



## City Research Online

### City, University of London Institutional Repository

---

**Citation:** Verrall, R. J. (2001). A Bayesian generalised linear model for the Bornhuetter-Ferguson method of claims reserving (Actuarial Research Paper No. 139). London, UK: Faculty of Actuarial Science & Insurance, City University London.

This is the unspecified version of the paper.

This version of the publication may differ from the final published version.

---

**Permanent repository link:** <https://openaccess.city.ac.uk/id/eprint/2276/>

**Link to published version:**

**Copyright:** City Research Online aims to make research outputs of City, University of London available to a wider audience. Copyright and Moral Rights remain with the author(s) and/or copyright holders. URLs from City Research Online may be freely distributed and linked to.

**Reuse:** Copies of full items can be used for personal research or study, educational, or not-for-profit purposes without prior permission or charge. Provided that the authors, title and full bibliographic details are credited, a hyperlink and/or URL is given for the original metadata page and the content is not changed in any way.

---

---



**A Bayesian Generalised Linear Model  
for the Bornhuetter-Ferguson Method  
of Claims Reserving**

by

R.J.Verrall

Actuarial Research Paper No. 139

November 2001

ISBN 1 901615 62 6

“Any opinions expressed in this paper are my/our own and not necessarily those of my/our employer or anyone else I/we have discussed them with. You must not copy this paper or quote it without my/our permission”.

# **A Bayesian Generalised Linear Model for the Bornhuetter-Ferguson Method of Claims Reserving**

**R.J.Verrall**

## **Abstract**

This paper shows how a Bayesian model within the framework of generalised linear models can be applied to claims reserving. It is shown that this approach is closely related to the Bornhuetter-Ferguson technique. The Bornhuetter-Ferguson technique has previously been studied by Benktander (1976) and Mack (2000), who advocated using credibility models. The present paper uses a fully Bayesian parametric model, within the framework of generalised linear models.

## **Keywords**

Bornhuetter-Ferguson method; Chain-ladder technique; Generalised linear models; Loss run-off triangles.

## **Address for correspondence**

Prof. R.J.Verrall,  
Department of Actuarial Science and Statistics,  
City University,  
Northampton Square,  
London. EC1V 0HB  
email: r.j.verrall@city.ac.uk

## 1. Introduction

The Bornhuetter-Ferguson method (Bornhuetter and Ferguson, 1972) has proved useful for certain classes of general insurance business. In particular, when the data are very unstable, a method such as the chain-ladder technique can produce unsatisfactory results. In order to stabilise the results, the Bornhuetter-Ferguson method uses an external initial estimate of ultimate claims. This is then used with the development factors of the chain-ladder technique, or something similar, to estimate outstanding claims. It is sometimes stated that the Bornhuetter-Ferguson method is a Bayesian method, because of the initial estimate of ultimate claims which is supplied as prior information. This method has been investigated by a number of authors, and the recent paper by Mack (2000) provides an excellent summary of this work. Mack (2000) gives details of a similar approach to that advocated in this paper, using a credibility theory approach, first suggested by Benktander (1976). The present paper is based very much on Generalised Linear Models, and the theory in this paper is not applicable to all sets of data (in particular, it may break down for negative incremental claims).

This paper shows how the Bornhuetter-Ferguson method is related to the Generalised Linear Models approach to claims reserving, using a Bayesian approach. The advantages of this are as follows:

- i) It provides further help for the actuary to understand what assumptions are made when the Bornhuetter-Ferguson method is used.
- ii) It clarifies the connection between the Bornhuetter-Ferguson method and the chain-ladder technique, particularly for the case when Generalised Linear Models are used.
- iii) It allows mean square prediction errors to be calculated and indicates how the Bornhuetter-Ferguson method could be incorporated into a DFA exercise.
- iv) It provides a range of Bayesian generalised linear models which could be used in a claims-reserving exercise, of which the Bornhuetter-Ferguson method and the chain-ladder technique are special cases.

The approach taken in this paper to the Bornhuetter-Ferguson method is based on the approach of Verrall (2000), in which it was shown that the chain-ladder technique can be expressed in a number of different ways as stochastic models. This paper uses the same framework and examines the Bornhuetter-Ferguson method, giving a number of insights into this method. The stochastic model developed in section 3 encompasses the Bornhuetter-Ferguson method, as it is usually implemented by actuaries, as a special case.

Without loss of generality, we assume that the data consist of a triangle of incremental claims:

$$\{C_{ij} : j = 1, \dots, n - i + 1; i = 1, \dots, n\}.$$

The cumulative claims are defined by:

$$D_{ij} = \sum_{k=1}^j C_{ik}$$

and the development factors of the chain-ladder technique are denoted by  $\{\lambda_j : j = 2, \dots, n\}$ .

No tail factors are applied, and claims are only forecast up to the latest development year ( $n$ ) so far observed. It would be possible to extend this to allow a tail factor, using the same methods, but no specific modelling is carried out of the shape of the run-off beyond the latest development year.

The paper is set out as follows. Section 2 describes briefly the Bornhuetter-Ferguson method, as currently used by actuaries. Section 3 defines a Bayesian generalized linear model which has the Bornhuetter-Ferguson method as a special case. Section 4 uses this model to examine the relationship between the Bornhuetter-Ferguson method and the chain-ladder technique, and section 5 contains some concluding comments.

## 2. The Bornhuetter-Ferguson method

The Bornhuetter-Ferguson method can be summarised as follows.

1. Obtain an initial estimate of ultimate claims for each accident year.
2. Estimate the proportion of ultimate claims that are outstanding for each accident year, using, for example, the chain-ladder technique.
3. Apply the proportion from 2 to the initial estimate of ultimate claims from 1, to obtain the estimate of outstanding claims.

The usual way of expressing this is as follows:

Let the initial estimate of ultimate claims for accident year  $i$  be  $M_i$

The estimate of outstanding claims for accident year  $i$  is

$$M_i \left( 1 - \frac{1}{\lambda_{n-i+2} \lambda_{n-i+3} \dots \lambda_n} \right) = M_i \frac{1}{\lambda_{n-i+2} \lambda_{n-i+3} \dots \lambda_n} (\lambda_{n-i+2} \lambda_{n-i+3} \dots \lambda_n - 1)$$

Thus,  $M_i \frac{1}{\lambda_{n-i+2} \lambda_{n-i+3} \dots \lambda_n}$  replaces the latest cumulative claims for accident year  $i$ , to which the usual chain-ladder parameters are applied to obtain the estimate of outstanding claims. For the chain-ladder technique, the estimate of outstanding claims is  $D_{i,n-i+1} (\lambda_{n-i+2} \lambda_{n-i+3} \dots \lambda_n - 1)$ .

Thus, it can be seen that the difference between the Bornhuetter-Ferguson method and the chain-ladder technique is the factor that is used to multiply the development factors. For the chain-ladder technique, this is  $D_{i,n-i+1}$  and for the Bornhuetter-Ferguson method, this is

$$M_i \frac{1}{\lambda_{n-i+2} \lambda_{n-i+3} \dots \lambda_n}.$$

## 3. A Generalised Linear Model for the Bornhuetter-Ferguson Method

This section defines a Bayesian generalised linear model which has the Bornhuetter-Ferguson method as a special case. The stochastic model is based on the generalized linear model for the chain-ladder technique defined by Renshaw and Verrall (1998), the (over-dispersed) Poisson model. The difference between the Poisson and the over-dispersed Poisson is the variance. For

the Poisson distribution, the variance is equal to the mean, while for the over-dispersed Poisson the variance is equal to  $\varphi \times$  the mean. In other words, if  $Y$  has an over-dispersed Poisson distribution with mean  $\mu$ , the variance of  $Y$  is  $\varphi\mu$ . This implies that  $\frac{Y}{\varphi}$  has a Poisson distribution with mean (and variance)  $\frac{\mu}{\varphi}$ .

The (over-dispersed) Poisson model for the chain-ladder technique can be written as follows

$$\begin{aligned} C_{ij} &\sim \text{independent over-dispersed Poisson, with } E[C_{ij}] = x_i y_j, \text{ and } \sum_{k=1}^n y_k = 1. \\ \text{i.e. } \frac{C_{ij}}{\varphi} &\sim \text{independent Poisson, with } E[C_{ij}] = \frac{x_i y_j}{\varphi}, \text{ and } \sum_{k=1}^n y_k = 1. \end{aligned} \quad (3.1)$$

$x_i = E[D_{in}]$ , expected ultimate cumulative claims (up to the latest development year so far observed). The column parameters,  $\left\{ y_j : j = 1, \dots, n; \sum_{j=1}^n y_j = 1 \right\}$ , can be interpreted as the proportions of ultimate claims which emerge in each development year. This model can be reparameterised as follows:

$$\frac{C_{ij}}{\varphi} \sim \text{Poisson, with } E[C_{ij}] = \frac{z_i y_j}{\varphi \sum_{k=1}^n y_k} \quad \text{and} \quad \sum_{k=1}^n y_k = 1. \quad (3.2)$$

In this case,  $z_i = E[D_{i,n-i+1}]$ , which is the expected value of cumulative claims up to the latest development year observed in accident year  $i$ .

In the chain-ladder model, no prior assumptions are made about the row parameters,  $\{x_i : i = 1, \dots, n\}$ . The key assumption of the Bornhuetter-Ferguson method is that there is prior knowledge about these parameters, and thus the Bornhuetter-Ferguson method uses a Bayesian approach. The prior information can be summarised as the following prior distributions for the row parameters:

$$x_i \sim \text{independent } \Gamma(\alpha_i, \beta_i)$$

This Bayesian model can be implemented (and indeed is probably best implemented) using an Markov chain Monte Carlo (MCMC) approach, through the software winBUGS (Spiegelhalter et al, 1996). However, it is useful to look at the predictive distribution for the data, in order to compare the Bornhuetter-Ferguson method with the chain-ladder technique. As in Verrall (2000), we consider just one row of data, for simplicity of exposition:

$$C_{i1}, C_{i2}, \dots, C_{i,n-i+1}.$$



As we are considering only one row of data, we drop the  $i$  suffix, and write the model for  $C_j$  given  $z(j)$  as

$$\frac{C_j}{\varphi} | z(j) \sim \text{Poisson, with mean } \frac{z(j)y_j}{\varphi S_j} \quad (3.3)$$

where  $S_m = \sum_{k=1}^m y_k$

and  $z(j) = E[D_j]$  is the expected value of aggregate claims up to development year  $j$ .

The label  $j$  has been attached to  $z$ , since the definition of  $z$  is different for each  $C_j$ .

$$\text{Now } z(j) = E[D_j] = E[D_{j-1}] + E[C_j] = z(j-1) + \frac{z(j)y_j}{S_j}$$

$$\text{and hence } z(j) = \frac{z(j-1)}{1 - \frac{y_j}{S_j}} = \frac{z(j-1)S_j}{S_{j-1}}.$$

Thus, the conditional distribution of  $C_j$  given  $z(j-1)$  is

$$\frac{C_j}{\varphi} | z(j-1) \sim \text{Poisson, with mean } \frac{z(j-1)y_j}{\varphi S_{j-1}}. \quad (3.4)$$

The following theorem gives the recursive distribution of  $z(j)$ , and is needed mainly for the corollary, which gives the predictive distribution of  $C_j$ .

### Theorem

$$z(j) | C_1, C_2, \dots, C_j \sim \Gamma\left(\alpha + \frac{D_j}{\varphi}, \frac{\beta\varphi + S_j}{\varphi S_j}\right), \text{ for } j = 1, 2, \dots, n.$$

### Proof

We prove this theorem by induction. Consider first the distribution of  $z(1) | C_1$ .

$$\frac{C_1}{\varphi} | z(1) \sim \text{Poisson, with mean } \frac{z(1)}{\varphi}, \text{ since } y_1 = S_1.$$

Note that  $z(n) = x$ , and that  $z(j-1) = \frac{z(j)S_{j-1}}{S_j}$ , and hence,  $z(1) = xS_1 = xy_1$ .

The prior distribution of  $x$  is  $x \sim \Gamma(\alpha, \beta)$ , and hence the prior distribution of  $z(1)$  is  $z(1) \sim \Gamma\left(\alpha, \frac{\beta}{y_1}\right)$ . A standard Bayesian prior-posterior analysis gives the distribution of  $z(1)$ , given  $C_1$ :

$$f(z(1) | C_1) \propto \left(\frac{z(1)}{\varphi}\right)^{\frac{C_1}{\varphi}} e^{-\frac{z(1)}{\varphi}} z(1)^\alpha e^{-\frac{\beta}{y_1} z(1)}$$

from which it can be seen that

$$z(1) | C_1 \sim \Gamma\left(\alpha + \frac{C_1}{\varphi}, \frac{\beta}{y_1} + \frac{1}{\varphi}\right)$$

$$\text{i.e. } z(1) | C_1 \sim \Gamma\left(\alpha + \frac{D_1}{\varphi}, \frac{\beta\varphi + S_1}{\varphi S_1}\right)$$

Hence the theorem is true for  $j = 1$ . Suppose it is true for  $j - 1$ :

$$z(j-1) | C_1, C_2, \dots, C_{j-1} \sim \Gamma\left(\alpha + \frac{D_{j-1}}{\varphi}, \frac{\beta\varphi + S_{j-1}}{\varphi S_{j-1}}\right)$$

Since  $\frac{C_j}{\varphi} | z(j-1) \sim \text{Poisson}$ , with mean  $\frac{z(j-1)y_j}{\varphi S_{j-1}}$ , a standard Bayesian analysis gives the posterior distribution of  $z(j-1)$  as

$$z(j-1) | C_1, C_2, \dots, C_{j-1}, C_j \sim \Gamma\left(\alpha + \frac{D_{j-1}}{\varphi} + \frac{C_j}{\varphi}, \frac{\beta\varphi + S_{j-1}}{\varphi S_{j-1}} + \frac{y_j}{\varphi S_{j-1}}\right).$$

$$\text{i.e. } z(j-1) | C_1, C_2, \dots, C_j \sim \Gamma\left(\alpha + \frac{D_j}{\varphi}, \frac{\beta\varphi + S_j}{\varphi S_{j-1}}\right).$$

Since we have a relationship between  $z(j)$  and  $z(j-1)$ , we can obtain the distribution of  $z(j)$ , conditional on the information received up to development year  $j$  by a straightforward transformation:

$$\text{If } z(j-1) \sim \Gamma(a, b) \quad \text{then } z(j) \sim \Gamma\left(a, \frac{bS_{j-1}}{S_j}\right).$$

Hence,

$$z(j)|C_1, C_2, \dots, C_j \sim \Gamma\left(\alpha + \frac{D_j}{\varphi}, \frac{\beta\varphi + S_j}{\varphi S_{j-1}} \frac{S_{j-1}}{S_j}\right)$$

$$\text{i.e. } z(j)|C_1, C_2, \dots, C_j \sim \Gamma\left(\alpha + \frac{D_j}{\varphi}, \frac{\beta\varphi + S_j}{\varphi S_j}\right)$$

which completes the recursive proof of the theorem.

### Corollary

The predictive distribution of  $\frac{C_j}{\varphi}$  is

$$\frac{C_j}{\varphi} | C_1, C_2, \dots, C_{j-1} \sim \text{Negative binomial, with parameters}$$

$$k = \alpha + \frac{D_{j-1}}{\varphi} \quad \text{and} \quad p = \frac{\beta\varphi + S_{j-1}}{\beta\varphi + S_j}.$$

### Proof

The predictive distribution of  $\frac{C_j}{\varphi}$  is

$$\begin{aligned} f\left(\frac{C_j}{\varphi} | C_1, C_2, \dots, C_{j-1}\right) &= \int f\left(\frac{C_j}{\varphi} | z(j-1)\right) f(z(j-1) | C_1, C_2, \dots, C_{j-1}) dz(j-1) \\ &= \int \frac{\left(\frac{z(j-1)y_j}{\varphi S_{j-1}}\right)^{\frac{C_j}{\varphi}} e^{-\frac{z(j-1)y_j}{S_{j-1}}} \left(\frac{\beta\varphi + S_{j-1}}{\varphi S_{j-1}}\right)^{\alpha + \frac{D_{j-1}}{\varphi}}}{\frac{C_j}{\varphi}! \Gamma\left(\alpha + \frac{D_{j-1}}{\varphi}\right)} z(j-1)^{\alpha + \frac{D_{j-1}}{\varphi} - 1} e^{-\left(\frac{\beta\varphi + S_{j-1}}{\varphi S_{j-1}}\right) z(j-1)} dz(j-1) \\ &= \frac{\left(\frac{y_j}{\varphi S_{j-1}}\right)^{\frac{C_j}{\varphi}} \left(\frac{\beta + S_{j-1}}{S_{j-1}}\right)^{\alpha + D_{j-1}}}{\frac{C_j}{\varphi}! \Gamma\left(\alpha + \frac{D_{j-1}}{\varphi}\right)} \int z(j-1)^{\alpha + \frac{D_{j-1}}{\varphi} + \frac{C_j}{\varphi} - 1} e^{-\left(\frac{\beta\varphi + S_{j-1}}{\varphi S_{j-1}} + \frac{y_j}{S_{j-1}}\right) z(j-1)} dz(j-1) \end{aligned}$$

$$\begin{aligned}
&= \frac{\left(\frac{y_j}{\phi S_{j-1}}\right)^{\frac{C_j}{\phi}} \left(\frac{\beta\phi + S_{j-1}}{\phi S_{j-1}}\right)^{\alpha + \frac{D_{j-1}}{\phi}} \Gamma\left(\alpha + \frac{D_j}{\phi}\right)}{\frac{C_j}{\phi}! \Gamma\left(\alpha + \frac{D_{j-1}}{\phi}\right) \left(\frac{\beta\phi + S_j}{\phi S_{j-1}}\right)^{\alpha + \frac{D_j}{\phi}}} \\
&= \frac{\Gamma\left(\alpha + \frac{D_j}{\phi}\right)}{\frac{C_j}{\phi}! \Gamma\left(\alpha + \frac{D_{j-1}}{\phi}\right)} \left(\frac{y_j}{\beta\phi + S_j}\right)^{\frac{C_j}{\phi}} \left(\frac{\beta\phi + S_{j-1}}{\beta\phi + S_j}\right)^{\alpha + \frac{D_{j-1}}{\phi}}
\end{aligned}$$

which is a Negative binomial distribution, with parameters

$$k = \alpha + \frac{D_{j-1}}{\phi} \quad \text{and} \quad p = \frac{\beta\phi + S_{j-1}}{\beta\phi + S_j}$$

#### 4. The relationship between the Bornhuetter-Ferguson method and the chain-ladder technique

This section compares the predictive distribution for the chain-ladder technique from Verrall (2000) and the predictive distribution for the Bayesian model derived above. Since they are both negative binomial distributions, we can compare them by looking at the means and variances. In particular, the means show clearly the differences in the assumptions made by the two approaches. We restore the row suffix,  $i$ .

For the chain-ladder technique, the mean of the predictive distribution is  $\frac{D_{i,j-1}y_j}{S_{j-1}}$ . Note that,

since  $\lambda_j = \frac{S_j}{S_{j-1}}$ , this can also be written as  $(\lambda_j - 1)D_{i,j-1}$ . Also, since  $D_{i,j} = D_{i,j-1} + C_{i,j}$ , the mean of the predictive distribution for aggregate claims is  $\lambda_j D_{i,j-1}$ .

The mean of the predictive distribution for  $\frac{C_j}{\phi}$  for the Bornhuetter-Ferguson method is

$$\frac{\left(\alpha_i + \frac{D_{i,j-1}}{\phi}\right) \frac{y_j}{\beta_i\phi + S_j}}{\frac{\beta_i\phi + S_{j-1}}{\beta_i\phi + S_j}} = \frac{\left(\alpha_i + \frac{D_{i,j-1}}{\phi}\right) y_j}{\beta_i\phi + S_{j-1}}$$

and hence the mean of the predictive distribution for  $C_j$  is

$$\begin{aligned} \varphi \frac{\left( \alpha_i + \frac{D_{i,j-1}}{\varphi} \right) y_j}{\beta_i \varphi + S_{j-1}} &= \left( \frac{S_{j-1}}{\beta_i \varphi + S_{j-1}} \frac{D_{i,j-1}}{S_{j-1}} + \frac{\beta_i \varphi}{\beta_i \varphi + S_{j-1}} \frac{\alpha_i}{\beta_i} \right) y_j \\ &= \left( Z_{ij} \frac{D_{i,j-1}}{S_{j-1}} + (1 - Z_{ij}) \frac{\alpha_i}{\beta_i} \right) y_j \end{aligned}$$

where  $Z_{ij} = \frac{S_{j-1}}{\beta_i \varphi + S_{j-1}}$ .

It can be seen that this is in the form of what is called by actuaries “a credibility formula”. In modern statistical terms, it is a natural trade off between 2 competing estimates for the row parameter. Note that  $y_j$  is the proportion of ultimate claims that emerge in development year  $j$ .

This is then multiplied by the prior mean of the ultimate claims,  $\frac{\alpha_i}{\beta_i}$ , for the Bornhuetter-

Ferguson method, or an estimate of ultimate claims from the data,  $\frac{D_{i,j-1}}{S_{j-1}}$ , for the chain-ladder

technique. We have here a combination of these two, and thus the stochastic model in this paper has the chain-ladder as one extreme (no prior information about the row parameters), and the Bornhuetter-Ferguson method as the other extreme (perfect prior information about the row parameters).

It is interesting to note that the Bornhuetter-Ferguson method assumes that there is perfect prior information about the row parameters, and does not use the data at all for this part of the estimation.

The mean of the predictive distribution can also be written as

$$\left( Z_{ij} D_{i,j-1} + (1 - Z_{ij}) \frac{\alpha_i}{\beta_i} S_{j-1} \right) \frac{y_j}{S_{j-1}}.$$

Since  $\lambda_j = \frac{S_j}{S_{j-1}}$  and  $S_n = 1$ ,  $\lambda_j \lambda_{j+1} \dots \lambda_n = \frac{S_j}{S_{j-1}} \frac{S_{j+1}}{S_j} \dots \frac{S_n}{S_{n-1}} = \frac{1}{S_{j-1}}$ . Hence, we may also write the mean as

$$\left( Z_{ij} D_{i,j-1} + (1 - Z_{ij}) \frac{\alpha_i}{\beta_i} \frac{1}{\lambda_j \lambda_{j+1} \dots \lambda_n} \right) \frac{y_j}{S_{j-1}} = \left( Z_{ij} D_{i,j-1} + (1 - Z_{ij}) \frac{\alpha_i}{\beta_i} \frac{1}{\lambda_j \lambda_{j+1} \dots \lambda_n} \right) (\lambda_j - 1).$$

It can then be seen that the two values used for the row parameter are the equivalent of those in section 2:

$$D_{i,j-1} \text{ and } \frac{\alpha_i}{\beta_i} \frac{1}{\lambda_j \lambda_{j+1} \dots \lambda_n} = M_i \frac{1}{\lambda_j \lambda_{j+1} \dots \lambda_n}.$$

The credibility factor,  $Z_{ij} = \frac{S_{j-1}}{\beta_i \varphi + S_{j-1}}$ , governs the trade-off between the prior mean and the data. Notice that the further through the development we are, the larger  $S_{j-1}$  is, and the more weight is given to the chain-ladder estimate. The choice of  $\beta_i$  is governed by the prior precision of the initial estimate for ultimate claims, and this should be chosen with due regard given to the over-dispersion parameter (an initial estimate of which could be obtained from the over-dispersed Poisson model of Renshaw and Verrall, 1998).

## 5. Conclusions

This paper has shown that the Bornhuetter-Ferguson method can be written as a Bayesian model within the framework of generalised linear models. It has also shown that the method as currently implemented by actuaries can be regarded, within the framework of generalised linear models, as an extreme case of a Bayesian model, which assumes perfect prior information about the row parameters. It would perhaps be more sensible to use a slightly less exact prior distribution in practice, and thus apply a model somewhere between the Bornhuetter-Ferguson method and the chain-ladder technique. The theory derived in this paper shows how the approach to Bornhuetter-Ferguson method described in Mack (2000) can be applied when a generalised linear model is used. The Bayesian model derived in this paper may break down if there are negative incremental claims values, and is therefore probably only suitable for paid data.

## References

- Benktander, G. (1976) *An approach to Credibility in Calculating IBNR for Casualty Excess Reinsurance*. In *The Actuarial Review*.
- Bornhuetter, R.L. and Ferguson, R.E. (1972) *The Actuary and IBNR* Proc. Cas. Act. Soc., LIX, pp 181-195
- Mack, T. (2000) *Credible Claims Reserves: The Benktander Method*. ASTIN Bulletin, Vol. 30, pp 333-347
- Renshaw, A.E. and Verrall, R.J. (1998) *A stochastic model underlying the chain ladder technique*. British Actuarial Journal 4, IV, pp 903-923
- Spiegelhalter, D.J., Thomas, A., Best, N.G. and Gilks, W.R. (1996) *BUGS 0.5: Bayesian Inference using Gibbs Sampling*, MRC Biostatistics Unit, Cambridge, UK.
- Verrall, R.J. (2000) *An Investigation into Stochastic Claims Reserving Models and the Chain-Ladder Technique*. Insurance: Mathematics and Economics, Volume 26, pp 91-99.

## DEPARTMENT OF ACTUARIAL SCIENCE AND STATISTICS

### Actuarial Research Papers since 2000

- 
- 122. Booth P.M. and Cooper D.R. The Tax Treatment of Pensions. April 2000. 36 pages.  
ISBN 1 901615 42 1
  - 123. Walsh D.E.P. and Rickayzen B.D. A Model for Projecting the number of People who will  
require Long-Term Care in the Future. Part I: Data Considerations. July 2000. 37 pages.  
ISBN 1 901615 43 X
  - 124. Rickayzen B.D. and Walsh D.E.P. A Model for Projecting the number of People who will  
require Long-Term Care in the Future. Part II: The Multiple State Model. July 2000. 27  
pages. ISBN 1 901615 44 8
  - 125. Walsh D.E.P. and Rickayzen B.D. A Model for Projecting the number of People who will  
require Long-Term Care in the Future. Part III: The Projected Numbers and The Funnel of  
Doubt. July 2000. 61 pages. ISBN 1 901615 45 6
  - 126. Cooper D.R. Security for the Members of Defined Benefit Pension Schemes. July 2000.  
23 pages. ISBN 1 901615 45 4
  - 127. Renshaw A.E. and Haberman S. Modelling for mortality reduction factors. July 2000.  
32 pages. ISBN 1 901615 47 2
  - 128. Ballotta L. and Kyprianou A.E. A note on the  $\alpha$ -quantile option. September 2000.  
ISBN 1 901615 49 9
  - 129. Spreeuw J. Convex order and multistate life insurance contracts. December 2000.  
ISBN 1 901615 50 2
  - 130. Spreeuw J. The Probationary Period as a Screening Device. December 2000.  
ISBN 1 901615 51 0
  - 131. Owadally M.I. and Haberman S. Asset Valuation and the Dynamics of Pension Funding with  
Random Investment Returns. December 2000. ISBN 1 901615 52 9
  - 132. Owadally M.I. and Haberman S. Asset Valuation and Amortization of Asset Gains and Losses  
in Defined Benefit Pension Plans. December 2000. ISBN 1 901615 53 7
  - 133. Owadally M.I. and Haberman S. Efficient Amortization of Actuarial Gains/Losses and Optimal  
Funding in Pension Plans. December 2000. ISBN 1 901615 54 5
  - 134. Ballotta L.  $\alpha$ -quantile Option in a Jump-Diffusion Economy. December 2000.  
ISBN 1 901615 55 3
  - 135. Renshaw A. E. and Haberman S. On the Forecasting of Mortality Reduction Factors.  
February 2001. ISBN 1 901615 56 1

136. Haberman S., Butt Z. & Rickayzen B. D. Multiple State Models, Simulation and Insurer Insolvency. February 2001. 27 pages. ISBN 1 901615 57 X
137. Khorasanee M.Z. A Cash-Flow Approach to Pension Funding. September 2001. 34 pages. ISBN 1 901615 58 8
138. England P.D. Addendum to "Analytic and Bootstrap Estimates of Prediction Errors in Claims Reserving". November 2001. 17 pages. ISBN 1 901615 59 6
139. Verrall R.J. A Bayesian Generalised Linear Model for the Bornhuetter-Ferguson Method of Claims Reserving. November 2001. 10 pages. ISBN 1 901615 62 6

### **Statistical Research Papers**

1. Sebastiani P. Some Results on the Derivatives of Matrix Functions. December 1995. 17 Pages. ISBN 1 874 770 83 2
2. Dawid A.P. and Sebastiani P. Coherent Criteria for Optimal Experimental Design. March 1996. 35 Pages. ISBN 1 874 770 86 7
3. Sebastiani P. and Wynn H.P. Maximum Entropy Sampling and Optimal Bayesian Experimental Design. March 1996. 22 Pages. ISBN 1 874 770 87 5
4. Sebastiani P. and Settimi R. A Note on D-optimal Designs for a Logistic Regression Model. May 1996. 12 Pages. ISBN 1 874 770 92 1
5. Sebastiani P. and Settimi R. First-order Optimal Designs for Non Linear Models. August 1996. 28 Pages. ISBN 1 874 770 95 6
6. Newby M. A Business Process Approach to Maintenance: Measurement, Decision and Control. September 1996. 12 Pages. ISBN 1 874 770 96 4
7. Newby M. Moments and Generating Functions for the Absorption Distribution and its Negative Binomial Analogue. September 1996. 16 Pages. ISBN 1 874 770 97 2
8. Cowell R.G. Mixture Reduction via Predictive Scores. November 1996. 17 Pages. ISBN 1 874 770 98 0
9. Sebastiani P. and Ramoni M. Robust Parameter Learning in Bayesian Networks with Missing Data. March 1997. 9 Pages. ISBN 1 901615 00 6
10. Newby M.J. and Coolen F.P.A. Guidelines for Corrective Replacement Based on Low Stochastic Structure Assumptions. March 1997. 9 Pages. ISBN 1 901615 01 4.
11. Newby M.J. Approximations for the Absorption Distribution and its Negative Binomial Analogue. March 1997. 6 Pages. ISBN 1 901615 02 2
12. Ramoni M. and Sebastiani P. The Use of Exogenous Knowledge to Learn Bayesian Networks from Incomplete Databases. June 1997. 11 Pages. ISBN 1 901615 10 3
13. Ramoni M. and Sebastiani P. Learning Bayesian Networks from Incomplete Databases. June 1997. 14 Pages. ISBN 1 901615 11 1



14. Sebastiani P. and Wynn H.P. Risk Based Optimal Designs. June 1997. 10 Pages.  
ISBN 1 901615 13 8
15. Cowell R. Sampling without Replacement in Junction Trees. June 1997. 10 Pages.  
ISBN 1 901615 14 6
16. Dagg R.A. and Newby M.J. Optimal Overhaul Intervals with Imperfect Inspection and Repair.  
July 1997. 11 Pages. ISBN 1 901615 15 4
17. Sebastiani P. and Wynn H.P. Bayesian Experimental Design and Shannon Information.  
October 1997. 11 Pages. ISBN 1 901615 17 0
18. Wolstenholme L.C. A Characterisation of Phase Type Distributions. November 1997.  
11 Pages. ISBN 1 901615 18 9
19. Wolstenholme L.C. A Comparison of Models for Probability of Detection (POD) Curves.  
December 1997. 23 Pages. ISBN 1 901615 21 9
20. Cowell R.G. Parameter Learning from Incomplete Data Using Maximum Entropy I: Principles.  
February 1999. 19 Pages. ISBN 1 901615 37 5
21. Cowell R.G. Parameter Learning from Incomplete Data Using Maximum Entropy II: Application  
to Bayesian Networks. November 1999. 12 Pages ISBN 1 901615 40 5
22. Cowell R.G. FINEX : Forensic Identification by Network Expert Systems. March 2001. 10  
pages. ISBN 1 901615 60X
23. Cowell R.G. When Learning Bayesian Networks from Data, using Conditional Independence  
Tests is Equivalent to a Scoring Metric. March 2001. 11 pages. ISBN 1 901615 61 8

## **Department of Actuarial Science and Statistics**

### **Actuarial Research Club**

The support of the corporate members

CGNU Assurance  
Computer Sciences Corporation  
Government Actuary's Department  
HCM Consultants (UK) Ltd  
KPMG  
PricewaterhouseCoopers  
Swiss Reinsurance  
Watson Wyatt Partners

is gratefully acknowledged.