Multiple State Models, Simulation and Insurer Insolvency

by

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We note that all models abstract to some extent from the real world and, as noted by Daykin et al (1994) and other authors, models are subject to three broad types of error or risk:

model error	- because models are not known with certainty and usually are only approximations to the real world
parameter error	- past observation data are limited in quantity and so parameters are not known with certainty.
process (or stochastic) error	- our target quantities will be subject to random fluctuations about the mean, even when the model and parameters are correct.

Our focus will be on the third component: process error. The pooling of insured units in a portfolio is a critical part of the insurance process and, as demonstrated by Cummins (1991), leads to a reduction in the relative level of variability in the portfolio i.e. the risk per unit. This is a statement about process error – the other two types of error are not controllable in a comparable way.

Before focusing on process error, we will make some comments at this stage about the 2 other types of error. Modelling a portfolio of disability insurance policies provides us with an example where model error could be important – there are, for example, at least 3 established methodologies for pricing and reserving of disability insurance products in use in the UK i.e. Manchester Unity, inception rates plus disability annuity approach and the multiple state model approach, although it is well argued that the multiple state model provides a framework within which the different models can be formulated (CMI Committee (1991), Haberman and Pitacco (1999)). Practitioners in the UK have tended to abandon the Manchester Unity approach because of the model error associated with it and now use one of the other two approaches. We would expect that the parameter error in disability insurance would be more significant than in life insurance because:

a) the underlying data are less reliable
b) the definition of sickness and of a claim are less clear cut, and are influenced by policy conditions;
c) sickness claims are often linked with economic conditions and there is scope for a moral hazard.

One of our objectives will be to measure the process error for a portfolio of independent disability insurance policies in a multiple state modelling context. A second objective will be to compare the process error in a model with random multiple state transitions and deterministic interest assumptions with that in a model with both random multiple state transitions and random investment returns. That is we will be able to compare, in a specific case, the magnitudes of the relative variability due to demographic risks and due to demographic and economic risks combined.

In recent years, stochastic modelling in the fields of life insurance and defined benefit pension funds has been an area of considerable development. We can identify the following themes:

a) demographic risks in life insurance for a single policy: Pollard and Pollard (1969);
b) demographic and economic risks in life insurance for a single policy: Pollard (1971, 1976), Boyle (1976), Panjer and Bellhouse (1980), Bellhouse and Panjer (1981), Giaccotto (1986), Dhaene (1989, 1992), Beekman and Fuelling (1990, 1991, 1993);

c) economic risks in an annuity portfolio and limiting distributions: Dufresne (1990), Beekman and Fuelling (1990, 1991, 1993), De Schepper and Goovaerts (1992), De Schepper et al (1994), Vanneste et al (1997), Milevsky (1997);

d) demographic and economic risks in a life insurance portfolio: Waters (1978), Frees (1990), Parker (1994a, 1994b, 1994c, 1996, 1997), Marceau and Gaillardetz (1999).

e) economic risks in a pension plan: Dufresne (1988, 1989, 1994), Haberman (1992, 1993a, 1993b, 1994, 1995, 1997), Zimbidis and Haberman (1993), Haberman and Sung (1994), Gerrard and Haberman (1996), Haberman and Wong (1997),Cairns and Parker (1997), Bédard (1999), Owadally and Haberman (1999a, 1999b), Haberman et al (2000);

f) demographic and economic risks in a pension plan: O'Brien (1986, 1987), Mandl and Mazurová (1996), Bilodeau (2000).

In life insurance, it is usual to consider the process risk as being relatively insignificant because claim amounts are fixed and portfolios tend to be large so that the law of large numbers and pooling lead to a reduction in process risk. However, Marceau and Gaillardetz (1999) have demonstrated that this effect operates differentially for different types of life insurance policy with the mortality process risk being more significant for temporary insurances than for endowment insurances. In disability insurance, there is variability in terms of claims incidence and size (i.e. measured by duration of time spent sick or disabled) and portfolios tend to be smaller than in life insurance, so that we might expect that liability process risk may not be negligible. This will be investigated in subsequent sections.

2. METHODOLOGY

We construct a simple asset-liability model of an insurer with PHI business only using a multiple state model. We simulate paths for individual policyholders occupying the healthy and sick states and moving between states and simulate investment returns using the widely used Wilkie model. As overall measures of risk, we will consider the probability of ruin for the portfolio and the levels of risk-based capital or risk-loaded premium to achieve a pre-specified probability of ruin. We will focus on two cases:

a) stochastic sickness experience and deterministic investment returns

b) stochastic sickness and investment experience

and make comparisons.

3. MODELS AND ASSUMPTIONS

3.1 Multiple State Model

To describe the transitions, we have used the three-state healthy-sick-dead model, with the states labelled 1, 2, 3 respectively. We consider an insured aged x at entry and let $S(x+t)$ denote the random state occupied by the insured at age $x+t$, $t \geq 0$. We define the conditional probabilities

$$ {}_t p_x^{ij} = \Pr[S(x+t) = j | S(x) = i] \tag{1} $$

for $i, j \in (1,2,3)$, assuming the Markov property. For the specific applications here, we will consider new policyholders who are healthy at entry (i.e. $S(x)$=1).

Then we define the transition intensities

$$ \mu_x^{ij} = \lim_{t \to 0} \left[\frac{{}_t p_x^{ij}}{t} \right] \tag{2} $$

For the functional form of the transition intensities we have adopted the graduated values obtained by CMI Committee (1991) from fitting to the 1975-78 experience, as these have wide currency in the UK (although a less complex set of graduations has been proposed by Renshaw and Haberman (1995)).

For the transition intensities from state 2 (sick), there is strong evidence of dependence on sickness duration, z, (i.e. length of stay in state 2) leading to a semi-Markov framework. However, we have ignored this level of complexity and used the corresponding graduated values corresponding to z = 17 weeks.

Our intention is to consider a portfolio of policies with a deferred period of 13 weeks. Where appropriate we have chosen the graduated transition intensities that are consistent with this selection. One of the features of the 1975-78 data (and other comparable data sets – see Renshaw and Haberman (1995)) is the reduced recovery rates during the first 4 weeks of claim. The choice of z = 17 weeks for the 13 week deferred week policies being considered here avoids this problem.

Thus, the functional forms are as follows:

$$ \mu_x^{12} = \exp(a_0 + a_1 x + a_2 x^2 + a_3 x^3) \tag{3} $$

where $a_0 = -2.722, a_1 = 0.1290, a_2 = -4.240 \times 10^{-3}$ and $a_3 = 3.888 \times 10^{-5}$

$$ \mu_x^{13} = b_0 + b_1 y + \exp(b_2 + b_3 y) \tag{4} $$

where $\quad y = \dfrac{x-70}{50}, b_0 = -4.652 \times 10^{-3},\ b_1 = -4.525 \times 10^{-3}, b_2 = -3.986, b_3 = 3.185$

$$ \mu_x^{21} = c_0 + c_1(x - c_2) \tag{5} $$

where $\quad c_0 = 3.086, c_1 = -0.0927, c_2 = 50.326$

$$ \mu_x^{23} = (d_0 + d_1(x-d_2) + d_3(x-d_2)^2)\, d_4 + d_5 \exp(d_6 x) \tag{6} $$

where $d_0 = 0.238,\ d_1 = -4.819\times10^{-3},\ d_2 = 0.326, d_3 = 9.587\times10^{-5},\ d_4 = 0.537$
$d_5 = 7.221\times10^{-3},\ d_6 = 2.435\times10^{-2}$.

3.2 Simulation and Thinning

If all policyholders are healthy at entry at age x say (i.e. in state 1), we will want to be able to simulate the path taken by each individual over the term of the policy. A convenient way to proceed is "thinning" as described by Ross (1990) and utilised by Jones (1997) for modelling continuing care retirement communities.
For a policyholder in state 1 at age x at time 0 and a policy with term n years we consider

$$\alpha = \max_{x \le t \le x+n} \left(\mu_t^{12} + \mu_t^{13}\right) \tag{7}$$

where the term in parentheses represents the transition intensity out of state 1 at age u, and α is chosen to be greater than or equal to this overall intensity throughout the age range $[x, x+n]$. We can generate event times of a Poisson process with rate α since the intervals between successive event times would be exponentially distributed with parameter α^{-1}. There are too many such event times because of the definition of α in (7) and so we "thin" out the jumps by simulating a random number that is uniform on (0,1) and accepting the first jump t_1 say if the random number is no greater than the ratio:

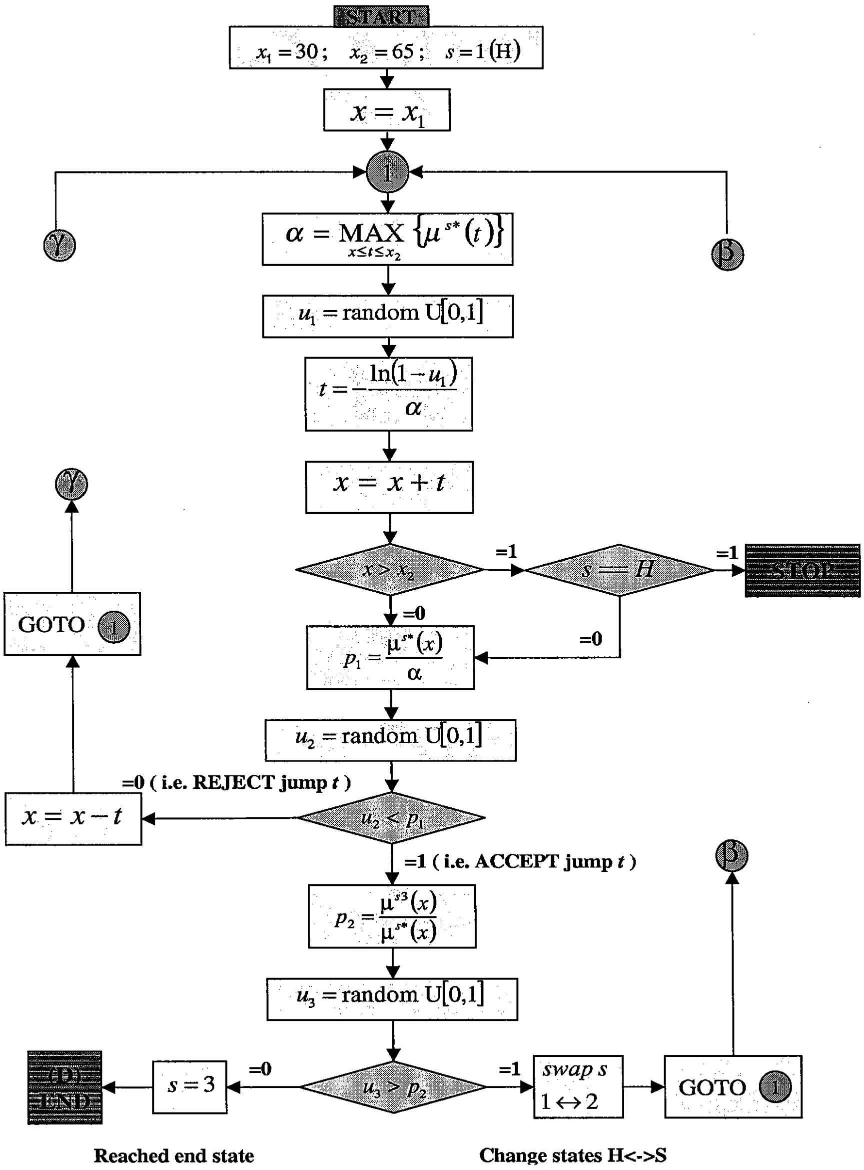
$$\frac{\mu_{x+t_1}^{12} + \mu_{x+t_1}^{13}}{\alpha}.$$

If t_1^* is the first accepted jump, then we need to establish whether the insured jumps to state 2 or state 3. This is effected by simulating a third random number that is uniform on (0,1) and if this random number is no greater than the ratio

$$\frac{\mu_{x+t_1^*}^{12}}{\mu_{x+t_1^*}^{12} + \mu_{x+t_1^*}^{13}}.$$

we accept the jump as being to the sick state (i.e. state 2).
Once the simulation has led to our deciding on the time and state of the next transition, we move forward to that next transition and determine α (in relation to age $x + t_1^*$) and proceed iteratively until we have a transition that takes the insured beyond time n or until death. The simulation and thinning process is summarised in the flow chart shown in Figure 1.
We note that there is a choice regarding a simulated jump time, t, which takes the path beyond time n: we can accept this either with or without testing it against a random number as in the thinning process. We have chosen the former on grounds of simplicity and have checked for certain cases whether the latter approach would markedly affect the results. Our conclusion is that there is little difference between the two approaches.

THINNING FLOW CHART



<u>Figure 1:</u> Logical Scheme of Simulation with Thinning.

3.3 Investment Returns Model

In order to generate random investment returns, we will use the widely accepted Wilkie model for equity returns and returns on index-linked government bonds. We use the structure of the model as in Wilkie (1995) and the initial parameter settings recommended therein. We consider an asset portfolio comprising 50% equities and 50% index-linked government bonds, with annual rebalancing so that the asset allocation strategy is static.
For the simulations where we investigate the effect of a deterministic interest rate assumption, we have used a fixed interest rate of 10.7% per annum. This is obtained from running 1,000 simulations of the Wilkie model over an appropriate time horizon and calculating the equivalent mean annual compounding interest rate. Minor adjustments have been made to allow for the effect of different time horizons for the projections, corresponding to different ages at entry (see later).

3.4 Policy Design

We consider a policy with terminal age 65 and a range of entry ages: 30, 40, 50 and 60. Premiums are payable while the policyholder is healthy or while the policyholder is sick but for a period less than the deferred period (i.e. a no claim exclusion period). Benefits are payable as an annuity while the policyholder is sick for a duration longer than the deferred period. Expenses are incorporated. Inflation on both premiums and expenses is also allowed for. Taxation is ignored. A fixed gross premium valuation basis is used throughout, together with a fixed risk discount rate to measure profitability.
The detailed assumptions are as follows and are intended to be representative of PHI contracts at the time of writing:

Deferred period:		13 weeks
Expenses:	Initial	£200 at inception of policy
	Renewal	£25 pa if the policyholder is healthy
		£95 pa if the policyholder is sick
	Claim	£200 at inception of the claim
Inflation:		of benefits and premiums 3% pa (fixed).
		of expenses 4% pa (fixed)
Benefits:		£8,000pa while policyholder is sick, but subject to the requirements of the deferred period.
Ages at entry:		30, 40, 50 or 60
Termination age:		65
Valuation basis:		gross premium prospective policy values using multiple state annuity values (see later). Valuation rate of interest: 6.5 %pa, otherwise the assumptions follow the premium basis.
Risk discount rate:		15% pa.

3.5 Cash Flow Model

For any policy year (t, t+1), the cash flow is assumed to occur mid-way through the year and is defined to be the difference between the simulated income and outgo i.e. for simulation j, the cash flow per cohort is:

$$CF_t^j = P_t^j - W_t^j - Y_t^j - Z_t^j - C_t^j \tag{8}$$

where for the policy year (t,t+1) and simulation j:

P_t^j = total premiums paid by all policyholders

W_t^j = total premium-related expenses

Y_t^j = total initial claim expenses (payable at the commencement of a claim)

Z_t^j = total regular claim expenses

C_t^j = total claim payments payable to policyholders who are sick and for whom the duration of sickness exceeds the deferred period of the policy.

In equation (8), the terms P_t^j and W_t^j are proportional to the simulated time spent in the healthy state or in the sick state if less than the deferred period. The term Y_t^j is proportional to the simulated number of new claims initiated in year (t, t+1).
The policy funds then accumulate as follows

$$A_{t+1}^j = A_t^j(1+r_t^j) + CF_t^j(1+r_t^j)^{1/2} \tag{9}$$

where A_t^j represents the accumulated assets at time t for simulation j and r_t^j is the simulated rate of return on investment for year (t, t+1). We assume that $A_o^j = 0$ so that there are zero initial assets per policy.
For the case where rates of return on investment are assumed to be non-stochastic, then we replace r_t^j by r_t (and in this case, as noted in section 3.3, we let $r_t = 0.107$ for all t).

3.6 Reserves and Profit Model

The deterministic reserves at time t per policy for a policyholder aged x at entry who is in state k at time t are defined as follows:

$$\begin{aligned} {}^kV_t = {} & c_t(1+e_1)^{1/2}\,\overline{a}_{t:\overline{N-t}|}^{k,2,1} + w_t(1+e_2)^{1/2}\,\overline{a}_{t:\overline{N-t}|}^{k,1,2} \\ & + y_t(1+e_2)^{1/2}\,\overline{a}_{t:\overline{N-t}|}^{k,2,2} - p_t(1+e_1)^{1/2}\,\overline{a}_{t:\overline{N-t}|}^{k,1,1} \end{aligned} \tag{10}$$

where p_t is the premium payable in year (t, t+1)

c_t is the claim payment for year $(t, t+1)$

y_t is the claim expense payment for year $(t,t+1)$

w_t is the premium-related expense payment for year $(t,t+1)$

e_1 is the assumed rate of inflation for premiums and claims

e_2 is the assumed rate of inflation for expenses

and the annuity functions are defined by

$$\bar{a}_{t:\overline{N-t}|}^{k,m,n} = \sum_{s=0}^{N-1-x-t} \left[l_{x+t+s+\frac{1}{2}}^{km} \left(\frac{1+e_n}{1+i} \right)^{s+\frac{1}{2}} \right] \quad \text{where } (k,m,n) \in \{1,2\} \tag{11}$$

and N is the termination age (65), i is the valuation rate of interest and $l_{x+t+s+\frac{1}{2}}^{km}$ are taken from the multiple state life table derived from the multiple state model (see Haberman and Pitacco (1999)) and represent the proportion of lives in state k at age $x+t$ who are in state m at age $x+t+s+\frac{1}{2}$. The l functions satisfy the recurrence relation

$$l_{x+t}^{km} = l_{x+t-1}^{k1} \cdot {}_1p_{x+t-1}^{1m} + l_{x+t-1}^{k2} \cdot {}_1p_{x+t-1}^{2m} \tag{12}$$

where the ${}_1p_y^{lm}$ are the 1 year conditional transition probabilities from the multiple state model. For the l functions at the midpoint of each year, we use the simple approximation:

$$l_{x+t+\frac{1}{2}}^{km} = \tfrac{1}{2}(l_{x+t}^{km} + l_{x+t+1}^{km}).$$

For the calculation of the ${}_1p_y^{lm}$ probabilities, we use the matrix method advocated by Cox and Miller (1965) and Haberman and Pitacco (1999) based on the assumption of constant transition intensities over each one month of age.

The profit for year t is then calculated from the cash flow and reserves. Letting TV_t^j be the total reserves at time t (for policyholders in states 1 and 2) for simulation j, then we define the profits for year $(t, t+1)$, accumulated to the end of the year, for simulation j as follows:

$$PRO_{t+1}^j = CF_t^j (1+r_t^j)^{\frac{1}{2}} + TV_t^j (1+r_t^j) - TV_{t+1}^j. \tag{13}$$

Then to assess profitability, we consider measures based on the net present value of profits and premiums where, for simulation j,

$$NPV\ PROFIT^j = \sum_{t=0}^{N-x} \frac{PRO_{t+1}^j}{(1+r_D)^{t+1}}$$

and

$$NPV\ PREM^{j} = \sum_{t=0}^{N-x} \frac{P_t^j}{(1+r_D)^{t+½}}$$

with r_D the chosen risk discount rate. This follows the now widely-used profit testing methodology: see for example Booth et al (1999).

3.7 Calculation of a "Break-Even" Premium

In order to calculate the "break-even" premium per policy, we consider a portfolio of 1,000 identical policies and run 500 simulations of the residual policy funds (using the stochastic asset model) at the termination of the contracts (i.e. at age 65) for a given initial premium. We then use iteration to derive a value for the initial premium such that

$$\Pr\,(A_{N-x} < 0) = 0.5 \qquad (14)$$

where A_S represents the accumulated policy assets at time *s*.

The values that are obtained are as follows:

Age at Entry	Initial Premium (£)
30	220
40	272
50	336
60	462

This approach is different from the conventional approach based on the equivalence principle where we equate expected present values of policy income and outgo. In future work, we will investigate the effect of different choices for the calculation method for the premium.
The implications of using (14) are that

a) the premium does not include any margin for profit or adverse experience
b) the company expects to "break even" at the termination of the contracts, assuming that all profits are retained internally
c) any projection that leads to ruin (of the portfolio) can be attributed to "process error" only.

4. SIMULATION RESULTS

Because of the limited space available, we will present only a selection of the results.
Figure 2 shows the simulated distribution of policy funds/assets for a portfolio of *n* policies issued at age 30 for the respective cases *n*=1,000 and 10,000, based on 500 simulations. These figures summarise the distribution at each age through the use of Box-plots with the 25th, 50th and 75th percentiles clearly identified. Figure 2 is based on the deterministic investment rate assumption. . It is clear that as *n* increases, the distributions become more compact.

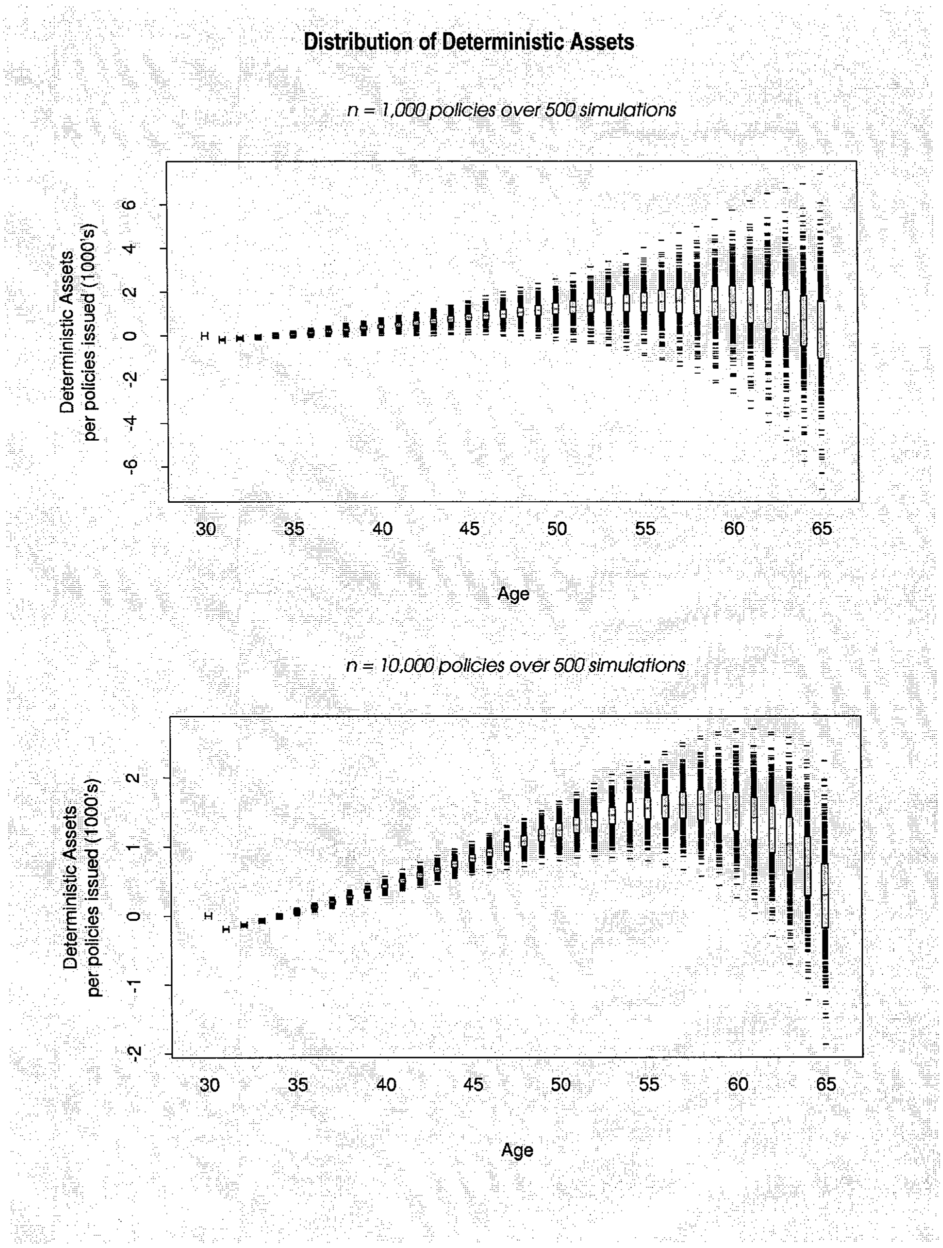


Figure 2: Deterministic Assets Distribution over age for 1,000 and 10,000 policies assumed at outset.

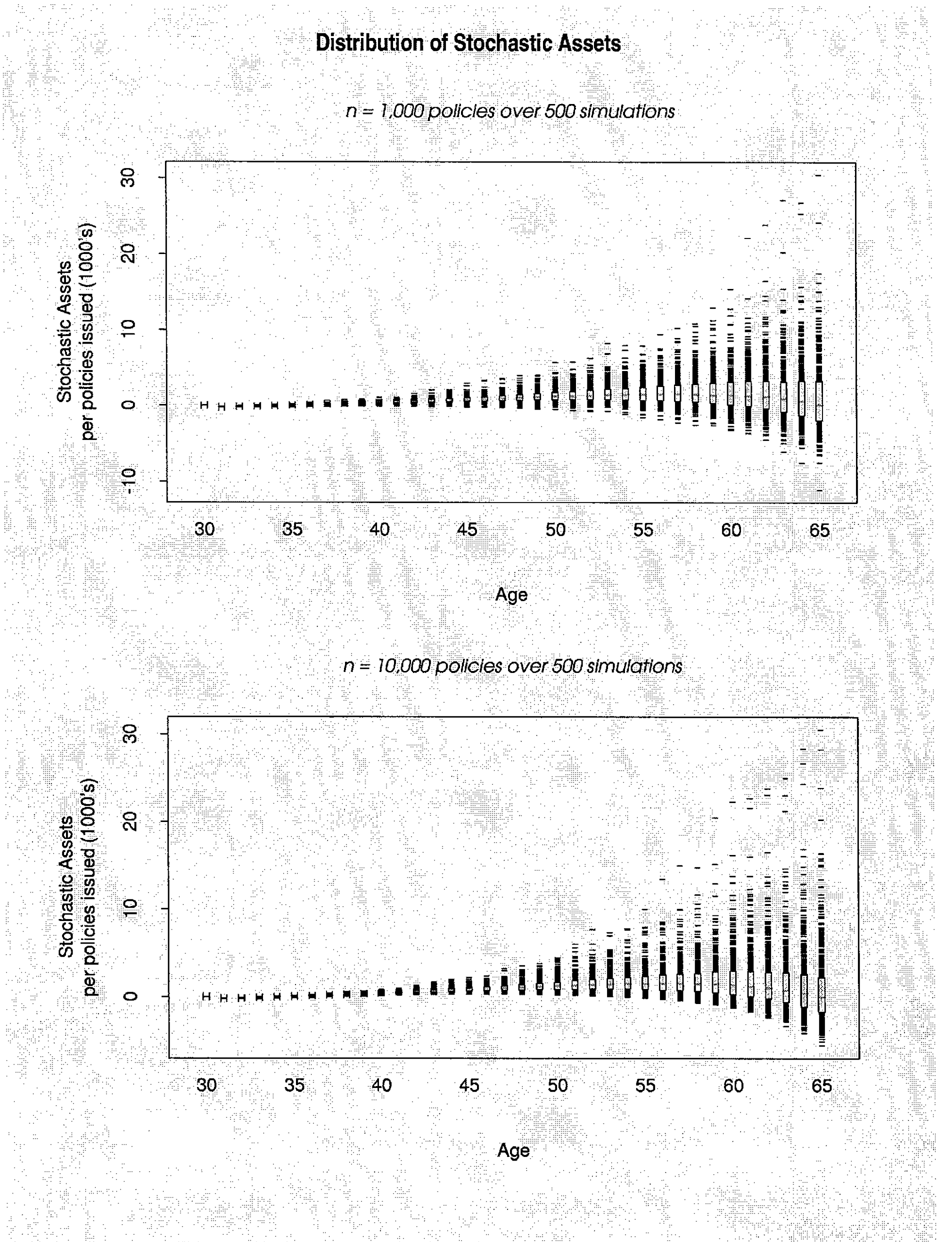


Figure 3: Stochastic Assets Distribution over age for 1,000 and 10,000 policies assumed at outset.

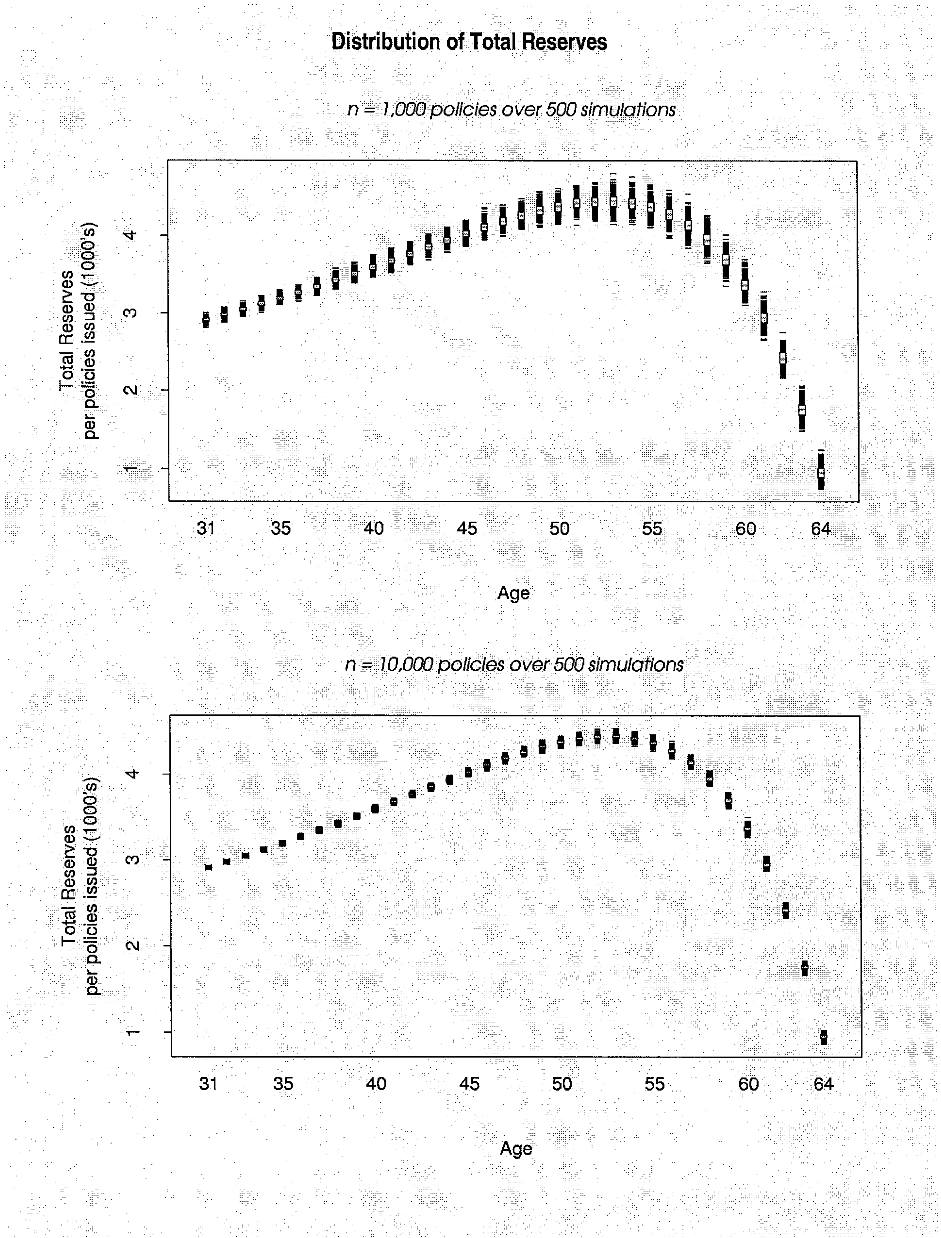


Figure 4: Distribution of Reserves over age for 1,000 and 10,000 policies assumed at outset.

Figure 3 repeats the presentation but allows for stochastic investment returns and now the distributions are much wider – as we would anticipate because of the extra source of variability present.

Figure 4 illustrates the progression of the simulated distribution of the reserves for the portfolio. We note the characteristic shape, with the distribution centred on a positive level at age 31 and progressing to a peak and then to zero by age 65 (as for a temporary life insurance portfolio). The reserves begin with a valuation strain because the premium and reserving assumptions are not identical.

Figures 5-8 illustrate the simulated distributions of the residual assets for portfolios of lives with initial ages 30, 40, 50 and 60 respectively, with the rates of investment returns generated from the Wilkie model as described earlier. Figure 9 corresponds to Figure 5 for the case of deterministic investment returns. The following features are noteworthy.

a) increasing the portfolio size from n=1,000 to n=10,000 when the asset model is deterministic leads to a reduction in the standard deviation of the residual assets by a factor of $\sqrt{10}$ (as we would expect from the pooling of risks), so that the distribution is sharper (Figure 9);

b) property a) is not exhibited in Figures 5-8. This is because pooling of risks does not reduce the impact of investment return variability which affects all policies in the portfolio simultaneously;

c) increasing the policy term, i.e. reducing the age at entry, leads to an increase in the skewness of the distribution. This is because the accumulation effect becomes stronger for longer terms and corresponds to the approximate lognormal character of the results from the Wilkie model. (Figures 5-8);

d) the probability of ruin estimates derived from the simulations are close to 0.5 for Figures 5-8. Given the criterion for determining the premium, we would anticipate such a result. Figure 9 leads to a lower probability of ruin (particularly for the case n=10,000), because these simulations use a deterministic asset model and lead to fewer unfavourable results.

Figure 10 presents the simulated distribution of the net present value of profits for a portfolio of policies with age at entry 30 based on the model with stochastic asset returns. Increasing the size of the portfolio makes little difference to the standard deviation of the distribution because of the dominant effect of investment return variability. The means in both cases are negative, because of the choice of r_D =15% per annum in computing the present values.

We now consider the effects of changing the initial assets and the premium per policy. For the case of a portfolio of policies with age at entry and n=1,000, Figure 11 presents the effect on the probability of ruin of increasing the initial assets from zero in the upper panel and the effect of moving from the break-even initial premium of £220 in the lower panel. In each case, the curves of the probability of ruin for the full stochastic model and for the model with deterministic asset returns are presented. We note the following features:

a) initial assets of zero correspond approximately to a probability of ruin of 0.5 for the stochastic case (and slightly lower for the deterministic case for the reason cited earlier);

b) increasing the initial assets per policy leads to a lower probability of ruin. This is more marked for the deterministic case where there is less overall variability;

c) an initial premium of £220 corresponds approximately to a probability of ruin of 0.5;

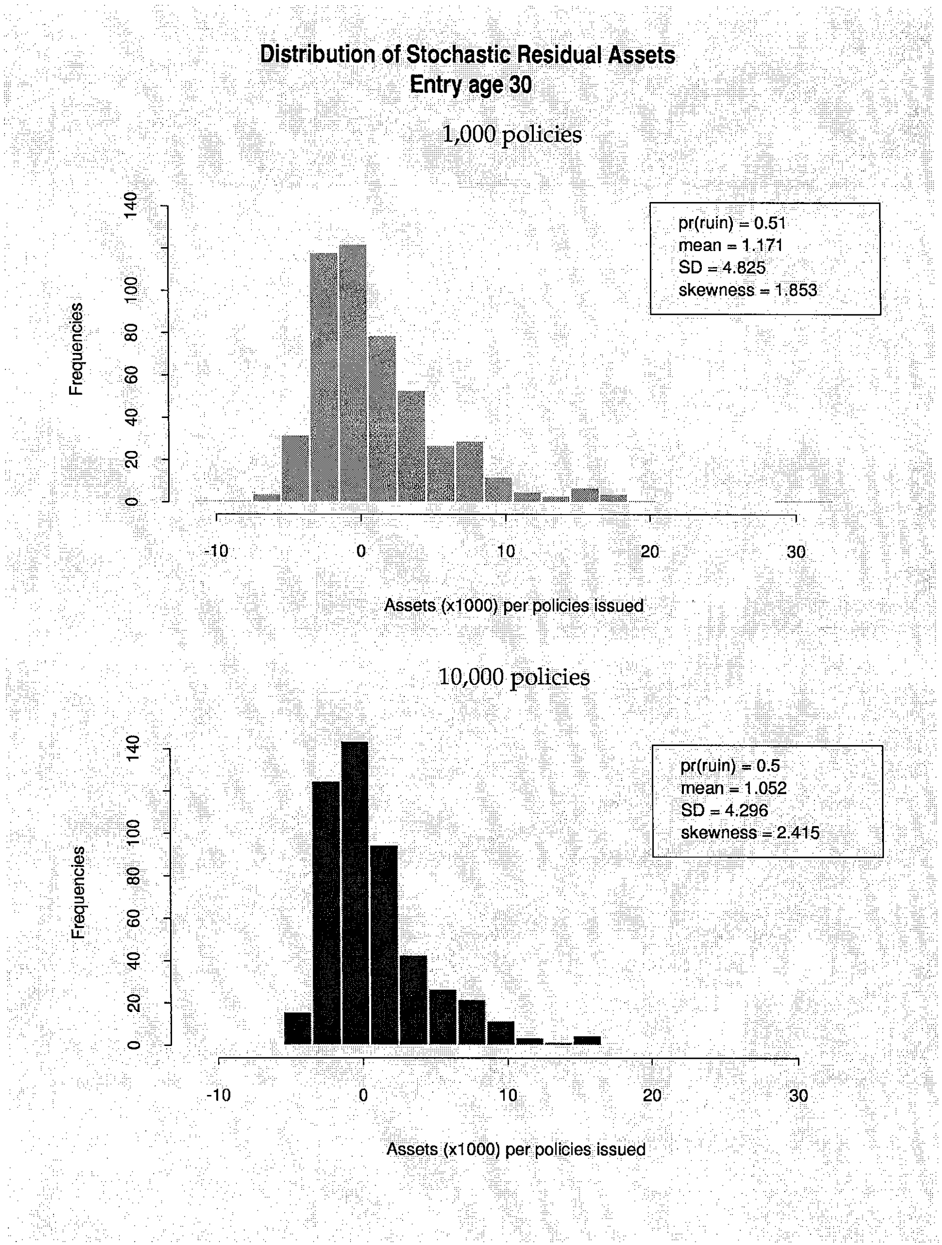


Figure 5: Stochastic Residual Assets Distribution corresponding to entry age 30.

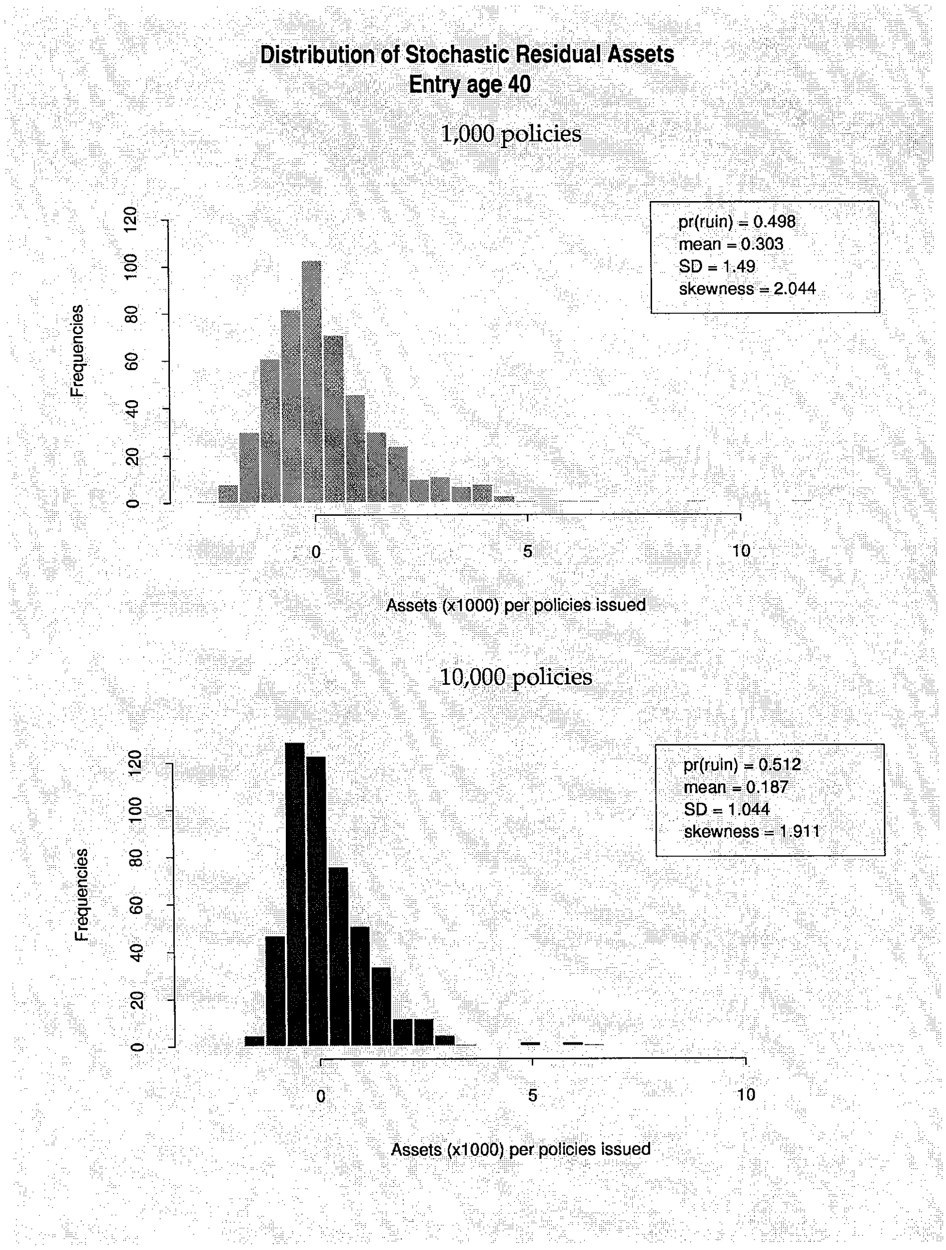


Figure 6: Stochastic Residual Assets Distribution corresponding to entry age 40.

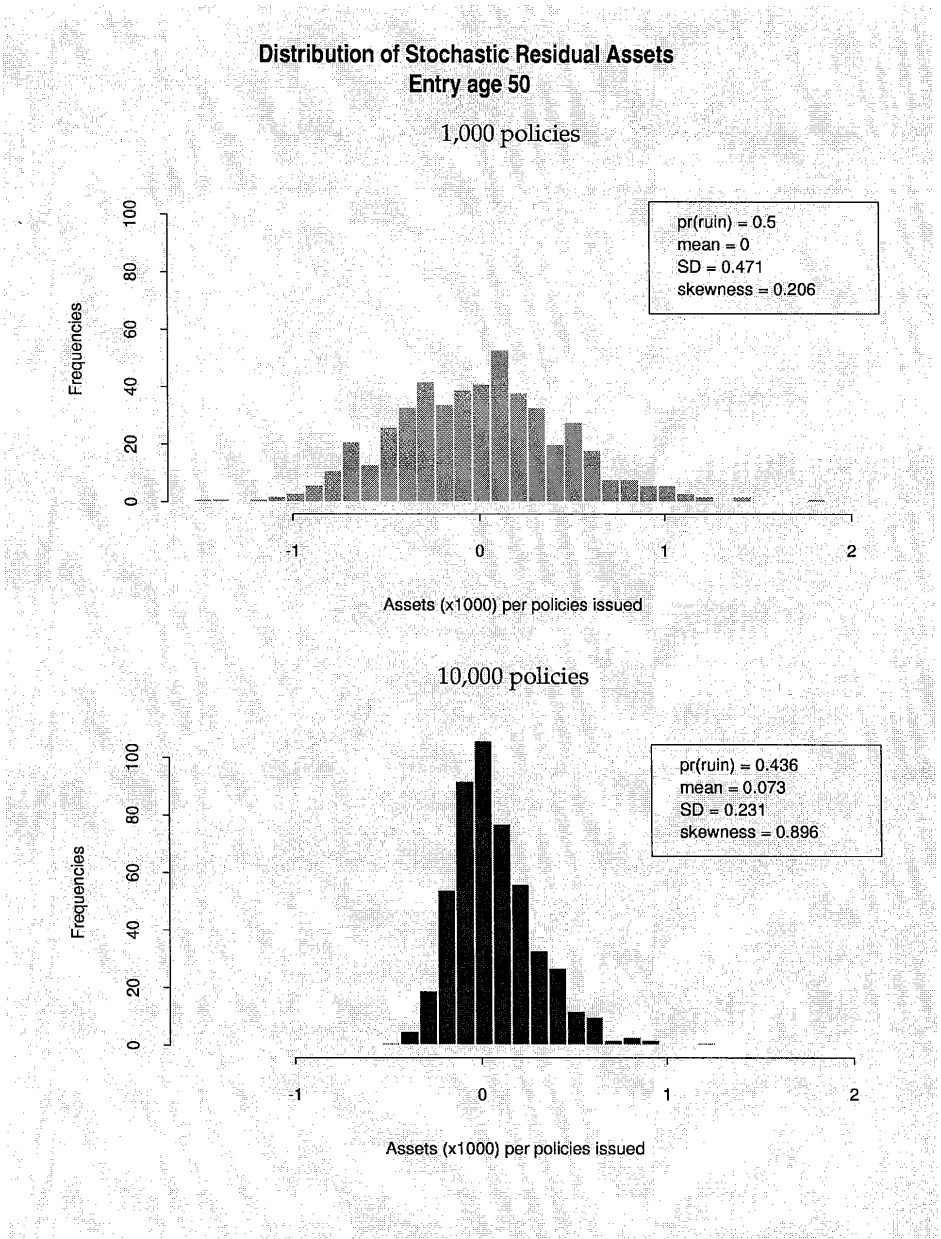


Figure 7: Stochastic Residual Assets Distribution corresponding to entry age 50.

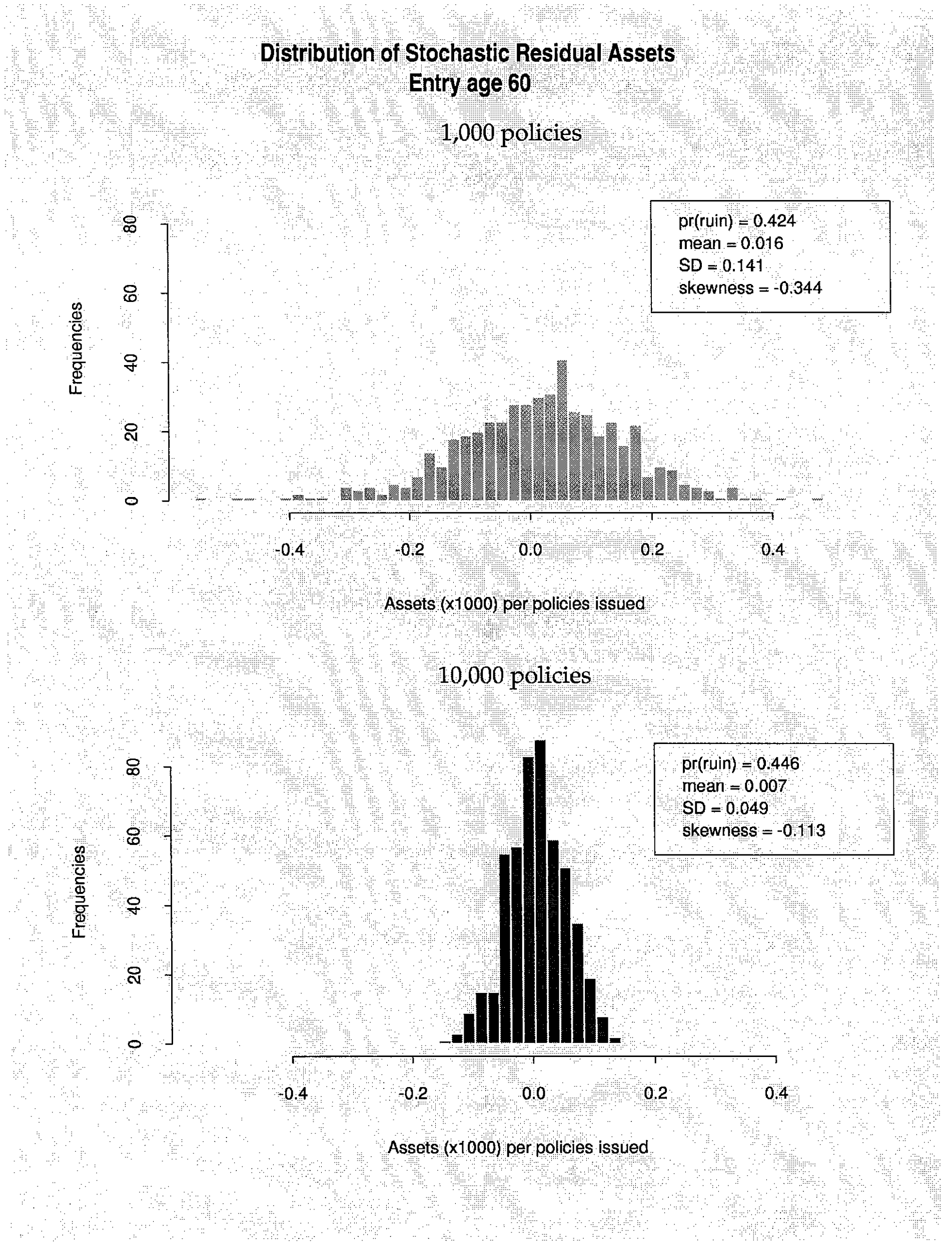


Figure 8: Stochastic Residual Assets Distribution corresponding to entry age 60.

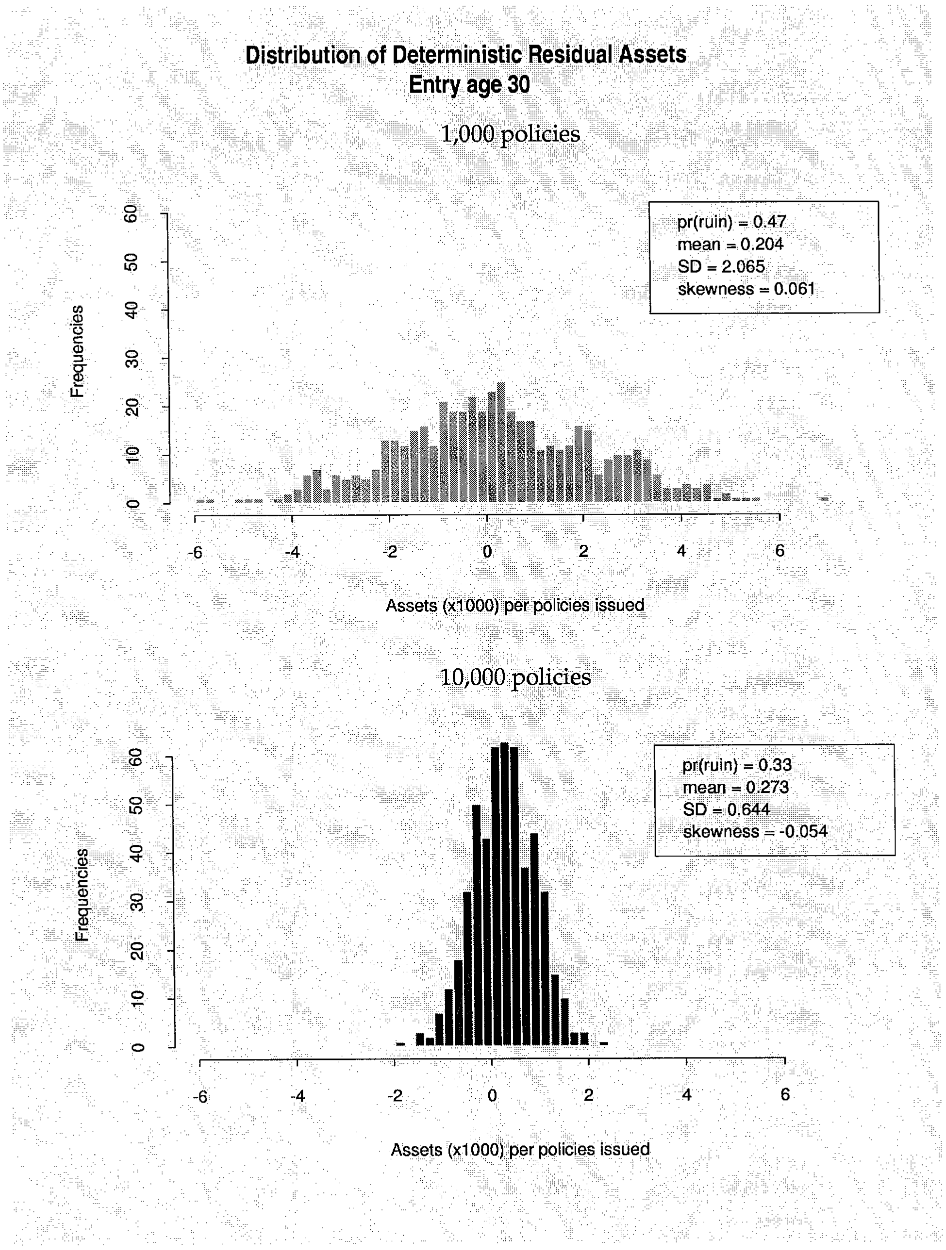


Figure 9: Deterministic Residual Assets Distribution corresponding to entry age 30.

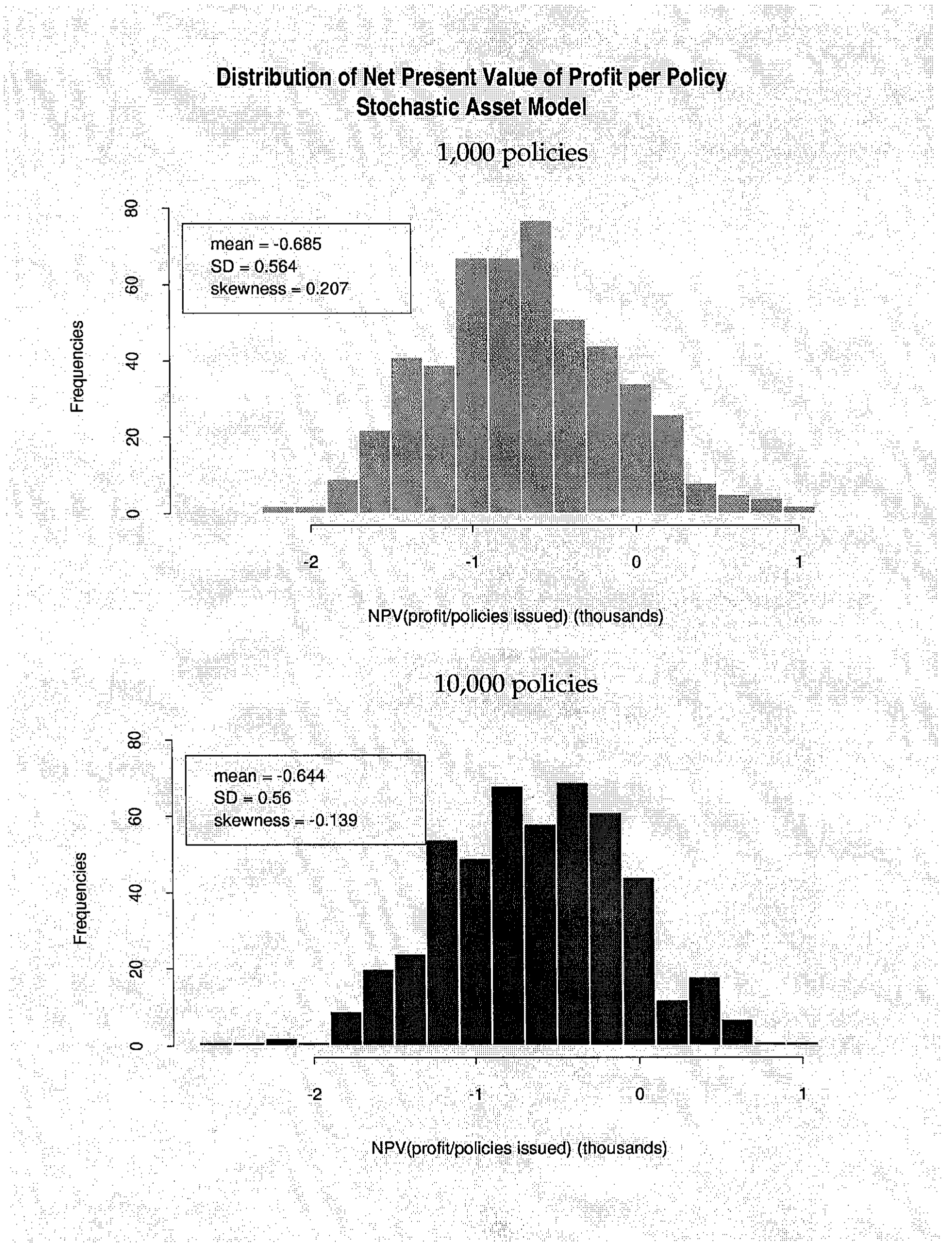


Figure 10: Distribution of Net Present Value of Profits based on Stochastic investment model for entry age 30.

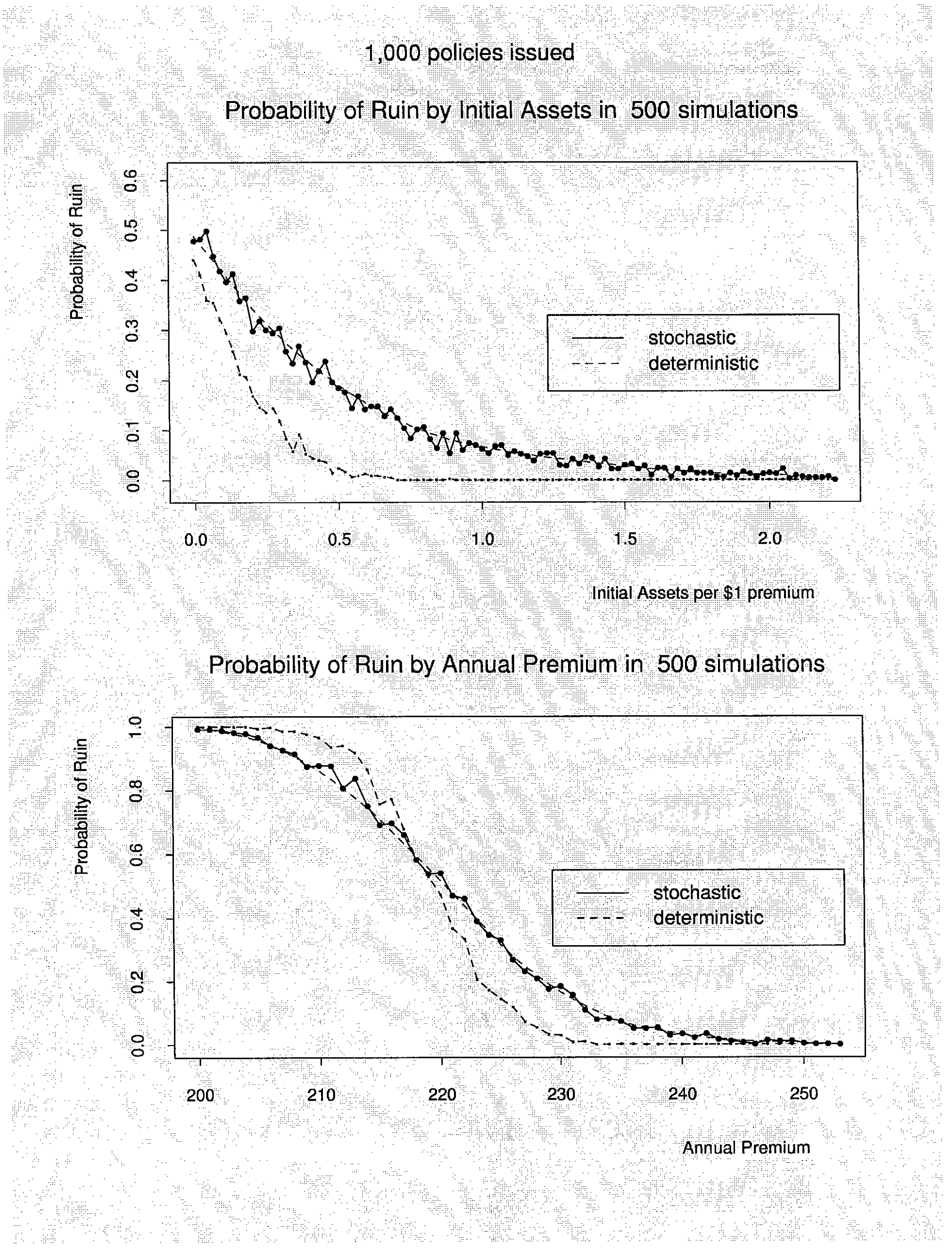


Figure 11: Probability of Ruin by initial assets and premium level for 1,000 policies assumed at outset.

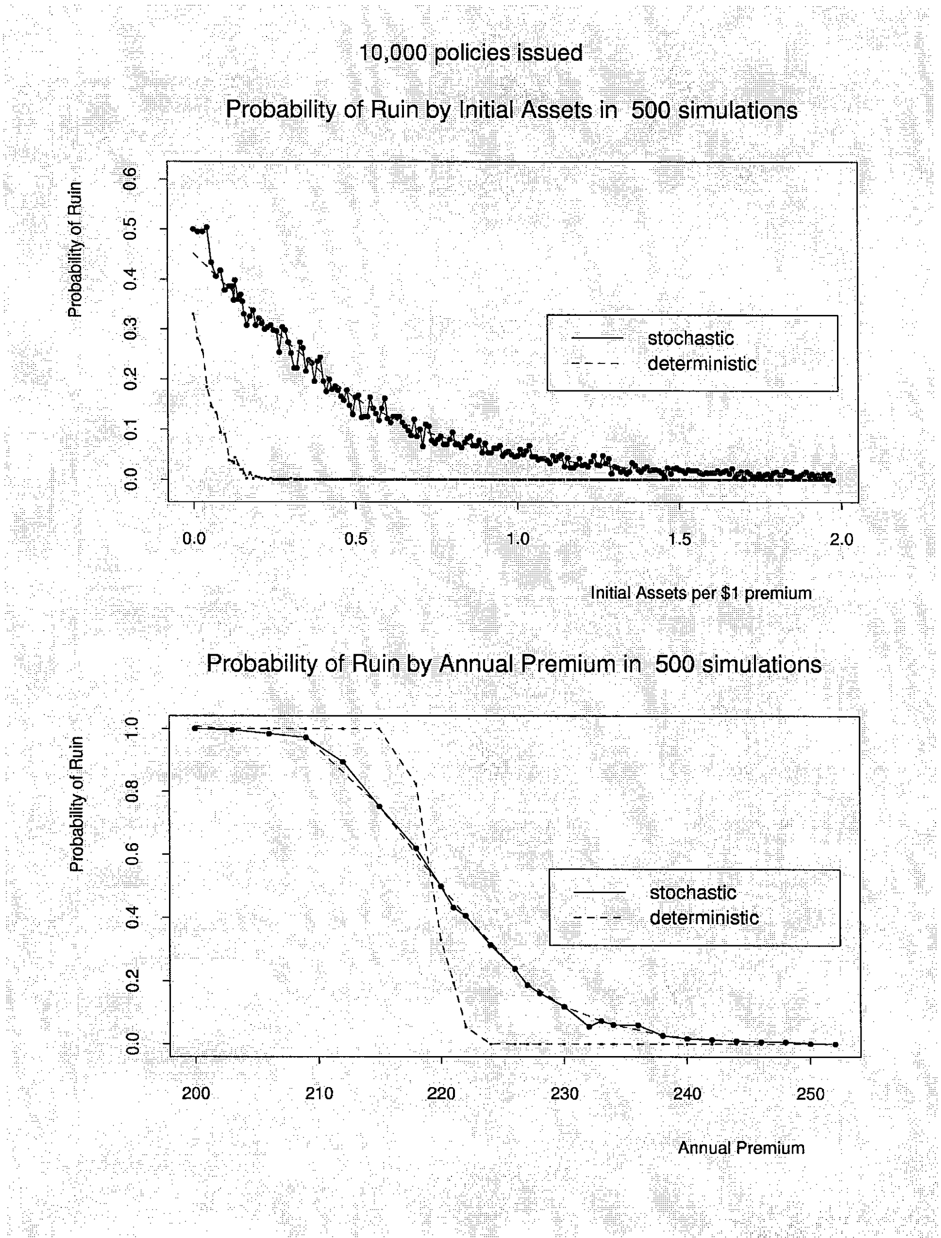


Figure 12: Probability of Ruin by initial assets and premium level for 10,000 policies assumed at outset.

d) increasing the premium leads to a lower probability of ruin, which, as in b), is more marked for the deterministic case;

e) decreasing the premium leads to a higher probability of ruin and we note that, unlike d), the extra variability from the stochastic case can help when the premium is inadequate and leads to a lower ruin probability.

Figure 12 refers to the case n=10,000 and we note the smoother progression of the curves. For the deterministic runs, the move from Figure 11 to 12 is particularly dramatic, with the tenfold increase in portfolio size leading to a reduction in "process risk" and a change in the shape of the curves.

An alternative method of presentation of probability of ruin is to compute the risk-based capital, which would be the initial assets $A_0(\varepsilon)$ required to produce a given level of ruin probability (ε). Table 1 presents some examples of $A_0(\varepsilon)$ for the cases of n=1,000 and 10,000; deterministic and stochastic asset models and a range of values of ε. The results show that:

(a) reducing ε leads to an increase in $A_0(\varepsilon)$;

(b) for the deterministic asset model, increasing n has a strong effect on $A_0(\varepsilon)$, implying the extent to which process risk is reduced;

(c) for the stochastic asset model, as noted earlier, the effect of increasing n has a much less strong effect on $A_0(\varepsilon)$ and hence on process risk.

Table 2 presents the results for $A_0(\varepsilon)$=0 for $\varepsilon = 5\%$ but allowing a proportionate risk loading 1+$\lambda(\varepsilon)$ to the premiums. Similar effects are demonstrated.

5. CONCLUSIONS

The results presented here (and more extensive investigations not reported) demonstrate that the demographic process error is reduced by pooling, as implied by the law of large numbers. We have shown that the economic process error is considerably more significant than the demographic process error for portfolios of disability income protection policies. We have also demonstrated the methodology for simulating in a multiple state context and shown how estimates for risk-loaded premiums or for risk-based capital can be obtained.

6. FURTHER WORK

We are investigating a range of enhancements to the model, including:

a) allowing for the dependence of the transitions intensities from the sick state on duration of sickness;

b) allowing the inflation rates to be random rather than fixed;

c) calculating the premium via the equivalence principle;

d) considering coherent risk measures, for example, expected adverse deviation or mean shortfall, rather than risk-based capital measures derived from percentiles (see Artzner et al (1999));

e) allowing for mixed liability portfolios, variable number of entrants, different asset allocation strategies;

f) consideration of parameter risk and comparison with process risk.

Table 1

Risk-Based Capital
Age at entry 30, Initial Premium £220

Initial assets required to produce a given ruin probability:
(Per £1 annual premium in year 1)

	Deterministic Asset Model		**Stochastic Asset Model**	
Pr (Ruin)	n=1,000	n=10,000	n=1,000	n=10,000
25%	0.148	0.025	0.374	0.328
10%	0.305	0.081	0.799	0.692
5%	0.402	0.101	1.224	1.020
1%	0.534	0.162	1.820	1.651

Table 2

Risk-Loaded Premium
Age at entry 30, Initial Premium £220

Additional premium required to produce a 5% ruin probability assuming 0 initial assets:
(Per £1 annual premium in year 1)

n	**Deterministic Asset Model**	**Stochastic Asset Model**
1,000	0.035	0.077
10,000	0.010	0.070

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