-, 35-, 40-, 45-, 50-, 55-, 60-64 yrs
calendar time at inception (t)	1975, 76, 77, ..., 94

with the further option of pooling the data into the five separate successive quadrennia 75-78, 79-82, ..., while at the same time relaxing the grouping with respect to x, prior to analysis. [Inset to follow]

For a specific deferred period d, we follow Renshaw & Haberman (1995) in targeting σ_{tx} within the generalised linear modelling framework by declaring independent overdispersed Poisson responses i_{tx} such that

$$m_{tx} = \mathrm{E}(i_{tx}) = e_{tx}\pi_{txd}\sigma_{tx},\ \mathrm{Var}(i_{tx}) = \phi m_{tx},$$

where the scale parameter ϕ is included in recognition of duplicates among the policies contributing to the data base. The associated deterministic structure is based on the log-link predictor relationship

$$\eta_{tx} = \log m_{tx} = \log(e_{tx}\pi_{txd}) + \log\sigma_{tx}$$

where η_{tx} denotes the linear predictor and $\log(e_{tx}\pi_{txd})$ the offset term.

Under this modelling formulation, when $\pi_{txd} = 1\ \forall\ t,x$ and specific d, σ_{tx} becomes τ_{tx} and we revert to the modelling of claim inception transition intensities. Here we are interested in modelling the trends in the sickness inception transitions and in comparing such trends with the trends established in the claim inception transitions (Renshaw & Haberman (1999)).

The role played by the (constant) scale parameter ϕ is of potential interest. Given that it impacts on the second moment properties and not on the first, it has no bearing on the values of the parameter estimates in the linear predictor, and hence on the fitted values $\hat{m}_{tx}$ (and $\hat{\sigma}_{tx}$ or $\hat{\tau}_{tx}$). Instead it forms a multiplicative factor $\sqrt{\phi}$ in the construction of the standard errors for the parameter estimates.

In the generalised linear modelling setting, it is customary to estimate ϕ as the model deviance D, divided by the model degrees-of-freedom. For the current choice of modelling distribution

$$D = \sum_{tx} d_{tx} = 2\sum_{tx}\left\{i_{tx}\log\frac{i_{tx}}{\hat{m}_{tx}} - (i_{tx} - \hat{m}_{tx})\right\}.$$

Also, given the presence of the free standing scale parameter ϕ in this analysis, we make use of Studentised deviance residuals

$$\mathrm{sign}(i_{tx} - \hat{m}_{tx})\sqrt{\frac{d_{tx}}{\hat{\phi}(1 - h_{tx})}}$$

throughout, where h_{tx} are the diagonal entries in the so called hat-matrix of the fit (eg McCullagh & Nelder (1989), Francis, Green & Payne (1993)). Typically here, values of h_{tx} are of the order of 0.05 and consequently have little impact on the values of such residuals in general, while the contribution from the scale parameter ϕ is analogous to the scaling of residuals under the more familiar normal $\mathrm{N}(\mu, \sigma^2)$ modelling assumption (scale parameter σ^2).

As with the targeting of τ (Renshaw & Haberman (1999)), bivariate polynomial structures

$$\log\sigma_{t'x} = \alpha_0 + \sum_{i=1}^{r}\alpha_i t'^i + \sum_{j=1}^{s}\beta_j x^j + \sum_{i=1}^{r}\sum_{j=1}^{s}\gamma_{ij}\, t'^i x^j$$

are fitted for separate DPs, where x is allocated values at the centre of the relevant age cell (as defined above) and $t' = (t - 1984.5)/9.5$ taking equally spaced values in the interval [-1, 1] corresponding to the calendar years t = 1975, 76,..., 94. Details of the fitted structures may be deduced from Table 5.1 in which the parameter estimates, their standard errors and 't-statistics' are tabulated. For completeness, we also tabulate details of the parameter estimates associated with the modelling of τ as reported in Renshaw & Haberman (1999). In the case of σ, the fitted structures comprise

DP1 quintic in t effects + cubic in x effects + 2nd & 3rd order product effects

DP4 quintic in t effects + cubic in x effects + 2nd order product effects

DP13 cubic in t effects + cubic in x effects

DP26 quintic in t effects + quadratic in x effects

which are identical in form to the case of τ with the exception of DP26 (cubic t effects plus cubic x effects). Each structure has been selected on the basis of exploratory data analysis involving an examination of deviance profiles, residual plots, and the statistical significance or otherwise of the associated parameter estimates. In particular, residuals plotted against both age and calendar year of sickness inception are especially informative. For economy of space, we tabulate the residuals for each DP in Table 5.2 instead of reproducing the plots. By scanning both the columns and rows of these rectangular tabulations (age by period), it is possible to make an assessment of the goodness-of-fit. With the possible exception of the years 1981, 1992 (DP1), the year 1982 (DP13 & DP26), and ages 18-24 (DP26), where negative residuals are over represented, the tabulations (and underlying plots) are supportive of the fitted model structures. Any such clusters of negative residuals imply that there is a tendency for the predicted sickness inception intensities to be overstated at these locations. Further, with the obvious exception of the case of DP26 at ages 18-24, it appears (from Figure 5.1) that these negative residuals occur at turning points in the predicted trends, which is a possible reflection of the rigidity of low order bivariate polynomial structures in accommodating sharp turning points. The apparent lack of fit at ages 18-24 for DP26 is attributable to the paucity of data in these cells, comprising relatively low exposures coupled mainly with zero reported transitions. As with this approach to modelling in general, any structures selected by these methods are demonstrably plausible, but are of necessity, not unique.

The role played here by the scale parameter ϕ is that of a token variance ratio, assumed to be *constant* over all the data cells (t, x). (The information necessary to compute the variance ratios for individual cells is not available). On the evidence of the typical set of variance ratios computed at each age for the 1979-82 male assured lives experience (Table 17.6, Forfar *et al.* 1988), estimated values, $\hat{\phi}$, in the range 1 to 2 are consistent with these tabulated values. While this is the case for three of the four deferred periods considered (Table 5.1), the estimated value, $\hat{\phi}$, for the DP1 experience is excessive. An examination of the individual deviance components reveals that this is due in no small part to relative large contributions from relatively few of the data cells. It is possible to identify these

cells by reference to Table 5.2. These are the cells associated with the largest residuals in absolute value. Note that these occur in roughly equal proportions of positives and negatives and are seemingly randomly scattered. In addition, we note that the value of the deviance and hence the estimated value, $\hat{\phi}$, can generally be reduced through the introduction of more complex predictor structures. Having considered this possiblity, we have concluded that we do not feel justified in doing this within the context of bivariate low degree polynomial predictor structures. We also remark, that as a guide to parameter retention when referring to Table 5.1, that if a *smaller* value $\tilde{\phi}$ of ϕ is imposed while at the same time retaining the same predictor giving rise to the estimated value $\hat{\phi}$ of ϕ, the resulting values of the t-statistics are *increased* by a factor of $\sqrt{(\hat{\phi}/\tilde{\phi})}$. That said, it is important to keep this issue in perspective, namely that it relates only to the second moment properties of the modelling process.

The resulting smoothed log sickness inception intensities generated by these models for each DP, plotted against calendar year for each age, are illustrated in the upper frames of Figures 5.1(a-d). Also by way of comparison, the matching smoothed log claim inception intensities reported in Renshaw & Haberman (1999) are reproduced in the respective lower frame of each of these figures.

The families of parallel curves in the case of the DP13 and DP26 experiences reflect the additive nature of the linear predictor in age and calendar period effects. For the DP1 and DP4 experiences the interactive nature of these curves is more pronounced in the DP1 experience compared with the DP4 experience. For the DP13 and DP26 experiences, with the exception of the extreme ages (below age 30 for claim inceptions and above age 55 for sickness inceptions), a noteworthy feature of these graphs concerns the reversal of the ordering of members of the same family of curves (age effects) when comparing log sickness with matching log claim inception intensities. This feature also has a strong presence in the DP4 experience, and, to a lesser extent, in the DP1 experience, although these cases are complicated by the interactive nature of the family of curves. In essence, claim inception intensities initially decrease before increasing with increasing age, approximately above the age of 30. On the other hand, sickness inception intensities decrease with increasing age, approximately below the age of 55 before increasing, subject to possible interaction effects for the DP1 experience in particular and, to a lesser extent, for the DP4 experience. We note that these patterns are largely consistent with the patterns reported in Table C16 (CMI Committee (1991)) for graduated values of σ_x based on the 1975-78 quadrennial male experience but are only partially consistent (for ages over 30) with the patterns in Table C17 for τ_x which increase with age over the whole of the age range for the 1975-78 quadrennial experience.

The cyclical appearance of these curves leads to the suggestion of an association with indices of the economic cycle, and a pausible causal link. In particular, there is a pronounced local minimum in the years 1981-82 (the depth of which is somewhat more extreme than that predicted by the model) coupled with a localised maximum in the years 1990-91. We note that Haberman & Walsh (1998) found difficulties in establishing a plausible causal link with a range of economic measures, albeit for a more restricted data set. This remains an area for future investigation.

6. ALTERNATIVE WAYS OF MODELLING σ_{tx}

The modelling process described in Section 5 gives rise to a choice of methods when targeting σ_{tx} (or τ_{tx}). Thus in Section 5, for a given deferred period d, by incorporating the probabilities π_{txd} into the offset, we have targeted σ_{tx} directly (so that τ_{tx} can then be targeted indirectly using the identity $\tau_{tx} = \pi_{txd}\sigma_{tx}$). Alternatively, by suppressing π_{txd} in the offset, we can target τ_{tx} directly, as in Renshaw & Haberman (1999), and subsequently use the inverse identity $\sigma_{tx} = \pi_{txd}^{-1}\tau_{tx}$ to target σ_{tx}, indirectly. For a given deferred period, let $\hat{\sigma}_{tx}$ and $\hat{\tau}_{tx}$ denote the respective fitted sickness and claim inception intensities generated by these two modelling approaches respectively: see Table 5.1 for details of the respective fits. It then follows from the very nature of the separate fitting processes, that $\hat{\sigma}_{tx}$ and $\hat{\tau}_{tx}$ will depart from the basic relationship $\hat{\tau}_{tx} = \pi_{txd}\hat{\sigma}_{tx}$. It is of interest to investigate the magnitude of this effect in order to gain a better understanding of the implications of these choices. It is apparent that such an investigation can only be conducted on an empirical level.

Given $\hat{\tau}_{tx}$ or $\hat{\sigma}_{tx}$ define $\overset{*}{\tau}_{tx}$ or $\overset{*}{\sigma}_{tx}$ respectively, such that

$$\overset{*}{\tau}_{tx} = \pi_{txd}\hat{\sigma}_{tx} \quad \text{when } \sigma_{tx} \text{ is the target of the model fitting exercise, and}$$
$$\overset{*}{\sigma}_{tx} = \pi_{txd}^{-1}\hat{\tau}_{tx} \quad \text{when } \tau_{tx} \text{ is the target of the model fitting exercise.}$$

Then the values

$$\frac{\overset{*}{\sigma}_{tx} - \hat{\sigma}_{tx}}{\hat{\sigma}_{tx}} \times 100\% \quad \left(= \frac{\hat{\tau}_{tx} - \overset{*}{\tau}_{tx}}{\overset{*}{\tau}_{tx}} \times 100\%\right) \tag{6.1}$$

provide a measure of the extent by which $\overset{*}{\sigma}$ and $\hat{\sigma}$ differ, (and by implication, the extent by which $\hat{\tau}$ and $\overset{*}{\tau}$ differ). Alternatively, the roles of the symbols $\hat{}$ and $*$ can be reversed in this ratio, leading to slightly different but equivalent measures of these effects. For practical purposes, it is sufficient to quote only one of these, and we present the former. The values based on the model structures reported in Table 5.1, matched for DP, are presented in Table 6.1, for alternate calendar years. On the basis of these figures it is possible to draw the broad conclusions that (a) there is a tendency for the results to diverge with increasing deferred period d: recall that the domain of the integral which determines π_{txd} in (2.1), and which lies outside the domain of the data set, increases with d, and that (b) the divergence of results is especially great in the age range 18-25 where the inception transition counts are generally low.

The same effects can also be investigated in more detail for specific quadrennia with the data classified by individual years of age $x = 18, 19, \ldots, 64$ rather than in grouped format (and the suffix t supressed): the same format as that used in the construction of the standard PHI model (CMI Committee (1991)) based on the 1975-78 quadrennium. By way of illustration, we concentrate on the 1991-94 quadrennium and fit the following polynomial age effects structures

$$\log\sigma_{x'} \text{ or } \log\tau_{x'} = \sum_{j=0}^{s}\beta_j x'^j$$

with $x' = (x - 41)/23$, ranging from -1 to 1. The values of π_{xd} are determined through the numerical evaluation of the integral expression (2.1) using the versions of formulae (4.1) and (4.2) for ρ_{xz} and ν_{xz} implied by the respective final columns of Tables 4.1 and 4.2. Details of the fitted structures may be summarised thus:

	DP1	*DP4*	*DP13*	*DP26*
target σ	cubic	quintic	quartic	quintic
target τ	quadratic	quintic	quartic	quintic

1991-94 quadrennial experience: fitted age effects

with the parameter details given in Table 6.2. The values of the ratio (6.1) based on these results are presented in Table 6.3 which are in agreement with the broad conclusions stated above.

7. SUMMARY

The same raw data are used to target both the claim inception intensity τ_{tx} and the sickness inception intensity σ_{tx}. For a given deferred period d, these intensities are related by the identity

$$\tau_{tx} = \pi_{txd}\,\sigma_{tx}$$

where π_{txd} is the probability that the time spent by an individual in the sick state exceeds the deferred period d in question. A knowledge of π_{txd} is essential when targeting σ_{tx} but is not essential when targeting τ_{tx}.

The evaluation of expression (2.1), *viz*

$$\pi_{txd} = \exp - \int_0^{d/52} (\rho_{t+u,x+u,u} + \nu_{t+u,x+u,u})\,du$$

is problematic in the sense that formulae for ρ and ν have to be assumed, within the limits of the integral, for the specific deferred d in question. The recovery transition intensity ρ is the dominant term in the integral. For the 1975-78 quadrennial experience (CMI Committee (1991)), four week 'run-in' periods in the DP4, DP13 and DP26 experiences have been identified, outside of which, a common formula for ρ is established across all deferred periods. This formula is then used to determine the contribution to the integral from ρ by extrapolating backwards to week zero. We have devised a method (Section 3) for testing these features and applied them to each of the five quadrennia 75-78, 79-82, 83-86, 87-90 and 91-94 with somewhat mixed results. While there is strong supportive evidence of persistent four week 'run-in' periods across all of the five quadrennia examined, the evidence for grouping the data outside these regions across all deferred periods is less compelling, to varying degrees, in the quadrennia subsequent to the initial 1975-78 quadrennium.

Bivariate polynomial structures have been used to model σ_{tx} as a function of calendar year t and

age x, for each deferred period d (Section 5), using values of π_{txd} based on the DP1 study of ρ_{txz} and the combined DP study of ν_{txz} (Renshaw & Haberman (1999)). Additionally, the patterns in the sickness inception intensity σ_{tx} have been compared and contrasted with those in the claim inception intensity τ_{tx}, matched for deferred period. Two noteworthy features, common to both inception intensities and all deferred periods, are a local minimum in the years 1981-82 and a local maximum in the years 1990-91.

An empirical investigation has been conducted (Section 6) into the differences which can arise in the fitted values of σ_{tx} under the two alternative modes of implementing the proposed model formulation. While these differences are shown to be very small over the bulk of the rectangular domain of the function σ_{tx}, there is a general tendency for the differences to increase with increasing deferred period. In addition, large differences are recorded for the youngest age grouping 18-24, where there is a paucity of data. The ultimate choice as to which method of targeting σ_{tx} to employ, may well be dictated by practical necessity. Thus, not only is the so-called 'direct' method, in which $\log \pi_{txd}$ features in the offset, intuitively appealing given the central role played by σ in the formulation of the PHI multi-state model, it also as the advantage of leading to a closed formula for σ_{tx}. This in turn has distinct computational advantages when it comes to the numerical computation of the associated probabilities and sickness benefits.

Finally it is noted that the Kolmogorov forward equations, underpinning the 'static' multiple state model based on the 1975-78 quadrennial experience (CMI Committee (1991)), generalise to allow explicitly for possible variation with calendar time over a longer time scale.

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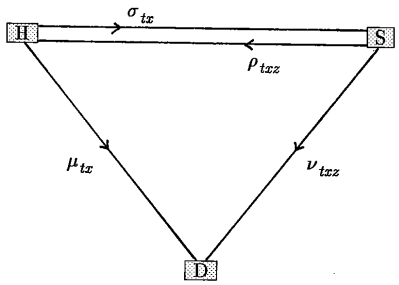
APPENDIX I

Consider the three state Markovian process depicted by



The process is defined by

$$\{Y(t, x), Z(t, x): x \geq 0, t \geq 0\}$$

where $Y(t, x)$ = H (for healthy), S (for sick) or D (for death), is a *state* variable, and $Z(t, x) \in \Re_{>0}$ is the duration of the sojourn so far in the current state $Y(t, x)$, for a life now aged x and time t. For example:

$$\{Y(t, x) = S, Z(t, x) = z\}$$

denotes that an individual is sick at age x, time t, and that the duration of the current sickness is z.

Define

$${}_s\mathrm{p}_{txz}^{jk} = \mathrm{P}(Y(t+s, x+s) = k \mid Y(t, x) = j \text{ and } Z(t, x) = z)$$

subject to the following implied rationalisation, summarised in tabular format below

	${}_s\mathrm{p}_{txz}^{jk}$		
$j \backslash k$	H	S	D
H	${}_s\mathrm{p}_{tx}^{HH}$	${}_s\mathrm{p}_{tx}^{HS}$	${}_s\mathrm{p}_{tx}^{HD}$
S	${}_s\mathrm{p}_{txz}^{SH}$	${}_s\mathrm{p}_{txz}^{SS}$	${}_s\mathrm{p}_{txz}^{SD}$
D	0	0	1

Also define

1) $${}_s\mathrm{p}_{tx}^{\overline{H}\,\overline{H}} = \mathrm{P}(Y(t+s, x+s) = \mathrm{H} \text{ and } Z(t+s, x+s) \geq s \mid Y(t, x) = \mathrm{H})$$

the probability that an individual remains healthy from (t, x) to $(t+s, x+s)$ given the individual was healthy at (t, x)

2) $${}_s\mathrm{p}_{txz}^{\overline{S}\,\overline{S}} = \mathrm{P}(Y(t+s, x+s) = \mathrm{S} \text{ and } Z(t+s, x+s) = z + s \mid Y(t, x) = \mathrm{S} \text{ and } Z(t, x) = z)$$

the probability that an individual remains sick from (t, x) to $(t+s, x+s)$ given the individual was sick at (t, x) with duration of sickness z; with the special case when $z = 0$ denoted by ${}_s\mathrm{p}_{tx0}^{\overline{S}\,\overline{S}} = {}_s\mathrm{p}_{tx}^{\overline{S}\,\overline{S}}$

3) $${}_{ws}\mathrm{p}_{tx}^{HS} = \mathrm{P}(Y(t+s, x+s) = \mathrm{S} \text{ and } Z(t+s, x+s) \leq w \mid Y(t, x) = \mathrm{H})$$

For this process, the transitions in a small interval of time $(s, s+\delta s]$ behave as follows:

$$P(Y(t+s+\delta s, x+s+\delta s) = S \mid Y(t+s, x+s) = H) = \sigma_{t+s,x+s}\delta s + o(\delta s)$$

$$P(Y(t+s+\delta s, x+s+\delta s) = D \mid Y(t+s, x+s) = H) = \mu_{t+s,x+s}\delta s + o(\delta s)$$

$$P(Y(t+s+\delta s, x+s+\delta s) = H \mid Y(t+s, x+s) = S \text{ and } Z(t+s, x+s) = z) = \rho_{t+s,x+s,z}\delta s + o(\delta s)$$

$$P(Y(t+s+\delta s, x+s+\delta s) = D \mid Y(t+s, x+s) = S \text{ and } Z(t+s, x+s) = z) = \nu_{t+s,x+s,z}\delta s + o(\delta s)$$

$$P(2 \text{ or more transitions}) = o(\delta s)$$

where

$$\lim_{\delta s \to 0} \frac{o(\delta s)}{\delta s} = 0.$$

1. Focus on

$${}_s p_{tx}^{\overline{H}\,\overline{H}} = P(Y(t+s, x+s) = H \text{ and } Z(t+s, x+s) \geq s \mid Y(t, x) = H).$$

Get

$${}_{s+\delta s}p_{tx}^{\overline{H}\,\overline{H}} = {}_s p_{tx}^{\overline{H}\,\overline{H}}(1 - \mu_{t+s,x+s}\delta s - \sigma_{t+s,x+s}\delta s) + o(\delta s)$$

which implies

$$\frac{\partial\, {}_s p_{tx}^{\overline{H}\,\overline{H}}}{\partial s} = - {}_s p_{tx}^{\overline{H}\,\overline{H}}(\mu_{t+s,x+s} + \sigma_{t+s,x+s}).$$

Integration gives

$${}_s p_{tx}^{\overline{H}\,\overline{H}} = \exp(-\int_0^s (\mu_{t+u,x+u} + \sigma_{t+u,x+u})\mathrm{d}u.$$

2. Focus on

$${}_s p_{txz}^{\overline{S}\,\overline{S}} = P(Y(t+s, x+s) = S \text{ and } Z(t+s, x+s) = z + s \mid Y(t, x) = S \text{ and } Z(t, x) = z).$$

Get

$${}_{s+\delta s}p_{txz}^{\overline{S}\,\overline{S}} = {}_s p_{txz}^{\overline{S}\,\overline{S}}(1 - \rho_{t+s,x+s,z+s}\delta s - \nu_{t+s,x+s,z+s}\delta s) + o(\delta s)$$

which implies

$$\frac{\partial\, {}_s p_{txz}^{\overline{S}\,\overline{S}}}{\partial s} = - {}_s p_{txz}^{\overline{S}\,\overline{S}}(\rho_{t+s,x+s,z+s} + \nu_{t+s,x+s,z+s}).$$

Integration gives

$${}_s p_{txz}^{\overline{S}\,\overline{S}} = \exp(-\int_0^s (\rho_{t+u,x+u,z+u} + \nu_{t+u,x+u,z+u})\mathrm{d}u.$$

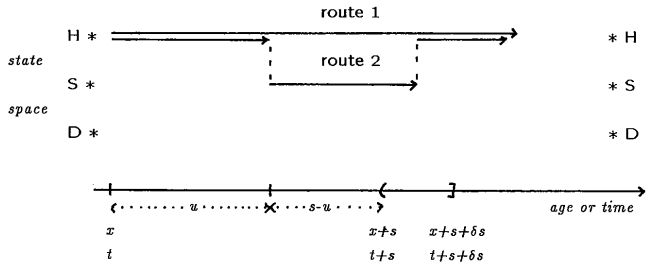
Note in particular

$$\pi_{txd} = {}_{d/52}p_{tx0}^{\overline{S}\,\overline{S}} = {}_{d/52}p_{tx}^{\overline{S}\,\overline{S}}.$$

3. Focus on

$$ {}_s\mathrm{p}_{tx}^{HH} = \mathrm{P}(Y(t+s,\, x+s) = \mathrm{H} \mid Y(t,\, x) = \mathrm{H}). $$

Consider



Get

$$ {}_{s+\delta s}\mathrm{p}_{tx}^{HH} = {}_s\mathrm{p}_{tx}^{HH}(1 - \mu_{t+s,x+s}\delta s - \sigma_{t+s,x+s}\delta s) \qquad \text{(contribution, route 1)} $$

$$ + \int_{u=0}^{s} {}_u\mathrm{p}_{tx}^{HH}\, \sigma_{t+u,x+u}\, {}_{s-u}\mathrm{p}_{t+u,x+u}^{\overline{S}\,\overline{S}}\, \rho_{t+s,x+s,s-u}\, \mathrm{d}u\delta s \qquad \text{(contribution, route 2)} $$

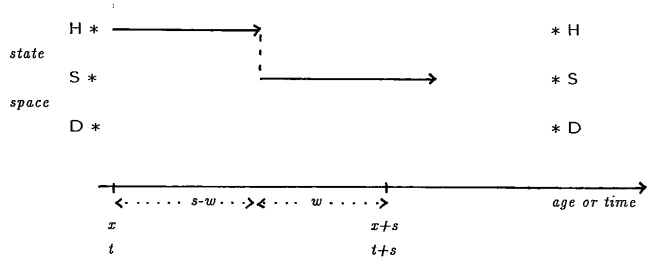
which implies:

$$ \frac{\partial\, {}_s\mathrm{p}_{tx}^{HH}}{\partial s} = -\, {}_s\mathrm{p}_{tx}^{HH}(\mu_{t+s,x+s} + \sigma_{t+s,x+s}) + \int_0^s {}_u\mathrm{p}_{tx}^{HH}\, \sigma_{t+u,x+u}\, {}_{s-u}\mathrm{p}_{t+u,x+u}^{\overline{S}\,\overline{S}}\, \rho_{t+s,x+s,s-u}\, \mathrm{d}u. $$

4. Focus on

$$ {}_{ws}\mathrm{p}_{tx}^{HS} = \mathrm{P}(Y(t+s,\, x+s) = \mathrm{S} \text{ and } Z(t+s,\, x+s) \leq w \mid Y(t,\, x) = \mathrm{H}). $$

Consider



Get

$$\frac{\partial\, {}_{ws}\mathrm{p}_{ix}^{HH}}{\partial w} = {}_{s-w}\mathrm{p}_{ix}^{HH}\, \sigma_{t+s-w,x+s-w}\, {}_{u}\mathrm{p}_{t+s-w,x+s-w}^{\overline{S}\,\overline{S}} \qquad 0 \le w < t$$

$$= 0 \qquad w > t.$$

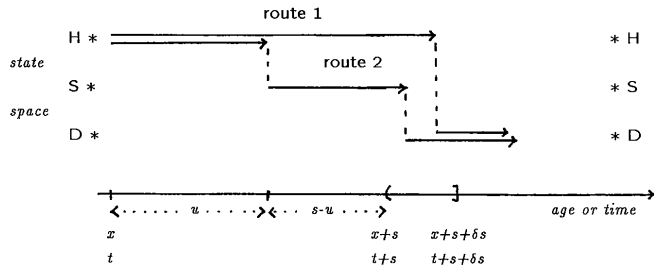
Integration implies

$${}_{ws}\mathrm{p}_{ix}^{HS} = \int_{max(0,s-w)}^{s} {}_{u}\mathrm{p}_{ix}^{HH}\, \sigma_{t+u,x+u}\, {}_{s-u}\mathrm{p}_{t+u,x+u}^{\overline{S}\,\overline{S}}\, \mathrm{d}u.$$

5. Focus on

$${}_{s}\mathrm{p}_{ix}^{HD} = \mathrm{P}(Y(t+s,\, x+s) = \mathrm{D} \mid Y(t,\, x) = \mathrm{H}).$$

Consider



Get

$$\frac{\partial\, {}_{s}\mathrm{p}_{ix}^{HD}}{\partial s} = {}_{s}\mathrm{p}_{ix}^{HH}\, \mu_{t+s,x+s} \qquad \text{(contribution, route 1)}$$

$$+ \int_{0}^{s} {}_{u}\mathrm{p}_{ix}^{HH}\, \sigma_{t+u,x+u}\, {}_{s-u}\mathrm{p}_{t+u,x+u}^{\overline{S}\,\overline{S}}\, \nu_{t+s,x+s,s-u}\, \mathrm{d}u. \qquad \text{(contribution, route 2)}$$

Integration implies

$${}_{s}\mathrm{p}_{ix}^{HD} = \int_{0}^{s} {}_{u}\mathrm{p}_{ix}^{HH}\, \mu_{t+u,x+u}\, \mathrm{d}u + \int_{0}^{s} {}_{u}\mathrm{p}_{ix}^{HH}\, \sigma_{t+u,x+u} \left\{ \int_{0}^{s-u} {}_{w}\mathrm{p}_{t+u,x+u}^{\overline{S}\,\overline{S}}\, \nu_{t+u+w,x+u+w,w}\, \mathrm{d}w \right\} \mathrm{d}u.$$

APPENDIX II

The integral expression

$$\pi_{txd} = \exp - \int_0^{d/52} (\rho_{t+v,x+v,v} + \nu_{t+v,x+v,v})\,dv$$

when

$$\log \rho_{t+z,x+z,z} = \mu + \alpha\sqrt{z} + \beta z + \gamma_1 t + \gamma_2 t\sqrt{z} + \gamma_3 tz + \theta x + \theta_2 x^2 + \theta_3 x^3 + \phi x\sqrt{z} + \psi xz$$

$$\log \nu_{t+z,x+z,z} = \mu + \gamma t + \theta x + \beta z + \sum_{j=1}^{n} \beta_j (z - z_j)_+$$

and all three of t, x, z are measured in years, may be evaluated as follows.

1) To determine the contribution from ρ, write

$$\log \rho_{t+z,x+z,z} = a_{tx} + b_{tx}\sqrt{z} + c_{tx} z$$

where

$$a_{tx} = \mu + \gamma_1 t + \theta x + \theta_2 x^2 + \theta_3 x^3, \quad b_{tx} = \alpha + \gamma_2 t + \phi x, \quad c_{tx} = \beta + \gamma_3 t + \psi xz.$$

Then

$$J(t,x;\, z) = \int_0^z \rho_{t+v,x+v,v}\,dv = k_{tx} \int_0^z \exp\left\{c_{tx}\left(\sqrt{v} + \frac{b_{tx}}{2c_{tx}}\right)^2\right\} dv$$

reducing to

$$J(t,x;\, z) = [\exp(a_{tx})\{\exp(b_{tx}\sqrt{z} + c_{tx} z) - 1\} - k_{tx} b_{tx} \int_{\frac{b_{tx}}{2c_{tx}}}^{\frac{b_{tx}}{2c_{tx}} + \sqrt{z}} \exp(c_{tx} s^2)\,ds]/c_{tx}$$

where

$$k_{tx} = \exp\left(a_{tx} - \frac{b_{tx}^2}{4c_{tx}}\right).$$

The definite integral is evaluated numerically by Simpson's rule. Since $c_{tx} > 0 \;\forall\; (t, x)$, it cannot be equated with the area under a standard normal variate (which in any event is evaluated numerically).

2) To determine the contribution from ν: for

$$z_j < z \le z_{j+1}, \;\; j = 0, 1, \ldots, n; \;\; 0 = z_0 < z_1 < z_2 < \ldots < z_{n+1} = \infty$$

let

$$\log \nu_{t+z,x+z,z} = A_{tx}^{(j)} + B^{(j)} z$$

where

$$A_{tx}^{(0)} = \mu + \gamma t + \theta x, \;\; B^{(0)} = \beta; \;\; A_{tx}^{(j)} = A_{tx}^{(j-1)} - \beta_j z_j, \;\; B^{(j)} = B^{(j-1)} + \beta_j, \; j = 1, 2, \ldots, n.$$

Then for $z_j < z \leq z_{j+1}$, define

$$I_j(t,x;\, z) = \int_{z_j}^{z} \exp(A_{tx}^{(j)} + B^{(j)} v)\, dv = \exp A_{tx}^{(j)} \{\exp(B^{(j)} z) - \exp(B^{(j)} z_j)\}/B^{(j)}$$

so that, for $z_g < z \leq z_{g+1}$

$$I(t,x;\, z) = \int_{0}^{z} \nu_{t+v,x+v,v}\, dv = \sum_{j=0}^{g-1} I_j(t,x;\, z_j) + I_g(t,x;\, z).$$

The summation term does not come into play when $g = 0$.

3) Hence the required result

$$\pi_{txd} = \{\exp\text{-}J(t,x;\, \tfrac{d}{52})\}\{\exp\text{-}I(t,x;\, \tfrac{d}{52})\}.$$

****

a) In the event that the expression for $\log \rho$ takes the form

$$\begin{aligned} \log \rho_{t+z',x+z',z'} &= \mu + \alpha' \sqrt{z'} + \beta' z' + \gamma_1 t + \gamma_2' t \sqrt{z'} + \gamma_3' t z' + \theta x + \theta_2 x^2 + \theta_3 x^3 + \phi' x \sqrt{z'} + \psi' x z' \\ &= a_{tx} + b_{tx}' \sqrt{z'} + c_{tx}' z' \end{aligned}$$

in which z' is expressed in weeks, the change of scale

$$z = z'/w \text{ with inverse } z' = wz \text{ where } w = 52$$

implies that

$$\alpha = \sqrt{w}\alpha',\ \gamma_2 = \sqrt{w}\gamma_2',\ \phi = \sqrt{w}\phi';\ b_{tx} = \sqrt{w} b_{tx}'$$
$$\beta = w\beta',\ \gamma_3 = w\gamma_3',\ \psi = w\psi';\ c_{tx} = w c_{tx}'$$

and

$$b_{tx} z = b_{tx}' z',\ c_{tx} z = c_{tx}' z',\ k_{tx} = \exp\Big(a_{tx} - \frac{b_{tx}'^2}{4 c_{tx}'}\Big).$$

b) In the event that the expression for $\log \nu$ takes the form

$$\log \nu_{t+z,x+z,z} = \mu' + \gamma' t' + \theta x + \beta' z' + \sum_{j=1}^{n} \beta_j' (z' - z_j')_+$$

where $t' = 1, 2, 3, 4, 5$ denote consecutive quadrennia, and z' is in weeks, the transformation

$$t = 4t' - 3 \ \Leftrightarrow\ t' = (t + 3)/4$$

mapping consecutive quadrennia into a run of consecutive matching years 1, 2, ..., 20; coupled with the change of scale

$$z = z'/w \text{ with inverse } z' = wz \text{ where } w = 52,$$

implies that

$$\mu = \mu' + 3\gamma'/4;\ \gamma = \gamma'/4;\ \beta = w\beta',\ \beta_j = w\beta_j'.$$

Table 3.1(a) Estimates $\hat{\eta}^{4}_{xz} = \hat{\alpha}_z$ and t-statistics by duration z, for quadrennia (all ages x)

	quadrennium				
duration	*75-78*	*79-82*	*83-86*	*87-90*	*91-94*
(weeks)					
4	1.1943	1.2711	1.4127	1.3104	1.2169
	(11.94)*	(11.82)*	(12.1)*	(12.8)*	(11.3)*
5	0.7065	0.5973	0.6622	0.7521	0.7424
	(7.27)*	(5.74)*	(6.21)*	(7.46)*	(6.76)*
6	0.3986	0.4759	0.5438	0.5057	0.7454
	(3.97)*	(4.65)*	(4.87)*	(5.01)*	(6.11)*
7	0.2198	0.3750	0.6446	0.4512	0.6766
	(2.12)*	(3.09)*	(5.21)*	(3.92)*	(5.51)*
8	0.0650	0.1864	-0.0756	0.2797	0.2695
	(0.54)	(1.34)	(-0.59)	(2.26)*	(2.04)*
9	0.0495	0.4593	0.1515	0.3431	0.2859
	(0.36)	(2.87)*	(1.06)	(2.62)*	(1.68)
10	0.1416	-0.0675	0.2883	0.2968	0.2361
	(0.90)	(-0.37)	(1.81)	(1.90)†	(1.25)
11-12	-0.0293	0.0004	0.2002	0.2824	-0.0794
	(-0.24)	(0.00)	(1.48)	(2.21)*	(-0.62)
13-14	-0.1597	0.2809	0.2979	-0.0573	0.1211
	(-1.00)	(1.78)	(1.97)†	(-0.34)	(0.72)
15-17	0.0919	0.1579	0.0175	-0.1857	-0.4223
	(0.64)	(1.03)	(1.11)	(-1.07)	(-2.60)*
18-21	-0.0521	-0.0954	-0.2090	-0.0991	0.0187
	(-0.31)	(-0.52)	(-1.14)	(-0.60)	(0.09)
22-25	0.0515	0.2199	0.0458	-0.4042	-0.9493
	(0.23)	(0.89)	(0.23)	(-1.66)	(-3.60)*
26-29	-0.2615	-0.2316	0.1316	0.3326	-0.1442
	(-1.03)	(-0.80)	(0.46)	(1.18)	(-0.60)
30-38	-0.4035	0.4652	-0.3558	-0.2823	-0.1758
	(-1.66)	(1.64)	(-1.35)	(-1.23)	(-0.71)
39-51	-0.3819	0.1488	-0.2254	-0.0386	-0.2736
	(-1.11)	(0.43)	(-0.93)	(-0.15)	(-1.02)
1- yr	-0.1617	-0.1058	0.0263	-0.9981	-0.1330
	(-0.75)	(-0.45)	(0.12)	(-3.63)*	(-0.66)
2-11yrs	-0.3571	0.0875	0.1441	-0.1483	-0.3549
	(-0.58)	(0.26)	(0.56)	(-0.50)	(-1.75)

(∗ statistically significant difference from zero, † of marginal statistical significance)

Table 3.1(b) Estimates $\hat{\eta}_{xz}^{13} = \hat{\alpha}_z$ and t-statistics by duration z, for quadrennia (all ages x)

	quadrennium				
duration *(weeks)*	*75-78*	*79-82*	*83-86*	*87-90*	*91-94*
13-14	0.8598	1.1436	1.2852	0.8478	1.1560
	(4.20)*	(5.83)*	(7.91)*	(4.39)*	(6.35)*
15-17	0.3303	0.5860	0.5665	0.4063	0.2665
	(2.34)*	(3.63)*	(3.82)*	(2.38)*	(1.71)
18-21	0.0358	0.1956	0.0248	-0.0142	0.1199
	(0.22)	(1.02)	(0.15)	(-0.09)	(0.72)
22-25	0.1150	0.6281	0.2030	-0.5101	-0.5193
	(0.59)	(2.45)*	(1.15)	(-2.34)*	(-2.20)*
26-29	-0.1894	0.1009	0.0617	-0.0042	0.0570
	(-0.75)	(0.34)	(0.27)	(-0.02)	(0.29)
30-38	-0.3341	0.8687	-0.1088	-0.3289	-0.2086
	(-1.56)	(3.16)*	(-0.49)	(-1.54)	(-1.11)
39-51	0.2814	0.2361	0.1934	-0.1729	-0.5156
	(0.94)	(0.78)	(0.90)	(-0.77)	(-2.51)*
1- yr	0.2775	0.0015	0.0375	-0.7759	-0.4309
	(1.41)	(0.01)	(0.21)	(-3.01)*	(-2.79)*
2-11yrs	-0.4474	-0.0408	0.3591	-0.7601	-0.4355
	(-1.07)	(-0.14)	(1.65)	(-3.62)*	(-2.77)*

(* denotes statistically significant difference from zero)

Table 3.1(c) Estimates $\hat{\eta}_{xz}^{26} = \hat{\alpha}_z$ and t-statistics by duration z, for quadrennia (all ages x)

	quadrennium				
duration *(weeks)*	*75-78*	*79-82*	*83-86*	*87-90*	*91-94*
26-29	0.3861	0.8504	0.9084	0.8706	0.8936
	(1.62)	(2.08)*	(2.17)*	(2.84)*	(3.14)*
30-38	-0.0322	0.7345	0.2116	-0.0694	0.3404
	(-0.15)	(2.61)*	(0.64)	(-0.34)	(1.52)
39-51	0.0418	0.4530	0.6085	0.1514	-0.0424
	(0.15)	(1.17)	(2.06)*	(0.66)	(-0.18)
1- yr	0.5234	0.7050	0.4076	-0.5860	0.0775
	(2.55)*	(2.64)*	(1.65)	(-2.42)*	(0.43)
2-11yrs	0.2077	0.6767	0.2834	-0.5710	-0.1099
	(0.58)	(2.22)*	(1.14)	(-2.89)*	(-0.60)

(* denotes statistically significant difference from zero)

Table 3.3 Estimates $\hat{\eta}^{4}_{x'z} = \hat{\alpha}_z + \hat{\beta}_{x'}$ and t-statistics, by duration z and age x', 1979-82

	age			
duration	*18-29*	*30-34*	*35-49*	*50-65*
(weeks)				
4	1.1088	1.5890	1.3610	1.1088
	(10.40)*	(10.64)*	(12.69)*	(10.40)*
5	0.4474	0.9276	0.6996	0.4474
	(4.35)*	(6.20)*	(6.71)*	(4.35)*
6	0.3298	0.8100	0.5820	0.3298
	(3.28)*	(5.51)*	(5.60)*	(3.28)*
7	0.2384	0.7186	0.4906	0.2384
	(2.05)*	(4.55)*	(4.02)*	(2.05)*
8	0.0283	0.5085	0.2805	0.0283
	(0.21)	(3.00)*	(2.08)*	(0.21)
9	0.3215	0.8017	0.5737	0.3215
	(2.12)*	(4.26)*	(3.73)*	(2.12)*
10	-0.2127	0.2675	0.0395	-0.2127
	(-1.22)	(1.33)	(0.22)	(-1.22)
11-12	-0.1351	0.3452	0.1171	-0.1351
	(-1.04)	(2.04)*	(0.87)	(-1.04)
13-14	0.1640	0.6442	0.4162	0.1640
	(1.10)	(3.50)*	(2.68)*	(1.10)
15-17	0.0329	0.5131	0.2851	0.0329
	(0.23)	(2.75)*	(1.93)	(0.23)
18-21	-0.2103	0.2700	0.0420	-0.2103
	(-1.21)	(1.29)	(0.24)	(-1.21)
22-25	0.1194	0.5997	0.3716	0.1194
	(0.52)	(2.31)*	(1.60)	(0.52)
26-29	-0.3520	0.1283	-0.0998	-0.3520
	(-1.30)	(0.44)	(-0.36)	(-1.30)
30-38	0.3870	0.8672	0.6392	0.2870
	(1.47)	(2.99)*	(2.39)*	(1.47)
39-51	0.0303	0.5106	0.2825	0.0303
	(0.09)	(1.51)	(0.88)	(0.09)
1- yr	-0.2501	0.2302	0.0021	-0.2501
	(-1.14)	(0.94)	(0.01)	(-1.14)
2-11yrs	-0.0288	0.4515	0.2234	-0.0288
	(-0.09)	(1.38)	(0.72)	(-0.09)

(* denotes statistically significant difference from zero)

Table 3.4 Estimates $\hat{\eta}^{13}_{x'z} = \hat{\alpha}_z + \hat{\beta}_{x'}$ and t-statistics, by duration z & age x', 1983-86

	age						
duration	*18-29*	*30-34*	*35-39*	*40-44*	*45-54*	*55-59*	*60-65*
(weeks)							
13-14	1.6756	1.5195	1.2743	1.4164	0.9913	1.3539	1.6756
	(8.63)*	(5.93)*	(6.23)*	(7.03)*	(5.87)*	(7.08)*	(8.63)*
15-17	0.9475	0.7915	0.5463	0.6884	0.2633	0.6259	0.9475
	(5.11)*	(3.30)*	(2.76)*	(3.55)*	(1.66)	(3.47)*	(5.11)*
18-21	0.3603	0.2042	-0.0410	0.1011	-0.3240	0.0386	0.3603
	(1.79)	(0.85)	(-0.19)	(0.50)	(-1.77)	(0.20)	(1.79)
22-25	0.5966	0.4406	0.1954	0.3375	-0.0876	0.2750	0.5966
	(2.90)*	(1.67)	(0.93)	(1.55)	(-0.48)	(1.39)	(2.90)*
26-29	0.3479	0.1919	-0.0533	0.0888	-0.3363	0.0263	0.3479
	(1.44)	(0.68)	(-0.21)	(0.35)	(-1.43)	(0.11)	(1.44)
30-38	0.2767	0.1206	-0.1246	0.0175	-0.4076	-0.0450	0.2767
	(1.18)	(0.42)	(-0.53)	(0.07)	(-1.86)	(-0.18)	(1.18)
39-51	0.5699	0.4138	0.1686	0.3107	-0.1144	0.2482	0.5699
	(2.51)*	(1.45)	(0.69)	(1.27)	(-0.54)	(1.07)	(2.51)*
1- yr	0.3659	0.2099	-0.0353	0.1068	-0.3184	0.0443	0.3659
	(1.86)	(0.84)	(-0.16)	(0.48)	(-1.72)	(0.22)	(1.86)
2-11yrs	0.6883	0.5323	0.2871	0.4292	0.0041	0.3667	0.6883
	(2.93)*	(1.79)	(1.15)	(1.63)	(0.02)	(1.74)	(2.93)*

(* denotes statistically significant difference from zero)

Table 4.1 Estimates and t-statistics, modelling ρ for specified quadrennia

	75-94	*75-78*	*79-82*	*83-86*	*87-90*	*91-94*
μ	2.7618	5.7524	3.7310	3.0168	2.8511	*
	6.58	30.45	4.74	4.10	3.31	
α	-9.9601E-1	-1.2064	-1.1325	-1.2422	-1.4525	-1.6170
	-19.43	-10.88	-10.22	-24.19	-13.56	-12.76
β	3.2393E-2	6.3075E-2	4.7333E-2	3.3063E-2	5.9901E-2	6.6951E-2
	7.15	5.48	5.00	27.41	7.69	8.83
θ	1.6973E-1	-3.0086E-2	1.2275E-1	1.7735E-1	2.0786E-1	4.1711E-1
	5.64	-5.60	2.17	3.30	3.39	36.30
ϕ	5.7027E-3	8.9599E-3	7.0503E-3	4.8709E-3	7.3811E-3	9.5578E-3
	5.79	3.74	3.11	4.77	3.34	3.73
ψ	-3.4675E-4	-9.6510E-4	-5.3929E-4	*	-5.3417E-4	-6.2393E-4
	-4.05	-3.70	-2.70		-3.17	-3.87
θ_2	-4.4399E-3	*	-3.5295E-3	-4.4978E-3	-5.2586E-3	-9.8314E-3
	-6.33		-2.66	-3.55	-3.69	-23.48
θ_3	2.9982E-5	-1.9764E-6	2.4205E-5	2.9881E-5	3.4947E-5	6.6818E-5
	5.66	-2.52	2.39	3.09	3.25	17.22
γ_1	4.2984E-2	*	*	*	*	*
	15.52					
γ_2	-2.6384E-2	*	*	*	*	*
	-16.04					
γ_3	1.3679E-3	*	*	*	*	*
	20.21					

Table 4.2 Parameter estimates and t-statistics, modelling ν for specified quadrennia

	75-94	*75-78*	*79-82*	*83-86*	*87-90*	*91-94*
μ	-4.1676	-3.3758	-4.8298	-4.0984	-5.4552	-5.9343
	-18.59	-8.35	-8.74	-11.44	-9.37	-7.81
θ	2.3393E-2	1.8262E-2	2.6243E-2	2.6055E-2	2.6239E-2	2.1733E-2
	8.78	2.62	3.76	4.37	4.69	3.41
β	1.0295E-1	3.4399E-2	1.4406E-1	2.6306E-2	1.5081E-1	2.2238E-1
	5.00	1.94	2.91	2.64	2.69	3.30
$\beta_j : z_j$=10.5	-8.8884E-1	*	-1.2731E-1	*	-1.2186E-1	-2.3039E-1
	-3.56		-2.12		-1.90	-3.41
$\beta_j : z_j$=15	*	-4.4799E-2	*	*	*	*
		-2.43				
$\beta_j : z_j$=28	-4.3287E-2	*	-6.6398E-2	6.8885E-2	-7.1999E-2	*
	-3.98		2.43	-3.71	-3.26	
$\beta_j : z_j$=45.5	2.1700E-2	*	4.2750E-2	3.7041E-2	3.5881E-2	*
	4.03		2.96	3.22	3.39	
$\beta_j : z_j$=182	7.8231E-3	1.2460E-2	8.3114E-3	5.2665E-3	7.6188E-3	8.0105E-3
	9.62	5.03	3.67	2.88	4.82	5.72
γ	-1.4287E-1	*	*	*	*	*
	-8.16					

Table 5.1 Inceptions: parameter estimates, standard errors, t-statistics

	targeting σ				*targeting τ*			
	DP1	*DP4*	*DP13*	*DP26*	*DP1*	*DP4*	*DP13*	*DP26*
α_0	-5.840	-4.604	-2.303	3.315	-5.428	-2.672	*	3.617
(s.e.)	6.165E-1	7.781E-1	1.181	4.782E-1	6.045E-1	7.915E-1		1.774
t-stat.	-9.47	-5.92	-1.95	6.93	-8.98	-3.38		2.04
α_1	2.648E-1	1.448	3.323E-1	1.078	-5.173E-1	5.530E-1	2.280E-1	7.005E-1
(s.e.)	3.062E-1	1.474E-1	7.414E-2	1.469E-1	2.973E-1	1.461E-1	7.110E-2	8.667E-2
t-stat.	0.86	9.82	4.48	7.34	-1.74	3.79	3.21	8.08
α_2	*	7.773E-1	*	*	*	7.389E-1	*	*
(s.e.)		1.514E-1				1.503E-1		
t-stat.		5.13				4.92		
α_3	-2.293	-1.245	-3.120E-1	-2.826	-2.301	-1.262	-3.261E-1	-5.962E-1
(s.e.)	2.520E-1	3.731E-1	1.059E-1	5.448E-1	2.432E-1	3.703E-1	1.013E-1	1.202E-1
t-stat.	-9.10	-3.34	-2.95	-5.19	-9.46	-3.41	-3.22	-4.96
α_4	-3.019E-1	-5.766E-1	*	*	-3.384E-1	-5.777E-1	*	*
(s.e.)	8.985E-2	1.583E-1			8.659E-2	1.571E-1		
t-stat.	-3.36	-3.64			-3.91	-3.68		
α_5	1.564	6.860E-1	*	1.916	1.570	6.947E-1	*	*
(s.e.)	2.064E-1	3.061E-1		4.478E-1	1.992E-1	3.038E-1		
t-stat.	7.58	2.24		4.28	7.88	2.29		
β_1	4.346E-1	3.231E-1	2.276E-1	-2.121E-1	2.378E-1	-1.713E-1	-4.582E-1	-8.180E-1
(s.e.)	4.418E-2	5.680E-2	8.353E-2	2.004E-2	4.325E-2	5.757E-2	9.750E-3	1.207E-1
t-stat.	9.84	5.69	2.72	-10.58	5.50	-2.98	-46.99	-6.78
β_2	-1.110E-2	-9.334E-3	-8.063E-3	1.947E-3	-5.512E-3	4.062E-3	1.038E-2	1.879E-2
(s.e.)	1.025E-3	1.333E-3	1.910E-3	2.062E-4	1.001E-3	1.347E-3	4.016E-4	2.663E-3
t-stat.	-10.83	-7.00	-4.22	9.44	-5.51	3.02	25.85	7.06
β_3	8.441E-5	7.707E-5	7.124E-5	*	4.170E-5	-2.352E-5	-6.573E-5	-1.250E-4
(s.e.)	7.686E-6	1.008E-5	1.414E-5		7.505E-6	1.017E-5	4.062E-6	1.910E-5
t-stat.	10.98	7.60	5.04		5.56	-2.31	-16.18	-6.54
γ_{11}	4.697E-2	-1.632E-2	*	*	6.051E-2	-4.199E-3	*	*
(s.e.)	1.362E-2	2.286E-3			1.324E-2	2.263E-3		
t-stat.	3.45	-7.14			4.57	-1.86		
γ_{12}	-6.286E-4	*	*	*	-6.737E-4	*	*	*
(s.e.)	1.493E-4				1.452E-4			
t-stat.	-4.21				-4.64			
γ_{21}	8.077E-3	*	*	*	8.091E-3	*	*	*
(s.e.)	1.844E-3				1.778E-3			
t-stat.	4.38				4.55			
$\hat{\phi}$	5.439	1.906	1.678	1.480	5.068	1.878	1.547	1.495

Table 5.2 Scaled deviance residuals by age and period

	1975	*1976*	*1977*	*1978*	*1979*	*1980*	*1981*	*1982*	*1983*	*1984*
18-24	0.98	1.46	0.18	-0.08	-0.31	-0.15	-1.12	-0.38	0.40	-0.09
25-29	0.54	0.56	-0.61	1.60	-0.36	0.51	-0.94	-0.12	-0.19	0.28
30-34	-0.56	-0.79	-1.53	-0.32	-0.72	0.50	-1.03	0.63	0.96	-0.01
35-39	-0.03	1.08	-0.56	-0.31	0.02	0.54	-1.57	-0.64	0.87	-0.23
40-44	0.53	-0.69	-1.17	0.29	0.58	-0.71	-0.36	-1.19	1.05	-0.07
45-49	0.82	0.53	-0.92	0.37	1.63	-0.09	-0.86	-1.11	-0.16	-1.20
50-54	-0.47	0.36	1.23	0.61	1.16	1.12	-2.25	-2.54	0.66	-1.03
55-59	-1.00	0.04	-1.28	0.83	-0.08	2.27	0.06	0.13	1.72	0.13
60-64	0.13	-0.78	-0.81	0.80	0.80	0.19	-0.29	-0.72	-0.15	-0.18

	1985	*1986*	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*	*1993*	*1994*
18-24	-0.31	-1.60	-1.81	-1.00	-0.29	-0.53	-0.55	-1.55	0.51	-0.39
25-29	0.76	-0.39	-1.29	0.70	-0.03	-2.45	-0.65	-1.15	-0.14	-0.34
30-34	1.46	0.92	2.33	1.59	1.41	0.48	0.54	-0.23	2.45	0.11
35-39	0.40	0.04	-1.74	-0.28	-0.15	0.87	0.11	-0.53	2.96	-0.56
40-44	0.70	0.24	-0.57	-0.13	1.04	-3.38	-1.50	-1.53	1.78	-0.94
45-49	1.20	0.97	0.47	-1.33	0.14	-1.90	-0.66	-0.22	0.49	-2.21
50-54	1.01	0.35	-0.66	0.97	0.97	-0.47	1.63	-0.22	1.36	-0.46
55-59	-1.06	-0.75	0.50	0.81	1.10	1.31	1.12	-0.37	0.44	0.71
60-64	-0.20	-0.98	-1.09	-0.97	0.44	-1.02	-0.81	-0.30	0.70	0.07

Deferred period 1 week

	1975	*1976*	*1977*	*1978*	*1979*	*1980*	*1981*	*1982*	*1983*	*1984*
18-24	-0.66	-0.64	0.18	1.28	1.37	-0.76	-0.28	0.13	0.63	0.70
25-29	-0.38	-1.03	-0.95	0.60	-0.13	0.18	-0.09	-0.33	0.38	-0.12
30-34	-0.15	1.30	-0.26	-0.50	-0.08	-0.06	-1.71	-1.76	1.59	1.08
35-39	0.72	0.67	2.03	0.27	1.36	-0.75	0.38	-0.23	0.22	1.35
40-44	1.20	-1.64	-0.60	0.98	0.01	0.56	-0.98	0.61	-2.27	1.08
45-49	-0.48	-1.24	1.55	-1.02	-0.36	-0.60	-0.19	1.57	-0.22	0.65
50-54	-0.18	0.05	0.96	-1.43	-0.50	0.93	-0.78	-1.43	0.77	-0.30
55-59	-0.78	-0.03	-0.65	0.72	0.05	0.25	0.51	0.06	-0.28	0.16
60-64	0.72	-0.66	-0.02	0.32	0.62	-0.43	-1.67	-1.25	-1.09	2.73

	1985	*1986*	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*	*1993*	*1994*
18-24	0.15	-2.15	-1.08	0.09	0.54	-0.73	0.92	2.13	-0.31	-1.58
25-29	0.50	-0.52	-0.52	-1.00	-1.82	-0.89	1.00	0.85	-0.57	-0.44
30-34	0.47	-1.31	0.54	0.34	0.80	-0.29	0.25	1.41	1.95	-0.65
35-39	-0.09	-0.40	-0.25	2.31	1.39	0.26	-0.40	1.86	-2.03	0.61
40-44	-0.06	-2.05	-1.83	1.12	-2.05	-1.19	-1.45	-0.13	-1.49	0.45
45-49	-0.95	0.50	-0.37	-0.93	-0.13	0.04	0.47	-1.60	-0.03	1.48
50-54	-0.12	-0.10	0.13	-0.05	-0.71	-0.64	0.94	0.54	0.71	-0.41
55-59	1.04	0.80	1.81	1.18	-0.33	0.57	2.07	-0.70	0.98	-0.32
60-64	0.03	-1.24	1.91	-1.65	0.38	-0.43	-1.84	-0.23	-0.16	-1.60

Deferred period 4 weeks

Table 5.2 (continued) Scaled deviance residuals by age and period

	1975	*1976*	*1977*	*1978*	*1979*	*1980*	*1981*	*1982*	*1983*	*1984*
18-24	-0.80	-0.08	-0.56	-0.79	0.63	-0.61	-2.08	-2.15	0.24	-0.29
25-29	0.20	-1.21	-0.05	-0.93	0.85	-0.56	0.69	-0.48	-0.63	1.34
30-34	-1.14	0.38	-1.33	-0.29	0.07	-0.41	1.15	-1.24	-0.39	-0.90
35-39	0.48	1.00	0.57	0.10	0.34	1.06	-1.02	-1.20	-0.72	0.56
40-44	0.87	0.65	-1.10	0.96	0.12	-1.24	0.27	-1.17	-0.11	0.93
45-49	0.87	-0.53	0.59	-0.14	-1.59	0.52	-2.18	-2.23	-0.17	0.62
50-54	-0.24	-1.22	0.89	0.43	1.12	-0.28	-0.35	-1.42	1.50	1.63
55-59	0.85	-0.67	-0.39	0.43	2.40	-0.10	-1.21	-1.02	0.90	-0.61
60-64	0.32	-0.32	1.34	0.45	1.53	1.33	-1.59	-0.81	-1.73	-0.53

	1985	*1986*	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*	*1993*	*1994*
18-24	-1.35	-0.20	0.90	0.47	1.73	-0.58	-0.42	-1.35	-1.17	-0.39
25-29	0.01	1.17	0.72	-0.03	0.01	0.25	1.10	0.09	-0.31	0.53
30-34	1.28	1.21	-1.32	0.76	0.42	0.34	-0.37	-0.48	1.14	0.53
35-39	0.52	0.09	0.01	1.04	-0.71	2.47	0.04	2.11	-0.99	1.07
40-44	1.30	0.42	0.15	-0.37	-0.51	-0.93	-0.76	-0.56	0.51	-1.84
45-49	-1.86	0.43	-1.25	-0.52	-0.62	-1.41	-0.36	-0.41	0.06	-2.09
50-54	1.29	1.15	0.17	-0.81	-1.44	0.94	-1.50	-0.87	1.21	2.40
55-59	1.00	1.60	0.77	-0.29	0.17	-0.50	0.65	1.89	0.43	0.67
60-64	0.84	-1.17	-0.42	-2.06	1.02	-1.85	-0.71	-0.22	0.28	-0.43

Deferred period 13 weeks

	1975	*1976*	*1977*	*1978*	*1979*	*1980*	*1981*	*1982*	*1983*	*1984*
18-24	0.03	-1.59	-0.36	-1.21	0.11	-0.57	-1.20	-1.12	-0.94	-0.99
25-29	-0.46	0.69	1.88	-0.37	0.50	1.52	0.95	-1.75	0.05	-0.07
30-34	0.44	-0.83	-0.38	0.26	0.06	1.40	-0.87	-1.52	-0.49	-0.29
35-39	1.69	-0.06	-0.61	0.86	-0.73	1.12	0.28	-1.39	0.69	2.13
40-44	-1.25	0.58	0.90	1.01	0.36	-0.50	0.08	-0.56	-0.66	0.47
45-49	0.25	-1.16	-0.84	0.09	-1.94	-1.21	-0.80	-0.83	-0.51	-0.42
50-54	0.10	-0.02	-0.77	1.21	-0.35	-0.75	-1.60	-2.02	-0.65	-0.46
55-59	-0.03	0.79	1.18	-1.27	0.61	1.08	-0.23	0.65	0.01	0.95
60-64	0.12	0.33	1.08	0.20	0.45	1.15	0.46	-0.63	0.03	-1.70

	1985	*1986*	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*	*1993*	*1994*
18-24	-0.99	-1.02	-1.08	-1.08	-1.06	-1.03	0.32	-0.29	0.33	-1.54
25-29	-0.76	-0.72	-1.75	0.37	0.13	0.73	-0.33	1.81	-0.66	-1.97
30-34	0.23	0.58	1.33	0.11	0.92	0.01	-0.11	0.55	0.93	1.01
35-39	0.70	-0.41	0.08	1.28	-0.54	0.12	0.73	0.19	-0.31	0.67
40-44	-0.18	-1.46	1.16	-0.79	-1.39	0.29	1.73	0.40	0.07	0.96
45-49	0.48	-1.35	-0.44	-2.12	2.04	-1.35	-0.08	0.14	-0.68	1.24
50-54	-0.16	-0.25	-1.89	-1.44	-0.22	-0.42	1.96	-0.44	-0.29	0.07
55-59	1.82	3.21	1.02	1.02	-0.35	-0.30	1.22	0.39	1.25	-1.74
60-64	0.25	-0.35	1.58	-1.69	-1.64	-1.91	-0.44	-1.49	1.43	-0.65

Deferred period 26 weeks

Table 6.1 Values of $(\overset{*}{\sigma}_{tx} - \hat{\sigma}_{tx})/\hat{\sigma}_{tx}$ expressed as percentages

	1975	*1977*	*1979*	*1981*	*1983*	*1985*	*1987*	*1989*	*1991*	*1993*	*1994*
18-24	1.8	0.0	-2.7	-5.5	-8.0	-10.1	-11.7	-12.9	-14.0	-15.4	-16.3
25-29	2.0	2.1	1.2	0.2	-0.7	-1.0	-0.8	-0.3	0.4	0.9	0.9
30-34	0.5	1.2	1.0	0.6	0.5	0.7	1.5	2.6	3.8	4.7	5.0
35-39	-1.5	-0.6	-0.6	-0.9	-1.0	-0.8	-0.1	0.9	1.9	2.5	2.6
40-44	-2.2	-1.2	-1.2	-1.6	-1.9	-1.9	-1.6	-1.0	-0.5	-0.5	-0.8
45-49	-1.0	0.0	-0.0	-0.5	-1.0	-1.2	-1.1	-0.9	-0.9	-1.4	-2.0
50-54	0.9	2.1	2.1	1.7	1.2	0.9	0.9	0.9	0.7	-0.1	-0.9
55-59	0.8	2.3	2.6	2.4	2.1	2.0	2.1	2.1	1.9	1.0	0.2
60-64	-4.5	-2.6	-1.9	-1.7	-1.6	-1.4	-1.0	-0.7	-0.7	-1.3	-2.0

Deferred period 1 week

	1975	*1977*	*1979*	*1981*	*1983*	*1985*	*1987*	*1989*	*1991*	*1993*	*1994*
18-24	1.7	-1.3	-4.1	-6.6	-8.8	-10.8	-12.5	-13.9	-15.3	-16.4	-16.7
25-29	5.0	4.1	3.3	2.8	2.6	2.7	3.1	3.7	4.5	5.7	6.4
30-34	2.8	2.7	2.6	2.8	3.2	4.0	5.0	6.1	7.5	9.1	10.2
35-39	-1.6	-1.6	-1.6	-1.4	-1.0	-0.4	0.3	1.1	2.1	3.2	3.9
40-44	-3.5	-3.6	-3.7	-3.8	-3.8	-3.6	-3.3	-3.1	-2.9	-2.6	-2.3
45-49	-1.4	-1.5	-1.8	-2.0	-2.2	-2.3	-2.4	-2.7	-3.0	-3.4	-3.5
50-54	2.7	2.8	2.7	2.5	2.4	2.2	1.9	1.4	0.8	0.1	-0.2
55-59	3.1	3.6	3.9	4.1	4.3	4.4	4.4	4.1	3.6	2.9	2.6
60-64	-7.7	-6.5	-5.6	-4.7	-4.0	-3.3	-2.8	-2.6	-2.6	-2.8	-2.9

Deferred period 4 weeks

	1975	*1977*	*1979*	*1981*	*1983*	*1985*	*1987*	*1989*	*1991*	*1993*	*1994*
18-24	-28.4	-28.9	-28.6	-27.6	-25.9	-23.5	-20.3	-16.4	-11.8	-6.2	-3.1
25-29	-5.2	-6.1	-5.9	-4.8	-2.6	0.5	4.7	9.9	16.3	23.9	28.2
30-34	2.6	1.1	0.8	1.5	3.2	5.9	9.5	14.1	19.9	26.7	30.6
35-39	0.8	-1.2	-2.1	-2.2	-1.3	0.3	2.8	6.0	10.0	14.8	17.4
40-44	-1.5	-3.9	-5.3	-6.0	-6.0	-5.3	-4.0	-2.1	0.2	3.1	4.7
45-49	1.6	-1.3	-3.3	-4.5	-5.1	-5.2	-4.8	-4.0	-2.8	-1.3	-0.4
50-54	9.5	6.1	3.6	1.8	0.6	-0.1	-0.4	-0.4	-0.1	0.4	0.7
55-59	15.8	12.1	9.1	6.9	5.2	4.0	3.1	2.5	2.1	1.8	1.7
60-64	9.2	5.5	2.6	0.2	-1.6	-3.1	-4.3	-5.3	-6.1	-6.9	-7.2

Deferred period 13 weeks

	1975	*1977*	*1979*	*1981*	*1983*	*1985*	*1987*	*1989*	*1991*	*1993*	*1994*
18-24	-30.2	-50.0	-45.8	-40.0	-40.3	-44.4	-46.0	-40.5	-27.8	-24.8	-38.1
25-29	22.9	-13.3	-7.4	1.3	-0.2	-8.0	-11.4	-2.9	17.2	21.7	-0.0
30-34	46.2	2.6	8.8	18.2	15.7	5.9	1.3	10.1	31.9	35.7	11.0
35-39	42.8	0.2	6.1	15.0	12.1	2.1	-3.0	4.6	24.1	26.5	2.8
40-44	31.0	-7.7	-2.0	6.3	3.5	-6.0	-11.1	-4.7	12.1	13.1	-8.5
45-49	24.8	-11.5	-5.6	2.6	-0.0	-9.3	-14.5	-8.7	6.8	6.9	-14.0
50-54	26.3	-9.8	-3.3	5.4	2.9	-6.6	-12.1	-6.4	8.9	8.3	-13.1
55-59	30.5	-6.4	0.8	10.2	7.7	-2.2	-8.0	-2.3	13.3	12.3	-10.2
60-64	26.8	-8.7	-1.6	7.8	5.4	-4.3	-10.1	-4.7	10.3	9.0	-13.0

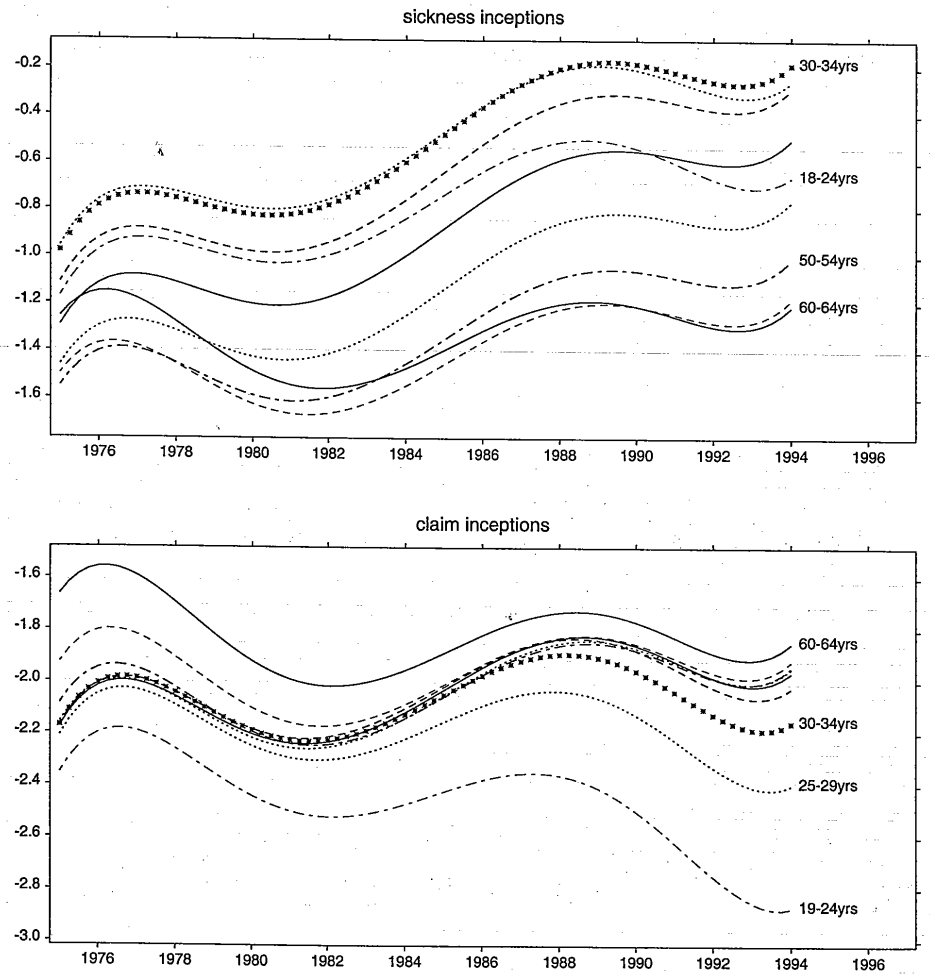
Deferred period 26 weeks

Table 6.2 Inceptions: 1991-94 quadrennial experience, parameter details

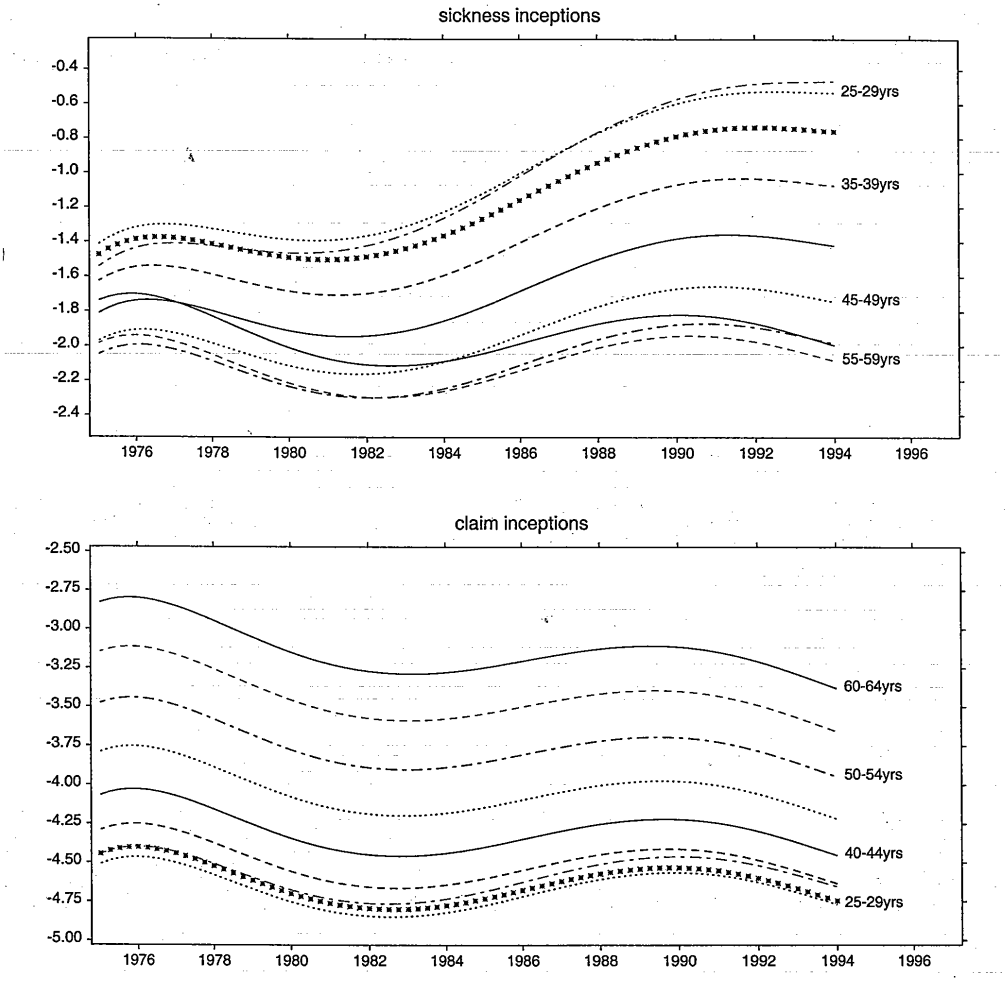
	targeting σ				*targeting* τ			
	DP1	*DP4*	*DP13*	*DP26*	*DP1*	*DP4*	*DP13*	*DP26*
β_0	-1.507E-1	-3.030E-1	-4.624E-1	*	-1.760	-3.718	-5.285	-5.307
(s.e.)	6.282E-2	6.945E-2	7.106E-2		5.899E-2	4.868E-2	1.021E-1	7.612E-2
t-stat.	-2.40	-4.36	-6.51		-29.8	-76.4	-51.8	-69.7
β_1	-5.105E-1	-2.575	-3.104	-3.539	8.869E-1	7.440E-1	1.152	9.341E-1
(s.e.)	2.226E-1	3.866E-1	3.081E-1	5.382E-1	1.107E-1	3.776E-1	1.488E-1	5.279E-1
t-stat.	-2.29	-6.66	-10.1	-6.58	8.01	1.97	7.74	1.77
β_2	-1.256	-1.993	*	-3.111	-4.412E-1	*	2.334	*
(s.e.)	2.608E-1	2.505E-1		3.416E-1	2.309E-1	*	8.959E-1	*
t-stat.	-4.81	-7.96		-9.11	-1.91	*	2.60	*
β_3	1.123	8.971	5.379	1.233E+1	*	4.006	*	5.416
(s.e.)	5.193E-1	1.857	7.941	2.713		1.818		2.608
t-stat.	2.16	4.83	6.77	4.55		2.20		2.08
β_4	*	*	-4.060	*	*	*	-2.782	*
(s.e.)			6.094E-1			*	1.407	
t-stat.			-6.66			*	-1.98	
β_5	*	-7.305	*	-9.124	*	-5.381	*	-6.480
(s.e.)		2.038		3.073		1.994		2.965
t-stat.		-3.58		-2.97		-2.70		-2.19
$\hat{\phi}$	2.528	1.488	1.099	1.519	2.457	1.430	1.124	1.490

Table 6.3 Values of $(\overset{*}{\sigma}_x - \hat{\sigma}_x)/\hat{\sigma}_x$ expressed as percentages: 1991-94 quadrennium

age	*DP1*	*DP4*	*DP13*	*DP26*
18	-16.6	2.1	794	5.2
20	-19.7	-1.6	219	-4.8
22	-17.9	-3.1	65.7	-9.1
24	-13.0	-2.3	14.6	-8.6
26	-6.5	-0.2	-3.3	-4.9
28	-0.0	2.0	-7.8	0.1
30	5.2	3.3	-6.5	4.5
32	8.1	3.2	-3.4	7.1
34	8.5	1.9	-0.7	7.5
36	7.0	-0.0	0.6	6.1
38	4.2	-1.9	0.5	3.9
40	1.0	-3.1	-0.4	1.9
42	-1.7	-3.2	-1.2	0.8
44	-3.7	-2.2	-1.3	0.8
46	-4.6	-0.5	-0.6	1.6
48	-4.4	1.4	1.0	2.8
50	-3.3	3.0	3.0	3.7
52	-1.3	3.4	4.6	3.5
54	1.0	2.3	4.8	1.9
56	3.3	0.0	2.8	-0.7
58	4.9	-2.7	-2.4	-3.1
60	5.4	-4.3	-10.8	-3.2
62	4.2	-2.2	-22.1	3.0
64	0.8	7.8	-35.1	23.0

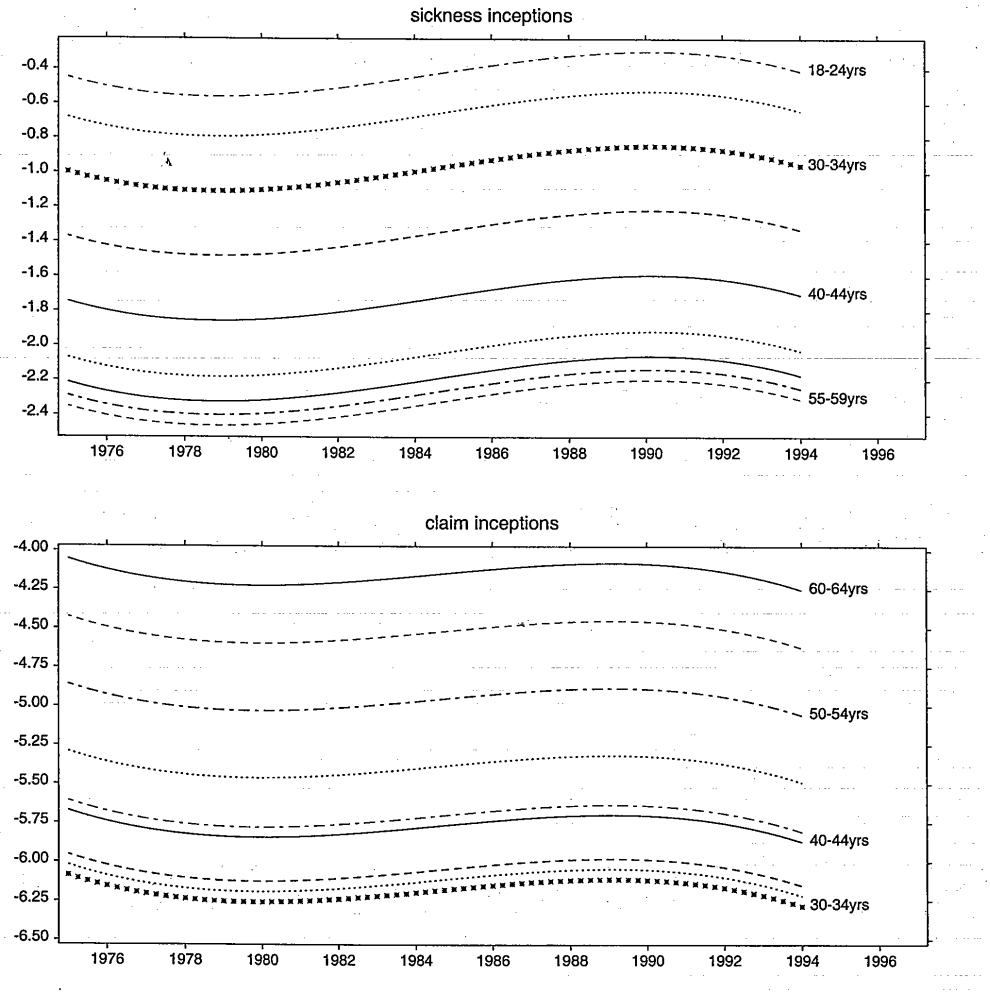


Log predicted inception intensities vs period, by age, DP1
Figure 5.1(a)



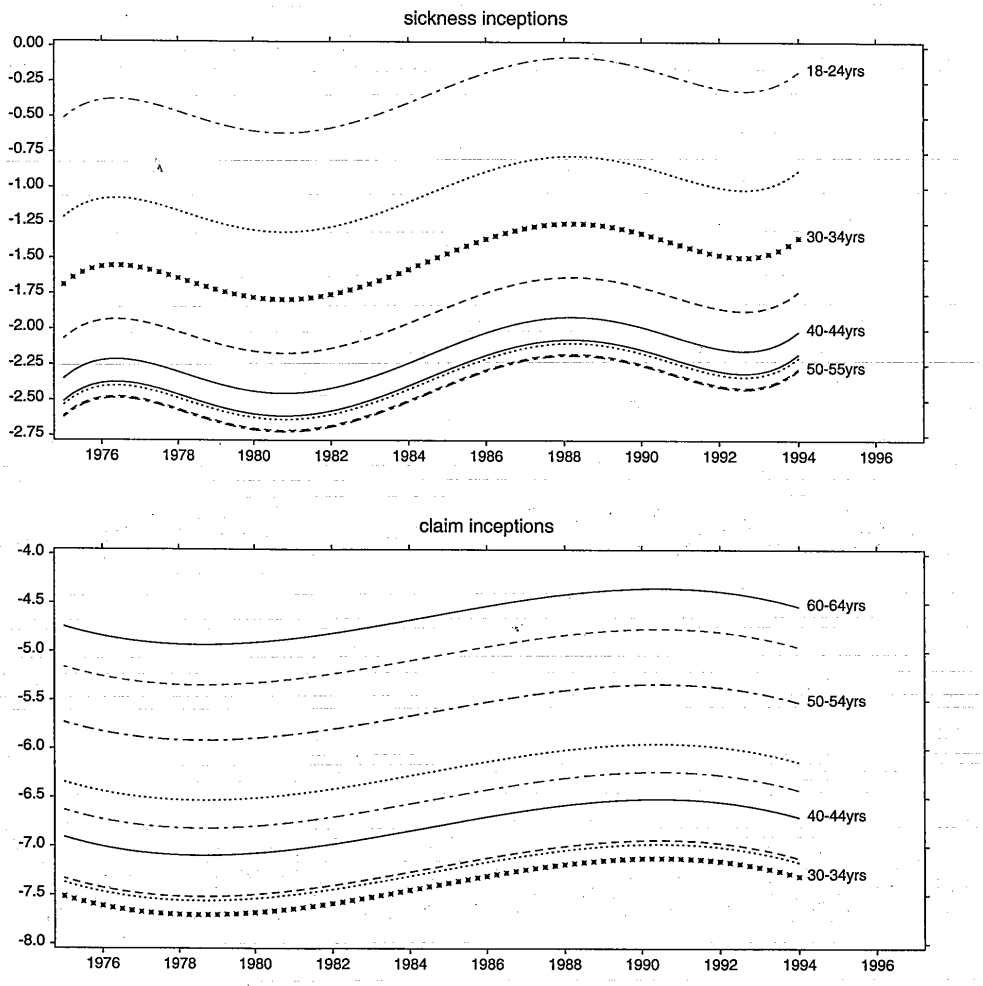
Log predicted inception intensities vs period, by age, DP4

Figure 5.1(b)



Log predicted inception intensities vs period, by age, DP13

Figure 5.1(c)



Log predicted inception intensities vs period, by age, DP26

Figure 5.1(d)

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