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The Decomposition of Disease and Disability Life Expectancies in England, 1992-2004.

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Decomposition of Disease and Disability Life Expectancies in England, 1992-2004

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Non-Technical Summary

At present, evaluation of health programs is based principally on life expectancy due to the easier availability of mortality than morbidity data. Whereas mortality data are routinely collected by national registers, morbidity statistics are generally obtained through *ad hoc* surveys. Since morbidity and mortality are complementary aspects of a population's health, a good measure of morbidity life expectancy (or life expectancy with diseases which are not necessarily limiting) should comprise both. By using the Sullivan method (1971), which estimates the number of years with morbidity through the product of the total number of person-years (from life tables) and morbidity prevalence rates, an index resulting from both morbidity and mortality can be derived.

Any morbidity life expectancy is influenced by a range of diseases whose effect is unknown unless decomposition techniques are applied to determine their individual contributions. Identifying the main conditions which trigger the most significant numbers of years spent with morbidity would help to assess and target areas where the most intervention is needed. It is, therefore, worth investigating.

The project aimed to apply a decomposition method to investigate the effect of the disease distribution on changes in morbidity life expectancies over time and on differences between the sexes.

England was selected for the study since the Health Survey for England ("HSE") includes detailed information on morbidity. The study was carried out on the period 1991-2005. Mortality and population counts were provided by the Office for National Statistics ("ONS") whilst disease counts were drawn from the HSE.

The morbidity life expectancies computed in the study comprised the standard measure of disability life expectancy (i.e. time spent living with a limiting long term illness) and a wider measure of morbidity life expectancy that also takes into account diseases which do not limit daily activities. Whilst data for the latter were available for the whole period, data for the former were only available from 1997 since it was only from that year that individuals were asked whether any long term illness they had was limiting.

Reported diseases and disabilities were aggregated into categories. These reflected a combination of trauma, chronic and long term conditions, as well as infectious diseases and acute episodes. The categories were infections, neoplasm, cardiovascular diseases, respiratory or other chronic diseases (i.e. digestive, musculoskeletal, and mental diseases), and other acute diseases (i.e. endocrine, blood, genitourinary, skin, eye and ear diseases).

In line with the official figures released by ONS, morbidity life expectancies were computed using 3-year data periods (i.e. 1991-1993, 1994-1996,....., 2003-2005). Therefore, the first central year in the analysis was 1992 and the last was 2004. Based on the survey data availability, disease and disability life expectancies were computed for 1991-2005 and 1997-2005, respectively.

The decomposition method provided by Nussleder and Looman (2004) was applied to disentangle changes to the morbidity life expectancies occurring over time and differences between the sexes.

Until now, the method had only been applied to obtain the effects attributable to each main disease separately. The focus on single diseases, as opposed to co-morbidity, can hide, however, significant

information in an ageing population such as that in the UK; at older ages people might suffer from multiple disease conditions which are not accounted for by decomposition performed purely on single disease categories. Therefore, for this research, the reported diseases drawn from the HSE were categorised into two exclusive groups: "single diseases" and "co-morbidity diseases".

Table 1 sets out a summary of some important results between 1998 and 2004.

Table 1: Changes in various measures of life expectancy at age 16 between 1998 and 2004

Life expectancies	Males			Females		
Life expectancies	1998	2004	Difference	1998	2004	Difference
Total life expectancy	59.7	61.5	1.8	64.5	65.7	1.2
Years spent without diseases	29.4	29.6	0.2	28.3	28.5	0.2
Years spent without disabilities	44.4	46.1	1.7	46.1	47.0	0.9
Years spent with disease	30.3	31.9	1.6	36.2	37.2	1.0
Years spent with disabilities	15.3	15.4	0.1	18.4	18.7	0.3

Table 1 suggests the following:

For males:

- of the 1.8 years of increased life expectancy, 0.1 years were disability free and 1.7 years were with disability
- of the 1.8 years of increased life expectancy, 0.2 years were spent without disease and 1.6 years with disease
- of the extra 1.6 years spent with disease, 0.2 years were due to a change in morbidity. This change resulted from a 1.6 years increase from multiple diseases and 1.4 years decline from single diseases.

For females:

- of the 1.2 years of increased life expectancy, 0.9 years were disability free and 0.3 years were with disability
- of the 1.2 years of increased life expectancy, 0.2 years were spent without disease and 1.0 years with disease
- the extra 1.0 years spent with disease was due entirely to increased life expectancy. The morbidity component as a whole did not contribute because there was a 1.5 years increase from co-morbidity and a 1.5 years decline from single diseases.

The results imply that, in recent years:

- a. life expectancy has been increasing for both sexes but the increase has been greater for males
- b. life expectancy with disease has increased more for males than females
- c. most of the additional years are being spent with diseases which are not limiting
- d. in the disease state, more years are being spent with co-morbidity as opposed to a single disease

The research also found that the co-morbidity category "cardiovascular, respiratory or other chronic diseases, and other acute diseases" was a significant cause for increasing both disabled and disease life expectancies.

The above trends have significant implications for health policy in the UK to the extent that they imply additional health costs and greater care needs coupled with reductions in work-related activity due to ill health. A further stage in this research would, therefore, be to seek to quantify these effects and investigate possible policies that might limit or stem their future progress.

Significantly, in future, ONS intends to decompose disability life expectancy and publish the relevant health statistics as a result of this project.

Full Research Report

Background

There is an emerging debate about how increasing longevity influences health and whether the additional years of life are spent in good health, or with disability or disease. A population that spends more years in disease or disability will have the effect of increasing the demands made upon health and social care services. Since most disability and disease occurs in the last years of life most of these extra demands will come from the older population, which is expected to increase from around 11m to 16m over the next decade. However, despite increasing life expectancy, there are competing theories that seem to point in different directions as to whether a greater or lesser proportion of life is being spent in ill-health (Karlsson et al 2006).

Part of the problem is definitional i.e. what is meant by 'disability', and this can frustrate comparative studies and often lead to divergent conclusions. A confounding problem is that is possible to suffer from a long term disease such as hypertension or diabetes without necessarily being 'disabled' to the extent that it restricts activities of daily living and thus disrupts normal life. A person with a long term disease will nevertheless make demands on health services, but not necessarily on social care. A person suffering from a disability, perhaps caused by a long term limiting illness, could on the other hand be expected to make demands upon both health and social care services. It therefore seems important to be able to disaggregate these effects, and track them through time to see how they change, either for better or worse. As a result of better measurement, society should be able to improve its knowledge and understanding not only about trends but also the long term effectiveness of different policies and interventions aimed, for example, at prolonging life and curing disease.

The purpose of this paper is to explore available data in order to unpick some of these issues and make appropriate estimates of their effects using data from the UK. At present, evaluation of health programs is based principally on life expectancy due to the easier availability of mortality than morbidity data. Whereas vital statistics are routinely collected by national registers, morbidity statistics are generally obtained through ad hoc surveys. In England and Wales the largest surveys collecting morbidity information are the General Household Survey (GHS) and the Health Survey for England (HSE).

Since morbidity and mortality are complementary aspects of a population's health, a good measure of health should comprise both. By using the Sullivan method (1971), which estimates the number of years with morbidity through the product of the total number of person-years (from life tables) and morbidity prevalence rates, an index resulting from both morbidity and mortality can be derived. On these grounds, the Sullivan method is a valuable tool for the verification and monitoring of a population's health and in the UK is applied to compute activity-limiting long-term illness life expectancy (generally called "disability life expectancy" - see Kelly et al 2000; Breakwell & Bajekal 2006).

Disability life expectancy in the UK is obtained through responses to survey questions. There is substantial controversy in the literature over the use of self-reported disability on the grounds that a respondent may

inflate the severity of his/her health problems in order to justify labour force non-participation and disability benefits (Benitez-Silva et al 2004). Also, disability rates do not include morbidity conditions which are not limiting but nevertheless require regular access to health services. As people might spend more years with "not limiting" diseases, morbidity measures which provide a wider coverage of a population's health are desirable depending on their purpose.

Any morbidity life expectancy (or life expectancy with long-term illness but not necessarily limiting) is influenced by a range of diseases whose effect is unknown unless decomposition or cause-elimination techniques are applied to determine their individual contributions in comparable units (in this case years). Recent research has, for instance, found that respiratory diseases are responsible for a significant proportion of serious morbidity in the UK and that, as a result, more resources may be needed to tackle this burden (Chung et al 2002). Identifying the main conditions which trigger the greatest numbers of years spent with morbidity would help to assess and target areas where the most intervention is needed and is, therefore, worth investigating. A decomposition method which enables the identification of the main causes influencing disability life expectancy is available. Up to now, the method has only been applied to obtain the effects attributable to each main disease separately. This was the approach adopted by Nussleder and Looman (2004) when they applied it to Dutch data.

The focus on single diseases, as opposed to co-morbidity, can hide, however, significant information in an ageing population such as that in the UK; at older ages people might suffer, in fact, from multiple disease conditions (Cornoni-Huntley et al 1991; Guralnik 1996; Rijken et al 2005) which are not accounted for by decomposition performed on single disease categories.

In response to these key issues, the project aimed to compute, along with the standard measure of disability life expectancy, a measure that could provide a more objective and wider coverage of a population's health that takes into account the fact that diseases may not be 'limiting". Furthermore, we wished to investigate changes in disability life expectancy which are due to co-morbidity conditions as well as single diseases.

Objectives

The initial aims of the project were to:

- a. Compute disability-free life expectancy in England using the HSE from 1991 to 2001
- b. Apply the decomposition method of Nussleder and Looman (2004) to explain changes in this measure over time for each sex
- c. Identify the single diseases which account for the changes.

The first aim was altered in four ways:

- 1. The availability of surveys and mortality rates up to 2005, rather than just 2001, led to the period of analysis being extended. This enabled a more complete investigation of the population's health in England to be carried out
- 2. "Disability life expectancy" was preferred to "disability-free life expectancy" as a measure since the former can be computed using disability prevalence rates and, therefore, better reflects changes in morbidity conditions
- 3. Along with "disability life expectancy", an additional measure of morbidity life expectancy was computed: "disease life expectancy". This was expected time spent with a disease which was not necessarily "limiting" in nature
- 4. The HSE did not include until 1997 a question needed for the computation of disability life expectancy; hence this measure was computed from 1997 onwards rather than 1991 as planned.

The second aim was extended by investigating differences between the sexes as well as changes within each sex over time.

The third aim was expanded by identifying the effect which co-occurring diseases, as well as single diseases, had on each morbidity life expectancy measure.

Method

Data

In order for the Sullivan method (1971) to be applied, mortality rates and prevalence morbidity rates are required. Hence, the data needed to comprise death, population and morbidity counts by sex and age. Mortality and population counts were provided by the Office for National Statistics (ONS) whilst morbidity counts were drawn from the HSE.

Sample sizes by sex for each survey year are shown in Table 1. The survey includes questions on the occurrence of long-term and limiting long-term illness, and on the occurrence of conditions that require medicine to be taken regularly.

Table 1. Health Survey for England between 1991 and 2005. Sample size for aged 16 and over.

HSE surveys	Males	Females
1991	1,492	1,750
1992	1,868	2,150
1993	8,416	9,271
1994	7,178	8,631
1995	7,335	8,720
1996	7,486	8,957
1997	3,898	4,684
1998	7,193	8,715
1999	3,558	4,240
2000	4,266	6,215
2001	6,966	8,681
2002	4,543	5,788
2003	6,602	8,234
2004	2,879	3,825
2005	4,629	5,674

Source: HSE

Respondents with a long-term illness could list up to six illnesses, which were categorised according to the classification adopted for deaths.

The prescribed medicine classification was used to categorise the respondents within disease groupings and the information was later used to investigate the morbidity conditions having the greatest influence on morbidity life expectancy. The prescribed medicine classification was available for every year of the survey except 1996.

The counts from the questions on long-term illness and medicine were used to obtain the wider measure of morbidity, "disease life expectancy". The question on limiting illness, used for the computation of disability life expectancy, was included for the first time in 1997.

The long-term illnesses and the diseases treated with the prescribed medicine were aggregated in five broad categories. These reflect a combination of trauma, chronic and long term conditions, as well as infectious diseases and acute episodes and were informed by the terminology used in the HSE. The categories are infections, neoplasm, cardiovascular diseases, respiratory or other chronic diseases (i.e. digestive, musculoskeletal, and mental diseases), and other acute diseases (i.e. endocrine, blood, genitourinary, skin, eye and ear diseases). These categories represented the single diseases used in the decomposition analysis.

Co-morbidity was obtained by combining the five disease categories and, for the decomposition analysis, the co-morbidities having the greatest effect on disease life expectancy were kept:

- "cardiovascular" and "respiratory or other chronic diseases"
- "cardiovascular" and "other acute diseases"
- "respiratory or other chronic diseases" and "other acute diseases"
- "cardiovascular" and "respiratory or other chronic diseases" and "other acute diseases"
- the remaining co-morbidity conditions.

Each respondent to the HSE survey was included only in one category (either single or multiple diseases).

Methodology

The Sullivan method requires morbidity prevalence rates and person-years from life tables, whose product provides the person-years with morbidity. In line with the official figures released by the Government Actuary's Department and the ONS, life tables were computed using 3-year data periods (i.e. 1991-1993, 1994-1996,....., 2003-2005). Therefore, the first central year in the analysis was 1992 and the last was 2004. Disease and disability rates were similarly obtained using 3-year survey periods. Based on the survey data availability, disease and disability life expectancies covered the periods 1991-2005 and 1997-2005 respectively.

The decomposition method (Nussleder & Looman 2004) was applied to disentangle changes to the morbidity life expectancies occurring over time for each sex and differences in morbidity life expectancies between the sexes. The method separates the mortality and morbidity contributions to morbidity life expectancy. For example, a change over time is decomposed according to the following formula:

$$\delta L_{morb} = \delta L \left(\frac{\pi_t + \pi_{t+n}}{2} \right) + \delta \pi \left(\frac{L_t + L_{t+n}}{2} \right)$$
 (1)

Where L_{morb} represents the person-years with morbidity, L represents the total number of person-years, π is the prevalence morbidity rate, and δ is the symbol for a change over time (of for a difference between the sexes depending on the decomposition purpose). t refers to the first time point of the decomposition and t+n to the second time point of the analysis; in a decomposition between the sexes t and t+n refers to the male and female populations. It is important to note that, in all of the analysis which follows, the first component of the morbidity life expectancy shown in equation (1) is the "mortality component" and the second is the "morbidity component".

The decomposition was applied to explain:

- the changes in both disabled life expectancy and disease life expectancy between 1992 and 2004, 1992 and 1998, and 1998 and 2004
- the differences in these measures between the sexes in 1992, 1998 and 2004.

All analyses were performed using STATA (a statistical computer package).

Results

Life expectancy with and without disease and disability

Table 2 shows for each sex the total life expectancy, the disease and disability life expectancies at age 16 and the variation which occurred between 1992 and 2004.

In 12 years the total life expectancy increased by 3.1 years for males and 2.0 years for females. For both sexes the change was greater in the most recent period (1998-2004) when males gained 1.8 years and females 1.2 years.

Life expectancy with diseases increased more rapidly than total life expectancy and this pattern was more evident for males than females (5.2 vs 2.6 years). The greatest change occurred in the earlier period (1992-1998) when males gained 3.6 years and females 1.7 years.

These findings suggest that total and disease life expectancies did not increase at the same rate and the majority of the increase occurred in different time periods from one another (i.e. 1992-1998 for disease life expectancy and 1998-2004 for total life expectancy).

Between 1998 and 2004 the number of years with disability increased by 0.1 years for males and 0.3 years for females compared to a change of 1.6 and 0.9 years in disease life expectancy. Hence, if the additional measure of disease life expectancy had not been computed, morbidity in England would have been underestimated by 1.5 years for males and 0.6 years for females.

It appears that the proportion of life spent with disease is considerable and much higher than for disability life expectancy. For example, amongst females, the disease life expectancy was approximately 56 per cent of total life expectancy during the period 1992 to 2004, whereas disability life expectancy was about 28 per cent of the total life expectancy in 1998 and in 2004. Males tended to spend a smaller proportion of their lives with disability or disease than females (approximately 48 per cent and 25 per cent, respectively).

It should be noted that, because the HSE is carried out on households, people residing in institutional care homes are not included. This might cause life expectancies with disease or disability to be underestimated. However, based on past research, which found that allowing for the institutional population does not alter the disability-free life expectancy at birth significantly (Bebbington and Darton 1996), any underestimation is likely to be small.

Table 2. Total, disease and disability life expectancies at age 16 by sex and year.

Years	Males			Females			
	Total LE	Disease LE	Disability LE	Total LE	Disease LE	Disability LE	
1992	58.4	26.7	-	63.7	34.5	-	
		[26.2 - 27.2]			[34.0 - 35.0]		
1993	58.7	27.7	-	63.9	35.2	-	
		[27.3 - 28.1]			[34.8 - 35.6]		
1994	58.8	28.4	-	63.9	35.5	-	
		[28.1 - 28.8]			[35.2 - 35.9]		
1995	59.1	-	-	64.1	-	-	
1996	59.2	-	-	64.2	-	-	
1997	59.5	-	-	64.4	-	-	
1998	59.7	30.3	15.3	64.5	36.2	18.4	
		[29.9 – 30.8]	[14.9 – 15.7]		[35.8 – 36.7]	[18.0 – 18.8]	
1999	60.0	30.4	15.0	64.7	36.4	16.9	
		[29.9 - 30.8]	[14.6 – 15.4]		[35.9 - 36.8]	[16.5 – 17.3]	
2000	60.3	30.6	14.9	64.9	36.9	17.1	
		[30.2 - 31.1]	[14.5 – 15.3]		[36.5 - 37.3]	[16.7 – 17.5]	
2001	60.6	32.0	15.1	65.2	39.0	17.4	
		[31.6 - 32.4]	[14.7 – 15.5]		[38.6 - 39.4]	[17.0 – 17.8]	
2002	60.9	32.3	15.5	65.3	38.9	18.8	
		[31.9 - 32.7]	[15.1 – 15.9]		[38.5 - 39.3]	[18.4 – 19.2]	
2003	61.2	31.9	15.6	65.5	37.6	18.9	
		[31.4 - 32.4]	[15.2 – 16.1]		[37.1 - 38.0]	[18.5 – 19.3]	
2004	61.5	31.9	15.4	65.7	37.2	18.7	
		[31.5 - 32.4]	[15.0 – 15.8]		[36.7 – 37.6]	[18.3 – 19.2]	
Change 1992-2004	3.11	5.26		2.00	2.65		
Change 1992-1998	1.30	3.62		0.80	1.72		
Change 1998-2004	1.81	1.64	0.07	1.20	0.93	0.32	

Decomposition of changes to disease life expectancies

Changes in the decomposition of disease life expectancy over time are shown in Table 3. Each change was separated into the mortality and morbidity effects, and the latter was further decomposed into single and co-occurring disease components. A negative sign for the morbidity component is actually a positive feature: it suggests individuals are spending less time with disease than at the earlier time point.

For males, the disease life expectancy increased by 5.3 years between 1992 and 2004; 2.9 years of this change were due to increased morbidity and 2.4 to increased life expectancy. Most of the morbidity increase occurred in the earlier sub-period (i.e. 1992-1998). Similar comments can be made about females.

Table 3. Changes in disease life expectancy over time. Mortality and morbidity effects.

Changes	Males		Females			
	1992 vs 2004	1992 vs 1998	1998 vs 2004	1992 vs 2004	1992 vs 1998	1998 vs 2004
Total change	5.26	3.62	1.64	2.65	1.72	0.93
Decomposition						
Mortality change	2.40	0.98	1.42	1.59	0.62	0.97
Morbidity change	2.86	2.64	0.22	1.06	1.10	-0.04
Decomposition of morbidity						
Single diseases	-0.39	1.04	-1.43	-2.57	-1.04	-1.53
Co-morbidity	3.25	1.60	1.65	3.63	2.14	1.49

The results for single and co-occurring diseases are presented in Figure 1.

For males, between 1992 and 2004 the years spent with single diseases decreased with the exception of those spent with cancer which rose by 0.2 years and those spent with "respiratory or other chronic diseases" which rose by 0.7 years. Amongst co-morbidities, the biggest disease life expectancy increase occurred for the category comprising "cardiovascular diseases", "respiratory or other chronic diseases", and "other acute diseases" (0.9 years).

For females, the biggest change in disease life expectancy occurred for the co-morbidity category comprising "cardiovascular", "respiratory or other chronic diseases", and "other acute diseases" which increased by 1.4 years.

MALES Resp or CDV & CDV & Chronic & CDV & other Other co-CDV chronic Other acute Chronic Other Other Chronic & diseases diseases Infections Cancer Other diseases diseases morb. 1.75 1.50 1.25 1.00 0.75 0.50 Change in years 0.25 0.00 -0.25 -0.50 -0.75 -1.00 -1.25 -1.50 1998 vs 2004 -1.75 1992 vs 1998 -2.00 ■ 1992 vs 2004 -2.50 -SINGLE DISEASES COMORBIDITIES **FEMALES** Resp and CDV & CDV & Chronic & CDV & CDV chronic Chronic Chronic & Other co-Other acute Other Other diseases Infections diseases diseases diseases Other morb. 1.75 1.50 1.25 1.00 0.75 0.50 Change in years 0.25 0.00 -0.25 -0.50 -0.75 -1 00 -1.25 -1.50 □1998 vs 2004 -1 75 □ 1992 vs 1998 -2.00 ■- 1992 vs 2004 -2.25 COMORBIDITIES SINGLE DISEASES

Figure 1. Decomposition of the morbidity effect accounting for changes in disease life expectancy.

Note: In the co-morbidity categories the abbreviation "Chronic" refers to "Respiratory or other chronic" diseases

Decomposition was also used to explain differences between the sexes. A positive sign for the morbidity category means that the morbidity rates are larger for females than males whilst a positive sign for the mortality category indicates that females survive longer than males.

The decomposition between females and males was performed for 1992, 1998 and 2004. Table 4 shows that in 1992 females spent 7.8 years longer with disease than males; the gap dropped to 5.2 years in 2004.

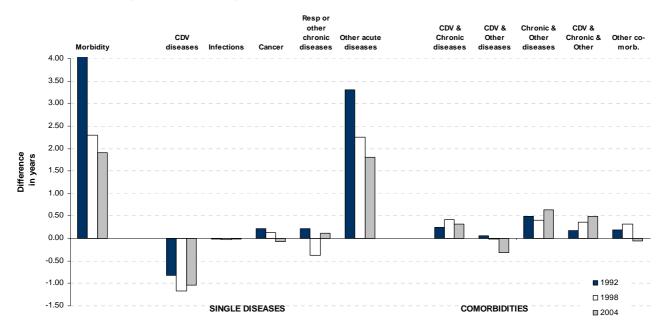
In terms of reported diseases, the greater number of years spent with diseases by females was caused by higher prevalence rates of "other acute diseases", which accounted for 3.3 years in 1992 and 1.8 years in 2004 (Figure 2).

Table 4. Differences between the sexes (females vs males) in disease life expectancy.

Mortality and morbidity effects.

Differences	1992	1998	2004
Total difference	7.84	5.93	5.23
Total difference	7.04	5.93	5.25
Decomposition			
Mortality difference	3.77	3.63	3.30
Morbidity difference	4.07	2.30	1.93
Decomposition of morbidity			
Single diseases	2.90	0.82	0.85
Co-morbidity	1.17	1.48	1.08

Figure 2. Decomposition of the morbidity effect accounting for differences in disease life expectancy between the sexes (females vs males)



Source: own analysis on ONS and HSE data

Decomposition of changes to disability life expectancy

Between 1998 and 2004, disability life expectancy gained about 0.1 years for males and 0.3 years for females (Table 5); these changes result from the counterbalancing values of mortality and disability.

For males, the mortality component increased by 0.8 years whilst the disability component decreased by 0.7 years causing a total change of 0.1 years in disability life expectancy.

For females, the mortality component increased by 0.6 years and the disability went down by 0.3 years leading to a change of 0.3 years.

Therefore, for both sexes most of the increase in life expectancy was spent without disabilities.

Analysing the decomposition of morbidity into their single and co-occurring disability components, the morbidity component was negative overall because the decrease from the single disability component more than offset the increase from the co-morbidity component. Between 1998 and 2004 the life spent with multiple disabilities increased by 0.5 years for males and 0.9 years for females. However, because of a larger decline of life spent with single diseases, there was an overall improvement in the quality of survival.

Table 5. Change in disability life expectancy over time, 1998-2004. Mortality and morbidity effects.

Changes	Males	Females
Total change	0.07	0.32
Decomposition		
Mortality change	0.83	0.62
Morbidity change	-0.76	-0.30
Decomposition of morbidity		
Single diseases	-1.25	-1.20
Co-morbidity	0.49	0.90

Decomposition into disability causes shows that the biggest change between 1998 and 2004 occurred for the life expectancy with "respiratory and other chronic diseases" which declined by about 0.6 years for both sexes (Figure 3). This is an encouraging finding.

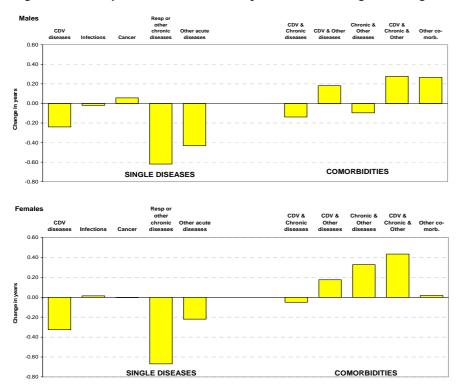


Figure 3. Decomposition of the disability effect accounting for changes in disability life expectancy.

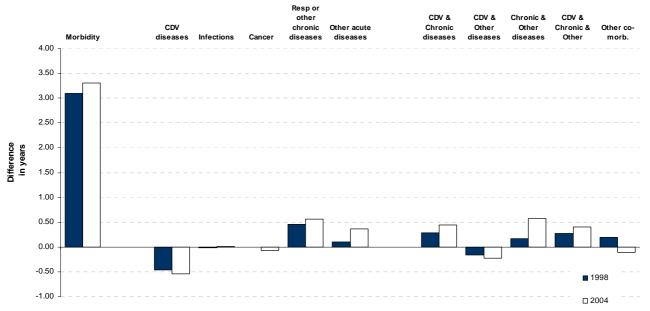
Regarding differences between the sexes, in 1998 and 2004 females spent about three more years with disability than males (Table 6). Most of the increase is attributable to pure survival, with the rest mainly due to the effect of co-morbidity. Life expectancy with single and co-occurring disabilities were higher for females than males.

Table 6. Differences between the sexes (females vs males) in disease life expectancy. Mortality and morbidity effects.

Differences	1998	2004
Total difference	3.1	3.3
Decomposition		
Mortality difference	2.3	1.9
Morbidity difference	0.8	1.4
Decomposition of morbidity		
Single diseases	0.1	0.3
Co-morbidity	0.7	1.1

By partitioning the single and co-morbidity categories into the reported disability categories (Figure 4), it transpired that females lived from 0.1 to 0.6 years longer than males with any disability apart from "cardiovascular diseases" and "cardiovascular and other acute diseases", where the prevalence rates were higher for males.

Figure 4. Decomposition of the disability effect accounting for differences in limiting long-term illness life expectancy between the sexes (females vs males).



Source: own analysis on ONS and HSE data

Discussion

We have discussed two types of life expectancy with disease: "disability life expectancy" which is the years spent with long term conditions that affect daily activities and "disease life expectancy" which is the years spent with diseases that may or may not be limiting daily activities. These measures were used to analyse changes over time, and to attribute changes to specific categories of disease. The results appear to suggest important trends which have implications both for the health of the population and for health policy.

The findings point to extension of life together with expansion of time spent with disease. However, because morbidity life expectancy time increasing or decreasing is a result of both mortality and morbidity prevalence rates, any conclusions about morbidity trends should follow from decomposition into the two components.

Through the application of the Nussleder and Looman method (2004) on changes in disease life expectancy, the study found that an increasing amount of time was spent with disease between 1992 and 1998; whereas it was fairly stable between 1998 and 2004 - the majority of the change in disease life expectancy reflected life extension.

Increases, reductions or stability in morbidity life expectancy stem from the pattern of the contributing diseases. In this study, these are given by the broad categories "single diseases" and "co-occurring diseases" and by the specific diseases contained within those two categories. The results suggest that morbidity

expansion between 1992 and 1998 was mainly driven by an increase in the co-morbidity component whilst morbidity stability between 1998 and 2004 was a result of the trends in single and co-occurring diseases offsetting one another.

The increase in the co-morbidity contribution to disabled life expectancy can be explained by the fact that the population is ageing. Older people are more likely than younger people to have several co-existing health problems (Cornoni-Huntley et al 1991; Guralnik 1996; Rijken et al 2005).

Between 1998 and 2004, the morbidity component did not have the same effect on disease life expectancy as it did on disability life expectancy. Despite the increase in both measures of morbidity life expectancies, the morbidity component was practically stable for disease life expectancy and declined for disability life expectancy. The findings suggest that, as life expectancy increases, the number of years spent with morbidity (computed through the Sullivan method) increases; this accords with a study forecasting future disability levels (Jagger et al. 2006).

Co-morbidity provided a larger contribution to differences between the sexes in disability life expectancy than disease life expectancy. This is consistent with the higher life expectancy of females and with the fact that disability is more prevalent at older ages.

In conclusion, it is emphasised that, compared to the Nusselder and Looman study (2004), the total morbidity effect was split into the separate contributions from single and co-occurring diseases. Since time spent with co-morbidities has been increasing and time spent with single morbidities has been decreasing estimates unadjusted for co-morbidity would have overestimated the effect of single diseases.

The study proposed a measure (i.e. disease life expectancy) which provides a wider coverage of morbidity conditions than the disability life expectancy and a more objective measure of morbidity than those based on self assessed health conditions.

Table 7 sets out a summary of the various life expectancy measures discussed in this work between 1998 and 2004. It suggests that:

- For males, disability-free life expectancy increased by 1.7 years. This implies that of the 1.8 years in increased life expectancy, 0.1 years were disability free and 1.7 years were spent with disability
- For females, disability-free life expectancy increased 0.9 years. This implies that of the 1.2 years in increased life expectancy, 0.9 years were disability free and 0.3 years were spent with disability.
- Disease-free life expectancy for males increased by 0.2 years. Therefore, of the 1.8 years of increased life expectancy, 0.2 years were spent without disease and 1.6 years with disease
- Disease-free life expectancy for females also increased by 0.2 years. Therefore, of the extra 1.2 years of life expectancy, 0.2 years were spent without disease and 1.0 years with disease.
- For males, of the extra 1.6 years spent with disease, 0.2 years were due to a change in morbidity. This change resulted from 1.6 years increase from multiple diseases and 1.4 years decline from single diseases.

- For females, the extra 1.0 years spent with disease was due entirely to increased life expectancy. The morbidity component as a whole did not contribute because there was a 1.5 years increase from comorbidity and a 1.5 years decline from single diseases.

Table 7 Changes in various measures of life expectancy between 1998 and 2004

Life annual and	Males		Females			
Life expectancies	1998	2004	Difference	1998	2004	Difference
Total life expectancy	59.7	61.5	1.8	64.5	65.7	1.2
Years spent without diseases	29.4	29.6	0.2	28.3	28.5	0.2
Years spent without disabilities	44.4	46.1	1.7	46.1	47.0	0.9
Years spent with disease	30.3	31.9	1.6	36.2	37.2	1.0
Years spent with disabilities	15.3	15.4	0.1	18.4	18.7	0.3

These results imply that:

- life expectancy is increasing for both males and females but the increase is larger for males
- life expectancy with disease has increased more for males than for females
- most of the additional years are being spent with diseases which are not limiting
- in the disease state, more years are being spent with co-morbidity as opposed to a single disease.

These trends, if accurately reflected in the data and methodology, thus have significant implications for health policy in the UK to the extent that they imply additional health costs and greater care needs, coupled with reductions in work related activity due to ill health. A further stage in this research therefore would be to seek to quantify these effects and investigate possible policies that might limit or stem their future progress.

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References

Benitez-Silva H., M. Buchinsky, H. Man Chan, S. Cheidvasser, & J. Rust (2004) How large is the bias in self-reported disability, *Journal of Applied Econometrics*, 19, 649-670.

Breakwell C., & M. Bajekal (2005). Review of sources and methods to monitor healthy life expectancy, *Health Statistics Quarterly*, 26, 17-22.

Case A., & C. Paxson (2005). Sex differences in Morbidity and Mortality, Demography, 42 (2), 189-214.

Chung F., N. Barnes, M. Allen, R. Angus, P. Corris, A. Knox, J. Miles, A. Morice, J. O'Reilly, & M. Richardson (2004)., Assessing the burden of respiratory disease in the UK, *Respiratory Medicine*, 96 (12), 963-975.

Cornoni-Huntley J.C., D.J. Foley, & J. Guralnik (1991). Co-Morbidity Analysis: A Strategy for Understanding Mortality, Disability and Use of Health Care Facilities of Older People, *International Journal of Epidemiology*, 20 (1), S8-S17.

Dalstra J.A.A., A.E. Kunst, C. Borrell, E. Breeze, E. Cambois, G. Costa, J.J.M. Geurts, E. Lahelma, H. Van Oyen, N.K. Rasmussen, E. Regidor, T. Spadea, & J.P. Mackhenbach (2005), Socioeconomic differences in the prevalence of common chronic diseases: and overview of eight European countries, *International Journal of Epidemiology*, 34, 316-326.

Guralnik J.M (1996). Assessing the impact of Comorbidity in the Older Population, *Annals of Epidemiology*, 6 (5), 376-380

Jagger C., R. Matthews, N. Spier, C. Brayne, A. Comas-Herrera, T. Robinson, J. Lindesey, & P. Croft (2006). Compression or Expansion of Disability?, Kings' Fund, *Wanless Social Care Review*.

Karlsson M., Mayhew L., Plumb R. & B. Rickayzen (2006) Future costs for long-term care: Cost projections for long-term care for older people in the United Kingdom, *Health Policy*, 75, 187-213

Kehoe R., S.Y. Wu, M.C. Leske, L.T. Chylack (1994), Comparing self-reported and physician-reported medical history, *American Journal of Epidemiology*, 139, 813-818.

Kelly S., A. Baker, & S. Gupta (2000). Healthy life expectancy in Great Britain, 1980–96, and its use as an indicator in United Kingdom Government strategies, *Health Statistics Quarterly*, 7, 32-37.

Nusselder W., & C.W.N. Looman (2004). Decomposition of differences in health expectancy by cause, *Demography*, 41 (2), 315-334.

Rijken M., M. Kerkhof, J. Dekker, & F. Schellevis (2005). Co-morbidity of chronic diseases, *Quality of Life Research*, 14 (1), 45-55.

Sullivan D.F (1971). A single index of mortality and morbidity, HSMHA Health Reports, 86 (4): 347-354.



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