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A Model for Projecting the Number of People Who Will Require Long-Term Care in the Future

Part II: The Multiple State Model

by

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A MODEL FOR PROJECTING THE NUMBER OF PEOPLE WHO WILL REQUIRE LONG-TERM CARE IN THE FUTURE

II: The Multiple State Model

BY B. D. RICKAYZEN AND D. E. P. WALSH

ABSTRACT

A multiple state model has been developed for projecting the number of people in the UK with disabilities over the next forty years. In this paper, we describe the model in detail. We then discuss the way in which the transition rates incorporated within the model could change over time to reflect recent trends in healthy life expectancy.

KEYWORDS

Long term care; Multiple state model; Healthy life expectancy

INTRODUCTION

We have developed a model to project the number of people in the United Kingdom who will be disabled over the next forty years. In Part I, we describe the various data sources for the model (Walsh and Rickayzen, 2000). In Part II, we describe the model (section 4) and discuss the way in which trends in healthy life expectancy data can be incorporated within the model (section 5).

4. A TRANSITION RATE MODEL

4.1 *Outline*

In Part I, section 2, we explained that we used the OPCS survey of disability in Great Britain (Martin et al, 1998) to provide the initial data for the number of people with disabilities. We did this by combining the prevalence rate data from the OPCS survey with the number of males and females at each age to give us the number of people at each of the ten disability levels, and the number healthy, in 1986. We need a transition rate model to project this population forward. Each year some people will show improvements in their abilities, some will show no change, some will deteriorate and some will die.

There are many possible transitions, all of which may depend on age and year. We have separate models for males and females. One thing we do not allow for in our model is duration: the probability that a transition takes place is taken to apply to all people in a particular sex/age/year/disability category; we do not take into account how or when someone arrived in that category.

All of the probabilities we use are annual. So, for example, a process that involves deteriorating from healthy to a category 3 disability and then deteriorating further to category 4 during the same year will be regarded as a single healthy-to-category 4 transition.

Thus, the model is a discrete time multiple state model. For a full description of such models and discussion of applications to disability insurance, see Haberman and Pitacco (1999).

4.2 *Mortality*

4.2.1 *Overall mortality*

We use the Government Actuary's Department (GAD) central population projection for the period 1996 to 2036 (Government Actuary 1998). This gives the projected total number of deaths each year at each age. Our model matches these numbers exactly. Note that the GAD projections include migration as a transition and we also include migration so that the numbers match.

In theory, it might be reasonable not to reproduce the GAD projected population. Future death rates will be closely related to the prevalence of disability in the future. Since we are producing a new model for the prevalence of disability it would be possible to use it to derive the number of deaths in each future year under certain assumptions about the link between mortality and disability. However, we decided that it would be undesirable to produce a population

projection which differed from the GAD central projection. Thus, we use the GAD central projection as a constraint on the output of our model.

Since the prevalence rate data apply to 1985 and 1986 there is a ten year gap to fill before the start of the period covered by the current GAD projection model. (We actually assume that the prevalence rate data all apply to 1986.) During these years we use mortality rates which are interpolated between those of English Life Tables No.14 (ELT14) (OPCS, 1987) which are taken to apply in 1980 and the GAD 1996 rates. These rates are used to determine the total population each year (working back from 1996) and also the total number of deaths each year during the ten year period.

4.2.2 *The dependence of mortality on disability*

The mortality rate is higher for people in the severe disability categories (see Table 7, for example) and we split the total mortality into two components in order to model this. One of the components applies equally to all healthy and disabled people of a given age and sex in a particular year. The other component is higher for people with severe disabilities.

This second component was set by reference to the US data described in Part I, section 2.2. Note that the US data only relate to people over 65. The US data were useful in suggesting an overall "shape" for the dependence of mortality on disability and how this relates to age. We have not attempted to include any of the detail from the US data in our model. The features that we incorporate in our model regarding the disability-related component of mortality are:

- There is only weak age dependence (above age 65) in the disability-related addition to healthy mortality.
- The extra mortality is low at younger ages. This is needed because applying the 65+ rates to the disabled population aged around 35 produces too many deaths. In fact the number of disabled people dying would be more than the total number of deaths according to the GAD model.
- There is no extra mortality compared with healthy people for those with disabilities in category 5 and lower. The description of these disabilities suggests they are not life threatening conditions.
- The extra mortality increases linearly starting with category 6. The US data do not fully support this, but we feel that we do not have enough information to justify a more complex category dependence.
- The maximum extra annual mortality is 0.20. The US data suggest a figure closer to 0.15 for the difference in mortality between the worst disability category and the not-disabled category (see discussion of Table 10). We choose a higher figure as a result of discussions with people with experience of managing insurance claims: their view is that the life expectancy of people with the types of disability which apply to OPCS disability category 10 are very short indeed and that a large addition to mortality is appropriate (Ainslie & Laurie, 1998). The 0.20 increase applies to elderly people in category 10. (So, for these people, "q_x" is 0.20 higher than for healthy people of the same age.)
- The model is the same for males and females.

Once the extra mortality has been chosen, the other mortality component is determined by the requirement that the total number of deaths should match the GAD projection numbers.

The formula we use to express the extra mortality for someone aged x in disability category n (where $n = 0$ means healthy) is:

$$ExtraMort(x, n) = \frac{0.20}{1 + 1.1^{50-x}} \cdot \frac{\text{Max}(n - 5, 0)}{5}$$

The form of "reciprocal of one plus an exponential" is the same as we use for modelling deterioration (see section 4.3). The choice of the pivotal age 50 and the steepness factor 1.1 effect the extra mortality at younger ages. The following table shows illustrative values for this function at a range of ages and disability levels.

Table 17. Annual addition to mortality due to disability

Age	Category 6	Category 8	Category 10
20	0.00	0.01	0.01
30	0.01	0.02	0.03
40	0.01	0.03	0.06
50	0.02	0.06	0.10
60	0.03	0.09	0.14
70	0.03	0.10	0.17
80	0.04	0.11	0.19
90	0.04	0.12	0.20
100	0.04	0.12	0.20
110	0.04	0.12	0.20

The extra mortality might change with time. We discuss trends in section 5.

4.3 *Deterioration*

4.3.1 *Outline*

Healthy people can become disabled and the condition of disabled people can become worse. Both of these come under the heading of deterioration. In our model, deterioration is allowed from any state to any more severely disabled state. This results in a huge number of transitions. Owing to the complexity, the model for deterioration is split into three parts, which are dealt with in the next three sections. One part relates to the probability of a healthy person becoming disabled, another relates to the distribution of the severity of new disabilities amongst previously healthy lives and the final part relates to deterioration amongst people who are already disabled.

There are parameters for each part of the deterioration model. The parameter values are chosen so that the transition rate model is able to reproduce the prevalence rate data closely.

In making the comparison between the observed prevalence rates and those produced by the model, we start with twenty-year olds with disabilities matching the OPCS rates. This population is projected forward to produce the model prevalence rates at higher ages. The transition model includes mortality and improvements in health as well as deterioration but these other components are fixed separately — they are not chosen for their ability to reproduce the prevalence rate data.

Note that this approach assumes that there is a stationary population, i.e. transition rates have been constant in the past. This is clearly not correct. We discuss this problem in Part III, section 8.2. For convenience, we use a single mortality table during this comparison process (rather than using time dependent rates). The mortality table we use is ELT14.

Table 18 presents the ability of the model to reproduce the crude prevalence data. It shows the difference between the disability prevalence rates according to the OPCS survey (i.e. those shown in Table 2) and those produced by the transition rate model.

Table 18a. Difference in prevalence rate for females, *data – model* (per thousand)

		OPCS Disability Category									
Age	Able	1	2	3	4	5	6	7	8	9	10
20–29	–2	0	0	0	1	0	1	0	0	0	0
30–39	4	0	–3	–1	1	0	0	0	0	0	0
40–49	5	0	–3	–3	1	–1	0	0	1	0	0
50–59	–5	2	–2	0	2	2	0	0	2	2	–1
60–69	0	0	3	–1	0	0	–3	–2	1	3	–2
70–79	4	–4	7	0	–2	–6	3	–4	3	0	–1
80+	–1	0	0	1	0	0	0	0	0	0	0

Table 18b. Difference in prevalence rate for males, *data – model* (per thousand)

		OPCS Disability Category									
Age	Able	1	2	3	4	5	6	7	8	9	10
20–29	–3	0	1	1	1	0	1	0	0	0	0
30–39	2	0	–2	0	0	0	0	0	0	–1	0
40–49	3	–1	–1	–1	0	0	1	0	0	–1	0
50–59	–5	0	0	2	0	1	1	0	1	0	0
60–69	–10	3	8	1	0	–1	–2	–1	1	1	1
70–79	15	–8	9	0	–5	–8	1	–4	2	–2	0
80+	0	0	0	0	0	0	0	0	0	0	0

Note that the structure of the data that we are trying to model (i.e. those shown in Table 2) is very complex. The prevalence rates do not vary smoothly across categories and the dependence of the prevalence rates on age is quite different for the low disability and high disability categories.

Both parts of Table 18 are encouraging as the differences between the data and the model are not large. There are some systematic errors but there appear to be no major problems at the highest categories, which are the more important categories as far as care costs are concerned.

The reason why the values in the table are small is that the model of deterioration is complex. There may be a case for simplifying the model and accepting a poorer fit to the data.

4.3.2 *The probability of becoming disabled*

We use formulae to express the probability of becoming disabled. The probability of becoming disabled is primarily constrained by the observed proportion of people who have no

disability. There are only seven age bands for the published disability survey data but we find that a complex model is needed to provide a good fit to the data. The formula we use has four parameters for females and there is an additional one for males. For females, the formula we use is:

$$NewDisab(x) = A + \frac{D - A}{1 + B^{C-x}}$$

where the four parameters are A, B, C and D and *NewDisab(x)* is the probability that a female aged x becomes disabled in a year. We note that this formula is logistic in form and was first proposed by Perks for the graduation of mortality rates (See Benjamin & Pollard, 1993).

For males, the formula we use is:

$$NewDisab(x) = \left(A + \frac{D - A}{1 + B^{C-x}} \right) \times \left(1 - \frac{1}{3} \cdot \exp\left[-\left(\frac{x - E}{4} \right)^2 \right] \right),$$

where the additional parameter is E.

The parameter A is the limit of the probability of becoming disabled at young ages. D is the limit of the probability of becoming disabled that would apply at extremely high ages. The pair of parameters B and C determine how rapidly the probabilities change between the two extreme values. The extra parameter, E, gives the age at which there is a "kink" in the *NewDisab(x)* function.

Figure 1 shows the logarithm (base 10) of the annual probability of becoming disabled, for males and females. The parameter values used in the figure are the same as were used to produce Table 18.

Figure 1: (log) Annual Probability of becoming Disabled

The shapes of the curves in the figure are quite complex. The continual oscillations, which show females having the higher probability of becoming disabled at young ages, followed by males in their fifties, followed by further changes, may be traced directly to the data. Tables 2a and 2b show that the prevalence rate of the "able" category is higher for males up to ages in the fifties, higher for females in their sixties, higher for males in their seventies and higher for females over eighty. To get sufficient flexibility in the shapes, an extra parameter was added for fitting the males' data (it is not needed for the females' data).

The parameter values that we use are given in Table 19.

Table 19. Parameter values for $NewDisab(x)$

Parameter	Males	Females
A	0.0017	0.0017
B	1.1063	1.0934
C	93.5111	103.6000
D	0.6591	0.9567
E	70.3002	(Not used)

The behaviour of the $NewDisab(x)$ formula above the age of 85 or so is not well constrained. Since the highest age group in the data we use to constrain the model includes all people over 80 and these have an average age of around 85, the probability of becoming disabled could be very different at the highest ages without noticeably changing the prevalence rates in the crude data.

We define the probability of becoming disabled in such a way that it only applies to people who survive the year. This was done for numerical convenience: because mortality and disability rates become high at old ages some technique is needed to avoid total transition probabilities exceeding 1. The device of defining transitions in sequence, i.e. with mortality first, followed by deterioration (which is followed by improvement), achieves this.

4.3.3 *The severity of new disabilities*

Someone becoming disabled from healthy may enter any of the ten disability categories. The relative probability of joining each category may change with age, with the likelihood that the disability is severe increasing for older people. The transition rate model has three parameters covering this age dependence.

An examination of the prevalence rates at ages over 80 in Table 2a shows that the progression is erratic. The rate is higher in category 1 than in category 2, category 2 has a lower rate than category 3 and so on. In fact the rate in every category is either higher than in both the neighbouring categories or lower than in both. A simple model cannot replicate such a pattern. We decided to adopt a model which could reproduce the observed pattern closely. This involves having a separate parameter to represent the "width" of each category. This approach is not unreasonable given the complex definitions used for each category. Because of the complexity of the definitions, some categories may include more people than others — this is the aspect of the disability prevalence rates that the width parameters are intended to mimic.

The formula for the probability that a person who becomes disabled at age x will have a disability in severity category n is given by:

$$Severity(x, n) = W(n) \cdot f(x)^{n-1} / Scale(x)$$

$$f(x) = A + \frac{1 - A}{1 + B^{C-x}}$$

$$Scale(x) = \sum_{n=1}^{10} W(n) \cdot f(x)^{n-1}$$

The category widths are given by $W(n)$. The *Scale* term ensures that the probabilities add up to 1 and its inclusion means that we can arbitarily set W(1)=1. The three parameters relating to the age dependence are A, B and C (they are distinct from the parameters used in the formulae for $NewDisab(x)$).

Figure 2 illustrates the age dependence of the relative severity of new disabilities. The figure shows the probability that someone newly disabled will be in category 6 or worse at the end of the year. The shapes of the curves are similar for other categories.

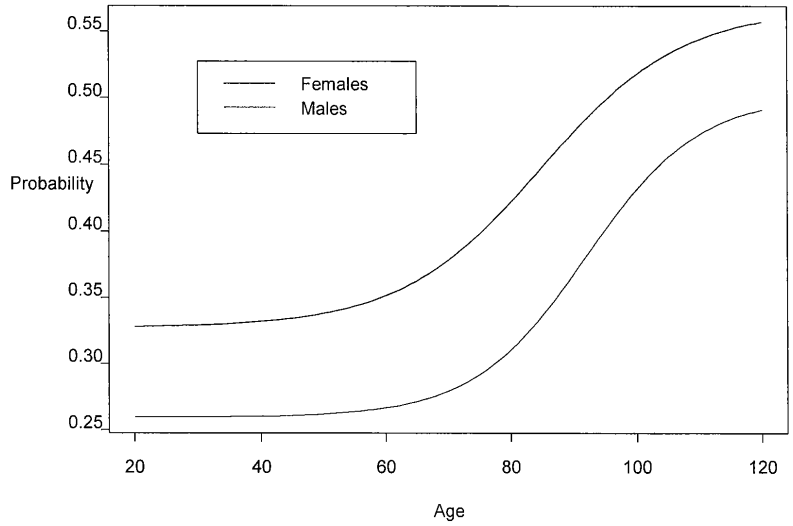

Figure 2: Probability of a new disability being category 6 or worse

Figure 2 shows that there is a difference between the probabilities for females and males. This was derived from fitting the prevalence rate data. However, it would be plausible to find slightly poorer fits in which there was little difference between the probabilities for males and females (by, for example, changing the likelihood of disabled people deteriorating).

The parameters we use are given in Table 20. Note that $W(1) = 1$ is fixed for both males and females.

Table 20. Parameter values for *Severity*

Parameter	Males	Females
A	0.8246	0.8180
B	1.1146	1.0911
C	91.7127	85.5099
$W(2)$	0.5250	0.6823
$W(3)$	0.4632	0.8166
$W(4)$	0.4622	0.6656
$W(5)$	0.6066	1.1749
$W(6)$	0.4205	1.0426
$W(7)$	0.6299	1.4203
$W(8)$	0.6370	0.9399
$W(9)$	0.9004	1.2222
$W(10)$	0.4874	1.0674

4.3.4 *Deterioration from disabled states*

People in any disability category can get worse and their new disability level could be any of the more severe categories. These transitions are included by relating them to the probability of deteriorating from healthy (i.e. becoming disabled). We use the following rule: the probability of someone in disability category m deteriorating to category n is F^m times the probability that a healthy person deteriorates to category n. This may be expressed by the following pair of equations:

$$Deteriorate(x, m, n) = Deteriorate(x, 0, n) \times F^m$$
$$Deteriorate(x, 0, n) = NewDisab(x) \times Severity(x, n).$$

The parameter F is required to be greater than 1 in order to reflect the fact that disabled people are more likely to become severely disabled than healthy people. For males we use $F = 1.1561$ and for females we use $F = 1.1830$.

4.3.5 *The fitting procedure*

We have said in section 4.3.1 that the parameters are set so that the transition rate model can generate a set of prevalence rates that closely matches the OPCS disability survey prevalence rates. In this section we specify how we define "close matching" and how we obtain a satisfactory fit.

We are trying to model prevalence rates for seven age bands and ten disability categories. This gives 70 "cells". We simply try to minimise the sum of the absolute values of the differences between the prevalence rates in the data and the prevalence rates produced by our model. In other words, we take the numbers in a table like 18a or 18b, remove the minus signs

and add them up. (The prevalence rate in the "able" category is automatically 1,000 minus the sum of the other ten prevalence rates at each age, and we do not include it in our error statistic.) Other statistics could have been chosen. For example, extra weighting could have been given to the high ages or high disability categories, or some weights relating to the uncertainties in the cells could have been used.

Our deterioration model has 17 parameters for fitting the females' data and 18 for fitting the males' data. It is difficult to obtain an optimal fit to the data when there are so many parameters to be considered. This is especially true when there are so many local minima encountered in the fitting process. However, we believe that, overall, we have obtained a good fit to the data.

We make some more comments on the fitting process and the uncertainties involved in Part III, section 8.

4.4 *Improvements*

As we noted in Part I, section 2.2, there is evidence that a significant number of disabled people improve to some extent. The US data show some dramatic improvements. It is not clear whether these represent recoveries from long-term disabilities or from temporary disabilities caused by, say, breaking a bone.

We decided not to include a full range of improvements in our transition rate model and, instead, we have adopted a simple assumption: all people, at all ages and in all disability categories have a 10% chance of improving by one category over the course of a year. The figure of 10% is broadly consistent with the UK data (Goddard, 1998). The approach is not consistent with what is shown by the US data unless those data include some short-term disabilities.

The 10% probability only applies to those who survive the year and do not deteriorate during the year.

5. ASSUMED TRENDS

In Part I, section 2.3 we discussed data relating to trends in healthy life expectancy (HLE). We want our projection model to be able to reproduce trends similar to those indicated by the data. In the projection model, trends are included by changing transition rates over time. The procedure we adopt to identify which transition rate changes correspond to the observed HLE trends is:

- Calculate the healthy life expectancies in 1986 using the definitions of "healthy" which are related to the disability categories of the OPCS survey (Martin et al, 1988).
- Project the population forward for ten years using a range of assumptions for changes to the transition rates.
- Calculate the healthy life expectancies in 1996 for the various projections.
- Compare the changes in HLE produced by the model with those shown by the data and decide which trends to continue with for projections up to the year 2036.

The reason why we stop in 1996 is because that is the year when the current GAD population projections start. The projection model becomes more complicated when it is built

around the GAD population model because it must include migration. The year 1996 is therefore a natural break point for the projections.

Table 21 shows the healthy life expectancies in 1986. To calculate these, prevalence rates and a life table are needed. We have used the prevalence rates produced by the transition rate model described in section 4. (The model provides prevalence rates at individual ages unlike the published data that give the rates in ten-year age bands.) The life table is an interpolation between ELT14 and the life table corresponding to the 1996 mortality rates in the GAD population projection model. The ELT14 table is taken to apply to 1980.

Table 21. Life expectancies in 1986

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	7.70	56.95%	12.58	93.09%	0.93
	70	5.17	49.19%	9.57	90.98%	0.95
	75	3.07	38.40%	6.98	87.24%	1.02
	80	1.56	25.99%	4.88	81.19%	1.13
	85	0.68	15.09%	3.26	72.20%	1.26
Females	65	9.14	52.84%	15.61	90.21%	1.69
	70	6.24	45.60%	11.99	87.65%	1.69
	75	3.89	37.16%	8.78	83.85%	1.69
	80	2.15	27.85%	6.03	78.03%	1.70
	85	1.03	18.47%	3.87	69.13%	1.73

The format of Table 21 is used many times in this paper. The columns have the following meanings.

- HLE(0) is the life spent free from any disability. It is measured in years. The numbers should be compared with those for 1985 in Table 11a. The reason why the healthy life expectancies in Table 21 differ from those in Table 11a is that there were more people disabled at high ages according to the OPCS disability survey (Martin et al, 1988) than there were according to the General Household Survey (see Part I, section 2.1 for an explanation). HLE trend data (i.e. the values in Table 11a) suggest that this quantity should increase over time.
- HLE(0)/*e* is the ratio of the time spent free from any disability to the life expectancy *e*. These ratios should be compared with those in Table 11b. They should stay roughly constant over time. Note, however, that the healthy life expectancies in Table 11 are for people aged 65 and 75, so they do not show what trends may have been happening at higher ages.
- HLE(7) is the time (in years) spent free from severe disability. Here, "severe disability" means the OPCS categories worse than category 7. These values should be compared with those for 1985 in Table 12a or Table 12d. The values cannot be expected to match exactly because the definitions of severe disability are different. Using category 7 as the borderline gives a match to Table 12 that is generally better than using category 6 or 8. HLE trend data suggest that this quantity should increase over time.
- HLE(7)/*e* is the ratio of the time spent free of severe disability to the future life expectancy. It should be compared with Table 12b or Table 12e. The ratio appears to have been increasing for males. It may have been either increasing or constant for females.

- DLE(7) is the severely disabled life expectancy, i.e. total life expectancy minus HLE(7). It should be compared with Table 12c or Table 12f. The comparison shows up the mismatch between the OPCS category 7 definition of severe disability and the ADL based definition that was used for Table 12. The mismatch is poorer for females than males. Note that for females aged 65 DLE(6) = 2.55 years and DLE(8) = 1.11years, while for females aged 75 DLE(6) = 2.46 years and DLE(8) = 1.13 years. The evidence for trends relating to severely disabled life expectancy is unclear, as we discussed in Part I, section 2.3. Some data indicate that it has been falling and others indicate it has been rising.

The comments made above about trends indicated by the data for severe disabilities relate to the ADL based definition of severe disability. As we noted in Part I, section 2.3, different definitions of severe disability show different trends (see Table 15). In terms of inability to manage steps and stairs, the time spent disabled has been roughly constant for males but has lengthened for females. In terms of mobility outdoors, there appears to have been deterioration for both males and females.

The following definitions of life expectancies have been used. Let l_x be the number of lives aged x in a life table and let $l_x^{(n)}$ be the number of lives who are healthy or who have a disability of category n or less. This means that $l_x^{(0)} < l_x^{(1)} < \ldots < l_x^{(9)} < l_x^{(10)} = l_x$. Then, we define:

Complete expectation of life: $$\mathring{e}_x = \frac{1}{l_x} \cdot \left\{ \left(\sum_{y \geq x} l_y \right) - \frac{l_x}{2} \right\},$$

Complete expectation of life spent in disability categories *0-n* inclusive:

$$\mathrm{HLE}(x,n) = \frac{1}{l_x} \cdot \left\{ \left(\sum_{y \geq x} l_y^{(n)} \right) - \frac{l_x^{(n)}}{2} \right\}$$

Complete expectation of life spent in disability categories more severe than n:

$$\mathrm{DLE}(x,n) = \mathring{e}_x - \mathrm{HLE}(x,n)$$

It should be noted that in Table 21, and the tables which follow, we have used the following abbreviated expressions: "HLE(0)", "HLE(7)" and "e" for HLE(0,x), HLE(7,x) and $\mathring{e}$, respectively.

Changes in the total life expectancy directly affect the healthy life expectancies. The changes in total life expectancy between 1986 and 1996 depend only on the mortality rates in those two years and not on the transition models or trends. These life expectancies are given in Table 22.

Table 22. Life expectancy (years)

		1986	1996
Males	65	13.51	14.57
	70	10.51	11.37
	75	8.01	8.67
	80	6.01	6.49
	85	4.52	4.85
Females	65	17.30	17.93
	70	13.68	14.27
	75	10.47	11.03
	80	7.73	8.23
	85	5.59	6.04

The transition rate model has six components — total mortality, extra mortality due to disability, the probability of becoming disabled, the severity of new disabilities, the extra likelihood of disabled people deteriorating as compared with healthy people, and improvements in health. Changes in any of these can affect healthy life expectancies.

For overall mortality we have adopted the central projection of the latest GAD model. We have not explored the effect of varying this. The GAD projection assumes reductions in the rates of mortality and therefore an increase in life expectancy. If there are no changes to disability prevalence rates, this leads to increases in disabled life expectancy. It also leads to a decrease in the ratio of healthy life expectancy to total life expectancy because the disability prevalence rates are highest at the high ages, so the extra years being gained are *ceteris paribus* years of below average health.

If the extra mortality due to disability declines, perhaps as a result of medical breakthroughs or an improvement in care provision for disabled people, then people will live longer once they become disabled. If there are no other changes, in particular no reduction in the number of people becoming disabled and no increase in the probability of people recovering from their disabilities, then people will spend a greater proportion of their lives with a disability. This would cause the disabled life expectancy and especially the severely disabled life expectancy to rise.

If the opposite happened, i.e. improvements in mortality rates applied more to the total population than to the disabled population, the effect on disabled life expectancy would be to tend to reduce it. It seems unlikely that there could be a substantial widening of the difference between the mortality rates of disabled people and the mortality rates of healthy people. This is because there is not very much "room" for improvement in the mortality rates of healthy people, so that a significant widening would actually require the mortality rate for disabled people to get worse over time.

We show the effect of changes in the level of extra mortality in two models. In one (model E) the gap between the mortality of healthy people and the mortality of severely disabled people widens and in the other (model F) it narrows. The way the trends are implemented is to replace the quantity 0.20 in the equation for *ExtraMort* (x, n) (see section 4.2.2) in year t by the expression $0.20 + \Delta \cdot (t - 1986) / 10$. In model E, $\Delta = 0.02$ and in model F, $\Delta = -0.02$.

Neither of these trends could continue indefinitely. Where Δ is positive it will eventually lead to a worsening of the mortality of disabled people. Where Δ is negative it will eventually lead to the mortality of disabled people being less than that of healthy people.

We consider one other model for changes in mortality, this is called model Q. This model is quite different from the rest of our models and we discuss it separately at the end of this section.

If fewer people become disabled then this will tend to increase the healthy life expectancy and decrease the disabled life expectancy. We introduced the probability *NewDisab* in section 4.3.2 to represent the probability of becoming disabled. We can change the parameters in this function to effect changes in the probability of becoming disabled.

We use expressions such as "1 in 10" to describe the changes made to *NewDisab*. A rate of 1 in 10 means that the probabilities that apply to someone aged x in year t will also apply to someone aged $x + 1$ in year $t + 10$, to someone aged $x + 2$ in year $t + 20$ and so on (so, for example, the probability that a 71 year-old becomes disabled in 2010 is the same as the probability that a 70 year-old becomes disabled in 2000). Table 23 indicates what a rate of 1 in 10 means in terms of percentage reductions in the probability of becoming disabled. The table shows, for example, $R - 1$ (expressed as a percentage) where R is the ratio of the probability of a sixty year old becoming disabled in year t to the probability of a sixty year old becoming disabled in year $t + 1$. The probabilities in year t are determined by the parameters in Table 19.

Table 23. Annual reduction in the probability of becoming disabled implied by a "1 in 10" change in *NewDisab(x)*

Age	Males	Females
20	0.19%	0.22%
30	0.39%	0.40%
40	0.64%	0.59%
50	0.83%	0.73%
60	0.90%	0.81%
70	0.74%	0.82%
80	0.81%	0.79%
90	0.59%	0.69%
100	0.34%	0.52%

$NewDisab(x)$ is assumed to affect all of the probabilities of deterioration, including the deterioration from one disabled state to another more severely disabled state (see section 4.3.4). Hence a reduction in $NewDisab(x)$ will reduce the number of people who become severely disabled in two ways: fewer people become disabled and fewer of these deteriorate to severe categories.

Trends in the probability of becoming disabled are included in most of the models we consider. The trends are expressed as rates such as "1 in 10" in Table 24.

The severity of new disabilities is one of the components of our transition rate model. If the average severity of new disabilities reduces, this should have a greater impact on severely disabled life expectancy than on disabled life expectancies based on a lower disability threshold.

As there is some indication from the healthy life expectancy trend data that there has been an increase in the proportion of life spent free from severe disability but no increase in the proportion of life spent free of all disability (see Part I, section 2.7), this component could help the model to match the observed trends.

We include trends in the *Severity* formula in the same way as in the $NewDisab(x)$ formula. That is we introduce changes at a rate of 1 in 10, say, so that the probabilities that apply to someone aged x in year t apply also to someone aged $x + 1$ in year $t + 10$ and so on (so that the distribution of the severity of new disabilities for 71 year olds in 2010, for example, is the same as the distribution of the severity of new disabilities for 70 year olds in 2000). Trends in *Severity* are included in models G to J. The trends are given as rates, such as "1 in 10", in Table 24.

In section 4.3.4 we introduced a parameter, F, which relates the probability of deterioration for a person who is disabled to the probability that a healthy person becomes disabled. If this parameter decreases then fewer people should become severely disabled. Its effect should therefore be similar to making new disabilities less severe. Trends in this parameter are included in models K to M.

Changes to the parameter F of the deterioration-from-disabled model are incorporated in a different way to changes in *NewDisab(x)* or *Severity*. Since F ought to be at least 1, we have used the following form for changes to F:

$$F(t) = 1 + \left[F(1986) - 1\right] \times \alpha^{t-1986}.$$

The value of F in 1986 is 1.156 for males and 1.183 for females (see section 4.3.4). When $\alpha = 0.99$ the value of $F(1996)$ is 1.141 for males and 1.166 for females, and the value of F(2036) is 1.094 for males and 1.111 for females. The trends in F are given in table 24 in terms of α. (The absence of a trend means $\alpha = 1$.)

A reduction in the relative likelihood of deterioration for a disabled person might be the result of the targeting of health care resources towards people who are already disabled.

The other ingredient in the transition rate model is the probability that disabled people improve slightly. We consider one model, model P, in which the probability of a disabled person improving increases steadily from 10% per year in 1986 to 12% per year in 1996.

We have considered seventeen combinations of trends in the transition rate model. The trends which we have assumed are listed in Table 24. The trends assumed in model Q are in a different form to those used for the other models and these are discussed separately after we have commented on the changes in healthy life expectancy produced by the other models. A dash indicates that no trends are included for the component.

Table 24. Transition rate trends in the models

Model	*ExtraMort*	*NewDisab*	*Severity*	*Deteriorate*	*Improve*	To 2036?
A	—	—	—	—	—	Yes
B	—	1 in 20	—	—	—	Yes
C	—	1 in 10	—	—	—	Yes
D	—	1 in 5	—	—	—	Yes
E	+2%	1 in 10	—	—	—	No
F	–2%	1 in 10	—	—	—	No
G	—	—	1 in 10	—	—	No
H	—	1 in 10	1 in 10	—	—	No
I	—	1 in 10	1 in 5	—	—	No
J	—	1 in 10	1 in 2	—	—	No
K	—	—	—	0.99	—	Yes
L	—	1 in 20	—	0.99	—	Yes
M	—	1 in 10	—	0.99	—	Yes
N	—	1 in 5	—	0.99	—	Yes
O	—	1 in 10	—	0.97	—	No
P	—	1 in 10	—	—	+2%	No
Q	Down	Down	—	—	—	Yes

The meaning of the final column will be explained after the healthy life expectancies have been discussed.

Table 25 is in sixteen parts, one each for models A to P. Each part shows the healthy life expectancies in 1996 as a result of using the trend models A to P between 1986 and 1996. The format of each table is the same as Table 21 which relates to healthy life expectancies in 1986. By comparing Table 21 with Table 25, we can establish how the trend assumptions contained within models A to P alter the computed healthy life expectancy figures between 1986 and 1996.

Table 25A. Life expectancies in 1996, Model A

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.10	55.58%	13.48	92.50%	1.09
	70	5.44	47.87%	10.27	90.31%	1.10
	75	3.24	37.30%	7.50	86.50%	1.17
	80	1.64	25.26%	5.22	80.43%	1.27
	85	0.71	14.71%	3.47	71.52%	1.38
Females	65	9.32	51.95%	16.07	89.63%	1.86
	70	6.38	44.71%	12.41	86.97%	1.86
	75	4.00	36.31%	9.16	83.04%	1.87
	80	2.23	27.12%	6.35	77.11%	1.88
	85	1.08	17.94%	4.11	68.16%	1.92

Table 25B. Life expectancies in 1996, Model B

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.17	56.08%	13.51	92.70%	1.06
	70	5.52	48.50%	10.30	90.58%	1.07
	75	3.30	38.09%	7.54	86.89%	1.14
	80	1.68	25.90%	5.25	80.91%	1.24
	85	0.74	15.17%	3.50	72.12%	1.35
Females	65	9.40	52.42%	16.11	89.88%	1.82
	70	6.45	45.21%	12.45	87.28%	1.81
	75	4.06	36.84%	9.20	83.44%	1.83
	80	2.28	27.64%	6.39	77.63%	1.84
	85	1.11	18.40%	4.16	68.84%	1.88

Table 25C. Life expectancies in 1996, Model C

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.24	56.58%	13.54	92.90%	1.04
	70	5.58	49.10%	10.33	90.85%	1.04
	75	3.37	38.88%	7.57	87.28%	1.10
	80	1.72	26.55%	5.28	81.38%	1.21
	85	0.76	15.62%	3.53	72.70%	1.32
Females	65	9.48	52.87%	16.16	90.12%	1.77
	70	6.52	45.71%	12.49	87.58%	1.77
	75	4.12	37.37%	9.24	83.83%	1.78
	80	2.32	28.16%	6.43	78.14%	1.80
	85	1.14	18.87%	4.20	69.51%	1.84

Table 25D. Life expectancies in 1996, Model D

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.38	57.53%	13.59	93.27%	0.98
	70	5.71	50.24%	10.39	91.35%	0.98
	75	3.51	40.45%	7.64	88.03%	1.04
	80	1.81	27.86%	5.34	82.31%	1.15
	85	0.80	16.55%	3.58	73.86%	1.27
Females	65	9.64	53.76%	16.24	90.58%	1.69
	70	6.66	46.68%	12.58	88.17%	1.69
	75	4.23	38.40%	9.33	84.59%	1.70
	80	2.40	29.19%	6.51	79.13%	1.72
	85	1.19	19.79%	4.27	70.81%	1.76

Table 25E. Life expectancies in 1996, Model E

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.27	56.72%	13.56	93.09%	1.01
	70	5.60	49.27%	10.36	91.10%	1.01
	75	3.39	39.07%	7.60	87.62%	1.07
	80	1.74	26.75%	5.32	81.88%	1.18
	85	0.77	15.83%	3.56	73.41%	1.29
Females	65	9.52	53.07%	16.21	90.39%	1.72
	70	6.55	45.94%	12.54	87.92%	1.72
	75	4.15	37.63%	9.29	84.27%	1.74
	80	2.34	28.44%	6.48	78.72%	1.75
	85	1.16	19.15%	4.24	70.27%	1.79

Table 25F. Life expectancies in 1996, Model F

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.22	56.42%	13.51	92.69%	1.06
	70	5.56	48.93%	10.30	90.59%	1.07
	75	3.36	38.69%	7.54	86.92%	1.13
	80	1.71	26.33%	5.25	80.87%	1.24
	85	0.75	15.42%	3.49	71.98%	1.36
Females	65	9.44	52.67%	16.11	89.84%	1.82
	70	6.49	45.48%	12.44	87.24%	1.82
	75	4.09	37.11%	9.19	83.39%	1.83
	80	2.29	27.88%	6.38	77.56%	1.85
	85	1.12	18.58%	4.15	68.73%	1.89

Table 25G. Life expectancies in 1996, Model G

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.10	55.58%	13.49	92.57%	1.08
	70	5.44	47.87%	10.28	90.41%	1.09
	75	3.24	37.30%	7.52	86.65%	1.16
	80	1.64	25.26%	5.24	80.67%	1.25
	85	0.71	14.71%	3.49	71.88%	1.36
Females	65	9.31	51.95%	16.09	89.72%	1.84
	70	6.38	44.70%	12.42	87.09%	1.84
	75	4.00	36.30%	9.17	83.20%	1.85
	80	2.23	27.11%	6.36	77.32%	1.87
	85	1.08	17.93%	4.13	68.43%	1.91

Table 25H. Life expectancies in 1996, Model H

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.24	56.57%	13.55	92.96%	1.03
	70	5.58	49.10%	10.34	90.94%	1.03
	75	3.37	38.88%	7.58	87.42%	1.09
	80	1.72	26.54%	5.30	81.61%	1.19
	85	0.76	15.62%	3.54	73.04%	1.31
Females	65	9.48	52.87%	16.17	90.21%	1.76
	70	6.52	45.70%	12.51	87.70%	1.76
	75	4.12	37.36%	9.26	83.98%	1.77
	80	2.32	28.15%	6.45	78.35%	1.78
	85	1.14	18.86%	4.21	69.77%	1.82

Table 25I. Life expectancies in 1996, Model I

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.24	56.57%	13.55	93.02%	1.02
	70	5.58	49.10%	10.35	91.03%	1.02
	75	3.37	38.88%	7.59	87.55%	1.08
	80	1.72	26.54%	5.31	81.82%	1.18
	85	0.76	15.62%	3.56	73.38%	1.29
Females	65	9.48	52.86%	16.19	90.29%	1.74
	70	6.52	45.70%	12.53	87.81%	1.74
	75	4.12	37.35%	9.28	84.13%	1.75
	80	2.32	28.14%	6.46	78.55%	1.77
	85	1.14	18.84%	4.23	70.04%	1.81

Table 25J. Life expectancies in 1996, Model J

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.24	56.57%	13.58	93.20%	0.99
	70	5.58	49.09%	10.38	91.27%	0.99
	75	3.37	38.87%	7.63	87.93%	1.05
	80	1.72	26.54%	5.35	82.42%	1.14
	85	0.76	15.62%	3.61	74.33%	1.25
Females	65	9.47	52.84%	16.23	90.54%	1.70
	70	6.52	45.68%	12.57	88.13%	1.69
	75	4.12	37.33%	9.33	84.57%	1.70
	80	2.31	28.11%	6.51	79.14%	1.72
	85	1.14	18.82%	4.28	70.84%	1.76

Table 25K. Life expectancies in 1996, Model K

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.10	55.57%	13.50	92.65%	1.07
	70	5.44	47.86%	10.29	90.52%	1.08
	75	3.23	37.29%	7.53	86.81%	1.14
	80	1.64	25.24%	5.25	80.94%	1.24
	85	0.71	14.70%	3.51	72.32%	1.34
Females	65	9.31	51.93%	16.12	89.91%	1.81
	70	6.37	44.68%	12.46	87.34%	1.81
	75	4.00	36.28%	9.21	83.55%	1.81
	80	2.23	27.07%	6.41	77.84%	1.82
	85	1.08	17.89%	4.18	69.22%	1.86

Table 25L. Life expectancies in 1996, Model L

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.17	56.07%	13.53	92.84%	1.04
	70	5.51	48.48%	10.32	90.78%	1.05
	75	3.30	38.08%	7.56	87.19%	1.11
	80	1.68	25.88%	5.28	81.40%	1.21
	85	0.73	15.15%	3.54	72.90%	1.31
Females	65	9.39	52.39%	16.16	90.15%	1.77
	70	6.45	45.18%	12.50	87.64%	1.76
	75	4.06	36.81%	9.26	83.93%	1.77
	80	2.27	27.60%	6.45	78.34%	1.78
	85	1.11	18.35%	4.22	69.88%	1.82

Table 25M. Life expectancies in 1996, Model M

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.24	56.57%	13.56	93.03%	1.02
	70	5.58	49.09%	10.35	91.04%	1.02
	75	3.37	38.87%	7.60	87.57%	1.08
	80	1.72	26.53%	5.31	81.85%	1.18
	85	0.76	15.60%	3.56	73.47%	1.29
Females	65	9.48	52.85%	16.21	90.38%	1.72
	70	6.52	45.68%	12.54	87.93%	1.72
	75	4.12	37.33%	9.30	84.31%	1.73
	80	2.31	28.12%	6.49	78.83%	1.74
	85	1.14	18.82%	4.26	70.52%	1.78

Table 25N. Life expectancies in 1996, Model N

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.38	57.53%	13.61	93.40%	0.96
	70	5.71	50.23%	10.41	91.53%	0.96
	75	3.51	40.43%	7.66	88.30%	1.01
	80	1.81	27.85%	5.37	82.75%	1.12
	85	0.80	16.53%	3.62	74.58%	1.23
Females	65	9.63	53.73%	16.29	90.83%	1.64
	70	6.66	46.66%	12.62	88.49%	1.64
	75	4.23	38.37%	9.38	85.04%	1.65
	80	2.40	29.15%	6.57	79.78%	1.66
	85	1.19	19.74%	4.33	71.77%	1.70

Table 25O. Life expectancies in 1996, Model O

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.24	56.55%	13.59	93.27%	0.98
	70	5.58	49.07%	10.39	91.37%	0.98
	75	3.37	38.84%	7.64	88.07%	1.03
	80	1.72	26.50%	5.37	82.67%	1.13
	85	0.76	15.57%	3.63	74.79%	1.22
Females	65	9.47	52.81%	16.29	90.83%	1.64
	70	6.51	45.63%	12.63	88.52%	1.64
	75	4.11	37.27%	9.39	85.13%	1.64
	80	2.31	28.04%	6.59	80.01%	1.64
	85	1.13	18.73%	4.36	72.28%	1.67

Table 25P. Life expectancies in 1996, Model P

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.32	57.11%	13.55	93.02%	1.02
	70	5.65	49.68%	10.35	91.00%	1.02
	75	3.42	39.48%	7.59	87.48%	1.09
	80	1.76	27.13%	5.30	81.67%	1.19
	85	0.78	16.12%	3.55	73.10%	1.30
Females	65	9.55	53.26%	16.19	90.27%	1.74
	70	6.58	46.12%	12.52	87.77%	1.74
	75	4.17	37.79%	9.27	84.07%	1.76
	80	2.35	28.56%	6.46	78.45%	1.77
	85	1.16	19.20%	4.22	69.91%	1.82

Model A includes no trends in the transition rates other than in the overall mortality. This results in a reduction in the ratio of life expectancy free of any disability to total life expectancy. This contradicts the data contained within Table 11. The severely disabled life expectancy increases. This appears to contradict the healthy life expectancy data, at least where disability is defined in terms of ADLs (see Table 12c).

Models B to D all include a reduction in the probability of becoming disabled (and consequently the probability of any other deterioration). The size of the reduction is greatest for

D and least for B. With the 1 in 10 reduction in model C, the HLE(0)/*e* ratio is roughly constant as the data suggest. With the 1 in 5 reduction of model D the severely disabled life expectancy — DLE(7) — stays roughly constant.

Models E and F include changes to the extra mortality due to disability as well as the same change in $NewDisab(x)$ as model C. In model E the gap between the mortality rates for healthy and disabled people widens, causing people to live for a shorter time in the disabled categories. This pushes up healthy life expectancy relative to model C. For the opposite reasons, disabled life expectancy is increased in model F. The differences between the healthy life expectancies from models E and F compared with model C are not large.

We do not think that a larger positive value of Δ than the 0.02 used in model E is plausible because it would lead to a worsening of the mortality rates for disabled people. A negative value of Δ tends to produce results opposite to those indicated by the data as it serves to reduce the proportion of life spent without severe disability.

In models G, H, I and J trends were included in the component of the transition rate model dealing with the severity of new disabilities. These trends make hardly any impact on the healthy life expectancies. Model G produces almost the same results as model A (which had no trends). Model H produces almost the same results as model C (which had the same trend in $NewDisab(x)$ as model H but no trend in $Severity(x,n)$). Neither model I nor model J causes much change in the life expectancies shown. This is despite the fact that model J is very extreme. A rate of change of 1 in 2 would mean that the severities which applied to the new disabilities of someone aged 75, say, in 1996 would be the same as those which had applied to someone aged 70 just ten years earlier.

In models K, L, M, N and O trends were included in the relative likelihood of disabled people deteriorating compared with the probability of a healthy person becoming disabled. In the first four models (K to N) the trend parameter α is set at 0.99. These four models are the same as A, B, C and D (in order) except for the inclusion of this trend. The effect of including the trend is not very large. The effect of including a trend with $\alpha = 0.99$ lies between that of models I and J, which had trends in $Severity(x,n)$ of 1 in 5 and 1 in 2 respectively. The combination of $\alpha = 0.99$ and a change of 1 in 10 in $NewDisab(x)$ is weaker than just having a change of 1 in 5 in $NewDisab(x)$ (i.e. model M produces results between those of models C and D).

Choosing a lower value of α in model O does lead to a reduction in DLE(7). This is the only model to produce this feature. This illustrates just how difficult it is to find reasonable models which can lead to a reduction in the time spent severely disabled when there is an ageing of the population.

Model P is the one that includes an increasing probability that disabled people will become less disabled. The effect is not great. The reduction in the time spent severely disabled is lower than that caused by slowing down the deterioration of disabled people to the extent that was done in model M. The increase in the ratio of life free of any disability to the total life expectancy is, however, larger than in model M.

We decided to proceed with projecting eight of the first sixteen models up to 2036. The choices are indicated in the final column of Table 24. The ones which we did not proceed with were:

- Models E and F — because using a negative Δ tends to produce changes in healthy life expectancy opposite to those observed, and a positive Δ has implausible implications for the mortality rates of disabled people.
- Models G, H, I and J — because changing $Severity(x,n)$ has a similar effect to slowing down the deterioration of disabled people. There is no point in continuing with both types of model and we think that slowing down deterioration may be more likely to happen than changing the severity of new disabilities.
- Model O — because setting the parameter Δ to 0.97 implies a rapid change in the likelihood of deterioration from disabled states. The model was included at the initial stage in an attempt to find one set of parameters that actually leads to a decrease in the time spent severely disabled.
- Model P — because increasing the rate of improvement has little impact on the time spent severely disabled (the increase from 10% to 12% over 10 years would have led to a doubling of the rate of improvement over the full projection, so it is a strong assumption).

The final model that we have considered, model Q, includes trends in a different form to those used in any of the other models. In this model we have set the trends to match, as far as is possible, the assumptions used in the central model of Nuttall et al (1994). In their central model, two trend assumptions were chosen:

- The probability of becoming disabled reduces by 0.5% per year
- Improvements in mortality from the disabled states were 50% more rapid than the improvements in mortality incorporated in the GAD population projection model.

It is straightforward to incorporate the first of these two trends. The change of 0.5% per year in the probability of becoming disabled is somewhat lower than that implied by the "1 in 10" rate of change — see Table 23.

However, there is one complication with introducing this change. In our model, all deterioration probabilities are linked to the probability of becoming disabled. Nuttall et al (1994) did not include transitions between disabled states, so the authors did not have to decide whether there should be any changes in these transitions to accompany the change in the probability of becoming disabled. There are two options for treating these other types of deterioration:

Option 1: to reduce the other transition probabilities at the same rate as the reductions in the probability of becoming disabled.

Option 2: to make no changes to the other transition probabilities.

The implication of using Option 2 is that, in the future, someone in disability category 1 would be less likely to become severely disabled than would someone who is healthy. This is clearly counter-intuitive. Consequently, we have chosen Option 1.

There are several complications introduced by trying to implement Nuttall et al's trend in the improvement in mortality for disabled people.

One complication is that population mortality does not improve at all ages in every year according to the latest GAD central population projection. There are several ages and years when the mortality rates are projected to become worse. Some of this is due to AIDS (this is at low

ages and is probably not important). But this worsening of experience also occurs sporadically at the oldest ages. It is not clear what would be meant by an "improvement" of 1.5 times that of an adverse trend. For reasons of convenience, we have just multiplied all changes in q_x by 1.5 whether they were positive or negative.

In implementing projection Model Q, there are some situations where it is impossible to include all the constraints being imposed on the projected mortality experience: the GAD model constrains the total number of deaths; the assumption for the improvements in the mortality of disabled people fixes the proportion of disabled people who die; common sense requires that the mortality rate for disabled people cannot be lower than that of healthy people; and no mortality rate can be negative. Problems can arise in two ways. On some occasions, this number of disabled deaths is higher than the total number of deaths. On other occasions, the number of deaths required of healthy people to make up the difference between the two figures is very high. It can mean that the mortality rate of healthy people has to be higher than that of disabled people (and sometimes over 100%).

In order to avoid these problems, we have imposed the following constraints. We require always that the GAD population is matched exactly. We also require that the mortality rate from healthy is never negative and never greater than the mortality rate from disabled. These restrictions means that it is necessary sometimes to stray away from the Nuttall et al (1994) trend.

In our model, the mortality of people with low levels of disability (up to category 5) is no different from the mortality of healthy people. Hence we cannot have the mortality of these people improving at a rate in excess of the improvement in mortality from healthy.

Table 26 shows the life expectancies in 1996 according to the model just described, Model Q.

Table 26. Life expectancies in 1996, Model Q

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.13	55.76%	13.44	92.27%	1.13
	70	5.47	48.09%	10.24	90.05%	1.13
	75	3.26	37.57%	7.48	86.18%	1.20
	80	1.66	25.52%	5.20	80.03%	1.30
	85	0.72	14.88%	3.45	71.04%	1.41
Females	65	9.36	52.23%	16.05	89.52%	1.88
	70	6.42	45.00%	12.39	86.84%	1.88
	75	4.03	36.56%	9.13	82.83%	1.89
	80	2.24	27.26%	6.32	76.76%	1.91
	85	1.08	17.92%	4.08	67.56%	1.96

This model produces one of the lowest ratios of HLE(0)/*e* of any of the models considered and the highest values of any of the models for DLE(7). The trends are more "pessimistic" than no trend at all (i.e. model A) in the sense that projections using these trends will lead to a large proportion of lives being severely disabled. The ratio of life expectancy free of any disability to the total life expectancy would have fallen since 1986 according to these trends. This appears to contradict the data.

6. *Conclusion*

In Part II of this paper, we have described, in detail, the multiple state model which we have developed to project the number of people with disabilities in the UK over the next 40 years. We have decided upon nine alternative sets of trend assumptions to be incorporated within the model to demonstrate a range of plausible projected numbers. We discuss the results obtained from each set of assumptions in Part III.

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