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Unitarity methods for scattering in two dimensions

Valentina Forini, Lorenzo Bianchi and Ben Hoare

Abstract The standard unitarity-cut method is applied to several massive two-dimensional models, including the world-sheet $AdS_5 \times S^5$ superstring, to compute $2 \rightarrow 2$ scattering S-matrices at one loop from tree level amplitudes. Evidence is found for the cut-constructibility of supersymmetric integrable models, while for models without supersymmetry (but integrable) the missing rational terms can be interpreted as a shift in the coupling.

1 Discussion

Unitarity-based methods, whose use in four dimensions has been crucial for an efficient evaluation of scattering amplitudes [1] in non-abelian gauge theories as well as gravity theories [2], have never really been applied in two dimensions [1]. The aim of our work [7] (we refer the reader to the independent results of [8]) has been to initiate the use of unitarity methods in the perturbative study of the S-matrix for *massive two-dimensional* field theories. Limiting ourselves to the use of *standard* unitarity (therefore placing on shell only two internal lines [2]) we present a formula for the one-loop $2 \rightarrow 2$ scattering amplitude built directly from the corresponding on-shell tree-level amplitudes.

As reviewed below, we have applied our method to various models, finding enough evidence to postulate that *supersymmetric, integrable two-dimensional theories should be cut-constructible via standard unitarity methods*. For bosonic theories with integrability, we find agreement with perturbation theory up to a finite shift in

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[1] For the three-dimensional case see [3, 4, 5, 6].

[2] This is nothing but the application of the optical theorem. The case where the loop amplitude is subdivided into more than two pieces is referred to as *generalized* unitarity.

the coupling [3]. We also successfully apply our method to the light-cone gauge-fixed sigma-model for the $AdS_5 \times S^5$ superstring, where - importantly - standard perturbation theory seems to fail in evaluating the S-matrix beyond the leading order due to regularization issues.

Natural extensions of our analysis would be the generalization to both higher loops [4] and higher points, as well as the evaluation of rational contributions in the case of scattering of particles with different masses, interesting for example for the $AdS_3 \times S^3 \times M^4$ world-sheet S-matrix [5].

2 Two-particle S-matrix from unitarity cuts at one loop

In two dimensions, the two-body scattering process of a translational-invariant field theory is described via the four-point amplitude

$$
\langle \Phi^P(p_3)\Phi^Q(p_4) | \mathbb{S} | \Phi_M(p_1)\Phi_N(p_2) \rangle = \mathscr{A}_{MN}^{PQ}(p_1, p_2, p_3, p_4)
$$
$$
\equiv (2\pi)^2 \delta^{(2)}(p_1 + p_2 - p_3 - p_4) \widetilde{\mathscr{A}}_{MN}^{PQ}(p_1, p_2, p_3, p_4) , \qquad (1)
$$

where $\mathbb{S}$ is the scattering operator, the fields Φ have on-shell momenta p_i (for us, all the particles have equal non-vanishing mass set to unity) and can carry flavor indices. Importantly, the energy-momentum conservation δ-function satisfies

$$
\delta^{(2)}(p_1 + p_2 - p_3 - p_4) = J(p_1, p_2) \left(\delta(\mathrm{p}_1 - \mathrm{p}_3)\delta(\mathrm{p}_2 - \mathrm{p}_4) + \delta(\mathrm{p}_1 - \mathrm{p}_4)\delta(\mathrm{p}_2 - \mathrm{p}_3) \right) ,
$$
$$
(2)
$$

which accounts for the fact that in $d = 2$ there is no phase space, and the only thing particles can do is either preserve or exchange their momenta. Above, p is the spatial momentum, the Jacobian $J(p_1, p_2) = 1/(\partial \varepsilon_{\mathrm{p}_1}/\partial \mathrm{p}_1 - \partial \varepsilon_{\mathrm{p}_2}/\partial \mathrm{p}_2)$ depends on the dispersion relation ε_{p} (the on-shell energy associated to p) for the theory at hand, and spatial momenta are assumed to be ordered $\mathrm{p}_1 > \mathrm{p}_2$. The S-matrix elements relevant for the description of the $2 \to 2$ scattering in the two-dimensional case are then defined [6] as

$$
S_{MN}^{PQ}(p_1, p_2) \equiv \frac{J(p_1, p_2)}{4\varepsilon_1 \varepsilon_2} \widetilde{\mathscr{A}}_{MN}^{PQ}(p_1, p_2, p_1, p_2) . \qquad (3)
$$

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[3] It would be interesting to analyze models which are just supersymmetric and not integrable.

[4] Two-loop logarithmic contributions to the world-sheet scattering matrix for several backgrounds of interest were evaluated in [8].

[5] This implies an extension to the case of different masses of the t-channel prescription we describe in the next section, and would complete the analysis of [8] where the logarithmic part was computed up to two loops.

[6] Without loss of generality, one can consider in (1) the amplitudes associated to the first product of δ-functions $\delta(\mathrm{p}_1 - \mathrm{p}_3)\delta(\mathrm{p}_2 - \mathrm{p}_4)$. The denominator in (3) is required to make contact with the standard definition of the S-matrix in two dimensions.

In applying the standard unitarity rules (derived from the optical theorem) [9] to the one-loop four point amplitude (1) one considers *two-particle cuts*, obtained by putting two intermediate lines on-shell. The contributions that follow to the imaginary part of the amplitude are therefore given by the sum of s- t- and u- channel cuts illustrated in Fig. 1, explicitly

$$\mathscr{A}^{(1)PQ}_{MN}(p_1,p_2,p_3,p_4)|_{s-cut} = \int \frac{d^2l_1}{(2\pi)^2} \int \frac{d^2l_2}{(2\pi)^2} \, i\pi\delta^+(l_1{}^2-1) \, i\pi\delta^+(l_2^2-1)$$
$$\times \mathscr{A}^{(0)RS}_{MN}(p_1,p_2,l_1,l_2)\mathscr{A}^{(0)PQ}_{SR}(l_2,l_1,p_3,p_4)\quad(4)$$

$$\mathscr{A}^{(1)PQ}_{MN}(p_1,p_2,p_3,p_4)|_{t-cut} = \int \frac{d^2l_1}{(2\pi)^2} \int \frac{d^2l_2}{(2\pi)^2} \, i\pi\delta^+(l_1{}^2-1) \, i\pi\delta^+(l_2{}^2-1)$$
$$\times \mathscr{A}^{(0)SP}_{MR}(p_1,l_1,l_2,p_3)\mathscr{A}^{(0)RQ}_{SN}(l_2,p_2,l_1,p_4)\quad(5)$$

$$\mathscr{A}^{(1)PQ}_{MN}(p_1,p_2,p_3,p_4)|_{u-cut} = \int \frac{d^2l_1}{(2\pi)^2} \int \frac{d^2l_2}{(2\pi)^2} \, i\pi\delta^+(l_1{}^2-1) \, i\pi\delta^+(l_2{}^2-1)$$
$$\times \mathscr{A}^{(0)SQ}_{MR}(p_1,l_1,l_2,p_4)\mathscr{A}^{(0)RP}_{SN}(l_2,p_2,l_1,p_3)\quad(6)$$

where $\mathscr{A}^{(0)}$ are tree-level amplitudes and a sum over the complete set of intermediate states R,S (all allowed particles for the cut lines) is understood. Notice that tadpole graphs, having no physical two-particle cuts, are by definition ignored in this procedure.
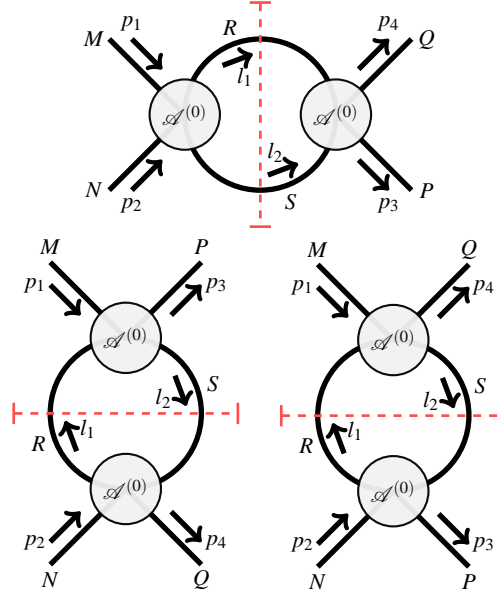
Fig. 1 Diagrams representing s-, t- and u-channel cuts contributing to the four-point one-loop amplitude.

To proceed, in each case one uses (1) and the momentum conservation at the vertex involving the momentum p_1 to integrate over l_2, e.g. for the *s*-channel

$$\widetilde{\mathscr{A}}^{(1)PQ}_{MN}(p_1,p_2,p_3,p_4)|_{s-cut} = \int \frac{d^2l_1}{(2\pi)^2} \, i\pi\delta^+(l_1{}^2-1) \, i\pi\delta^+((l_1-p_1-p_2)^2-1)$$
$$\times \widetilde{\mathscr{A}}^{(0)RS}_{MN}(p_1,p_2,l_1,-l_1+p_1+p_2) \widetilde{\mathscr{A}}^{(0)PQ}_{SR}(-l_1+p_1+p_2,l_1,p_3,p_4), \quad (7)$$

The simplicity of the two-dimensional kinematics and of being at one loop plays now its role, since in each of the integrals the set of zeroes of the δ-functions is a discrete set, and the cut loop-momenta are frozen to specific values [7]. This allows us to pull out the tree-level amplitudes with the loop-momenta evaluated at those zeroes [8]. In what remains, following standard unitarity computations [9], we apply the replacement $i\pi\delta^+(l^2-1) \longrightarrow \frac{1}{l^2-1}$ (*i.e.* the Cutkowsky rule in reverse order) which sets loop momenta back off-shell, thus reconstructing scalar bubbles. This allows us to rebuild, from its imaginary part, the cut-constructible piece of the amplitude and, via (3) [9], of the S-matrix. It then follows that a candidate expression for the one-loop S-matrix elements is given by the following simple sum of products of two tree-level amplitudes [10]

$$S^{(1)PQ}_{MN}(p_1,p_2) = \frac{1}{4(\varepsilon_2\,p_1-\varepsilon_1\,p_2)} \Big[\tilde{S}^{(0)RS}_{MN}(p_1,p_2) \tilde{S}^{(0)PQ}_{RS}(p_1,p_2) I_{p_1+p_2} \quad (8)$$
$$+\tilde{S}^{(0)SP}_{MR}(p_1,p_1) \tilde{S}^{(0)RQ}_{SN}(p_1,p_2) I_0 + \tilde{S}^{(0)SQ}_{MR}(p_1,p_2) \tilde{S}^{(0)PR}_{SN}(p_1,p_2) I_{p_1-p_2} \Big]$$

where the coefficients are given in terms of the bubble integral

$$I_p = \int \frac{d^2q}{(2\pi)^2} \frac{1}{(q^2-1+i\varepsilon)((q-p)^2-1+i\varepsilon)} \quad (9)$$

and read explicitly

$$I_{p_1+p_2} = \frac{i\pi - \text{arsinh}(\varepsilon_2\,p_1-\varepsilon_1\,p_2)}{4\pi i\,(\varepsilon_2\,p_1-\varepsilon_1\,p_2)}, \qquad I_0 = \frac{1}{4\pi i}, \qquad I_{p_1-p_2} = \frac{\text{arsinh}(\varepsilon_2\,p_1-\varepsilon_1\,p_2)}{4\pi i\,(\varepsilon_2\,p_1-\varepsilon_1\,p_2)} \;.$$

A few importants remarks are in order:

(a) Since the unitarity-cut procedure only ensures the correctness of logarithmic terms (in general, of those terms associated to branch-cut singularities, typ-

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[7] At two loops, to constrain completely the four components of the two momenta circulating in the loops one needs four cuts, each one giving an on-shell δ-function. Two-particle cuts at two loops would result in a manifold of conditions for the loop momenta.

[8] This is like using $f(x)\delta(x-x_0) = f(x_0)\delta(x-x_0)$, where $f(x)$ are the tree-level amplitudes in the integrals.

[9] This corresponds to the choice $p_3 = p_1$, $p_4 = p_2$.

[10] In (8), $\tilde{S}^{(0)}(p_1,p_2) = 4(\varepsilon_2\,p_1 - \varepsilon_1\,p_2)S^{(0)}(p_1,p_2)$ and the denominator on the right-hand side comes from the Jacobian $J(p_1,p_2)$ assuming a standard relativistic dispersion relation (for the theories we consider, at one-loop this is indeed the case).

ically logarithms or polylogarithms), the proposal (8) and its fermionic general-ization [7] crucially need to be tested on known examples [11].

(b) The t-channel cut requires a prescription, since if one first uses the δ-function identity (2) to fix, for example, $p_1 = p_3$ and $p_2 = p_4$ the corresponding integral is ill-defined. To avoid this ambiguity we follow the prescription that we should only impose the δ-function identity at the end [12]. Furthermore, if we choose the alternative solution of the conservation δ-function in (5), namely $\ell_2 = \ell_1 + p_4 - p_2$, the coefficient of $I(0)$ in (8) would be different, which leads to the consistency condition on the tree-level S-matrix [13]

$$\tilde{S}^{(0)SP}_{MR}(p_1,p_1)\,\tilde{S}^{(0)RQ}_{SN}(p_1,p_2) = \tilde{S}^{(0)PS}_{MR}(p_1,p_2)\,\tilde{S}^{(0)QR}_{SN}(p_2,p_2)\;. \qquad (10)$$

We have checked this for the tree-level S-matrices of all the field theory models treated below.

(c) As they only involve the scalar bubble integral in two dimensions, the re-sult (8) following from our procedure is inherently *finite*. No additional regular-ization is required and the result can be compared directly with the $2 \rightarrow 2$ particle S-matrix (following from the finite or renormalized four-point amplitude) found using standard perturbation theory. Of course, this need not be the case for the original bubble integrals before cutting – due to factors of loop-momentum in the numerators. These divergences, along with those coming from tadpole graphs, which we did not consider, should be taken into account for the renormaliza-tion of the theory. We have not investigated this issue, since all the theories we consider below are either UV-finite or renormalizable.

To explore the validity of the procedure outlined we have considered both relativistic and non-relativistic (world-sheet field theory for the AdS$_5 \times S^5$ superstring) models.

3 Relativistic models

In the *relativistic, bosonic case*, we looked at a class of generalized sine-Gordon models [10, 11], theories defined by a gauged WZW model for a coset G/H plus a potential, whose classical integrability can be demonstrated through the existence of a Lax connection. Considering the coset $G/H = \mathrm{SO}(n+1)/\mathrm{SO}(n)$, where asymp-totic excitations are a free $\mathrm{SO}(n)$ vector with unit mass (which is the case we have considered in our general procedure), this class includes the sine-Gordon ($n=1$) and complex sine-Gordon ($n=2$) models, for which the exact S-matrices are known [12, 13]. In all the cases the one-loop S-matrix got via unitarity cuts agrees – up to

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[11] Because its bubble integral I_0 can only contribute to rational terms, the t-channel contribution has been neglected in [8], where all rational terms were determined from symmetry considerations.

[12] In some sense this is natural as, in general dimensions, quantum field theory amplitudes have the form (1), while the δ-function identity (2) is specific to two dimensions.

[13] See [7] for the generalization to the case which includes fermions.

a term proportional to the tree-level S-matrix which can be interpreted as a scheme-dependent shift in the coupling [14] – with the one known from perturbation theory. Importantly, the latter includes one-loop corrections coming from a gauge-fixing procedure which integrates out unphysical fields [15] and results in contributions to the one-loop S-matrix which restore various properties of integrability.

As for *relativistic, supersymmetric models*, which have checked the procedure on theories obtained as Pohlmeyer reductions of the Green-Schwarz action for the Type IIB superstring on $AdS_5 \times S^5$[16, 17], $AdS_3 \times S^3$ [18] and $AdS_2 \times S^2$ [16] which can be seen as supersymmetric generalizations of the bosonic models considered above [15]. These reduced theories are all classically integrable, demonstrated by the existence of a Lax connection, and conjectured to be UV-finite [20]. The tree-level and one-loop S-matrices for these theories were computed in [21, 22], while the exact S-matrices have been conjectured using integrability techniques in [23] for the reduced $AdS_2 \times S^2$ model, [22] for the reduced $AdS_3 \times S^3$ model and [24] for the reduced $AdS_5 \times S^5$ model. In all the cases considered, the agreement is *exact* and no additional shift of the coupling is needed. The presence of the super-symmetry, albeit deformed, may provide an explanation for this, with shifts arising from bosonic loops cancelled by shifts from fermionic loops. Importantly, in the reduced $AdS_3 \times S^3$ standard perturbative computation a contribution coming from a one-loop correction needs to be added so that the S-matrix satisfies the Yang-Baxter equation. It is this S-matrix that the unitarity technique matches. This is then another example of how unitarity methods applied to a classically integrable theory seem to provide a *quantum integrable* result. This seems to suggest a relationship between integrable quantization and unitarity techniques which would be interesting to investigate further.

4 $AdS_5 \times S^5$ superstring world-sheet theory

We have finally considered the case of the light-cone gauge-fixed superstring on $AdS_5 \times S^5$ and its world-sheet S-matrix [16]. Assuming the quantum integrability of the full world-sheet theory and using the global symmetries the *exact* world-sheet S-matrix has been uniquely determined [26] up to an overall phase, or dressing factor [27]. The determination of the latter exploited the non-relativistic generalization

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[14] In the sine-Gordon case the agreement is exact. For $n \geq 2$ the shift in the coupling is by the dual Coxeter number of the group $G = SO(n)$, a structure that appears regularly in the quantization of WZW and gauged WZW models, where k is the quantized level (see for example [14]).

[15] The reduced $AdS_2 \times S^2$ theory is in fact given by the $\mathcal{N} = 2$ supersymmetric sine-Gordon model. The reduced $AdS_3 \times S^3$ and $AdS_5 \times S^5$ theories have a non-local $\mathcal{N} = 4$ and $\mathcal{N} = 8$ supersymmetry respectively, which manifests as a q-deformation of the S-matrix symmetry algebra. We have also checked that the unitarity-cutting procedure matches the perturbative result at one-loop in the $\mathcal{N} = 1$ supersymmetric sine-Gordon model [19].

[16] Notice that this is a *non-relativistic* model, as seen quantizing it perturbatively and noticing that the choice of a flat Minkowski worldsheet metric is incompatible with Virasoro constraints (see for example [25]).

of the crossing symmetry [28, 29] as well as perturbative data both from the string and gauge theory sides [30, 31]. Relaxing the level-matching condition and taking the limit of infinite light-cone momentum (decompactification limit), the world-sheet theory becomes a massive field theory defined on a plane, with well-defined asymptotic states and S-matrix. The scattering of the world-sheet excitations has been studied at tree-level in [32], while one-loop [33] and two-loop [34] results have been carried out only in the simpler near-flat-space limit [35] where interactions are at most quartic in the fields. These studies have also explicitly shown some consequences of the integrability of the model, such as the factorization of the many-body S-matrix and the absence of particle production in the scattering processes [36].

The tree level matrix elements were evaluated in [32] in the generalized uniform light-cone gauge (showing therefore an explicit dependence on the parameter a labeling different light-cone gauge choices [37]) at leading order in perturbation theory, where the small parameter is the inverse of the string tension $g = \frac{\sqrt{\lambda}}{2\pi}$. After having explicitly verified that the tree-level matrix elements above verify the fermionic generalization of the consistency relation (10), we could safely use them as an input of our procedure and get the one-loop S-matrix for the light-cone gauge-fixed sigma model [17]. As a first result, an overall phase could be resummed at the one-loop order, which show the expected gauge dependence [39]. As mentioned above, because of the complicated structure of interactions of the light-cone gauge-fixed sigma model, the perturbative S-matrix is known beyond the leading order [33, 34] only in the kinematic truncation known as near-flat-space limit [35]. Therefore, to test the validity of the unitarity method, we needed to compare our one-loop result to the corresponding limit of the exact world-sheet S-matrix. This was achieved by considering the matrix elements derived in [26] for a single $SU(2|2)$ sector together with the dressing phase, here needed at next-to-leading order in the $1/\sqrt{\lambda}$ expansion [18].

In comparing the exact S-matrix with the one found via unitarity cuts [19] we found

$$
\left(S_{AB}^{CD}\right)_{\text{exact}} = e^{\frac{i}{4g}\left(([A]+2[B]-[C]-2)\text{p}_1+([B]-2[C]-[D]+2)\text{p}_2\right)} e^{\varphi_{a=0}(\text{p}_1,\text{p}_2)} \left(S_{AB}^{CD}\right)_{\text{cut}} + \mathcal{O}(1/g^3) .
\tag{11}
$$

From (11) we see that we have agreement up to a phase whose argument is linear in momenta. This is not surprising, as it simply amounts to moving from the string frame to the spin-chain frame [40, 41]. As argued already at the tree level [32], such terms should not affect the physical spectrum following from inputting the S-matrix into the asymptotic Bethe equations.

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[17] Notice that the non-relativistic dispersion relation $\varepsilon(\text{p}) = \sqrt{1 + \frac{\lambda}{\pi^2}\sin^2\frac{\text{p}}{2}}$ [38, 26], when expanded in the near-BMN limit $\text{p} \to \zeta\text{p}$, corresponding to the perturbative regime, leads to a relativistic energy $\varepsilon_i = \sqrt{1 + \text{p}_i^2}$.

[18] In the comparison with the world-sheet calculation all dimensional quantities (such as the spin-chain length and the momenta) should be rescaled via a factor of $\sqrt{\lambda}/(2\pi)$ [32], for us $\text{p} \to \zeta\text{p}$.

[19] This is done in the so-called constant-J gauge $a = 0$.

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