Supplement to "Forecasting Using Heterogeneous Panels with Cross-Sectional Dependence"

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This document contains the material which are not reported in the paper in order to save space. These are the detailed description of the parameter estimation methods (Section S1) and additional Monte Carlo evidence obtained by considering a dynamic prediction model (Section S2).

S1 Estimation Methods

Consider the stationary predictive panel data model used in the paper. The h-steps ahead variable $y_{i,t+h}$, $h \geq 0$, $i = 1, 2, \ldots, n$, $t = 1, 2, \ldots, T$, is given by

$$y_{i,t+h} = \alpha_i + \boldsymbol{\beta}_i' \mathbf{x}_{it} + \boldsymbol{\gamma}_i^{y\prime} \mathbf{f}_t^y + u_{i,t+h}, \tag{S1}$$

$$u_{i,t+h} = \sum_{j=1}^{n} r_{ij} \varepsilon_{j,t+h}, \tag{S2}$$

where $\mathbf{x}_{it} = (x_{i1t}, x_{i2t}, \ldots, x_{ik_xt})'$ is a $(k_x \times 1)$ vector of observed individual-specific regressors which can include predetermined variables, $\boldsymbol{\beta}_i = (\beta_{i1}, \beta_{i2}, \ldots, \beta_{ik_x})'$ represents the corresponding $(k_x \times 1)$ slope parameters, r_{ij} are unknown spatial weights, ε_{it} is an error term which is uncorrelated over time and individuals. $\mathbf{f}_t^y$ is a vector of unobservable common factors of size m_y, $\boldsymbol{\gamma}_i^y$ are the associated $(m_y \times 1)$ factor loadings. α_i are the unit specific time-invariant effects. Unless otherwise specified, $\boldsymbol{\beta}_i$, $\boldsymbol{\gamma}_i^y$ and α_i are assumed to be fixed parameters.

S1.1 Estimators and Predictors with No Common Factors

The estimators which assume no common factors in the data generating process (DGP) set $\boldsymbol{\gamma}_i^{y\prime}\mathbf{f}_t^y = 0$. In the paper, these estimators are used only in the empirical application and not in simulations. The predictions using these estimators take the form

$$\widehat{y}_{i,T+h|T} = \widehat{\alpha}_i + \widehat{\boldsymbol{\beta}}_i'\mathbf{x}_{iT}, \tag{S3}$$

where for any estimator $\widehat{\boldsymbol{\beta}}_i$ the intercepts are estimated by $\widehat{\alpha}_i = \frac{1}{T-h}\sum_{t=h+1}^{T}\left(y_{i,t} - \widehat{\boldsymbol{\beta}}_i'\mathbf{x}_{i,t-h}\right)$. In the case of a pooled estimator $\widehat{\boldsymbol{\beta}}_i = \widehat{\boldsymbol{\beta}}$.

Let us define the matrices of deviations from the means as $\widetilde{\mathbf{X}}_{i.,-h} = \mathbf{M}_e\mathbf{X}_{i.,-h}$, $\widetilde{\mathbf{y}}_{i.} = \mathbf{M}_e\mathbf{y}_{i.}$, $\mathbf{X}_{i.,-h} = (\mathbf{x}_{i1}', \mathbf{x}_{i2}', \ldots, \mathbf{x}_{i,T-h}')'$, $\mathbf{y}_{i.} = (y_{i,h+1}, y_{i2}, \ldots, y_{iT})'$ and $\mathbf{M}_e = \mathbf{I}_{T-h} - (T-h)^{-1}\mathbf{e}_{T-h}\mathbf{e}_{T-h}'$, where $\mathbf{e}_{T-h}$ is a vector of ones of length $T-h$. The *OLS* estimator of the unit specific intercepts (*Ind. OLS*) is given by

$$\widehat{\boldsymbol{\beta}}_{OLS,i} = \left(\widetilde{\mathbf{X}}_{i.,-h}'\widetilde{\mathbf{X}}_{i.,-h}\right)^{-1}\widetilde{\mathbf{X}}_{i.,-h}'\widetilde{\mathbf{y}}_{i.}. \tag{S4}$$

The predictions labeled by *Ind. OLS* are obtained by replacing (S4) in (S3).

Assume that the coefficients $\boldsymbol{\beta}_i$ follow a random coefficient model

$$\boldsymbol{\beta}_i = \boldsymbol{\beta} + \boldsymbol{\delta_i}, \quad \boldsymbol{\delta}_i \sim \text{IID}(\mathbf{0}, \boldsymbol{\Omega}_\delta), \tag{S5}$$

where $\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots, \beta_{k_x})'$, $\boldsymbol{\delta}_i = (\delta_{i1}, \delta_{i2}, \ldots, \delta_{ik_x})'$, $\boldsymbol{\delta_i}$ are distributed independently of u_{jt} and $\mathbf{x}_{jt}$, for each $i, j = 1, \ldots, n$. Chamberlain (1982) and Pesaran and Smith (1995) suggest estimating the average coefficient vector $\boldsymbol{\beta}$ by the mean group (*MG*) estimator as

$$\widehat{\boldsymbol{\beta}}_{MG} = n^{-1}\sum_{i=1}^{n}\widehat{\boldsymbol{\beta}}_{OLS,i}. \tag{S6}$$

Its asymptotic variance can be estimated by

$$\widehat{\text{Var}}\left(\widehat{\boldsymbol{\beta}}_{MG}\right) = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left(\widehat{\boldsymbol{\beta}}_i - \widehat{\boldsymbol{\beta}}_{MG}\right)\left(\widehat{\boldsymbol{\beta}}_i - \widehat{\boldsymbol{\beta}}_{MG}\right)'. \tag{S7}$$

Swamy (1970) derives the feasible *GLS* estimator of the average of the slope coefficients $\boldsymbol{\beta}_i$ under random coefficients assumption. The estimator of $\boldsymbol{\beta}$ is given by

$$\widehat{\boldsymbol{\beta}}_{SW} = \sum_{i=1}^{n}\mathbf{W}_i\widehat{\boldsymbol{\beta}}_{OLS,i}, \tag{S8}$$

with $\mathbf{W}_i = \left[\sum_{j=1}^{n}\left(\widehat{\boldsymbol{\Omega}}_\delta + \widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{\beta}}_{OLS,j}}\right)^{-1}\right]^{-1}\left(\widehat{\boldsymbol{\Omega}}_\delta + \widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{\beta}}_{OLS,i}}\right)^{-1}$ where $\widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{\beta}}_{OLS,i}} = \widehat{\sigma}_i^2\left(\widetilde{\mathbf{X}}_{i.,-h}'\widetilde{\mathbf{X}}_{i.,-h}\right)^{-1}$, $\widehat{\sigma}_i^2 = \frac{1}{T-h}\widehat{\mathbf{e}}_{i.}'\widehat{\mathbf{e}}_{i.}$, $\widehat{\mathbf{e}}_{i.} = \widetilde{\mathbf{y}}_{i.} - \widetilde{\mathbf{X}}_{i.,-h}\widehat{\boldsymbol{\beta}}_{OLS,i}$ and $\widehat{\boldsymbol{\Omega}}_\delta = n\widehat{\text{Var}}(\widehat{\boldsymbol{\beta}}_{MG})$. The variance of this estimator can be estimated by

$$\widehat{\text{Var}}\left(\widehat{\boldsymbol{\beta}}_{SW}\right) = \left[\sum_{i=1}^{n}\left(\widehat{\boldsymbol{\Omega}}_{\delta} + \widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{\beta}}_{OLS,i}}\right)^{-1}\right]^{-1}. \tag{S9}$$

Conditional on $\boldsymbol{\beta}_i$, the *Ind. OLS* estimator is the best linear unbiased predictor (BLUP) of the random coefficient $\boldsymbol{\beta}_i$ under the assumption of no cross-sectional dependence (CD). Lee and Griffiths (1979) show that, unconditionally, the feasible BLUP of $\boldsymbol{\beta}_i$ is given by

$$\widehat{\boldsymbol{\beta}}_{SW,i} = \widehat{\boldsymbol{\beta}}_{SW} + \widehat{\boldsymbol{\Omega}}_{\delta}\widetilde{\mathbf{X}}'_{i.,-h}\left(\widehat{\sigma}_i^2\mathbf{I}_{T-h} + \widetilde{\mathbf{X}}_{i.,-h}\widehat{\boldsymbol{\Omega}}_{\delta}\widetilde{\mathbf{X}}'_{i.,-h}\right)^{-1}\left(\widetilde{\mathbf{y}}_{i.} - \widetilde{\mathbf{X}}_{i.,-h}\widehat{\boldsymbol{\beta}}_{SW}\right). \tag{S10}$$

The predictions which are called *Ind. GLS* are obtained by replacing (S10) in (S3).

The usual fixed effects (*FE*) estimator of the average slope parameter $\boldsymbol{\beta}$ which is used in the application is

$$\widehat{\boldsymbol{\beta}}_{FE} = \left(\sum_{i=1}^{n}\widetilde{\mathbf{X}}'_{i.,-h}\widetilde{\mathbf{X}}_{i.,-h}\right)^{-1}\sum_{i=1}^{n}\widetilde{\mathbf{X}}'_{i.,-h}\widetilde{\mathbf{y}}_{i.}. \tag{S11}$$

In the case of slope heterogeneity, the variance of this estimator can be estimated by

$$\widehat{\text{Var}}\left(\widehat{\boldsymbol{\beta}}_{FE}\right) = n^{-1}\mathbf{Q}_0^{-1}\boldsymbol{\Lambda}_0\mathbf{Q}_0^{-1}, \tag{S12}$$

where

$$\mathbf{Q}_0 = n^{-1}\sum_{i=1}^{n}\left(\frac{\widetilde{\mathbf{X}}'_{i.,-h}\widetilde{\mathbf{X}}_{i.,-h}}{T-h}\right), \tag{S13}$$

$$\boldsymbol{\Lambda}_0 = \sum_{i=1}^{n}\left(\frac{\widetilde{\mathbf{X}}'_{i.,-h}\widetilde{\mathbf{X}}_{i.,-h}}{T-h}\right)\widehat{\boldsymbol{\Omega}}_{\delta}\left(\frac{\widetilde{\mathbf{X}}'_{i.,-h}\widetilde{\mathbf{X}}_{i.,-h}}{T-h}\right). \tag{S14}$$

The predictions, called *FE*, are obtained by replacing (S11) in (S3).

Under general conditions, the assumption $\boldsymbol{\gamma}_i^{y\prime}\mathbf{f}_t^y = 0$ and that the explanatory variables are strictly exogeneous, $\widehat{\boldsymbol{\beta}}_{OLS,i}$ is consistent for $\boldsymbol{\beta}_i$ as $T \to \infty$ and $\widehat{\boldsymbol{\beta}}_{MG}$, $\widehat{\boldsymbol{\beta}}_{SW}$ and $\widehat{\boldsymbol{\beta}}_{FE}$ are consistent for $\boldsymbol{\beta}$ as $n, T \to \infty$. If the explanatory variables contain weakly exogeneous regressors, Pesaran and Smith (1995) show that $\widehat{\boldsymbol{\beta}}_{FE}$ is inconsistent in the case of heterogeneous slope parameters.

S1.2 Estimators and Predictors with Common Factors

The usual two-way fixed effects (*2WFE*) estimator assumes the existence of common factors, however, it is also assumed that their loadings are homogeneous across panel units. We use the standard formulation for this estimator and its variance. The details can be found, for instance, in Baltagi (2013). To forecast using the *2WFE*, we use the coefficient on the last time dummy in the sample as the future value. Baltagi (2008) reviews alternative methods to forecast with the *2WFE* estimator.

The predictions using the estimators which take into account the common factors and their heterogeneous can be written as

$$\widehat{y}^{P}_{i,T+h|T} = \widehat{\alpha}_i + \widehat{\boldsymbol{\beta}}'_i \mathbf{x}_{iT} + \widehat{\boldsymbol{\gamma}}^{y\prime}_i \widehat{\mathbf{f}}^{y}_{T}, \tag{S15}$$

where the subscript $P \in \{R, A\}$ signifies one the two prediction approaches, the RBA or the AVA. The intercepts are estimated as before by $\widehat{\alpha}_i = \frac{1}{T-h}\sum_{t=h+1}^{T}\left(y_{i,t} - \widehat{\boldsymbol{\beta}}'_i \mathbf{x}_{i,t-h}\right)$ and in the case of a pooled estimator $\widehat{\boldsymbol{\beta}}_i = \widehat{\boldsymbol{\beta}}$.

When the unobserved common factors $\mathbf{f}^{y}_{t}$ are correlated with the explanatory variables $\mathbf{x}_{it}$ the estimators in the previous subsection turn inconsistent. We use two methods to estimate the unobserved common factors in order to achieve consistency. The first one uses simple cross-sectional averages of some observed variables and the second one uses principal component (PC) methods on observed or estimated variables.

Pesaran (2006) proposes to use the cross-sectional averages of the dependent variable and the independent variables as proxies for the common factors. The estimator is built on two crucial assumptions. First, it is assumed that the explanatory variables have a factor structure given by

$$\mathbf{x}_{it} = \mathbf{a}^{x}_{i} + \boldsymbol{\Gamma}^{x\prime}_{i}\mathbf{f}_t + \mathbf{v}_{it}. \tag{S16}$$

Second, the following rank condition[1] on the factor loadings is satisfied

$$\text{rank}(\bar{\mathbf{C}}) = m \leq k_x + 1, \tag{S17}$$

where $\bar{\mathbf{C}} = n^{-1}\sum_{i=1}^{n}\mathbf{C}_i$ and

$$\mathbf{C}_i = \begin{pmatrix} \gamma^{y}_{i} & \boldsymbol{\Gamma}^{x}_{i} \end{pmatrix} \begin{pmatrix} 1 & \mathbf{0} \\ \boldsymbol{\beta}_i & \mathbf{I}_{k_x} \end{pmatrix}.$$

The estimator for the individual parameters, called *Ind. CCE* is given by

$$\widehat{\boldsymbol{\beta}}_{CCE,i} = \left(\mathbf{X}'_{i.,-h}\mathbf{M}_{H_1}\mathbf{X}_{i.,-h}\right)^{-1}\mathbf{X}'_{i.,-h}\mathbf{M}_{H_1}\mathbf{y}_{i.}, \tag{S18}$$

where $\mathbf{M}_{H_1} = \mathbf{I}_{T-h} - \mathbf{H}_1(\mathbf{H}'_1\mathbf{H}_1)^{-}\mathbf{H}'_1$, $\mathbf{H}_1 = (\mathbf{e}_{T-h}, \bar{\mathbf{Z}})$, $\bar{\mathbf{Z}} = (\bar{\mathbf{z}}'_{.1}, \bar{\mathbf{z}}'_{.2}, \ldots, \bar{\mathbf{z}}'_{.,T-h})'$, $\bar{\mathbf{z}}'_{.t} = n^{-1}\sum_{i=1}^{n}\mathbf{z}'_{it}$ and $\mathbf{z}_{it} = (y_{it}, \mathbf{x}'_{i,t-h})'$, $t = h+1, \ldots, T$.

In general, one can use cross-sectional averages of any variable correlated with the common factors. As an alternative we use the estimator called *Ind. CCEX* given by

$$\widehat{\boldsymbol{\beta}}_{CCEX,i} = \left(\mathbf{X}'_{i.,-h}\mathbf{M}_{H_2}\mathbf{X}_{i.,-h}\right)^{-1}\mathbf{X}'_{i.,-h}\mathbf{M}_{H_2}\mathbf{y}_{i.}, \tag{S19}$$

[1]See Karabiyik et al. (2017) for a discussion on this topic.

where $\mathbf{M}_{H_2} = \mathbf{I}_{T-h} - \mathbf{H}_2(\mathbf{H}_2'\mathbf{H}_2)^{-}\mathbf{H}_2'$, $\mathbf{H}_2 = (\mathbf{e}_{T-h}, \bar{\mathbf{W}})$, $\bar{\mathbf{W}} = (\bar{\mathbf{w}}'_{.1}, \bar{\mathbf{w}}'_{.2}, \ldots, \bar{\mathbf{w}}'_{.,T-h})'$, $\bar{\mathbf{w}}'_{.t} = n^{-1}\sum_{i=1}^{n} \mathbf{w}'_{it}$ with $\mathbf{w}_{it}$ being a vector of observed exogeneous or weakly exogeneous variables. The predictions which are called *Ind. CCE* and *Ind. CCEX* are obtained by replacing (S18) and (S19) in (S15), respectively. In our Monte Carlo simulations we used the right hand side variables and an additional exogeneous variable to compute this estimator as described in the paper. In the empirical application this estimator uses all available variables in our data set, except the dependent variable. The advantage of this estimator is that it does not contain endogenous variables, hence, it is expected to perform better in terms of bias in the case of small n.

Chudik and Pesaran (2015) show that, when the model contains weakly exogeneous variables the estimator (S18) cannot account for all the dependencies between the unobserved common factors and the right hand side variables. In this case sufficient number of lags of the cross-sectional averages of the dependent and independent variables should be added in the construction of the matrix $\mathbf{H}$. In our Monte Carlo simulations we added one lag of these cross-sectional averages to correct the *Ind. CCE* estimator. When there are numerous explanatory variables in the model, this can cause a serious loss of degrees of freedom. As *Ind. CCEX* already contains the cross-sectional averages of an additional variable, to avoid this problem, we do not correct this estimator.

Song (2013) considers an iterative PC estimator of the individual parameters based on the pooled estimator of Bai (2009). This estimator is defined by the following two nonlinear equations:

$$\widehat{\boldsymbol{\beta}}_{IPC,i} = \left(\mathbf{X}'_{i.,-h}\mathbf{M}_{H_3}\mathbf{X}_{i.,-h}\right)^{-1}\mathbf{X}'_{i.,-h}\mathbf{M}_{H_3}\mathbf{y}_{i.}, \tag{S20}$$

$$\left[\frac{1}{nT}\sum_{i=1}^{n}\left(\mathbf{y}_{i.} - \mathbf{X}_{i.,-h}\widehat{\boldsymbol{\beta}}_{IPC,i}\right)\left(\mathbf{y}_{i.} - \mathbf{X}_{i.,-h}\widehat{\boldsymbol{\beta}}_{IPC,i}\right)'\right]\widehat{\mathbf{F}}^{y} = \widehat{\mathbf{F}}^{y}\widehat{\mathbf{V}}_{nT}, \tag{S21}$$

where $\mathbf{M}_{H_3} = \mathbf{I}_{T-h} - \mathbf{H}_3(\mathbf{H}_3'\mathbf{H}_3)^{-}\mathbf{H}_3'$, $\mathbf{H}_3 = (\mathbf{e}_{T-h}, \widehat{\mathbf{F}}^{y})$, $\widehat{\mathbf{V}}_{nT}$ is a diagonal matrix containing the m_y largest eigenvalues of the matrix in the brackets on the left hand side of the equation and $\widehat{\mathbf{F}}^{y}$ are the corresponding eigenvectors. The rows of these eigenvectors serve as estimates of the common factors $\mathbf{f}_t^{y\prime}$. To obtain the final estimator of the slope parameters, one can iterate between these two equations until numerical convergence.

The consistency of the PC estimator require stronger conditions than the boundedness of the of the spatial weight matrix $\mathbf{R} = [r_{ij}]$. As shown by Song, the estimator is consistent only if $\mathrm{E}(u_{it}u_{js}) = \sigma_{ij,ts}$, $|\sigma_{ij,ts}| \leq \bar{\sigma}_{ij}$ for all (t,s) and $|\sigma_{ij,ts}| \leq \bar{\tau}_{ts}$ for all (i,j) such that $\frac{1}{n}\sum_{i,j=1}^{n}\bar{\sigma}_{ij} \leq M$, $\frac{1}{T}\sum_{t,s=1}^{T}\bar{\tau}_{ts} \leq M$, $\frac{1}{nT}\sum_{i,j,t,s=1}|\sigma_{ij,ts}| \leq M$. These conditions limit the serial and cross-sectional dependence and heteroskedasticity in the error terms.

A simpler estimator can be obtained by estimating the common factors from the explanatory variables, assuming that they have a common factor structure. Such an estimator is

$$\widehat{\boldsymbol{\beta}}_{PCX,i} = \left(\mathbf{X}'_{i.,-h}\mathbf{M}_{H_4}\mathbf{X}_{i.,-h}\right)^{-1}\mathbf{X}'_{i.,-h}\mathbf{M}_{H_4}\mathbf{y}_{i.}, \tag{S22}$$

where $\mathbf{M}_{H_4} = \mathbf{I}_{T-h} - \mathbf{H}_4(\mathbf{H}_4'\mathbf{H}_4)^{-}\mathbf{H}_4'$, $\mathbf{H}_4 = (\mathbf{e}_{T-h}, \widehat{\mathbf{F}}^x)$ with $\widehat{\mathbf{F}}^x$ being $\sqrt{T-h}$ times the first m_x eigenvectors of the matrix $\sum_{i=1}^n \mathbf{X}_{i.,-h}\mathbf{X}'_{i.,-h}$. This estimator uses the estimates only the common factors contained in the explanatory variables. As shown by Bai and Ng (2002) the consistent estimation of the common factors using the explanatory variables require weak serial and cross-sectional dependence and heteroskedasticity conditions as above.

In the case that the DGP for the dependent variable contains additional common factors, this estimator does not remove them. Alternatively, one can use

$$\widehat{\boldsymbol{\beta}}_{PCX2S,i} = \left(\mathbf{X}'_{i.,-h}\mathbf{M}_{H_5}\mathbf{X}_{i.,-h}\right)^{-1}\mathbf{X}'_{i.,-h}\mathbf{M}_{H_5}\mathbf{y}_{i.}, \tag{S23}$$

where $\mathbf{M}_{H_5} = \mathbf{I}_{T-h} - \mathbf{H}_5(\mathbf{H}_5'\mathbf{H}_5)^{-}\mathbf{H}_5'$, $\mathbf{H}_5 = (\mathbf{e}_{T-h}, \widehat{\mathbf{F}}^x, \widehat{\mathbf{F}}^u)$ with $\widehat{\mathbf{F}}^u$ being $\sqrt{T-h}$ times the first m_y eigenvectors of the matrix $\sum_{i=1}^n \left(\mathbf{y}_{i.} - \mathbf{X}_{i.,-h}\widehat{\boldsymbol{\beta}}_{PCX,i}\right)\left(\mathbf{y}_{i.} - \mathbf{X}_{i.,-h}\widehat{\boldsymbol{\beta}}_{PCX,i}\right)'$. The predictions which are called *Ind. IPC*, *Ind. PCX* and *Ind. PCX2S* are obtained by replacing (S20), (S22) and (S23) in (S15), respectively.

For the individual estimators given in (S18), (S19), (S20), (S22), (S23), we can compute the MG estimates of the average slope parameter as in (S6) and their variances can be estimated in the same spirit as (S7). For instance, we have

$$\widehat{\boldsymbol{\beta}}_{CCEMG} = n^{-1}\sum_{i=1}^n \widehat{\boldsymbol{\beta}}_{CCE,i}, \tag{S24}$$

$$\widehat{\text{Var}}\left(\widehat{\boldsymbol{\beta}}_{CCEMG}\right) = \frac{1}{n(n-1)}\sum_{i=1}^n \left(\widehat{\boldsymbol{\beta}}_{CCE,i} - \widehat{\boldsymbol{\beta}}_{CCEMG}\right)\left(\widehat{\boldsymbol{\beta}}_{CCE,i} - \widehat{\boldsymbol{\beta}}_{CCEMG}\right)'. \tag{S25}$$

Another way to estimate the average slope parameter is to use the pooled estimators. The pooled estimator of Bai (2009) is obtained from the following two equations by iteration:

$$\widehat{\boldsymbol{\beta}}_{IPCP} = \left(\sum_{i=1}^n \mathbf{X}'_{i.,-h}\mathbf{M}_{H_6}\mathbf{X}_{i.,-h}\right)^{-1}\sum_{i=1}^n \mathbf{X}'_{i.,-h}\mathbf{M}_{H_6}\mathbf{y}_{i.}, \tag{S26}$$

$$\left[\frac{1}{nT}\sum_{i=1}^n \left(\mathbf{y}_{i.} - \mathbf{X}_{i.,-h}\widehat{\boldsymbol{\beta}}_{IPCP}\right)\left(\mathbf{y}_{i.} - \mathbf{X}_{i.,-h}\widehat{\boldsymbol{\beta}}_{IPCP}\right)'\right]\widetilde{\mathbf{F}}^y = \widetilde{\mathbf{F}}^y\widetilde{\mathbf{V}}_{nT}, \tag{S27}$$

where $\mathbf{M}_{H_6} = \mathbf{I}_{T-h} - \mathbf{H}_6(\mathbf{H}_6'\mathbf{H}_6)^{-}\mathbf{H}_6'$, $\mathbf{H}_6 = (\mathbf{e}_{T-h}, \widetilde{\mathbf{F}}^y)$, $\widetilde{\mathbf{V}}_{nT}$ is a diagonal matrix containing the m_y largest eigenvalues of the matrix in the brackets and $\widetilde{\mathbf{F}}^y$ are the corresponding eigenvectors. The pooled counterparts of other estimators given in (S18), (S19), (S22), (S23) are obtained in a standard way. For instance, *CCEP* and its heterogeneity-robust variance are computed by

$$\widehat{\boldsymbol{\beta}}_{CCEP} = \left(\sum_{i=1}^{n} \mathbf{X}'_{i.,-h} \mathbf{M}_{H_1} \mathbf{X}_{i.,-h} \right)^{-1} \sum_{i=1}^{n} \mathbf{X}'_{i.,-h} \mathbf{M}_{H_1} \mathbf{y}_{i.}, \tag{S28}$$

$$\widehat{\mathrm{Var}} \left(\widehat{\boldsymbol{\beta}}_{CCEP} \right) = n^{-1} \mathbf{Q}_1^{-1} \boldsymbol{\Lambda}_1 \mathbf{Q}_1^{-1}, \tag{S29}$$

where

$$\mathbf{Q}_1 = n^{-1} \sum_{i=1}^{n} \left(\frac{\mathbf{X}'_{i.,-h} \mathbf{M}_{H_1} \mathbf{X}_{i.,-h}}{T-h} \right), \tag{S30}$$

$$\boldsymbol{\Lambda}_1 = \sum_{i=1}^{n} \left(\frac{\mathbf{X}'_{i.,-h} \mathbf{M}_{H_1} \mathbf{X}_{i.,-h}}{T-h} \right) \widehat{\boldsymbol{\Omega}}_{\delta} \left(\frac{\mathbf{X}'_{i.,-h} \mathbf{M}_{H_1} \mathbf{X}_{i.,-h}}{T-h} \right). \tag{S31}$$

where $\widehat{\boldsymbol{\Omega}}_{\delta} = n^{-1} \left(\widehat{\boldsymbol{\beta}}_{CCE,i} - \widehat{\boldsymbol{\beta}}_{CCEMG} \right) \left(\widehat{\boldsymbol{\beta}}_{CCE,i} - \widehat{\boldsymbol{\beta}}_{CCEMG} \right)'$.

When we have a dynamic model, as in the case of individual estimators, we modified the $CCEP$ estimator by adding the lags of the related cross-sectional averages in the construction of the matrix $\mathbf{M}_{H_1}$. The estimator $CCEPX$ is left unmodified for the same reasons mentioned above in the discussion of the *Ind. CCEX* estimator.

S2 Additional Monte Carlo Simulations For Dynamic Models

In the paper the simulation exercise focused on a static DGP for the variable of interest y_{it}. Here, we report results from a robustness check considering a dynamic model for the dependent variable as it is important to explore the consequences of having a lagged dependent variable in a forecasting study. We generated samples from the following DGP:

$$y_{i,t+h} = \alpha_i + \beta_{i1} y_{i,t} + \beta_{i2} x_{it} + \gamma_{i1} f_{1t} + \gamma_{i2} f_{2t} + u_{i,t+h}, \tag{S32}$$

$$x_{it} = a_i + \gamma_{i21} f_{1t} + \gamma_{i23} f_{3t} + v_{it}, \tag{S33}$$

where $i = 1, 2, \ldots, n$, $t = 1, 2, \ldots, T$, x_{it} is the observed explanatory variable, f_{jt}, $j = 1, 2, 3$, are the unobserved common factors with loadings γ_{ijk}, α_i and a_i are the fixed effects, β_{ij} are the slope coefficients. All variables and parameters except the coefficients β_{i1} are generated as in the paper. In all experiments, the parameter is generated from a uniform distribution with bounded support as $\beta_{i1} \sim \mathrm{IIDU}(0.05, 0.95)$. We report the results for 1-step-ahead forecasts, hence h is set to 1. For the AVA approach we use the additional variable x_{i3t} as generated in the paper and the estimators *Ind. CCEX* and *CCEPX* use the cross-sectional average of this variable as in the static model.

S2.1 Results

The results on the prediction performance of different estimators combined with the two forecast approaches for the case of a dynamic DGP are given in Tables S1-S8. Just as in the case reported in the paper, the forecast performance of each estimator is superior using the AVA compared to the RBA. Since in the case of a dynamic model the main theoretical expectation is that pooled estimators perform poorly, we focus on the comparison of the individual estimators and the pooled estimators. The results of the paper are confirmed with a few additional interesting results which we summarize as follows.

Table S1 reports the results for the case of low spatial dependence and low factor dependence (Case *1*). The results show that, all individual estimators have lower relative RMSEs compared to their pooled counterparts. With a static model in Case *1*, it was found that most of the individual estimators outperform the pooled estimators even when $T = 20$, the smallest size considered. The exceptions were *Ind. PCX* and its pooled version. In the case of a dynamic model, when $n, T = 20$ the relative RMSE of *Ind. PCX* is 0.920 and its pooled counterpart has a relative RMSE equal to 0.936 with the AVA approach. This shows that in a dynamic model individual estimator is always better. For other estimators the relative gain from using individual estimators is greater. An interesting finding is that the performance of these two estimators converge when n increases for a fixed T. For instance, when $T = 20$ and $n = 100$ the relative RMSEs for the *Ind. PCX* and its pooled version are 0.904 and 0.907, respectively, in the case of AVA approach. Whereas, as T gets larger the difference is more pronounced.

In the case of a dynamic predictive model, once more the conclusions do not change in the case of higher factor dependence, Case *2*, reported in Table S2. Whereas, when the degree of spatial dependence is higher some changes are observed. This is Case *3* reported in Table S3. Again with only one exception, individual estimates outperform the pooled counterparts. The exception is *Ind. IPC* and its pooled version as before. When $n, T = 20$ with a small difference pooled estimator performs better than the individual estimate. Their relative RMSEs are 0.1014 and 0.999, respectively. However, this exception is valid only for the AVA. For the RBA still the individual estimator provides better forecasts than the pooled estimator. With the RBA their relative RMSEs are 1.117 and 1.154, respectively.

Most of these results are confirmed in the case of high heterogeneity (see Tables S5-S8). The only exceptions are *Ind. IPC* and its pooled version once more. To summarize, in the case of a dynamic model the AVA is still the preferred forecasting approach over the RBA in all cases and individual estimators are preferred to pooled estimators.

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Table S1: Relative RMSE (Dynamic Model) – Low Heterogeneity, Case *1*: Low Spatial Dependence & Low Factor Dependence

	Individual									*Pooled*							
T \ n	Residual Based Approach				Auxiliary Variables Approach					Residual Based Approach				Auxiliary Variables Approach			
	20	30	50	100	20	30	50	100		20	30	50	100	20	30	50	100
Ind. CCE									*CCEP*								
20	1.012	0.992	0.984	0.985	0.824	0.793	0.820	0.747	20	1.146	1.125	1.175	1.112	0.916	0.892	0.971	0.846
30	0.999	0.988	0.989	0.976	0.791	0.793	0.810	0.831	30	1.098	1.168	1.150	1.188	0.863	0.926	0.937	1.001
50	1.002	0.996	0.985	0.978	0.846	0.834	0.773	0.801	50	1.122	1.159	1.150	1.181	0.937	0.960	0.906	0.971
100	0.996	0.993	0.979	0.980	0.799	0.809	0.817	0.809	100	1.103	1.155	1.174	1.190	0.883	0.938	0.977	0.994
Ind. CCEX									*CCEPX*								
20	1.000	0.988	0.971	0.981	0.815	0.789	0.807	0.744	20	1.145	1.125	1.175	1.112	0.916	0.892	0.971	0.846
30	0.997	0.979	0.980	0.968	0.790	0.786	0.802	0.824	30	1.098	1.168	1.150	1.188	0.863	0.925	0.936	1.001
50	0.991	0.984	0.978	0.969	0.837	0.824	0.766	0.792	50	1.123	1.159	1.150	1.181	0.937	0.959	0.906	0.971
100	0.989	0.981	0.967	0.969	0.794	0.800	0.808	0.799	100	1.103	1.155	1.174	1.190	0.882	0.938	0.977	0.993
Ind. IPC									*IPCP*								
20	1.003	0.983	0.971	0.981	0.829	0.793	0.810	0.746	20	1.153	1.129	1.174	1.113	0.918	0.893	0.979	0.845
30	0.988	0.975	0.975	0.968	0.789	0.789	0.802	0.826	30	1.103	1.175	1.150	1.191	0.863	0.918	0.940	0.996
50	0.975	0.970	0.972	0.967	0.836	0.820	0.764	0.792	50	1.134	1.164	1.151	1.180	0.938	0.960	0.906	0.975
100	0.964	0.967	0.961	0.967	0.782	0.794	0.805	0.798	100	1.110	1.157	1.178	1.188	0.882	0.939	0.977	0.998
Ind. PCX									*PCPX*								
20	1.091	1.051	1.011	0.984	0.920	0.872	0.865	0.796	20	1.162	1.134	1.188	1.101	0.936	0.908	0.972	0.859
30	1.071	1.031	1.014	0.985	0.889	0.862	0.864	0.859	30	1.105	1.179	1.153	1.194	0.885	0.934	0.945	0.986
50	1.109	1.051	1.000	0.981	0.950	0.902	0.824	0.839	50	1.167	1.185	1.155	1.186	0.953	0.971	0.917	0.978
100	1.078	1.037	0.996	0.980	0.904	0.881	0.863	0.839	100	1.128	1.174	1.190	1.199	0.907	0.956	0.987	0.996
Ind. PCX2S									*PCPX2S*								
20	1.047	1.007	0.978	0.974	0.852	0.807	0.816	0.752	20	1.162	1.134	1.188	1.101	0.921	0.893	0.962	0.844
30	1.022	0.990	0.983	0.967	0.814	0.799	0.809	0.824	30	1.105	1.179	1.153	1.194	0.872	0.919	0.933	0.980
50	1.051	1.002	0.978	0.964	0.872	0.837	0.771	0.793	50	1.167	1.185	1.155	1.186	0.940	0.958	0.904	0.966
100	1.019	0.994	0.968	0.967	0.818	0.811	0.810	0.800	100	1.128	1.174	1.190	1.199	0.892	0.940	0.974	0.988

Table S2: Relative RMSE (Dynamic Model) – Low Heterogeneity, Case *2*: Low Spatial Dependence & High Factor Dependence

	Individual									*Pooled*								
	Residual Based Approach				Auxiliary Variables Approach					Residual Based Approach				Auxiliary Variables Approach				
n \ T	20	30	50	100	20	30	50	100		20	30	50	100	20	30	50	100
Ind. CCE									*CCEP*								
20	0.986	0.972	0.975	0.978	0.734	0.734	0.821	0.681	20	1.078	1.064	1.102	1.068	0.788	0.795	0.901	0.746
30	0.973	0.976	0.980	0.973	0.659	0.731	0.805	0.849	30	1.045	1.092	1.092	1.110	0.709	0.813	0.879	0.941
50	0.965	0.978	0.979	0.976	0.771	0.802	0.751	0.844	50	1.049	1.089	1.087	1.111	0.824	0.873	0.831	0.934
100	0.964	0.977	0.974	0.982	0.688	0.760	0.799	0.854	100	1.039	1.088	1.098	1.117	0.742	0.837	0.886	0.951
Ind. CCEX									*CCEPX*								
20	0.963	0.963	0.949	0.971	0.721	0.727	0.803	0.675	20	1.079	1.064	1.102	1.068	0.788	0.795	0.901	0.746
30	0.970	0.958	0.961	0.955	0.656	0.719	0.794	0.841	30	1.045	1.093	1.092	1.110	0.709	0.812	0.879	0.941
50	0.943	0.955	0.963	0.953	0.759	0.787	0.740	0.832	50	1.049	1.090	1.087	1.111	0.824	0.873	0.831	0.934
100	0.950	0.954	0.948	0.956	0.679	0.746	0.786	0.839	100	1.039	1.088	1.098	1.117	0.742	0.836	0.886	0.951
Ind. IPC									*IPCP*								
20	0.974	0.963	0.948	0.971	0.737	0.735	0.809	0.677	20	1.081	1.066	1.095	1.069	0.791	0.797	0.910	0.748
30	0.961	0.957	0.955	0.953	0.659	0.726	0.795	0.843	30	1.051	1.105	1.086	1.114	0.708	0.808	0.885	0.943
50	0.941	0.944	0.955	0.950	0.763	0.787	0.738	0.832	50	1.046	1.089	1.087	1.107	0.829	0.877	0.833	0.942
100	0.928	0.940	0.942	0.953	0.673	0.743	0.785	0.839	100	1.041	1.086	1.099	1.111	0.743	0.839	0.888	0.958
Ind. PCX									*PCPX*								
20	1.158	1.104	1.030	1.006	0.886	0.848	0.865	0.751	20	1.101	1.077	1.110	1.060	0.819	0.817	0.905	0.765
30	1.145	1.087	1.043	1.003	0.833	0.836	0.865	0.875	30	1.057	1.120	1.096	1.123	0.744	0.829	0.888	0.931
50	1.182	1.098	1.025	0.996	0.922	0.890	0.811	0.869	50	1.084	1.113	1.092	1.120	0.848	0.888	0.844	0.938
100	1.161	1.084	1.029	0.995	0.861	0.860	0.857	0.865	100	1.058	1.105	1.119	1.126	0.776	0.858	0.900	0.950
Ind. PCX2S									*PCPX2S*								
20	1.024	0.990	0.956	0.965	0.756	0.742	0.804	0.680	20	1.101	1.077	1.110	1.060	0.790	0.792	0.894	0.742
30	1.001	0.974	0.965	0.953	0.682	0.729	0.794	0.835	30	1.057	1.120	1.096	1.123	0.711	0.803	0.875	0.924
50	1.014	0.976	0.963	0.947	0.786	0.794	0.741	0.825	50	1.084	1.113	1.092	1.120	0.824	0.869	0.827	0.929
100	0.986	0.969	0.950	0.953	0.702	0.754	0.784	0.833	100	1.058	1.105	1.119	1.126	0.745	0.835	0.881	0.946

Table S3: Relative RMSE (Dynamic Model) – Low Heterogeneity, Case *3*: High Spatial Dependence & Low Factor Dependence

	Individual									*Pooled*							
	Residual Based Approach				Auxiliary Variables Approach					Residual Based Approach				Auxiliary Variables Approach			
n \ T	20	30	50	100	20	30	50	100		20	30	50	100	20	30	50	100
Ind. CCE									*CCEP*								
20	1.033	1.018	1.013	1.006	0.941	0.911	0.923	0.860	20	1.118	1.115	1.148	1.102	0.983	0.964	1.017	0.921
30	1.018	1.008	1.012	1.003	0.923	0.907	0.925	0.916	30	1.078	1.136	1.125	1.166	0.951	0.984	0.999	1.033
50	1.032	1.021	1.006	1.000	0.947	0.942	0.886	0.901	50	1.099	1.130	1.123	1.147	0.986	1.009	0.967	1.015
100	1.025	1.014	0.996	0.994	0.919	0.917	0.906	0.896	100	1.074	1.120	1.134	1.145	0.948	0.991	1.009	1.021
Ind. CCEX									*CCEPX*								
20	1.064	1.041	1.013	1.009	0.964	0.926	0.923	0.861	20	1.120	1.116	1.150	1.102	0.982	0.963	1.016	0.920
30	1.042	1.016	1.015	1.003	0.939	0.912	0.927	0.915	30	1.079	1.137	1.126	1.166	0.951	0.983	0.999	1.032
50	1.038	1.023	1.008	0.998	0.953	0.943	0.887	0.898	50	1.100	1.130	1.124	1.147	0.986	1.009	0.966	1.014
100	1.028	1.012	0.992	0.990	0.922	0.916	0.903	0.892	100	1.075	1.121	1.135	1.145	0.948	0.991	1.009	1.021
Ind. IPC									*IPCP*								
20	0.991	0.989	0.990	0.997	0.925	0.900	0.910	0.855	20	1.123	1.112	1.145	1.101	0.988	0.968	1.025	0.924
30	0.987	0.983	0.988	0.992	0.904	0.892	0.913	0.908	30	1.078	1.142	1.120	1.167	0.953	0.981	1.006	1.033
50	0.981	0.984	0.988	0.986	0.922	0.921	0.875	0.891	50	1.110	1.132	1.124	1.145	0.988	1.013	0.968	1.020
100	0.985	0.986	0.979	0.984	0.894	0.897	0.892	0.888	100	1.085	1.123	1.138	1.145	0.949	0.991	1.008	1.023
Ind. PCX									*PCPX*								
20	1.117	1.082	1.041	1.013	1.014	0.969	0.949	0.889	20	1.154	1.134	1.174	1.101	0.999	0.976	1.017	0.929
30	1.082	1.053	1.037	1.013	0.990	0.953	0.954	0.930	30	1.090	1.154	1.139	1.177	0.967	0.991	1.004	1.021
50	1.106	1.061	1.019	1.001	1.014	0.979	0.915	0.918	50	1.142	1.156	1.131	1.154	0.999	1.017	0.975	1.017
100	1.072	1.041	1.007	0.994	0.978	0.957	0.930	0.911	100	1.099	1.137	1.147	1.153	0.966	1.003	1.015	1.023
Ind. PCX2S									*PCPX2S*								
20	1.062	1.031	1.009	0.999	0.957	0.920	0.918	0.861	20	1.154	1.134	1.174	1.101	0.985	0.964	1.011	0.919
30	1.033	1.014	1.007	0.996	0.935	0.909	0.922	0.908	30	1.090	1.154	1.139	1.177	0.956	0.980	0.998	1.017
50	1.071	1.029	1.000	0.987	0.967	0.941	0.884	0.893	50	1.142	1.156	1.131	1.154	0.991	1.009	0.967	1.011
100	1.045	1.018	0.990	0.985	0.934	0.919	0.900	0.890	100	1.099	1.137	1.147	1.153	0.959	0.995	1.008	1.018

Table S4: Relative RMSE (Dynamic Model) – Low Heterogeneity, Case *4*: High Spatial Dependence & High Factor Dependence

	Individual									*Pooled*							
	Residual Based Approach				Auxiliary Variables Approach					Residual Based Approach				Auxiliary Variables Approach			
n \ T	20	30	50	100	20	30	50	100		20	30	50	100	20	30	50	100
Ind. CCE									*CCEP*								
20	1.010	0.995	0.994	0.993	0.846	0.829	0.888	0.777	20	1.081	1.074	1.099	1.069	0.879	0.872	0.950	0.826
30	0.993	0.990	0.997	0.990	0.790	0.820	0.888	0.902	30	1.046	1.086	1.090	1.109	0.816	0.881	0.943	0.979
50	0.992	0.999	0.992	0.988	0.857	0.886	0.834	0.900	50	1.050	1.086	1.080	1.103	0.886	0.936	0.894	0.975
100	0.992	0.994	0.983	0.986	0.797	0.846	0.863	0.899	100	1.036	1.082	1.090	1.104	0.825	0.903	0.933	0.980
Ind. CCEX									*CCEPX*								
20	1.020	1.007	0.981	0.992	0.854	0.836	0.881	0.775	20	1.082	1.074	1.099	1.069	0.879	0.871	0.950	0.826
30	1.011	0.987	0.991	0.981	0.799	0.818	0.885	0.898	30	1.046	1.087	1.090	1.108	0.816	0.880	0.943	0.979
50	0.986	0.991	0.986	0.975	0.855	0.881	0.829	0.893	50	1.051	1.087	1.081	1.103	0.886	0.936	0.894	0.974
100	0.988	0.983	0.969	0.971	0.795	0.840	0.856	0.890	100	1.037	1.082	1.090	1.105	0.824	0.903	0.933	0.980
Ind. IPC									*IPCP*								
20	0.975	0.971	0.963	0.984	0.834	0.820	0.873	0.772	20	1.087	1.074	1.093	1.070	0.882	0.874	0.956	0.828
30	0.966	0.967	0.968	0.974	0.777	0.810	0.876	0.896	30	1.049	1.097	1.086	1.113	0.817	0.877	0.948	0.979
50	0.952	0.961	0.971	0.968	0.839	0.868	0.821	0.889	50	1.053	1.089	1.082	1.100	0.889	0.938	0.895	0.980
100	0.950	0.962	0.959	0.967	0.778	0.829	0.851	0.889	100	1.041	1.083	1.093	1.101	0.824	0.904	0.932	0.984
Ind. PCX									*PCPX*								
20	1.155	1.107	1.037	1.011	0.955	0.912	0.919	0.825	20	1.108	1.087	1.111	1.064	0.900	0.886	0.952	0.839
30	1.123	1.078	1.039	1.008	0.912	0.897	0.925	0.917	30	1.056	1.113	1.096	1.122	0.840	0.891	0.947	0.968
50	1.146	1.079	1.019	0.994	0.964	0.942	0.871	0.911	50	1.085	1.107	1.085	1.109	0.903	0.943	0.902	0.974
100	1.114	1.062	1.014	0.989	0.913	0.911	0.899	0.903	100	1.054	1.095	1.105	1.112	0.848	0.916	0.940	0.977
Ind. PCX2S									*PCPX2S*								
20	1.043	1.012	0.981	0.982	0.860	0.835	0.875	0.776	20	1.108	1.087	1.111	1.064	0.880	0.869	0.945	0.822
30	1.016	0.993	0.986	0.976	0.806	0.820	0.879	0.891	30	1.056	1.113	1.096	1.122	0.818	0.873	0.940	0.964
50	1.033	1.001	0.981	0.967	0.868	0.880	0.826	0.885	50	1.085	1.107	1.085	1.109	0.888	0.933	0.892	0.969
100	1.008	0.991	0.967	0.967	0.808	0.841	0.851	0.884	100	1.054	1.095	1.105	1.112	0.829	0.902	0.929	0.976

Table S5: Relative RMSE (Dynamic Model) – High Heterogeneity, Case *1*: Low Spatial Dependence & Low Factor Dependence

	Individual									*Pooled*							
T	Residual Based Approach				Auxiliary Variables Approach					Residual Based Approach				Auxiliary Variables Approach			
n	20	30	50	100	20	30	50	100		20	30	50	100	20	30	50	100
Ind. CCE									*CCEP*								
20	1.013	0.992	0.984	0.985	0.831	0.804	0.832	0.755	20	1.183	1.167	1.220	1.155	0.964	0.948	1.028	0.898
30	0.999	0.988	0.990	0.976	0.796	0.799	0.819	0.842	30	1.142	1.211	1.194	1.235	0.914	0.977	0.991	1.059
50	1.003	0.996	0.985	0.978	0.856	0.845	0.783	0.815	50	1.166	1.201	1.194	1.227	0.991	1.012	0.960	1.031
100	0.996	0.993	0.979	0.980	0.806	0.821	0.829	0.826	100	1.148	1.199	1.220	1.235	0.937	0.995	1.036	1.055
Ind. CCEX									*CCEPX*								
20	1.000	0.988	0.971	0.981	0.822	0.799	0.818	0.751	20	1.183	1.168	1.220	1.155	0.964	0.948	1.027	0.897
30	0.997	0.979	0.980	0.968	0.795	0.791	0.811	0.835	30	1.142	1.211	1.194	1.235	0.913	0.976	0.991	1.059
50	0.992	0.984	0.978	0.969	0.847	0.834	0.776	0.807	50	1.166	1.201	1.195	1.228	0.991	1.012	0.960	1.031
100	0.989	0.981	0.967	0.969	0.801	0.812	0.820	0.816	100	1.148	1.199	1.220	1.235	0.937	0.995	1.036	1.055
Ind. IPC									*IPCP*								
20	1.004	0.985	0.974	0.981	0.834	0.803	0.822	0.753	20	1.191	1.171	1.220	1.158	0.966	0.949	1.037	0.899
30	0.988	0.977	0.975	0.969	0.793	0.795	0.812	0.838	30	1.148	1.220	1.195	1.239	0.914	0.970	0.996	1.056
50	0.974	0.970	0.972	0.967	0.844	0.830	0.774	0.807	50	1.179	1.206	1.196	1.227	0.992	1.013	0.961	1.037
100	0.964	0.967	0.962	0.967	0.790	0.806	0.817	0.816	100	1.156	1.202	1.224	1.234	0.936	0.996	1.037	1.061
Ind. PCX									*PCPX*								
20	1.090	1.051	1.012	0.984	0.925	0.880	0.874	0.802	20	1.201	1.178	1.234	1.146	0.983	0.961	1.026	0.909
30	1.071	1.031	1.014	0.986	0.894	0.867	0.871	0.869	30	1.150	1.224	1.199	1.243	0.935	0.984	0.998	1.042
50	1.109	1.051	0.999	0.981	0.956	0.910	0.832	0.850	50	1.212	1.230	1.201	1.235	1.005	1.022	0.970	1.036
100	1.078	1.037	0.996	0.981	0.909	0.890	0.873	0.852	100	1.176	1.222	1.239	1.246	0.960	1.011	1.044	1.055
Ind. PCX2S									*PCPX2S*								
20	1.047	1.008	0.978	0.973	0.858	0.816	0.826	0.758	20	1.201	1.178	1.234	1.146	0.969	0.947	1.018	0.895
30	1.021	0.990	0.983	0.969	0.818	0.804	0.817	0.835	30	1.150	1.224	1.199	1.243	0.922	0.970	0.987	1.037
50	1.051	1.002	0.978	0.964	0.880	0.847	0.781	0.807	50	1.212	1.230	1.201	1.235	0.994	1.010	0.957	1.025
100	1.019	0.995	0.968	0.968	0.825	0.823	0.822	0.816	100	1.176	1.222	1.239	1.246	0.946	0.997	1.032	1.049

Table S6: Relative RMSE (Dynamic Model) – High Heterogeneity, Case *2*: Low Spatial Dependence & High Factor Dependence

	Individual									*Pooled*							
	Residual Based Approach				Auxiliary Variables Approach					Residual Based Approach				Auxiliary Variables Approach			
n \ T	20	30	50	100	20	30	50	100		20	30	50	100	20	30	50	100
Ind. CCE									*CCEP*								
20	0.986	0.971	0.974	0.978	0.746	0.750	0.834	0.704	20	1.096	1.084	1.123	1.089	0.821	0.835	0.937	0.793
30	0.973	0.976	0.980	0.972	0.670	0.743	0.821	0.862	30	1.065	1.112	1.112	1.131	0.746	0.849	0.919	0.978
50	0.965	0.978	0.979	0.976	0.786	0.819	0.769	0.863	50	1.069	1.108	1.108	1.133	0.861	0.912	0.871	0.976
100	0.964	0.977	0.973	0.982	0.702	0.778	0.817	0.871	100	1.060	1.109	1.119	1.138	0.780	0.878	0.929	0.990
Ind. CCEX									*CCEPX*								
20	0.963	0.963	0.949	0.971	0.733	0.743	0.817	0.698	20	1.096	1.084	1.123	1.088	0.821	0.834	0.937	0.793
30	0.969	0.958	0.961	0.955	0.667	0.731	0.810	0.854	30	1.065	1.112	1.112	1.131	0.745	0.848	0.918	0.977
50	0.942	0.955	0.963	0.953	0.774	0.804	0.757	0.851	50	1.070	1.109	1.108	1.133	0.861	0.912	0.871	0.976
100	0.950	0.954	0.948	0.955	0.692	0.764	0.805	0.856	100	1.060	1.109	1.120	1.138	0.780	0.878	0.929	0.990
Ind. IPC									*IPCP*								
20	0.976	0.964	0.949	0.970	0.751	0.751	0.823	0.700	20	1.098	1.085	1.116	1.091	0.825	0.837	0.947	0.796
30	0.961	0.956	0.954	0.953	0.670	0.738	0.811	0.857	30	1.072	1.123	1.107	1.136	0.745	0.845	0.925	0.981
50	0.941	0.944	0.955	0.950	0.777	0.804	0.756	0.851	50	1.067	1.108	1.108	1.128	0.866	0.916	0.874	0.985
100	0.928	0.941	0.942	0.953	0.686	0.761	0.804	0.856	100	1.061	1.106	1.120	1.133	0.781	0.881	0.931	0.998
Ind. PCX									*PCPX*								
20	1.160	1.103	1.031	1.005	0.895	0.858	0.876	0.769	20	1.119	1.097	1.133	1.081	0.849	0.852	0.939	0.809
30	1.144	1.087	1.043	1.004	0.841	0.844	0.876	0.886	30	1.078	1.140	1.118	1.145	0.778	0.863	0.925	0.966
50	1.182	1.098	1.025	0.997	0.931	0.903	0.824	0.884	50	1.107	1.134	1.113	1.141	0.882	0.924	0.881	0.978
100	1.159	1.085	1.029	0.996	0.870	0.873	0.870	0.879	100	1.080	1.128	1.142	1.147	0.812	0.896	0.939	0.987
Ind. PCX2S									*PCPX2S*								
20	1.025	0.991	0.957	0.964	0.767	0.756	0.817	0.701	20	1.119	1.097	1.133	1.081	0.822	0.831	0.931	0.788
30	1.001	0.973	0.966	0.954	0.692	0.740	0.809	0.848	30	1.078	1.140	1.118	1.145	0.748	0.839	0.914	0.961
50	1.015	0.977	0.963	0.948	0.799	0.811	0.757	0.844	50	1.107	1.134	1.113	1.141	0.861	0.908	0.867	0.971
100	0.987	0.970	0.950	0.953	0.714	0.771	0.801	0.850	100	1.080	1.128	1.142	1.147	0.783	0.876	0.924	0.985

Table S7: Relative RMSE (Dynamic Model) – High Heterogeneity, Case *3*: High Spatial Dependence & Low Factor Dependence

	Individual									*Pooled*							
	Residual Based Approach				Auxiliary Variables Approach					Residual Based Approach				Auxiliary Variables Approach			
n \ T	20	30	50	100	20	30	50	100		20	30	50	100	20	30	50	100
Ind. CCE									*CCEP*								
20	1.033	1.018	1.013	1.006	0.945	0.917	0.928	0.865	20	1.140	1.141	1.176	1.128	1.010	0.997	1.050	0.952
30	1.018	1.008	1.012	1.003	0.926	0.910	0.930	0.923	30	1.103	1.162	1.151	1.196	0.979	1.014	1.031	1.069
50	1.032	1.021	1.006	1.000	0.953	0.947	0.892	0.909	50	1.127	1.155	1.151	1.176	1.021	1.040	0.999	1.051
100	1.025	1.014	0.996	0.994	0.923	0.924	0.913	0.905	100	1.103	1.149	1.164	1.174	0.981	1.027	1.045	1.059
Ind. CCEX									*CCEPX*								
20	1.064	1.041	1.013	1.009	0.967	0.932	0.927	0.866	20	1.142	1.142	1.177	1.128	1.010	0.997	1.049	0.952
30	1.042	1.016	1.015	1.002	0.942	0.915	0.933	0.921	30	1.104	1.162	1.151	1.196	0.979	1.013	1.031	1.069
50	1.038	1.023	1.008	0.998	0.959	0.948	0.893	0.907	50	1.128	1.156	1.151	1.176	1.020	1.039	0.999	1.051
100	1.028	1.012	0.992	0.990	0.926	0.924	0.910	0.901	100	1.103	1.149	1.164	1.174	0.980	1.026	1.045	1.058
Ind. IPC									*IPCP*								
20	0.992	0.989	0.991	0.997	0.928	0.905	0.915	0.860	20	1.146	1.137	1.174	1.129	1.015	1.001	1.058	0.956
30	0.987	0.984	0.988	0.993	0.906	0.895	0.919	0.915	30	1.104	1.168	1.146	1.197	0.982	1.012	1.039	1.071
50	0.982	0.984	0.988	0.986	0.927	0.926	0.881	0.900	50	1.141	1.157	1.151	1.175	1.021	1.044	1.001	1.058
100	0.985	0.986	0.979	0.984	0.898	0.905	0.899	0.897	100	1.115	1.153	1.168	1.174	0.981	1.027	1.045	1.061
Ind. PCX									*PCPX*								
20	1.115	1.084	1.041	1.012	1.015	0.973	0.953	0.893	20	1.176	1.161	1.203	1.128	1.025	1.007	1.048	0.959
30	1.081	1.052	1.037	1.013	0.992	0.955	0.958	0.935	30	1.117	1.182	1.167	1.207	0.995	1.020	1.035	1.056
50	1.106	1.062	1.019	1.001	1.018	0.982	0.919	0.925	50	1.172	1.183	1.159	1.184	1.031	1.046	1.006	1.052
100	1.072	1.042	1.007	0.994	0.981	0.963	0.936	0.918	100	1.131	1.168	1.179	1.183	0.998	1.037	1.050	1.059
Ind. PCX2S									*PCPX2S*								
20	1.061	1.033	1.009	0.999	0.959	0.926	0.922	0.865	20	1.176	1.161	1.203	1.128	1.012	0.997	1.043	0.950
30	1.032	1.013	1.006	0.998	0.937	0.911	0.927	0.915	30	1.117	1.182	1.167	1.207	0.985	1.010	1.029	1.054
50	1.071	1.029	1.000	0.987	0.972	0.946	0.890	0.901	50	1.172	1.183	1.159	1.184	1.025	1.039	0.999	1.047
100	1.044	1.018	0.990	0.985	0.937	0.925	0.907	0.898	100	1.131	1.168	1.179	1.183	0.992	1.030	1.044	1.055

Table S8: Relative RMSE (Dynamic Model) – High Heterogeneity, Case *4*: High Spatial Dependence & High Factor Dependence

	Individual									*Pooled*							
T	Residual Based Approach				Auxiliary Variables Approach					Residual Based Approach				Auxiliary Variables Approach			
n	20	30	50	100	20	30	50	100		20	30	50	100	20	30	50	100
Ind. CCE									*CCEP*								
20	1.010	0.994	0.994	0.993	0.852	0.842	0.897	0.796	20	1.094	1.089	1.115	1.085	0.900	0.902	0.976	0.862
30	0.993	0.990	0.997	0.990	0.797	0.830	0.895	0.914	30	1.061	1.101	1.105	1.126	0.841	0.909	0.967	1.009
50	0.992	0.999	0.992	0.987	0.866	0.894	0.845	0.912	50	1.067	1.101	1.097	1.119	0.913	0.960	0.923	1.004
100	0.992	0.994	0.982	0.985	0.806	0.859	0.875	0.910	100	1.052	1.098	1.106	1.121	0.852	0.933	0.963	1.009
Ind. CCEX									*CCEPX*								
20	1.019	1.007	0.981	0.992	0.860	0.849	0.889	0.794	20	1.095	1.089	1.115	1.085	0.900	0.902	0.975	0.861
30	1.010	0.987	0.991	0.981	0.806	0.827	0.892	0.911	30	1.061	1.102	1.105	1.125	0.841	0.908	0.967	1.009
50	0.986	0.991	0.985	0.975	0.865	0.889	0.841	0.906	50	1.067	1.101	1.097	1.119	0.913	0.960	0.923	1.004
100	0.988	0.983	0.969	0.971	0.804	0.852	0.869	0.901	100	1.052	1.098	1.107	1.121	0.852	0.932	0.963	1.009
Ind. IPC									*IPCP*								
20	0.976	0.972	0.964	0.983	0.842	0.833	0.882	0.790	20	1.099	1.089	1.109	1.086	0.903	0.904	0.982	0.864
30	0.966	0.967	0.968	0.975	0.784	0.818	0.883	0.908	30	1.064	1.112	1.101	1.130	0.842	0.904	0.972	1.010
50	0.952	0.961	0.971	0.968	0.849	0.876	0.833	0.901	50	1.069	1.103	1.098	1.116	0.916	0.962	0.924	1.010
100	0.950	0.962	0.959	0.967	0.787	0.841	0.864	0.900	100	1.057	1.099	1.109	1.117	0.852	0.934	0.963	1.013
Ind. PCX									*PCPX*								
20	1.154	1.106	1.038	1.010	0.958	0.921	0.925	0.839	20	1.121	1.103	1.128	1.080	0.919	0.914	0.976	0.872
30	1.122	1.077	1.039	1.010	0.917	0.903	0.930	0.926	30	1.072	1.128	1.113	1.140	0.864	0.916	0.970	0.997
50	1.146	1.080	1.018	0.994	0.970	0.948	0.880	0.921	50	1.104	1.124	1.102	1.126	0.928	0.966	0.929	1.003
100	1.113	1.062	1.014	0.990	0.919	0.920	0.908	0.913	100	1.071	1.113	1.123	1.129	0.874	0.943	0.968	1.005
Ind. PCX2S									*PCPX2S*								
20	1.043	1.012	0.982	0.981	0.865	0.847	0.883	0.793	20	1.121	1.103	1.128	1.080	0.901	0.899	0.970	0.858
30	1.016	0.993	0.985	0.976	0.812	0.828	0.885	0.902	30	1.072	1.128	1.113	1.140	0.843	0.900	0.964	0.994
50	1.033	1.000	0.981	0.967	0.877	0.888	0.837	0.897	50	1.104	1.124	1.102	1.126	0.915	0.956	0.920	1.000
100	1.008	0.991	0.968	0.967	0.816	0.853	0.863	0.895	100	1.071	1.113	1.123	1.129	0.856	0.931	0.959	1.004